École doctorale n° 405 : Économie, Management, Mathématiques, Physique et Sciences Informatiques (EM2PSI)
Théorie Économique, Modélisation et Applications (THEMA)

Thèse de doctorat

pour l'obtention du titre de
Docteur en Sciences Économiques
délivré par

CY Cergy Paris Université

Three Essays on Measuring the Cost of Children

présentée et soutenue publiquement par

Anderson VIL

le 10 Décembre 2024

préparée sous la direction de **M. Olivier DONNI**

Jury

Président du Jury :
M. Laurens CHERCHYE Professeur, KU Leuven

Directeur de thèse :
M. Olivier DONNI Professeur, CY Cergy Paris Université

Rapporteure / Rapporteur :
M. Olivier BARGAIN Professeur, Université de Bordeaux
Mme. Elena STANCANELLI Directrice de recherche CNRS, Paris School of Economics

Examinatrice / Examinateur :
Mme. Marion GOUSSE Associate Professor, ENSAI
M. Alexandros THELOUDIS Associate Professor, Tilburg University

Thèse de doctorat

Three Essays on Measuring the Cost of Children

présentée et soutenue publiquement par

Anderson VIL

le 10 Décembre 2024

préparée sous la direction de **M. Olivier DONNI**

Jury

Président du Jury :	
M. Laurens CHERCHYE	Professeur, KU Leuven
Directeur de thèse :	
M. Olivier DONNI	Professeur, CY Cergy Paris Université
Rapporteure / Rapporteur :	
M. Olivier BARGAIN	Professeur, Université de Bordeaux
Mme. Elena STANCANELLI	Directrice de recherche CNRS, Paris School of Economics
Examinatrice / Examinateur :	
Mme. Marion GOUSSE	Associate Professor, ENSAI
M. Alexandros THELOUDIS	Associate Professor, Tilburg University

À mes parents,
en mémoire de mes grands-parents partis durant cette thèse,
et en hommage au courage et à la résilience du peuple haïtien.

Remerciements

L'œuvre intellectuelle est le fruit d'un arbre collectif. Je ne peux ici mentionner toutes les personnes envers qui je me sens redevable. Ma bibliographie en est le reflet. Je me contente donc de remercier toutes celles et ceux qui m'ont accompagné, de près ou de loin, dans la construction de cet édifice. Je dois avouer que cette section n'a pas été facile à écrire, sans pour autant être la plus ardue.

Ma première pensée va à mon directeur de thèse, Olivier Donni, à qui j'exprime toute ma gratitude pour la confiance qu'il m'a accordée en acceptant de diriger ce travail. Son encadrement a su marier une grande liberté d'exploration avec la rigueur nécessaire pour évaluer les fruits de mes recherches. Je tiens à souligner combien sa confiance m'a été précieuse, surtout dans les moments de doute. Il m'a permis de poursuivre cette aventure avec sérénité. Au-delà de la direction, il a accepté d'être mon co-auteur, une générosité que j'apprécie profondément. Même si, soyons raisonnables, je ne prétends pas voler la place du professeur Bargain — les frères Olivier(s) sont indétrônables !

Je souhaite également remercier Professeurs Olivier Bargain, Elena Stancanelli, Laurens Cherchye, Marion Goussé et Alexandros Theloudis d'avoir accepté de faire partie de mon comité de thèse. J'ai été agréablement surpris d'apprendre que professseurs Bargain et Stancanelli seraient les rapporteurs de cette thèse. J'ai tant appris des travaux de M. Bargain, et aujourd'hui, c'est un honneur qu'il soit mon rapporteur. En tant que membre de mon comité de suivi de thèse, professeure Stancanelli m'a donné des conseils et des recommandations précieux. J'espère qu'à la lecture de ce travail, vous verrez des progrès. Un merci particulier à vous deux pour avoir accepté généreusement ce rôle. Je garde aussi en mémoire ma rencontre avec professeure Goussé, qui a laissé une impression durable. Un merci spécial à professeur Theloudis, non seulement pour votre présence au sein du comité, mais aussi pour votre article "Consumption Inequality across Heterogenous Families", qui m'a beaucoup éclairé récemment sur l'importance de l'hétérogénéité des préférences dans la transmission des inégalités de revenus à la consommation. Enfin, je ne peux cacher mon admiration pour Laurens, qui m'a accompagné lors de mon séjour à Leuven. Il m'a tant appris et a toujours fait preuve de bienveillance, même face à mes questions dont les réponses devaient parfois lui sembler tellement évidentes.

Et comment pourrais-je oublier l'équipe THEMA ! Je commence par celles qui ne sont plus là : Imen et Cécile. Un merci tout particulier pour les services précieux que vous m'avez rendus. Mes chères Lisa et Yolande, honnêtement, je manque de mots pour exprimer ma gratitude. Vous avez rendu ma vie tellement plus simple durant ces longues années. Toujours présentes, toujours

souriantes, et presque toujours avec une solution à la clé.

Je souhaite aussi exprimer ma gratitude à l'ANR et l'Ecole doctorale EM2PSI pour leur encadrement et le financement de ma thèse. Je remercie également Eric Danan et Olivier Charlot qui ont organisé les conditions matérielles à sa réalisation.

Je tiens aussi à remercier tous mes professeur(e)s de Cergy ainsi qu'aux chercheurs du THEMA. Un remerciement tout particulier à Pierre André, Pascal Belan, Guillaume Chapelle, Gabriel Desgranges, Aristide Houndetougan, Laurence Jacquet, Arnaud Lefranc, Pauline Morault, Andras Niedermayer, Nathalie Picard, Regis Renault, Thepthida Sopraseult, Thomas Tregouët et Peter Vida. Merci pour vos précieux conseils, vos échanges enrichissants, et tout ce que vous m'avez appris. Un merci spécial à Fabian, qui fut d'abord mon professeur et est désormais un ami. Ce travail est aussi le fruit de nos discussions. Merci pour ton soutien constant. Je voudrais accorder une attention particulière à Fanny, Sébastien et Mariona pour leurs précieux conseils sur le job market. Vous êtes une véritable source d'inspiration, que vous le sachiez ou non.

Je n'oublie pas non plus les professeur(e)s qui m'ont honoré en me confiant leurs travaux dirigés. Un grand merci à vous. Ce fut un réel plaisir de collaborer avec vous. Vous m'avez permis de faire un métier que j'aime profondément.

Enfin, comment ne pas remercier mes étudiants et étudiantes, qui ont accepté de partager avec moi ce pain de la connaissance ? Je vous l'ai souvent dit, je m'inscris dans la perspective de Paulo Freire, celle de la dialectique maître-élève. Combien de fois ai-je appris à vos côtés, alors même que vous pensiez que j'étais le "maître" ! Merci pour vos questions, pour votre soif d'apprendre qui, bien des fois, a nourri la mienne.

Ma reconnaissance va également à Valentin, mon frère et ami fidèle, ainsi qu'à Steve, Catherine, et la famille Vayssié, les premiers à m'avoir accompagné dès mon arrivée en France en 2017. Valentin, tu as toujours été à mes côtés et m'as soutenu sans faille. Steve, regarde-moi aujourd'hui : j'ai bien grandi, et je n'ai plus si peur du froid.

À tous mes anciens et nouveaux collègues doctorants, vous avez fait de cette aventure bien plus qu'un simple parcours académique. Ce voyage aurait été un désert aride sans vos éclats de vie, vos rires et blagues au bureau et ces discussions animées autour d'une table. Chacun d'entre vous a contribué à enrichir ces années, et vos traits uniques resteront gravés dans ma mémoire.

Les presque Dr(e)s. Ben et Mél, mes profs d'anglais : Mél, avec ton fameux "fais des phrases plus courtes !", tu incarnes la détermination qui inspire. Ben, ton humour, ton amitié, et les pizzas (sans oublier que tu as probablement lu ma thèse plus que moi) font de toi un allié précieux.Ivan, my "Spanish lessons" haven't always given me the right words, but thankfully, we have our Real Madrid... and this portrait which, according to Zein, our ray of sunshine, doesn't quite do me justice ! Nicos, mon pasteur préféré, je ne suis pas prêt pour le mariage. Samar, moque-toi moins, ou tu regretteras mon départ ! Thierry, mes modèles n'auraient jamais survécu sans ton talent en maths. Martin, tu prévois de soutenir en allemand ? Aurélie, ton sourire et ta bienveillance illuminent tout autour de toi. Mira, je te laisse quelqu'un pour répondre à tes innombrables questions. Omar, une cafétière s'impose. Et Jean, un chapelier est en chemin, comme promis ! Et Maxime, dis-moi, y a-t-il un slot pour présenter au PhD seminar ? Sois en fier, cher organisateur !

Les Pythonistes. Lucas, ta générosité et tes leçons de Python m'ont conquis. Je suis fan de toi mon ami. Romu, grâce à toi, j'ai brillé sur un devoir et, au-delà, j'ai appris des argots.

Les Aventuriers. Even from miles away, your friendship stays close to my heart. Aisha, c'est pour quand notre prochain verre ? Ha, I'll make sure to visit you at your place.

Les dorénavant Dr(e)s. Gabie, bien plus qu'une collègue, tu es une amie précieuse qui m'a soutenu dans chaque étape. Arnaud et Ulysse, mes frères, vous êtes au cœur de cette aventure. Herman, merci pour les notions de maths. Que dire des mots doux de Modou ! Fatou, ton éclat est incomparable. Margaux, l'exceptionnelle, je rêve de m'organiser comme toi, mais c'est impossible. José, avec toi, je me sens toujours un peu moins bête. Patrick, l'Haïtien qui ne dit pas son nom. Eloïse, tu as prouvé qu'il n'y a pas que la macro dans la vie, il y a la danse aussi. Samimi, prépare la playlist, car la fête nous attend après la soutenance. À Nate, Boris, Flore, Salimeh, Julien, Ibrahim et Sangwon, votre présence a aussi marqué ce voyage de manière unique.

To my friends from ESSEC : Ahmet, Huali, Mahault, Quynh-Nga, Rajarshi, Raphael, Thomas, Xiaowei, and Xin, you have been the architects of some of the most unforgettable moments of this journey.

Les désirants d'être Dr(e)s. Eben-Ezer et Saranath, échanger avec vous est toujours un plaisir, entre sérieux et bonne humeur. Marylou, Ubaid et Yanis, votre envie de finir avant même de commencer est franchement inspirante.

I would also like to thank my friends from Leuven, whom I had the pleasure of meeting during my stay : Wietse, Gaston, Alexander, Andrew, and Gianluca. A special thanks goes to Professor Vermeulen, who generously hosted me for a research visit in Leuven.

À mes amis : Bernard, Berlony, Camilien, Carlos, Colbert, Claudy, Iuiu, Jonathan, Jude, ma petite Jul, Kendios, Kenson, Kerby, Litem, Love, Makelot, Manou, Obed, Omane, Patrick, Romain, Sépho, Shelove, Tania, Olriche, Val, Valéry, ainsi qu'à mes amis GRATS, merci pour votre soutien précieux, chacun à votre manière. Dritche, ton soutien indéfectible, ton écoute et ta bienveillance ont illuminé des moments plus sombres. Et enfin, à ma petite Mimi : tu as été un rayon de soleil dans ma vie ces derniers mois, apportant force et réconfort. Tu es inestimable.

Une pensée profonde pour ma famille. À Manmi et Papi, pour votre soutien inébranlable et tout ce que vous avez investi en moi. Après tout, suivant mes calculs, je ne vous coûte pas si cher que ça ! À ma tendre cousine Mirlande, mes cousins John-Peter et Patchouco, mes tantes et oncles, et mes trois trésors inestimables, Tika, Tete et Kyse : vous êtes mes piliers, mes ancrages. Votre amour et vos encouragements ont illuminé chaque étape de cette aventure.

Enfin, à la mémoire du peuple haïtien, traversant peut-être l'un des chapitres les plus douloureux de son histoire. À Bdelle, partie en mon absence, à ma tante Mamoune, ma grand-mère et mon grand-père, qui ont quitté ce monde au cours de cette thèse. Vos absences ont marqué ce chemin, mais vos présences restent gravées dans mon cœur.

Abstract

This thesis consists of three chapters, each dedicated to a specific aspect of the study of the cost of children. The first chapter, primarily empirical, examines the allocation of resources within single-parent households using British data. The following two chapters adopt a theoretical approach while integrating empirical elements. The second chapter models the impact of children on women's labor supply, relying on the equivalence scale methodology. The third chapter focuses on the full cost of children (both monetary and time-related) and the value of parental time, applying the theoretical model to American data.

The first chapter analyzes the cost of children in single-parent households. Specifically, it extends the collective model of Bargain, Donni, and Hentati (2022) to single-adult households and addresses identification issues specific to this context. It then estimates the consumption shares allocated to single parents and their children. Using three sets of UK Expenditure Surveys, two major findings emerge : first, models based on couples tend to underestimate the cost of children in single-parent households due to structural differences ; second, family size significantly affects the resources allocated to children in low-income families, a less pronounced effect in high-income families.

The second chapter develops a theoretical framework using equivalence scales adapted to labor supply, proposing a general technological function that integrates both the financial and time costs of children without specific consumption data. Empirical results from a sample of single women in the U.S. indicate that, for single mothers, monetary effects dominate time effects. Additionally, the median total cost per child is approximately $17,060.

The third chapter proposes a structural framework to measure children's full cost, accounting for both parental time and monetary expenditures. This model differentiates between childcare activities perceived as work and those considered as leisure. A key aspect of our approach is that the price of parental time is not simply equal to the wage but is determined by the substitutability between parental time and purchased childcare services. Empirical analysis based on U.S. working couples' data shows that mothers perceive 68% of this time as work, compared to 53% for fathers. Furthermore, a significant portion of the cost of children borne by parents is non-monetary, underscoring the importance of incorporating time dimensions into the evaluation of parental costs.

Keywords : Collective Model, Consumption demand, Economies of scale, Equivalence scales, Identification, Labor supply, Leisure demand, Price of time, Resource sharing, Shadow price.

Résumé

Cette thèse contient trois chapitres, chacun consacré à un aspect spécifique de l'étude du coût des enfants. Le premier chapitre, principalement empirique, examine l'allocation des ressources au sein des familles monoparentales en utilisant des données britanniques. Les deux chapitres suivants adoptent une approche théorique tout en intégrant des éléments empiriques. Le deuxième chapitre modélise l'impact des enfants sur l'offre de travail des femmes, en s'appuyant sur la méthodologie des échelles d'équivalence. Le troisième chapitre se concentre sur le coût total des enfants (tant monétaire et temporel) et sur la valeur du temps parental, en appliquant le modèle théorique à des données américaines.

Le premier chapitre analyse le coût des enfants dans les familles monoparentales. Plus précisément, il étend le modèle collectif de Bargain, Donni, and Hentati (2022) aux ménages monoparentaux et aborde les questions d'identification spécifiques à ce contexte. Il estime ensuite les parts de consommation allouées aux parents célibataires et à leurs enfants. En utilisant trois ensembles d'enquêtes sur les dépenses au Royaume-Uni, deux conclusions majeures émergent : premièrement, les modèles basés sur les couples ont tendance à sous-estimer le coût des enfants dans les ménages monoparentaux en raison de différences structurelles ; deuxièmement, la taille de la famille affecte significativement les ressources allouées aux enfants dans les familles à faibles revenus, un effet moins marqué dans les familles à hauts revenus.

Le deuxième chapitre présente un cadre théorique basé sur des échelles d'équivalence adaptées à l'offre de travail. Il propose une fonction générale qui intègre les coûts financiers et temporels des enfants, sans utiliser de données spécifiques sur la consommation. Les résultats empiriques d'un échantillon de femmes célibataires aux États-Unis indiquent que, pour les mères célibataires, les effets monétaires dominent les effets temporels. De plus, le coût total médian par enfant est d'environ 17 060 $.

Le troisième chapitre propose un cadre structurel pour mesurer le coût total des enfants, prenant en compte à la fois le temps parental et les dépenses monétaires. Ce modèle différencie les activités de garde d'enfants perçues comme du travail de celles considérées comme du loisir. Un aspect clé de notre approche est que le prix du temps parental n'est pas simplement égal au salaire, mais est déterminé par la substituabilité entre le temps parental et les services de garde d'enfants achetés. L'analyse empirique basée sur les données de couples américains qui travaillent montre que les mères perçoivent 68 % de ce temps comme du travail, contre 53 % pour les pères. En outre, une part importante du coût des enfants supporté par les parents est non monétaire, soulignant l'importance d'intégrer les dimensions temporelles dans l'évaluation des coûts parentaux.

Mots-clés : Demande de consommation, Demande de loisir, Échelle d'équivalence, Économie d'échelle, Identification, Offre de travail, Modèle collectif, Prix de l'ombre, Prix du temps, Règle de partage.

Contents

Acknowledgements **i**

Abstract **iv**

Résumé **v**

List of Tables **v**

List of Figures **vi**

General Introduction **1**

1 Children Costs in a One-Adult Household: Empirical Evidence from the UK **10**

1.1 Introduction . . . 11

1.2 Theoretical Framework . . . 14

1.2.1 The Consumption Behavior of a Single-Adult without Children . . . 14

1.2.2 The Consumption Behavior of a Single-Adult with Children . . . 15

1.2.3 Identification . . . 18

1.3 Empirical Implementation . . . 21

1.3.1 Econometric Specification . . . 21

1.3.2 Estimation Strategy and Instruments . . . 23

1.4 Data . . . 25

1.4.1 Sample Selection . . . 25

1.4.2 Sum up the Data . . . 26

1.5 Estimation Results and Discussion . . . 27

1.5.1 Budget Share Equations . . . 27

1.5.2 Resource Share Equations . . . 27

1.5.3 The Two-Child Limit: Blessing or Burden? . . . 29

1.5.4 Intra-household Resource Allocation . . . 29

1.5.5 Comparison with OECD-modified Equivalence Scale . . . 32

1.5.6 Sensitivity Analysis . . . 33

1.6 Conclusion . . . 35

Appendices **36**
1.A Identification Proof 36
1.B Additional Estimation Results 37
1.C Informal Investigation 38
1.D Additional Figures 39

2 How to Incorporate Children into Labor Supply Equations? An Equivalence-Scale-Based Approach **49**
2.1 Introduction 50
2.2 The Model 52
2.2.1 The General Model 52
2.2.2 A Tractable Model 57
2.2.3 Extension to a Couple with Children 58
2.3 Estimation 59
2.3.1 Empirical Specification 59
2.3.2 Estimation Method 60
2.3.3 The Cost of Children 61
2.4 Data 62
2.4.1 Descriptive statistics 63
2.4.2 Endogeneity and Preliminary estimations 63
2.5 Estimation Results and Discussion 64
2.5.1 Parameter estimates of the wage equations 64
2.5.2 Parameter estimates of the baseline labor supply equations 65
2.5.3 Parameter estimates of the children cost equations 68
2.6 Conclusion 71

Appendices **73**
2.A Marginal effects of w and y 73
2.B Supplementary Data Information 73
2.B.1 Definition of covariates in regressions 73
2.B.2 Composition of different sources of income (source: PSID codebook) 74
2.C Further Results 75
2.D Additional Figures 76

3 The Household Demand for Leisure, the Price of Time and the Full Cost of Children: A Structural Model and Evidence from the PSID **80**
3.1 Introduction 81
3.2 The Model 83
3.2.1 Decision-making in a couple with children 83
3.2.2 A More Tractable Model: The Separation Principle 86
3.2.3 Identification 87
3.3 Empirical Specification and Estimation Method 90

3.3.1 The Childcare Technology . . . 90
3.3.2 Spouses' Preferences and the Full Cost of Children . . . 91
3.3.3 Estimation Method . . . 94
3.4 Data and Estimation Results . . . 95
3.4.1 Sample selection . . . 95
3.4.2 Estimates of the Childcare Technology . . . 98
3.4.3 Leisure demand equations . . . 99
3.5 Conclusion . . . 103

Appendices **105**
3.A Proofs . . . 105
3.A.1 Proof of Proposition 1. . . . 105
3.A.2 Proof of Proposition 2. . . . 106
3.A.3 Proof of Proposition 3. . . . 107
3.B The Likelihood Function . . . 110
3.B.1 Likelihood function of the demand for parental childcare time . . . 110
3.B.2 Likelihood Function of demand for market childcare services . . . 113
3.B.3 Data description. . . . 113
3.B.4 Definition of covariates in regressions. . . . 114
3.C Additional Results . . . 115
3.D Correction factor Results . . . 117

General Conclusion **118**

Bibliography **121**

List of Tables

1.1 Descriptive statistics from the FES 1978-2020: Single adults and single parents 26
1.2 Estimated paramaters of the individual resource shares and individual prices 28
1.3 Children resource share estimates 30
1.4 Robustness tests 33
1.5 Estimated paramaters of the individual resource shares: further results 34
1.6 Results for budget share equations 37
1.7 Estimates of the difference of the average cost of children by parent 38
1.8 Estimates of the difference in the average cost of children by the gender of child/children 38
1.9 Estimates of the average cost of children 39

2.1 Descriptive Statistics 64
2.2 Estimated parameters of the net total expenditures 65
2.3 Estimated parameters of the wage equations 66
2.4 Estimated parameters of the labor equations 67
2.5 Estimated quartiles of the distribution of marginal effects hourly wages and net total expenditures on labor supply 68
2.6 Estimated parameters of the children cost equations 69
2.7 Distribution of unpartnered women's price of time and simulated wage 69
2.8 Cost of children 70
2.9 Estimated parameters of the net total expenditures 75
2.10 Estimated quartiles of the distribution of marginal effects hourly wages and net total expenditures on labor supply 76

3.1 Descriptive statistics 96
3.2 Estimated parameters of childcare time 98
3.3 Estimated parameters of childcare expenses 99
3.4 Marginal Budget Shares 100
3.5 Estimated parameters of the cost function of children 101
3.6 Cost of children computed at the median point of the sample 103
3.7 Estimated Demand Full Leisure Equations 115
3.8 Estimated parameters of the cost function of children 115
3.9 Cost of children computed at the median point of the sample 116

List of Figures

1.1 Children resource share by total expenditures 30
1.2 Share of parents total expenditures devoted to children 31
1.3 Histogram of children's share 39
1.4 Relative price indexes for male and female clothing between 1978 and 2020 40
1.5 Share of parents' total expenditures devoted to boys 41
1.6 Share of parents' total expenditures devoted to children in families with only boys . . 41
1.7 Cost of boys borne by each single parent 42
1.8 Cost of children borne by each single parent in families with only boys 42
1.9 Share of parents' total expenditures devoted to girls 43
1.10 Share of parents' total expenditures devoted to children in families with only girls . . 43
1.11 Cost of girls borne by each single parent 44
1.12 Cost of children borne by each single parent in families with only girls 44
1.13 Share of parents' total expenditures devoted to children of mixed-gender 45
1.14 Share of parents' total expenditures devoted to children in families with only mixed-gender siblings 45
1.15 Cost of children of mixed-gender borne by each single parent 46
1.16 Cost of children borne by each single parent in families with only mixed-gender siblings 46
1.17 Share of parents' total expenditures devoted to same-gender children 47
1.18 Share of parents' total expenditures devoted to children in families with only same-gender siblings 47
1.19 Cost of children of same-gender borne by each single parent 48
1.20 Cost of children borne by each single parent in families with only same-gender siblings 48

2.1 Expenditures on children by family size 71
2.2 Wage Distribution of Working Women by Fertility Status 76
2.3 Wage Distribution of Working Women by Race 77
2.4 Labor Supply Distribution Among Women by Fertility Status 77
2.5 Labor Supply Distribution Among Women by Race 78

3.1 Distributions of time allocation shares across households 97
3.2 Family expenditures on a child, by income level and age of the youngest child 104
3.3 Family expenditures on a child, by income level and age of the youngest child 117

General Introduction

In many political systems, the state assumes a role in ensuring that families with children are not placed at a financial disadvantage. This aligns with the broader social contract, where citizens expect the government to protect vulnerable groups, such as children, through redistributive policies. Accurate measurement of the cost of raising children strengthens the state's ability to fulfill this responsibility.

Children, as social beings endowed with rights, inevitably impose economic costs on their parents. To conceptualize their role in the economic sphere, some economists have described them as goods that bring satisfaction to parents (Weiss and Willis, 1985, Blundell, Chiappori, and Meghir, 2005, Cherchye, Rock, and Vermeulen, 2012), as long-term investments (Heckman and Carneiro, 2003, Cunha and Heckman, 2007, Heckman and Masterov, 2007, Heckman et al., 2010), or as public goods generating both positive and negative externalities (Folbre, 1994). Despite the diversity of theoretical perspectives, it is undeniable that raising and caring for a child involves significant expenditures.

For a long time, economists have sought to develop methods for evaluating the cost of children. Early approaches, dominant before the 1940s, aimed to answer the following question: "How much income does a family with children need, compared to a childless family, to achieve an equivalent standard of living?" [1] (Whiteford, 1985, Browning, 1992, Nelson, 1993). These methods, known as "budgetary standards", involved defining a basket of goods deemed necessary to ensure a minimum standard of living for the child and estimating its cost.

These approaches were criticized for their normative nature, relying on subjective judgments about the goods and services deemed necessary to achieve a certain standard of living. Despite these criticisms, they have significantly influenced national and international statistics, serving as the basis for defining equivalence scales. For example, in 1995, the Australian Minister for Social Security tasked the Social Policy Research Centre (SPRC) with developing indicative budget standards, including estimates of the cost of children in different family configurations. These estimates remain the reference for budgetary standards in Australia today. Similarly, in 1963, the United States developed an equivalence scale

[1] Browning (1992) highlights four types of questions that arise in studies regarding the cost of children. Firstly, the positive question, which asks how children affect the expenditure patterns of a household. Secondly, the needs question, which investigates how much income a family with children requires compared to a childless family. Thirdly, the expenditure question, focusing on how much parents actually spend on their children. Finally, the iso-welfare question, which explores how much income a family with children needs to be as well off as a family without children. .

based on daily nutritional needs, which was used as the official poverty measure and is still in use today.

Other methods in the literature face a methodological and conceptual shift in their approach. Instead of addressing the question of needs, these methods aim to answer a slightly different question: "How much income does a family with children need to be as well off as a childless family?" This distinction is essential as it shifts the analysis toward a comparison of living standards between households with and without children, going beyond the simple satisfaction of basic needs.

The Engel and Rothbarth methods are two of the most commonly used approaches in this context. The Engel method, inspired by the work of Ernst Engel (1895), is based on the idea that the share of income spent on food is a good indicator of a household's standard of living. Engel posited that, as household income or size increases, the proportion of income dedicated to food decreases. Thus, the proportion of income spent on food is used to compare the living standards of households with and without children. The cost of children is then estimated by adjusting household income until the share of food expenditure returns to its pre-child level.

However, this method has several limitations. It tends to overestimate the cost of children, as it focuses solely on food expenditures (Nicholson, 1976). Since children primarily consume food and clothing, adjusting income to restore the share of the food budget to its pre-child level may result in overcompensation (Nicholson, 1976). Watts (1967) introduced the iso-prop method, which incorporates other consumer goods, such as housing and clothing, to address this bias.

Another limitation of the Engel method is its assumption that the proportion of food expenditure is a reliable indicator of overall well-being. This implies that two households with the same food budget share have the same level of well-being, regardless of family composition, which is debatable.

The Rothbarth method (1943) takes a different approach. It is based on the idea that household expenditures on adult goods (such as alcohol, tobacco, or adult clothing) reflect their well-being. When children are introduced into the household, parents reduce their spending on adult goods to meet the needs of their children. By observing this reduction, the Rothbarth method allows for an indirect estimation of the cost of children.

Although this method is often considered more realistic than the Engel method, it also has limitations. It does not account for the income effect associated with the presence of children, nor does it consider economies of scale. Additionally, the choice of adult goods is sometimes problematic. For example, using items like alcohol and tobacco to evaluate well-being may seem inadequate, particularly for families who do not consume these products. Deaton and Muellbauer (1986) pointed out that these goods are not very sensitive to income changes, making the income effects poorly defined. In several studies (Dunbar, Lewbel, and Pendakur, 2013, Bargain, Donni, and Hentati, 2022, among others), including the first chapter of this thesis, adult clothing is used as an adult good, as it is well-defined in household surveys.

Finally, contemporary family economics literature recognizes that households are composed of individuals with potentially divergent preferences. See, e.g., Chiappori (1988, 1992), Vermeulen (2002), Blundell et al. (2005), Donni and Chiappori (2011), Bargain and Donni (2012a), Cherchye et al. (2012), Dunbar et al. (2013), Browning et al. (2013, 2014), Bargain et al. (2022), Lechene et al. (2022).

Consumption decisions therefore result from a process of negotiation between household members, each seeking to maximize their own well-being while taking shared resources into account. This development has led to the redefinition of equivalence scales based on collective models, which are theoretically more robust because they are grounded in individual measures of well-being. However, most of these studies focus on couples (Blundell et al., 2005, Bargain and Donni, 2012a, Dunbar et al., 2013, 2021, Penglase, 2021, Bargain et al., 2022), raising the question of their applicability to single-parent families, a question that the first chapter of this thesis titled "Children costs in a one-adult household: empirical evidence from the UK" aims to address.

To what extent are child cost estimates derived from couple households applicable to single-parent families? Does the two-child limit policy effectively account for resource distribution in low-income, single-parent households? To address these questions, I adapt the collective model developed by Bargain et al. (2022) to one-adult households to investigate how changes in the parent and children's characteristics translate into changes in individual-level allocation in one-adult households. The model infers expenditures on children by analyzing traditional consumer surveys, with a particular focus on adult clothing expenditures and socio-demographic variables.

The primary objective of this chapter is to measure the cost of children borne by single parents. In this chapter, a single parent refers to someone who cares for and raises one or more children alone, without the presence of a spouse or partner in their household. To achieve this, we model the consumption behavior of two distinct groups: single adults without children and single parents. For childless single adults, the model assumes that utility is derived from the consumption of adult-specific goods and a composite good. The goal is to define the demand function for the adult-specific good. This is accomplished by deriving the adult-good budget function from the optimization process for that group.

In contrast, for single parents, the model assumes that they behave altruistically and that both the parent and child have distinct preferences. As in the case of multi-person households, some goods possess public characteristics, necessitating a consumption technology that accounts for the joint consumption of these public goods. Additionally, parents allocate a portion of their resources to their children's consumption while retaining the remainder for themselves. This mechanism is captured through the sharing rule. [2] In this study, we assume that the sharing rule depends on both the parent's and child's characteristics. The parent then maximizes their utility subject to this adjusted budget constraint. From this solution, we derive the demand function for the adult-specific good. An

[2]The collective models, such as those developed by Chiappori (1988, 1992), posit that households make efficient decisions, regardless of the internal decision-making process. These models are built on the premise of Pareto efficiency, where no individual within the household can be made better off without making someone else worse off. The second fundamental theorem of these models suggests that household decisions can be understood through a theoretical two-step process. In the first step, individuals within the household share their resources according to a pre-determined sharing rule, which reflects the bargaining power of each member. In the second step, each individual uses their allocated resources to independently select the bundle of goods they wish to consume. The sharing rule refers to the allocation of resources in the initial stage and is directly linked to bargaining power. The greater an individual's bargaining power, the larger the proportion of resources they receive.

interesting feature of this model lies in the fact that the adult-good demand function is sufficient for retrieving the cost of children, as the sharing rule allows us to estimate how resources are divided between parent and child.

However, identifying the sharing rule and consumption technology requires additional assumptions. Two approaches are considered: either imposing assumptions on the comparison of preferences across different household structures (Bargain et al., 2010, Lise and Seitz, 2011, Bargain and Donni, 2012a, Browning et al., 2013, Bargain et al., 2022) or incorporating distribution factors (Attanasio and Lechene, 2014). Since the latter is rarely available, we adopt the first strategy, further strengthened by an exclusion restriction.

The demand equations are estimated separately for men and women, as this study focuses on unpartnered adults. Specifically, we estimate the demand functions for individuals without children and for those with children. Using data from three sets of expenditure surveys in the United Kingdom (UK), covering the period from 1978 to 2020, the full structural model—comprising individual preferences, the sharing rule, and the shadow price—is estimated in a single step.

Our findings align with the Rothbarth hypothesis, which posits that the presence of children reduces adult-specific consumption. Additionally, we estimate the sharing rule and investigate potential differences in the costs borne by single parents compared to couples. The results reveal significant disparities: child costs estimated from couple-based models (Bargain et al., 2022) tend to underestimate the costs faced by single parents. This is potentially due to structural differences between the two household types (Nieuwenhuis and Maldonado, 2018). Our results also indicate that in low-income families, the proportion of resources allocated to children is much larger compared to wealthier families, where household size plays a less significant role.

This chapter makes several contributions to the literature. First, it provides the first empirical estimates of child costs in single-parent households, revealing potential underestimation when applying estimates derived from couples (Bargain et al., 2022). Second, the findings raise critical questions about the effectiveness of the UK's two-child limit policy, particularly in its impact on larger, low-income families. These contributions broaden the discussion on fertility and welfare programs (Kearney, 2004, Milligan, 2005, Brewer et al., 2012, Cohen et al., 2013, Laroque and Salanié, 2014, González and Trommlerová, 2023).

Although equivalence scales have been extensively studied (Bourguignon and Browning, 1991, Bradbury, 1994, Apps and Rees, 2001, Bradbury, 2008, Bargain et al., 2010, Bargain and Donni, 2012a, Dunbar et al., 2013, Penglase, 2021, Bargain et al., 2022), they offer a relatively limited view of the true cost of children for parents. The costs of raising children can be divided into two main types: direct costs and indirect costs. Direct costs include the expenses directly attributed to raising a child, such as spending on food, clothing, housing, healthcare, education, and so one. However, to estimate the full cost of children, indirect costs must also be included. These represent lost income due to reduced working hours or delayed career advancement, often impacting mothers the most (Adda et al., 2017). Together, these direct and indirect costs provide a more complete estimate of the full cost of raising children. However, traditional approaches to equivalence scales have primarily been anchored in consumption models, therefore overlook the labor supply dimension, which would better capture the reallocation of the family resources (time and money).

Existing models that explore the impact of children on labor supply often rely on non-structural frameworks (Angrist and Evans, 1998). Moreover, current structural models typically focus on the substitution effect associated with raising children (Blau and Robins, 1988, Connelly, 1992, Averett et al., 1997, Baker et al., 2008, Bernal, 2008, Apps et al., 2016). Specifically, when a woman has a child, the time she dedicates to her child competes with the time available for work, increasing the opportunity cost of participating in the labor market. However, having a child also entails additional expenses, prompting parents to work more to offset the higher financial demands associated with raising children. This income effect is generally not considered in the current literature of labor supply. The second chapter entitled "How to incorporate children into labor supply equations? An equivalence-scale-based approach" seeks to fill that gap by presenting a labor supply model that incorporates both the substitution and income effects of children.

The second chapter's primary objective is twofold: first, to develop a simple, theory-based method for including children into labor supply models, and second, to estimate the full cost of children. To achieve this, we focus on two groups: single women without children and single mothers. By modeling their respective labor supply behaviors, the goal is to quantify the full cost of children.

We posit that a mother behaves altruistically, in line with Becker's framework, and assume distinct preferences for both the mother and her child within a collective household model. The model explicitly incorporates children's time and monetary costs, which directly affect labor supply. The mother must allocate her available time between childcare and labor, dedicating a portion of her income to children's expenses. This allocation reduces the time available for paid work and decreases the disposable income left for her own consumption. The household maximizes overall utility under three constraints: a standard budget constraint, a childcare technology constraint, and a non-negativity constraint. The childcare constraint assumes that the mother's time and market-based childcare are imperfect substitutes, meaning that more parental time cannot fully compensate for less market-based care, and vice versa.

Using the second fundamental theorem to our collective model, the outcome of the household decision can be reached with a two-step procudure. In the first step, the mother minimizes the full cost of childcare by optimizing the allocation of her time versus market-provided childcare, as well as her expenditures on children. This allows us to derive both the time and monetary costs associated with children. In the second step, the mother maximizes her own utility subject to her adjusted budget constraint, conditional on the full cost of children and the time spent on childcare. The resulting solution provides the labor supply function of the mother. A key feature of this function is that it adjusts the worker's full income to account for the direct monetary costs of raising children.

To identify the structural parameters of the model, we make several assumptions. Following the equivalence scale literature, particularly collective household consumption models, we assume that preferences remain stable between women with and without children (Bargain et al., 2010, Bargain and Donni, 2012a, Bargain et al., 2022). Specifically, we assume that the marginal rate of substitution between labor and consumption is identical for these two groups. Additionally, a proportionality assumption is introduced, derived from the collective consumption literature, which posits that childcare expenditures are a linear function of exogenous income (Bargain and Donni, 2012a, Menon et al., 2012, Dunbar et al., 2013, Bargain et al., 2022, Lechene et al., 2022). The model also assumes that the price of a mother's time devoted to childcare is proportional to her wage rate. This reflects the opportunity cost of time, where higher wages imply a higher cost of time spent on childcare instead of paid labor.

This chapter further provides a measure of the full cost of children, encompassing both monetary and time dimensions. The full cost of children is identified by observing how labor supply changes in response to children's presence and needs. The model uses the woman's wage rate as the price of time for working mothers and computes a reservation wage for non-working mothers to estimate the time and monetary costs of childcare.

To illustrate the model's practical application, we use a dataset from the 2019 Panel Study of Income Dynamics (PSID), focusing on single women. We found that mothers' working hours are more responsive to wage changes than those of childless women. In addition, our findings indicate that on average, single mothers allocate 443 hours annually to childcare, with approximately 44% of their non-labor income directed toward child-related expenses. The median annual total cost of children is estimated at $17,060, a figure consistent with U.S. Department of Agriculture estimates (Lino et al., 2017). Finally, we found that the monetary effect of children dominates the time effect. This is potentially because single mothers value each income opportunity more highly due to the immediate financial need to care for children.

This chapter contributes to the literature by introducing a novel method for identifying women's preferences and the full cost of children through labor supply equations. It extends the proportionality hypothesis from collective demand models (Bargain and Donni, 2012a, Dunbar et al., 2013, Bargain et al., 2022, Lechene et al., 2022) and presents a simplified childcare technology model that captures children's impact on labor supply using three parameters, similar to the Independent-of-the-Base equivalence scales in demand models (Lewbel, 1989a,b). Our model shows that the effect of children on labor supply is influenced by two competing forces: the need to reduce working hours due to the time demands of children (substitution effect) and the potential increase in working hours due to higher monetary needs (income effect). The overall impact is ambiguous and depends on the balance between these two effects.

Notably, the methodology developed in this chapter allows for the estimation of the full cost of children without requiring data on childcare time or child-specific consumption. This is particularly valuable in contexts where such data is unavailable but where there is a need for effective policymaking related to children. The method provides an alternative means of deriving the cost of children by leveraging existing labor supply data. Our contribution complements existing research on the impact of formal and informal childcare expenses on maternal labor supply (Blau and Robins, 1988, Connelly, 1992, Averett et al., 1997, Baker et al., 2008, Bernal, 2008, Apps et al., 2016). Lastly, our approach integrates equivalence scales with labor supply models, which is a relatively rare contribution in the literature (Hurd and Pencavel, 1981).

Cherchye, De Rock, and Vermeulen (2015) pointed out a critical challenge in applying Becker's time allocation model. They highlight an identification problem arising from the fact that nonmarket goods (e.g., clean homes, child care) are typically unobserved, as are the prices or "values" of different time uses. In Becker's model, households decide how to allocate their time and financial resources toward the production of nonmarket goods. In the case of childcare, this involves the time parents spend with their children (time input) and the money spent on items such as toys, books, and school supplies (market goods input). A common approach is to assume that the time households devote to nonmarket activities is valued at the market wage. For example, if a parent could earn \$20 per hour by working, the time spent on childcare is also assumed to be worth \$20 per hour. However, this assumption prevents us from discerning whether differences in childcare behavior stem from differences in preferences (e.g., some parents may value spending more time with their children) or differences in productivity (e.g., some parents may be more efficient at providing quality childcare in less time).

Furthermore, assuming time is valued at the wage rate has two other implications. Firstly, it suggests that a spouse can freely allocate time between paid work and childcare, without external constraints, which is often unrealistic. For instance, if a parent wants to work 40 hours per week but can only secure part-time employment for 20 hours, the remaining time may be spent on childcare—not by choice, but due to a lack of available work. Secondly, it implies that childcare time is perfectly substitutable to paid work. Yet, Cosaert and Hennebel (2023) show that a large fraction of childcare is perceived as leisure. Similarly, Hallberg and Klevmarken (2003) found that childcare time is among the most enjoyable activities.

This prompts us to turn towards a third chapter in this dissertation, co-authored with Olivier Donni, entitled "The household demand for leisure, the price of time and the full cost of children: a structural model and evidence from the PSID". This chapter addresses the following question: How can we measure the full cost of children, taking into account both monetary and time-related costs? In doing so, this chapter also explores the following subsidiary question: How can we assign a value to the time parents spend on childcare? To answer these questions, we present a collective labor supply model for households with children. We assume that parents make decisions about how to allocate their time (between paid work, childcare, and leisure) and how to allocate their financial resources (between consumption for themselves and spending on their children). Through childcare technologies, parents are expected to provide a certain level of childcare to ensure their child's well-being.

While the second chapter deals with the price of time by assuming it equals the wage rate for working individuals, this chapter treats childcare time as having a dual nature. Here, time spent on childcare can be viewed either as a leisure-like or labor-like activity. The intuition behind this is that parents might value their childcare time differently—some may see it as rewarding, hence leisure-like, while others see it as labor-like, adding to their workload. Therefore, the price of childcare time will be lower than the wage rate if parents derive leisure-like satisfaction from spending time with their children. This represents a departure from the standard literature, as the model assumes childcare time is not perfectly substitutable with market work. In simpler terms, spending an hour on childcare is not the same as spending an hour at work. The cost of childcare depends on the degree of substitutability between parental care and market childcare.

The substitution between parental childcare and market childcare depends on the level of process benefits derived from childcare by parents. If parents derive significant process benefits (leisure-like utility) from spending time with their children, they will prefer to spend more time with their children rather than pay for market services. On the other hand, if the process benefits are small (i.e., parents view childcare as labor-like), they will be more inclined to substitute their time with external services.

In the model, parental time is valuable because it could be spent working and earning wages. For households with higher wages, the opportunity cost of spending time on childcare (in terms of foregone earnings) is higher. Therefore, households with higher parental wages are more likely to outsource childcare to external services.

The model assumes that households face a childcare constraint that must be met, either through parental time or external services. However, when the number of children increases, there may be economies of scale in childcare, meaning that the additional time or money required for each additional child is less than the cost of caring for the first child.

To identify the full cost of children, we decentralize the decision-making process of the household. The decentralization process is used in several papers in the literature (Lise and Seitz, 2011, Bargain and Donni, 2012a, Bargain et al., 2022). In our case, the first stage determines the price of childcare time by minimizing the cost of childcare. The second stage looks at how parents allocate the rest of their income to maximize utility, after accounting for the childcare cost. Further, we assume that parents' preferences for leisure and consumption are unchanged by the presence of children.

We test the model using data from the 2019 PSID, focusing on dual-earner couples with and without children. First, we estimate the childcare technology and determine the price of parental childcare time. Second, we estimate the full leisure demand functions that account for the leisure-like component of childcare activities. Our findings reveal that mothers perceive approximately 68% of the time they spend on childcare as labor-like, while for fathers, this figure drops to 50%. Our findings indicate that the shadow cost of children is the most significant financial burden borne by parents. [3] Moreover, our results suggest that children are viewed as a luxury good, suggesting that wealthier families prioritize quality over quantity.

This paper makes several important contributions to the literature. First, our model allows us to endogenously determine the price of childcare time and estimate the full cost of raising children—a first in the field.

Additionally, our model innovatively breaks down childcare time into two components: "pure childcare" activities and leisure-like (or labor-like) activities that directly impact parents' utility. This leads to an interesting feature, as the parent's leisure incorporates the leisure-like aspect of childcare time. In this sense, our contribution is also relevant to the limited studies on process benefits—the utility parents derive from spending time with their children (Graham and Green, 1984, Kerkhofs and Kooreman, 2003, Cosaert and Hennebel, 2023).

To conclude, the three chapters comprised in this dissertation explore the cost of children (direct and indirect costs), both from a theoretical and empirical perspectives. The first chapter, primarily empirical, examines the cost of children borne by single-parent households using data from the UK.

[3]The shadow cost represents the cost that is not actually paid by the parent.

The second chapter proposes an equivalence scale method adapted to labor supply models. The third chapter focuses on the full cost of children (both monetary and time-related) and the value of parental time, applying the theoretical model to American data.

Chapter 1

Children Costs in a One-Adult Household: Empirical Evidence from the UK

Anderson VIL *

November 25, 2024

Abstract: This paper addresses two critical questions for family and economic policy. Are estimates of the cost of children based on two-parent households generalizable to single-parent families? Does the “two-child limit” policy—restricting family benefits to low-income parents with a maximum of two children—contribute to child poverty? In this paper, I propose a collective consumption model for one-adult households, apply it to data from the Family Expenditure Survey (FES) in the UK, and present two key findings. First, child cost estimates derived from two-parent households tend to underestimate by 5.3 percentage points those incurred by single parents. Second, in low-income families, household size plays a crucial role in determining the proportion of resources allocated to children, a factor less relevant for higher-income families. This suggests that the “two-child limit” policy would likely exacerbate inequalities and increase child poverty within larger families.

JEL codes: C30, D11, D12, D36, D63, I31, J12, J13.
Keywords: Collective Model, Economies of scale, Identification, Resource sharing, Shadow price.

*THEMA, CY Cergy Paris Université, Cergy-Pontoise, France. anderson.vil@cyu.fr

1.1 Introduction

The cost of raising children is a significant factor in shaping family transfer policies, particularly in the UK, where child poverty remains a pressing issue. Recent poverty statistics reveal that the number of children living in relative poverty (after housing costs) increased from 3.6 million in 2010 to 4.2 million by 2022.[1] This represents about 29% of all children in the UK, highlighting a concerning upward trend in child poverty over the decade. This alarming trend has sparked considerable attention in the national media and underscores the need for well-designed family transfer systems. Achieving equity in tax deductions and social policies for single and coupled parents requires a deep understanding of the financial pressures unique to each family type. In two-parent households, resource pooling and shared decision-making may alleviate some economic pressures (DeLeire et al., 2005, Nieuwenhuis and Maldonado, 2018). By contrast, single parents, whose well-being is more directly tied to that of their children, often bear a greater financial strain. Therefore, cost estimates based on two-parent households overlook these structural differences, potentially underestimating the financial strain on single parents and leading to policies that fail to meet their specific needs. Despite this, much of the existing literature focuses on two-parent households, raising concerns about the applicability of such findings to single-parent families. See, e.g., Bradbury (1994, 2008), Bourguignon (1999), Apps and Rees (2001), Blundell, Chiappori, and Meghir (2005), Bargain, Donni, and Gbakou (2010), Bargain and Donni (2012a), Dunbar, Lewbel, and Pendakur (2013, 2021), Adda, Dustmann, and Stevens (2017), Penglase (2021), Bargain, Donni, and Hentati (2022).

An additional policy concern in the UK revolves around the "two-child limit" in welfare provisions, which restricts benefits to the first two children in a family. This policy can disproportionately affect low-income families, particularly those with more than two children. Moreover, this policy risks deepening inequalities between children from low- and high-income families, if larger families in lower-income brackets face increasing financial disadvantage.

This paper leverages existing methods in the literature to identify and estimate the consumption shares of single parents and their children.[2] Specifically, it investigates whether standard resource shares (computed for 2-parent households) accurately measure the individual well-being of single parents. More importantly, the paper examines how family size influences the allocation of resources to children across income levels. For this purpose, I extend the collective model of

[1] The report is available on the Child Poverty Action Group website.

[2] In this chapter, single parents are defined as follows: a) widowed, divorced, or separated parents with dependent children living in their household; b) a child is defined as any biological child of the household head aged 0 to 15 years. By "living in the household", we mean that the children regularly spend at least four nights per week at the parent's residence. This definition ensures the inclusion of households where children reside primarily, which is critical for analyzing costs.

Bargain, Donni, and Hentati (2022, hereafter BDH) to a setting with single adult households and discuss identification in this environment. The approach infers expenditures on children from traditional consumer surveys by examining adult clothing expenditures and socio-demographic variables. This study constructs and estimates a static model of intra-household allocation to explore how changes in parent and child characteristics affect resource distribution. The suggested household consumption framework has three main components: (1) an additive utility function, assuming the parent is altruistic towards their children; (2) a consumption technology describing how households convert purchased goods into individual consumption; and (3) a sharing rule that determines the distribution of individual resource shares, defined as the fraction of household's total resources devoted to each member. The model is estimated on a sample of one-adult households with and without children from the UK Family Expenditure Survey (henceforth FES) from 1978 to 2020.[3] It is important to clarify that this analysis neither measures child consumption nor, more broadly, child welfare.[4] Instead, it focuses on estimating the costs incurred by single parents in raising children.

The UK welfare system historically provided benefits for every child in a household. However, following the Conservative government's victory in the 2015 general election, a new policy known as the "two-child limit" was introduced in the budget. This policy, which came into effect on 6 April 2017, prevents families with a third child born after 6 April 2017, from receiving benefits for this child.

For example, a low-income family with three children, all born before 6 April 2017, would still receive Universal Credit or Child Tax Credit payments for all three children. In contrast, a similar family with three children, where at least one was born after that date, would only receive payments for the first two children.

The rationale for the Conservatives' introduction of the "two-child limit" policy was to promote fairness by aligning the financial decisions of households receiving benefits with those of families who rely solely on income from work. The idea is that means-tested benefits should reflect similar financial choices, encouraging families to think about their financial resources

[3]Family Expenditure Survey (FES) has been replaced by Expenditure and Food Survey (EFS) in 2001, then Living Costs and Food Survey from 2008 onwards. For convenience, all three are referred to as FES in this paper.

[4]For example, in the case of a single-parent household, financial transfers from the other parent, who resides outside the household, may contribute to the child's consumption, as children may live under shared custody arrangements even if they primarily reside with one parent. Regarding well-being, it is worth noting that this study does not incorporate certain important considerations, such as the potential effects on a child's well-being of having separated parents. These issues, while significant, fall outside the scope of this model and its objectives. See Browning (1992) and Folbre (2008) for an in-depth analysis of how to conceptualize the cost of children. Research on child development includes numerous articles addressing external investment in children. Those interested in delving deeper into this issue should refer to the works of Costas Meghir on Early Childhood Interventions. Also, the consideration of cognitive skills investment and its lasting effects on children is explored by Cunha et al. (2010), Del Boca et al. (2014, 2016), and Agostinelli and Wiswall (2016), among others.

when deciding on family size.

However, this policy is likely one driver of the recent increase in relative child poverty rates among larger families. A key focus of the paper is to investigate why the “two-child limit” policy is likely exacerbating child poverty among larger and low-income families. [5]

The primary contribution of this paper is the initial estimation of the cost of children in single-parent households. It reveals a potential underestimation of this cost when these families are assessed using estimates derived from couples (BDH). Furthermore, this paper contributes to the literature on the UK’s two-child welfare program by questioning its effectiveness for larger, low-income families, while broadening the discussion on fertility and welfare programs (Kearney, 2004, Milligan, 2005, Brewer, Ratcliffe, and Dsmith, 2012, Cohen, Dehejia, and Romanov, 2013, Laroque and Salanié, 2014, González and Trommlerová, 2023). Finally, by including single fathers in the analysis, this study also lays a basis for comparing the child-related costs between single mothers and single fathers. [6]

This research is made possible by the availability of relatively large sample data on single-household expenditures. However, as expenditure surveys typically provide consumption data at the household level, addressing the issue of equivalence scales presents significant challenges. Our empirical findings can be summarized as follows. First, estimates of child-related costs derived from couple households appear to underestimate these costs for single-parent households by 5.3 percentage points. Second, family size emerges as a critical determinant of child-related economic burden in low-income families, whereas it plays a negligible role in high-income families. Specifically, a larger number of siblings significantly disadvantages children in low-income households. This suggests that the “two-child limit” policy would likely worsen disparities and increase child poverty in larger and low-income families. Finally, our results reveal notable differences in resource allocation between single mothers and single fathers, as well as differences due to the size and gender composition of siblings, which overall provide strong support in favor of economies of scale in childcare. On average, the cost of a child represents 33% of total expenditures for single fathers and 22% for single mothers. This puzzling result likely stems from single mothers receiving higher alimony payments from their ex-husbands compared to single fathers from their ex-wives.

The remainder of this paper is organized into five parts. The first presents the theoretical model. The second outlines the empirical framework. The third describes the data. The fourth

[5] Recent statistics indicate that, while relative child poverty rates after housing costs have decreased for families with one or two children since 2015, they have actually risen for families with three or more children (Latimer and Waters, 2024).

[6] Prior studies on single parents have predominantly centered on mothers. See, e.g., Edin and Lein (1997), Meyer and Rosenbaum (2000, 2001), Schoeni and Blank (2000), Grogger (2001), Blank and Schoeni (2003), Blundell and Hoynes (2004), Meyer and Sullivan (2004, 2008), Winship and Jencks (2004), and DeLeire et al. (2005).

reports and discusses the empirical results, and the last section concludes.

1.2 Theoretical Framework

This section presents a static household consumption model following BDH, starting with single individuals as a foundation for analyzing unpartnered adults with children.

1.2.1 The Consumption Behavior of a Single-Adult without Children

In this section, I model the consumption behavior of a single-adult household without children. Each household is assumed to have a well-behaved utility function, $U(x^a, x^c)$, which is twice continuously differentiable, strictly increasing, and strictly concave over two goods: an assignable good x^a and a composite good x^c.[7] Individual utility is further influenced by preference-driven factors, which are incorporated into the budget share function in the empirical section. Preferences are assumed to be stable, enabling predictions about household behavior.

Each individual purchases x^a quantities of private assignable goods and x^c quantities of composite goods.[8] Thus, each individual faces the budget constraint as follows:

$$x^a p + x^c = y \tag{1.1}$$

where y denotes the total household expenditure and p the price of the assignable good. The market price of the composite good is normalized to one.

At this stage, the optimization program of the household member is as follows:

$$\max_{x^a, x^c} \quad u(x^a, x^c) \text{ subject to (1)} \tag{1.2}$$

The solution of this program allows expressing the demand functions for the assignable good as:

$$\omega = g(p, y) \tag{1.3}$$

where $\omega = px^a/y$. It is worth noting that $U(.)$ is strictly increasing, then ω must exhaust the

[7] An assignable good refers to a good that is consumed by a specific individual and cannot be shared or jointly consumed with others (e.g., clothing). The composite good represents any good other than the assignable one. Thus, the distinction between private and assignable goods could be omitted in the case of an unpartnered adult without children, as all goods are consumed privately, eliminating any potential confusion regarding individual consumption. However, I retain this distinction to maintain clarity and consistency, particularly because the demand function under consideration is specifically related to assignable goods.

[8] Non-durable goods are excluded from the analysis, as is standard in the literature. Therefore, if the fraction of purchased goods that remain unconsumed is small, the quantities purchased can be considered equivalent to the quantities consumed.

total expenditure.

1.2.2 The Consumption Behavior of a Single-Adult with Children

In this section, I consider a household consisting of an adult and their children. [9] The parent is assumed to be altruistic in the Beckerian sense, meaning they derive utility not only from their own consumption but also from their child's well-being. In this case, each single parent has a well-behaved utility function $W[u(x^a, x^c), u_k(x^k)]$ that contains two components - the first sub-utility derived from their own consumption u and the other one from their representative child's consumption u_k.

I consider an additive utility function that takes the following form:

$$W = u(x^a, x^c) + \delta(n)u_k(x^k) \tag{1.4}$$

where x^a represents goods consumed exclusively by the parent, while x^c and x^k represent composite goods for the parent and for the children, respectively. [10] The parameter $\delta(n)$ reflects how resources allocated to the child change as the number of children increases and can be interpreted as the weight the parent places on the child's consumption (Bargain and Donni, 2012b). [11] Alternatively, it can be viewed as a measure of parental altruism.

For simplicity, household income is given with no time-allocation decisions or household production considerations.[12] Household income is entirely allocated to purchasing q^a quantities of assignable goods and Q quantities of composite goods. Thus, y represents total expenditures rather than total income. The household budget constraint is expressed as follows:

$$q^a p + Q = y \tag{1.5}$$

Here, q^a and Q denote, respectively, the purchased quantities of the household's exclusive goods and household composite goods.

[9] In contrast to Penglase (2021), the model treats foster and non-foster children indiscriminately. Penglase (2021) explicitly separates the two groups of children, focusing on whether there is differential treatment in the allocation of resources for the consumption of foster and non-foster children. At this point, no distinction is made regarding the characteristics of the children. My assumption is limited to the child residing with either the father or the mother and younger than 16 years old.

[10] Assignable good and exclusive good are used interchangeably as well as single parent and lone parent. See Browning, Chiappori, and Lewbel (2013) for more details about exclusive and assignable goods.

[11] When there are no children ($n = 0$), then $\delta = 0$, and the model reduces to a standard single-adult consumption model. Thus, children influence household decisions through the utility their parents derive from their well-being. The Pareto weight $\lambda(n)$ may depend on factors such as prices, income, its sources (e.g., government transfers), and other variables, which could influence the resource share allocated to children. For simplicity of notation, I abstract from these considerations at this stage.

[12] A broader perspective on this topic is addressed by Apps and Rees (2001) and Cherchye et al. (2012).

Several remarks can be made. First, children's consumption is included in Q. Second, there are two types of goods: an adult exclusive good x^a, such as adult clothing, and other goods that are non-assignable to adults, x^c and x^k. Third, household composite goods consist of both private non-assignable goods and public goods. Finally, household survey data typically do not track individual consumption within a household. Therefore, information on composite goods provides limited insights into the share of resources allocated to children. However, observing adult-exclusive goods can reveal relevant aspects of household behavior.

An assignable good or exclusive good is purely private. That is, for any household's demographic structure, the consumption of an exclusive good reflects precisely the household's expenditure. Thus:

$$q^a = x^a \tag{1.6}$$

However, in a household with at least two members—an adult and a child—some goods have public properties, meaning their consumption cannot be accurately captured by their purchased quantities alone. Two main approaches address economies of scale. The Independence of Base (IB) assumes that the cost savings from shared consumption are independent of both prices and the household's total expenditure. This approach suits studies based on cross-sectional data from a single or two years (Lewbel and Pendakur, 2008, Bargain and Donni, 2012a, Dunbar et al., 2013). Conversely, the Barten scales introduce a transformation in the price vector, allowing the cost savings from shared consumption to vary with the household's composition and the type of good. Given the multi-year dataset with varying prices, I favor the latter approach. In this framework, purchased quantities are transformed into higher consumption levels with a transformation rate dependent on three exogenous variables. This assumption follows the work of (Browning, Chiappori, and Lewbel, 2013, hereafter BCL).

Assumption 1 *(Barten prices). For each adult living in a household with $n > 0$, there exists a scalar-valued, differentiable function $\pi(y, p, n)$ such that household purchases of composite goods satisfy:*

$$Q = \pi(y, p, n)x^c + x^k \tag{1.7}$$

The function $\pi(y, p, n)$ represents shadow prices for the parent. To ensure identification, I normalize the shadow price for children to one. The shadow price serves as a deflator that measures the cost savings experienced by adult due to household economies of scale (Bargain and Donni, 2012a). Instead of using the market purchases Q to produce composite goods that contribute to utility, the household effectively achieves an increased quantity of market goods x^c through sharing. A classic example of such a good is heating. The interpretation of $\pi(y, p, n)$

leads to three distinct scenarios. If $\pi(y,p,n) = 1$ for $n > 0$, goods are purely private.[13] The parent's shadow price depends on the presence of children. If the parent prefers public goods because of the children, π will be less than one; otherwise, it will be greater than one.

Substituting Equations (1.6) and (1.7) into the household budget constraint (1.5), we get:

$$x^a p + \pi(y,p,n)x^c + x^k = y \tag{1.8}$$

Parents maximize their utility subject to the new budget constraint (1.8). In a household consisting of one adult and children, the parent's decisions are automatically Pareto efficient. This outcome is derived from the assumption that the adult acts as a dictator within the household, making all consumption decisions on behalf of the children. [14]

The trade-off that needs to be done will happen in allocating resources for the parent and child consumption. Given budget and technology constraints, parents cannot improve the child's well-being without diminishing their own. The household allocation can be derived from the following optimization program:

$$\begin{aligned} &\max_{x^a,x^c,x^k} u(x^a,x^c) + \delta(n)u_k(x^k) \\ &\text{s.t.} \quad x^a p + \pi(y,p,n)x^c + x^k = y \end{aligned} \tag{1.9}$$

where $\delta(n)$ represents the weight the parent assigns to the child, depending on the number of children. The budget constraint shows total expenditures on both adult and child consumption.

Adopting an additive utility function simplifies the transition to a decentralized approach. In the first stage, the resource distribution between the parent and the child is determined by solving:

$$\max_{\phi,\phi_k} \quad \nu\left(\frac{p}{\pi}, y\frac{\phi}{\pi}\right) + \delta(n)\nu_k(y\phi_k) \quad \text{s.t.} \quad \phi + \phi_k = 1 \tag{1.10}$$

where ν and ν_k are the indirect sub-utility functions of the parent and child, respectively, and ϕ and ϕ_k represent the share of total expenditures allocated to the parent and child.
The second stage solves the parent's decision problem:

$$\max_{x^a,x^c} \quad u(x^a,x^c) \quad \text{s.t.} \quad x^a p + \pi(y,p,n)x^c = y \cdot \phi(y,p,n) \tag{1.11}$$

[13] This explains why it is unnecessary to explicitly introduce the function π in Equation 1.1.

[14] Dauphin, El Lahga, Fortin, and Lacroix (2011) found that children, particularly those aged 16 and older, may have some degree of decision-making power within households. However, the present study focuses on children aged 15 or younger. At this age, children are neither expected to contribute to household income nor to play a significant role in household decision-making. Consequently, it is reasonable to assume that children in this age group hold no bargaining power within the household.

where $\phi(y, p, n) \leq 1$ and $n > 0$. The term $\phi_i(y, p, n_i)$ represents the fraction of resources the parent retains for their consumption, with $1 - \phi$ allocated to the child. If no children are present, $\phi = 1$, meaning the parent keeps the entire budget, as in a single-adult household.

Finally, the budget share equation is given by:

$$\frac{\omega}{\phi(y, p, n)} = g\left(\frac{p}{\pi(y, p, n)}, y\frac{\phi(y, p, n)}{\pi(y, p, n)}\right) \tag{1.12}$$

where $\omega = px^a/y$. The demand function highlights why detailing the child's utility function is unnecessary, as it does not dictate the model's outcome.

1.2.3 Identification

An important question in the model of consumer behaviour under study is regarding the sharing function and economies of scale and how to recover them. Overall, the answer to this question lies in the preference stability assumption (typically the state of individual preferences from childless individuals to single parents), the observation of exclusive goods, and the non-linearity of the Engel curve.

As is standard in the literature, I assume that the preferences of individuals with identical characteristics over exclusive goods remain unchanged regardless of family status. See, e.g., BCL, BDH. In this context, the preferences of single individuals and single parents for exclusive goods are considered similar. This assumption allows for the estimation of sharing parameters between single parents and children using the demand functions of single individuals, as their indifference curves remain unaffected by the presence of children. Therefore, any shifts in consumption patterns among single parents are attributed to changes in household composition rather than to alterations in individual preferences when moving from childless individuals to parents.

Chiappori and Ekeland (2009) mentioned that identification requires estimating at least three goods. However, Bourguignon (1999) and Bourguignon et al. (2009) demonstrated that having an assignable good suffices to recover the sharing rule and reach identification. Assignable goods such as clothing are central in several studies. See, e.g., BCL, Bargain and Donni (2012a), and BDH, among others. I exploit the existence of observable assignable goods (clothing in that case) to identify the model's structural elements.

Prais and Houthakker (1971) provided evidence supporting the use of nonlinear Engel curves and the inclusion of socio-demographic characteristics as control variables. Identification in this context requires that the demand equations exhibit non-linearities in log total expenditures. However, this is not a major concern, as budget share equations are typically non-linear, as demonstrated by Banks, Blundell, and Lewbel (1997). Finally, identification in this framework is based on two normalization conditions. First, in households without children, the market price of the composite good is normalized to one. Second, in such households, the adult retains the entirety of their budget. Consequently, the model for a childless household represents a special case of the more general household model that includes children. The following proposition summarizes the main result of identification.

Proposition 1 *Let the demand functions for an exclusive good, respectively, for single individuals and single parents be defined as:*

$$\omega = g(z_\omega, p, y),$$
$$\omega = g(z_\omega, \pi(p, z_\pi, y, n) \cdot p, \phi(p, z_\phi, y, n) \cdot y),$$

where $\pi(p, z_\pi, y, n)$ represents the price transformation à la Barten, and $\phi(p, z_\phi, y, n)$ denotes the sharing rule. Here, p is the price of the exclusive good, y represents total expenditures, n is the number of children, and z_ω, z_π, z_ϕ are sociodemographic variables associated with ω, π, and ϕ, respectively.

The functions $\pi(p, z_\pi, y, n)$ and $\phi(p, z_\phi, y, n)$ can be generically identified if any of the following conditions hold:

1. *At least one variable in z_ω is excluded from both z_π and z_ϕ.*
2. *π and ϕ are independent of y (total expenditures).*
3. *π and ϕ are independent of p (prices).*
4. *$\pi(p, z_\pi, y, n)$ and $\phi(p, z_\phi, y, n)$ are known up to some parameters (semi-parametric identification).*
5. *$\pi(p, z_\pi, y, n) = \pi_1(p, z_\pi, y) \cdot \pi_2(n)$, with $\pi_2(1) = 1$, and $\phi(p, z_\phi, y, n) = \phi_1(p, z_\phi, y) \cdot \phi_2(n)$, with $\phi_2(1) = 1$.*

The proof is given in the Appendix 1.A. Here, we give the intuition for each of the points.

1. z_ω refers to a set of variables that impact the demand for the exclusive good ω, but not necessarily the shadow price π or sharing rule ϕ. These variables may include sociodemographic factors such as age, education, or personal preferences. If there is a variable in z_ω (say, education) that only affects the demand for the exclusive good but not the shadow price or sharing rule, this creates an exclusion restriction. By observing how demand responds to changes in this variable, one can isolate its effect on demand while keeping π and ϕ fixed. For instance, if one observes that a parent's level of education influences their demand for clothing (the exclusive good) without affecting the allocation of their budget between themselves and their children (ϕ) or the adjusted price due to the presence of children (π), this allows for control over education's impact on ω and facilitates the estimation of π and ϕ. This point, together with Point 3, is utilized to identify the sharing rule and shadow prices in the empirical application.

2. In this case, the shadow price π and sharing rule ϕ do not change with total expenditures y. Therefore, whether the household's total expenditure is £1000 or £2000, π and ϕ remain unchanged. If one finds that the parent's allocation of resources between themselves and their child remains constant regardless of changes in the household's overall budget, it is not necessary to account for variations in y when estimating π and ϕ, thereby simplifying their identification. [15]

3. Suppose the price of the exclusive good (such as clothing) increases, but this does not affect how the parent allocates their budget between themselves and their child (ϕ). In this case, π and ϕ remain unchanged, allowing for their separate estimation, independent of the effect of price changes on demand.

4. If the functional forms of π and ϕ are known up to some parameters, one can focus on estimating those unknown parameters rather than trying to figure out the entire functional forms of π and ϕ. For example, if it is known that ϕ depends on the number of children, but the exact magnitude of this dependence is unknown, one might assume a specific functional form for $\phi = \Phi(n)$, where Φ could be a simple linear or logistic function. The task would then be to estimate the parameter that determines how much ϕ decreases as additional children are introduced.

5. By assuming that π and ϕ can be separated into two components, one simplifies their structure. This factorization allows us to estimate the impact of having children (n) separately from the impact of prices and sociodemographics. For example, π might be written as $\pi_1(p, z, y) \cdot \pi_2(n)$, where π_1 reflects the effects of prices and demographics, and $\pi_2(n)$ captures the effect of the number of children. This approach simplifies the estimation of π, as the problem is broken down into two smaller parts: one that depends on p, z and y, and another that depends only on n. The condition $\pi_2(1) = 1$ ensures that, in the

[15] The sharing rule is a key ingredient in my model. One of the alternative to identify the sharing rule is the "Independence of the Base" assumption made by Dunbar, Lewbel, and Pendakur (2013). Essentially, Dunbar et al. (2013) demonstrate that the sharing rule within a household can be recovered using distribution factors, provided it does not depend on total expenditure—in other words, the share an individual receives within the household is independent of the household's overall consumption. However, they allow it to depend on closely related factors such as wealth. The crucial assumption here is that resource shares do not depend on total expenditure. This assumption, while essential for identification, cannot be tested within their framework. Cherchye, De Rock, Lewbel, and Vermeulen (2015) propose a method that allows for testing this assumption and arrive at a surprising conclusion: resource shares do not depend on the household's full income. Full income is defined as the sum of both spouses' maximum potential labor income and non-labor income (excluding savings and spending on durables), with leisure factored in. Although Dunbar et al. (2013) do not directly test their "Independence of the Base" as sumption, they illustrate their methodology by examining the impact of access to credit on within-household consumption allocation in Malawi. Their findings reveal that the effect of credit on resource shares varies significantly based on the type of credit—microcredit and agricultural credit tend to reduce resources allocated to children—and on the recipient of the credit. Specifically, credit received by women tends to shift resources away from men and toward children and women themselves.

absence of children, the shadow price π does not alter the market price p.

1.3 Empirical Implementation

This section outlines the empirical methodology in two steps: first, specifying the model, followed by addressing the endogeneity issue.

1.3.1 Econometric Specification

The empirical specification examines a demand system which is quadratic in logarithmic expenditure, as used in studies such as Browning et al. (1994) and BDH. This quadratic parameterization addresses the limitation posed by the linearity hypothesis, wherein marginal budget shares are independent of the expenditure level. To capture the unobserved heterogeneity, I introduce an error term, ϵ_i, which accounts for optimization errors and other unobserved factors that influence budget allocation, but remain unaddressed by the model.

$$\omega_i = \alpha_i z_i + \beta_i \ln p_i + \gamma_i \ln y_i + \eta_i (\ln y_i)^2 + \epsilon_i \tag{1.13}$$

for $i = f, m$, where α_i, β_i, γ_i and η_i are the parameters to be estimated. The vector z_i is a linear function of a set of covariates, including education level, adult age and its square, year and its square, a set of dummies for labor force participation, home ownership and region of residence.[16] Notably the equations are gender-specific, with separate estimations for men and women.

When the individual has no children, the equation simplifies from (1.12) to (1.3). To distinguish between single parents and childless adults, I introduce a dummy variable, $\mathscr{I}_i$, which equals 1 if the adult is a parent and 0 otherwise. The stochastic structure of the budget share equations for single adults and single parents is then expressed as follows:

$$\text{If} \quad \mathscr{I}_i = 0, \quad \text{then} \quad \epsilon_i = \omega_i - \alpha_i z_i - \beta_i \ln p_i - \gamma_i \ln y_i - \eta_i (\ln y_i)^2 \tag{1.14}$$

$$\text{If} \quad \mathscr{I}_i = 1, \quad \text{then} \quad \epsilon_i = \frac{\omega_i}{\phi_i} - \alpha_i z_i - \beta_i \ln\left(\frac{p_i}{\pi_i}\right) - \gamma_i \ln\left(\frac{\phi_i y_i}{\pi_i}\right) - \eta_i \left[\ln\left(\frac{\phi_i y_i}{\pi_i}\right)\right]^2 \tag{1.15}$$

To model how parental and child attributes influence child-related costs through parental

[16]The regions for which dummy variables have been defined are described in the sample selection subsection.

resource shares, I employ a logistic function. [17] This specification aligns with prior studies by Browning et al. (1994), Lise and Seitz (2011), and BDH, among others.

$$\phi(p, n) = \frac{e^{\Phi_i(s,\kappa)}}{1 + e^{\Phi_i(s,\kappa)}} \tag{1.16}$$

Here, the price is used as an exclusion condition to ensure the identification of the sharing rule, as demonstrated in Proposition 1 (point 3). Unlike previous studies, I refrain from applying a Taylor expansion to linearize the sharing rule. Although an error term could account for unobserved heterogeneity, I follow the standard approach of modeling Φ_i as a deterministic function of, respectively, parents and children attributes, say s and k:

$$\Phi(\boldsymbol{s}, \boldsymbol{\kappa}) = \boldsymbol{s}'\Delta_s + \boldsymbol{\kappa}'\Delta_\kappa \tag{1.17}$$

Here, Δ_s and Δ_k are vectors of parameters, with s including a constant and four covariates which are the adult's education level, age, labor market status, and the logarithm of total expenditures. The vector κ consists of child-related variables, such as the number of children and its square, the average age of the children, the proportion of boys, and a dummy for presence of siblings of same gender, with the latter three multiplied by the number of children. This specification assumes that the allocation of resources to children depends on both the parent's socio-demographic factors (s) and the children's attributes (κ). Following BDH, κ is assumed to be independent of total expenditures.

Recall that the decision-making process governing resource allocation is assumed not to be subject to children's wishes, where bargaining power considerations are irrelevant. Nonetheless, the model still includes a sharing rule that determines how parental and children characteristics may drive the distribution of resources within the household. For example, older children may incur higher costs, and the presence of more children may increase total expenditures on them.

Finally, shadow prices vary with total expenditures and the number of children, specified as:

$$\pi_i(y, n) = \Pi_i \ln y_i \sqrt{n_i} \tag{1.18}$$

where π_i is a parameter to be estimated. For simplicity, the constant term is assumed to be zero.

[17] This formulation ensures that the parent's total expenditures transferred to children cannot exceed or fall below permissible levels.

1.3.2 Estimation Strategy and Instruments

The demand equations for men and women are estimated separately, given that our study focuses on unpartnered adults. Specifically, for each gender, we estimate the demand equations for individuals without children (1.14) and for those with children (1.15), ensuring identification through the hypothesis of stable preferences. The full structural model, which includes individual preferences, the sharing rule, and the shadow price, is estimated in a single step.

For each adult individual, we estimate the budget share devoted to clothing, defined as the ratio of weekly clothing expenditures to total weekly expenditures on non-durable goods.[18] The demand equations' covariates, previously enumerated, include relevant demographic and economic factors. For parents, we further estimate the sharing rule parameters, enabling us to infer the cost of raising children. The identification of the sharing rule is strengthened by the exclusion restriction, as detailed in Proposition 1.[19] In this context, we exclude several variables — house owner, prices, year and its square and region — from the sharing rule function, while controlling for the presence of siblings to capture the potential economies of scale among children.

Our model addresses two potential sources of endogeneity. The first arises from measurement error in total expenditures, which can result from the infrequency of purchases or recall errors in household surveys. Such errors may induce a correlation between total expenditures and the error term in the budget share function, leading to biased estimates. To correct for this, we follow the approach of Dunbar, Lewbel, and Pendakur (2013, hereafter DLP[1]), using total income as an instrumental variable for total expenditures. In this context, total income is uncorrelated with the consumption allocation error within a given time period, but correlated with total expenditures, making it a valid instrument. Total income also serves to address endogeneity stemming from recall errors, as long as income measurement errors are orthogonal to consumption recall errors, and income remains correlated with total expenditures.

As previous studies suggest, expectations regarding marriage can significantly influence fertility decisions (Nakamura and Nakamura, 1992, Apps and Rees, 2001).[20] However, the econometric model includes a decent set of controls that help mitigate this issue. Furthermore,

[18] In the data, we observe consumption expenditures on clothing for adults (men and women), as well as consumption expenditures on children's clothing. Additionally, we observe the consumption of composite goods. However, the consumption of private goods by adults is sufficient to infer the cost of children. We do not estimate the budget share allocated to composite goods, as the budget shares must add up to one.

[19] Please refer to point 1 in Proposition 1.

[20] Single women can easily have children without resorting to adoption or assisted reproductive technologies. Consequently, single parents and childless individuals may have fundamentally different preferences regarding children, which undermines the assumption of stable preferences across marital statuses. This selection issue presents a challenge to inferring single parents' preferences from those of observably similar childless singles.

recent empirical evidence by Bargain, Lacroix, and Tiberti (2022) demonstrates that the predictions regarding individual resource shares, particularly when using assignable goods like clothing, perform satisfactorily under the assumption of stable preferences.

To set the instruments suitably, I write the budget share equations (1.14) and (1.15) as a unique budget share equation. To do this, multiply equation (1.14) by $(1 - \mathscr{I}_i)$ if single individual and equation (1.15) by $\mathscr{I}_i$ if single parent to obtain:

$$\epsilon_i = (1 - \mathscr{I}_i)\Big[\omega_i - \alpha_i z_i - \beta_i \ln p_i - \gamma_i \ln y_i - \eta_i (\ln y_i)^2\Big] + \mathscr{I}_i\Bigg[\frac{\omega_i}{\phi_i} - \alpha_i z_i - \beta_i \ln\left(\frac{p_i}{\pi_i}\right) - \gamma_i \ln\left(\frac{\phi_i y_i}{\pi_i}\right) - \eta_i\left(\ln\left(\frac{\phi_i y_i}{\pi_i}\right)\right)^2\Bigg]$$

Rearranging the right-hand side and obtains:

$$\omega_i = \alpha_i z_i + \beta_i \ln p_i + \gamma_i \ln y_i + \eta_i (\ln y_i)^2 + \mathscr{I}_i A_i + \epsilon_i \tag{1.19}$$

with

$$A_i = \beta_i \ln\left(\frac{1}{\pi_i}\right) + \ln\left(\frac{\phi_i}{\pi_i}\right)\left[\gamma_i + \eta_i \ln\left(\frac{y_i^2 \phi_i}{\pi_i}\right)\right] - \omega_i \frac{1 - \phi_i}{\phi_i}.$$

To deal with endogeneity issues, I estimate the system of no simultaneous budget share equations by setting the iterated Two Stage Least Square Method.[21] The nonlinear estimators are iterated until the estimated parameters and error/orthogonality condition covariance matrices settle.

I use all the exogenous variables as instruments, except total expenditures which are instrumented by total income. Furthermore, I set as instruments the product $\mathscr{I}_i$ and a second-order polynomial of all the exogenous variables that enter A_i and total income. This yields 19 instruments for each equation.

To obtain adequate initial values, I first estimate the budget shares on clothing equations for individuals without children (1.14). These initial estimates serve as starting points for the estimation of the complete model, which includes individuals with (1.15) and without children (1.14). By using the simpler model (without children) first, we can efficiently derive starting values for the parameters of the full system.

For more efficient and robust estimation of the full system, we leverage the cross-equation

[21] Recall that the female budget share equation is estimated separately from the male's one as household decisions are unilateraly taken.

covariance matrix obtained from the first step as the initial matrix for the iterative procedure. This matrix captures the relationships between the residuals (errors) across the different equations in the system and helps to inform the estimation process. By using it as the starting point, the iterative 2SLS estimation methods can converge more quickly and accurately, improving the overall efficiency of the estimation.

1.4 Data

This section presents the sample selection process and summarizes the descriptive statistics.

1.4.1 Sample Selection

To measure the cost of children in single-parent households, I use data from the UK Family Expenditure Survey (FES) for the period 1978–2020. [22] The FES was replaced by the Expenditure and Food Survey (EFS) in 2001, which later became the Living Costs and Food Survey (LCF) in 2008. [23] These surveys provide detailed socio-economic information on households, including income, expenditure patterns, and regional location.

Over the entire period, the sample comprises data on 135,642 households, including single individuals, couples with and without children, and unpartnered parents. The adults range from 18 to 60 years of age. For the empirical analysis, I focus on childless adults and single parents aged up to 55. I further restrict the sample by excluding households with negative total expenditures, outliers in expenditure data, and cases with missing key information. This results in a final sample of 40,079 households: 13,921 single males, 10,726 single females, 1,644 single fathers, and 13,788 single mothers. Notably, single fathers represent only 11% of single-parent households. Among parents, 57% of fathers and 51% of mothers have only one child.

The empirical analysis focuses on budget shares for clothing, using only non-durable goods, as expenditures on durable goods do not accurately capture consumption expenditures. The demand system includes two exclusive goods—adult male and female clothing—alongside a composite good, which represents all other omitted goods to ensure total budget shares sum to one. Prices for all goods are measured annually at the national level.

The covariates include adult socio-demographic variables such as educational attainment, age, labor force participation, and homeownership. For children, I consider the number of children, their average age, and the proportion of boys, along with a dummy variable for same-gender

[22] I thank Olivier Bargain for providing the first wave of data used in the initial versions of this paper.

[23] For simplicity, I refer to these surveys collectively as FES. They have been used previously by Lise and Seitz (2011) and BDH.

siblings to account for economies of scale. Education is measured as years of schooling completed, while labor force participation and homeownership are captured through binary variables. I also include year and weekly total expenditures in pounds. To control for regional variation, I include twelve regions of Great Britain: Northern, Northern Ireland, York and Humberside, East Midlands, West Midlands, East Anglia, Greater London, South-East, North Western, South Western, Wales, and Scotland.

Table 1.1 – *Descriptive statistics from the FES 1978-2020: Single adults and single parents*

		Single Women	Single Men	Single Mother			Single Father		
				Children					
				1	2	3	1	2	3
Expenditure data									
Female clothing	Weekly expenditure (in £)	9.36	-	7.43	6.10	5.18	-	-	-
		(17.82)		(14.74)	(13.10)	(11.35)			
	Percentage of zeros	0.43	-	0.44	0.47	0.48			
		(0.50)		(0.50)	(0.50)	(0.50)			
Male clothing	Weekly expenditure (in £)	-	5.25	-	-	-	4.30	3.76	1.35
			(15.10)				(11.78)	(10.90)	(4.51)
	Percentage of zeros	-	0.72	-	-	-	0.71	0.71	0.84
			(0.45)				(0.46)	(0.46)	(0.37)
Total weekly expenditure		105.72	111.60	126.32	132.76	135.05	144.94	150.62	143.62
		(73.99)	(82.06)	(86.89)	(86.28)	(86.39)	(90.02)	(96.48)	(75.64)
Individual and household characteristics									
Women's labor participation		0.71	-	0.50	0.43	0.29	-	-	-
		(0.45)		(0.50)	(0.50)	(0.45)			
Men's labor participation		-	0.65	-	-	-	0.55	0.52	0.39
			(0.48)				(0.50)	(0.50)	(0.49)
Women's education (in years)		12.43	-	11.70	11.57	11.27	-	-	-
		(3.40)		(2.39)	(2.25)	(2.04)			
Mens's education (in years)		-	12.28	-	-	-	11.32	11.46	11.31
			(3.44)				(2.18)	(2.19)	(2.11)
Women's age		39.10	-	34.84	33.90	33.33	-	-	-
		(11.15)		(9.17)	(7.02)	(5.92)			
Men's age		-	38.32	-	-	-	38.39	37.10	35.95
			(10.20				(9.14)	(7.90)	(7.05)
House owner		0.52	0.50	0.28	0.28	0.19	0.46	0.46	0.27
		(0.50)	(0.50)	(0.45)	(0.45)	(0.39)	(0.50)	(0.50)	(0.45)
Average age of children		-	-	7.81	7.85	7.82	8.81	8.02	8.04
				(4.84)	(3.73)	(3.09)	(5.26)	(4.10)	(3.27)
Proportion of boys		-	-	0.51	0.50	0.52	0.58	0.52	0.55
				(0.50)	(0.35)	(0.30)	(0.49)	(0.35)	(0.31)
Number of observations		10726	13921	7038	4629	1577	941	505	150

Notes: This table presents the mean values of the variables used in the study, with standard deviations shown in parentheses. It provides separate statistics for single women, single men, single mothers (with one, two, or three children), and single fathers (with one, two, or three children), enabling a comparative analysis across these groups. The number of observations for each group is shown at the bottom. Expenditures are in 1987 pounds.

1.4.2 Sum up the Data

Table 1.1 reports descriptive statistics of the sample for the main variables, facilitating a preliminary analysis in the Rothbarth sense. Here are the following analyzes of clothing spending by adults. Descriptive statistics provide evidence of a reduction in adult clothing expenses due to the presence of children, regardless of the adults' gender. As illustrated in the first two columns, women and men living alone spend on average respectively £9.4 and £5.3 on clothing per week. These expenditures decrease to £7.4 and £4.3, respectively, for single mothers and single fathers with a child, representing respective declines of 21% and 19%. Furthermore, the more children parents have, the lower their clothing expenses. For instance, the average weekly expenditure on clothing for fathers drops significantly, reaching a minimum of £1.4 (£5.2 for mothers). These findings echo Rothbarth's view, as household size appears to diminish parents' welfare derived from consumption. Finally, Table 1.1 also reports the high proportion of zero values for adult clothing expenses. This pattern supports the notion that infrequent purchases introduce endogeneity in total expenditure, as noted by Keen (1986).

1.5 Estimation Results and Discussion

This section describes and analyzes findings related to the budget share equation detailed above.

1.5.1 Budget Share Equations

Table 1.6 in Appendix 1.B presents results from the budget share equations. I estimate clothing budget share equations separately for men and women using the iterative two-stage least squares method. The results indicate that socio-demographic preference parameters do not always affect the budget share for both genders in the same way. My findings partially confirm those of BDH. For women, the clothing budget share decreases with education and age but increases at a certain age. This age-related trend is significant for both genders. Additionally, the results suggest that, all else being equal, male homeowners spend less on clothing compared to non-homeowners.

1.5.2 Resource Share Equations

A key focus of this study is the effect of children on parental resource shares. Table 1.2 presents the results, showing how resource allocation reflects both parental and child-related characteristics. As discussed, ϕ_i represents the parent's retained resources, with $\phi_k = 1 - \phi_i$ allocated to children. A negative coefficient in the sharing function indicates an increase in child-related resource allocation.

The results indicate that children have an augmenting effect on parental resources (both fathers and mothers). Specifically, the negative sign of the intercept suggests that the cost of children rises significantly with the number of children, while the resources allocated per child decrease as family size increases ($\hat{\kappa}_{\text{Number of children}}$). These findings are consistent with previous studies, including Bargain and Donni (2012a), DLP[1], Penglase (2021), and BDH. Further, the results indicate that older children impose higher costs on parents. Although most parameters related to children's characteristics for fathers are not statistically significant, the signs of the variables remain consistent with those observed for mothers.

Table 1.2 – *Estimated paramaters of the individual resource shares and individual prices*

		With Siblings		Without Siblings	
		Women	Men	Women	Men
Parent characteristics					
$\boldsymbol{s}$	Intercept	1.831***	1.007	1.780***	0.932
		(0.449)	(1.063)	(0.452)	(1.044)
	Education	0.004	-0.031	0.005	-0.026
		(0.020)	(0.064)	(0.020)	(0.064)
	Age (in years)	0.007	-0.040**	0.007	0.037**
		(0.008)	(0.018)	(0.008)	(0.018)
	Labor participation	-0.167	0.304***	-0.175	0.219
		(0.155)	(0.254)	(0.159)	(0.247)
	Log total expenditures	-0.061	1.928***	-0.115	1.930***
		(0.675)	(0.597)	(0.701)	(0.646)
Children characteristics					
$\boldsymbol{\kappa}$	Intercept	-0.752***	-1.088***	-0.675***	-0.915**
		(0.143)	(0.423)	(0.139)	(0.380)
	Number of children	0.092***	0.104	0.080***	0.067
		(0.025)	(0.082)	(0.024)	(0.075)
	Age (in years)	-0.013**	-0.003	-0.013**	0.001
		(0.006)	(0.017)	(0.006)	(0.017)
	Proportion of boys	-0.054	-0.136	-0.043	-0.129
		(0.042)	(0.145)	(0.039)	(0.128)
	Same-gender siblings	0.068**	0.130		
		(0.034)	(0.111)		
Shadow prices					
$\boldsymbol{\Pi}$	Log total expenditures	-0.535	0.833	-0.567	0.805
		(0.327)	(0.560)	(0.332)	(0.587)
Sample size		25 514	15 565	25 514	15 565
(Number of free parameters, Instruments)		(33,43)	(33,43)	(32,42)	(32,42)

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parentheses. This table presents the impact of parental and child characteristics on individual resource shares. Note that the child-related variables are multiplied by the number of children, making the intercept the key parameter of interest. The dependent variable is the budget share allocated to adult clothing expenditures. The demand equation for individuals without children (1.14) is estimated simultaneously using the 2SLS method along with the equation for individuals with children (1.15). The demand system for women is estimated separately from that of men, based on the assumption of preference stability, which posits that an adult's utility reflects the same relative preferences for the exclusive good as a single individual of the same gender. Columns (1) and (3) report the results for females - and columns (2) and (4) that of males.

As expected, the parameter for same-gender siblings is positive. Specifically, the coefficient indicates that mothers retain a larger share of total expenditures when the household includes siblings of the same gender, suggesting potential economies of scale. A similar trend is observed for fathers, although this parameter is not statistically significant. An illustrative example involves same-gender siblings close-in-age siblings who ofter share clothing. This variable thus captures both family size effects and the influence of gender composition.

1.5.3 The Two-Child Limit: Blessing or Burden?

Given the variability in family expenditures, Figure 1.1 illustrates the per-child resource shares across different points in the household expenditure distribution, divided into 20 vigintiles. Focusing on the second panel, the resource shares per child at the bottom of the distribution show significant divergence: for low-incomefamilies with one child, the share hovers around 30%, while for low-income families with two and three children, it ranges from 40% to 67%, respectively. This suggests that single-child households with limited means can allocate more resources per child compared to families with multiple children. However, as total parental expenditures increase, the per-child resource share converges to approximately 12%, indicating a more uniform distribution of resources per child in wealthier households, regardless of family size.

This graph conveys several key insights. First, it highlights that there exists a minimum expenditure threshold below which government intervention is crucial to ensure the well-being of children. In other words, parents whose income falls below or near this threshold should be targeted by social policy measures tailored to the specific needs of children. Typically, there is a baseline level of consumption, independent of the number of children. For instance, consider a single father earning £1500 per month, with no access to family benefits and fixed subsistence expenditures of £1400. If he has one child, that child receives the remaining £100. However, if he has two or more children, they must share the £100 between them, as the parent's minimum subsistence expenditures leave only £100 for all the children combined. Conversely, the graph also shows that children in affluent households experience nearly uniform levels of material well-being, irrespective of family size. Thus, for wealthier parents, the number of children has little impact on the resources allocated to each child, whereas in low-income families, having more siblings significantly disadvantages children.

1.5.4 Intra-household Resource Allocation

Table 1.3 presents the average cost of children. Our findings indicate that single mothers and single fathers allocate 21.9% and 33.4% of their resources to their children, respectively. In comparison, estimates from studies on couples with children, such as those by BDH, using the same dataset from 1978 to 2007, indicate that in households with one child, mothers allocate 16.6% of resources, while fathers allocate 11.6%. These results suggest that couple-based estimates may underestimate the cost of children borne by single parents. Specifically, child cost estimates for mothers in couples, as derived from BDH, tend to underestimate by 5.3 percentage points the costs incurred by single mothers. This pattern is consistent with the structural

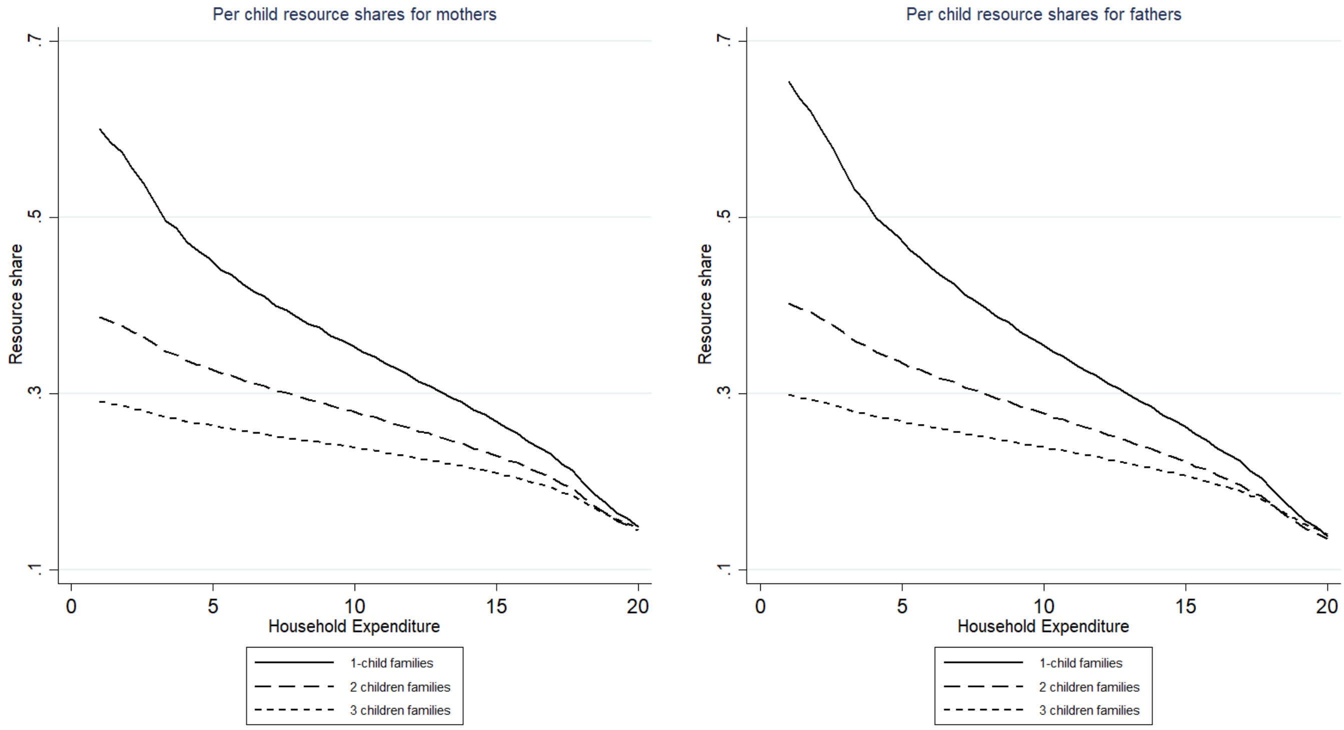

Figure 1.1 – *Children resource share by total expenditures*

Notes: This figure plots the per-child resource shares allocated by parents across different points of the household expenditure distribution. The x-axis shows the distribution of total household expenditures divided into 20 vigintiles, ranging from the 1st to the 20th. The y-axis displays the per-child resource shares for mothers (left panel) and fathers (right panel). The solid line represents households with one child, the dashed line indicates households with two children, and the densely dashed line corresponds to households with three children.

Table 1.3 – *Children resource share estimates*

	Single Mothers			Single Fathers		
Number of children	Mean	Lower bound	Upper bound	Mean	Lower bound	Upper bound
1	0.219	0.218	0.219	0.334	0.319	0.349
	(0.119)	(0.019)	(0.020)	(0.231)	(0.221)	(0.242)
2	0.324	0.323	0.325	0.467	0.446	0.487
	(0.034)	(0.033)	(0.034)	(0.238)	(0.225)	(0.254)
3	0.421	0.419	0.424	0.611	0.575	0.646
	(0.044)	(0.043)	(0.046)	(0.222)	(0.199)	(0.250)
Sample size		13 788			1 644	

Notes: Standard deviations are in parentheses. This table reports the average expenditure on children. I use the structural parameters associated to s and κ to compute ϕ_k for each parent, then I take the average to obtain $\bar{\phi}_k$.

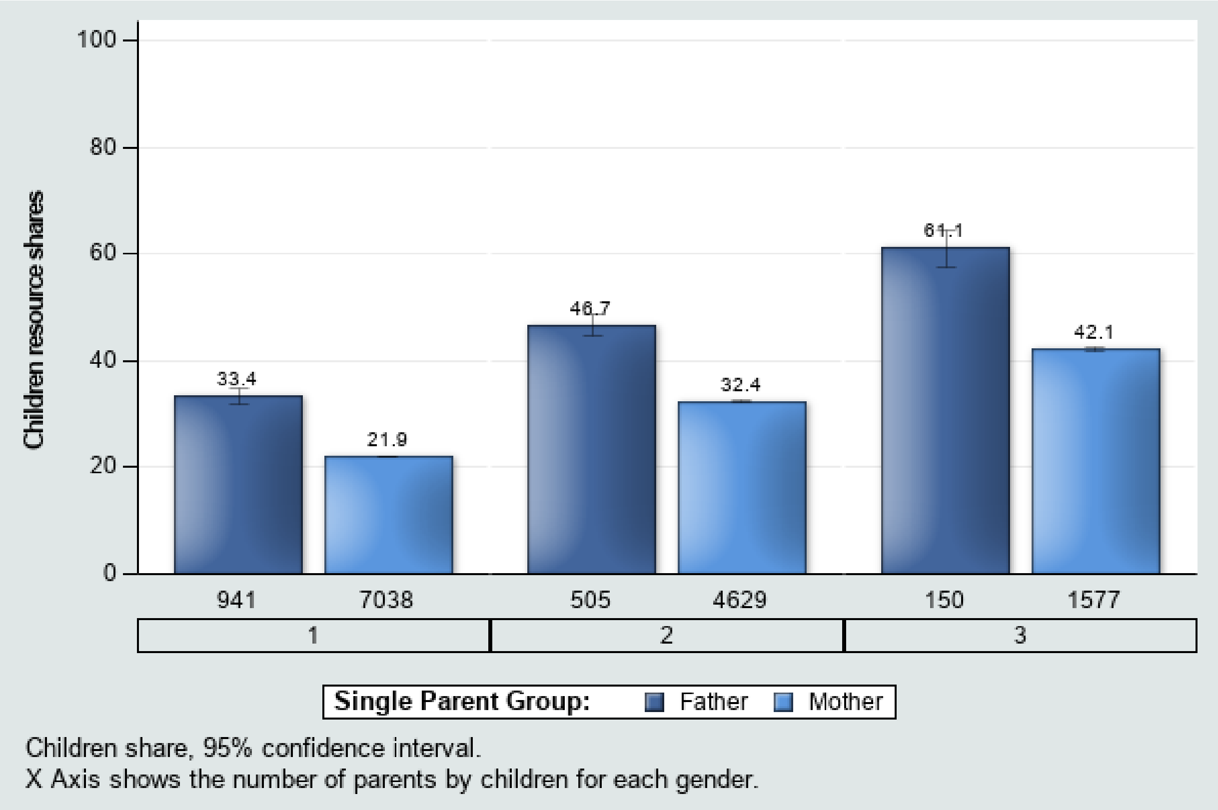

Figure 1.2 – *Share of parents total expenditures devoted to children*

Notes: This figure illustrates parental expenditures on children, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

differences between these two types of households, as highlighted by DeLeire et al. (2005) and Nieuwenhuis and Maldonado (2018).

The differences in child costs between single-parent and two-parent households can be attributed to variations in altruism, structural differences, and economic factors. First, evidence from studies such as BDH suggests that fathers in two-parent households may allocate fewer resources to children relative to mothers. In contrast, single parents, including single fathers, often display greater altruism toward their children, given their sole responsibility for the children's well-being.

Second, single-parent households differ fundamentally from two-parent households in terms of resource pooling, decision-making processes, and access to external support networks. For instance, single parents typically have less flexibility to share responsibilities or combine income, which may influence the allocation of resources to children.

Third, two-parent households are better positioned to benefit from economies of scale in shared goods, such as housing and utilities, thereby reducing the per capita cost of raising children. Conversely, single parents face higher per capita costs due to limited opportunities for shared consumption.

Figure 1.2 plots the parental expenditures on children conditional on family size. We note that, regardless of the number of children, the cost is consistently higher for fathers than for mothers. This finding is somewhat unexpected.[24] The smaller sample size of single fathers, combined with the likelihood that they represent a highly selected population group with distinct characteristics, motivations, and life histories compared to single mothers, poses challenges to drawing firm conclusions from this result. Two potential explanations for the observed differences are as follows. First, fathers are more likely to transfer resources to mothers for the benefit of their children than the reverse. This asymmetry in resource transfers could reduce the financial burden of raising children for single mothers. Second, fathers with dependent children may be less inclined than mothers to seek out and utilize existing child-related benefits. If mothers, on average, are more likely to access family benefits for children, this would effectively lower the cost of raising children for single mothers.

Furthermore, we observe that the cost of children follows a nonlinear pattern. A similar trend applies to mothers, possibly indicating a notable decrease in children's resource shares. For instance, the cost attributed to children for a father with three children falls short of doubling that of a father with a unique child.

1.5.5 Comparison with OECD-modified Equivalence Scale

The OECD-modified equivalence scale is widely used in income comparisons to adjust for household size and composition. Under this scale, children under 14 are assigned a value of 0.3 and children 14 and over a value of 0.5 relative to the first adult, who is assigned a value of 1.0. The estimates of children resource shares can be compared directly to the child resource shares implied by the OECD scale.

For example, in a single-parent household with one child under 14, the OECD scale implies that the child's share of household resources would be $0.3/1.3 = 23\%$, which is close to my estimate for single mothers (21.9%) but lower than my estimate for single fathers (33.4%). For two children under 14, the OECD equivalence scale would imply a total child share of resources of $0.6/1.6 = 37.5\%$, which again aligns closely with my estimates for single mothers with two children (32.4%) but is lower than my estimate for single fathers with two children (46.7%). The OECD scale provides a simplified view of resource allocation, whereas my model captures actual household spending patterns, which can vary by factors such as gender and income. Although the OECD scales align with our estimates, our method is essential for capturing the influence of parent and child characteristics on expenditures and for accurately distinguishing the share of child-related costs borne by mothers and fathers.

[24] Cherchye et al. (2012) suggest that empowering fathers may benefit children more than empowering mothers.

Table 1.4 – *Robustness tests*

Models		Female				Male			
		Sargan statistics	LR-type statistics	Degrees of freedom	p-value	Sargan statistics	LR-type statistics	Degrees of freedom	p-value
Reference model		20.79		10		6.63		10	
Models with	linear time trend in s	17.46	3.33	1	0.07	5.62	1.00	1	0.32
	linear time trend in κ	19.06	1.73	1	0.19	4.19	2.44	1	0.12
	prices of clothing in s	18.45	2.34	1	0.13	3.50	3.13	1	0.08
	prices of clothing in κ	19.58	1.21	1	0.27	3.49	3.14	1	0.08
	cubic term in Engel curves	20.61	0.14	1	0.93	3.12	3.51	1	0.06
Models without	economies of scale	29.79	0.60	1	0.44	9.02	2.39	1	0.12
	log total expenditures in s	23.51	2.72	1	0.10	10.45	3.83	1	0.05

Notes: This table reports Sargan and LR-type statistics for various specification of the model. The first column in each panel for both females and males shows the Sargan statistics, which are the objective function value times the number of observations. The LR-type statistics in the second column in each panel are computed as the absolute value of the difference between the Sargan statistics of the baseline model and those of the respective alternative model. It is worth noting that the objective function calculation for the alternative models is conducted using the identical baseline model weighting matrix.

1.5.6 Sensitivity Analysis

I implement three procedures to test the robustness of the results. First, I introduce seven variants of the model. Second, I test for overidentifying restrictions. Lastly, I estimate the model on a restricted sample of households. The core results exhibit qualitative consistency, albeit less pronounced in significance.

One of the primary objectives of this paper is to assess whether the sharing rule function can provide accurate estimates of the cost of children over time. I assume that the sharing rule depends on both parent and child characteristics. The first two specifications I estimate introduce time progressively into the s (parent characteristics) and κ (child characteristics) components. Table 1.4 presents the results of Sargan's test and LR-type statistics. The null hypothesis—that the sharing rule is unaffected by a linear time trend in either s or κ—is not rejected at conventional significance levels. Therefore, the determinants of s and κ remain stable over time. In this context, year serves as a relevant variable for identifying the sharing rule (see Proposition 1). Consequently, shifts in child resources are unlikely to be driven by time through parent or child characteristics.

The second robustness check incorporates the price of clothing into the sharing rule function to account for potential variability.[25] The results indicate that prices have an insignificant effect on individual resource shares. The next test assesses the sensitivity of the results by adding a third-order term to the Engel curves. The p-values (0.93 for females and 0.06 for males) do not reject the null hypothesis in the sharing rule equation at conventional significance levels.

The results from the final set of specifications are reported in the second panel of Table

[25] A graph illustrating the relative price trends is provided in Appendix 1D.

Table 1.5 – *Estimated paramaters of the individual resource shares: further results*

		I-Simplified		II-Only Mixed Gender Siblings		III-Only Working Individuals	
		Women	Men	Women	Men	Women	Men
Parent characteristics							
$\boldsymbol{s}$	Intercept	2.327***	2.242	1.825***	-0.860	1.830***	-0.259
		(0.543)	(1.479)	(0.463)	(1.357)	(0.552)	(2.503)
	Education	0.002	-0.021	0.011	0.066	0.008	0.004
		(0.018)	(0.075)	(0.020)	(0.113)	(0.026)	(0.195)
	Age (in years)	0.005	0.038*	0.004	0.076***	-0.002	0.056
		(0.007)	(0.020)	(0.007)	(0.025)	(0.009)	(0.045)
	Labor	-0.096	0.323	-0.135	0.810***	-	-
		(0.136)	(0.309)	(0.158)	(0.305)	-	-
	Log total expenditures	0.273	1.846**	0.274	2.922***	0.093	2.900***
		(0.682)	(0.763)	(0.614)	0.855	(1.338)	(0.895)
Children characteristics							
$\boldsymbol{\kappa}$	Intercept	-1.314***	-2.581	-0.713***	-1.442***	-0.694***	0.394
		(0.393)	(1.650)	(0.134)	(0.557)	(0.198)	(1.517)
	Number of children	0.211***	0.448	0.090***	0.177*	0.067*	-0.192
		(0.085)	(0.406)	(0.024)	(0.099)	(0.037)	(0.387)
	Age (in years)	-0.012**	-0.006	-0.012*	-0.052**	-0.011	-0.003
		(0.005)	(0.020)	(0.007)	(0.027)	(0.009)	(0.044)
	Proportion of boys	-0.057	-0.126	-0.134*	0.627	-0.020	-0.186
		(0.039)	(0.157)	(0.069)	(0.410)	(0.062)	(0.495)
	Same-sex siblings	0.097**	0.207	-	-	0.091*	-0.509
		(0.040)	(0.132)			(0.051)	(0.451)
Shadow prices							
$\boldsymbol{\Pi}$	Log total expenditures	-0.312	0.763	-0.337	1.311*	-0.269	1.075**
		(0.396)	(0.670)	(0.351)	(0.693)	(0.737)	(0.442)
Sample size		24514	15565	21713	15268	13685	9823
Sargan statistics		13.285	5.508	15.461	9.153	15.173	11.422
(Number of free parameters, Instruments)		(33,38)	(33,38)	(32,42)	(32,42)	(31,38)	(31,38)

Notes: $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. This table reports additional results on the sharing rule to test the robustness of the baseline model. The three models estimate the clothing demand equations separately for men and women. Model I uses a limited number of instruments by removing the second-degree polynomials for the variables included in the A_i function. Model II estimates the demand equations on a sample where parents with same-gender children are excluded. Model III uses a sample of individuals who are employed.

1.4. First, I empirically test the hypothesis of economies of scale within households using LR-type statistics, which measure the difference between the Sargan statistics of constrained and unconstrained models. Under the null hypothesis, both models (with and without economies of scale) are equivalent. However, the findings do not support the theoretical assumption of economies of scale in households and may also suggest potential issues with the functional form of the economies of scale function. Finally, I find evidence supporting the inclusion of log total expenditures in the $\boldsymbol{s}$ part of the sharing rule function, although this approach lacks strong empirical validation in the female sample.

Table 1.5 provides additional robustness checks. Given the relatively small sample size, particularly for single fathers, the estimates may suffer from overidentification bias. To address this, I re-estimate the model using fewer instruments, removing second-order polynomials for the exogenous variables in A_i. The results in Model I show that the core conclusions remain consistent, though coefficient estimates and standard errors increase. In Model II, I exclude parents with children of the same gender, and the results are comparable to those of the benchmark model. The fifth and final columns present the results when only individuals participating in the labor market are included. While the significance is reduced, the qualitative conclusions remain consistent.

1.6 Conclusion

Several models have attempted to assess the cost of children for parents, but most focus exclusively on two-parent households, overlooking the increasing prevalence of single-parent families in OECD countries (Nieuwenhuis and Maldonado, 2018). In this paper, I adapt the collective approach to better capture the decision-making processes of single parents. The primary objective is to estimate the cost of children borne by single parents. To do so, I employ a consumption model, leveraging the stability of preferences and the observation of adult exclusive goods to retrieve information from the sharing rule function in one-adult households.

Using a sample of single adults with and without children from the UK Family Expenditure Survey (FES) spanning 1978–2020, we find that standard resource shares, typically calculated for two-parent households, may not fully apply to single-parent families. Specifically, our findings suggest that the costs incurred by single parents are underestimated by 5.3 percentage points when using these measures. Furthermore, our results highlight that family size significantly influences the allocation of resources to children in low-income households, whereas it has little impact in high-income families. In other words, having more siblings disadvantages children in low-income families, but this factor becomes irrelevant in wealthier households. These findings suggest that parents operate with a minimum expenditure threshold, below which public intervention through family allowance policies is necessary to ensure children's needs are met, particularly for families with incomes below this critical level. This result also demonstrates why the two-child limit is likely to contribute to child poverty in larger and low-income families.

The main limitation of this paper concerns the potential endogeneity of having children. Fertility decisions are often influenced by expectations about marriage, which can introduce a selection bias that challenges the assumption of stable preferences across different marital statuses. While this issue could be addressed by employing revealed preference techniques, as proposed by Cherchye et al. (2015) to estimate bounds on household sharing, this approach remains a topic for future research.

Appendices

1.A Identification Proof

Proof 1. Let's write $z_\omega = (z_{\omega 0}, z_{\omega 1})$ where $z_{\omega 1} \notin z_\pi$ and $z_{\omega 1} \notin z_\phi$. Then consider two values of $z_{\omega 1}$, say $z^1_{\omega 1}$ and $z^2_{\omega 1}$. This provides a system of two equations with two unknowns:

$$\omega(\bar{p}, z_\omega, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n}) = g(\bar{p}, z^1_{\omega 1}, \pi(\bar{p}, \bar{z}_\pi, \bar{y}, \bar{n}) \cdot \bar{p}, \phi(\bar{p}, \bar{z}_\phi, \bar{y}, \bar{n}) \cdot \bar{y})$$

$$\omega(\bar{p}, z_\omega, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n}) = g(\bar{p}, z^2_{\omega 1}, \pi(\bar{p}, \bar{z}_\pi, \bar{y}, \bar{n}) \cdot \bar{p}, \phi(\bar{p}, \bar{z}_\phi, \bar{y}, \bar{n}) \cdot \bar{y})$$

Under some regularity conditions, this system of two equations generally has a unique solution for $\pi(\bar{p}, \bar{z}_\pi, \bar{y}, \bar{n})$ and $\phi(\bar{p}, \bar{z}_\phi, \bar{y}, \bar{n})$, and another for each choice of $(\bar{p}, \bar{z}_\pi, \bar{z}_\phi, \bar{y}, \bar{n})$.

2-3. Combine (2) and (3), the proof of this statement is similar to the previous one. [26]

4. Let's consider choosing a parametric specification for the sharing function, specifically a linear form that depends on k parameters. There are k degrees of freedom, representing the k identifiable parameters. The idea is that we need k equations to determine the unknown parameters.

5. Let

$$\omega = g(z_\omega, \pi_1(p, z_\pi, y) \cdot \pi_2(n) \cdot p, \phi_1(p, z_\phi, y) \cdot \phi_2(n) \cdot y)$$

By varying the values of y and n, we might obtain the following equations:

$$\omega = g(z_\omega, \pi_1(\bar{p}, \bar{z}_\pi, y_1) \cdot \pi_2(n_1) \cdot \bar{p}, \phi_1(\bar{p}, \bar{z}_\phi, y_1) \cdot \phi_2(n_1) \cdot y_1)$$

$$\omega = g(z_\omega, \pi_1(\bar{p}, \bar{z}_\pi, y_1) \cdot \pi_2(n_2) \cdot \bar{p}, \phi_1(\bar{p}, \bar{z}_\phi, y_1) \cdot \phi_2(n_2) \cdot y_1)$$

$$\omega = g(z_\omega, \pi_1(\bar{p}, \bar{z}_\pi, y_2) \cdot \pi_2(n_1) \cdot \bar{p}, \phi_1(\bar{p}, \bar{z}_\phi, y_2) \cdot \phi_2(n_1) \cdot y_2)$$

$$\omega = g(z_\omega, \pi_1(\bar{p}, \bar{z}_\pi, y_2) \cdot \pi_2(n_2) \cdot \bar{p}, \phi_1(\bar{p}, \bar{z}_\phi, y_2) \cdot \phi_2(n_2) \cdot y_2)$$

[26] The complete proof for the statement 2 is given by DLP[1], online appendix and (Penglase, 2021, online appendix).

The above example shows a set of 4 equations with 4 unknowns. Then we can identify the sharing function as well as the economies of scales. This completes the proof.

□

1.B Additional Estimation Results

Table 1.6 – *Results for budget share equations*

	With siblings				Without siblings			
	Women's budget equation		Men's budget equation		Women's budget equation		Men's budget equation	
Parameters	Est. val.	Std. err.	Est. val.	Std. err.	Est. val.	Std. err.	Est. val.	Std. err.
Intercept	0.190***	(0.014)	0.150***	(0.013)	0.190***	(0.014)	0.150***	(0.013)
Education	-0.001*	(0.000)	0.000	(0.000)	-0.001*	(0.000)	0.000	(0.000)
Age (in years)	-0.004***	(0.001)	-0.004***	(0.001)	-0.004***	(0.001)	-0.004***	(0.001)
Age2 (in years)	0.004***	(0.001)	0.004***	(0.001)	0.004***	(0.001)	0.004***	(0.001)
Year	0.904***	(0.329)	1.051***	(0.379)	0.902***	(0.329)	1.062***	(0.379)
year2	-0.904***	(0.328)	-1.048***	(0.378)	-0.902***	(0.328)	-1.059***	(0.378)
House owner	-0.000	(0.002)	-0.003*	(0.002)	-0.000	(0.002)	-0.003*	(0.002)
Labor participation	0.004	(0.003)	0.001	(0.002)	0.004	(0.003)	0.001	(0.002)
Region:								
Norhern	-0.001	(0.004)	-0.008	(0.005)	-0.001	(0.004)	-0.008	(0.005)
York & Humberside	-0.000	(0.004)	-0.016***	(0.004)	-0.000	(0.004)	-0.016***	(0.004)
East Midlands	0.003	(0.004)	-0.021***	(0.005)	0.003	(0.004)	-0.021***	(0.005)
East Anglia	-0.001	(0.004)	-0.018***	(0.005)	-0.001	(0.004)	-0.018***	(0.005)
Greater London	0.002	(0.004)	-0.018***	(0.004)	0.001	(0.004)	-0.018***	(0.004)
South-East	-0.001	(0.004)	-0.021***	(0.004)	-0.001	(0.004)	-0.021***	(0.004)
South-West	-0.003	(0.004)	-0.024***	(0.004)	-0.003	(0.004)	-0.023***	(0.004)
Wales	-0.003	(0.004)	-0.014***	(0.005)	-0.003	(0.004)	-0.014***	(0.005)
West-Midlands	0.001	(0.004)	-0.016***	(0.004)	0.001	(0.004)	-0.016***	(0.004)
North-West	-0.000	(0.004)	-0.019***	(0.004)	-0.000	(0.004)	-0.019***	(0.004)
Scotland	-0.003	(0.004)	-0.015***	(0.004)	-0.003	(0.004)	-0.015***	(0.004)
Log relative price	-0.001	(0.004)	0.011	(0.007)	-0.001	(0.004)	0.011	(0.007)
Log total expenditures	0.013	(0.012)	0.016*	(0.008)	-0.014	(0.012)	0.015*	(0.009)
(Log total expenditures)2	-0.014*	(0.008)	-0.001	(0.006)	-0.014	(0.009)	-0.001	(0.006)
Sample size	24 514		15 565		24 514		15 565	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parentheses. This table presents partial results from the demand equations for women - columns (1-2) and (5-6) - and men - columns (3-4) and (7-8). The demand equation for individuals without children (1.14) is estimated simultaneously using the 2SLS method along with the equation for individuals with children (1.15). The demand system for women is estimated separately from that of men, based on the assumption of preference stability, which posits that an adult's utility reflects the same relative preferences for the exclusive good as a single individual of the same gender.

Table 1.7 – *Estimates of the difference of the average cost of children by parent*

Parents	Method	N	95% LC Mean	Mean	95% UC Mean	95% LC SDV	SDV	95% UC SDV
Panel 1: Unweighed mean								
I-Fathers		1596	0.397	0.409	0.421	0.242	0.250	0.259
II-Mothers		13244	0.342	0.344	0.346	0.095	0.096	0.097
Diff(I-II)	Pooled	-	0.059	0.065	0.071	0.121	0.122	0.123
Diff(I-II)	Satterthwaite	-	0.053	0.065	0.077	-	-	-
Panel 2: Weighed mean								
I-Fathers		1596	0.433	0.445	0.457	0.300	0.310	0.321
II-Mothers		13244	0.378	0.379	0.381	0.124	0.125	0.127
Diff(I-II)	Pooled	-	0.059	0.066	0.072	0.154	0.156	0.158
Diff(I-II)	Satterthwaite	-	0.053	0.066	0.078			

Notes: N, LC, UC and SDV mean respectively sample size, Lower Confidence, Upper confidence and Standard Deviation. DF for Degree of Freedom.

Table 1.8 – *Estimates of the difference in the average cost of children by the gender of child/children*

Parents	Method	N	95% LC Mean	Mean	95% UC Mean	95% LC SDV	SDV	95% UC SDV
Panel 1: Cost of boys								
I-Fathers		1596	0.410	0.423	0.435	0.244	0.253	0.262
II-Mothers		13244	0.350	0.352	0.353	0.098	0.099	0.100
Diff(I-II)	Pooled	-	0.065	0.071	0.078	0.124	0.125	0.126
Diff(I-II)	Satterthwaite	-	0.059	0.071	0.084	-	-	-
Panel 2: Cost of girls								
I-Fathers		1596	0.380	0.392	0.404	0.237	0.246	0.254
II-Mothers		13244	0.335	0.336	0.338	0.091	0.092	0.093
Diff(I-II)	Pooled	-	0.050	0.056	0.062	0.117	0.118	0.120
Diff(I-II)	Satterthwaite	-	0.044	0.056	0.068			

Notes: See the notes to Table 1.7.

1.C Informal Investigation

I present a linear regression model to estimate the share of total resources devoted to children on both parent and children characteristics. The objective is simply to explore and confirm the existing correlation between parental preferences and the average cost of children.

While caution is needed in interpreting these results as causal effects, asserting that these findings validate a highly pronounced correlation between individual characteristics (parent and children) and the average cost of children remains valid. Furthermore, these estimates corroborate the signs of the different coefficients obtained in the structural model estimation.

Table 1.9 – *Estimates of the average cost of children*

		Women		Men	
	Parameters	Est. value	Std. Err.	Est. value	Std. Err.
z	Intercept	0.055***	(0.001)	0.338***	(0.012)
	Education	-0.002***	(0.000)	0.004***	(0.001)
	Age (in years)	-0.001***	(0.000)	-0.007***	(0.000)
	Labor	0.017***	(0.000)	-0.062***	(0.002)
	Log total expenditures	-0.096***	(0.000)	-0.367***	(0.002)
k	Number of children	0.209***	(0.001)	0.182***	(0.011)
	(Number of children)^2	-0.021***	(0.000)	-0.013***	(0.003)
	Age (in years)	0.003***	(0.000)	0.001**	(0.000)
	Proportion of boys	0.013***	(0.000)	0.027***	(0.003)
	Same-sex siblings	-0.026***	(0.000)	-0.028***	(0.003)
Sample size		13 244		1 596	

Notes: $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. This table presents estimates of the share of total resources allocated to children. Results for single mothers are shown in the Women column, while results for single fathers are shown in the Men column. The average cost of children is estimated separately for each group.

1.D Additional Figures

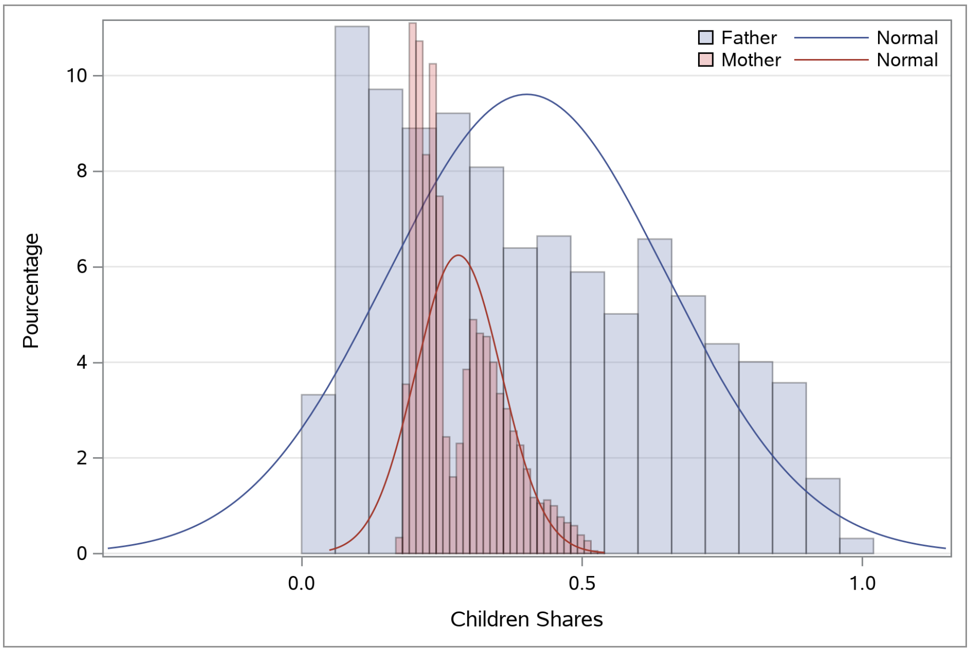

Figure 1.3 – *Histogram of children's share*

Note: This figure plots the density of the cost of children across parents. Based on the sharing rule estimates, the mean share of resources devoted to children is 0.28 and 0.40 respectively for mothers and fathers.

Figure 1.3 represents the distribution of resource devoted to children by parents. The resource shares devoted to children by fathers appear to be more evenly distributed across a broader range. The histogram indicates that fathers allocate varying levels of resources to children, with a noticeable peak around 0.4. Mothers resources allocated to children are much more concentrated in the lower range of the distribution, peaking sharply around 0.1. There is a steep drop-off, and very few mothers seem to allocate shares beyond 0.3.

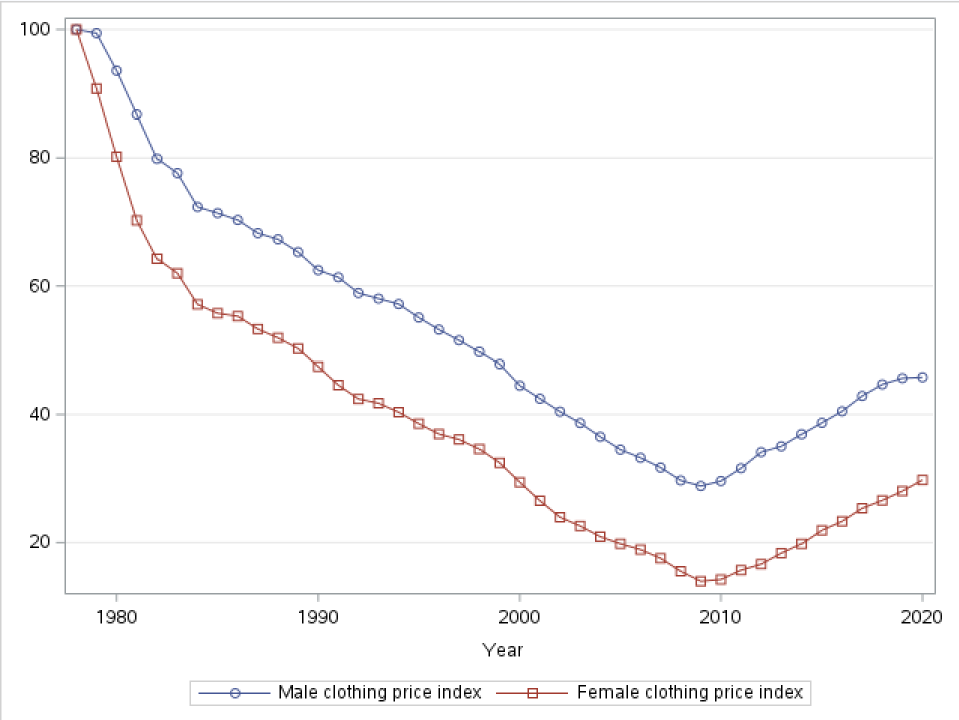

Figure 1.4 – *Relative price indexes for male and female clothing between 1978 and 2020*
Note: This figure illustrates the relative prices of male and female clothing from 1978 to 2020, represented by indices set to 100 in both 1978 and 2020, respectively..

Figure 1.4 presents the relative price trends for male and female clothing, with both indices set to 100 in the years 1978 and 2020, respectively. Both male and female clothing prices experienced a significant decline from 1978 to approximately 2010. However, female clothing prices fell more sharply than male clothing prices during this period. For example, by the mid-1980s, the index for female clothing had dropped to below 60, while male clothing was still above 70. By the early 2000s, female clothing prices reached their lowest point, falling to nearly 20 on the index, compared to approximately 30 for male clothing. From around 2010 onwards, both indices show a recovery, with male and female clothing prices gradually increasing.

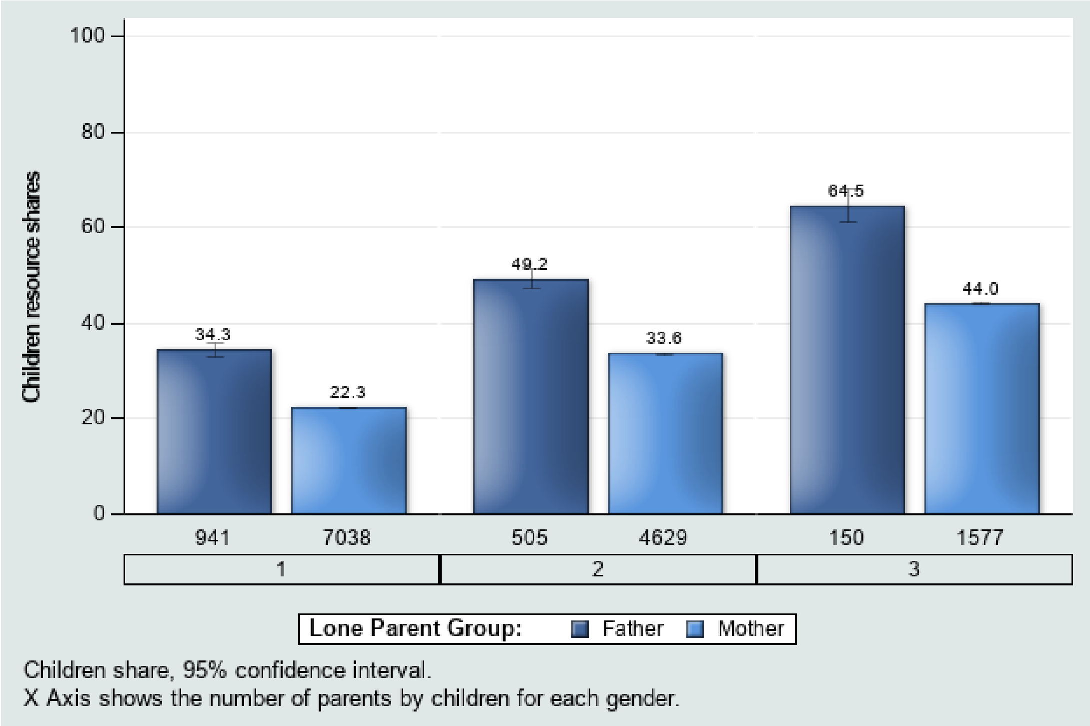

Figure 1.5 – *Share of parents' total expenditures devoted to boys*

Notes: This figure illustrates parental expenditures on boys, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

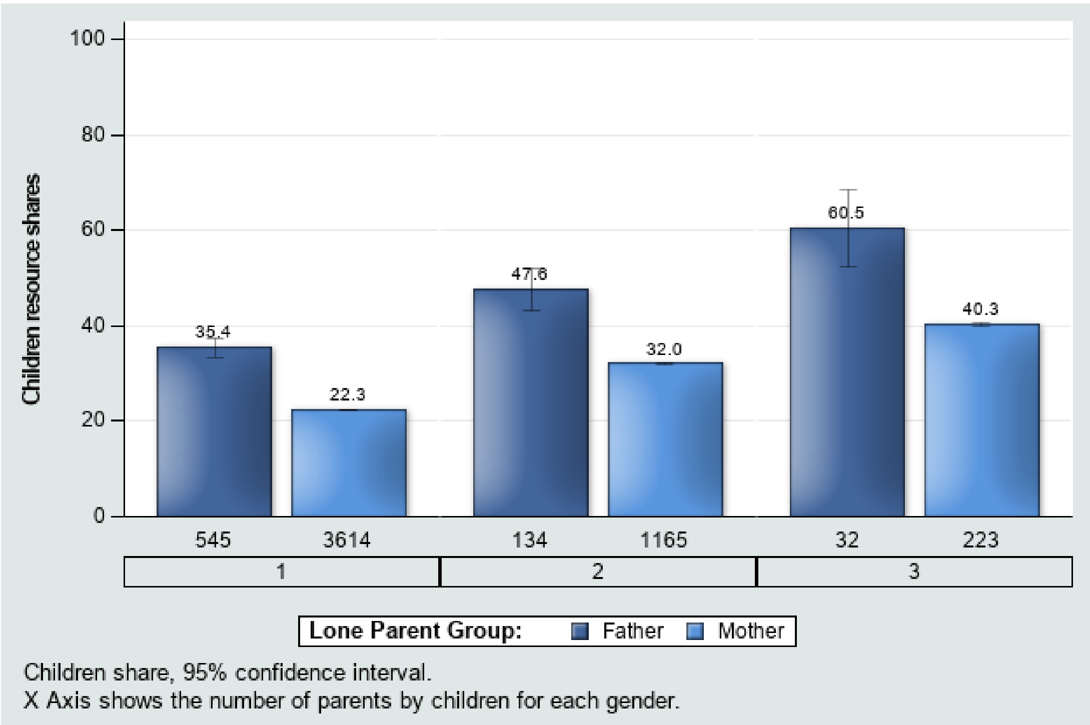

Figure 1.6 – *Share of parents' total expenditures devoted to children in families with only boys*

Notes: This figure illustrates parental expenditures on boys in families with only male children, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

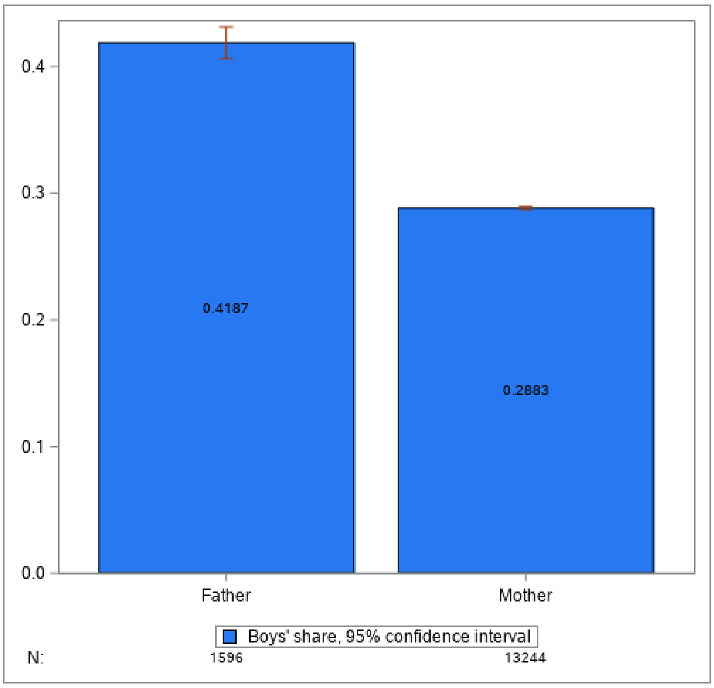

Figure 1.7 – *Cost of boys borne by each single parent*

Notes: This figure compares the average expenditure on boys by parents, based on their gender.

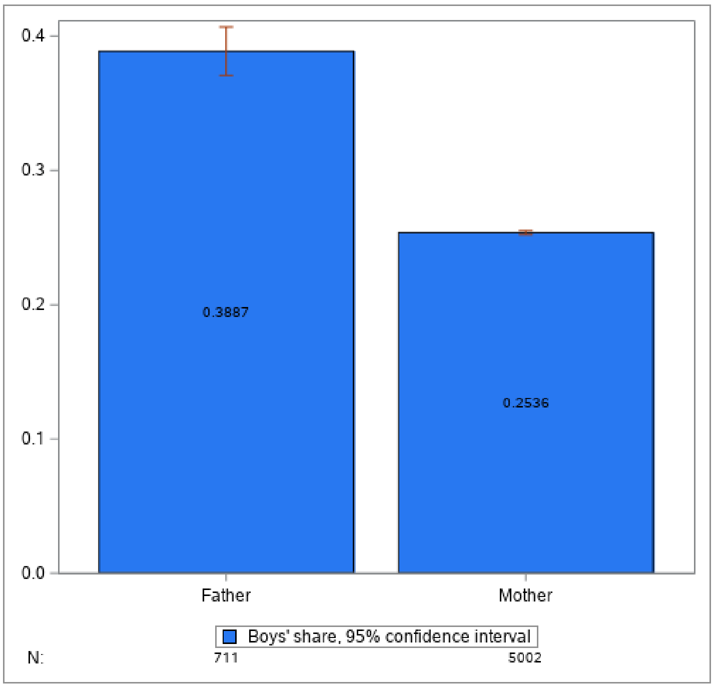

Figure 1.8 – *Cost of children borne by each single parent in families with only boys*

Notes: This figure compares the average expenditure on boys by parents in families with only male children, based on their gender.

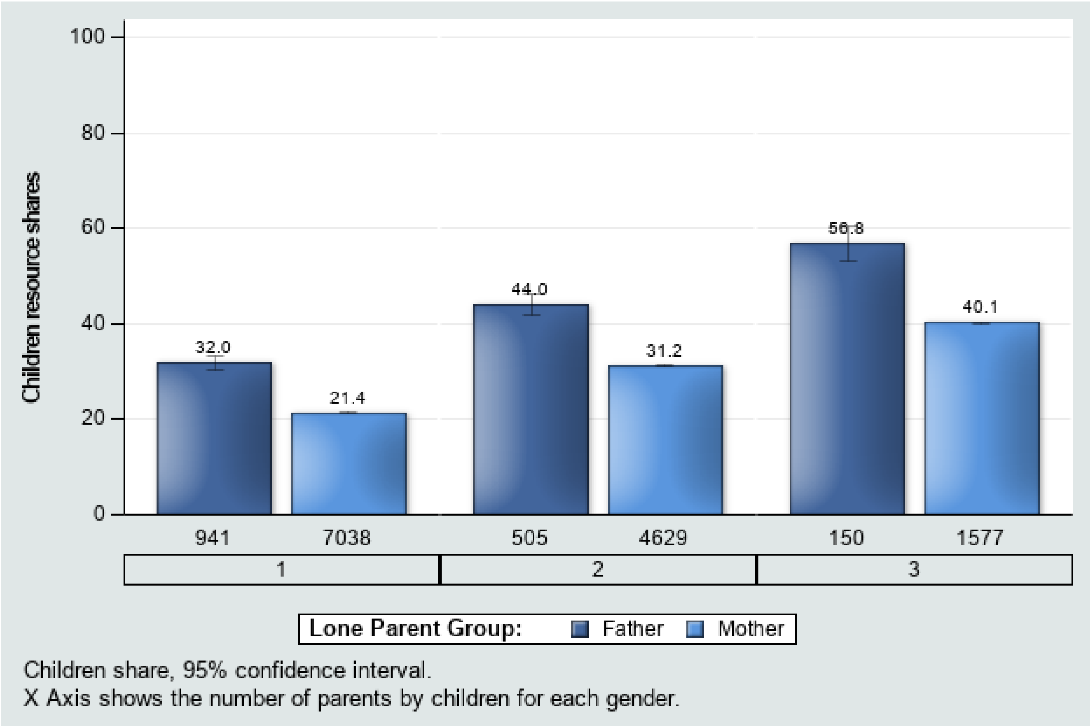

Figure 1.9 – *Share of parents' total expenditures devoted to girls*

Notes: This figure illustrates parental expenditures on girls, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

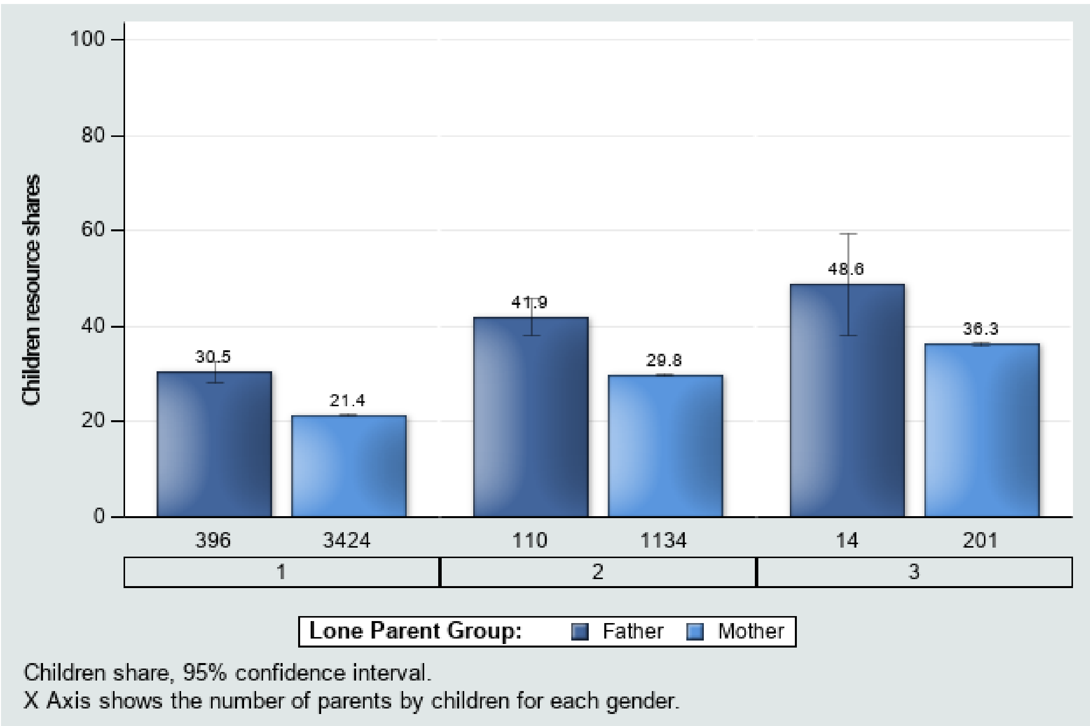

Figure 1.10 – *Share of parents' total expenditures devoted to children in families with only girls*

Notes: This figure illustrates parental expenditures on girls in families with only male children, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

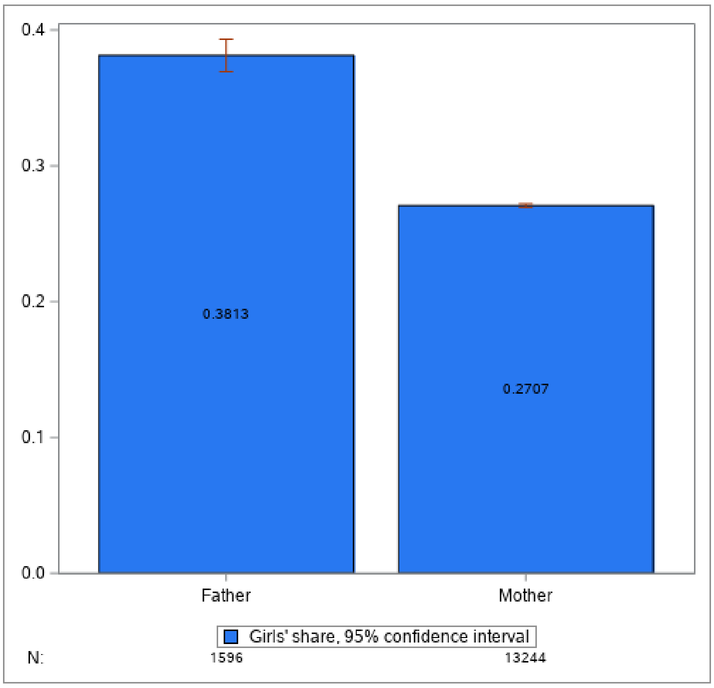

Figure 1.11 – *Cost of girls borne by each single parent*

Notes: This figure compares the average expenditure on girls by parents, based on their gender.

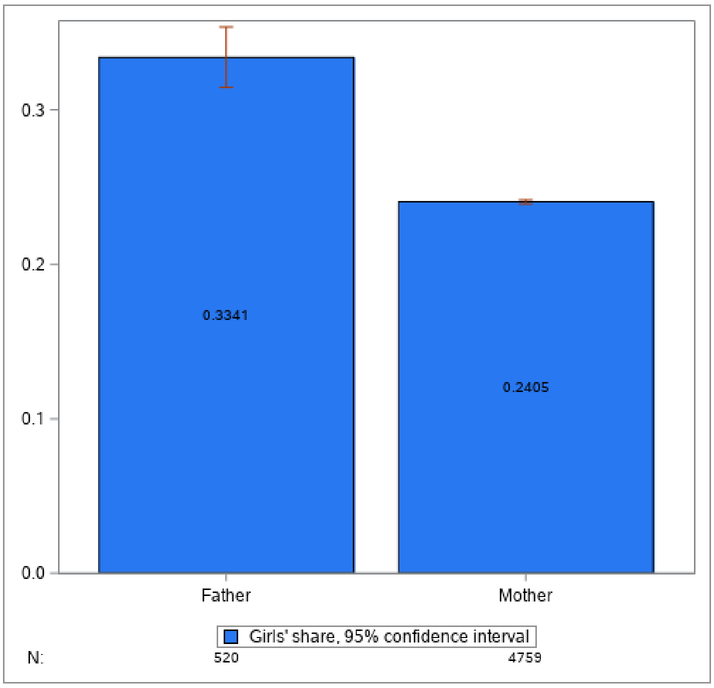

Figure 1.12 – *Cost of children borne by each single parent in families with only girls*

Notes: This figure compares the average expenditure on girls by parents in families with only male children, based on their gender.

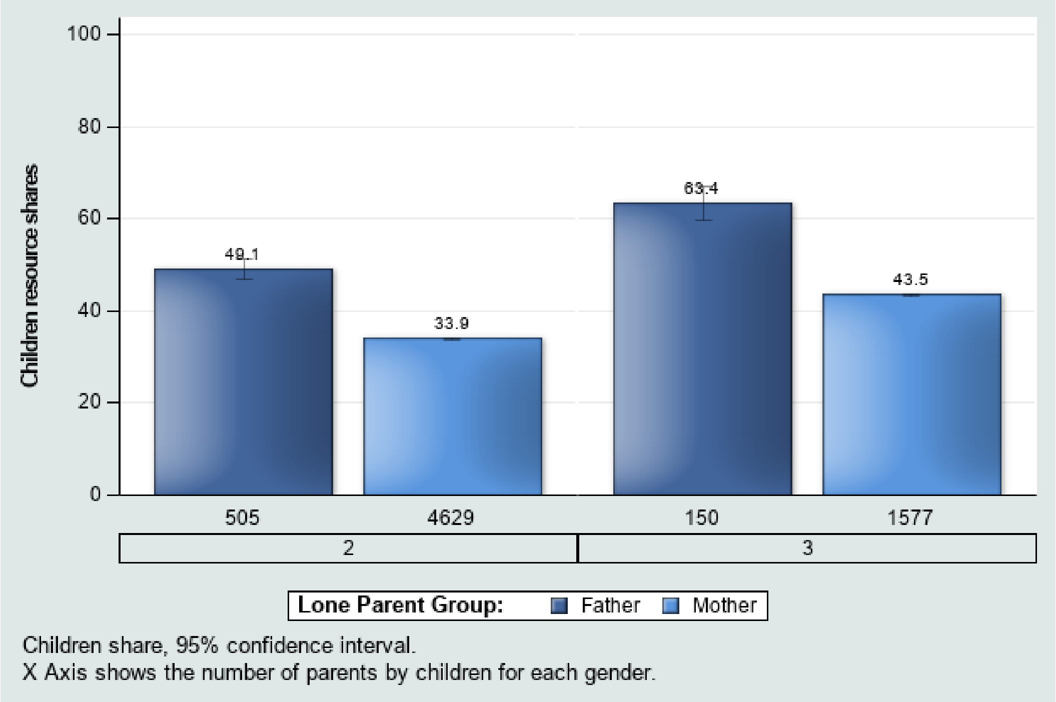

Figure 1.13 – *Share of parents' total expenditures devoted to children of mixed-gender*

Notes: This figure illustrates parental expenditures on mixed-gender children, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

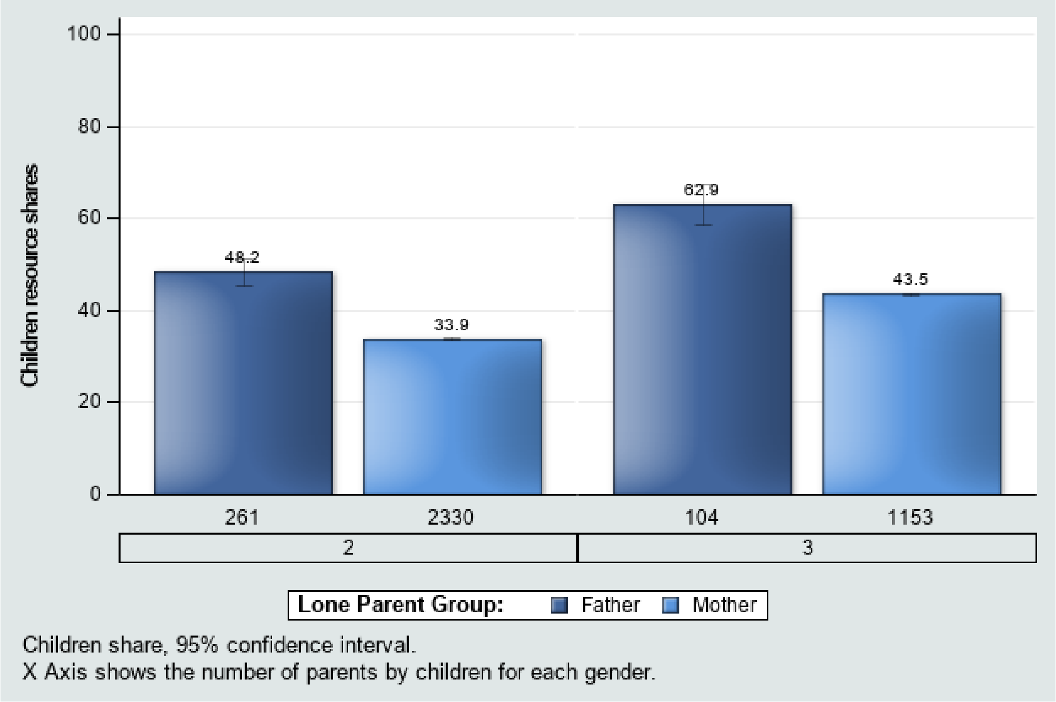

Figure 1.14 – *Share of parents' total expenditures devoted to children in families with only mixed-gender siblings*

Notes: This figure illustrates parental expenditures on mixed-gender children in families with only mixed-gender siblings, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

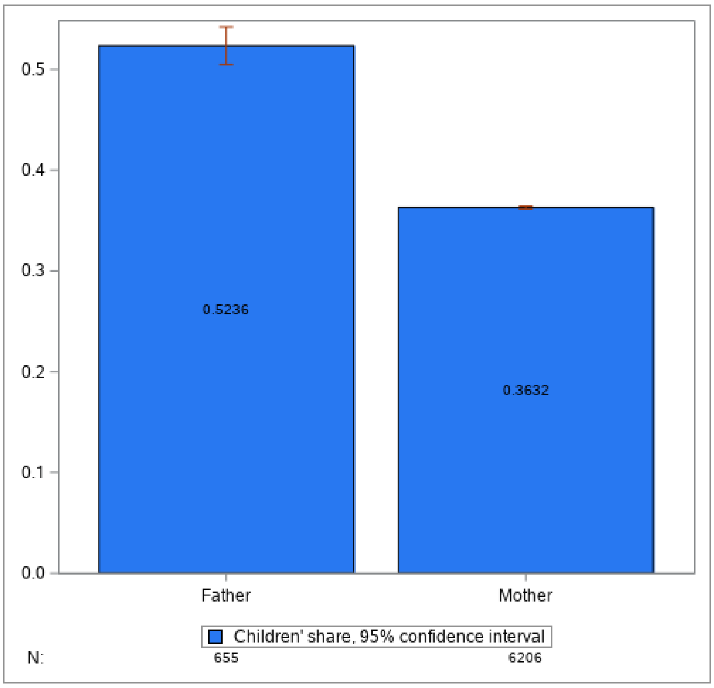

Figure 1.15 – *Cost of children of mixed-gender borne by each single parent*

Notes: This figure compares the average expenditure on mixed-gender children by parents, based on their gender.

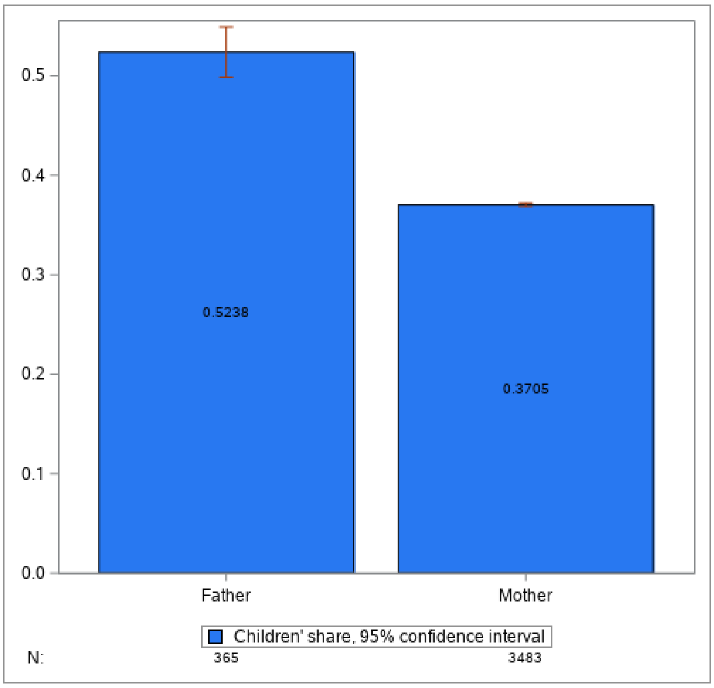

Figure 1.16 – *Cost of children borne by each single parent in families with only mixed-gender siblings*

Notes: This figure compares the average expenditure on mixed-gender children by parents in families with only mixed-gender siblings, based on their gender.

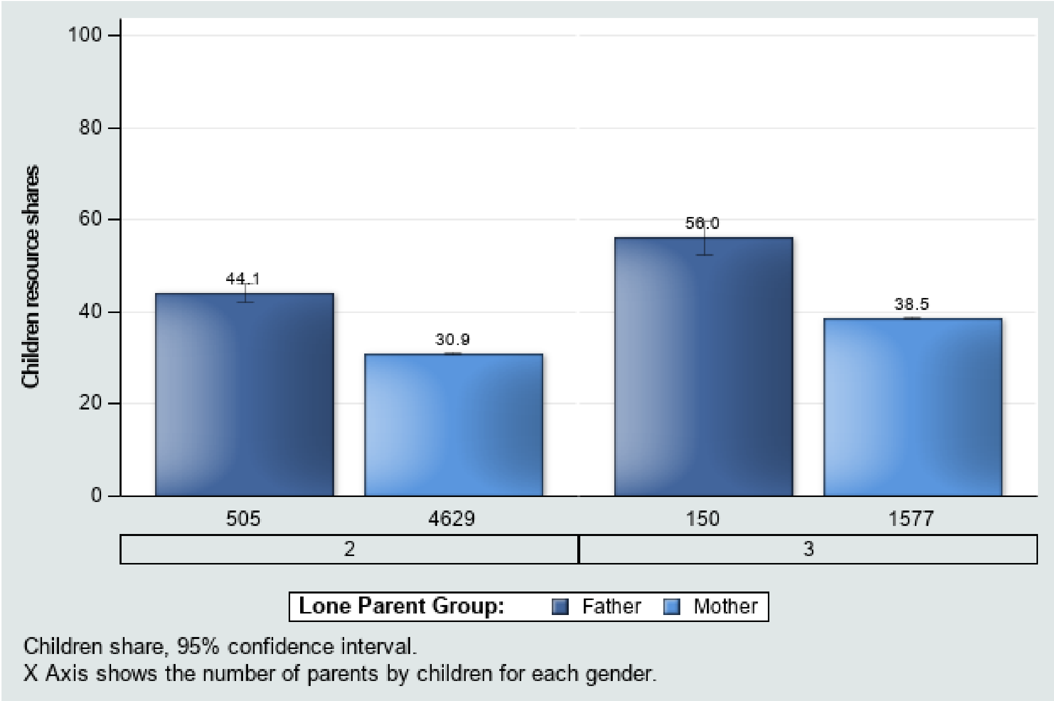

Figure 1.17 – *Share of parents' total expenditures devoted to same-gender children*

Notes: This figure illustrates parental expenditures on same-gender children, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

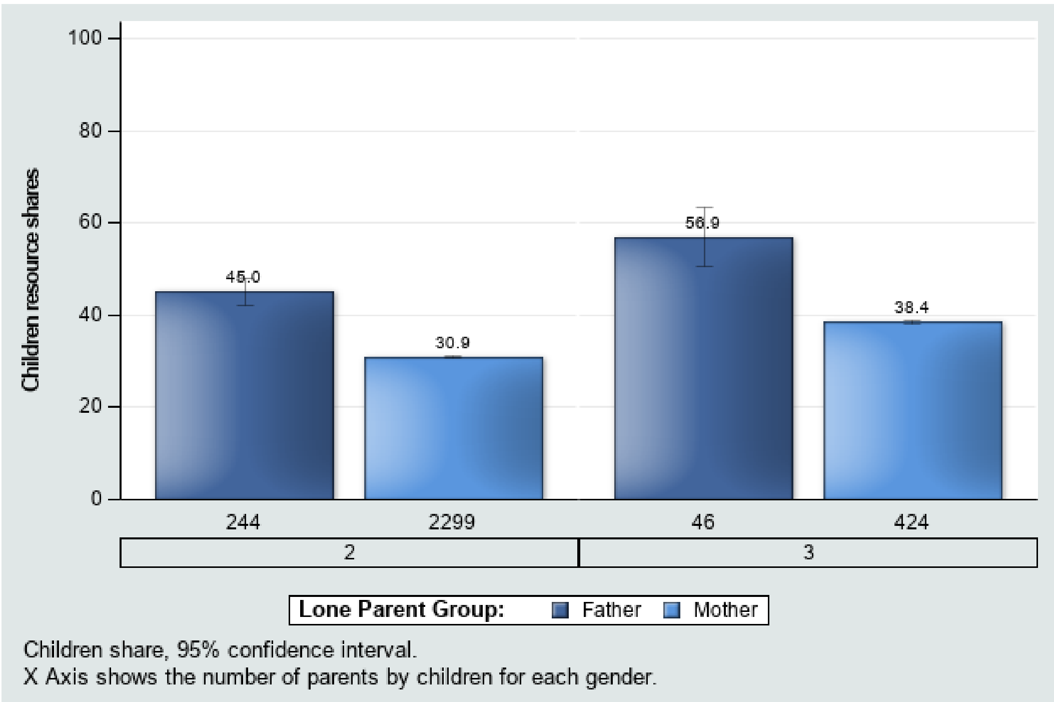

Figure 1.18 – *Share of parents' total expenditures devoted to children in families with only same-gender siblings*

Notes: This figure illustrates parental expenditures on same-gender children in families with only same-gender siblings, conditional on family size. The x-axis represents the number of parents, categorized by the number of children.

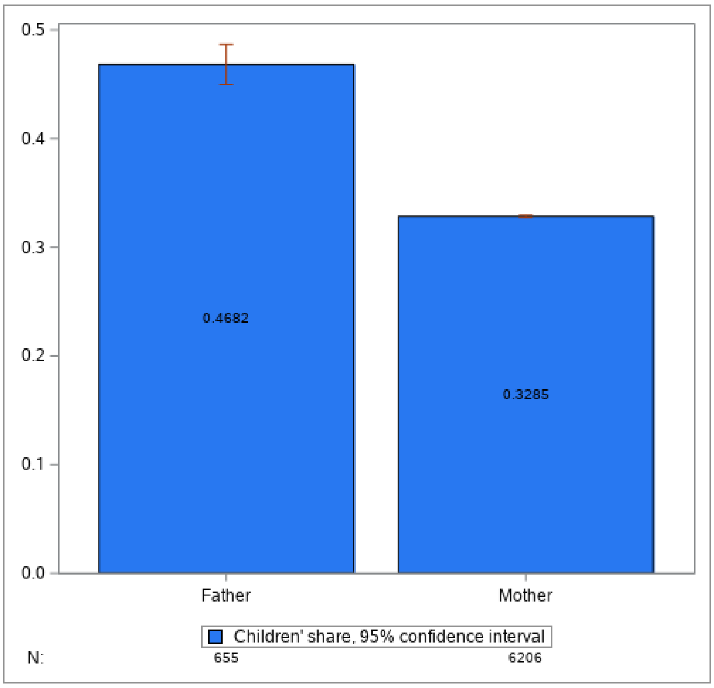

Figure 1.19 – *Cost of children of same-gender borne by each single parent*

Notes: This figure compares the average expenditure on same-gender children by parents, based on their gender.

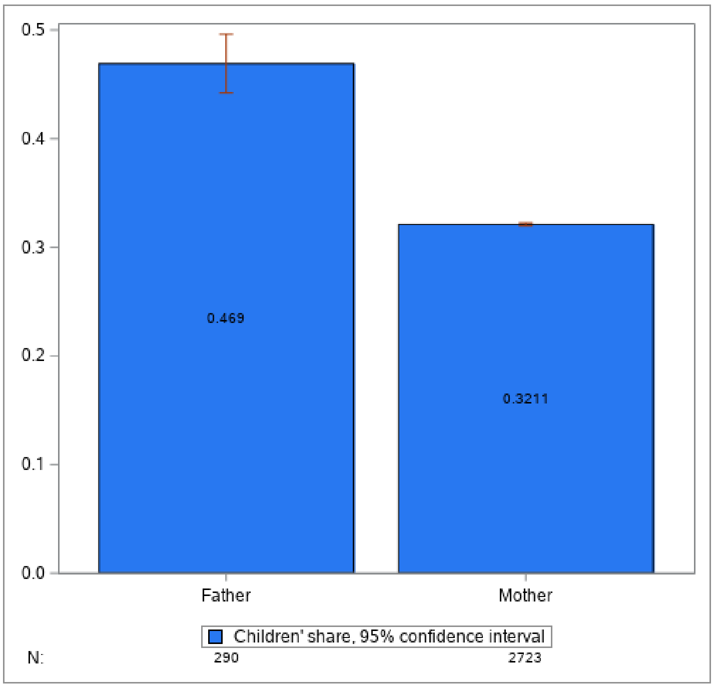

Figure 1.20 – *Cost of children borne by each single parent in families with only same-gender siblings*

Notes: This figure compares the average expenditure on same-gender children by parents in families with only same-gender siblings, based on their gender.

Chapter 2

How to Incorporate Children into Labor Supply Equations? An Equivalence-Scale-Based Approach

Olivier DONNI,[*,†] **Anderson VIL**[‡,§]

November 25, 2024

Abstract: We provide a novel method for modeling and estimating the impact of chil dren on labor supply decisions. We rely on the literature on equivalence scales and collective models. Our approach takes into account both the time and monetary costs of raising children. Using semiparametric restrictions on indi vidual preferences, we identify the cost of children from the curvature of the labor supply equations with low data requirements. We apply the model to PSID data from the USA and investigate the women's labor supply sensitivity, the full cost of children, and the women's price of time. We find that women's labor supply is highly sensitive to wage variations. In addition, we show that mothers allocate, on average, 44% of their net total expenditures to children. Finally, by comparing our findings to those derived from other studies using children's expense data, our model provides relatively consistent predictions regarding the cost of children.

JEL codes: I30, I31, J21, J22, J31.

Keywords: Collective Model, Equivalence scales, Identification, Labor Supply, Price of time, Resource sharing.

[*]THEMA, CY Cergy Paris Université, Cergy-Pontoise, France. olivier.donni@cyu.fr

[‡]THEMA, CY Cergy Paris Université, Cergy-Pontoise, France. anderson.vil@cyu.fr

2.1 Introduction

A substantial body of economic literature has shown that the presence of children significantly affects the labor market behavior of mothers, and to a lesser extent, fathers. [1] As Browning (1992) aptly noted, "typically, any measure of female labor supply (for example, participation or hours if participating) is negatively correlated with any measure of young children (for example, the number of preschool children or the presence of an infant)." Despite this very well-established fact, there is no theoretical framework guiding the incorporation of children into labor supply models. In these models, the number of children — sometimes broken down by age and sex — is generally treated as a simple control variable alongside other socio-demographic factors. This stands in stark contrast with the case of demand equations, where a well-developed theoretical framework based on equivalence scales provides a more robust basis for analysis. See Browning, 1992 or Lewbel, 1999 for a survey of the equivalence scales literature.

The primary objective of this paper is to develop a straightforward, theory-consistent method for incorporating children into labor supply models. The key challenge is to capture the impact of children in a wholistic – yet reasonably realistic – manner. Our approach is designed to enable the effective estimation of labor supply models using traditional workforce surveys, without relying on direct observations of childcare time or monetary costs associated with children. To achieve this, we develop a model labor supply for single adults, both with and without children, featuring three key elements: (a) both the adult and the child (if present) have distinct preferences, with the adult being altruistic toward the child, (b) the worker's full income is adjusted for the direct monetary costs of raising children (using a function that represents expenditures for children), and (c) the time devoted to childcare is determined by a childcare technology and subtracted from the worker's total available time. These components collectively define the total cost of children for the parent, encompassing both time and financial costs. This approach is inspired by recent collective models of consumption, where parents and children are characterized by their own preferences and where resources are shared between them (Bargain and Donni, 2012a, Cherchye, Rock, and Vermeulen, 2012, Dunbar, Lewbel, and Pendakur, 2013, Bargain, Donni, and Hentati, 2022, Lechene, Pendakur, and Wolf, 2022). [2]

Our theoretical results can be summarized as follows. First, we demonstrate that workers'

[1] See Angrist and Evans, 1998, Lundberg and Rose, 2000, Michaud and Tatsiramos, 2011, Cools, Markussen, and Strøm, 2017, for a non-exhaustive list.

[2] See also the foundational work by Bourguignon and Browning (1991) for collective models with children and the contributions by Blundell et al. (2005), as well as Cherchye et al. (2012), which present sophisticated models that require rich data for estimation.

preferences and the total cost of children can be identified from the observation of labor supply equations. This is achieved by assuming a relatively general form of childcare technology while the direct monetary cost of children is a linear function of exogenous incomes, a generalization of the proportionality hypothesis used in certain collective demand models (Bargain and Donni, 2012a, Dunbar et al., 2013, Bargain et al., 2022, Lechene et al., 2022) . Second, we introduce a simplified childcare technology that allows the impact of children on labor supply to be captured by three constants that can be estimated. This simplified approach is less data-intensive compared to more general technologies, making it analogous to the incorporation of fixed costs (Cogan, 1981) or to the use of equivalence scales in demand models. More precisely, these constants make adjustments to labor supply and to household resources, akin to the Independent-of-the-Base equivalence scales (Lewbel, 1989a,b, Blundell and Lewbel, 1991, Blackorby and Donaldson, 1993). Our approach stands out as one of the rare efforts to integrate equivalence scales with labor supply models. [3] Finally, we extend our framework to include couples, both with and without children.

Our contribution can be compared to several studies, many of them relatively dated, that have examined the impact of formal and informal childcare costs on mothers' labor supply decisions (Blau and Robins, 1988, Connelly, 1992, Michalopoulos et al., 1992, Ribar, 1992, 1995, Cleveland et al., 1996, Averett et al., 1997, Powell, 1997, Doiron and Kalb, 2005, Kornstad and Thoresen, 2007, Baker et al., 2008, Bernal, 2008, Apps et al., 2016). These studies, like ours, recognize that the presence of children affects the opportunity cost of parents' time, as the time spent on childcare directly reduces the time available for work, but they differ from ours in that they do not fully account for the income effect associated with children and do not focus on measuring the total cost of children. More importantly, they generally require richer data that includes direct information on parents' childcare time.

To illustrate our theoretical results, we consider a sample of single women, both with and without children, from the 2019 PSID data. Using the Heckman (1974)'s method to account for the participation decision, we estimate a labor supply equation that integrates the simplified childcare technology. We then compute the total cost for children using the wage rate as the price of time for working mothers and computing a shadow price as the price of time for nonworking mothers. Our results show that single mothers spend, on average, 443 hours annually on childcare, allocating 44% of their non-labor income to this purpose. The total annual median cost of children is estimated at $17,060. When compared with U.S. Department of Agriculture

[3] One of notable exception is the labor supply model developed by Hurd and Pencavel (1981), which uses Barten scales to transform prices and wages.

estimates, our model yields cost predictions that are relatively consistent.

The remainder of this paper is as follows. Section II presents our theoretical model. Section III specifies the empirical model. Section IV describes the data. Section V outlines and discusses our empirical results. Section VI concludes.

2.2 The Model

Our analysis adopts a static model of labor supply and focuses on a single woman (or more broadly a single adult) with or without children.

2.2.1 The General Model

The case of a Woman with Children. We begin by examining the case of a woman with children. Her labor time is represented by h and her childcare time by t, while c denotes her consumption of a composite good. In addition, the consumption of children is denoted by m and the number of children, assumed to be exogenously given, by n. To capture her preferences, we assume a utility function of the form:

$$u_f(h+t, c) + \lambda(n)u_c(m), \tag{2.1}$$

where $u_f(\cdot)$ denotes the woman's sub-utility, $u_c(\cdot)$ denotes the children's sub-utility, and $\lambda(n)$ is a weight that represents mothers' altruism. Specifically, the woman's sub-utility decreases with total labor supply but increases with consumption, while the children's sub-utility increases with their consumption. Both sub-utilities are strictly concave in their respective arguments. Before proceeding further, we would like to make a few remarks. Firstly, the mother's caregiving behavior is consistent with Becker's altruism (or 'caring'), as commonly employed in the literature on collective models. Secondly, the mother's utility function is additive. While this may appear restrictive, our results remain valid even with a more general form of separability in the utility function, such as $U(u_f(h+t, c), u_c(m), \lambda(n))$ for some aggregating function $U(\cdot)$. Thirdly, the children's sub-utility reflects the utility of an average or representative child. Finally, labor and childcare time are assumed to be perfectly substitutable, which is perhaps the most restrictive assumption in our model. Despite this, such an assumption is frequently employed in the literature for its practical convenience (Gronau, 1977, Donni, 2008, Donni and Matteazzi, 2012, 2018). It is indispensable in our case.[4]

The household budget constraint is given by

$$y + hw \geq c + m + k \tag{2.2}$$

where w denotes the woman's wage rate, y her other exogenous incomes, and k her expenditure on childcare time (i.e., the cost of purchasing daycare services). The price of consumption is normalized to one. The childcare technology constraint is given by

$$f(t, n) + k \geq T_c(n), \tag{2.3}$$

where $T_c(n)$ can be interpreted as the time necessary for caring for the n children, and $f(t, n)$ is a function increasing in t. To start, let us consider an extreme case where $T_c(n)$ represents 24 hours per day for a single child. In that case, the mother thus has to allocate 24 hours to childcare, either by providing it herself or by purchasing it from the market. If the mother's time and market-based childcare are perfectly substitutable, this constraint can be expressed as $t + k \geq 24$ where k represents the cost of purchased formal childcare, assuming the price per hour is normalized to one. Although intuitive, however, this interpretation is overly restrictive. First, childcare arrangements can lead to inefficiencies related to transportation or administrative tasks. Second, in-home childcare may result in diminishing returns due to parental fatigue. Third, the quality of care provided at home may differ from that available through formal childcare services. More realistically, we thus assume imperfect substitutability between the mother's time and market-based childcare. In addition, $T_c(n)$ should be understood not merely as the number of hours required for childcare, but in a broader context that accounts for these factors.

The optimization problem of the mother can thus be represented as:

$$\max_{h,t,m,c,k} u_f(h + t, c) + \lambda(n)u_c(m) \tag{2.4}$$

subject to the budget constraint (3.1), the childcare technology constraint (3.3), and non-negativity constraints. This problem can be decomposed into two stages. In the first stage, the mother chooses the optimal combination of her childcare time and market childcare time in

[4]The main consequence of this assumption is that the price of time spent on leisure is equal to the price of time spent on childcare. The case where these prices of time are distinct is explored in a companion paper (Donni and Vil, 2024a).

order to minimize the total cost:

$$e_t(w,n) = \min_{t,k}(tw+k) \text{ subject to } f(t,n)+k \geq T_c(n)$$

where $e_t(w,n)$ is the time-cost of children. Concurrently, she also chooses the optimal level of expenditure m for chidren, denoted as $m = e_m(y,w,n)$. Here, the model implicitly assumes a sharing rule between the mother's consumption and the child's consumption. This rule reflects the mother's altruistic preferences for her children, captured by the parameter $\lambda(n)$. This is the same model used by Bargain and Donni (2012b), but it incorporates additional constraints and is applied within the framework of labor supply theory. In the second stage, she maximizes her sub-utility taking $e_t(w,n)$, $e_m(y,w,n)$ and $t(w,n)$ as given. The optimization problem is:

$$\max_{h,c} u_f(h+t(w,n),c) \text{ subject to } y+(h+t(w,n))w \geq c+e_m(y,w,n)+e_t(w,n) \tag{2.5}$$

The solution of this optimization problem is of the form:

$$h(y,w,n) = F(w, y-e_t(w,n)-e_m(y,w,n)) - t(w,n), \tag{2.6}$$

where $F(\cdot)$ is a traditional "Marshallian" labor supply function or the baseline labor supply function that represents the woman's utility only. Structurally, the expression (2.6) is analogous to a labor supply model that incorporates both a fixed monetary cost and a fixed time cost, as outlined by Cogan (1981). These fixed costs influence working time in two primary ways: by shifting the baseline labor supply function and by shifting the mother's available resources. These two effects operate differently: the first tends to reduce the hours worked, while the second tends to increase them through an income effect. Consequently, the theoretical impact of children on working time is indeterminate. [5]

The Case of a Woman without Children. We now briefly consider the case of a childless woman and we assume that the utility is of the form:

$$u_f = \psi(u_f(h,c))$$

where $\psi(\cdot)$ is an increasing transformation of the sub-utility of women with children. Stated differently, the marginal rate of substitution between labor and consumption is identical for

[5] The first effect likely predominates for women, as suggested by the numerous studies cited in the introduction. The second effect may explain why men often increase their working hours when children are present.

women, regardless of whether they have children or not. If her utility function is maximized under the budget constraint $y + hw \geq c$, then her labor supply function is of the form:

$$h(y, w, n) = F(w, y). \tag{2.7}$$

Our identification strategy is based on the idea that the function $F(\cdot)$ is the same for women, regardless of whether they have children or not, as assumed by Bargain and Donni (2012a) and Bargain et al. (2022). However, additional structure is necessary.

In several recent contributions to collective models of consumption (Bargain and Donni, 2012a, Dunbar et al., 2013, Penglase, 2021, Lechene et al., 2022), the assumption that the cost of children is proportional to total household income is frequently employed. Menon et al. (2012) provide evidence using disaggregated data that this assumption holds for children's expenditures. For labor supply models, nonetheless, this assumption seems to be less appropriate. In consumption models, total household income includes both labor and non-labor incomes. Consequently, proportionality in this context implies that $e_m = b(w, n) \cdot (hw + y) = a(w, n) + b(w, n) \cdot y$ where $a(w, n) = b(w, n) \cdot hw$ for some functions $a(w, n)$ and $b(w, n)$. The linearity property for labor supply models can thus be seen as a natural generalization of the proportionality property used in consumption models.

Assumption A. Expenditures on the childcare good is a linear function of exogenous incomes. That is, $e_m(y, w, n) = a_m(w, n) + y \cdot b_m(w, n)$, where $a_m(w, n)$ and $b_m(w, n)$ are functions, with $b_m(w, n) \in (0, 1)$.

Bargain and Donni (2012b) demonstrate that linearity is automatically achieved when the indirect utility functions corresponding to $u_f(\cdot)$ and $u_c(\cdot)$ in (2.1) are of the CARA form. The constant $a_m(w, n)$ is then a linear function of the degree of altruism $\lambda(n)$.

Then, using this assumption, it can be shown that the main structural components of the model are identified. The sum of time and monetary costs is the total cost of children.

Proposition. *Assume A. If $h'_y \neq 0$ and $h''_y \neq 0$ and some technical conditions, the total cost of children can be recovered from the observation of the female market labor supply function. If $h'_y \neq 0$ and $h''_y = 0$, the function $b_m(w, n)$ can be recovered.*

Proof. The function $F(\cdot)$ can be recovered from the market time behavior of childless women. Then deriving the labor supply function gives:

$$h'_y(w, y, n) = F'_2(w, y - e_t(w, n) - a_m(w, n) - y \cdot b_m(w, n)) \times (1 - b_m(w, n)). \tag{2.8}$$

where the left-hand side is known, and F_2' is the derivative of $F(\cdot)$ with respect to the second argument. We then consider two cases.

i) **Linear Case**

The labor supply function is linear in y, that is, it is of the form:

$$h(w,y,n) = \alpha(w) + \beta(w) \cdot (y - e_t(w,n) - a_m(w,n) - y \cdot b_m(w,n)) - t(w,n) \tag{2.9}$$

where $\alpha(w)$ and $\beta(w)$ are functions of w only. From (2.8), the function $b(w,n)$ is directly identified:

$$b_m(w,n) = 1 - \frac{h_y'(w,y,n)}{\beta(w)}.$$

Incorporating this expression in (2.9) gives:

$$\beta(w) \cdot e_t(w,n) + \frac{\partial e_t}{\partial w}(w,n) = \beta(w) \cdot a_m(w,n) + \theta(w,n)$$

where

$$t(w,n) = \frac{\partial e_t}{\partial w}(w,n)$$

from Shephard's lemma, and

$$\theta(w,n) = \alpha(w) + y \cdot h_y'(w,y,n) - h(w,y,n)$$

is a known function. This is a linear differential equation. For any n, the general solution is:

$$e_t(w,n) = \frac{\int^w \mu(t)\left(\beta(t) \cdot a_m(t,n) + \theta(t,n)\right) \mathrm{d}t + c(n)}{\mu(w)}$$

where $\mu(w) = \exp\left(\int^w \beta(t)\mathrm{d}t\right)$, $a(w,n)$ is an unknown function, and $c(n)$ is an integration constant (or an integration function in the present case).

ii) **General Case**

The labor supply function is not linear in y. Thus, we have:

$$\frac{h_y'(w,y,n)}{1 - b_m(w,n)} = F_2'(w, y - e_t(w,n) - a_m(w,n) - y \cdot b_m(w,n)), \tag{2.10}$$

from (2.8). Since $F_2' \neq 0$ for any (w,n), we can invert this expression and obtain:

$$G_2'\left(w, \frac{h_y'(w,y,n)}{1 - b_m(w,n)}\right) = y - e_t(w,n) - a_m(w,n) - y \cdot b_m(w,n). \tag{2.11}$$

for some function G_2' inverse of F_2', i.e., $G_2' = 1/F_2'$. Hence, differentiating (2.11) with respect to y gives:

$$G_2''\left(w, \frac{h_y'(w,y,n)}{1-b_m(w,n)}\right) \times h_y''(w,y,n) = (1-b_m(w,n))^2, \tag{2.12}$$

where $G_2'' = -F_2''/(F_2')^2$. This equation defines $b_m(w,n)$. If the following condition is satisfied for any (w,n),

$$\left|\frac{1}{2}G_2'''\left(w, \frac{h_y'(w,y,n)}{1-b(w,n)}\right) \times \frac{h_y'(w,y,n) \times h_y''(w,y,n)}{(1-b(w,n))^3}\right| < 1,$$

where $G_2''''' = (2(F_2'')^2 - F_2'''F_2')/(F_2')^4$, then the equation is a contraction with respect to $(1-b(w,n))^2$, ensuring that $b(w,n)$ is uniquely defined for any (w,n), with $b_m(w,n) \in (0,1)$. This condition is sufficient, though not necessary. Once $b_m(w,n)$ is defined, substituting its value back into (2.11) gives $e_t(w,n) + a_m(w,n)$. Finally, from (2.6), we obtain the childcare time function. □

This result indicates that the total cost of children can be determined without needing information on childcare time or parental expenditure on children. However, if the baseline labor supply function $F(\cdot)$ is linear with respect to its second argument, the identification of the cost of children is incomplete. Consequently, the identification hinges on the nonlinearity of $F(\cdot)$. The main challenge, therefore, lies in accurately estimating the necessary second-order derivatives of labor supply functions from standard data sets.

2.2.2 A Tractable Model

To develop a more manageable model, we suggest a straightforward childcare technology. Specifically, we assume that the childcare cost function is linear in w, expressed as:

$$e_t(w,n) = a_t(n) + b_t(n)w$$

where $a_t(n)$ and $b_t(n)$ are functions of n only. The underlying childcare technology can be described as follows:

$$\begin{aligned} f(t,n) &= t \qquad \text{if } t \leq b_t(n) \\ &= +\infty \quad \text{if } t > b_t(n) \end{aligned}$$

Intuitively, the mother has a set number of hours she can devote to childcare, and she cannot exceed this limit, regardless of her wage rate. The childcare time function is perfectly rigid and given by $t(w,n) = b_t(n)$. Similarly, the expenditure for children is simplified as follows:

$$e_m(y,n) = a_m(n) + b_m(n)y.$$

Incorporating the childcare time function and the childcare expenditure function into the labor supply function, we obtain:

$$h(y, w, n) = F(w, y - a(n) - b_t(n)w - b_m(n)y) - b_t(n). \tag{2.13}$$

where $a(n) = a_t(n) + a_m(n)$. As it can be easily shown, the cost of children can even be identified if the function $F(\cdot)$ is linear with respect to its second argument. In the linear case, we obtain:

$$h(w, y, n) = \alpha_0 + \alpha_1 w + \alpha_2(y - a(n) - b_t(n)w - b_m(n)y) - b_t(n). \tag{2.14}$$

Hence, the presence of children modifies the constant, the slope with respect to the wage rate and the slope with respect to exogenous income. If $\alpha_1 > 0$ and $\alpha_2 < 0$, as is typically assumed, children amplify the effect of the wage rate while diminishing the impact of exogenous income. [6] This suggests that the Slutsky positivity condition remains satisfied. This specification is similar to demand equations using Independent-of-the-Base equivalence scales. In our empirical analysis, we will adopt this simplified model with a slightly more general form for the baseline labor supply function.

2.2.3 Extension to a Couple with Children

The previous model primarily applies to a single woman with children, but it can be adapted to represent the behavior of a married woman as well. For instance, in the unitary approach, the couple's utility function is given by:

$$u(h_f + t_f, h_m + t_m, c) + \lambda(n)u_c(m).$$

where h_f, h_m denote the wife's and husband's labor time and t_f, t_m denote the wife's and husband's childcare time. [7] The childcare technology is then generalized to:

$$f(t_f, t_m, n) + k \geq T_c(n). \tag{2.15}$$

[6] This holds true in a broader context. However, if the money cost of children varies as a function of the wage rate, the impact of the wage rate on working hours may become unclear.

[7] Here, consumption c represents the combined consumption of both spouses. This aggregation is not restrictive and can be justified by the Hicks aggregation theorem.

From this, it can easily be shown that the wife's working time function becomes:

$$h_f(y, w_f, w_m, n) = F(w_f, w_m, y - e_t(w_f, w_m, n) - e_m(w_f, w_m, n)) - t(w_f, w_m, n), \tag{2.16}$$

where w_f, w_m are the husband's and wife's wage rate, respectively. In this adapted model, all previously derived results remain applicable. Specifically, the total cost of children, which is supported by the couple as a whole, can be identified from the observation of the sole wife's labor supply function. The unitary approach offers the advantage of simplicity, particularly by focusing on the total cost of children. Its main limitation is that it does not allow for the identification of how the total cost is distributed between the parents. [8]

If the husband's working time is fixed — a not entirely unreasonable assumption Donni (2007) – the labor supply function (2.6) can also be adapted to represent the behavior of a married woman. In this case, exogenous income is viewed as the sum of the man's income and non-labor income. While this approach is common when estimating female labor supply functions, it represents a challenge here: if the husband's working time is constrained, his wage rate no longer accurately reflects the price of his time spent on childcare. In the empirical section, we will explore how to value time when a worker's hours are constrained in the market.

2.3 Estimation

In this section, we will focus on the case of single women, with the simplified childcare technology. We will begin by presenting the empirical specification, followed by a discussion of the estimation method.

2.3.1 Empirical Specification

For the baseline labor supply equation, we have selected a quadratic specification, following Brown et al. (1976), as follows:

$$h = \alpha_0 z_h + \alpha_1 w + \alpha_2 w^2 + \alpha_3 y + \alpha_4 y^2 + \alpha_5 yw + \rho\nu + \sigma_\varepsilon \varepsilon \tag{2.17}$$

where α_0, α_1, α_2, α_3, α_4, α_5, ρ and σ_ε are parameters, ε and ν are normally distributed disturbances with zero mean and unit variance, and z_h are additional explanatory variables

[8] In the absence of distribution factors, the unitary approach is only marginally more restrictive than the collective approach. The key requirement is that the base labor supply functions must satisfy the Slutsky conditions, which essentially reduces to a positivity condition when considering only a single labor supply function.

(including a constant). Because of the flexibility of the functional form, leisure is not globally normal, and the Slutsky positivity condition is not globally satisfied. This issue may not be problematic, as the domain where these conditions hold could be extensive.

For the market wage equation, we adopt the following specification:

$$w = \beta z_w + \sigma_\nu \nu \tag{2.18}$$

where β and σ_ν are parameters, and z_w are explanatory variables (including a constant). The disturbance ν enters the labor supply equation so that the wage rate is endogenous if $\rho \neq 0$.

We finally adopt the simplified childcare technology. (i) The time cost of children is of the form:

$$e_t(w) = (a_t + b_t w) \times \delta(n), \tag{2.19}$$

where a_t and $b_t > 0$ are constants and $\delta(n)$ is a function of the number of children, satisfying $\delta(0) = 0$ and, for normalization, $\delta(1) = 1$. The function $\delta(n)$ can be seen as a measure of the economies of scale in childcare; hence, we have chosen: $\delta(n) = \sqrt{n}$. Then, from Shephard's Lemma, we obtain: $t(w, n) = b_t \times \delta(n)$. (ii) The money cost of children is of the form:

$$e_m(y, w) = (a_m + b_m y) \times \delta(n), \tag{2.20}$$

where a_m and $b_m \in (0, 1)$ are constants. Incorporating these expressions into the baseline labor supply function (2.17) gives:

$$\begin{aligned} h \;=\; & \alpha_0 z_h + \alpha_1 w + \alpha_2 w^2 + \alpha_3 (y - (a + b_t w + b_m y)\delta(n)) \\ & + \alpha_4 (y - (a + b_t w + b_m y)\delta(n))^2 \\ & + \alpha_5 (y - (a + b_t w + b_m y)\delta(n)) w - b_t \delta(n) + \rho\nu + \sigma_\varepsilon \varepsilon \end{aligned} \tag{2.21}$$

where $a = a_t + a_m$. All the parameters of (2.21) are identified, as it can be easily checked.

2.3.2 Estimation Method

The parameters are estimated by maximizing the log-likelihood function. To simplify the notation, we compactly write the labor supply equation as $h = h_0(w) + \rho\nu + \sigma_\varepsilon \varepsilon$ and the wage equations as $w = w_0 + \sigma_\nu \nu$, where $h_0(w)$ and w_0 have obvious definitions. Then the decision to participate is defined by:

$$\begin{aligned} h &= h_0(w) + \rho \frac{w - w_0}{\sigma_\nu} + \sigma_\varepsilon \varepsilon \\ w &= w_0 + \sigma_\nu \nu \qquad\qquad \text{if } \varepsilon > -\frac{h_0(w)}{\sigma_\varepsilon} - \rho \frac{w - w_0}{\sigma_\varepsilon \sigma_\nu} \end{aligned}$$

and

$$h = 0 \qquad \text{if } \varepsilon \leq -\frac{h_0(w_0 + \sigma_\nu \nu)}{\sigma_\varepsilon} - \rho \frac{\nu}{\sigma_\varepsilon}$$

The contribution to likelihood for women who do not work is:

$$\int_{-\infty}^{+\infty} \Phi\left(-\frac{h_0(w_0 + \sigma_\nu \nu)}{\sigma_\varepsilon} - \rho\frac{\nu}{\sigma_\varepsilon}\right) \phi(\nu) \cdot \mathrm{d}\nu$$

where Φ and ϕ are, respectively, the cumulative probability function and the density probability function of the standardized normal distribution. The contribution to likelihood for women who do work is:

$$\frac{1}{\sigma_\varepsilon \sigma_\nu} \cdot \phi\left(\frac{w - w_0}{\sigma_\nu}\right) \cdot \phi\left(\frac{h - h_0(w)}{\sigma_\varepsilon} - \rho\frac{w - w_0}{\sigma_\nu \sigma_\varepsilon}\right).$$

Summing the logarithm of these expressions over all observations gives the log-likelihood function, which is maximized to estimate the parameters.

2.3.3 The Cost of Children

The total cost of children — though not its time and money components separately — can be estimated from the parameters. To do this, we must distinguish between whether the mother is working or not. For working mothers, the total cost of children is simply given by

$$e_t(w) + e_m(y) = (a + b_t w + b_m y) \times \delta(n), \tag{2.22}$$

where the wage rate serves as a measure of the price of time. However, for nonworking mothers, the wage rate can no longer be considered as a measure of the price of time. Instead, the price of time, denoted as w_R, corresponds to the reservation wage, which is determined by inverting the labor supply equation with $h = 0$. The equation can then be expressed as follows:

$$Aw_R^2 + Bw_R + C = 0$$

where

$$\begin{aligned}
A &= \alpha_2 + \alpha_4 b_t^2 \delta(n)^2 - \alpha_5 b_t \delta(n) \\
B &= \alpha_1 - \alpha_3 b_t \delta(n) - 2\alpha_4 (y - (a + b_m y)\delta(n)) b_t \delta(n) + \alpha_5 (y - (a + b_m y)\delta(n)) \\
C &= \alpha_0 z_h + \alpha_3 (y - (a + b_m y)\delta(n)) + \alpha_4 (y - (a + b_m y)\delta(n))^2 - b_t \delta(n) + \rho\nu + \sigma_\varepsilon \varepsilon
\end{aligned}$$

If $B^2 - 4AC > 0$, this equation has two solutions for w_R. Only one of these solutions satisfies the Slutsky positivity condition, given by $2Aw_R + B > 0$ (which ensures that the slope of the labor supply function along the participation frontier is positive). This solution is defined as:

$$w_R = -\frac{B + \sqrt{B^2 - 4AC}}{2A}.$$

If $A < 0$ and $B > 0$, which seems reasonable, the price of time is positive.

Since C depends on disturbances ν and ε, which can vary from $-\infty$ to $+\infty$, there will inevitably be instances where $B^2 - 4AC < 0$ for some individuals, making the price of time impossible to compute. While this may not be a major concern, as such instances are likely to be rare, it does mean that the moments of the probability distribution of the price of time cannot be reliably calculated. Nevertheless, we can still estimate certain aspects of this probability distribution, such as quantiles, to gain insight into its variability.

2.4 Data

Our structural labor supply model will examine a sample of households drawn from the widely used Panel Study of Income Dynamics (PSID). The PSID is a longitudinal survey of the University of Michigan that reports income, employment trajectories, and other variables over time. This dataset provides extensive information on housing, employment, income, wealth, and detailed information on individual and family characteristics. In contrast with previous surveys, the 2019 wave contains household expenditures for various goods and services. This information is used to construct the variable net total expenditures, which we employ in the empirical analysis in place of exogenous income. The net total expenditures are determined by subtracting labor income from household total expenditures, as detailed below.

The initial sample consisted of 8,195 households. Our primary focus is on single women, both with and without children, aged 20 to 60, which allows us to circumvent the complexities associated with modeling couples, particularly due to the limited flexibility in men's working hours. Moreover, focusing on single women offers significant value, as they represent a group frequently overlooked in research, despite being particularly vulnerable to poverty. Interestingly, unlike married women, single women with children (at least in the PSID) exhibit a higher participation in the labor market than their childless counterparts. Finally, for single mothers, we further restricted our sample to those with no more than two children. Given that our theoretical model accounts for daycare expenditures, we excluded households where the youngest child was older than six. After eliminating observations with relevant missing data, our final sample consisted of 1,270 households, with 987 single women and 283 single mothers.

2.4.1 Descriptive statistics

In Table 1, we present summary statistics for the key variables employed in this research. Single women, on average, work 1,634 hours annually, with a participation rate of approximately 85%, showing slight variations based on whether they have children. Interestingly, single mothers have a higher participation rate than their childless counterparts, despite earning a lower hourly wage. This suggests that the income effect associated with having children is substantial for single mothers. Furthermore, the average net total expenditures also reveal distinct patterns: while childless women generally manage to cover their expenses with their earnings, single mothers often find their wages insufficient to meet their household's total expenditures. [9]

The other striking fact in the data is the high percentage of Black single mothers, who make up two-thirds of this demographic. Kearney (2023) previously documented this pattern, highlighting that Black children are significantly less likely to live with married parents compared to their White, Asian, and Hispanic counterparts. As of 2019, while over 60% of non-Black children lived with married parents — reaching as high as 88% among Asian children — only 38% of Black children did so (Kearney, 2023). Finally, the data reveals that working women tend to be younger and more educated than their non-working peers.

2.4.2 Endogeneity and Preliminary estimations

A key endogeneity concern needs to be addressed. Household net total expenditures may not be orthogonal to the structural disturbance in labor supply. For example, if measurement errors influence the decision to work more or fewer hours, this could create a correlation between net total expenditures and the error term in the labor supply equation. To address this issue, we adopt the approach of Donni and Matteazzi (2018), substituting the household net total expenditures in the labor supply equation with its fitted value from a first-stage regression. The natural instruments used are various sources of household income, as these are exogenous determinants of work hours and net total expenditures (Blundell and MaCurdy, 1999). The most important results are reported in Table 2. [10] We find that, with the exception of total asset income and its square, all sources of income have a statistically significant effect.

[9] The expenditures reported in the PSID are not exhaustive and do not cover the full range of household expenditures.

[10] See Appendix 2.C for the entire results.

Table 2.1 – *Descriptive Statistics*

	All Women	Working Women	Non-Working Women	Childless Women	Single Mothers
Labor market participation	84.57%	100%	0%	83.08%	89.75%
	(36.14%)	-	-	(37.51%)	(30.38%)
Annual hours worked	1 634	1 933	0	1 639	1 620
	(981)	(749)	-	(1 004)	(897)
Hourly wages	17.54	20.74	-	18.78	13.22
	(16.91)	(16.48)	-	(18.14)	(10.56)
Total net expenditures	-226	-4 047	2 071	-3 007	9 475
	(27 409)	(27 508)	(14 288)	(29 242)	(16 412)
Age	40.24	38.91	47.53	42.19	33.44
	(11.76)	(11.43)	(10.90)	(11.70)	(9.19)
Years of education	13.64	13.91	12.18	13.85	12.90
	(2.40)	(2.33)	(2.23)	(2.36)	(2.40)
Black	60.24%	58.38%	70.41%	58.06%	67.85%
	(48.96%)	(49.32%)	(45.76%)	(49.37%)	(46.79%)
Hispanic	8.66%	9.31%	5.10%	7.30%	13.43%
	(28.14%)	(29.07%)	(22.06%)	(26.02%)	(34.16%)
Number of children	0.35	0.37	0.25	0	1.56
	(0.69)	(0.70)	(0.62)	-	(0.50)
Sample size	1270	1074	196	987	283

Notes: This table presents the mean values of the variables used in the study, with standard deviations shown in parentheses. The variables include labor market participation, annual hours worked, hourly wages, total net expenditures, age, years of education, race (Black and Hispanic), and number of children. The table provides separate statistics for all women, working women, non-working women, childless women, and single mothers, allowing for a comparative analysis across these groups. The sample size for each category is indicated at the bottom.

2.5 Estimation Results and Discussion

To maximize the log-likeligood function, we use simulation techniques, generating draws from Halton sequences in accordance with the procedure outlined by Train (2009). In this section, we report the main results for the baseline model and its alternatives. In the baseline model, the endogeneity of total expenditures is adressed by using its fitted value as previously explained. As alternatives, we also consider a model that uses the control function approach to address endogeneity, as well as a model where endogeneity is not controlled. We begin by presenting the parameter estimates of the wage equation. We then provide a comprehensive discussion of the parameter estimates of the labor supply equation. Finally, we focus on the implications of our results for the cost of children.

2.5.1 Parameter estimates of the wage equations

The estimated parameters of the wage equations are reported in Table 3. Column I shows the coefficients of the wage equations by instrumenting net total expenditures by its fitted values.

Table 2.2 – *Estimated parameters of the net total expenditures*

Variables	Estimate	SE
Intercept	64.171***	(10.968)
Total asset income	-0.259	(0.309)
(Total asset income)^2	0.001	(0.003)
Total welfare income	1.540***	(0.379)
(Total welfare income)^2	-0.025**	(0.012)
Total transfer income	1.982***	(0.418)
(Total transfer income)^2	-0.034***	(0.014)
Total other income	0.439***	(0.053)
(Total other income)^2	-0.002***	(0.000)
Sample Size	1,270	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parenthesis. Household net total expenditures are the dependent variable and express in thousands of dollars. Other covariates including in the model which do not appear in Table 2.2 are the following: women's level of education, women's age and its square, and dummies for black and hispanic women.

Column II does the same using the control function approach, while column III provides the coefficients without adressing endogeneity issues. We find that highly educated women earn more in the labor market for all wage equations. Specifically, each additional year of schooling is associated with a significant increase in hourly wages by approximately three dollars. We also observe that age has a diminishing effect on wages, indicating that while experience is valued, its impact lessens as individuals age. In line with previous studies, the coefficient associated with black women is significantly negative, implying potential labor market discrimination. These results remain consistent across different specifications.

2.5.2 Parameter estimates of the baseline labor supply equations

In Table 4, we report the estimated parameters of the baseline labor supply equations. The annual working hours are measured in units of 1,000 hours. In each column, the coefficient for net total expenditures is significantly different from zero, while the coefficient for its square is small and not significant. Moreover, the coefficient for the interaction between net total expenditures and the wage rate is also small but significant. This finding indicates that net total expenditures have a significant negative linear impact on annual working hours, which is sufficient to identify the simplified childcare technology. However, given that nonlinearity is crucial for identifying the general childcare technology, estimating the general model may prove much more difficult. This result aligns with those previously documented by Chiappori et al.

Table 2.3 – *Estimated parameters of the wage equations*

Variables	Wage Equation (I)	(II)	(III)
α_0	-40.68***	-15.28**	-14.98***
	(7.739)	(7.400)	(7.354)
α_{educ}	2.936***	1.457***	1.475***
	(0.223)	(0.213)	(0.212)
α_{age}	0.974***	0.680**	0.653**
	(0.388)	(0.370)	(0.368)
α_{age^2}	-1.073**	-0.358	-0.332
	(0.475)	(0.453)	(0.450)
α_{black}	-4.860***	-3.945***	-4.015***
	(1.084)	(1.035)	(1.028)
$\alpha_{hispanic}$	2.812	-0.011	0.039
	(1.845)	(1.772)	(1.760)
σ_{ν}	16.74***	16.18***	16.07***
	(0.442)	(0.382)	(0.377)
Instrumental variable	2SLS	CF	
Sample size	1270	1270	1270
Objective Function Value	-6194	-6025	-5986

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parenthesis. The baseline model is Model I where we instrument the net total expenditures by its fitted values derived from a classic linear regression using the OLS method. In Model II, we use the residual of the first-stage regression of net total expenditures as instrument. In Model III, we perform the estimation without correcting for endogeneity of net total expenditures.

(2002), Fernandez-Val (2003), Blundell et al. (2007), Bloemen (2010), Donni and Matteazzi (2012, 2018).

As expected, the baseline labor supply is responsive to wage variations. The linear effect of the wage is significantly different from zero and quite large, while the interaction terms are small. Specifically, focusing on the linear effect, a \$10 increase in the hourly wage for a childless woman results in an approximate 290-hour increase in her annual working time. Regarding socio-demographic variables, age has a negative impact on working time, whereas the dummy variables for Black and Hispanic individuals show a positive effect. In the baseline model, the estimated correlation between the disturbances of the labor supply equation and the wage equation is also positive. This positive correlation aligns with economic expectations, as unobserved factors that increase wages (e.g., higher skills or productivity) are likely to encourage greater labor supply.

Given that wages and net total expenditures enter labor supply equations quadratically, we calculate the marginal effects of these variables for each observation and each realization of

Table 2.4 – *Estimated parameters of the labor equations*

Variables	Labor Supply Equation (I)	(II)	(III)
α_0	1.685***	0.355	1.289**
	(0.428)	(1.010)	(0.620)
α_{age}	-0.014	0.009	0.012
	(0.023)	(0.056)	(0.034)
α_{age^2}	-0.001	-0.072	-0.048
	(0.028)	(0.066)	(0.040)
α_{black}	0.120*	0.331*	0.201*
	(0.068)	(0.179)	(0.110)
$\alpha_{hispanic}$	0.271***	0.150	0.212
	(0.105)	(0.254)	(0.155)
α_{wage}	0.029***	0.072***	0.013
	(0.006)	(0.021)	(0.013)
α_{wage^2}	-0.000***	0.000***	0.000***
	(0.000)	(0.000)	(0.000)
α_y	-0.051***	-0.033***	-0.032***
	(0.006)	(0.002)	(0.002)
α_{y^2}	0.000	0.000	-0.000
	(0.000)	(0.000)	(0.000)
α_{wy}	0.001***	0.000***	0.000**
	(0.000)	(0.000)	(0.000)
ρ	0.241***	-2.193***	-1.201***
	(0.100)	(0.345)	(0.207)
σ_ϵ	0.934***	0.822***	0.800***
	(0.025)	(0.020)	(0.019)
Instrumental variable	2SLS	CF	
Sample size	1270	1270	1270
Objective Function Value	-6194	-6025	-5986

Notes: See the notes in Table 2.3. We measure hours of work in units of 1000 hours.

the error terms. The quartiles of the distribution of marginal effects across different samples are presented in Table 5. For the full sample, the median marginal effects of wages and net total expenditures on working time are 0.020 and -0.824, respectively. Our findings with a sample of single women align with what is typically observed among married women (Bloemen, 2010): net total expenditures significantly negatively impact women's working time, even among single women, while wage rates positively influence their working time. The differences between women with and without children are particularly interesting. Compared to childless women, mothers' working hours are more responsive to changes in wage rates. Specifically, a $10 increase in hourly wages leads to an additional 330 working hours per year for mothers, nearly double the 170 additional hours for childless women. Blau and Kahn (2007) observed this pronounced responsiveness among married women, likely driven by economic necessity — a factor that is particularly relevant for single mothers, who often work part-time and have fewer alternative income sources. [11] This observation is perfectly consistent with our theoretical model where rising wage rates increase the cost of children and, consequently, decrease the resources of the household. A similar pattern can be observed with the effect of net total expenditures, which is lower in absolute value for mothers compared to childless women — another prediction of our theoretical model.

Table 2.5 – *Estimated quartiles of the distribution of marginal effects hourly wages and net total expenditures on labor supply*

Direct marginal on labor supply of	Full sample			Women without children			Women with children		
	P25	P50	P75	P25	P50	P75	P25	P50	P75
Panel 1: Price of time									
Hourly wages	0.010	0.020	0.030	0.008	0.017	0.024	0.026	0.033	0.039
Net total expenditures	-1.239	-0.824	-0.421	-1.373	-0.985	-0.655	-0.410	-0.286	-0.201
Panel 2: Simulated wages									
Hourly wages	0.010	0.021	0.031	0.007	0.019	0.028	0.025	0.033	0.041
Net total expenditures	-1.128	-0.729	-0.427	-1.227	-0.873	-0.575	-0.448	-0.298	-0.187
Sample size		1270			987			283	

Notes: In Panel I, we compute the marginal effects using the price of time, while in Panel II, we use simulated wages. The price of time for women active in the labor market is equivalent to their market hourly wages.

2.5.3 Parameter estimates of the children cost equations

In this section, we examine the equation for the cost of children as described in (2.22). As previously noted, the parameter a aggregates the constants of the time and monetary costs, which cannot be estimated separately. The parameters b_t and b_m correspond to the effect of wage rates in the time cost component and the effect of exogenous incomes in the monetary cost component, respectively, and are expected to be positive. In Table 6, we first observe that all the parameter estimates are positive, as required. In column I, the estimate for b_m amount to 0.443, indicating that, in a one-child household, the child receives approximately \$443 for every \$1,000 increase in net total expenditures. In a two-child household, the total amount allocated to both children together increases to about $\$626 \approx \$443 \times \sqrt{2}$. The estimate for b_t is also equal to 0.443, implying that, on average, the mother of a single child devotes approximately 443

[11] In Appendix, we present the marginal effects conditional on the number of children. The estimates indicate that mothers with one young child are less responsive to wage variations than those with two young children.

hours per year to childcare. These results are in line with those by Bruins (2017). The results presented in columns II and III appear inconsistent.

Table 2.6 – *Estimated parameters of the children cost equations*

	Children Cost Equation		
	(I)	(II)	(III)
a	6.597***	5.931*	9.287***
	(3.171)	(3.575)	(3.734)
b_t	0.443***	0.360*	0.472***
	(0.189)	(0.194)	(0.193)
b_m	0.443***	0.023	0.085
	(0.135)	(0.096)	(0.092)
Instrumental variable	2SLS	CF	
Sample size	1270	1270	1270
Objective Function Value	-6194	-6025	-5986

Notes: See the notes in Table 2.3.

As previously discussed, we also calculated the price of time and the total cost of children for both working and non-working women. These results are presented in Table 7 and are consistent across all samples. Our analysis then focuses on the column corresponding to the full sample. The panels 1 and 2 of the table provide estimates of the price of time and the simulated hourly wages. The simulated hourly wages represent the offered market wage, while the price of time can be interpreted as the reservation wage. It is important to recall that for working women, the price of time is equal to their hourly wage.

Table 2.7 – *Distribution of unpartnered women's price of time and simulated wage*

	Full sample			Women without children			Women with children		
	P25	P50	P75	P25	P50	P75	P25	P50	P75
Panel 1: Price of time									
All women	11.87	17.58	25.72	13.13	19.66	27.45	9.31	13.04	16.95
Working women	11.03	16.60	25.69	12.09	17.94	28.42	8.79	12.54	17.14
Non-working women	17.79	23.46	25.75	20.89	24.25	26.32	13.50	14.61	16.28
Panel 2: Simulated wages									
All women	11.00	16.54	23.41	11.53	17.49	24.54	9.13	13.96	17.29
Working women	11.65	17.34	24.38	13.14	18.99	25.17	9.40	14.05	18.47
Non-working women	8.21	11.22	16.50	8.72	11.25	16.77	6.75	10.32	14.95

Notes: This table presents the distribution of the price of time and simulated wages for unpartnered women. Panel 1 reports the price of time across different groups, while Panel 2 shows the simulated wages for the same groups. The first column provides the distribution for the full sample, the second column focuses on women without children, and the third column highlights single mothers.

Our findings indicate that non-working women have a reservation wage exceeding their offered market wage, whereas working women have a reservation wage lower than their simulated hourly wage. Specifically, the median price of time for a non-working woman is \$23.46, which is double the median wage offered in the labor market. In contrast, for a woman actively participating in the labor market, the median wage stands at \$17.34, slightly more than the value she assigns to her time outside of work. Essentially, non-working women value their time at nearly twice the rate of the simulated hourly wages. These estimates suggest that women's nonparticipation is primarily driven by a high reservation wage combined with a low market wage. This pattern holds true even when considering women's fertility status. Overall, these findings are consistent with economic theory. It is also noteworthy that childless women have a higher price of time compared to single mothers. This finding contradicts the predictions of Cogan (1981), who argued that, due to the fixed costs associated with children, the reservation wage for mothers should be higher than that of women without children. This seemingly unexpected result may be related to the fact that mothers have a higher participation rate than childless women, as previously seen in the descriptive statistics. Single mothers might be incentivized by more immediate financial obligations linked to the exclusive care of their children, leading them to assign a higher value to each income opportunity. The monetary effect of children dominates the time effect.

Table 2.8 – *Cost of children*

Cost of children	All mothers			Working mothers			Non-working mothers		
	P25	P50	P75	P25	P50	P75	P25	P50	P75
Panel 1: Price of time									
All children	13.15	17.24	22.45	12.95	16.90	21.87	16.45	21.81	25.99
1 child	11.47	13.93	17.40	11.40	13.68	17.44	14.50	16.22	16.80
2 children	16.47	20.77	24.90	16.18	19.79	24.21	21.10	24.69	26.84
Panel 2: Simulated wages									
All children	14.18	17.06	20.92	14.26	17.01	20.31	14.18	17.60	22.51
1 child	12.29	13.49	16.37	12.29	13.57	16.55	12.19	12.68	14.18
2 children	17.00	19.45	23.22	16.98	19.31	23.22	17.15	21.70	24.87

Notes: This table presents the cost of children, calculated using both the price of time and simulated wages. Panel 1 shows the cost based on the price of time, while Panel 2 reports the cost using simulated wages. The cost is categorized by all mothers, working mothers, and non-working mothers. The values are measured in thousands of dollars.

To conclude this section, our assessment of the cost of children is reported in Table 8. The cost is computed with two approaches: first, by using the price of time for non-working women, and second, by using the simulated hourly wages. The results show that the cost of children, measured in thousands of dollars, is statistically similar across both approaches. For the full sample of mothers, the median cost of children is estimated at $\$17,240$. Although this figure may seem high, it appears reasonable when considering that it includes the opportunity cost of time, an aspect generally overlooked by other measures. For comparison, expenditures on children in 2019 for single-parent families calculated using our method and those in 2015 published in a study by the U.S. Department of Agriculture (USDA, hereafter) are presented in Figure 1. Our estimates are slightly higher, which aligns with expectations, as our study includes households where the youngest child is aged 6 or younger, whereas the USDA study considers children up to 5 years old.[12]

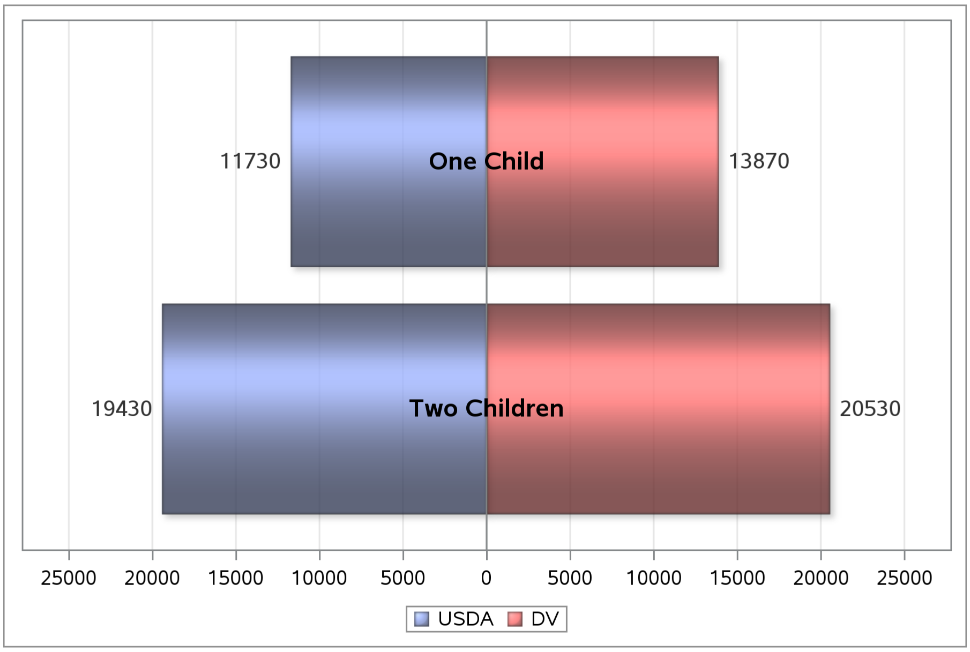

Figure 2.1 – *Expenditures on children by family size*

Notes: The figure plots our estimated child-rearing expenses for single mothers in 2019 compared to those for single mothers in 2015 by USDA's. The expenditures are in dollars. We focus on single mothers with one child aged 6 years or younger and those with two children, where the younger one is no older than 6. In contrast, the USDA focuses on single-parent families with one child aged 5 years or younger and those with two children whose youngest is 6 years old or younger.

2.6 Conclusion

The key contribution of this paper is the development of equivalence scales specifically tailored to labor supply models, which is crucial given the substantial impact children have on work decisions. Our approach explicitly assumes that children modify parents' labor supply functions in a simple and wholistic way through both income and price effects. This approach allows us to estimate the total cost of children, including its time component, without relying on direct data about time use or disaggregated consumption. Using a sample of single women, with and without children, from the PSID, we then estimate a labor supply model and recover

[12]The USDA has provided estimates of child-related expenses from birth through age 17, with additional detailed estimates for younger age groups. We compare our results with the estimates from the age group of children closest to the one on which our study is based. The USDA study is authored by Lino, Kuczynski, Rodriguez, and Schap (2017).

the cost of children. Our analysis shows that, on average, single mothers dedicate 443 hours annually to their children. For each increase in net total expenditures, approximately 44% is allocated to their children. The total cost of a child is estimated to be around $17,240 per year. When compared with USDA findings, our model delivers relatively consistent estimates of the expenses associated with raising children.

To obtain more reliable estimates of the cost of children, richer data would be preferable, including detailed information on the time parents devote to their children and the money they spend on them. Our primary objective, however, was to illustrate how children should be integrated into a labor supply model. We demonstrate that simply using a control variable for the number of children in the regression is insufficient. Children affect both the impact of wages and exogenous income on working hours, while also altering the intercept of the labor supply equation. Furthermore, we argue that measures of the cost of children based solely on consumption data are inadequate, as the largest component of the total cost may be the opportunity cost of childcare.

The primary limitation of our framework is its static nature. Labor supply decisions can have long-term consequences, such as the loss of human capital, which adds to the total cost of children. Integrating our framework into a more comprehensive, intertemporal model would address this issue, but that remains a task for future research.

Appendices

2.A Marginal effects of w and y

$$\frac{\partial h}{\partial w} = (\alpha_1 - \alpha_3\kappa_1\delta(n)) + (2\alpha_2 - \alpha_5\kappa_1\delta(n))w + (\alpha_5 - 2\alpha_4\kappa_1\delta(n))\left[y - (\kappa_0 + \kappa_1 w + \kappa_2 y)\right]$$

$$\frac{\partial h}{\partial y} = (1 - \kappa_2)\left[(\alpha_3 + \alpha_5 - 2\alpha_4\kappa_1\delta(n))w - 2\alpha_4\left((\kappa_0\delta(n) - (1 - \kappa_2)y)\right)\right]$$

2.B Supplementary Data Information

2.B.1 Definition of covariates in regressions

Here, we provide detailed definition of the covariates in the regressions.

— **Total Expenditures**: include expenditures on food at home, food delivered, food eaten out, hospital, doctor bills, prescriptions, health insurance, mortgage, rent, utilities, telephone and internet, homeowners insurance, property taxes, household repairs, household furnishings, vehicle loans, vehicle leases, vehicle down payments, auto insurance, additional vehicle expenses, vehicle repairs, gasoline, parking, bus, taxi, other transportation expenses, education, childcare, clothing, trips, other recreation, and computing expenses.

— **Wage rate**: represents the hourly earnings in dollars and cents.

— **Education**: indicates the highest grade of school completed by the woman.

— **Age**: refers to the woman's age in 2019.

— **Black**: a dummy variable equal to one if woman is Black, African-American, or "Negro", and zero otherwise.

— **Hispanic**: a dummy variable equal to one if woman is Mexican, Mexican-American, Chicano, Puerto Rican, Cuban, or other Spanish, Hispanic, Latino, and zero otherwise.

— **Number of children**: the number of household members under 7 years of age.

2.B.2 Composition of different sources of income (source: PSID codebook)

- **Total asset income** refers to the combined income from both labor and assets for women in 2018. It includes the following:
  - Labor portion of income from women's business activities in 2018.
  - Asset portion of income from women's business activities in 2018.
  - Income women earned from rent in 2018.
  - Income women received from dividends in 2018.
  - Interest income women earned in 2018.
  - Income from trust funds and royalties received by women in 2018.
  - Labor portion of income from women's work in unincorporated businesses in 2018.
  - Asset portion of business income for the spouse or partner, as reported in 2018.
- **Total welfare income** refers to the various sources of financial assistance received by women in 2018, measured in whole dollars. It includes:
  - Income from TANF (Temporary Assistance for Needy Families) or other state programs.
  - Supplemental Security Income (SSI).
  - Income from other welfare programs.
  - Veterans Administration pension payments.
  - Income from other retirement pay and pensions.
  - Income from annuities.
  - Income from IRAs (Individual Retirement Accounts).
  - Other retirement income.
  - Unemployment compensation, including strike benefits.
  - Income from workers' compensation.
- **Total transfer income** refers to various forms of financial support women received in 2018, measured in whole dollars. It includes:
  - Income from child support.
  - Income from alimony.
  - Financial assistance received from relatives.
  - Financial help received from non-relatives or friends.
  - Income from miscellaneous transfers.
- **Total other income** refers to the sum of various income sources for all other family unit (FU) members, as well as women, in 2018, measured in whole dollars. It includes:

— Labor income for all other FU members in 2018.
— Asset income for all other FU members in 2018.
— TANF (Temporary Assistance for Needy Families) income for all other FU members.
— Supplemental Security Income (SSI) for all other FU members in 2018.
— Other welfare income for all other FU members in 2018.
— Veterans Administration pension income for all other FU members in 2018.
— Retirement, pension, and annuity income for all other FU members in 2018.
— Unemployment compensation for all other FU members in 2018.
— Workers' compensation for all other FU members in 2018.
— Child support received by all other FU members in 2018.
— Financial help from relatives received by all other FU members in 2018.
— Other transfer income received by all other FU members in 2018.
— Women's Social Security income in 2018.
— Social Security income for all other FU members in 2018.

2.C Further Results

Table 2.9 – *Estimated parameters of the net total expenditures*

Variables	Estimate	SE
Intercept	64.171***	(10.968)
Education	-2.927***	(0.310)
Age	-1.558***	(0.547)
Age^2	1.707***	(0.663)
Black	4.669***	(1.550)
Hispanic	1.811	(2.659)
Total asset income	-0.259	(0.309)
(Total asset income)^2	0.001	(0.003)
Total welfare income	1.540***	(0.379)
(Total welfare income)^2	-0.025**	(0.012)
Total transfer income	1.982***	(0.418)
(Total transfer income)^2	-0.034***	(0.014)
Total other income	0.439***	(0.053)
(Total other income)^2	-0.002***	(0.000)
Sample Size	1,270	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parenthesis. Household net total expenditures are the dependent variable and express in thousands of dollars. Other covariates including in the model which do not appear in Table 2.2 are the following: women's level of education, women's age and its square, and dummies for black and hispanic women.

Table 2.10 – *Estimated quartiles of the distribution of marginal effects hourly wages and net total expenditures on labor supply*

Cost of children	Women with 1 child			Women with 2 children		
	P25	P50	P75	P25	P50	P75
Panel 1: Price of time						
Hourly wages	0.024	0.032	0.036	0.028	0.035	0.041
Net total expenditures	-0.475	-0.371	-0.268	-0.313	-0.244	-0.171
Panel 2: Simulated wages						
Hourly wages	0.021	0.031	0.038	0.028	0.035	0.043
Net total expenditures	-0.538	-0.399	-0.283	-0.320	-0.251	-0.162
Sample size		126			157	

Notes: See the notes in Table 2.5.

2.D Additional Figures

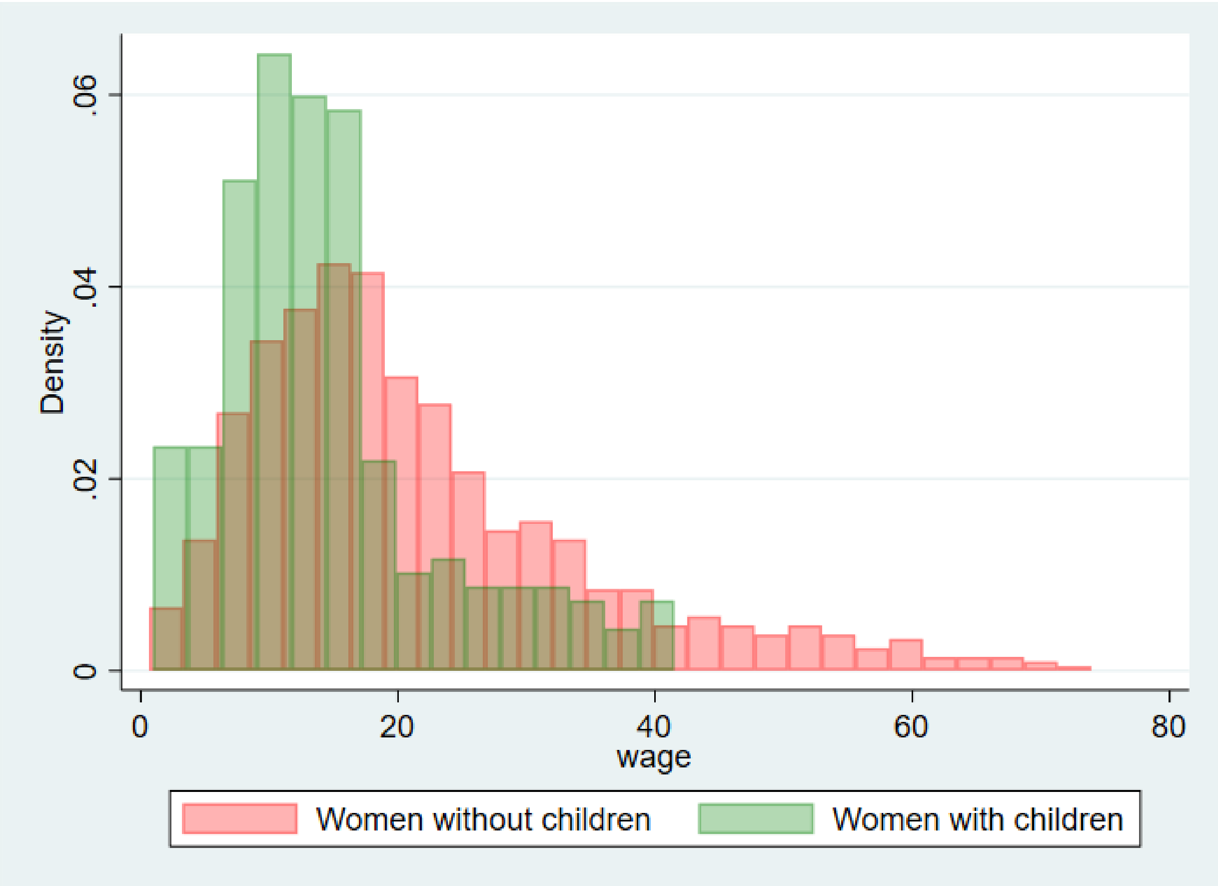

Figure 2.2 – *Wage Distribution of Working Women by Fertility Status*

Notes: This figure shows the distribution of wages for working women, conditional on their fertility status.

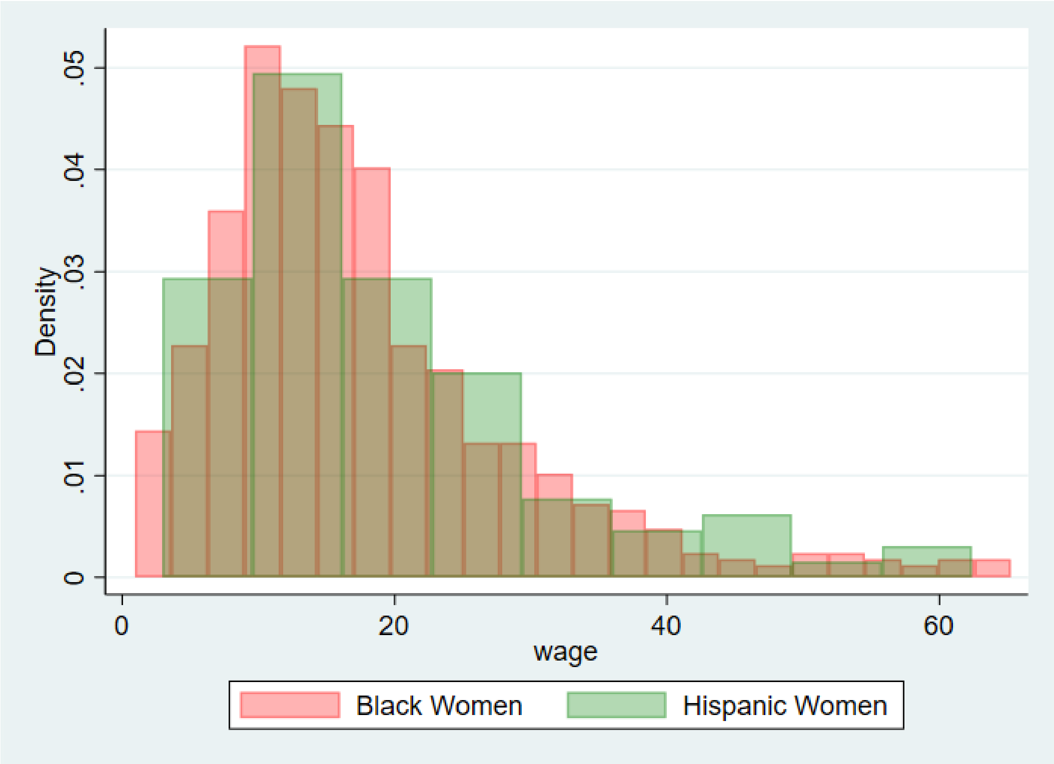

Figure 2.3 – *Wage Distribution of Working Women by Race*

Notes: This figure shows the distribution of wages for working women conditional on their race.

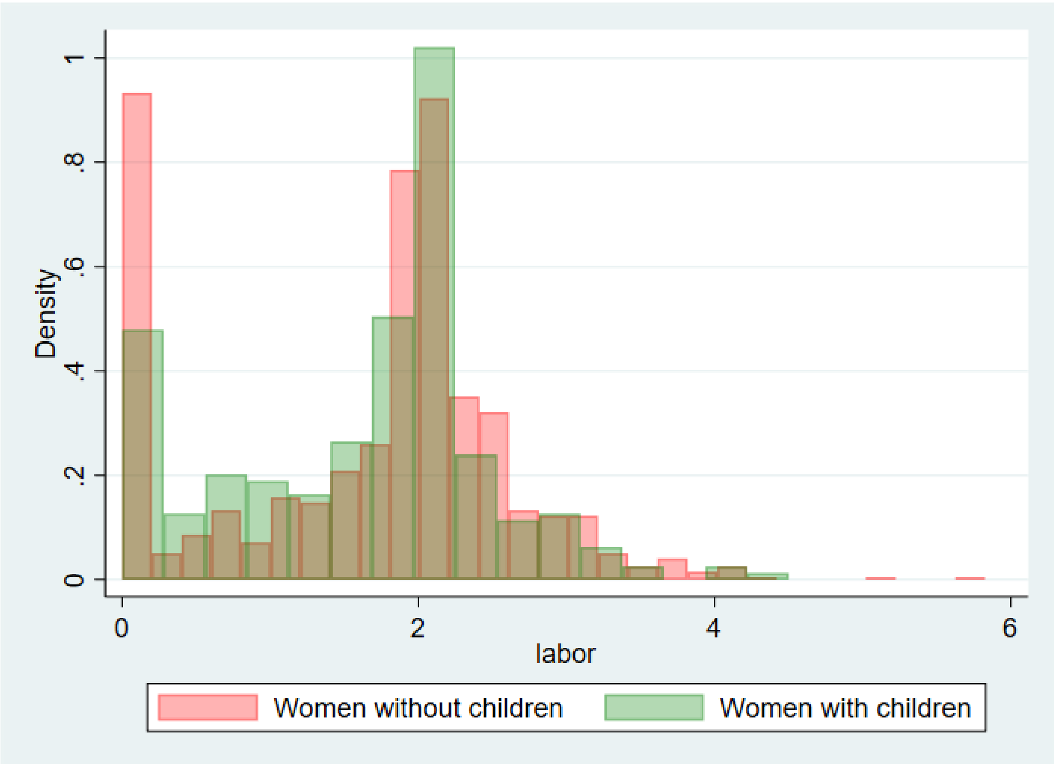

Figure 2.4 – *Labor Supply Distribution Among Women by Fertility Status*

Notes: This figure shows the distribution of women's market hours, conditional on their fertility status.

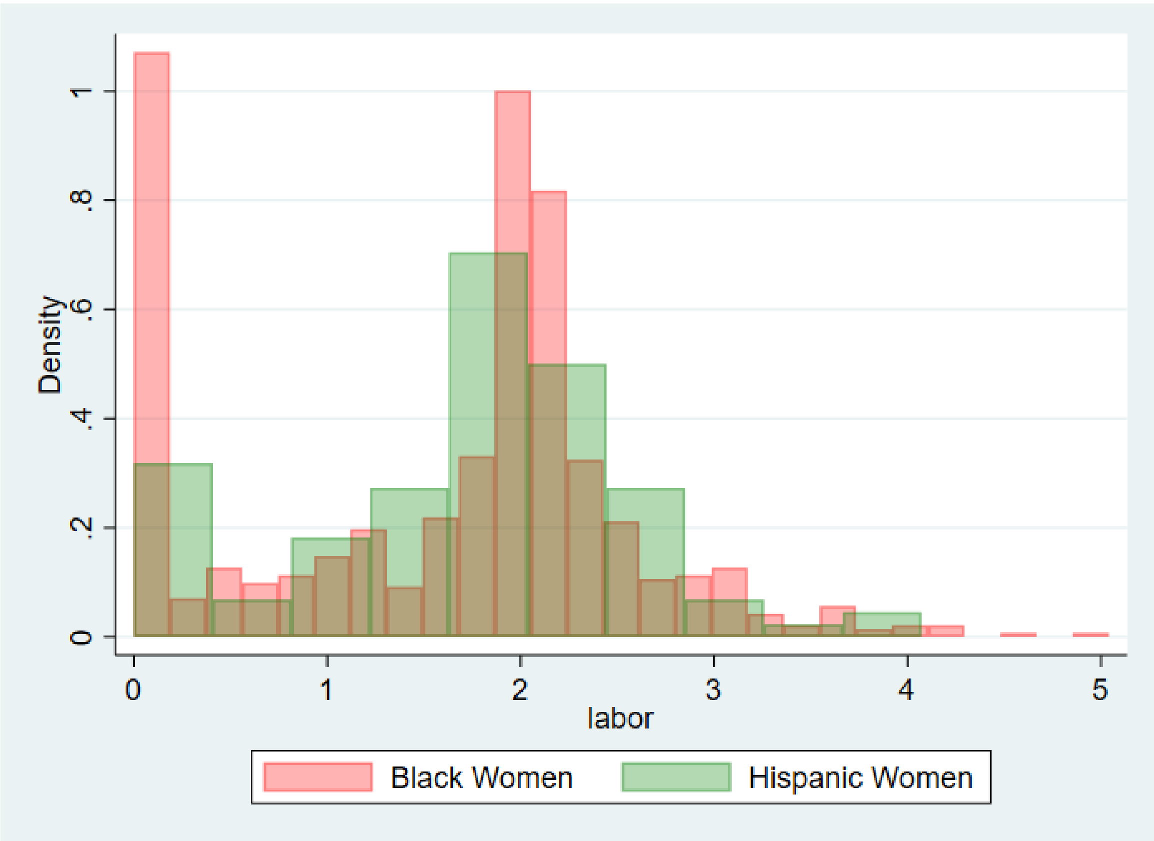

Figure 2.5 – *Labor Supply Distribution Among Women by Race*

Notes: This figure shows the distribution of women's market hours, conditional on their race.

Chapter 3

The Household Demand for Leisure, the Price of Time and the Full Cost of Children: A Structural Model and Evidence from the PSID

Olivier DONNI,[*,†] **Anderson VIL**[‡,§]

November 25, 2024

Abstract: We propose a method to assess the full cost of children by endogenously identifying the price of parental time, which is not assumed to equal the parent's wage rate. Instead, the price of time depends on how parents perceive their time spent with children. Our model builds on a collective labor supply framework for working couples, incorporating distinct individual preferences and childcare technology. Using 2019 PSID data, we find that 68% of mothers' childcare time is perceived as work, compared to 53% for fathers. Additionally, our results show that a significant share of the full cost of children is non-monetary.

JEL codes: I30, I31, J21, J22, J31.

Keywords: Collective Model, Identification, Labor Supply, Leisure Demand, Price of time, Resource sharing.

[*]THEMA, CY Cergy Paris Université, Cergy-Pontoise, France. olivier.donni@cyu.fr

[‡]THEMA, CY Cergy Paris Université, Cergy-Pontoise, France. anderson.vil@cyu.fr

3.1 Introduction

The cost of children is a critical parameter used to implement many economic policies or to calculate inequality measures. Economists have long developed methods to infer from survey data what parents spend for children (see Bargain and Donni, 2012a, Dunbar, Lewbel, and Pendakur, 2013, Bargain, Donni, and Hentati, 2022, for recent applications based on Rothbarth-like methods). However, simply identifying expenditures dedicated to children is insufficient. The full cost of children can indeed be decomposed into a monetary cost (the purchase of goods and services that contribute to children's well-being) and a time cost (the value of the time parents and other persons dedicate to children).[1] The evaluation of the time cost is particularly complicated and has been largely overlooked, with only a few exceptions. Gustafsson and Kjulin (1994) focus on assessing the value of parental time allocated to children, while neglecting the monetary aspects. They value time either by using the parent's wage rate or by the price of equivalent services. In contrast, Apps and Rees (2001) estimate the full cost of children, accounting for both monetary and time costs, through a structural model. Their model assumes linear homogeneity in childcare technology, identical preferences between parents, and the determination of the price of time by parents' wage rates. Similarly, Colombino (2000) constructs a structural model where parental time is also valued at the wage rate. Finally, Bradbury (2008) provides a theoretical framework and numerical illustrations, inferring the full cost of children from variations in parents' leisure.[2]

One of the main challenges in evaluating the full cost of children is assigning a value to childcare time. Typically, the price of time a spouse dedicates to any productive activity is assumed to align with their wage rate. However, this approach relies on two strong hypotheses: (a) the working time of the spouse has to be freely chosen, without any constraints such as non-participation in the labor market, and (b) the time dedicated to the activity is perfectly substitutable to market working time, with a marginal rate of substitution equal to one. If these conditions do not hold, then the price of childcare time is endogenously determined, depending on the preferences and technologies of the spouses. In the context of childcare, however, the second condition is particularly problematic. For example, Cosaert and Hennebel (2023) show that a large fraction of childcare time is perceived as leisure by parents. Similarly, Hallberg and Klevmarken (2003) find that childcare time is among the most enjoyable activities. To describe

[1] To be comprehensive, other long-term costs may include reduced career advancement opportunities (see Korenman and Neumark, 1990, Waldfogel, 1998, Budig and England, 2001, Dechter, 2014, Glauber, 2018).

[2] Koulovatianos et al. (2009) use an alternative identifying strategy to recover the full cost of children which is based on subjective questions.

situations where well-being arises from an activity regardless of its intended outcome, Juster and Stafford (1991) introduce the concept of "process benefits".

The aim of this paper is to evaluate the full cost associated with raising children, using a structural framework capable of delineating the price of parents' childcare time. To achieve this, we develop a collective labor supply model for a couple with children. Each parent is assumed to have a distinct utility function that depends on his or her working time, childcare time, and consumption with the crucial feature that childcare time is imperfectly substitutable for working time. More generally, childcare time can be viewed for parents as either a leisure-like activity that increases utility — reflecting the concept of process benefits — or a labor-like activity that decreases it. In addition to influencing utility, childcare time is also integrated into a childcare technology alongside other monetary inputs. Consequently, the price of a parent's childcare time is not simply equal to his or her wage; instead, it is determined by the substitutability rate between parental childcare time and the external childcare services purchased on the market. To complete the model, we assume that parents incur expenses for their children according to a predetermined rule. The full cost of children is finally assumed to comprise both the value of parental time devoted to the pure childcare activity (and not its effect on parents' utility) and the direct expenses for children.

Our main result is that the full cost of children can be identified from observed behavior. Specifically, our model allows us to break down each unit of childcare time into a "pure childcare" activity and a leisure-like (or labor-like) activity that directly affects the parents' utility. We then estimate this model using a sample of dual-earner couples, both with and without children, from the 2019 wave of the Panel Study of Income Dynamics (PSID). This dataset offers detailed information on time allocation, expenditures (including external childcare services), and socio-demographic variables. Our empirical analysis proceeds in two stages. First, we estimate the childcare technology and determine the price of parental childcare time, enabling us to calculate the time cost of children. Second, we define a composite measure of full leisure that includes the leisure-like (or labor-like) component of childcare activities. We then estimate full leisure demand functions to derive the full cost of raising children. We find that, of the time spent on child care, 68% of the tasks performed by mothers can be perceived as labor time, while this percentage drops to 50% for fathers. For a full-time working couple with two children, the median full cost of raising children is estimated at approximately $1,358 per week. However, the amount parents actually pay is approximately $418 per week. Moreover, our results suggest that children are viewed as a luxury good, suggesting that wealthier families prioritizing quality over quantity.

To the best of our knowledge, this is the first study to endogenously determine prices of childcare time and to estimate the full cost of children based on such prices. This approach contrasts with prior studies, which either rely on wages to value time or focus solely on consumption data, addressing only the monetary component of the cost of children. Our method, while similar to Rothbarth-like approaches in that it compares the demand for specific adult goods between couples with and without children, incorporates several key innovations. The adult good we analyze is spouses' leisure, which we define to include a portion of childcare time, with this fraction determined by a childcare technology. Unlike Bargain and Donni (2012a) and Bargain et al. (2022), we do not rely on data from single individuals and, unlike Dunbar et al. (2013), we avoid imposing strong conditions on individual preferences. The trade-off is that our method requires a more comprehensive dataset with detailed information on both time allocated to childcare and expenditures on external childcare services, as well as variation in prices, specifically parents' wages. Our contribution can also be related to the few studies that have attempted to estimate process benefits, though not specifically focused on childcare. Graham and Green (1984) were pioneers in this area, yet they did not address issues related to identification.[3] Kerkhofs and Kooreman (2003) conducted a thorough examination of the identification problems associated with process benefits, ultimately reaching rather pessimistic conclusions, while also providing empirical results.[4] Cosaert and Hennebel (2023), using a nonparametric approach, appear to be the only ones who explicitly consider process benefits in the context of childcare.

The rest of this paper is as follows. Section 2 presents the theoretical model. Section 3 outlines the empirical specification and the estimation method. Section 4 describes data and the empirical results. Finally, section 5 concludes.

3.2 The Model

3.2.1 Decision-making in a couple with children

We first consider a two-adult household, consisting of a wife (W) and a husband (H), with their children, who make decisions about leisure, child-care, and consumption in a static framework.[5] The wife's and husband's leisure time and consumption are respectively denoted by l_W, l_H, c_W and c_H. The wife's and husband's childcare time are respectively denoted by t_W and t_H. Spouses have specific preferences for how they allocate their time and consumption.

[3] Graham and Green (1984) use a Cobb-Douglas specification for the household production function in two-adult households. They also introduced a specification for the jointness functions, enabling an analytical solution to the household optimization problem.

[4] See also Gørtz (2011) for an application with Danish data.

[5] The terms "wife" and "husband" are used here for simplicity, and the partners are not necessarily married.

More precisely, each spouse I has a utility function of the form:

$$u_I = u_I(l_I, t_I, c_I),$$

where $u_I(\cdot)$ is a differentiable function, strictly increasing in l_I and c_I, and strongly concave in l_I, t_I and c_I, with $I = W$ and H. In this specification, spouses can be described as "egoistic", meaning their utility depends solely on their own consumption and time use, without consideration for the consumption or time use of their spouse or children. However, all the results immediately extend to the case of "altruistic" agents in a Beckerian sense, as discussed by Donni and Chiappori (2011). It is also noteworthy that child-care time directly enters utility functions, acting as either a leisure-like activity if $\partial u_I/\partial t_I > 0$ or a labor-like activity if $\partial u_I/\partial t_I < 0$ and capturing the aforementioned process benefits.[6]

The household choices are subject to different constraints. The traditional household budget constraint, assuming the prices for both the parents' and children's consumption are normalized to one, is given by

$$Y - c_W - c_H - c_K - l_W w_W - l_H w_H - t_W w_W - t_H w_H - m \geq 0 \tag{3.1}$$

where $Y = T \cdot (w_W + w_H) + y$ is the household full income, T the total time endowment of each spouse, y other nonlabor income, w_W and w_H the spouses' wage rates, c_K the children's consumption of goods and services, and m the money dedicated to purchase external childcare services. The child-welfare constraint is conveniently written as:

$$c_K - f(w_W, w_H, Y, n) \geq 0 \tag{3.2}$$

where $f(\cdot)$ is any positive, differentiable function of the exogenous variables and n is the number of children.[7] This function represents the share of household full income that is dedicated by parents to children for their direct expenses. This share may reflect the degree of parental altruism, though the specific allocation mechanism between parents and children is not explicitly

[6] This feature generalizes the model proposed by Donni and Vil (2024b). Most labor supply models that incorporate domestic production assume perfect substitutability between non-market working time and market working time (Gronau, 1977, Donni, 2008). This assumption is particularly strong and often unrealistic when applied specifically to childcare time rather than general non-market working time.

[7] The number of children n can also be understood, mutatis mutandis, as a vector including all the characteristics of the children, and not only their number.

modeled. Finally, the childcare constraint is given by

$$m - g\left(t_W, t_H, n\right) \geq 0 \tag{3.3}$$

where $g(\cdot)$ is a positive function, differentiable, strictly decreasing and strictly concave in its first two arguments (this condition guarantees that isoquant between m and t_W or t_H are decreasing and convex with respect to the origin) and decreasing in its last argument. The childcare constraint can be viewed as a technological constraint that must be fulfilled to ensure that children receive supervision round the clock. One limitation should be noted. This specification does not explicitly account for the possibility of parents jointly caring for children, nor does it consider the potential increased profitability for children that may result from joint care.

The decision process is assumed to lead to Pareto efficient outcomes. For a couple with children, the optimization problem of the household is thus:

$$\max_{l_W, l_H, t_W, t_H, c_W, c_H, m, c_K} \phi u_W(l_W, t_W, c_W) + (1-\phi) u_H(l_H, t_H, c_H) \tag{$\bar{\mathrm{P}}$}$$

subject to the budget constraint (3.1), the child-welfare constraint (3.2), the child-care constraint (3.3) and some non-negativity constraints that are not explicit here, where ϕ is a Pareto weight that may generally depend on all the exogenous variables. The Pareto weight determines the location on the Pareto frontier. If $\phi = 0$, then the household behaves as though the husband always gets his way, whereas, if $\phi = 1$, it is as if the wife is the effective dictator. If ϕ is constant, the optimization problem simplifies to the maximization of a separable household utility function, consistent with the unitary approach. If ϕ is a function of wage rates and other exogenous incomes — reflecting the idea that these variables are indicators of bargaining power [8] — then the optimization problem corresponds to the collective approach stricto sensu. Finally, the utility obtained by the children is not explicitly modeled here, but it is implicitly determined by the constraints and may depend on m, t_H and t_W.

As is common with collective models (see Donni and Chiappori, 2011), the optimization problem can be decentralized. If we focus on interior solutions, we have the following result.

Proposition 1. *The optimal allocation of time and consumption in Problem $\bar{\mathrm{P}}$ can be seen as the solution of a decentralized decision process. More precisely, there exists a pair of functions (θ_W, θ_H) of (w_W, w_H, Y, n) such that:*

[8] The bargaining weight may also be a function of distribution factors, i.e., variables that influence bargaining power without affecting the budget constraint.

(a) *the childcare cost is minimized:*

$$e(w_W, w_H, Y, n) = \min_{t_W, t_H} \left(t_W \theta_W w_W + t_H \theta_H w_H + g\left(t_W, t_H, n\right)\right), \tag{P$_1$}$$

and the remaining income is shared between parents, with the wife receiving κ_W *and the husband receiving* κ_H*, where*

$$\kappa_W + \kappa_H = Y - f(w_W, w_H, Y, n) - e(w_W, w_H, Y, n) = Y_R$$

and Y_R *is the residual full income after covering the full cost of children;*

(b) *each spouse maximizes her or his own utility function subject to a budget constraint:*

$$\max u_W(l_W, t_W, c_W) \text{ subject to } \kappa_W - c_W - l_W w_W - t_W(1-\theta_W)w_W \geq 0, \tag{P$_2$}$$

$$\max u_H(l_H, t_H, c_H) \text{ subject to } \kappa_H - c_H - l_H w_H - t_H(1-\theta_H)w_H \geq 0. \tag{P$_3$}$$

The proofs are in Appendix A. Intuitively, one unit of childcare time by spouse I is valued $\theta_I w_I$ as a pure childcare activity and $(1-\theta_I)w_I$ as a leisure-like activity if $\theta_I < 1$ (or as a labor-like activity if $\theta_I > 1$). If $\theta_I = 0$, the unit of childcare time is valued exactly as leisure.[9] If $\theta_I = 1$, there is no process benefits; similar to market labor time, childcare time simply reduces leisure time. The price of pure childcare time $w_I^* = \theta_I w_I$ and the price of leisure-like time $\bar{w}_I^* = (1-\theta_I)w_I$ are similar to Lindahl prices found in public economics (see also Donni, 2007, 2009), and their sum equals w_I. The time cost of children, defined as $e(w_W, w_H, Y, n)$, is evaluated using these prices. If childcare time is seen as a leisure-like activity for parents, meaning that w_I^* is lower than w_I, then the time cost tends, ceteris paribus, to be smaller. The full cost is defined as the sum of the time cost and the monetary cost.

3.2.2 A More Tractable Model: The Separation Principle

The key-point in this result is the equalization of the time devoted to the pure child-care activity (solution of P_1) and to the leisure-like (or labor-like) activity (solution of P_2 or P_3), achieved through appropriately decomposing the price of child-care time into $\theta_I w_I$ and $(1-\theta_I)w_I$. Therefore, the decision process is not strictly speaking two-staged or sequential. To obtain a two-stage decision process, we can adopt the approach proposed by Graham and Green (1984)

[9] This situation cannot occur for an interior solution if the derivative of the function $g\left(t_W, t_H, n\right)$ with respect to t_W or t_H is strictly positive. More broadly, if $\theta_I \leqslant 0$, there will necessarily be a boundary solution.

and Donni and Matteazzi (2018) and impose more structure on utility functions as follows:

U.1 Each spouse I has a utility function of the form:

$$u_I(l_I, t_I, c_I) = u_I(l_I + \varphi_I(t_I), c_I)$$

with $I = W$ or H, where $L_I = l_I + \varphi_I(t_I)$ can be viewed as the "full leisure" of spouse I for some differentiable and concave functions $\varphi_I(\cdot)$, satisfying $\varphi_I(0) = 0$.

This additional structure preserves the core properties of our model while making it suitable for empirical estimation. Specifically, the price of the leisure-like activity is determined by the first-stage choice of childcare time, as detailed in the following proposition.

Proposition 2. *Assume that utility functions are of the form U.1. Then the optimal allocation of time and consumption in Problem $\bar{P}$ is sequential. Firstly, the childcare cost is minimized in Problem P_1 with $\theta_W = 1-\varphi'_W(t_W)$ and $\theta_H = 1-\varphi'_H(t_H)$, giving $t^*_H = t_H(w_W, w_H, n)$ and $t^*_W = t_W(w_W, w_H, n)$ as optimal levels of childcare time. Secondly, each spouse maximizes her or his own utility function subject to a budget constraint in Problems P_2 and P_3 with $\theta_W = 1-\varphi'_W(t^*_W)$ and $\theta_H = 1-\varphi'_H(t^*_H)$, giving $L^*_W = L_W(w_W, \kappa_W + \pi_W)$ and $L^*_H = L_H(w_H, \kappa_H + \pi_H)$ as optimal levels of full leisure, with $\pi_W = \varphi_W(t^*_W)w_W - t^*_W(1-\theta_W)w_W$ and $\pi_H = \varphi_H(t^*_H)w_H - t^*_H(1-\theta_H)w_H$.*

This result suggests that the prices of childcare time are determined exclusively by the solutions of the cost minimization process, and not by individual preferences (except for the functions φ_I), implying that childcare activities are separated from consumption activities. [10] In this specification, the function π_I can be viewed as a profit function, where profit is derived from the production of leisure, with a price of w_I, using childcare time as an input, with a price of $\overline{w}^*_I = (1-\theta_I)w_I$. In the empirical application, we will adopt an even more simplified formulation in which the functions $\varphi_I(t_I)$ are linear in t_I, so that the functions π_I reduce to zero.

3.2.3 Identification

The primary objective of the present investigation is to show how the full cost of children, and its decomposition into time cost and monetary cost, can be identified. While the two-stage decision-marking is not strictly required, it does simplify the estimation procedure, as will be discussed later, and plays a crucial role in identifying the monetary cost of children.

[10] This concept is known as the separation principle in agricultural economics Benjamin (1992).

As is usual in the literature on the identification of collective models, we first assume that individual consumptions c_W, c_H and c_K are not observed by the economist. Instead, only their sum $c_W + c_H + c_K$ is observable. The other dependant variables, l_W, t_W, l_H, t_H and m, are observed as functions of the exogenous variables. We then define the optimal solution of the cost minimization problem as m^*, t_W^* and t_H^* as functions of (w_W, w_H, Y, n). The first identification result is the following.

Proposition 3. *The child-care cost can be recovered from* m^* *,* t_W^* *, and* t_H^* *as functions of* (w_W, w_H, Y, n)*. The technology* g *and the prices* θ_W *and* θ_H *can be recovered as well.*

This result does not rely on the sequential nature of the decision-making process and can be obtained without assuming that the utility functions take the specific form outlined in U.1. It could even be extended relatively easily to scenarios where one or both parents are not working, and wage rates are not observed. Intuitively, the prices of pure childcare time, $w_H^* = \theta_H w_H$ and $w_W^* = \theta_W w_W$, identified under this proposition, correspond to the marginal rate of substitution between t_W and t_H on the one hand, and m on the other..

The other structural components of the model, and in particular, the consumption of children c_K can be identified as well. The core idea is to compare the behavior of couples with children to those without children, under the assumption that adults' preferences are separable from fertility choices. Formally, we assume that the adults' preferences can be extended as follows:

A.2 Each spouse I has a child-conditional utility function of the form:

$$U_I(L_I, c_I; n) = F(u_I(L_I, c_I); n),$$

where $F(\cdot)$ is some transformation increasing in its first argument.

That is, the number of children does not modify the marginal rate of substitution between L_I and c_I. While such assumption is certainly strong, it is indispensable to measure the cost of children. It underpins earlier studies on equivalence scales and continues to play a central role in more recent studies, including those based on the Rothbarth-like methods developed by Bargain and Donni (2012a), Bargain et al. (2022) and Dunbar et al. (2013). It should be emphasized that the notion of stable preferences — essential for identification — is conceivable only when utility functions are of the form U.1.

To begin with, we assume that the technology and the prices θ_W and θ_H are known. This is the application of Proposition 3. We then recall $\theta_I = 1 - \varphi_I'$. Therefore, the functions $\varphi_I(t_I)$ can be recovered by integrating the functions $1 - \theta_I(t_I)$, with the boundary condition $\varphi_I(0) = 0$. The full leisure times can then be computed as $L_I = l_I + \varphi_I(t_I)$ as well as the utility profit functions π_I. If the stability assumption holds, the full leisure demand functions for a couple without children are given by

$$L_W = L_W(w_W, \kappa_W), \tag{3.4}$$

$$L_H = L_H(w_H, \kappa_H), \tag{3.5}$$

where $L_W = l_W$, $L_H = l_H$ and $\kappa_W + \kappa_H = Y$ while, for a couple with children, they are given by

$$L_W = L_W(w_W, \kappa_W + \pi_W), \tag{3.6}$$

$$L_H = L_H(w_H, \kappa_H + \pi_H), \tag{3.7}$$

where $L_W = l_W + \varphi_W(t_W)$ and $L_H = l_H + \varphi_H(t_H)$ and $\kappa_W + \kappa_H = Y - f - e$. From traditional results in the literature (Chiappori, 1988, 1992, or Chiappori, Fortin and Lacroix, 2002, for instance), the full leisure demand functions as well as the sharing functions κ_W and κ_H can be identified from a sample of childless couples provided that certain regularity conditions are satisfied. More precisely, these functions can be recovered up to a unique constant. This constant can be determined using additional information — for example, by examining the behavior of single individuals, as shown by Lise and Seitz (2011). Crucially, however, the exact identification of the full cost of children is achieved independently of this constant. To show this, we assume that the functions L_W and L_H are recovered up to a constant (using the results of Chiappori, 1988, 1992 for instance) and that $\partial L_W/\partial \kappa_W \neq 0$ and $\partial L_H/\partial \kappa_H \neq 0$, so that the full leisure demand functions can be inverted. From the inversion of (3.4) and (3.5), the sharing functions can be written as:

$$\kappa_W = G_W(w_W, L_W) + k, \tag{3.8}$$

$$\kappa_H = G_H(w_H, L_H) - k, \tag{3.9}$$

where k is the undefined constant, and G_W and G_H are known functions that are determined only by spouses' preferences. From the inversion of (3.4) and (3.5), we also have:

$$\kappa_W + \pi_W = G_W(w_W, L_W) + k, \tag{3.10}$$

$$\kappa_H + \pi_H = G_H(w_H, L_H) - k, \tag{3.11}$$

where the functions G_W and G_H are the same as in (3.8) and (3.9) because of the stability assumption. Summing and rearranging with the budget constraint give:

$$f = Y - e + \pi_W + \pi_H - G_W(w_W, L_W) - G_H(w_H, L_H).$$

where all the terms on left-hand side are known. Consequently, the monetary cost of children can be identified from the budget constraint, even if the constant k remains unknown. Except for how the price of pure childcare time is determined, this approach is similar to the one suggested by Bradbury (2008). It is also important to note that we make no assumptions regarding how the full cost is divided between the parents. It could be predominantly borne by either the mother or the father, or it could be equally shared.

Given that estimating a collective labor supply model—irrespective of the issue of children—is often challenging, we will introduce some minor simplifications in the empirical section. These simplifications are intended to focus on the key components of the model necessary for identification, while minimizing any loss of generality.

3.3 Empirical Specification and Estimation Method

To estimate the preceding model, we will specify functional forms for the child-care technology and individual preferences.

3.3.1 The Childcare Technology

The concept of childcare technology reflects the need for parents to coordinate in caring for their children. Assume that childcare requires 24 hours a day. If parents can coordinate perfectly, the time they each dedicate to childcare is perfectly substitutable. That is, $m = \alpha^* - \beta^*_W t_W - \beta^*_H t_H$ for some parameters α^*, β^*_W and β^*_H. If their schedules impose constraints that limit each parent to certain disjoint periods of the day, coordination becomes more complex, and there may be overlaps. In such cases, the time they spend with the children becomes more complementary rather than substitutable. To illustrate this, we suppose that the childcare technology is of the following form:

$$m = \alpha^* - \beta^*_W \left(\frac{(t_W - \delta^*_W)^{\gamma_W} - 1}{\gamma_W} \right) - \beta^*_H \left(\frac{(t_H - \delta^*_H)^{\gamma_H} - 1}{\gamma_H} \right) \tag{3.12}$$

where α^*, δ^*_W, δ^*_H, β^*_W, β^*_H, γ_W, and γ_H are parameters, with β^*_W, $\beta^*_H > 0$ and γ_W, $\gamma_H < 1$ to satisfy the positivity, monotonicity and concavity conditions previously mentioned.

The parameters δ^*_W and δ^*_H are baseline levels of parental time contributions from mothers and fathers, respectively, while the parameters β^*_W and β^*_H measure the effectiveness of the time in reducing market childcare expenditures. The parameters γ_W and γ_H control the curvature of the effect of parental time on reducing childcare expenditures, indicating how the marginal effect of additional parental time changes as more time is devoted. These parameters are related to the substitution of parents' childcare time. If $\gamma_W = \gamma_H = \gamma$, this specification is similar to a CES (constant elasticity of substitution) technology, though it is simpler to apply. More precisely, the elasticity of substitution between $(t_W - \delta^*_W)$ and $(t_H - \delta^*_H)$ is given by $1/(\gamma - 1)$.

If $\gamma \to 1$, then t_W and t_H are perfectly substitutable. If $\gamma \to -\infty$, then t_W and t_H are perfectly complementary, meaning the isoquants take the shape of right angles.

The solution of the cost minimization problem is:

$$t_W = \delta_W^* + \left(\frac{\beta_W^*}{w_W^*}\right)^{1/(1-\gamma_W)}, \tag{3.13}$$

$$t_H = \delta_H^* + \left(\frac{\beta_H^*}{w_H^*}\right)^{1/(1-\gamma_H)}, \tag{3.14}$$

where w_W^* and w_H^* are the price of spouses' pure childcare time as previously explained, which is supposed to be proportionate to hourly wage rates, that is, $w_W^* = \theta_W w_W$ and $w_H^* = \theta_H w_H$ for some positive parameters θ_W and θ_H.

If $\delta_W^* = \delta_W(\boldsymbol{z}_W^t) + v_W$ and $\delta_H^* = \delta_H(\boldsymbol{z}_H^t) + v_H$, where $\boldsymbol{z}_W^t$ and $\boldsymbol{z}_H^t$ are control variables and v_W and v_H are zero-mean disturbances, we obtain:

$$t_W = \delta_W(\boldsymbol{z}_W^t) + \left(\frac{\beta_W}{w_W}\right)^{1/(1-\gamma_W)} + v_W \tag{3.15}$$

$$t_H = \delta_H(\boldsymbol{z}_H^t) + \left(\frac{\beta_H}{w_H}\right)^{1/(1-\gamma_H)} + v_H, \tag{3.16}$$

where $\beta_W = \beta_W^*/\theta_W$ and $\beta_H = \beta_H^*/\theta_H$. The regressors $\boldsymbol{z}_W^t$ and $\boldsymbol{z}_H^t$ include demographic characteristics such as the spouse's level of education, the number of children, as well as dummies for the presence of children under three years old and other adults in the household.

To incorporate unobservable heterogeneity in demand for external childcare services, we write: $\alpha^* = \alpha(\boldsymbol{z}^m) + u$, where u is a zero-mean disturbance. Incorporating (3.13) and (3.14) into (3.12) with $\beta_W^* = \beta_W\theta_W$ and $\beta_H^* = \beta_H\theta_H$ gives:

$$m = \alpha(\boldsymbol{z}^m) - \frac{\theta_W\beta_W}{\gamma_W}\left(\left(\frac{\beta_W}{w_W}\right)^{\frac{\gamma_W}{1-\gamma_W}} - 1\right) - \frac{\theta_H\beta_H}{\gamma_H}\left(\left(\frac{\beta_H}{w_H}\right)^{\frac{\gamma_H}{1-\gamma_H}} - 1\right) + u \tag{3.17}$$

The regressors $\boldsymbol{z}^m$ in the market childcare demand equation include the husband's and wife's education levels, the number of children, and dummy variables for the presence of children under three years old and other adults in the household. The variance-covariance matrix of (v_W, v_H, u) is assumed to be unconstrained. Finally, the time cost is defined as: $e = t_W\theta_W w_W + t_H\theta_H w_H + m$.

3.3.2 Spouses' Preferences and the Full Cost of Children

Since our focus is on the cost of children rather than the allocation of resources between spouses, we adopt a very simple collective model. To begin with, we note that $\varphi_W(t_W) =$

$(1-\theta_W)t_W$ and $\varphi_H(t_H) = (1-\theta_H)t_H$ in spouses' utility functions as θ_W and θ_H are constant, as previously said. Consequently, the full leisure of both spouses is defined as $L_W = l_W + (1-\theta_W)t_W$ and $L_H = l_H + (1-\theta_H)t_H$. The term $\theta_I t_I$ can be interpreted as the fraction of childcare time that is labor, and the term $(1-\theta_I)t_I$ as the fraction that is pure leisure. The linear specification also garantees that the utility profit functions π_W and π_H are always equal to zero and can therefore be ignored.

We then assume that adults' preferences regarding full leisure time and consumption are represented by the following Stone-Geary direct utility function:

$$u(c_I, L_I) = a_I^* \ln(L_I - A_I) + b_I^* \ln(c_I - B_I)$$

with $a_I^*, b_I^* > 0$ and $a_I^* + b_I^* = 1$, by normalization, with $I = W, H$. The parameters a_I^* and b_I^* represent the marginal budget shares, while the parameters A_I and B_I, denote the subsistence level for c_I and L_I. We can interpret A_I as what Goodin et al. (2008) call "discretionary leisure". If this function is maximized with respect to the budget constraint $\kappa_I - c_I - L_I w_I \geq 0$, we obtain the corresponding indirect utility function:

$$v_I(w_I, \kappa_I) = a_I^* \ln\left(\frac{\kappa_I - A_I w_I - B_I}{w_I}\right) + b_I^* \ln\left(\kappa_I - A_I w_I - B_I\right).$$

The full leisure demand functions are obtained by applying the Roy's identity to the preceding indirect utility function:

$$L_I = a_I^* \frac{\kappa_I - A_I w_I - B_I}{w_I} + A_I.$$

The Stone-Geary specification, with this specific cardinalization, produces a particularly simple form for the sharing rule. The first-stage maximization program that determines the distribution of resources among spouses is given by

$$\begin{aligned}\max_{\kappa_W, \kappa_H} \quad & \phi\left[a_W^* \ln\left(\frac{\kappa_W - A_W w_W - B_W}{w_W}\right) + b_W^* \ln\left(\kappa_W - A_W w_W - B_W\right)\right] \\ & + (1-\phi)\left[a_H^* \ln\left(\frac{\kappa_H - A_H w_H - B_H}{w_H}\right) + b_H^* \ln\left(\kappa_H - A_H w_H - B_H\right)\right]\end{aligned}$$

subject to $\kappa_W + \kappa_H = Y_R$, where $\phi \in (0,1)$ is a parameter. If the solution is interior, we have:

$$\begin{aligned}\kappa_W &= \phi_W Y_R - (1-\phi_H)\left(A_H w_H + B_H\right) + (1-\phi_W)\left(A_W w_W + B_W\right) \\ \kappa_H &= \phi_H Y_R + (1-\phi_H)\left(A_H w_H + B_H\right) - (1-\phi_W)\left(A_W w_W + B_W\right)\end{aligned}$$

where $\phi_W = \phi$ and $\phi_H = 1-\phi$. To account for heterogeneity, we write the parameter a_I^* as follows:

$$a_I^*(\boldsymbol{x}_I) = \frac{\exp(\boldsymbol{a}_I \boldsymbol{x}_I)}{1 + \exp(\boldsymbol{a}_I \boldsymbol{x}_I)}$$

where $\boldsymbol{x}_I$ denotes a vector of sociodemographic characteristics specific to each member (i.e., a constant, the number of years of schooling, age and its square), and $\boldsymbol{a}_I$ a vector of parameters to be estimated. The monetary cost of children is simply assumed to be proportional to full income, excluding net expenditure, adjusted by a scale factor that accounts for the number of children:

$$f = k \times (w_W T + w_H T) \times n^\tau,$$

where $k \in (0,1)$ and τ are parameters.[11] The parameter τ is expected to be less than 1, reflecting the idea that economies of scale reduce the average cost per child as the number of children increases. The proportional specification is widely used in the literature (Bargain and Donni, 2012a, Dunbar et al., 2013). Finally, we add a zero-mean disturbance ϵ_I in each equation to account for unobservable heterogeneity, measurement errors and optimisation errors[12] to obtain:

$$L_W = a_W^*(\boldsymbol{x}_W) \frac{\kappa_W - A_I w_W - B_W}{w_W} + A_W + \epsilon_W \tag{3.18}$$

$$L_H = a_H^*(\boldsymbol{x}_H) \frac{\kappa_H - A_I w_H - B_H}{w_H} + A_H + \epsilon_H. \tag{3.19}$$

The Stone-Geary specification is chosen for its simplicity and convenience. Specifically, it implies that the response to variations in full income is linear, which is typically the most general response that can be robustly estimated in labor supply models. The linearity — while it generally complicates the identification of the sharing functions — does not impede the identification of the full cost of children. In our specific case, only the sum of the parameters $B_W + B_H$ and the product of the parameters $a_W \phi_W$ and $a_H \phi_H$ can be identified, rather than the separate parameters B_W, B_H, a_W, a_H, ϕ_W and ϕ_H.[13] Hence, we define the parameter

[11] Given the significant heterogeneity in net total expenditure, we also estimate a cost function where net total expenditure is included linearly as an alternative approach:

$$f = k \times (w_W T + w_H T + y) \times n^\tau$$

.

[12] Disturbances could be directly incorporated into parameters A_W, A_H or B, but this way of proceeding tends to be somewhat artificial here.

[13] Identification of the sharing functions (up to the standard additive constant) is achievable if ϕ_W and ϕ_H.are functions instead of parameters. This is true, in particular, if there are distribution factors entering these functions. Such identification is unlikely to be robust, though. Please refer to Appendix B for further details on identification.

$B = B_W + B_H$ and set $\phi_W = \phi_H = 1/2$. With this specification, an increase in the full cost of children has exactly the same effect as an increase in the full income. [14]

3.3.3 Estimation Method

The five equations can be estimated simultaneously, but one important consideration should be noted. In the data, parents occasionally report allocating no hours per week to childcare, and a significant portion of them also report zero expenditure on paid childcare services. The censored nature of these variables complicates the estimation process. To simplify, the five-equation system can alternatively be estimated recursively, which is also advantageous because it allows for the use of different samples in each stage of the estimation procedure. In a first stage, the three-equation system for the childcare technology, represented by (3.15), (3.16) and (3.17), is estimated as follows. The parameter γ_I is first set to -1.5 — a value supported by the data. The two equations (3.15) and (3.16) are then estimated as a bivariate Tobit model by maximizing the log-likelihood function to obtain $\widehat{\beta}_W$, $\widehat{\beta}_H$, $\widehat{\gamma}_W$, $\widehat{\gamma}_H$, and the residuals $\widehat{v}_W$ and $\widehat{v}_H$. These estimates are then substituted into (3.17), resulting in the following expression:

$$m = \alpha - \theta_W \frac{\widehat{\beta}_W}{\widehat{\gamma}_W} \left(\left(\frac{\widehat{\beta}_W}{w_W} \right)^{\frac{\widehat{\gamma}_W}{1-\widehat{\gamma}_W}} - 1 \right) - \theta_H \frac{\widehat{\beta}_H}{\widehat{\gamma}_H} \left(\left(\frac{\widehat{\beta}_H}{w_H} \right)^{\frac{\widehat{\gamma}_H}{1-\widehat{\gamma}_H}} - 1 \right) + \widehat{u}, \tag{3.20}$$

where $\widehat{u} = \rho_W \widehat{v}_W + \rho_H \widehat{v}_H + \varepsilon$. The equation (3.20) is finally estimated as a Tobit model by maximizing the log-likelihood function to obtain $\widehat{\alpha}$, $\widehat{\theta}_W$, $\widehat{\theta}_H$, $\widehat{\rho}_W$ and $\widehat{\rho}_H$. [15] In a second stage, both full leisure times are computed using $\widehat{\theta}_W$ and $\widehat{\theta}_H$ and the two-equation system for spouses' preferences, represented by (3.18) and (3.19), is estimated by the SUR (Seemingly Unrelated Regression) method. Identifying the effect of full income in such models is often challenging. However, it may be easier to obtain precise estimates here because of the larger variations in full income caused by the variation in time costs. Moreover, variations in full leisure time are also likely more important than those captured by traditional measures of leisure.

To ensure model consistency with a life-cycle framework, we opted to use net total expenditures instead of nonlabor income, as the latter does not account for savings (Blundell and MaCurdy, 1999). However, there are many reasons to believe that net total expenditures might

[14] We recall that when the Pareto weights are constant, the collective approach simplifies to the unitary approach. In our case, this simplification is not overly restrictive, as previously explained, but it implies the "pooling" of full income and the total cost of chilldren.

[15] The log-likelihood functions are presented in Appendix 3.B.

be endogenous. Primarily, labor supply and net total expenditures are likely jointly determined, meaning that household spending is influenced by the amount of labor supplied, and vice versa. More importantly, measurement errors in working time and therefore in leisure time are, by construction, reflected in net total expenditures. Following Mroz (1987), we use a second-order polynomial in age and education to instrument for net total expenditures. Additional instruments include father's education, mother's education, religion, and dummies for region and homeownership.

3.4 Data and Estimation Results

In this section, we first provide an overview of the data, followed by a presentation of the empirical results.

3.4.1 Sample selection

Our empirical analysis is based on the 2019 wave of the Panel Study of Income Dynamics (PSID). The PSID, a nationally representative longitudinal survey, offers extensive information on various subjects, such as income, wealth, demographics, and household members' allocation of time including leisure time, personal care time and time spent on childcare. The 2019 wave also includes detailed information on disagregated expenditures of the household, including parents' spending on external childcare services. This wave covers 9,569 households. For our analysis, we concentrate on non student, working heterosexual couples with and without children, both males and females, aged 22 to 60. We exclude households having children aged more than 7 years old. Additionally, we exclude observations where both parents declaring zero childcare time. After filtering out observations with incomplete data and outliers, our final sample consists of $1,533$ households, including 782 households with children and 751 households without children. Appendix D offers additional details on our dataset and a detailed description of the key variables construction.

Descriptive statistics for weekly expenditures, time allocation, and socio-demographic variables are provided in Table 1. The top panel highlights household expenditures, revealing that the average net expenditure is negative, suggesting that households typically spend more than they earn. This discrepancy likely arises because the PSID does not capture all typical household expenditures. Moreover, for parents, the average weekly childcare expenses amount to \$72.27, with a high standard deviation indicating considerable variation in spending. This variation is partly due to the fact that a significant portion of parents, approximately 40%, do not use paid childcare services. The middle panel of the table shows a clear distinction in time allocation between husbands and wives. On average, mothers dedicate nearly a quarter of their

Table 3.1 – *Descriptive statistics*

	Pooled data		Childless couples		Partnered parents	
	Husband	Wife	Husband	Wife	Husband	Wife
Expenditures ($ per week)						
Net total expenditures	-961.25		-1023.96		-900.56	
	(1577.2)		(1794.92)		(1331.44)	
Expenditures on childcare	36.72		-		72.27	
	(100.09)		-		(130.97)	
Time use (hours per week)						
Market labor	45.29	37.90	45.07	39.31	45.51	36.54
	(11.32)	(11.53)	(11.21)	(11.20)	(11.43)	(11.69)
Leisure time	55.97	55.24	69.64	75.40	42.75	35.74
	(22.56)	(27.65)	(11.21)	(11.20)	(22.91)	(24.69)
Childcare time	13.44	21.56	-	-	26.44	42.43
	(20.65)	(28.02)	-	-	(22.26)	(25.67)
Socioeconomic variables						
Age (years)	40.70	39.24	45.79	44.47	35.78	34.18
	(10.20)	(10.03)	(11.03)	(10.89)	(6.14)	(5.58)
Education level (years)	13.91	14.55	13.79	14.34	14.02	14.74
	(2.57)	(2.38)	(2.52)	(2.50)	(2.61)	(2.24)
Hourly wage rate	32.20	26.76	33.21	26.09	31.23	27.41
	(30.97)	(30.44)	(36.88)	(36.24)	(23.88)	(23.49)
Non-labor income	262.15		329.17		197.28	
	(956.33)		(975.57)		(933.378)	
Number of children	1.06		-		2.08	
	(1.23)		-		(0.93)	
Age of the youngest child	1.44		-		2.84	
	(1.91)		-		(1.78)	
Presence of other adults	0.18		0.29		0.08	
	(0.39)		(0.45)		(0.28)	
Nb of observations	1533		751		782	

Notes: This table shows the mean values of the variables used in this study with standard deviations in parentheses. It includes data on expenditures, time use, and socioeconomic characteristics for husbands and wives in pooled data, childless couples, and partnered parents. The number of observations for each group is listed at the bottom.

week to childcare, though the time spent varies significantly among them. Mothers typically invest substantially more time in childcare, averaging 42.43 hours per week, compared to fathers, who average 26.44 hours per week. Finally, the bottom panel of the table shows that men and women exhibit similar average age as well as average number of years of education. For couples with children, the average number of children aged 7 or younger is 2.

Figure 1 plots the distributions of wife's share in various time allocations. Panel A illustrates the proportion of the mother's time dedicated to childcare relative to the combined childcare time of both parents. The average share shows that mothers spend significantly more time caring for their children than fathers, accounting for approximately 63% of the total childcare

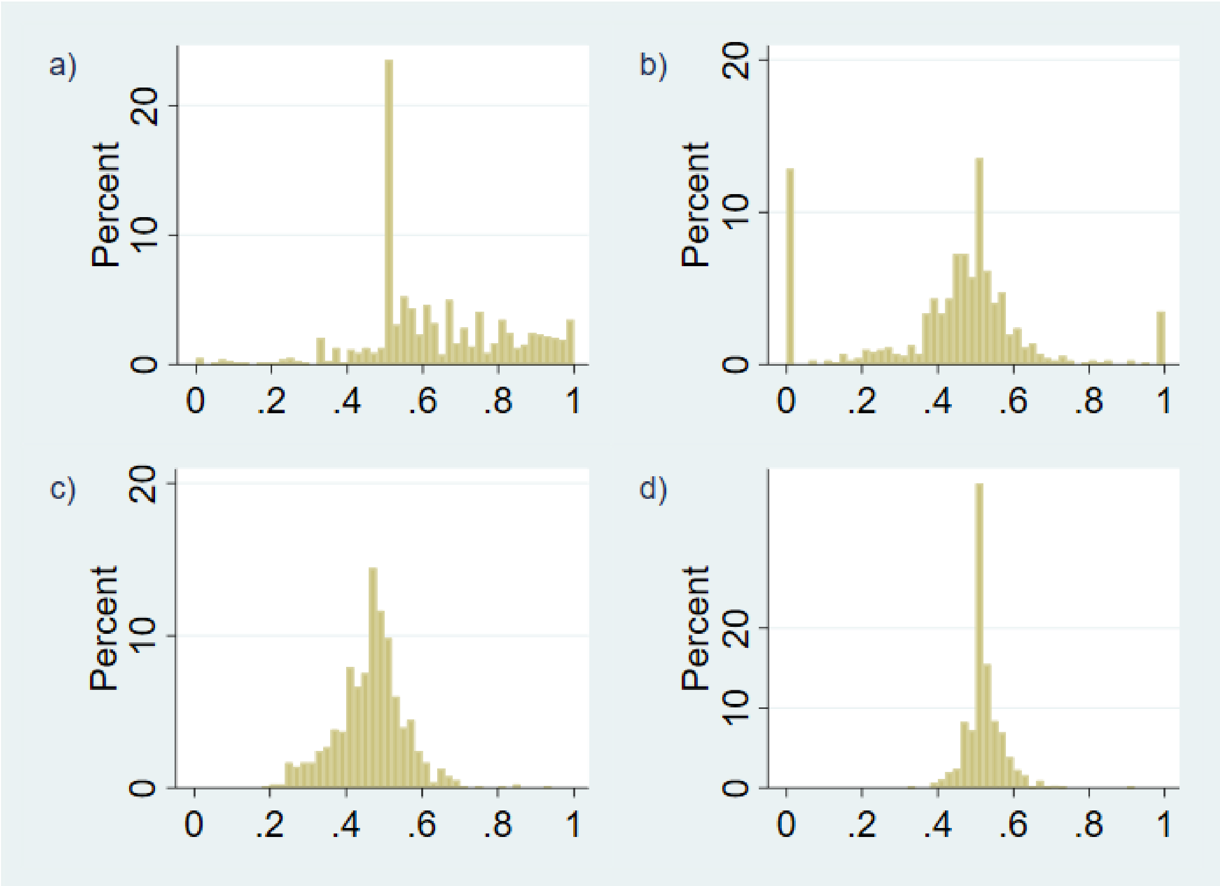

Figure 3.1 – *Distributions of time allocation shares across households*

Notes: This figure plot the distributions of time allocation shares across households. a) Childcare time $\frac{t_W}{t_W+t_H}$ (mean:.63, std:.19); b) leisure time $\frac{l_W}{l_W+l_H}$ (mean:.43, std:.22); c) full leisure for parents $\frac{L_W}{L_W+L_H}$ (mean:.47, std:.09); d) leisure for childless couples $\frac{L_W}{L_W+L_H}$ (mean:.52, std:.05).

time within the couple. One of the primary factors driving the difference between Panels B and C is how time spent with children is perceived. Panel B captures pure leisure, and given societal norms, mothers are more likely to engage in household or childcare duties, resulting in less pure leisure time compared to fathers. In contrast, Panel C incorporates the enjoyment or satisfaction derived from childcare activities (referred to as "process benefits"), which reduces the time imbalance. This suggests that mothers may perceive part of their time spent with children as relaxing, increasing their full leisure time, even though their actual "free" leisure time (without responsibilities) remains lower. Lastly, Panel D represents the distribution of leisure for childless couples, where the most striking difference compared to Panel C is the sharp concentration of leisure equality around 0.5.

3.4.2 Estimates of the Childcare Technology

In this section, we present the estimated parameters of the three-equation system for the childcare technology. In Table 2, the estimated parameters for two variations of the childcare time equations (3.15)-(3.16) are displayed, with the variations arising from different definitions of childcare time. Our findings indicate that having children under the age of three increases the amount of time parents spend on childcare by approximately 5 hours per week for fathers and 7 hours per week for mothers. Conversely, the presence of other adults in the household tends to reduce the amount of time parents spend on childcare. Moreover, the relationship between wages and the time dedicated to childcare is negative, as indicated by the positive parameter β_i. The residual error correlation $\rho^t = 0.38$ suggests unobserved factors similarly affect both parents' childcare decisions. For instance, a busy workweek or a child's illness might equally impact the provision of time both parents dedicate to childcare. This correlation may also reflect interactions or complementarity in childcare decisions between parents, such as coordination in the allocation of childcare time.

Table 3.2 – *Estimated parameters of childcare time*

	Dependent variables: Parental childcare time			
	Husbands	Wives	Husbands	Wives
	1		2	
δ_0	17.355***	15.084***	17.676**	11.102***
	(0.812)	(0.944)	(0.755)	(0.869)
δ_i [education]	0.171***	0.739***	0.116***	0.925***
	(0.040)	(0.052)	(0.037)	(0.048)
δ_i [Number of children]	-0.443***	2.048***	-0.298***	2.360**
	(0.105)	(0.116)	(0.097)	(0.104)
δ_i [dummy for presence of child aged less than 3]	5.176***	6.687***	5.086***	5.991***
	(0.207)	(0.232)	(0.192)	(0.209)
δ_i [dummy for presence of other adults]	-3.902***	-1.326**	-4.283***	-0.699
	(5.770)	(0.616)	(0.539)	0.545
β_i	1893.849***	5229.472***	1463.519***	3991.266***
	(243.594)	(299.405)	(196.001)	(231.939)
σ_i	23.142***	26.245***	21.579***	23.752***
	(0.088)	(0.117)	(0.080)	(0.090)
ρ^t	0.383***		0.352***	
	(0.004)		(0.004)	
Log Likelihood	-6974.265		-6827.565	
Sample Size	782		778	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parentheses. In (1), the parental childcare time is defined as $\min(t_i, 168 - h_i - \bar{s})$, where 168 represents the total hours available in a week, h_i stands for the weekly market hours, and $\bar{s}$ indicates the average weekly sleep time. In (2), the parental childcare time is defined as $\min(t_i, 168 - h_i - \bar{s} - p_i)$, where p_i indicates the average personal care time. We address censored data using the Tobit model, with γ_i being constrained.

The estimated parameters of paid childcare demand are presented in Table 3. With the linear specification for process benefits, the parameters θ_W and θ_H can be understood as the proportion of childcare time that is considered as pure labor time by the mother and the father respectively. Our findings indicate that, depending on the specification, 68% or 75% of the time

Table 3.3 – *Estimated parameters of childcare expenses*

	Dependent variables: Childcare expenditures	
	(1)	(2)
α_0	2725.987***	2293.488***
	(325.569)	(277.202)
α_1 [$education_1$]	5.674*	5.422*
	(3.068)	(3.083)
α_2 [$education_2$]	7.598**	7.394*
	(3.810)	(3.816)
α_3 [Number of children]	18.376	17.826
	(13.671)	(13.671)
α_4 [dummy for presence of child aged less than 3]	-111.825**	-112.341**
	(48.285)	(48.013)
α_5 [dummy for presence of other adults]	-3.590	-3.036
	(6.773)	(6.776)
θ_H	0.530***	0.620***
	(0.125)	(0.139)
θ_W	0.678***	0.748***
	(0.105)	(0.116)
ρ_H^m	0.701**	0.670**
	(0.294)	(0.309)
ρ_W^m	-1.277***	-1.421***
	(0.270)	(0.294)
σ	149.809***	149.374***
	(4.847)	(4.815)
Log likelihood	-3237.42	-3234.084
Sample Size	782	778

Notes: See Notes in Table 3.2. Childcare expenses per week are the dependent variable and express in thousands of dollars.

mothers spend with their children can be classified as labor, with the remainder being akin to leisure activities. In contrast, for fathers, only about half of their time spent with children is perceived as labor. Alternatively, these parameters can be interpreted as the degree of parental productivity in domestic tasks. For parents with the same wage rate, a high θ indicates greater productivity at the equilibrium. For example, a mother with a θ_W of 0.68 is more productive in domestic tasks compared to a father with a θ_H of 0.53.

In Table 3, we observe that the coefficient ρ_H^m is positive, indicating that fathers who spend more time on childcare are more likely to invest in paid childcare services. In contrast, the negative coefficient ρ_W^m suggests that mothers who dedicate more time to childcare are less inclined to use paid services. Thus, our findings illustrate a complementary relationship between paternal childcare time and expenses, with maternal childcare time potentially substituting for paid childcare.

3.4.3 Leisure demand equations

This section sequentially presents the results on marginal budget shares and the cost of children, all derived from the estimation of full leisure demand equations. The parameter estimates of the full leisure demand equations are provided in Appendix F.

Marginal budget shares

Table 4 provides the estimates of marginal budget shares for individuals' full leisure time. Panel A specifically presents the marginal budget shares computed at the median values for education levels, age, and the dummy variable indicating the presence of children under three years old. Panel B details these shares respectively for childless couples and parents. The first column reports results based on the full leisure equation defined as $l_i + (1 - \hat{\theta}_i t_i)$, where leisure (l_i) is calculated as $168 - h_i - \bar{s} - t_i$, with $\bar{s}$ representing the average weekly sleep time. The second column presents results for an alternative definition of the full leisure equation, where leisure (l_i) incorporating (L_i) is computed as $168 - h_i - \bar{s} - t_i - p_i$, with p_i denoting weekly adult personal care.

All estimated marginal budget shares a_I^* are positive and below one, satisfying the model's regularity condition. Focusing on column (1), our results suggest that wives exhibit a stronger preference for full leisure compared to husbands. This finding is consistent for the alternative specification. Further, we note that parents are likely to derive more utility from an additional unit of full leisure relative to consumption than childless couples do. This might be because parents have less leisure time available due to childcare, making the leisure they do get more impactful on their overall well-being.

Table 3.4 – *Marginal Budget Shares*

	Husbands	Wives	Husbands	Wives
	(1)		(2)	
Panel A: Median of the sample				
	0.076***	0.156***	0.077***	0.156***
	(0.011)	(0.012)	(0.012)	(0.013)
Panel B: Median of the sample conditional on fertility status				
No children	0.039***	0.064***	0.033***	0.061***
	(0.014)	(0.013)	(0.014)	(0.013)
With children	0.092***	0.178***	0.096***	0.181***
	(0.010)	(0.011)	(0.011)	(0.012)

Notes: ***$p < 0.01$. Standard errors are in parentheses. In the top panel, we calculated the marginal budget shares a_I^* for a representative couple within the sample, using median values for education, age, squared age, as well as the median of the dummy variable indicating the presence of children under six years old. The bottom panel presents the marginal shares for families specifically for couples without children and couples with children. In columns (1), the dependent variable is full leisure (L_i), defined as $l_i + (1 - \hat{\theta}_i t_i)$, where leisure (l_i) is $168 - h_i - \bar{s} - t_i$, with $\bar{s}$ denoting the avererage weekly sleep hours. In column (2), full leisure is defined as $l_i + (1 - \hat{\theta}_i t_i)$, with leisure calculated as $168 - h_i - \bar{s} - t_i - p_i$, where p_i denoting adult personal care.

Measures of the full cost of children

Tables 5 outlines the estimated parameters for the child-related consumption function. In the first two columns, the dependent variable is derived from the first definition of full leisure, whereas the alternative full leisure definition is applied in the last columns.[16] In columns (1) and (3), the cost function is defined as $f = k \times (w_1 T + w_2 T) \times n^{\tau}$. In contrast, columns (2) and (4) incorporate net total expenditures into the cost function as $f = k \times (w_1 T + w_2 T + y) \times n^{\tau}$.

Table 3.5 – *Estimated parameters of the cost function of children*

	Definition 1 of Leisure		Definition 2 of Leisure	
	(1)	(2)	(3)	(4)
k	0.075***	0.114***	0.068**	0.117***
	(0.025)	(0.029)	(0.026)	(0.032)
τ	-0.592*	-0.317	-0.558	-0.245
	(0.342)	(0.194)	(0.383)	(0.194)
Total time endowment	168	168	168	168
Sample size	1533	1533	1526	1526
Sargan statistics	3051	3051	3036	3036

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. This table presents the estimated parameters for the child-related consumption function and economies of scale. In columns (1-2), the dependent variable is full leisure (L_i), defined as $l_i + (1 - \hat{\theta}_i t_i)$, where leisure ($l_i$) is calculated as $168 - h_i - \bar{s} - t_i$. Columns (3-4) employ an alternative definition of full leisure, where (l_i) which incorporates full leisure is defined as $168 - h_i - \bar{s} - t_i - p_i$, with p_i representing adult personal care. Columns (1) and (3) uses the cost function specification $f = k(w_1 T + w_2 T)n^{\tau}$. In contrast, columns (2) and (4) incorporate net total expenditures linearly into the cost function as $f = k(w_1 T + w_2 T + y)n^{\tau}$. It is noteworthy that the sample size decreases to 1,526 couples in the last two columns due to missing data on adult personal care.

Results from column (1) show that, for a one-child family, the cost of raising the child is around 8% of the potential combined parental labor income.[17] When accounting for net total expenditures in the cost function, the parameter k increases to 11%. These findings remain consistent when using the alternative definition of the full leisure.

We turn to the estimate of the correction factor. The negative value of τ implies that as the number of children n increases, the per-child cost decreases. Specifically, the rate at which costs decrease is slightly faster than an inverse square root. The $\tau = -0.59$ suggests strong economies of scale. The cost per child decreases significantly as more children are added. This could reflect shared resources, bulk purchasing, or other factors that reduce the marginal cost per child.

Figure 4 in Appendix F plots the evolution of the cost of children as the number of children

[16] Both definitions are presented in 3.4.3.

[17] It should be noted that the value of T is irrelevant. See Tables 8 and 9 in Appendix F.

increases. Let us assume that the parents work for the total weekly hours of 168 hours as in column (1). If $n = 4$, the cost function would be:

$$f = 0.08(w_1 \times 168 + w_2 \times 168) \times 4^{-0.59} \approx 0.08(w_1 \times 168 + w_2 \times 168) \times 0.44$$

This results in a per-child cost of approximately 3.52% of the combined potential income, which is significantly lower than the cost per child when there is only one child. Consequently, for a family with four children, the full cost amounts to around 14% of the combined potential income.

Table 6 presents point estimates of children's expenditures. Columns (1) and (3) show the cost per child for a median couple working the full weekly time. Columns (2) and (4) estimate per-child costs for a couple working full time at the minimum wage of $7.25. Columns (1-2) include both consumption goods and paid childcare, while columns (3-4) extend these estimates to include the value of parental childcare time.

For a median couple, the weekly full cost of raising children is estimated at $1,358 per week, while the actual amount paid by parents (what we call the direct cost of children) is $418 per week. When both parents earn the federal minimum wage ($7.25), the weekly direct cost decreases to $151. In a family with four children and both parents earn the federal minimum wage, the amount per child paid by parents is $110 per week, while the full cost amounts to $1050. Our findings indicate that the shadow cost of children is the most significant financial burden borne by parents.

We also estimated the cost of children by considering different values for the total labor supply. The model adjusts accurately and provides results consistent with expectations. This indicates that the cost of raising children can be reliably estimated from potential income. These results are presented in Appendix F.

Children as Luxuries: Wealthier Parents Prioritize Quality Over Quantity - Figure 2 plots the expenditure on a child according to different income levels and the age of the youngest child in the family. The results show a positive correlation between child-rearing expenses and household income. As family income increases, parents are likely to spend significantly more on their children. This can include higher-quality education, extracurricular activities, healthcare, and better living conditions. [18] The increased spending reveals that children, in this context, may be seen as luxury goods. This aligns with Becker's quantity-quality theory, which posits that wealthier families prioritize the quality of children over the quantity (Becker, 1960).

[18] According to Pierre Bourdieu's theory, parents from higher socio-economic backgrounds invest heavily in their children's education and activities to accumulate cultural capital. This capital is crucial for maintaining and enhancing social status.

Table 3.6 – *Cost of children computed at the median point of the sample*

	Direct cost of children		Full cost of children	
	(1)	(2)	(3)	(4)
Panel A: Median of the sample				
	417.95**	150.72***	1357.96***	1090.72***
	(173.10)	(53.87)	(173.10)	(53.87)
Panel B: Median of the sample conditional on the number of children				
1	614.76***	211.95***	1554.76***	1151.96***
	(196.70)	(61.21)	(196.70)	(61.21)
2	417.95**	150.72***	1357.96***	1090.72***
	(173.10)	(53.87)	(173.10)	(53.87)
3	335.17**	124.96**	1275.17***	1064.72***
	(167.60)	(52.14)	(167.60)	(52.14)
4	287.38*	110.09**	1227.39***	1050.09***
	(162.30)	(50.49)	(162.30)	(50.49)
Total time endowment	168	168	168	168
Minimum wage		✓		✓

Notes:* $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parentheses. This table presents the point estimates of the total cost of children. Child-related consumption expenses are modeled as $k(w_1T + w_2T)n^\tau$ and evaluated at the median wage for columns (1) and (3), and at the minimum wage of $7.25 for columns (2) and (4).

3.5 Conclusion

In this paper, we have developed a model to evaluate the full cost of children, including both a monetary component and a time component. The crucial point of our study is that the price of the time used to evaluate the time cost of children is not necessarily equivalent to the parents' wage rate. Instead, this price depends on how parents perceive their time spent with their children — whether they view it more as leisure or as labor. To the best of our knowledge, this is the first study to make this important distinction and to present estimates of the full cost of children based on this distinction.

For clarity and simplicity, the model is expressed in terms of the demand for leisure, though the results can be interpreted in terms of labor supply. Estimating this model requires detailed data on both the time parents devote to their children and their expenditures on external childcare services. We applied the model to data from the 2019 PSID, which provides all the necessary information. Our findings indicate that the price of childcare time for mothers is approximately 68% of their wage rate, while for fathers, it is roughly 53% of their wage rate. For a couple with two children, the median full cost of children is estimated at approximately $1,358 per week, with the median direct monetary cost estimated at approximately $418 per

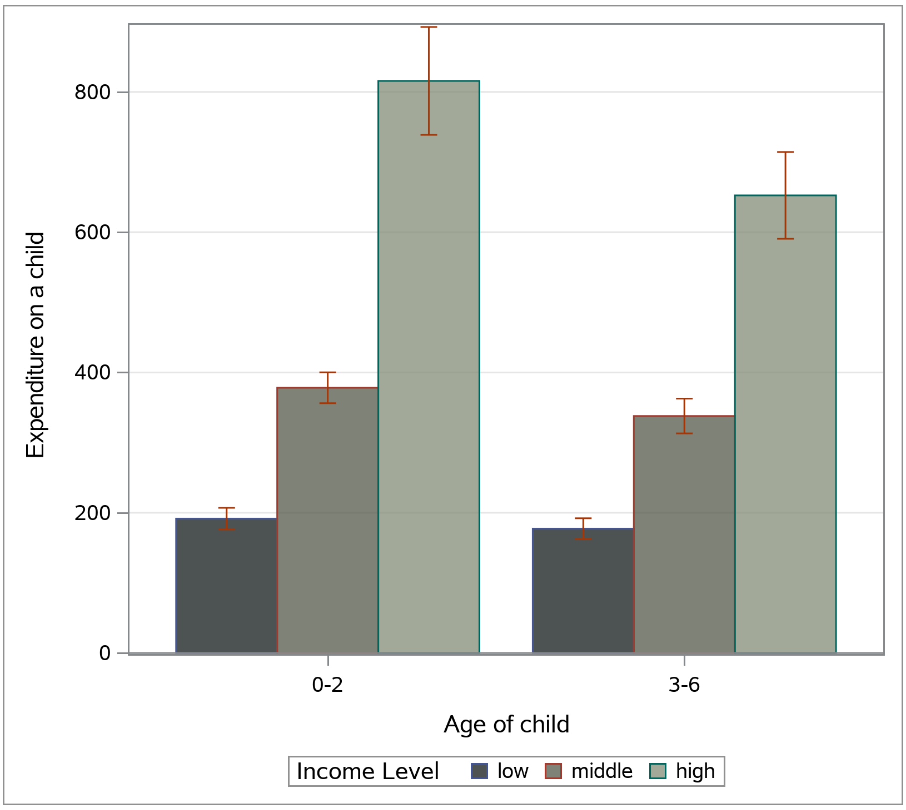

Figure 3.2 – *Family expenditures on a child, by income level and age of the youngest child*

Notes: The bars in the chart are clustered by income level, as indicated by the legend at the bottom. The dark grayish olive bars represent low-income households, with annual incomes ranging from \$10,100 to \$81,600. The grayish olive bars correspond to middle-income households, with annual incomes between \$81,723 and \$128,000. The light grayish olive bars represent high-income households, with annual incomes from \$128,026 to \$1,275,000. The height of each bar reflects the average expenditure within each category. Specifically, each bar shows the average amount spent on a child by families of a particular income level and for a given age group of the youngest child.

week. Thus, our results show that the shadow cost represents the most significant part of the cost of children. Finally, our results indicate that wealthier families tend to prioritize the quality of investment in their children over having more children.

The main limitation of this paper stems from the "corner solution" issue arising in households that do not spend on formal childcare services. For these families, we lack sufficient data to precisely estimate the price of childcare time, as certain couples never engage with the formal childcare market. Future research could address this limitation by incorporating observed market prices for formal childcare services as a benchmark, allowing us to infer time prices even for families that rely exclusively on informal care. Additionally, it is important to acknowledge that our estimates do not account for the long-term costs associated with raising children, such as education investments or parental time's impact on children's future outcomes. While these long-term considerations are beyond the scope of this study, they represent a promising avenue for future research.

Appendices

3.A Proofs

3.A.1 Proof of Proposition 1.

The first order conditions of the optimization problem $\bar{\text{P}}$ are:

$$
\begin{aligned}
l_W &: \phi\frac{\partial u_W}{\partial l_W} - \lambda_1 w_W = 0 && (3.21)\\
t_W &: \phi\frac{\partial u_W}{\partial t_W} - \lambda_1 w_W - \lambda_3 g'_W = 0 && (3.22)\\
c_W &: \phi\frac{\partial u_W}{\partial c_W} - \lambda_1 = 0 && (3.23)\\
l_H &: (1-\phi)\frac{\partial u_H}{\partial l_H} - \lambda_1 w_H = 0 && (3.24)\\
t_H &: (1-\phi)\frac{\partial u_H}{\partial t_H} - \lambda_1 w_H - \lambda_3 g'_H = 0 && (3.25)\\
c_H &: (1-\phi)\frac{\partial u_H}{\partial c_H} - \lambda_1 = 0 && (3.26)\\
m &: \lambda_3 - \lambda_1 = 0 && (3.27)\\
k &: \lambda_2 - \lambda_1 = 0 && (3.28)
\end{aligned}
$$

where λ_1, λ_2 and λ_3 are the Lagrange Multipliers for (3.1), (3.2) and (3.3), respectively. From (3.27) and (3.28), we have: $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$. From (3.22) and (3.25), write:

$$
\begin{aligned}
g'_W + \theta_W w_W &= 0 && (3.29)\\
g'_H + \theta_H w_H &= 0 && (3.30)
\end{aligned}
$$

where

$$
\begin{aligned}
\theta_W &= 1 - \frac{1}{\lambda_W w_W}\frac{\partial u_W}{\partial t_W}\\
\theta_H &= 1 - \frac{1}{\lambda_H w_H}\frac{\partial u_H}{\partial t_H}
\end{aligned}
$$

compute at the solution of the optimization problem, where $\lambda_W = \lambda/\phi$ and $\lambda_H = \lambda/(1-\phi)$. This system of two equations constitutes the first-order conditions of the optimization problem P_1. From (3.21)-(3.23), write:

$$\begin{aligned} \frac{\partial u_W}{\partial l_W} - \lambda_W w_W &= 0 \qquad (3.31) \\ \frac{\partial u_W}{\partial t_W} - \lambda_W w_W (1-\theta_W) &= 0 \qquad (3.32) \\ \frac{\partial u_W}{\partial c_W} - \lambda_W &= 0 \qquad (3.33) \end{aligned}$$

where

$$1 - \theta_W = 1 + \frac{g'_W}{w_W}.$$

This system of three equations constitutes the first-order conditions of the optimization problem P_2, where λ_W is the Lagrange Multiplier. Similarly, write:

$$\begin{aligned} \frac{\partial u_H}{\partial l_H} - \lambda_H w_H &= 0 \qquad (3.34) \\ \frac{\partial u_H}{\partial t_H} - \lambda_H w_H (1-\theta_H) &= 0 \qquad (3.35) \\ \frac{\partial u_H}{\partial c_H} - \lambda_H &= 0 \qquad (3.36) \end{aligned}$$

where

$$1 - \theta_H = 1 + \frac{g'_H}{w_H}.$$

This system of three equations constitutes the first-order conditions of the optimization problem P_3, where λ_H is the Lagrange Multiplier. □

3.A.2 Proof of Proposition 2.

From the first-order conditions of Problem P_1, we obtain:

$$\theta_I w_I + \frac{\partial g}{\partial t_I}(t^*_W, t^*_H, n) = 0 \qquad (3.37)$$

with $I = W$ and H. Replacing θ_I by $1 - \varphi'_I(t^*_I)$ gives:

$$(1 - \varphi'_I(t^*_I))w_I + \frac{\partial g}{\partial t_I}(t^*_W, t^*_H, n) = 0. \qquad (3.38)$$

This system of equations can be solved with respect to t_W^* and t_H^*. From the first-order conditions of Problems P_2 and P_3 with respect to l_I, t_I and c_I, respectively, we also obtain:

$$\begin{aligned} \frac{\partial u_I}{\partial L_I}(L_I^*, c_I^*) &= \mu_I w_I, \\ \frac{\partial u_I}{\partial L_I}(L_I^*, c_I^*) \cdot \varphi_I'(t_I^*) &= \mu_I (1-\theta_I) w_I, \\ \frac{\partial u_I}{\partial c_I}(L_I^*, c_I^*) &= \mu_I, \end{aligned}$$

where μ_I is the Lagrange Multiplier of the spouse's budget constraint. If $\varphi_I'(t_I^*) = 1 - \theta_I$, the second first-order condition is redundant and can be eliminated. The budget constraint $\kappa_I - c_I - l_I w_I - t_I(1-\theta_I)w_I \geq 0$ is then written as:

$$\kappa_I - c_I - L_I w_I + \varphi_I(t_I^*) w_I - t_I^*(1-\theta_I) w_I \geq 0.$$

Using this budget constraint and defining $\pi_I = \varphi_I(t_I^*) w_I - t_I^*(1-\theta_I) w_I$, the first and the third first-order equations can be solved to give L_I^* and c_I^*, with $I = W$ and H. $\square$

3.A.3 Proof of Proposition 3.

Given that

$$m^*(w_W, w_H, Y, n) = g(t_W^*(w_W, w_H, Y, n), t_H^*(w_W, w_H, Y, n), n), \tag{3.39}$$

we have

$$\frac{\partial m^*}{\partial w_W} = g_W' \cdot \frac{\partial t_W^*}{\partial w_W} + g_H' \cdot \frac{\partial t_H^*}{\partial w_W} \tag{3.40}$$

$$\frac{\partial m^*}{\partial w_H} = g_W' \cdot \frac{\partial t_W^*}{\partial w_H} + g_H' \cdot \frac{\partial t_H^*}{\partial w_H} \tag{3.41}$$

for any n, where g_W' and g_H' are the derivatives of g with respect to its first two arguments. Since the function g is strictly concave, the determinant of the matrix

$$\begin{pmatrix} \frac{\partial t_W^*}{\partial w_W} & \frac{\partial t_H^*}{\partial w_W} \\ \frac{\partial t_W^*}{\partial w_H} & \frac{\partial t_H^*}{\partial w_H} \end{pmatrix} \tag{3.42}$$

is non-zero. Thus, this system of equations can be solved for $g_W'(t_W, t_H, n)$ and $g_H'(t_W, t_H, n)$. By integrating $g_W'(t_W, t_H, n)$ and $g_H'(t_W, t_H, n)$, we can recover $g(t_W, t_H, n)$ up to an additive function of n. This remaining function can be determined by the boundary condition in

(3.39). The prices of childcare time are then given by $\theta_W w_W = -g'_W(t^*_W, t^*_H, n)$ and $\theta_H w_H = -g'_H(t^*_W, t^*_H, n)$ from the first-order conditions of the cost minimization problem. Combining all these results allows us to recover the time cost. □

Further Identification Results

Let us incorporate the expressions:

$$\begin{aligned}
\kappa_W &= \phi_W Y_R - (1-\phi_H)(A_H w_H + B_H) + (1-\phi_W)(A_W w_W + B_W) \\
\kappa_H &= \phi_H Y_R + (1-\phi_H)(A_H w_H + B_H) - (1-\phi_W)(A_W w_W + B_W)
\end{aligned}$$

into the following equations:

$$L_W = a_W \frac{\kappa_W - A_W w_W - B_W}{w_W} + A_W$$

$$L_H = a_H \frac{\kappa_H - A_H w_H - B_H}{w_H} + A_H$$

We obtain:

$$L_W w_W = a_W \phi_W Y_R - a_W (1-\phi_H) A_H w_H + A_W (1 - a_W \phi_W) w_W - a_W (\phi_W B_W + (1-\phi_H) B_H)$$

$$L_H w_H = a_H \phi_H Y_R - a_H (1-\phi_W) A_W w_W + A_H (1 - a_H \phi_H) w_H - a_H (\phi_H B_H + (1-\phi_W) B_W)$$

We define the following reduced parameters:

$$\begin{aligned}
\Pi_1 &= a_W \phi_W \\
\Pi_2 &= -a_W (1-\phi_H) A_H \\
\Pi_3 &= A_W (1 - a_W \phi_W) \\
\Pi_4 &= -a_W (\phi_W B_W + (1-\phi_H) B_H) \\
\Pi_5 &= a_H \phi_H \\
\Pi_6 &= -a_H (1-\phi_W) A_W \\
\Pi_7 &= A_H (1 - a_H \phi_H) \\
\Pi_8 &= -a_H (\phi_H B_H + (1-\phi_W) B_W) \\
\phi_W + \phi_H &= 1
\end{aligned}$$

with structural parameters: $a_W, a_H, A_W, A_H, B_W, B_H, (\phi_W, \phi_H)$.

$$
\begin{aligned}
\Pi_1 &= a_W \phi_W \\
\Pi_2 &= -a_W \phi_W A_H \\
\Pi_3 &= A_W \left(1 - a_W \phi_W\right) \\
\Pi_4 &= -a_W \left(\phi_W B_W + \phi_W B_H\right) \\
\Pi_5 &= a_H (1 - \phi_W) \\
\Pi_6 &= -a_H (1 - \phi_W) A_W \\
\Pi_7 &= A_H \left(1 - a_H (1 - \phi_W)\right) \\
\Pi_8 &= -a_H \left((1 - \phi_W) B_H + (1 - \phi_W) B_W\right) \\
\phi_W + \phi_H &= 1
\end{aligned}
$$

Thus, the identification of the structural parameters can be derived as follows:

$$
\begin{aligned}
\Pi_1 &= a_W \phi_W \\
A_H &= -\frac{\Pi_2}{\Pi_1} \\
-\frac{\Pi_5 \Pi_3}{\Pi_6} &= \left(1 - a_W \phi_W\right) \\
\Pi_4 &= -a_W \left(\phi_W B_W + \phi_W B_H\right) \\
\Pi_5 &= a_H (1 - \phi_W) \\
A_W &= -\frac{\Pi_6}{\Pi_5} \\
-\frac{\Pi_1 \Pi_7}{\Pi_2} &= \left(1 - a_H (1 - \phi_W)\right) \\
\Pi_8 &= -a_H \left((1 - \phi_W) B_H + (1 - \phi_W) B_W\right) \\
\phi_W + \phi_H &= 1
\end{aligned}
$$

$$
\begin{aligned}
\Pi_1 &= a_W \phi_W \\
A_H &= -\frac{\Pi_2}{\Pi_1} \\
-\frac{\Pi_5 \Pi_3}{\Pi_6} &= (1 - \Pi_1) \\
\Pi_4 &= -\Pi_1 \left(B_W + B_H\right) \\
\Pi_5 &= a_H (1 - \phi_W) \\
A_W &= -\frac{\Pi_6}{\Pi_5} \\
-\frac{\Pi_1 \Pi_7}{\Pi_2} &= (1 - \Pi_5) \\
\Pi_8 &= -\Pi_5 \left(B_H + B_W\right) \\
\phi_W + \phi_H &= 1
\end{aligned}
$$

Finally, we can identify the structural parameters A_H, A_W, $(B_H + B_W)$, $a_H(1 - \phi_W)$, and $a_W \phi_W$.

3.B The Likelihood Function

3.B.1 Likelihood function of the demand for parental childcare time

The generic form of the bivariate censored demand of childcare model for each observation $i = 1, ..., s$ is given by:

$$t^*_{Wi} = z'_{Wi} \Delta_W + \upsilon_{Wi}$$

$$t^*_{Hi} = z'_{Hi} \Delta_H + \upsilon_{Hi}$$

Here, t^*_{Wi} and t^*_{Hi} are the latent variables, z_{Wi} and z_{Hi} are the vectors of explanatory variables including wages, Δ_W and Δ_H are the vector of parameters to be estimated, and υ_{Wi} and υ_{Hi} are the error terms. The observed dependent variables t_{Wi} and t_{Hi} might be censored at zero, depending on the behavior of the latent variables t^*_{Wi} and t^*_{Hi}. The observed variables are defined as:

$$t_{Wi} = \begin{cases} t^*_{Wi} & \text{if } t^*_{Wi} > 0 \\ 0 & \text{if } t^*_{Wi} \leq 0 \end{cases}$$

$$t_{Hi} = \begin{cases} t^*_{Hi} & \text{if } t^*_{Hi} > 0 \\ 0 & \text{if } t^*_{Hi} \leq 0 \end{cases}$$

Let assume that the latent variables (t^*_{Wi}, t^*_{Hi}) follow a bivariate normal distribution:

$$\begin{pmatrix} t^*_{Wi} \\ t^*_{Hi} \end{pmatrix} \sim N(\mu, \Sigma)$$

where

$$\mu = \begin{pmatrix} z'_{Wi}\Delta_W \\ z'_{Hi}\Delta_H \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sigma^2_W & \rho^t\sigma_W\sigma_H \\ \rho^t\sigma_W\sigma_H & \sigma^2_H \end{pmatrix}$$

We assume that the disturbances υ_{Wi} and υ_{Hi} have a joint normal distribution with mean μ and standard deviations σ_W and σ_H, and correlation ρ^t.

The likelihood for each observation $i = 1, ..., s$ needs to take into account the four potential censoring outcomes for the pair (t_{Wi}, t_{Hi}):

1. **Both t_{Wi} and t_{Hi} are Observed (uncensored).**

The likelihood is computed from a bivariate normal density:

$$\ell_i = \phi(t_{Wi}, t_{Hi}; \mu, \Sigma)$$

where $\phi_2(t_{Wi}, t_{Hi}; \mu, \Sigma)$ is the density function for a bivariate normal distribution with mean μ and matrice of variance-covariance Σ. The expression of the bivariate pdf is the following:

$$\phi_2(t_{Wi}, t_{Hi}; \mu, \Sigma) = \frac{\exp\left\{-\frac{1}{2(1-\rho^{t^2})}\left[\frac{(t_{1i}-\mu_1)^2}{\sigma_1^2} - 2\rho^t\frac{(t_{Wi}-\mu_W)(t_{2i}-\mu_H)}{\sigma_W\sigma_H} + \frac{(t_{Hi}-\mu_H)^2}{\sigma_H^2}\right]\right\}}{2\pi\sigma_W\sigma_H\sqrt{1-\rho^{t^2}}}$$

2. **t_{Wi} is Censored and t_{Hi} is Observed.**

The likelihood function for this scenario involves the conditional probability that $t^*_{Wi} \leq 0$ given $t^*_{Hi} = t_{Hi}$. Since t_{Wi} is censored at 0, we need to integrate the joint density function $\phi(t_{Wi}, t_{Hi}; \mu, \Sigma)$ over the possible values of t^*_{Wi}, from $-\infty$ to 0.

We want to compute:

$$\Pr(t^*_{Wi} \leq 0 | t^*_{Hi} = t_{Hi})$$

Using the properties of the bivariate normal distribution, the conditional distribution of $\mathbb{T}^*_{Wi}|\mathbb{T}^*_{Hi} = t_{Hi}$ is normal with:

$$\mu_{W|H} = z'_{Wi}\Delta_W + \rho^t\frac{\sigma_W}{\sigma_H}(t_{Hi} - z'_{Hi}\Delta_H)$$

$$\sigma^2_{W|H} = \sigma^2_W(1 - \rho^{t^2})$$

Therefore,

$$\Pr(t^*_{Wi} \leq 0 | t^*_{Hi} = t_{Hi}) = \Phi\left(\frac{-\mu_{W|H}}{\sigma_{W|H}}\right)$$

where Φ is the standard normal CDF. The likelihood can be rewritten as:

$$\ell_i = \int_{-\infty}^{0} \phi_H(\upsilon_W, t_{Hi}; \mu, \Sigma) d\upsilon_W$$

3. **t_{Wi} is Observed and t_{Hi} is Censored.**

   Similar to the above but for t_{Hi}. Compute:

$$\Pr(t^*_{Hi} \leq 0 | t^*_{Wi} = t_{Wi})$$

The conditional distribution of $\mathbb{T}^*_{Hi} | \mathbb{T}^*_{Wi} = t_{Wi}$ is normal with:

$$\mu_{H|W} = z'_{Hi}\Delta_H + \rho^t \frac{\sigma_H}{\sigma_W}(t_{Wi} - z'_{Wi}\Delta_W)$$

$$\sigma^2_{H|W} = \sigma^2_H(1 - \rho^{t^2})$$

Therefore,

$$\Pr(t^*_{Hi} \leq 0 | t^*_{Wi} = t_{Wi}) = \Phi\left(\frac{-\mu_{H|W}}{\sigma_{H|W}}\right)$$

The likelihood can be rewritten as:

$$\ell_i = \int_{-\infty}^{0} \phi_H(t_{Wi}, \upsilon_H; \mu, \Sigma) d\upsilon_H$$

4. **Both t_{Wi} and t_{Hi} are Censored**[19]

   The likelihood is computed as:

$$\ell_i = \int_{-\infty}^{0} \int_{-\infty}^{0} \phi_H(\upsilon_W, \upsilon_H, \mu, \Sigma)\, d\upsilon_W\, d\upsilon_H$$

This double integral represents the probability that both $t^*_{1i} \leq 0$ and $t^*_{Hi} \leq 0$, which is calculated using the bivariate normal CDF:

$$\ell_i = \Phi_H\left(\frac{-z'_{Wi}\Delta_W}{\sigma_W}, \frac{-z'_{Hi}\Delta_W}{\sigma_W}, \rho^t\right)$$

[19] This part will not be included in the analysis, as we exclude households where both parents report having allocated zero hours per week to childcare.

3.B.2 Likelihood Function of demand for market childcare services

The latent model associated with the paid childcare services can be written as:

$$m^* = h(w_W, w_H; z^m) + \alpha_u u \tag{3.43}$$

The demand for childcare services is observed only if the couple-parent spends a positive amount on acquiring childcare services. Hence we have:

$$m^* > 0 \qquad \text{if } u < \frac{m - h(w_W, w_H; z^m)}{\alpha_u}$$

and

$$m^* \leq 0 \qquad \text{if } u \leq \frac{-h(w_W, w_H; z^m)}{\alpha_u}$$

Then the contribution to likelihood for the couple-parent who does not acquire paid childcare services is:

$$\int_{-\infty}^{+\infty} \Phi\left(\frac{-h(w_W, w_H; z^m)}{\alpha_u}\right) \cdot \mathrm{d}u$$

where Φ is the cumulative probability function of the standardized normal distribution.

The contribution to likelihood for the couple-parent who does acquire paid childcare services is:

$$\phi\left(\frac{m - h(w_W, w_H; z^m)}{\alpha_u}\right).$$

Summing the logarithm of these expressions over all observations gives the log-likelihood function, which is maximized to estimate the parameters.

Supplementary Data Information

3.B.3 Data description.

The model requires two variables related to time: parental childcare time and leisure time. The PSID inquires about typical weekly parental time spent caring for children as follows: "In a typical week, how many hours [do you/does [he/she]] spend [c]aring for or looking after children?" Childcare hours are excluded if they are considered part of the parent's job. For some households, the sum of the weekly time reported for childcare and the time spent in the labor market exceeds 168 hours. To address these discrepancies, we redefine the time variable as follows: $\min(t_i, 168 - h_i - \bar{s})$, where 168 represents total hours available in a week, h_i stands for the weekly market hours, and $\bar{s}$ indicates the average weekly sleep time. Willoughby et al.,

(2023) show that the average sleep time for American households is estimated at 6.9 hours per day, amounting to a total of 48.3 hours per week.

The PSID asks also for usual weekly leisure hours: “In a typical week, how many hours [do you/does [he/she]] spend) [d]oing leisure activities for enjoyment, for example, watching TV, doing physical activities that (you enjoy/[he/she] enjoys), going online, or spending time with friends?". However, following the literature, we construct the weekly leisure hours variable as net total of hours available: $168 - h_i - \bar{s} - t_i$. As outlined in the text, our model's final stage is analyzing the demand for full leisure. This variable is constructed using the estimated parameter $\hat{\theta}_i$, which is obtained during the first stage of estimation through the analysis of demand for paid childcare services. The full leisure variable is defined as $l_i + (1 - \hat{\theta}_i t_i)$, where $(1 - \hat{\theta}_i t_i)$ corresponds to the part of parental childcare time perceived as leisure.

Finally, the PSID asks about the childcare expenditures in the following: “How much did you (and your family living there) pay for child care in 2018?" We then divide the annual childcare expenditures by 52 to obtain the weekly childcare expenditures.

The redefined variables for parental childcare time show means of 27.25 and 43.71 hours for fathers and mothers, respectively, compared to 33.76 and 55.45 hours in the original PSID data. This indicates that the redefinition has lowered the average time spent on childcare, likely by adjusting or removing extreme values, resulting in more consistent and less variable data. The significantly reduced variances and capped maximum values further support this. Despite these adjustments, the median and mode remain unchanged, suggesting that the redefinition primarily impacted the upper end of the distribution without altering the central tendency. This pattern is consistent across both fathers and mothers.

3.B.4 Definition of covariates in regressions.

Here, we provide detailed definition of the covariates in the regressions.

— **Wage rate**: represents the hourly earnings in dollars and cents.

— **Education of Husband/Wife**: indicates the highest grade of school completed by the husband or wife.

— **Age of Husband/Wife**: refers to the husband's or wife's age in 2019.

— **Presence of children aged less than 3**: a dummy variable equal to one if children under 3 years old are present, and zero otherwise.

— **Presence of other adults**: a dummy variable equal to one if other adults, aside from the husband and wife, are present in the household, and zero otherwise.

— **Number of children**: the number of children in the household.

3.C Additional Results

Table 3.7 – *Estimated Demand Full Leisure Equations*

	Husbands	Wives	Husbands	Wives
	(1)		(2)	
$B_H + B_W$	-719.458***		-780.473***	
	(102.2)		(108.4)	
A_I	74.063***	79.153***	66.266***	68.059***
	(1.008)	(1.031)	(0.936)	(1.000)
a_I [*intercept*]	-3.966**	-5.194***	-4.10**	-5.162***
	(1.729)	(1.164)	(2.109)	(1.306)
a_I [*education*]	0.044**	-0.017	0.045**	-0.030
	(0.021)	(0.018)	(0.023)	(0.021)
a_I [*age*]	0.118	0.246***	0.140	0.237***
	(0.095)	(0.067)	(0.120)	(0.076)
a_I [age^2]	-0.002*	-0.004***	-0.003	-0.004***
	(0.001)	(0.001)	(0.002)	(0.001)
a_I [dummy for presence of child aged less than 3]	-0.670**	-0.212***	-0.333**	-0.198**
	(0.322)	(0.080)	(0.130)	(0.093)
Sample size	1533		1526	

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Standard errors are in parentheses. This table reports the estimated parameters for the demand for full leisure. In column (1), we use the first definition of full leisure, while column (2) presents results based on the second definition.

Table 3.8 – *Estimated parameters of the cost function of children*

	Definition 1 of Leisure				Definition 2 of Leisure			
	(1)	(2)	(3)	(4)	(5)	6)	(7)	(8)
k	0.075***	0.104***	0.126***	0.114***	0.068**	0.094**	0.114**	0.117***
	(0.025)	(0.016)	(0.042)	(0.029)	(0.026)	(0.037)	(0.044)	(0.032)
τ	-0.592*	-0.592*	-0.592***	-0.317	-0.558	-0.558	-0.558	-0.245
	(0.342)	(0.342)	(0.342)	(0.194)	(0.383)	(0.383)	(0.383)	(0.194)
Total time endowment	168	121	100	168	168	121	100	168
Sample size	1533	1533	1533	1533	1526	1526	1526	1526
Sargan statistics	3051	3051	3051	3051	3036	3036	3036	3036

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. This table presents the estimated parameters for the child-related consumption function and economies of scale. In columns (1-4), the dependent variable is full leisure (L_i), defined as $l_i + (1 - \hat{\theta}_i t_i)$, where leisure ($l_i$) is calculated as $168 - h_i - \bar{s} - t_i$. Columns (5-8) employ an alternative definition of full leisure, where (l_i) which incorporates full leisure is defined as $168 - h_i - \bar{s} - t_i - p_i$, with p_i representing adult personal care. Columns (1-3) and (5-7) uses the cost function specification $f = k(w_1T + w_2T)n^{\tau}$. In contrast, columns (4) and (8) incorporate net total expenditures linearly into the cost function as $f = k(w_1T + w_2T + y)n^{\tau}$. Additionally, columns (1), (4), (5), and (8) use the weekly total time endowment of 168 hours in the cost function. Columns (2) and (6) apply a net weekly time endowment of 121 hours, accounting for sleep, while columns (3) and (7) assume an available weekly time of 100 hours. It is noteworthy that the sample size decreases to 1,526 couples in the last four columns due to missing data on adult personal care.

Table 8 reports the estimated parameters of the cost function. We first examine column (1) when potential income in the function f is substantially larger than that in columns (2-3). The results show that, for a one-child family, the cost of raising the child is 8% of the potential combined parental labor income. The parameter k, which estimates the proportion of parental income allocated to children consumption, increases to 10% in column (2) and to 13% in column

(3). This rise compensates for the decrease in working hours from 168 to 121 then 100, thereby maintaining a consistent total expenditure on children despite the reduction in available labor time. This shift in k is a necessary recalibration to sustain an equivalent financial allocation between the three settings, thus allowing for an accurate assessment of the cost of children based on the potential income of the parents. Table 9 provides an illustration of this.

Table 3.9 – *Cost of children computed at the median point of the sample*

	Total cost of children effectively paid				Total shadow cost of children			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Median of the sample								
	417.95**	417.95**	417.95**	150.72***	1357.96***	1357.96***	1357.96***	1090.72***
	(173.10)	(173.10)	(173.10)	(53.87)	(173.10)	(173.10)	(173.10)	(53.87)
Panel B: Median of the sample conditional on the number of children								
1	614.76***	614.76***	614.76***	211.95***	1554.76***	1554.76***	1554.76***	1151.96***
	(196.70)	(196.70)	(196.70)	(61.21)	(196.70)	(196.70)	(196.70)	(61.21)
2	417.95**	417.95**	417.95**	150.72***	1357.96***	1357.96***	1357.96***	1090.72***
	(173.10)	(173.10)	(173.10)	(53.87)	(173.10)	(173.10)	(173.10)	(53.87)
3	335.17**	335.17**	335.17**	124.96**	1275.17***	1275.17***	1275.17***	1064.72***
	(167.60)	(167.60)	(167.60)	(52.14)	(167.60)	(167.60)	(167.60)	(52.14)
4	287.38*	287.38*	287.38*	110.09**	1227.39***	1227.39***	1227.39***	1050.09***
	(162.30)	(162.30)	(162.30)	(50.49)	(162.30)	(162.30)	(162.30)	(50.49)
Total time endowment	168	121	100	168	168	121	100	168
Minimum wage				✓				✓

Notes: * $p < 0.10$, **$p < 0.05$, ***$p < 0.01$. This table presents the point estimates of the total cost of children. Child-related consumption expenses are modeled as $k(w_1T + w_2T)n^{\tau}$ and evaluated at the median wage for columns (1-3) and (5-7), and at the minimum wage of $7.25 for columns (4) and (8). In columns (1), (3), (5), and (7), we assume both spouses work full-time with labor market hours fixed at 35. In columns (1), (4-5), and (8), we use the weekly total time endowment of 168 hours in the cost function. In columns (2) and (6), we use the net weekly time available after sleep of 121 hours in the cost function. In columns (3) and (6), we assume an available weekly time of 100 hours in the cost function.

3.D Correction factor Results

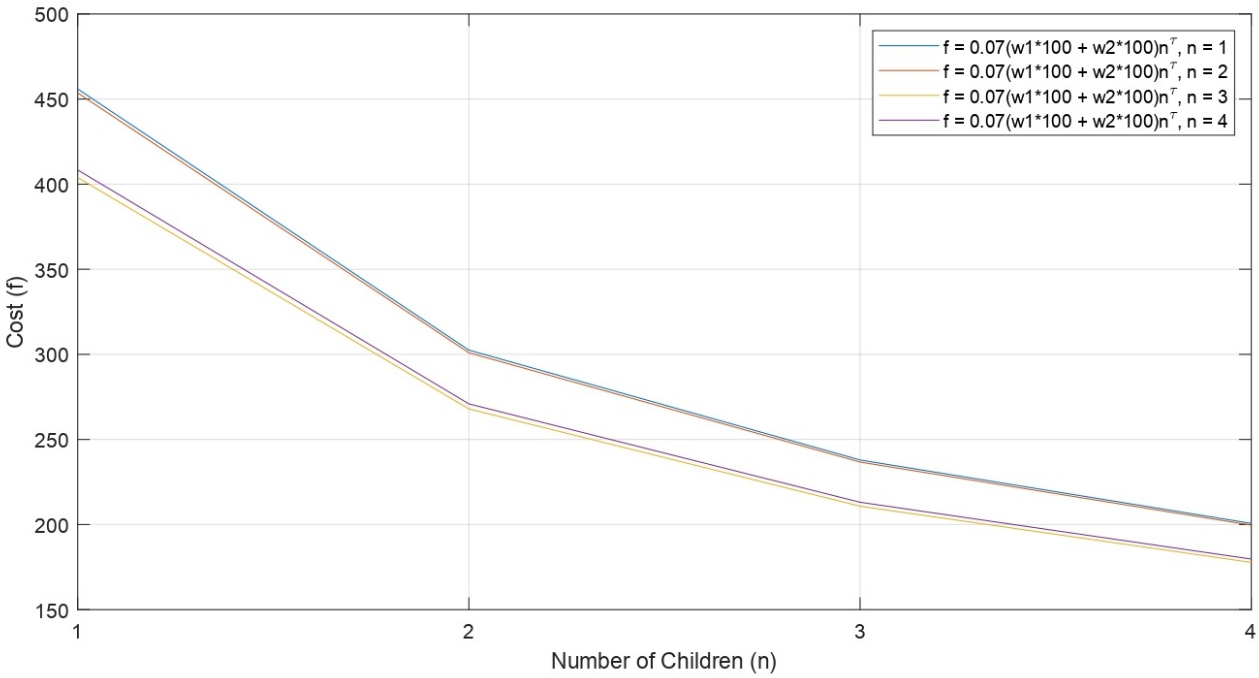

Figure 3.3 – *Family expenditures on a child, by income level and age of the youngest child*

Notes: This figure illustrates the evolution of the cost per child as the number of children increases. The blue, red, yellow, and purple lines represent the cost per child for a couple with one, two, three, and four children, respectively.

General Conclusion

This thesis presents a theoretical and empirical framework for assessing the cost of raising children across various household types, using multiple data sources. This dissertation is divided into three chapters, each explores distinct aspects of the financial and time-related costs associated with child-rearing.

Throughout these chapters, I contribute to several strands of literature by examining equivalence scales from both consumption theory and labor supply theory perspectives. From the consumption theory angle, the first chapter builds on the growing body of research on equivalence scales (Bargain et al., 2010, Bargain and Donni, 2012a, Browning et al., 2013, Dunbar et al., 2013, Bargain et al., 2022). Specifically, by leveraging existing methods in the literature (Bargain et al., 2022), my findings suggest that standard resource shares, typically calculated for two-parent households, are invalid measures of individual well-being in single-parent households. Additionally, I demonstrate that family size significantly affects the resources allocated to children in low-income families, a less pronounced effect in wealthier families. These findings underscore the necessity of public intervention, such as family allowance policies, to ensure that children's needs are met, particularly in households with incomes below a critical threshold. This analysis provides new insights into the longstanding debate regarding the effectiveness of the UK's Two-Child Limit policy (Kearney, 2004, Milligan, 2005, Brewer et al., 2012, Cohen et al., 2013, Laroque and Salanié, 2014, González and Trommlerová, 2023).

The first chapter opens several promising avenues for future research. One key area for further exploration is the strong evidence supporting economies of scale in childcare. This aspect could be further investigated using the Dunbar et al. (2013) model, in which the demand function for child-specific goods can be estimated. For example, the sharing of children's clothing within households may influence cost structures.

Moreover, the model in the first chapter focuses on the decision-making proces of single parents, including those who are separated, widowed, or single by choice. However, co-parents who share childcare responsibilities are likely to engage in a bargaining process. Extending the proposed model to study the costs of children in all single-parent households, including co-parenting situations, would be a logical next step.

From a labor supply theory perspective, my contributions are rooted in two distinct models.

The first is a theoretical framework using equivalence scales adapted for labor supply. From a methodological standpoint, although the literature offers general functions to capture equivalence scales (Lewbel, 1989a,b, Blundell and Lewbel, 1991, Blackorby and Donaldson, 1994, Browning et al., 2013), this chapter introduces a general technological function that integrates both the financial and time-related costs of children, without relying on specific consumption data. The second contribution is a collective labor supply model that represents the decision-making proces of dual-earner couples regarding time use and consumption. Notably, this model challenges traditional economic assumptions (Gustafsson and Kjulin, 1994, Colombino, 2000, Apps and Rees, 2001) by accounting for the fact that childcare time is not perfectly substitutable with market work. This distinction allows us to separate "pure childcare" from "process benefits."

The final two chapters also contribute to the literature by recovering the full cost of raising children. Overall, our findings suggest that the full cost of children can be identified through the observation of labor supply. Additionally, the third chapter examines the value of parental time and calculates the full cost of children based on these estimates.

While the last two chapters make substantial contributions, they also highlight several avenues for future research. One promising direction is to extend our static framework into a dynamic, intertemporal model. While the current approach consistently captures immediate labor supply decisions and their associated costs, incorporating the long-term consequences—such as potential human capital depreciation—would offer a more comprehensive view of the total cost of raising children.

In the third chapter, we assume perfect coordination between parents in their allocation of childcare responsibilities. In practice, parental roles and coordination are often more complex, with joint or overlapping caregiving tasks potentially influencing both time costs and process benefits. Future research could expand on this model by considering the effects of joint childcare.

Lastly, the model focuses on dual-earner households. Future studies could adapt the framework to analyze different family structures, exploring how the monetary and time costs of childcare differ in households with non-working parents.

Bibliography

Adda, J., C. Dustmann, and K. Stevens (2017). The career costs of children. *Journal of Political Economy 125*(2), 293–337.

Agostinelli, F. and M. Wiswall (2016). Estimating the technology of children's skill formation. Technical report, National Bureau of Economic Research.

Angrist, J. D. and W. N. Evans (1998). Children and their parents' labor supply: Evidence from exogenous variation in family size. *American Economic Association 88*(3), 450–477.

Apps, P., J. Kabátek, R. Rees, and A. van Soest (2016). Labor supply heterogeneity and demand for child care of mothers with young children. *Empirical Economics 51*(4), 1641–1677.

Apps, P. and R. Rees (2001). Household production, full consumption and the costs of children. *Labour Economics 8*(6), 621–648.

Attanasio, O. P. and V. Lechene (2014). Efficient responses to targeted cash transfers. *Journal of political Economy 122*(1), 178–222.

Averett, S. L., H. E. Peters, and D. M. Waldman (1997). Tax credits, labor supply, and child care. *Review of Economics and Statistics 79*(1), 125–135.

Baker, M., J. Gruber, and K. Milligan (2008). Universal child care, maternal labor supply, and family well-being. *Journal of political Economy 116*(4), 709–745.

Banks, J., R. Blundell, and A. Lewbel (1997). Quadratic engel curves and consumer demand. *Review of Economics and statistics 79*(4), 527–539.

Bargain, O. and O. Donni (2012a). Expenditure on children: A rothbarth-type method consistent with scale economies and parents' bargaining. *European Economic Review 56*(4), 792–813.

Bargain, O. and O. Donni (2012b). Targeting and child poverty. *Social Choice and Welfare 39*, 783–808.

Bargain, O., O. Donni, and M. Gbakou (2010). The measurement of child costs: evidence from ireland. *The Economic and Social Review 41*(1), 1–20.

Bargain, O., O. Donni, and I. Hentati (2022). Resource sharing in households with children: A generalized model and empirical evidence from the uk. *Journal of the European Economic Association 20*(6), 2468–2496.

Bargain, O., G. Lacroix, and L. Tiberti (2022). Intrahousehold resource allocation and individual poverty: Assessing collective model predictions using direct evidence on sharing. *The Economic Journal 132*(643), 865–905.

Becker, G. (1960). An economic analysis of fertility. *Universities-National Bureau. Demographic and economic change in developed countries.*

Benjamin, D. (1992). Household composition, labor markets, and labor demand: testing for separation in agricultural household models. *Econometrica: Journal of the Econometric Society*, 287–322.

Bernal, R. (2008). The effect of maternal employment and child care on children's cognitive development. *International Economic Review 49*(4), 1173–1209.

Blackorby, C. and D. Donaldson (1993). Adult-equivalence scales and the economic implementation of interpersonal comparisons of well-being. *Social Choice and Welfare 10*, 335–361.

Blackorby, C. and D. Donaldson (1994). Measuring the cost of children: a theoretical framework. *The measurement of household welfare*, 51–69.

Blank, R. M. and R. F. Schoeni (2003). Changes in the distribution of children's family income over the 1990's. *American Economic Review 93*(2), 304–308.

Blau, D. M. and P. K. Robins (1988). Child-care costs and family labor supply. *The Review of Economics and Statistics*, 374–381.

Blau, F. D. and L. M. Kahn (2007). Changes in the labor supply behavior of married women: 1980–2000. *Journal of Labor economics 25*(3), 393–438.

Bloemen, H. G. (2010). An empirical model of collective household labour supply with non-participation. *The Economic Journal 120*(543), 183–214.

Blundell, R., P.-A. Chiappori, T. Magnac, and C. Meghir (2007). Collective labour supply: Heterogeneity and non-participation. *The Review of Economic Studies 74*(2), 417–445.

Blundell, R., P.-A. Chiappori, and C. Meghir (2005). Collective labor supply with children. *Journal of political Economy 113*(6), 1277–1306.

Blundell, R. and H. W. Hoynes (2004). Has ’in-work’ benefit reform helped the labor market? In *Seeking a Premier Economy: The Economic Effects of British Economic Reforms, 1980-2000*, pp. 411–460. University of Chicago Press.

Blundell, R. and A. Lewbel (1991). The information content of equivalence scales. *Journal of econometrics 50*(1-2), 49–68.

Blundell, R. and T. MaCurdy (1999). Labor supply: A review of alternative approaches. *Handbook of labor economics 3*, 1559–1695.

Bourguignon, F. (1999). The cost of children: may the collective approach to household behavior help? *Journal of Population Economics 12*(4), 503–521.

Bourguignon, F. and M. Browning (1991). Bourguignon f., chiappori pa (1992):“collective models of household behaviour”, european economic review 36, pp 355–364. *European Economic Review 36*, 355–364.

Bourguignon, F., M. Browning, and P.-A. Chiappori (2009). Efficient intra-household allocations and distribution factors: Implications and identification. *The Review of Economic Studies 76*(2), 503–528.

Bradbury, B. (1994). Measuring the cost of children. *Australian economic papers 33*(62), 120–138.

Bradbury, B. (2008). Time and the cost of children. *Review of Income and Wealth 54*(3), 305–323.

Brewer, M., A. Ratcliffe, and S. Dsmith (2012). Does welfare reform affect fertility? evidence from the uk. *Journal of Population Economics 25*, 245–266.

Brown, C. V., E. Levin, and D. Ulph (1976). Estimates of labour hours supplied by married male workers in great britain. *Scottish Journal of Political Economy 23*(3), 261–277.

Browning, M. (1992). Children and household economic behavior. *Journal of Economic Literature 30*(3), 1434–1475.

Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994). Income and outcomes: A structural model of intrahousehold allocation. *Journal of political Economy 102*(6), 1067–1096.

Browning, M., P.-A. Chiappori, and A. Lewbel (2013). Estimating consumption economies of scale, adult equivalence scales, and household bargaining power. *Review of Economic Studies 80*(4), 1267–1303.

Browning, M., P.-A. Chiappori, and Y. Weiss (2014). *Economics of the Family*. Cambridge University Press.

Bruins, M. (2017). Tanf, child care and child wellbeing in sole parent families. *University of Oxford mimeo*.

Budig, M. J. and P. England (2001). The wage penalty for motherhood. *American sociological review 66*(2), 204–225.

Cherchye, L., B. De Rock, A. Lewbel, and F. Vermeulen (2015). Sharing rule identification for general collective consumption models. *Econometrica 83*(5), 2001–2041.

Cherchye, L., B. De Rock, and F. Vermeulen (2015). A simple identification strategy for gary becker's time allocation model. *Economics letters 137*, 187–190.

Cherchye, L., B. D. Rock, and F. Vermeulen (2012). Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information. *American Economic Review 102*(7), 3377–3405.

Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica: Journal of the Econometric Society 56*(1), 63–90.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of political Economy 100*(3), 437–467.

Chiappori, P.-A. and I. Ekeland (2009). The microeconomics of efficient group behavior: Identification 1. *Econometrica 77*(3), 763–799.

Chiappori, P.-A., B. Fortin, and G. Lacroix (2002). Marriage market, divorce legislation, and household labor supply. *Journal of political Economy 110*(1), 37–72.

Cleveland, G., M. Gunderson, and D. Hyatt (1996). Child care costs and the employment decision of women: Canadian evidence. *Canadian Journal of Economics*, 132–151.

Cogan, J. F. (1981). Fixed costs and labor supply. *Econometrica 49*(4), 945–963.

Cohen, A., R. Dehejia, and D. Romanov (2013). Financial incentives and fertility. *Review of Economics and Statistics 95*(1), 1–20.

Colombino, U. (2000). The cost of children when children are a choice. *Labour 14*(1), 79–95.

Connelly, R. (1992). The effect of child care costs on married women's labor force participation. *The review of Economics and Statistics*, 83–90.

Cools, S., S. Markussen, and M. Strøm (2017). Children and careers: How family size affects parents' labor market outcomes in the long run. *Demography 54*(5), 1773–1793.

Cosaert, S. and V. Hennebel (2023). Parental childcare with process benefits. *Economica 90*(357), 339–371.

Cunha, F. and J. Heckman (2007). The technology of skill formation. *American economic review 97*(2), 31–47.

Cunha, F., J. J. Heckman, and S. M. Schennach (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica 78*(3), 883–931.

Dauphin, A., A.-R. El Lahga, B. Fortin, and G. Lacroix (2011). Are children decision-makers within the household? *The Economic Journal 121*(553), 871–903.

Deaton, A. S. and J. Muellbauer (1986). On measuring child costs: With applications to poor countries. *Journal of Political Economy 94*(4), 720–744.

Dechter, E. K. (2014). Maternity leave, effort allocation, and postmotherhood earnings. *Journal of Human Capital 8*(2), 97–125.

Del Boca, D., C. Flinn, and M. Wiswall (2014). Household choices and child development. *Review of Economic Studies 81*(1), 137–185.

Del Boca, D., C. Flinn, and M. Wiswall (2016). Transfers to households with children and child development. *The Economic Journal 126*(596), F136–F183.

DeLeire, T. C., H. Levy, et al. (2005). *The Material Well-Being of Single Mother Households in the 1980s and 1990s: What Can We Learn from Food Spending?* Irving B. Harris Graduate School of Public Policy Studies, University of Chicago.

Doiron, D. and G. Kalb (2005). Demands for child care and household labour supply in australia. *Economic Record 81*(254), 215–236.

Donni, O. (2007). Collective female labour supply: Theory and application. *The Economic Journal 117*(516), 94–119.

Donni, O. (2008). Labor supply, home production, and welfare comparisons. *Journal of Public Economics 92*(7), 1720–1737.

Donni, O. (2009). A simple approach to investigate intrahousehold allocation of private and public goods. *The Review of Economics and Statistics 91*(3), 617–628.

Donni, O. and P.-A. Chiappori (2011). Nonunitary models of household behavior: a survey of the literature. *Household economic behaviors*, 1–40.

Donni, O. and E. Matteazzi (2012). On the importance of household production in collective models: evidence from us data. *Annals of Economics and Statistics/ANNALES D'ÉCONOMIE ET DE STATISTIQUE*, 99–125.

Donni, O. and E. Matteazzi (2018). Collective decisions, household production, and labor force participation. *Journal of Applied Econometrics 33*(7), 1064–1080.

Donni, O. and A. Vil (2024a). The household demand for leisure, the price of time and the full cost of children: A structural model and evidence from the psid. *ThEMA mimeo.*

Donni, O. and A. Vil (2024b). How to incorporate children into labor supply? a equivalence scale-based approach. *ThEMA mimeo.*

Dunbar, G. R., A. Lewbel, and K. Pendakur (2013). Children's resources in collective households: identification, estimation, and an application to child poverty in malawi. *American Economic Review 103*(1), 438–471.

Dunbar, G. R., A. Lewbel, and K. Pendakur (2021). Identification of random resource shares in collective households without preference similarity restrictions. *Journal of Business & Economic Statistics 39*(2), 402–421.

Edin, K. and L. Lein (1997). *Making ends meet: How single mothers survive welfare and low-wage work.* Russell Sage Foundation.

Fernandez-Val, I. (2003). Household labor supply: evidence for spain. *investigaciones económicas 27*(2), 239–275.

Folbre, N. (1994). Children as public goods. *The American Economic Review 84*(2), 86–90.

Folbre, N. (2008). *Valuing children: Rethinking the economics of the family.* Harvard University Press.

Glauber, R. (2018). Trends in the motherhood wage penalty and fatherhood wage premium for low, middle, and high earners. *Demography 55*(5), 1663–1680.

González, L. and S. K. Trommlerová (2023). Cash transfers and fertility: How the introduction and cancellation of a child benefit affected births and abortions. *Journal of Human Resources 58*(3), 783–818.

Goodin, R. E., J. M. Rice, A. Parpo, and L. Eriksson (2008). Discretionary time: A new measure of freedom.

Gørtz, M. (2011). Home production–enjoying the process or the product? *Electronic International Journal of Time Use Research 8*(1).

Graham, J. W. and C. A. Green (1984). Estimating the parameters of a household production function with joint products. *The Review of Economics and Statistics*, 277–282.

Grogger, J. (2001). The effects of time limits and other policy changes on welfare use, work, and income among female-headed families. nber working paper no. 8153. *National Bureau of Economic Research.*

Gronau, R. (1977). Leisure, home production, and work–the theory of the allocation of time revisited. *Journal of political economy 85*(6), 1099–1123.

Gustafsson, B. and U. Kjulin (1994). Time use in child care and housework and the total cost of children. *Journal of Population Economics 7*(3), 287–306.

Hallberg, D. and A. Klevmarken (2003). Time for children: A study of parent's time allocation. *Journal of Population Economics 16*, 205–226.

Heckman, J. (1974). Shadow prices, market wages, and labor supply. *Econometrica: journal of the econometric society*, 679–694.

Heckman, J. and P. Carneiro (2003). Human capital policy. nber. *Working Paper Series* (9495).

Heckman, J. J. and D. V. Masterov (2007). The productivity argument for investing in young children. *Review of Agricultural Economics 29*(3), 446–493.

Heckman, J. J., S. H. Moon, R. Pinto, P. A. Savelyev, and A. Yavitz (2010). The rate of return to the highscope perry preschool program. *Journal of public Economics 94*(1-2), 114–128.

Hurd, M. D. and J. H. Pencavel (1981). A utility-based analysis of the wage subsidy program. *Journal of Public Economics 15*(2), 185–201.

Juster, F. T. and F. P. Stafford (1991). The allocation of time: Empirical findings, behavioral models, and problems of measurement. *Journal of Economic literature 29*(2), 471–522.

Kearney, M. S. (2004). Is there an effect of incremental welfare benefits on fertility behavior?: A look at the family cap. *Journal of Human Resources 39*(2), 295–325.

Kearney, M. S. (2023). *The Two-Parent Privilege: How the decline in marriage has increased inequality and lowered social mobility, and what we can do about it.* Swift Press.

Keen, M. (1986). Zero expenditures and the estimation of engel curves. *Journal of Applied Econometrics 1*(3), 277–286.

Kerkhofs, M. and P. Kooreman (2003). Identification and estimation of a class of household production models. *Journal of Applied Econometrics 18*(3), 337–369.

Korenman, S. and D. Neumark (1990). Marriage, motherhood, and wages.

Kornstad, T. and T. O. Thoresen (2007). A discrete choice model for labor supply and childcare. *Journal of Population economics 20*, 781–803.

Koulovatianos, C., C. Schrder, and U. Schmidt (2009). Nonmarket household time and the cost of children. *Journal of Business & Economic Statistics 27*(1), 42–51.

Laroque, G. and B. Salanié (2014). Identifying the response of fertility to financial incentives. *Journal of Applied Econometrics 29*(2), 314–332.

Latimer, E. and T. Waters (2024). The two-child limit: poverty, incentives and cost [comment]. *Institute for Fiscal Studies*.

Lechene, V., K. Pendakur, and A. Wolf (2022). Ordinary least squares estimation of the intrahousehold distribution of expenditure. *Journal of Political Economy 130*(3), 681–731.

Lewbel, A. (1989a). Household equivalence scales and welfare comparisons. *Journal of Public Economics 39*(3), 377–391.

Lewbel, A. (1989b). Identification and estimation of equivalence scales under weak separability. *The Review of Economic Studies 56*(2), 311–316.

Lewbel, A. (1999). Consumer demand systems and household equivalence scales. *Handbook of Applied Econometrics Volume 2: Microeconomics*, 155–185.

Lewbel, A. and K. Pendakur (2008). Estimation of collective household models with engel curves. *Journal of econometrics 147*(2), 350–358.

Lino, M., K. Kuczynski, N. Rodriguez, and T. Schap (2017). Expenditures on children by families, 2015. Technical report.

Lise, J. and S. Seitz (2011). Consumption inequality and intra-household allocations. *The Review of Economic Studies 78*(1), 328–355.

Lundberg, S. and E. Rose (2000). Parenthood and the earnings of married men and women. *Labour Economics 7*(6), 689–710.

Menon, M., K. Pendakur, and F. Perali (2012). On the expenditure-dependence of children's resource shares. *Economics Letters 117*(3), 739–742.

Meyer, B. D. and D. T. Rosenbaum (2000). Making single mothers work: Recent tax and welfare policy and its effects. *National Tax Journal 53*(4), 1027–1061.

Meyer, B. D. and D. T. Rosenbaum (2001). Welfare, the earned income tax credit, and the labor supply of single mothers. *The quarterly journal of economics 116*(3), 1063–1114.

Meyer, B. D. and J. X. Sullivan (2004). The effects of welfare and tax reform: the material well-being of single mothers in the 1980s and 1990s. *Journal of public economics 88*(7-8), 1387–1420.

Meyer, B. D. and J. X. Sullivan (2008). Changes in the consumption, income, and well-being of single mother headed families. *American Economic Review 98*(5), 2221–2241.

Michalopoulos, C., P. K. Robins, and I. Garfinkel (1992). A structural model of labor supply and child care demand. *Journal of Human Resources*, 166–203.

Michaud, P.-C. and K. Tatsiramos (2011). Fertility and female employment dynamics in europe: the effect of using alternative econometric modeling assumptions. *Journal of Applied Econometrics 26*(4), 641–668.

Milligan, K. (2005). Subsidizing the stork: New evidence on tax incentives and fertility. *Review of Economics and statistics 87*(3), 539–555.

Mroz, T. A. (1987). The sensitivity of an empirical model of married women's hours of work to economic and statistical assumptions. *Econometrica 55*(4), 765–799.

Nakamura, A. and M. Nakamura (1992). The econometrics of female labor supply and children. *Econometric Reviews 11*(1), 1–71.

Nelson, J. A. (1993). Household equivalence scales: Theory versus policy? *Journal of Labor Economics 11*(3), 471–493.

Nicholson, J. L. (1976). Appraisal of different methods of estimating equivalence scales and their results. *Review of Income and Wealth 22*(1), 1–11.

Nieuwenhuis, R. and L. Maldonado (2018). *The triple bind of single-parent families: Resources, employment and policies to improve well-being.* Policy Press.

Penglase, J. (2021). Consumption inequality among children: Evidence from child fostering in malawi. *The Economic Journal 131*(634), 1000–1025.

Powell, L. M. (1997). The impact of child care costs on the labour supply of married mothers: Evidence from canada. *Canadian Journal of Economics*, 577–594.

Prais, S. J. and H. S. Houthakker (1971). *The analysis of family budgets*, Volume 4. CUP Archive.

Ribar, D. C. (1992). Child care and the labor supply of married women: Reduced form evidence. *Journal of human resources*, 134–165.

Ribar, D. C. (1995). A structural model of child care and the labor supply of married women. *Journal of labor Economics 13*(3), 558–597.

Schoeni, R. F. and R. M. Blank (2000). What has welfare reform accomplished? impacts on welfare participation, employment, income, poverty, and family structure. nber working paper no. 7627.

Train, K. E. (2009). *Discrete choice methods with simulation.* Cambridge university press.

Vermeulen, F. (2002). Collective household models: principles and main results. *Journal of Economic Surveys 16*(4), 533–564.

Waldfogel, J. (1998). The family gap for young women in the united states and britain: can maternity leave make a difference? *Journal of labor economics 16*(3), 505–545.

Watts, H. W. (1967). The iso-prop index: an approach to the determination of differential poverty income thresholds. *Journal of human resources*, 3–18.

Weiss, Y. and R. J. Willis (1985). Children as collective goods and divorce settlements. *Journal of Labor Economics 3*(3), 268–292.

Whiteford, P. (1985). *A Family's Needs: Equivalence Scales, Poverty, and Social Security.* Development Division, Department of Social Security.

Winship, S. and C. Jencks (2004). How did welfare reform affect material hardship among single mothers? evidence from the food security survey. *Unpublished Paper, Harvard University.*