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MONETARY POLICY, THE TERM STRUCTURE OF INTEREST RATES AND THE MACROECONOMY

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For my sister

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Abstract

This Ph.D. thesis was written with the ambition to better understand the role of interest rates as an instrument influencing the economy for the central bank. It contains three chapters focusing on the Euro Area, Japan, and the U.S. economy. The first chapter of this thesis analyzes a particular transmission channel of the sovereign debt purchase programme of the European Central Bank. It sheds light on the importance of the reward for interest rates uncertainty in transmitting this unconventional monetary policy to the economy. The second chapter of this Ph.D. thesis investigates the low growth - low inflation environment present in Japan since the 1990s, using observed and equilibrium interest rates. It shows that the different monetary policy regimes implemented by the Bank of Japan did not have an homogeneous impact on interest rates and that they were not sufficient to revive the Japanese economy. Finally, a third chapter demonstrates that the relationship between inflation and economic activity in the U.S., the Phillips curve, is not dead as opposed to the current common thinking. This chapter finds evidence that the apparent disappearance of the relationship is due to a more active U.S. Federal Reserve.

Summary

Entitled *Monetary policy, the term structure of interest rates and the macroeconomy*, this Ph.D. thesis was written with the ambition to better understand the role of interest rates as a monetary policy instrument driving the economy for the central bank. It contains three chapters in the field of Monetary Policy and Macro-Finance, focusing on the Euro Area, Japan, and the U.S. economy.

The first chapter of this thesis is entitled *Quantitative Easing and the term premium channel in the Euro Area.* This chapter analyzes a particular transmission channel of the sovereign bonds purchase programme of the European Central Bank, focusing on the impact on aggregated Euro Area macroeconomic variables. Especially, this empirical research paper sheds light on the importance of the bond term premium to transmit the effects of the QE programme. It gives a stance as to the relative importance of its channel compared to the overall yield channel. To do so, it combines state-of-arts techniques of term structure modeling and Bayesian time series analysis.

The second chapter of this Ph.D. thesis, written with two co-authors, investigates the low growth - low inflation environment present in Japan since the 1990s, through the yield curve gap. Entitled *Potential growth and the natural yield curve in Japan*, the research paper extends the concept of (short-term) natural rate of interest to medium and long-term maturities, so as to estimate the natural yield curve. By comparing it to the real sovereign yield curve, it explains the Japanese output and inflation gaps using a dynamic term structure model coupled with a semi-structural macroeconomic model. In a nutshell, this chapter shows that the different monetary policy regimes implemented by the Bank of Japan did not have an homogeneous impact on interest rates at all maturities, and that they were not sufficient to significantly revive the Japanese economy.

Finally, a third chapter shows that the U.S. price Phillips curve is not dead, as opposed to the current common thinking. Using a combination of New-Keynesian theory and VAR analysis, this chapter shows that the slope of the price Phillips curve – the structural relationship between price inflation and measures of real economic activity - is not flat, once filtered from all supply shocks, and not only cost-push shocks. The research paper also finds evidence that the apparent flattening of the curve is due to the so-called policy hypothesis, that the U.S. Federal Reserve has become a stricter inflation targeter. Written with two co-authors, this chapter is entitled *Inflation-output gap correlation and the slope of the New Keynesian Phillips curve.*

General Introduction

Interest rates are at the center of macroeconomic policy-making. Interest rates influence the economy, and the economy influences interest rates. Instrument at the service of the central bank, the policy rate shapes investment-saving decisions, which drives the economy's production. It is at the foundation of what is usually referred to as the *term structure of interest rates*: the observed investment-saving intersections observed for different horizons.

Until the 1990s, monetary policy was mostly about money supply and the policy rate. In the early 2000s in Japan, and after the Great Financial Crisis of 2008 in the U.S. and Euro Area, central banks started to implement what is called unconventional monetary policy: a kind of policy with other instruments than the policy rate. Reasons for that were the zero-lower bound constraint on nominal short-term rates, a will to act on the long-end of the yield curve, or a desire to absorb detrimental conditions in specific markets. Pre-empting Academics, such a shift in monetary policy-making lead to an abundance of research to try to better understand the new environment we were facing. This Ph.D. thesis is in line with this literature.

This thesis investigates three research questions about three different countries, that may seem disconnected at first sight. However, there is a key question acting as a guiding principle for the three chapters: how does monetary policy influence the macroeconomy through interest rates?

The first chapter of this thesis explores a particular transmission channel of the European Central Bank's large-scale asset purchase programme to the Euro Area economy. It looks into the little known term premium transmission channel of Quantitative Easing (QE), a type of unconventional monetary policy introduced in 2015 in the aftermaths of the Great Financial Crisis and the Eurozone sovereign debt crisis. The chapter analyzes the impact of a decrease in the government bond reward for risk, a component of long-term interest rates, following a bonds purchase by the central bank. Little theory is known about the transmission of unconventional monetary policy through the term premium. Current New-Keynesian models are unfortunately not flexible enough to simulate the impact of a QE policy through an endogenous yield curve. Still, some papers have identified the mechanisms through which QE affects long-term yields: it reduces the supply of risk available to market participants and forces them to re-balance their portfolio according to their risk profiles. This decrease in the long-end of the yield curve contributes to shaping investment-saving decisions at long horizons, thus affecting current economic conditions. Using advanced term structure modeling and time-series analysis, this first chapter captures the impact of the term premium channel of QE on real GDP and consumer prices. If its expansionary effects are quite similar to the ones of the overall impact on long-term yields, a historical analysis shows that the term premium channel of QE was actually not easing financial conditions enough to contribute positively to Euro Area consumer prices.

The second chapter of this thesis studies the investment-saving equilibrium rates in Japan through the different monetary policy regimes implemented by the Bank of Japan since the 1990s. The chapter extends the concept of short-term natural rate of interest to all maturities, as the rates for which investment equals savings at any horizon. It considers the mismatch between observed real rates and natural rates, the yield curve gap, in the perspective of the Japanese output and inflation gaps. The country has indeed experienced a *Lost Decade* (1991-2001), followed by a particularly active monetary policy period during which the central bank tried to revive an anemic economic growth and a subdued inflation. Thanks to a term structure model coupled with a semi-structural macroeconomic model, the chapter is able to capture the interactions between the yield curve gap, the output gap, and inflation, in an economy experiencing a prolonged period of recession and low levels of inflation. It makes several contributions to the modeling of the natural yield curve with macroeconomic fundamentals. It shows that the different monetary policy regimes implemented by the Bank of Japan did not have an homogeneous impact on interest rates at all maturities, and that they were not sufficient to significantly revive the Japanese economy despite the loose monetary conditions.

Finally, the third chapter of this Ph.D. thesis resuscitates the U.S. Price Phillips curve. The structural relationship between price inflation and measures of real economic activity has indeed recently raised people's attention, as many economists believe it dead. The slope of the Phillips curve has become flat in appearance, and inflation now seems to respond only modestly to real activity. Using a combination of New-Keynesian theory and time-series analysis, this chapter shows that the slope of the curve has not flattened much, once controlled for all supply shocks. The chapter argues that single equation estimates of the slope based on aggregate data are biased if supply shocks are important, as suggested by standard New Keynesian theory. A cleaned slope of the U.S. price Phillips curve is thus estimated and compared through different sub-samples, denying the apparent flattening of the inflation-output gap relation. Actually, results tend to show that the observed flattening is due to a more aggresive U.S. Federal Reserve in absorbing inflationary shocks. To a lesser extent, the chapter also finds evidence that the correlation between inflation and real economic activity declined because the relative importance of demand and supply shocks has changed.

How the central bank influences the economy through interest rates is a large question that would require more than a Ph.D. thesis to be addressed. Nevertheless, my hope is that these three chapters will contribute to improve our understanding of the interactions between interest rates and macroeconomic variables. Taking the cases of the Euro Area, Japan, and the U.S., I wish to shed light on the prominent role of the central banker in modern monetary policy-making.

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CHAPTER I

Quantitative Easing and the term premium channel in the Euro Area*

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Abstract

Long-term yields can be broken down into two components: a risk-neutral rate that embodies interest rate expectations, and a term premium that rewards investors for risk. The transmission of Quantitative Easing through this term premium is here analyzed for the asset purchase programme of the European Central Bank. The Euro Area sovereign bond term premium is first extracted from a term structure model to quantify the transmission of QE to the yield curve. It is then plugged into a Structural VAR with Euro Area macroeconomic variables to identify the transmission to the Economy. This SVAR model, in which shocks are identified using sign restrictions, provides a clear view about the importance of the little known term premium channel of QE.

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1 Introduction

The European Central Bank (ECB) introduced its first large-scale asset purchase programme in January 2015. Also called Quantitative Easing (QE), this programme mostly consisted in the purchase of a large amount of government bonds on the secondary market through the Public Sector Purchase Programme (PSPP). With its policy rates constrained by their effective lower bounds, the ECB injected a considerable amount of liquidity to act on medium to long-term yields.

A lot of papers in the literature have attempted to explain the different transmission channels of QE in the past ten years. A clear theoretical characterization is, for example, provided by Krishnamurthy & Vissing-Jorgensen (2011). This paper contributes to this literature by analyzing the term premium transmission channel of QE in the Euro Area. That is, the impact of the ECB asset purchase programme through the decrease in sovereign bond term premia.

Long-term yields can be broken down into two components: a risk-neutral rate that embodies interest rate expectations, and a term premium that rewards investors for risk. This bond term premium is a compensation given to investors to hold long-term bonds instead of rolling-over short-term risk-free bonds. It is a reward for interest rate risk (uncertainty about future interest rates), liquidity risk (uncertainty about the ability to exchange the bond on the market) and default risk (uncertainty about the solvency of the counterparty). Unobserved, the term premium is usually defined as the residual matching the difference between an interest rate and its associated risk-neutral rate, or interest rate expectations. The existence of such risk premium in the data is in contradiction of the *Pure Expectation hypothesis*, at stake in most linearized New-Keynesian models. Under this hypothesis, long-term yields are only explained by interest rate expectations and the term premium is nonexistent. Thus, in that framework both asset purchase and forward guidance can only drive interest rate expectations. However, there is a lot of empirical evidence towards time-varying and maturity-specific term premia in the sovereign bond market (Cohen, Hördahl & Xia (2018)). The need to depart from the *Pure Expectation hypothesis* to analyze the impact of unconventional monetary policy on medium to long-term yield is therefore necessary.

The term premium channel of QE works through the so-called portfolio-balance mechanism, though which investors re-balance the composition of their portfolio after selling government bonds to the central bank. Assuming that some investors have certain preferred-habitat demands (Vayanos & Vila (2009)), they are only willing to invest in a specific maturity of assets and therefore replace the bonds sold by other assets of equivalent maturity. Assuming that some other investors have different appetites for risk (Hamilton & Wu (2010)), they re-balance their portfolio with assets of equivalent risk profiles. Mix of the two, this portfolio-balance mechanism leads to a global drop in interest rates through a compression in term premia. This contraction in term premia is equivalent to a duration extraction by the central bank, that reduces the supply of risk available on the market by buying long-term risk-bearing bonds. This results in a stimulation of aggregate demand for goods and services, leading to upward pressures on consumer prices.

Little known, this term premium channel of QE is complementary to the so-called signaling channel of monetary policy, through which central bank communication and forward guidance operate. If the term premium channel of QE can only directly affect the term premium component of medium to long-term yields (the short-term rate is risk-free by definition), the signaling channel of QE refers to the communication of the central bank and can act on both the term premium and the risk-neutral rate. Isolating the term premium channel of QE therefore requires the estimation of the bond term premium and the identification of the shocks related to QE driving the risk premium.

There are different ways to estimate interest rate expectations to extract the term premium from observed yields (forecasts, surveys, models). Among them, term structure models provide plausible arbitrage-free estimations. However, not flexible enough to identify shocks, these term structure models alone often say little about the structural relationships between the term premium and macroeconomic variables. Quite the contrary, Structural VAR (SVAR) studies are well appropriate to estimate structural relations, but they are not suited to capture accurate bond term premia.[1] In this paper, I reconcile the two empirical strands of the literature by combining the use of a cutting-edge term structure model with an advanced VAR setting to analyze the term premium channel of QE to both the yield curve and aggregate macroeconomic variables.

I proceed in two steps. First, I extract the 10-year Euro Area nominal term premium from a shadow rate term structure model based on Wu & Xia (2016). Accounting for the zero-lower bound constraint in the pricing of short-term yields, this term structure model has become a popular tractable way to fit the yield curve. To make sure that the expected short-term rate matches market participants' perceptions, I anchor interest rate expectations with data from surveys, following Kim & Wright (2005) and Geiger & Schupp (2018). Estimated via maximum likelihood within a state-space model, this term structure model enables me to analyze the impact of all announcements related to the Public Sector Purchase Programme of the ECB on the term premium at a daily and monthly frequency. Second, I plug the term premium into a monthly Structural VAR including a series of amount of bonds to be purchased by the ECB, real equity prices, core consumer prices and real GDP. I identify a term premium QE shock thanks to zero and sign restrictions (Arias, Rubio-Ramirez & Waggoner (2018)), and Narrative sign restrictions (Antolín-Díaz & Rubio-Ramírez (2018)). Estimated with Bayesian methods, this SVAR model enables me to analyze how a term premium QE shock differs from a conventional yield QE shock. Using calibrated Impulse Response Functions (IRFs) and Historical Decompositions (HD), I am able to give a stance as to how the term premium channel of QE played a role in the Euro Area.

My findings are threefold. First, an asset purchase of one-percent of nominal GDP decreases both the 10-year term premium and the 10Y yield by 0.1%. I therefore find a strong evidence towards a transmission of QE through the portfolio-balance mechanism, as opposed to the signaling channel. However, this drop in the term premium is very transitory and goes reverse after seven months, confirming the importance of the flow effect of QE as opposed to the stock effect. Second, the one-percent purchase raises real equity prices by +0.48% on impact, but core consumer prices and real GDP only increase by +0.02% and +0.09% after 12 months. This quick reversal effect in the term premium following a purchase of one-percent of nominal GDP does not seem to be sufficient for aggregate demand to be stimulated enough. Third, if the contribution of the term premium channel of QE in driving real output fastly grew following the QE announcement, it only became positive after 6 quarters in mid-2016. From this point on, the contribution to consumer prices started growing, although it remained negative throughout the programme. It therefore seems that the transitory impact of QE through the term premium channel did not stimulate aggregate

[1] Conditional expectations of the short-term rate derived from a VAR model do not respect the arbitrage-free condition and can lead to very implausible results.

demand enough for both real GDP and consumer prices to significantly increase.

This paper is related to financial studies on the impact of QE on asset prices in the Euro Area: Altavilla, Carboni & Motto (2015), Andrade et al. (2016), and Georgiadis & Gräb (2016) for event studies, Koijen et al. (2016), De Santis (2016), and Dedola et al. (2017) for time-series analysis, and Eser et al. (2019) for a yield curve modeling approach. This study is also linked to research on the impact of QE on macroeconomic aggregates: Garcia Pascual & Wieladek (2016), Gambetti & Musso (2017) for a time-series approach, and Andrade et al. (2016), Mouabbi & Sahuc (2018), and Darracq Pariès & Papadopoulou (2019) for the theory. However, these paper either focus on the impact of QE on the yield curve or on the overall macroeconomic impact of asset purchse programmes. The only paper that identifies a term premium QE shock is Blattner & Joyce (2016), to the best of my knowledge. Focusing on the macroeconomic implications of a net debt supply shock, they nonetheless provide little analysis on the term premium channel of QE. Some other papers went close to it by either analyzing the impact of a term spread QE shock (Kapetanios, et al. (2012), Baumeister & Benati (2013)), or by using interest rate futures as a proxy for interest rate expectations (Weale & Wieladek (2016)).[2] In addition, current New-Keynesian theory also fails in featuring sizable time-varying endogenous term premia.[3] Thus, tricks are also often used, such as labeling transaction costs or market frictions as a term premium (Chen, Cúrdia & Ferrero (2012), Kiley (2014), Carlstrom, Fuerst & Paustian (2017)). Rarely in the macroeconomic literature have authors been able to capture bond premia originating from interest rate, liquidity and default risk. An exception might be Ireland (2015) that builds an affine term structure model with observable and unobservable macroeconomic factors. However, no link is made with QE in this paper. My paper intends to fill this gap in the literature.

The rest of the paper is organized as follows: **Section 2** extracts a bond term premium and analyzes the impact of QE-related announcements on this series. **Section 3** investigates in the transmission of the term premium channel of QE to the Economy, and **Section 4** concludes.

2 Bond term premium series extraction

The bond term premium is an unobserved component that is a reward for risk. It is a residual, that explains the gap between the observed yield and risk-free short-term interest rate expectations.

2.1 Definition of the term premium

I follow the literature in defining the term premium on an n-period zero-coupon bond $\kappa_{n,t}$ as the difference between the n-period zero-coupon yield $y_{n,t}$ and the risk-neutral rate $e_{n,t}$ under the physical probability measure $\mathbb{P}$:[4]

$$\kappa_{n,t} = y_{n,t} - e_{n,t} \tag{1}$$

[2] This cannot, however, distinguish clearly the term premium from the risk-neutral rate, as both the term spread and futures embed a term premium and an interest rate expectations component.

[3] Linearized models do not feature any term premium; a second-order approximation leads to a constant term premium. Only a third-order approximation gives a time-varying (although often small) term premium.

[4] Other definitions of the term premium can be found in Cochrane & Piazzesi (2008).

where $e_{n,t}$ is the risk-neutral rate, with:

$$e_{n,t} = \frac{1}{n} \sum_{j=0}^{n-1} E_t^{\mathbb{P}}(r_{t+j}) \tag{2}$$

That is, this term premium $\kappa_{n,t}$ is the difference in expected return between a buy and hold strategy for an n-period zero-coupon bond and an instantaneous rollover strategy at the risk-free rate. It is a reward for interest rate risk (uncertainty about future interest rates). As the data employed are synthetic Euro Area implied yields of all ratings, the term premium here also encompasses a potential compensation for liquidity risk (uncertainty about the ability to exchange the bond on the market), and default risk (uncertainty about the solvency of the counterparty). Such risks were not negligible for periphery countries during the Euro Area sovereign debt crisis. On the contrary, the risk-neutral rate $e_{n,t}$ is an average of the expected short-term rate at different horizons. It is a proxy for expectations about monetary policy.

To ensure that this risk-neutral rate matches the perceptions of market participants, I anchor interest rate expectations with data from surveys[5] as in Kim & Wright (2005) and Geiger & Schupp (2018):

$$r_{t+j}^{surveys} = E_t^{\mathbb{P}}(r_{t+j}) + \Sigma_{\varsigma}\varsigma_t \tag{3}$$

with $\varsigma_t \sim \mathcal{MVN}(0, I_J)$ a vector of observation errors.

Note that assuming $E_t^{\mathbb{P}}(r_{t+j}) \approx f_{j,j+1,t}^{\circ}$ with $f_{j,j+1,t}^{\circ}$ the observed one-period forward rate, implies that $\kappa_{n,t} = 0 \ \forall \ n$.[6] That is, proxying the expected short-term rate with forwards is inconsistent with having a non-zero term premium. This is simply because forwards are not pure expectations but also embed a term premium. Thus, such an approximation would not allow the term premium channel of QE to be isolated, which is why a term structure model is needed to obtain estimates of a non-zero term premium.

2.2 Shadow rate term structure model

In the Eurozone, QE was implemented at the effective lower bound, when there was no more scope for conventional monetary policy. In January 2015, the deposit facility rate was indeed at -0.20%, leaving little room for further rate cuts. Consequently, a term structure model employed to recover the yield curve during this period should account for the non-symmetric probability distribution in the pricing of the short-term rate near the effective lower bound. To do so, I introduce an arbitrage-free discrete-time Shadow Rate Term Structure Model (SRTSM) based on Wu & Xia (2016). In this class of models, the observed short-term rate is the maximum of the shadow rate and a lower bound, so that the short-term rate is bounded while the shadow rate can evolve freely into negative territory.

The (nominal) short-term interest rate r_t is defined as the maximum between the shadow rate s_t and a lower bound $\underline{r}_t$:

$$r_t = max(s_t, \underline{r}_t) \tag{4}$$

where the shadow rate is an affine function of a state vector X_t:

$$s_t = a_0 + b_0' X_t \tag{5}$$

[5] I use surveys of the 3-month Euribor three months ahead, that I adjust for spread with the Euro Area implied yield curve.

[6] The sum of forward premia is equal to the yield premium.

with $a_0 \in \mathbb{R}$ and $b_0 \in \mathbb{R}^N$ coefficients, and $X_t \in \mathbb{R}^N$ estimated through the Kalman filter.

The lower bound $\underline{r}_t$ is described by:

$$\underline{r}_t = min(\underline{r}_{t-1}, 0, Y_t^\circ, r_{t+1}^{surveys}) \tag{6}$$

with $\underline{r}_0 = 0$, Y_t° the vector of observed yields at time t and $r_{t+1}^{surveys}$ the one-period ahead expected short-term interest rate from surveys. As opposed to Wu & Xia (2016) and in the spirit of Lemke & Vladu (2017) and Geiger & Schupp (2018), the lower bound is not constant but time-varying. Equation (6) indeed implies that the lower bound is a decreasing function (i.e, it cannot go up). This better reflects the change in the perception of the lower bound by market participants over the period 2012-2019.

Standard in the term structure literature, the transition equation that governs the state vector under the physical probability measure $\mathbb{P}$ and the risk-neutral probability measure $\mathbb{Q}$ is:

$$X_t = \mu^{\mathbb{P},\mathbb{Q}} + \Theta^{\mathbb{P},\mathbb{Q}} X_{t-1} + \Sigma_\varepsilon \varepsilon_t \tag{7}$$

where $X_t \in \mathbb{R}^N$, $\mu^{\mathbb{P},\mathbb{Q}} \in \mathbb{R}^N$, $\Theta^{\mathbb{P},\mathbb{Q}} \in \mathcal{M}_{N\times N}(\mathbb{R})$, $\Sigma_\varepsilon \in \mathcal{M}_{N\times N}(\mathbb{R})$, and $\varepsilon_t \overset{iid}{\sim} \mathcal{MVN}(0, I_N)$.

In addition, parameters linking the risk-neutral and physical dynamics $\mathbb{Q}$ and $\mathbb{P}$ are related as follows:

$$\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \lambda_0 \tag{8}$$

$$\Theta^{\mathbb{Q}} = \Theta^{\mathbb{P}} - \lambda_1 \tag{9}$$

where $\lambda_0 \in \mathbb{R}^N$ and $\lambda_1 \in \mathcal{M}_{N\times N}(\mathbb{R})$.

Note that the market prices of risk parameters λ_0 and λ_1 are here left unrestricted. Imposing a restriction on these parameters will impact estimates of the term premium. Setting λ_0 and λ_1 at zero leads the risk-neutral and physical measures $\mathbb{Q}$ and $\mathbb{P}$ to coincide and the term premium to be null. This parametrization falls under the *strong form* of the *Expectation hypothesis*, i.e. investors require no term premium to hold long-term bonds. In the so-called Vasicek (1977) model, $\lambda_0 \neq 0$ and $\lambda_1 = 0$, so that the term premium is non-trivial but time-invariant and the model falls under the *weak form* of the *Expectation hypothesis*. On the other hand, my parametrization leads to a non-zero time-varying market price of risk and term premium, thus departing from the *Expectation hypothesis*, consistent with empirical findings (Cohen, Hördahl & Xia (2018)).

In this framework, bonds are priced recursively and the form of the n-period (nominal) zero-coupon bond yield $y_{n,t}$ under the risk-neutral probability measure $\mathbb{Q}$ is:

$$y_{n,t} = \frac{1}{n}\sum_{j=0}^{n-1} f_{j,j+1,t}$$

where

$$f_{n,n+1,t} \approx \underline{r}_t + \sigma_{s,n}^{\mathbb{Q}} \times g\left(\frac{a_n + b_n' X_t - \underline{r}_t}{\sigma_{s,n}^{\mathbb{Q}}}\right) \tag{10}$$

with $f_{n,n+1,t}$ the forward rate for a loan starting at date $t+n$ and maturing at date $t+n+1$, $g(x) = x\Phi(x) + \phi(x)$ with $\Phi(x)$ the cumulative distribution function of x and $\phi(x)$ its probability density function. Besides, recursive coefficients $a_n \in \mathbb{R}$, $b_n \in \mathbb{R}$, and $\sigma_{s,n}^{\mathbb{Q}} \in \mathbb{R}$ are defined in **Appendix A**. A complete derivation of the model is presented in **Appendix A** of Wu & Xia (2016).

Thus, the measurement equation giving the relation between the observed zero-coupon bond yields Y_t° and the model-implied zero-coupon bond yields Y_t is:

$$Y_t^\circ = Y_t + \Sigma_\varsigma \varsigma_t \tag{11}$$

where $Y_t^\circ = (y_{1,t}^\circ, ..., y_{n,t}^\circ,)'$, $Y_t = (y_{1,t}, ..., y_{n,t})'$ and $\varsigma_t = (\varsigma_{1,t}, ..., \varsigma_{n,t})'$ with $\varsigma_t \sim \mathcal{MVN}(0, I_J)$ the vector of observation errors[7] and $\Sigma_\varsigma \in \mathcal{M}_{J\times J}(\mathbb{R})$ with $\Sigma_\varsigma = diag(\sigma_\varsigma, ..., \sigma_\varsigma)$.

To consistently recover the yield curve, I focus on $J = 12$ yields with the following maturities: three- and six-month, one-, two-, three-, five-, seven-, ten-, fifteen-, twenty-, twenty-five-, and thirty-year. Additionally, data employed are end of the month observations of the Euro Area implied yield curve from September 2004 to December 2019.[8] A focus is made on the government bond curve, as the Public Sector Purchase Programme constitutes about 85% of the purchase, the rest being the corporate bonds (through the Corporate Sector Purchase Programme or CSPP), covered bonds, and asset backed securities.

Finally, the state-space model composed of transition Equation (7) and measurement Equation (11) is estimated via maximum likelihood through an extended Kalman filter.

2.3 Impact of QE on the term premium

Figure 1 plots the monthly 10Y term premium, risk-neutral rate and yield from 2004:M09 to 2019:M12. The counter-cyclicality of the term premium and pro-cyclicality of the risk-neutral rate during the Great Financial Crisis are striking. The former rose by about 1.30% between mid-2008 and mid-2009, while the latter dropped by about 1.40%. This pattern reflects the fact that investors are usually risk-averse in 'bad' times, so risk supply increases and its price goes down (its return goes up). It also reflects the fact that the ECB cut the deposit facility rate from 3.25% in July 2008 to 0.25% in April 2009; a measure that shaped medium to long-term interest rate expectations downward.

However, the term premium did not increase during the Eurozone sovereign debt crisis of 2012-2013. This can be attributed to the fact that the sharp increase in periphery countries' credit and liquidity risks (Italy, Portugal, Greece, and Spain mostly), was compensated by flight-to-quality movements toward AAA-rated bonds such as the German and Dutch ones. In addition, term premia may have been driven down by unconventional monetary policy measures implemented by the ECB in this period (Long-Term Refinancing Operations in particular).

Similarly, the current low level of the term premium since 2015 coincides with the QE period that started in January of this year. The recent negative level reflects the fact that investors are willing to pay a premium to secure their money in relatively risk-free investments, in an environment where the ECB is believed to be ready to do "whatever it takes" to preserve the euro.

Finally, the increase in the term premium in May-June 2015 corresponds to a short-lived deterioration in market liquidity (Cohen, Hördahl & Xia (2018)), while the increase in

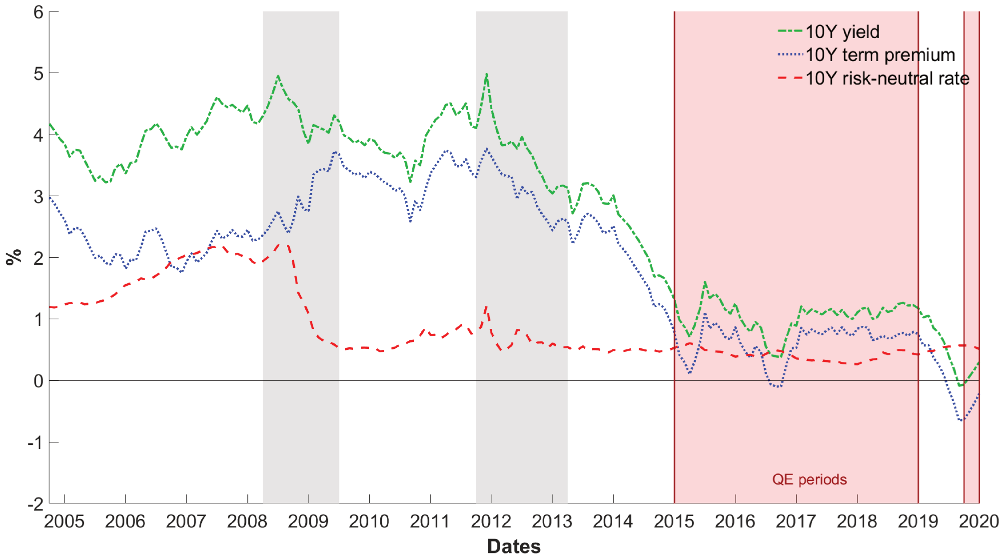
[7]Standard in the literature, this means that the variance of observed errors is assumed to be the same across all maturities.

[8]Data are available at: http://www.ecb.europa.eu/stats/money/yc/html/index.en.html

November-December 2016 coincides with the lower than expected extension of the asset purchase programme.

As a robustness exercise, term premium estimates from a standard gaussian affine model (GATSM) à la Duffie & Kan (1996) and Duffee (2002) estimated via maximum likelihood and from the famous Adrian, Crump & Moench (2013) model estimated via ordinary least square, are displayed in **Appendix C - Figure 13**. In addition, a version of the GATSM and of the SRTSM without anchoring short-term interest rate expectations is estimated. It turns out that all term premia estimates are very similar in their dynamics whatever the model used. Only a difference in the level arises when using the GATSM and SRTSM with non-anchored interest rate expectations. Overall, the estimate is quite robust to the specification used and there is little uncertainty as to the level of 10Y Euro Area government bond term premium during the QE period. My term premia estimates are also in line with those of Wu & Xia (2017), Lemke & Vladu (2017), and Cohen, Hördahl & Xia (2018).[9]

Figure 1: 10Y yield decomposition



10Y yield, risk-neutral rate and term premium between 2004:M9 and 2019:M12
Solid grey areas correspond to recession periods as defined by the CEPR
Solid red area corresponds to the QE periods

The impact of all monetary policy decisions related to the Public Sector Purchase Programme of the ECB is now studied to gauge the transmission of the announcements to the term premium. The daily term premium over the month of the PSPP-related announcements presented in **Table 1** is plotted on **Figure 2**.[10] A similar plot for the the 10Y yield is presented in **Appendix - C Figure 16** for information.

Strikingly, January 22nd and March 9th, 2015 saw major events that affected the term premium. On the former date, the asset purchase programme was introduced and the term premium went down by 0.12%. On the latter date, the programme started (with a release of important details) and the term premium dropped by 0.11%. On the other hand, December 8th, 2016 seems to be a date that contributed to an upward movement of the term premium:

[9] Dynamics are very similar even though levels slightly differ as they do not anchor interest rate expectations.

[10] The estimate comes from the SRTSM without anchored interest rate expectations, as surveys are only available at a monthly frequency.

the market trend reversed, as the PSPP programme was announced to be downsized from €80bn to €60bn from April 2017 on. As a consequence, the term premium went up by 0.09%, which is tantamount to a negative QE shock. However, it is not clear that this event was the main market mover of the month, nor for the other dates studied.

Some researchers (for example Middeldorp (2015) and De Santis (2016)) believe that QE was very strongly anticipated by market participants, so much so that the actual announcement date could be said to be August 2014, following President Draghi's Jackson Hole speech (Garcia Pascual & Wieladek (2016)). However, once again there is no sign here that this event was an obvious market mover for the term premium, even though it may have contributed to its overall downward trend around that period, with a decrease of 0.06% of the term premium following Draghi's speech. Therefore, I will only use January 22^{nd} and March 9^{th}, 2015 as input dates when identifying the term premium QE shock with the Narrative sign restrictions in **Section 3.3**. Unfortunately for my identification scheme, other news came out during the months that saw QE announcements. For example, January 22^{nd}, 2015 also saw an announcement related to the Long-Term Refinancing Operations of the ECB. It is assumed here that such events were less meaningful for the term premium than the QE programme. Overall, the choice of January 22^{nd} and March 9^{th}, 2015 is in line with Andrade et al. (2016) and Dedola et al. (2017),[11] who find a statistically significant impact on asset prices at these dates.

Besides, **Figure 3** plots the yield, term premium and risk-neutral curves one day before and one day after the QE announcement on January 22, 2015. As it can be seen, most of the decrease (70%) arose from the drop in the term premium curve, while the drop in the risk-neutral curve contributed by 30% to the 10 bps reduction in the the yield curve. Empirical findings therefore tend to show that the announcement of the QE programme was mostly transmitted to the yield curve by the term premium channel.

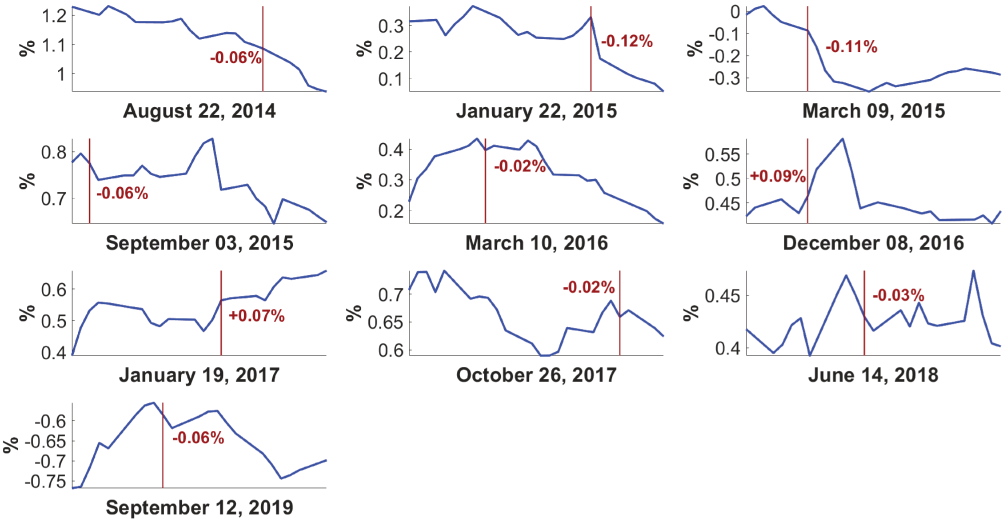
Table 1: Events related to the Public sector purchase programme of the ECB

Dates	Event
22/08/2014	Draghi Jackson Hole Speech
22/01/2015	Announcement of the Expanded APP
09/03/2015	Start and release of implementation details of APP
03/09/2015	Increase in PSPP issue share limit
10/03/2016	PSPP amount raised and CSPP announced
08/12/2016	PSPP recalibration
19/01/2017	Extension of the PSPP below the DFR
26/10/2017	PSPP recalibration
14/06/2018	PSPP recalibration
12/09/2019	New PSPP programme

APP stands for Asset Purchase Programme
PSPP stands for Public Sector Purchase Programme
CSPP stands for Corporate Sector Purchase Programme
DFR stands for Deposit Facility Rate
Source : ECB Press Release

[11] See Appendix B, table 1 in particular.

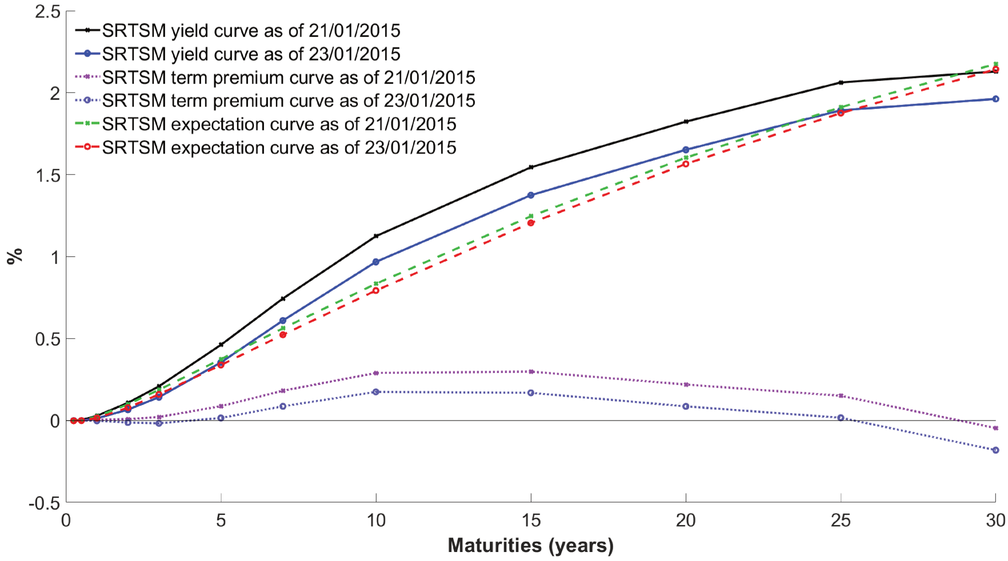
Figure 2: Daily 10Y term premium during months of PSPP-related events



Red vertical bars corresponds to announcement

Variation displayed in red corresponds to the difference between the term premium one day after and one day before the event

Figure 3: Curves fall before and after QE announcement



3 Term premium QE shock analysis

The term premium channel of QE transmits the impact of asset purchase to macroeconomic variables through the government bond term premium. By reducing the risk premium component of long-term yields, QE contributes to the global drop in interest rates and stimulates aggregate demand, as liquidity is injected into the financial system. In order to capture this term premium channel of QE to the Economy, a Structural VAR (SVAR) model in which shocks are identified using sign-restrictions is now introduced.

3.1 SVAR model

The structural representation of a vector Υ_t containing N endogenous variables of interest is:

$$B^{-1}\Upsilon_t = \mu + \sum_{k=1}^{p} Z_k \Upsilon_{t-k} + w_t \tag{12}$$

with $B \in \mathcal{M}_{N\times N}(\mathbb{R})$ and $Z_k \in \mathcal{M}_{N\times N}(\mathbb{R})$ matrices of structural parameters, $\mu \in \mathbb{R}$ a vector of intercepts, and $w_t \in \mathbb{R}^N$ the vector of structural shocks where $w_t \sim \mathcal{MVN}(0, I_N)$. It is assumed that all roots of the characteristics polynomial lie outside the unit disk, so that the VAR is stationary.[12][13]

The SVAR model will be estimated with 6 lags and a constant on monthly data ranging from January 2009 to December 2019. The idea is to focus on the post-GFC period, which also corresponds to the introduction of unconventional monetary policies. It is estimated using Bayesian methods with standard natural conjugate (Normal-Wishart) priors following Arias, Rubio-Ramirez & Waggoner (2018). Endogenous variables entering the SVAR are described in the next sub-section.

3.2 Term premium channel v.s. yield channel

To quantify the relative importance of a term premium QE shock on the economy, a comparison is made between the term premium channel of QE (i.e., the transmission of QE through the term premium), and the yield channel of QE (i.e., the transmission of QE through the overall corresponding yield). Thus, when studying the term premium channel of QE in this sub-section, Υ_t contains the 10-year term premium $\kappa_{10Y,t}$ from Equation (1), the *log* of the real equity prices eq_t, the *log* of the core consumer price level p_t, and *log*-real GDP y_t. Data used are described in **Appendix D**.[14]

$$\Upsilon_t = (\kappa_{10Y,t}, eq_t, p_t, y_t)' \tag{13}$$

As opposed, when studying the overall yield channel of QE, κ_{10Y} is swapped with the 10Y yield y_{10Y} from Equation (1).

To identify the term premium QE shock and yield QE shock, I employ a combination of zero restrictions and traditional sign restrictions on the impulse response functions.

The first identification scheme employed to analyse the term premium and yield channels of QE is presented in **Table 2**:

[12]In practice, all draws whose companion matrix lead to explosive eigenvalues are discarded.

[13]An obvious criticism of constant parameter SVARs is that they do not allow for a possible change in the underlying structural relationships over time. Yet, time-varying parameter (TVP) VARs are famous for their instability. TVP-VARs identified with sign restrictions have also recently been criticized for their inconsistency with Bayesian inference (Bognanni (2018)).

[14]Real GDP is interpolated at a monthly frequency using a *Denton* approach and the OECD Composite leading indicator (CLI) as an index.

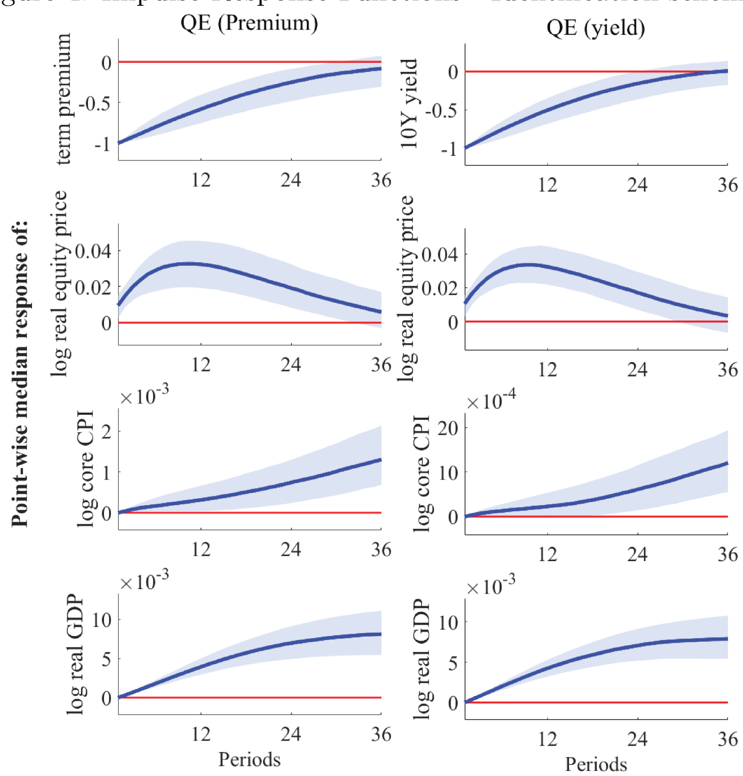
Table 2: Identification scheme I

	QE
κ_{10Y}/y_{10Y}	-
eq	$+$
p	$0+$
y	$0+$

Restrictions are imposed on impact, except for "0+" for which a zero restriction is imposed on impact and a positive restriction is imposed at period one.

In this setting, a QE shock is a shock that lowers the term premium (or the long-term yield) and boosts real equity prices. The impact of this shock on the macro variables is initially set to zero under the usual reasonable assumption that macroeconomic variables can only respond with a lag to a monetary policy shock. At period one, the intuitive assumption that real GDP and consumer prices must respond positively to this QE shock is based on Chen, Cúrdia & Ferrero (2012). In this paper, the authors build a DSGE model with bond market segmentation estimated for the U.S., in which the impact of QE on the term premium and on macroeconomic variables is analyzed. They find that both real GDP and inflation respond positively to a QE shock, as the term premium and the 10Y yield drop.

Figure 4: Impulse Response Functions - Identification scheme I



Point-wise median and 68% Bayesian credible set of 1,000 draws.

Figure 4 presents the impulse response functions associated to identification scheme I. The term premium QE shock and the yield QE shock are calibrated to be of size one percent for a comparative purpose.

As it can be observed, a once percent QE shock moves the 10Y term premium and 10Y yield for about 36 months. Quite unusual, this long-lasting impact on the yield curve contrasts with the bigger importance usually given to the flow effect of QE v.s. the stock effect.

This long-lasting impact will be challenged in identification scheme II, where the drop in the term premium and 10Y yield is associated with an announced amount of bonds to be purchased.

Moreover, the one-percent term premium QE shock and yield QE sock have relatively similar impacts on real equity prices, core consumer prices, and real GDP. Indeed, real equity prices hike to +0.95% (1.05%) on impact and reach their maximum at about 3.26% (3.36%) after 10 months through the term premium (yield) channel. Core CPI increase by +0.03% (+0.02%) after 12 months and +0.07% (+0.06%) after 24 months with the term premium (yield) channel. On the other hand, real GDP increases by +0.39% (+0.42%) after 12 months and by +0.70% (+0.71%) after 24 months with the term premium (yield) channel. However, this quantitatively similar direct impact of the two QE channels on the macro variables may not question the role of interest rate expectations in the investment-saving decision process. This may only reflect the reduced importance of the risk-neutral rate in driving the 10Y yield during the QE period, both in terms of volatility and level, as it can be seen on **Figure 1**.

An interesting comparison can be made with Ireland (2015), that builds an affine term structure model with observable and unobservable macroeconomic factors. If no link to QE is made in this paper, it is one of the only studies analyzing the impact of a risk premium shock on macroeconomic variables. Ireland (2015) finds that an increase of about 0.14% in the 5Y risk premium is associated with a drop of about 0.18% in the output gap and inflation after 12 months. In addition, Rudebusch, Sack & Swanson (2007) and Jardet, Monfort & Pegoraro (2013) find that a decline in the term premium is usually followed by faster GDP growth. Consistent with my findings, this provides support to the view that a reduction in the term premium leads the same qualitative impact as a drop in the short-term rate: the decrease in the term premium pushes long-term yields down, which fosters aggregate demands and finally puts upward pressures on consumer prices.

Nevertheless, one caveat of the previous identification scheme is that the term premium and real equity prices car move in opposite direction for other reasons not linked to monetary policy (news for example). Therefore, let us now add a series of announced amount of bonds to be purchased through the PSPP, b_t, in percentage of nominal GDP, to link the decrease in the yield curve with the asset purchase programme. As before, Υ also contains the 10-year term premium $\kappa_{10Y,t}$ or the 10Y yield y_{10Y}, the *log* of the real equity prices eq_t, the *log* of the core consumer price level p_t, and *log*-real GDP y_t:

$$\Upsilon_t = (b_t, \kappa_{10Y,t}, eq_t, p_t, y_t)' \tag{14}$$

The identification scheme II employed to analyse the term premium and yield channels of QE is presented in **Table 3**:

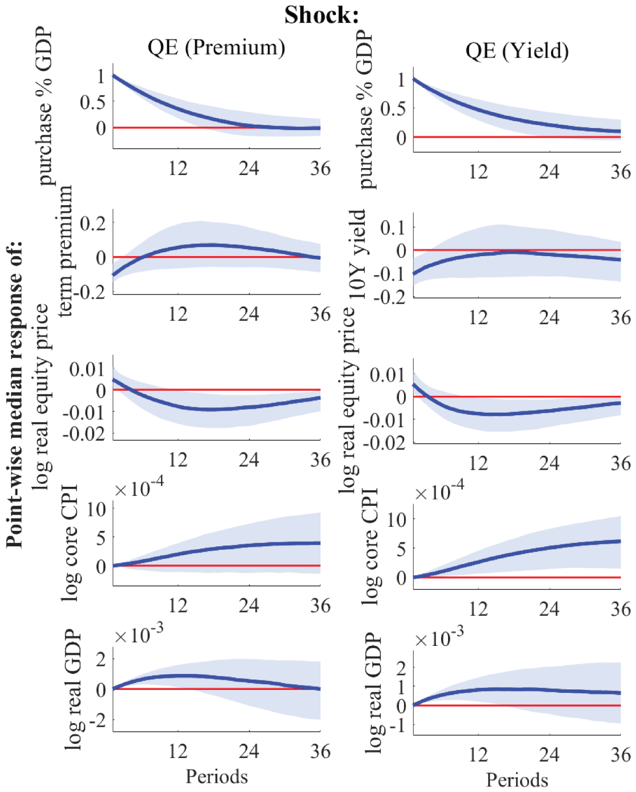
Table 3: Identification scheme II

	QE
b_t	+
κ_{10Y}/y_{10Y}	-
eq	+
p	0+
y	0+

Restrictions are imposed on impact, except for "0+" for which a zero restriction is imposed on impact and a positive restriction is imposed at period one.

Figure 5 presents the IRFs associated to identification scheme II for the term premium QE and the yield QE shocks, calibrated to be a bond purchase of one-percent of nominal GDP.

Figure 5: Impulse Response Functions - Identification scheme II



Point-wise median and 68% Bayesian credible set of 1,000 draws.

As opposed to the IRFs of identification scheme I, the impact of the QE shock on the term premium (10Y yield) is shorter-lived: it jumps from -0.10% to zero (-0.10% to -0.01%) after seven (twelve) months. Worse, the term premium actually stabilizes at +0.05% after 24 months. As a consequence, the positive impact of the term premium QE (yield QE) shock of +0.48% (+0.55%) on real equity prices quickly switches sign after five (four) months. The impact of QE is thus very transitory, whether it is through the term premium or yield channel. This confirms the importance of the flow effect v.s. the stock effect of QE on the market.

As a consequence, both the term premium QE and the yield QE shocks result in a relatively small impact on macroeconomic variables. Indeed, the median response of consumer prices only reaches +0.02% (+0.02%) in the case of the term premium (yield) channel after 12 months. After 24 months, it doubles to +0.04% (+0.05%). However, both QE channels raise real GDP a little bit higher: real economic activity goes up by +0.09% after 12 months and +0.05% after 24 months with the term premium channel, while it increases by +0.08% after 12 months and remains at the same level after 24 months with the yield channel.

With respect to empirical findings from the literature, Garcia Pascual & Wieladek (2016) find a peak impact of -0.93% for the 10Y yield, +0.11% for real GDP and +0.07% for core CPI, following a one-percent asset purchase announcement. They use an SVAR model identified with zero and sign-restrictions, in which they only identify the overall yield channel of QE. Besides, Gambetti & Musso (2017) use a time-varying parameter VAR model with stochastic volatility in which shocks are identified with timing, sign and magnitude restrictions. They find a maximum impact of +0.18% for real GDP and +0.36% for the Harmonized Consumer Price Index (HICP) through the overall yield channel of QE. The difference in the impact of the QE shocks on real GDP and the measures of consumer prices can be explained by the following. First, the bigger impact on core CPI found in Garcia Pascual & Wieladek (2016) is driven by an unusual long-lasting effect of asset purchase on the 10Y yield, that only goes back to zero after 40 months. This challenges the usual view that QE impacts interest rates through its flow effect, v.s. its stock effect. Contrastingly, a much shorter-lasting effect on the term premium and 10Y yield is found here, with a reversal effect after seven and twelve months. This ephemeral impact does not seem to stimulate aggregate demand and only a very small upward pressure on consumer prices appear. Second, Gambetti & Musso (2017) use the HICP and not the core CPI as a measure of prices. Known to be much more volatile because of large movements of food and energy prices on international markets, HICP measure may blur the dynamics of underlying prices.

In this paper, it is found that both the term premium QE and the yield QE shocks moved the term premium down in a very transitory manner, leading to a small increase in real GDP and core consumer prices. To better understand this relatively low impact of QE, the underlying forces driving the aggregate macroeconomic variables will now be analyzed from an historical perspective.

3.3 Historical perspective

As in identifications scheme II, the term premium channel of QE is analyzed using the following specification:

$$\Upsilon_t = (\kappa_{10Y,t}, eq_t, p_t, y_t, b_t)' \tag{15}$$

where $\kappa_{10Y,t}$ is the 10-year term premium, eq_t is the real equity prices, p_t is core consumer prices, y_t is real GDP, and b_t is the amount of bonds to be purchased through the PSPP.

In addition to the term premium QE shock, a risk shock, a demand shock, and a supply shock are identified in identification scheme III, so as to gauge the historical driving forces of the macroeconomic variables of interest. Zero and sign-restrictions employed are presented in **Table 4**:

Table 4: Identification scheme III

	QE Premium	Risk	Demand	Supply
κ_{10Y}	-	-	$\star$	$\star$
eq	+	-	+	+
p	0+	$\star$	+	-
y	0+	$\star$	+	+
b	+	0	0	0

$\star$ Means no restriction is imposed.
Restrictions are imposed on impact, except for "0+" ("0-") for which a zero restriction is imposed on impact and a positive (negative) restriction is imposed at period one.

As in Weale & Wieladek (2016), a risk shock happens with a decline in real equity prices and in the less risky government bond term premium. To this increase in uncertainty, the central bank can only react with a lag. Identifying this shock enables to distinguish between movements of the term premium that are due to QE and those due to flight-to-quality movements. Besides, standard demand and supply shocks are identified and their impact cannot generate a contemporaneous response of the central bank.

Little theory has been developed about the behavior of bond term premia to non-monetary policy shocks. A paper worth mentioning is however Rudebusch, Sack & Swanson (2007), that builds a New Keynesian macro-finance model solved at the third order to feature a time-varying endogenous term premium. They show that the term premium responds positively to a conventional monetary policy shock and to a government purchase shock, but negatively to a technology shock. However, no intuition is given as to the channels of transmission. In addition, such third-order models suffers from the fact that they can hardly be estimated to provide a measure of produced term premium. The produced simulated term premium does also not encompass any liquidity or default risk, in contrast to empirical evidence regarding Euro Area periphery bonds. For these reasons, no assumption is made on the response of term premium to the demand and supply shocks.

To sharpen the identification of the term premium QE shock, zero and traditional sign restrictions are coupled with Narrative sign restrictions. The Narrative sign restrictions approach was developed by Antolín-Díaz & Rubio-Ramírez (2018) and has since become very popular. This cutting-edge identification strategy helps pin down structural shocks using prior Narrative information. Here, I extend the Narrative sign restrictions approach beyond the authors' framework and I label them *extended* Narrative sign restrictions. In particular, I impose a restriction on the sign and relative amplitude of the QE shock and on contribution of the term premium QE shock to the historical decomposition of the term premium. I impose these restrictions for January 2015 and March 2015: the two most relevant QE-related event for the term premium, according to **Figure 2**. Details of the extended Narrative sign restrictions are presented in **Table 5**:

Table 5: Extended Narrative sign restrictions

22/01/2015	QE shock is positive.
	The size of the QE shock is the highest of the sample.
	The contribution of the QE shock to the HD of the term premium is negative.
	The contribution of the QE shock to the HD of the term premium is bigger than the contribution of the other shocks.
09/03/2015	QE shock is positive.
	The contribution of the QE shock to the HD of the term premium is negative.
	The contribution of the QE shock to the HD of the term premium is bigger than the contribution of the other shocks.

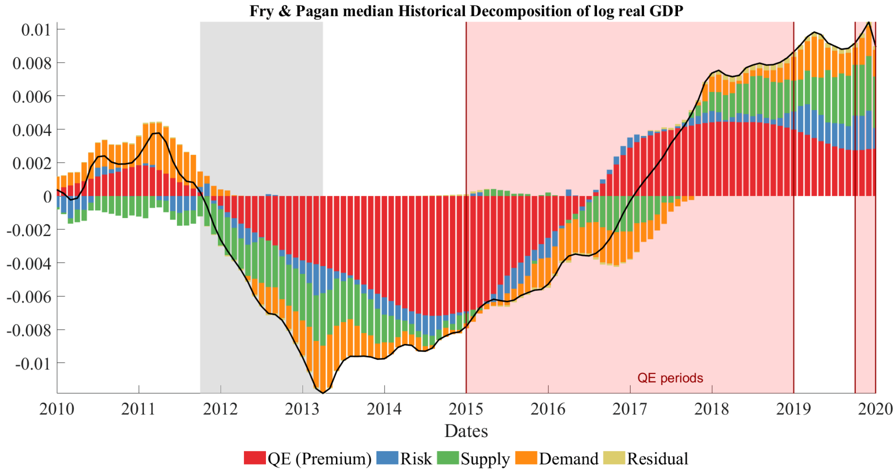
Historical decompositions of real GDP and core consumer prices are presented on **Figure 6** and **Figure 7** respectively, while IRFs are presented in **Appendix - Figure 17** for information purpose. To answer the usual critics regarding the potential inconsistency of using the point-wise median coming from different models to build the historical driving forces of a variables, the Fry & Pagan (2011) median-target is computed. It is optimized on each variable and each shock, and based on 500 draws.

Strikingly, the term premium channel of QE played a big role as one of the main driver of real economic activity in the sample studied. The contribution of the term premium QE shock is slightly positive before the sovereign debt crisis and largely negative between 2012 and mid-2016. This indicates two things: first, in the absence of a QE programme tensions on the term premium were contractionnary for real GDP. Second, the announcement of the QE programme in January 2015 was followed by an increasing impact of the term premium channel that turned positive mid-2016. This lag of 6 quarters before the contribution turns positive highlights the remaining tensions in the sovereign bond market in 2015-2016 (see **Figure 1**). From 2016:Q2 on, however, the term premium channel of QE had a considerable impact on real GDP and reached its maximum in 2018. Biggest driver of output during this period, the importance of the term premium QE shock then slowly decreases at the end of the sample. Noteworthy, the impact of the announcement of the "second" QE programme in September 2019 can slightly be perceived in the sample studied, with a progressive increase in the contribution of the term premium QE shock at the end of the sample.

Besides, the demand and supply shocks weighted down on economic activity between mid-2012 and 2017:Q3, overlapping the negative contribution of the term premium QE shock. The absence of sufficient demand from economic agents during this period a posteriori justifies the need for monetary policy to revive an anemic real output. The downsizing of the QE programme from 2018 on, eventually hands over to the growing demand at the end of the sample.

Finally, note that the contribution of the risk shock and residual shocks are negligible during the sample.

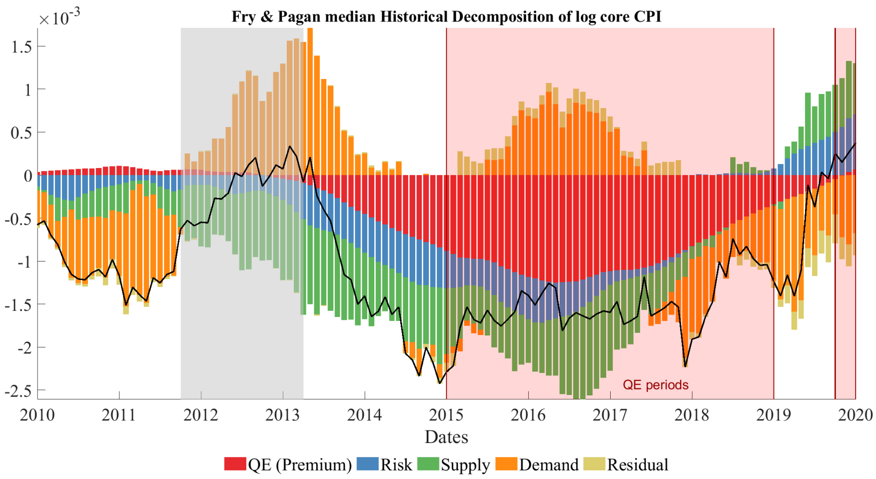
Figure 6: Historical Decomposition of real GDP - Identification scheme III



Historical decomposition is expressed as deviation from initial conditions
Solid grey area corresponds to a recession period as defined by the CEPR
Solid red area corresponds to the QE periods

As opposed, term premium QE shocks only played a negative role in driving core consumer prices in the sample, despite the small but positive contribution observable in the IRF of **Figure 5**. From this assessment, one can infer the following: the term premium channel of QE was expansionary for consumer prices, but the amplitude of the historical QE shocks was not big enough to end up in a positive contribution for prices. In fact, one can observe that this contribution starts rising in mid-2016, which coincides with the switch to a positive contribution of the term premium QE shocks in the historical decomposition of real GDP. That is, once the term premium channel of QE had a positive impact on aggregate demand, upward pressure on consumer prices started to appear. The overall negative contribution of the QE, risk, and supply shocks between 2014 and 2019 explain the sluggish inflation during that period (average of 0.40% of annualized MoM core-inflation between 2014 and 2019).

Figure 7: Historical Decomposition of core CPI - Identification scheme III



Historical decomposition is expressed as deviation from initial conditions
Solid grey area corresponds to a recession period as defined by the CEPR
Solid red area corresponds to the QE periods

Obviously, a potential omitted variable in the previous identification scheme is the short-term interest rate. If the ECB conventional monetary policy has mostly been inactive during the 2009-2020 period due to the zero-lower bound constraints, the 3M yields still fluctuated: it decreased from +1.26% in January 2009 to -0.56% in December 2019. In the next identification scheme presented in **Table 6**, I therefore include the 3M yield, and I identify a conventional monetary policy shock. For computational feasibility, Narrative sign-restrictions are however dropped.

Table 6: Identification scheme IV

	QE Premium	**MP**	**Risk**	**Demand**	**Supply**
$\boldsymbol{\kappa_{10Y}}$	-	$\star$	-	$\star$	$\star$
$\boldsymbol{r}$	0	-	0	$\star$	$\star$
$\boldsymbol{eq}$	+	+	-	+	+
$\boldsymbol{p}$	0+	0+	$\star$	+	-
$\boldsymbol{y}$	0+	0+	$\star$	+	+
$\boldsymbol{b}$	+	0	0	0	0

$\star$ Means no restriction is imposed.
Restrictions are imposed on impact, except for "0+" ("0-") for which a zero restriction is imposed on impact and a positive (negative) restriction is imposed at period one.

The conventional monetary policy shock identified here is standard: a decrease in the short-term rate raises real equity prices and the macroeconomic variables with a lag. It is not associated to an announcement of bonds to be purchased and the term premium is left unrestricted to prevent from over-identification. In addition, the term premium QE shock does not contemporaneously impact the short-term rate, to cleary distinguish unconventional and conventional monetary policy, in a context where the short-term rate is stuck at the zero-lower bound (Baumeister & Benati (2013)). For similar reasons, the risk shock leaves the short-term rate unaffected on impact.

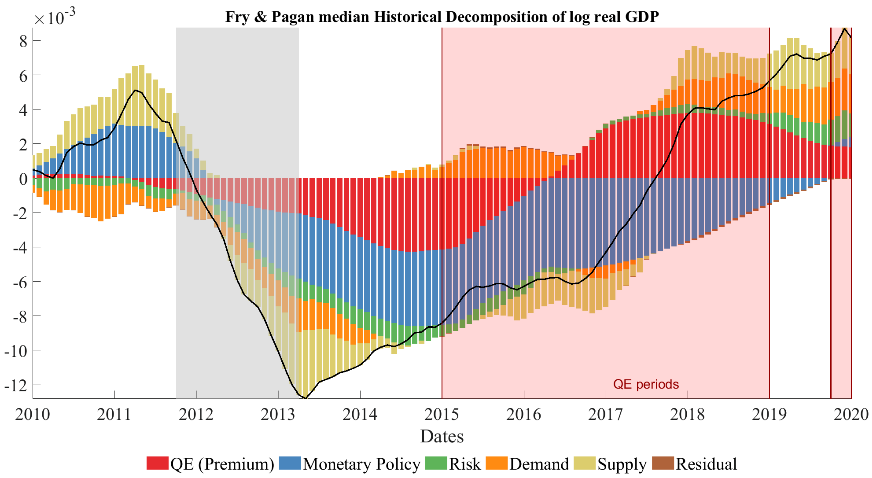
Fry & Pagan (2011) median-target historical decompositions of real GDP and core consumer prices based on 500 draws are presented on **Figure 8** and **Figure 9** respectively, while IRFs are presented in **Appendix - Figure 18** for information purpose.

Both in the case of real GDP and core CPI, the conventional monetary policy shock played a large negative role between 2013 and 2019 and eclipses a now smaller role of the term premium channel of QE compared to **Figure 7**. That is, conventional monetary policy was still restrictive despite the null or negative deposit facility rate, in an environment where the natural rate of interest was estimated to be strongly negative.[15] Otherwise stated, the interest rate gap may still have been positive during this period. Thus, the overall action of the central bank is negative.

Furthermore, the apparent failure of the term premium channel of QE to significantly stimulate aggregate demand sheds light to limits of acting on long-term yields when the short-end of the curve is stuck at its effective lower bound with a positive interest rate gap. Indeed, although not studied here, the role of long-term yields in the investment-saving process may somewhat be less important than the one of the short-term rate. Still, this may not eclipse the potential big impact of the other transmission channels of QE not examined here, such as the bank lending and exchange rate channels.

Finally, note that both for real GDP and consumer prices, the contribution of the risk, supply, demand, and residual shocks is relatively robust to the introduction of the 3M yield in the SVAR model.

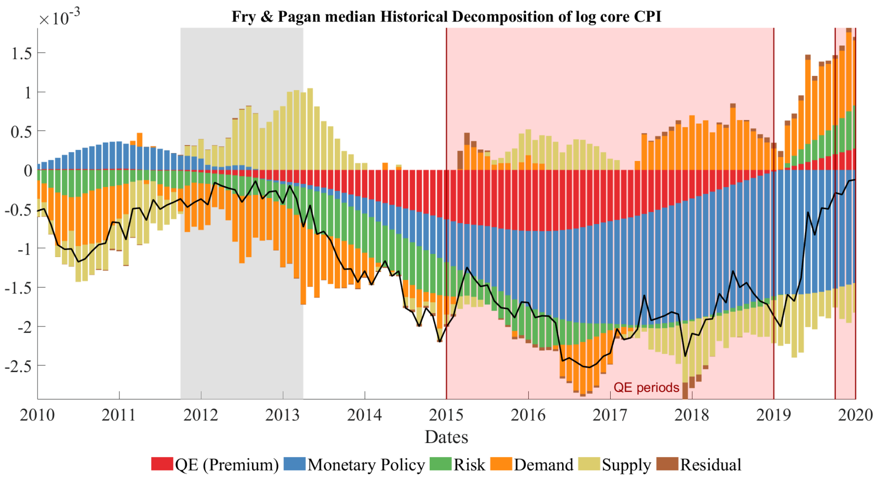
Figure 8: Historical Decomposition of real GDP - Identification scheme IV



Historical decomposition is expressed as deviation from initial conditions
Solid grey area corresponds to a recession period as defined by the CEPR
Solid red area corresponds to the QE periods

[15] Many papers find a large negative natural rate of interest in the Euro Area during the 2014-2019 period. See Brand, Bielecki & Penalver (2018) for a comprehensive summary of studies.

Figure 9: Historical Decomposition of core CPI - Identification scheme IV



Historical decomposition is expressed as deviation from initial conditions
Solid grey area corresponds to a recession period as defined by the CEPR
Solid red area corresponds to the QE periods

4 Conclusion

In this paper, the impact of the Public Sector Purchase Programme (PSPP) of the European Central Bank (ECB) is analyzed to evaluate its effect on the yield curve and on the Euro Area economy. A focus is made on the term premium transmission channel of QE. That is, the impact that a reduction in the government bond term premium has on aggregate macroeconomic variables following a bond purchase by the ECB. A comparison is made with the overall yield channel of QE to understand the specificities of this little known term premium channel.

To that aim, I first extract a term premium series from a shadow rate term structure model with anchored interest rate expectations. Priced at a daily frequency, this model is able to quantify the impact of each PSPP-related announcements on the term premium. I then plug the term premium in a monthly Bayesian Structural VAR with real equity prices, consumer prices, real GDP, and a series of amount of bonds to be purchased through the PSPP. I next identify a QE term premium shock thanks to zero restrictions, sign restrictions, and Narrative sign restrictions, that I extend beyond their original framework. Using Impulse Response Functions, I show that the term premium channel of QE behaves qualitatively and quantitatively like the overall yield channel: they both have small positive impact on macroeconomic variables. A purchase of one-percent of GDP reduces the term premium by 0.1% on impact and switches sign after seven months. This confirms the importance of the flow effect of QE as opposed to the stock effect. As a consequence, the increase in consumer prices and real GDP, by respectively +0.03% and +0.09% after 12 months, is very small. A historical analysis of the drivers of consumer prices also shows that the term premium QE shocks did not contribute positively to the price measure, even though the amplitude of their contribution grew in response to the programme. Whether it is because other transmission channels were at stake, because output was still too far from potential, or because prices are not reacting much to changes in real economic activity anymore remains an open question. I leave that for future research.

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Appendices

A Shadow rate term structure model

The form of the n-period zero-coupon bond yield $y_{n,t}$ under the risk-neutral probability measure $\mathbb{Q}$ is:

$$y_{n,t} = \frac{1}{n}\sum_{j=0}^{n-1} f_{j,j+1,t}$$

where

$$f_{n,n+1,t} \approx \underline{r}_t + \sigma^{\mathbb{Q}}_{s,n} \times g\left(\frac{a_n + b_n' X_t - \underline{r}_t}{\sigma^{\mathbb{Q}}_{s,n}}\right)$$

is the forward rate for a loan starting at date $t+n$ and maturing at date $t+n+1$,

with $g(x) = x\Phi(x) + \phi(x)$ where $\Phi(x)$ is the CDF of x and $\phi(x)$ the PDF. In addition,
$a_n = \delta_0 + \delta_1'\left(\sum_{j=0}^{n-1}\Theta^{\mathbb{Q}j}\right)\mu^{\mathbb{Q}} - \frac{1}{2}\delta_1'\left(\sum_{j=0}^{n-1}\Theta^{\mathbb{Q}j}\right)\Sigma_\varepsilon\Sigma_\varepsilon'\left(\sum_{j=0}^{n-1}\Theta^{\mathbb{Q}j}\right)'\delta_1 \;\; \in \mathbb{R}$
$b_n' = \delta_1'\Theta^{\mathbb{Q}n} \;\; \in \mathbb{R}^N$
$\sigma^{\mathbb{Q}}_{s,n} = [\sum_{q=0}^{n-1}\delta_1'(\Theta^{\mathbb{Q}})^q\Sigma_\varepsilon\Sigma_\varepsilon'(\Theta^{\mathbb{Q}})^{q\prime}\delta_1]^{1/2} \;\; \in \mathbb{R}$

B Narrative sign restrictions

In the following, I present the mathematical expressions of the *extended* Narrative sign restrictions that it is possible to impose on the structural shocks and on the historical decompositions. I start by the plain restrictions given in Antolín-Díaz & Rubio-Ramírez (2018) and then provide my extended version.

Class I $\oplus$ restriction on structural shock w_t^3 at date t_s takes the form:

$$S_{t_s}F(\Theta)e_3 > 0 \tag{B.1}$$

where e_3 is the third column of I_3

$F \in \mathcal{M}_{3(p+1)\times 3}(\mathbb{R})$ is such that $F = (B^{-1}, Z_1, ..., Z_p)'$ contains the structural parameters from Equation (12). Besides, $S_{t_s} \in \mathcal{M}_{1\times 3(p+1)}(\mathbb{R})$ is a matrix of full row rank such that:

$$S_{t_s} = \begin{pmatrix} \upsilon_{1,t_s} & \upsilon_{2,t_s} & \upsilon_{3,t_s} & \dots & -\upsilon_{1,t_s-p} & -\upsilon_{2,t_s-p} & -\upsilon_{3,t_s-p} \end{pmatrix} \tag{B.2}$$

Class I $\ominus$ restriction is simply $S_{t_s}F(\Theta)e_3 < 0$.

Class II Type 1 restriction on the historical decomposition of υ_3 at date t_s takes the form:

$$SF(\Theta)e_3 > 0 \tag{B.3}$$

where e_3 is the third column of I_3

$F \in \mathcal{M}_{3\times 3}(\mathbb{R})$ is such that:

$$F = \begin{pmatrix} 0 & |H_{\upsilon_3,w^2,t_s}| - |H_{\upsilon_3,w^1,t_s}| & |H_{\upsilon_3,w^3,t_s}| - |H_{\upsilon_3,w^1,t_s}| \\ |H_{\upsilon_3,w^1,t_s}| - |H_{\upsilon_3,w^2,t_s}| & 0 & |H_{\upsilon_3,w^3,t_s}| - |H_{\upsilon_3,w^2,t_s}| \\ |H_{\upsilon_3,w^1,t_s}| - |H_{\upsilon_3,w^3,t_s}| & |H_{\upsilon_3,w^2,t_s}| - |H_{\upsilon_3,w^3,t_s}| & 0 \end{pmatrix} \tag{B.4}$$

where H_{j,w^i,t_s} is the contribution of shock i to the historical decomposition of variable j at date t_s

In addition, $S \in \mathcal{M}_{(3-1)\times 3}(\mathbb{R})$ is a matrix of full row rank such that $S = \sum_{j=1}^{3-1} e_{j,3-1}e_{j,3}'$, where $e_{j,i}$ is the j^{th} column of I_i

Class II Type 2 restriction takes the form:

$$SF(\Theta)e_3 > 0 \tag{B.5}$$

where e_3 is the third column of I_3

$F = (F_1, F_2, F_3) \in \mathcal{M}_{3\times 3}(\mathbb{R})$ is such that:

$$F_1 = \begin{pmatrix} |H_{\upsilon_1,w^1,t_s}| - |H_{\upsilon_1,w^2,t_s}| - |H_{\upsilon_1,w^3,t_s}| \\ |H_{\upsilon_2,w^1,t_s}| - |H_{\upsilon_2,w^2,t_s}| - |H_{\upsilon_2,w^3,t_s}| \\ |H_{\upsilon_3,w^1,t_s}| - |H_{\upsilon_3,w^2,t_s}| - |H_{\upsilon_3,w^3,t_s}| \end{pmatrix}$$

$$F_2 = \begin{pmatrix} |H_{\upsilon_1,w^2,t_s}| - |H_{\upsilon_1,w^1,t_s}| - |H_{\upsilon_1,w^3,t_s}| \\ |H_{\upsilon_2,w^2,t_s}| - |H_{\upsilon_2,w^1,t_s}| - |H_{\upsilon_2,w^3,t_s}| \\ |H_{\upsilon_3,w^2,t_s}| - |H_{\upsilon_3,w^1,t_s}| - |H_{\upsilon_3,w^3,t_s}| \end{pmatrix}$$

$$F_3 = \begin{pmatrix} |H_{\upsilon_1,w^3,t_s}| - |H_{\upsilon_1,w^2,t_s}| - |H_{\upsilon_1,w^1,t_s}| \\ |H_{\upsilon_2,w^3,t_s}| - |H_{\upsilon_2,w^2,t_s}| - |H_{\upsilon_2,w^1,t_s}| \\ |H_{\upsilon_3,w^3,t_s}| - |H_{\upsilon_3,w^2,t_s}| - |H_{\upsilon_3,w^1,t_s}| \end{pmatrix} \tag{B.6}$$

$S = e_3'$ is the transpose of the third column of I_3

Now, what follows constitute my *extended* Narrative restrictions:

Class I Type 1 restriction takes the form:

$$|S_{t_s}F(\Theta)e_3| > |S_{t_s}F(\Theta)e_i| \;\; \forall\, i \;\in\; [\![1,2]\!] \tag{B.7}$$

Class I Type 2 is:

$$|S_{t_s}F(\Theta)e_3| > |S_{t_s}F(\Theta)e_1| + |S_{t_s}F(\Theta)e_2| \tag{B.8}$$

Class I Category A restriction is:

$$|S_{t_s}F(\Theta)e_3| > |S_{t_j}F(\Theta)e_3| \;\; \forall\, j \;\in\; [\![1,T]\!] \setminus \{s\} \tag{B.9}$$

With F and S as in Equation (B.1)

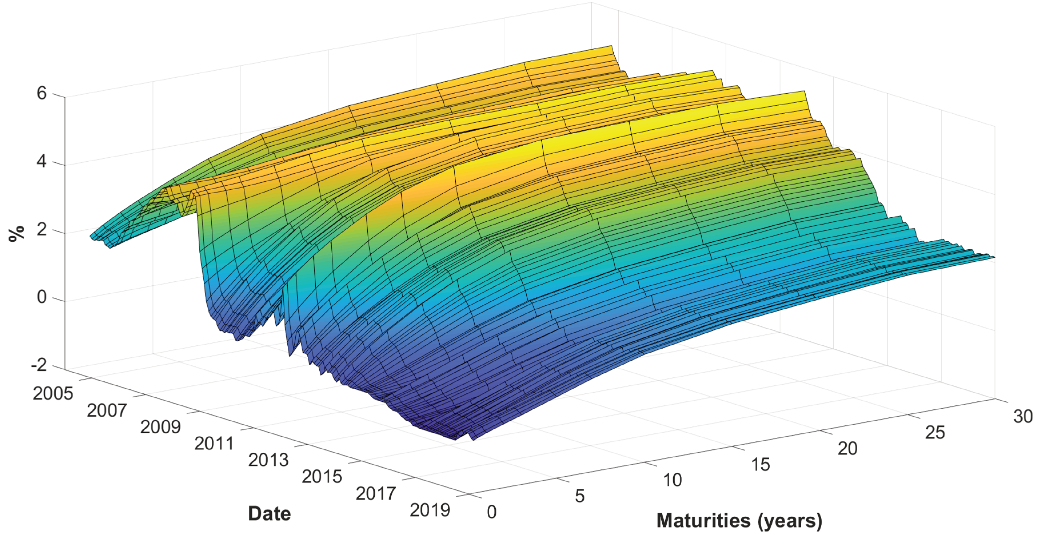
Moreover, **Class II** is $\oplus$ if $H_{\upsilon_3,w^3,t_s} > 0$ and $\ominus$ if $H_{\upsilon_3,w^3,t_s} < 0$.

Finally, **Class II Category A** is expressed by:

$$|H_{\upsilon_3,w^3,t_s}| > |H_{\upsilon_3,w^3,t_j}| \;\; \forall\, j \;\in\; [\![1,T]\!] \setminus \{s\} \tag{B.10}$$

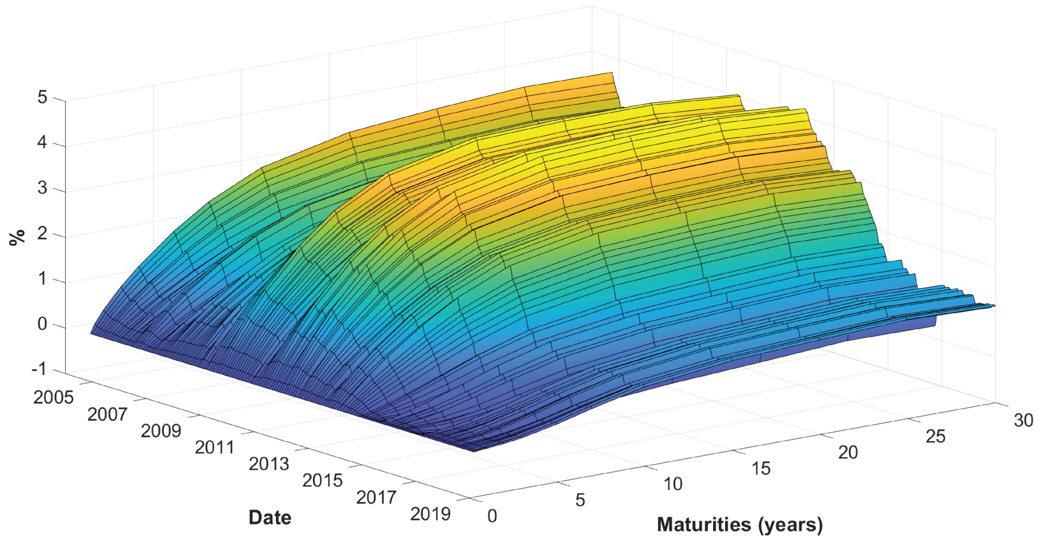
C Additional results

Figure 10: Fitted Yield Surface



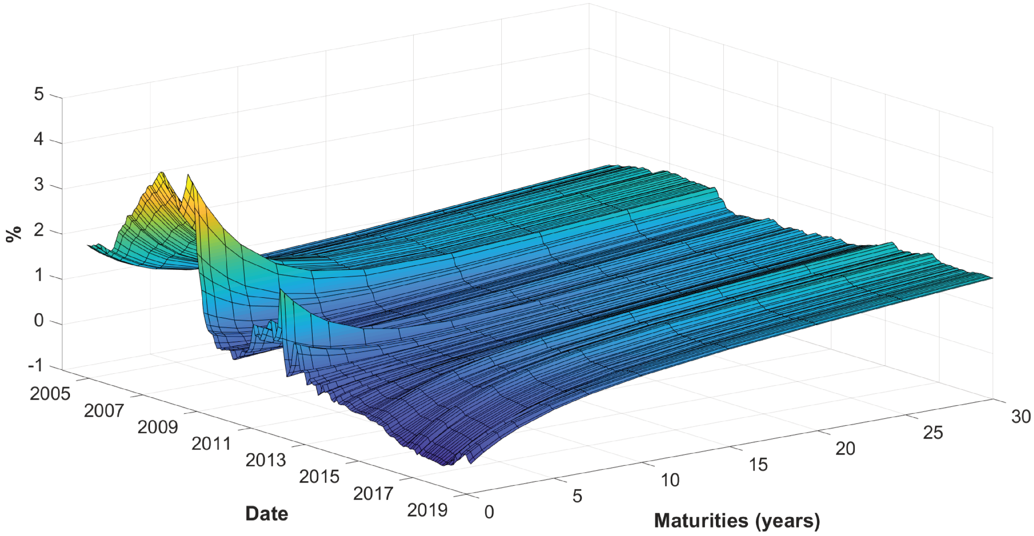
Fitted Yield surface from 2004:M9 to 2019:M12

Figure 11: Term Premium Surface



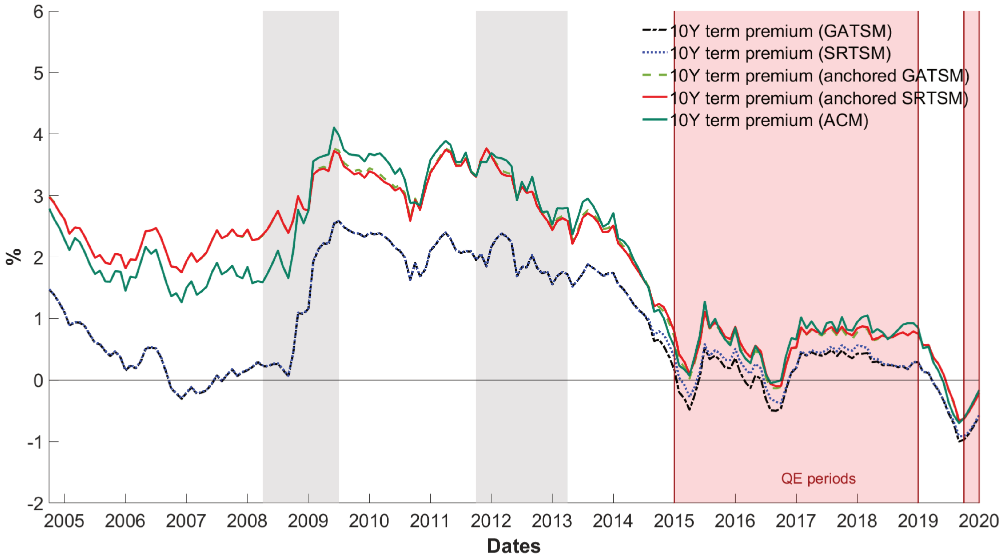
Term premium surface from 2004:M9 to 2019:M12

Figure 12: Risk-neutral rate Surface



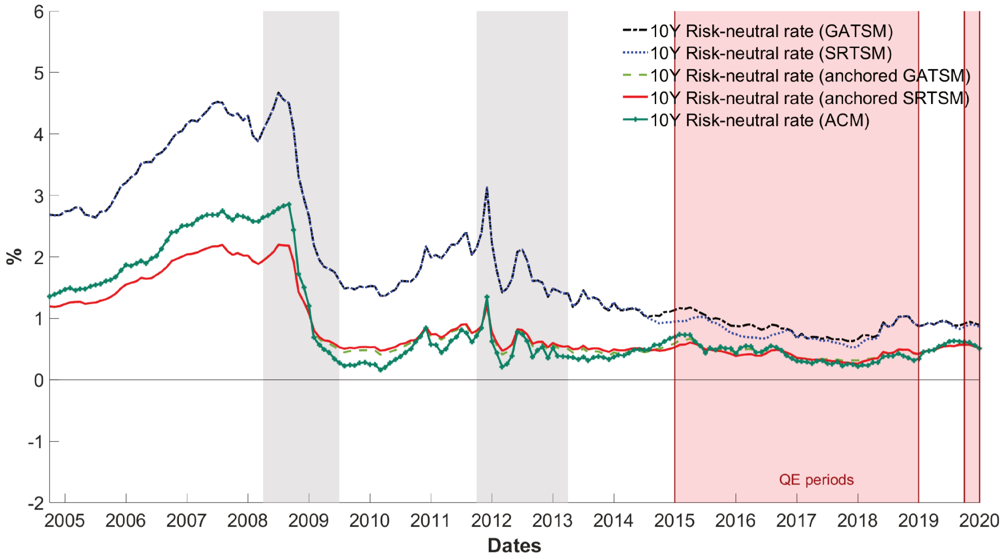
Risk-neutral rate surface from 2004:M9 to 2019:M12

Figure 13: Term premium comparison



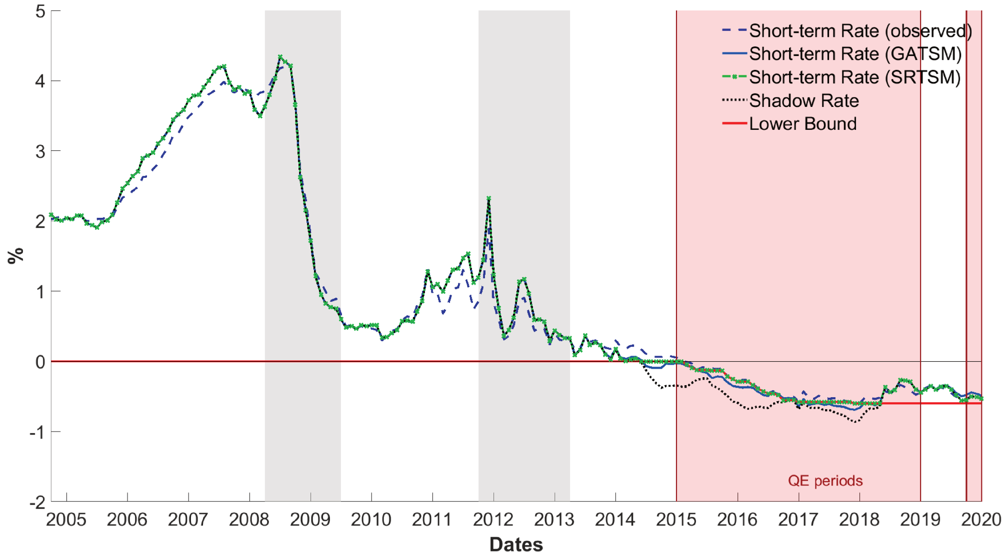
Term premium comparison from 2004:M9 to 2019:M12

Figure 14: Risk-neutral rate comparison



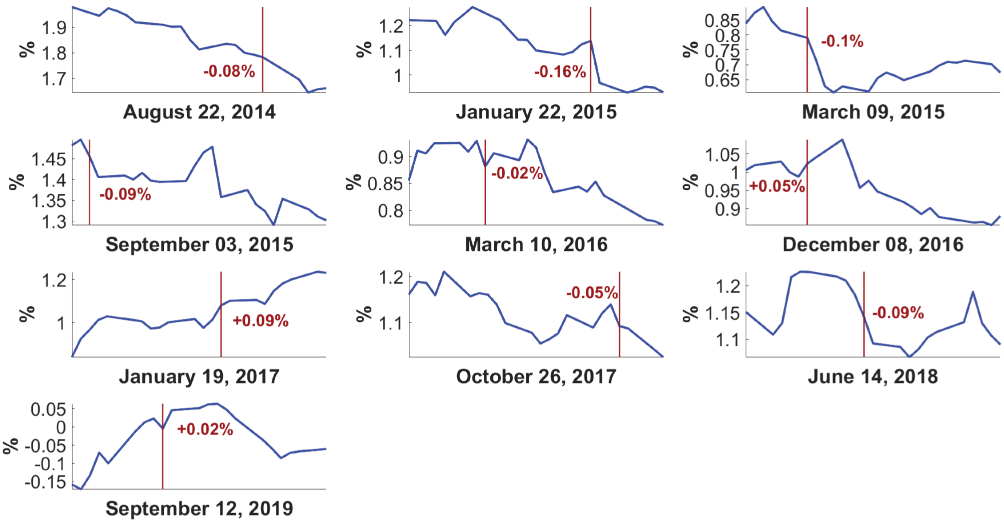
Risk-neutral rate comparison from 2004:M9 to 2019:M12

Figure 15: Shadow Rate



Shadow rate from 2004:M9 to 2019:M12
Solid grey area corresponds to a recession period as defined by the CEPR
Solid red area corresponds to the QE periods

Figure 16: Daily 10Y yield during months of key QE-related events



Red vertical bar corresponds to announcement

Table 7: Maximum Likelihood Estimation

Parameter	**Value**
${\mu_1}^{\mathbb{Q}}$	0.363
${\mu_2}^{\mathbb{Q}}$	0
${\mu_3}^{\mathbb{Q}}$	0
${\mu_1}^{\mathbb{P}}$	0.043
${\mu_2}^{\mathbb{P}}$	-0.122
${\mu_3}^{\mathbb{P}}$	0.100
${\theta_{11}}^{\mathbb{Q}}$	1.007
${\theta_{12}}^{\mathbb{Q}}$	0
${\theta_{13}}^{\mathbb{Q}}$	0
${\theta_{21}}^{\mathbb{Q}}$	0
${\theta_{22}}^{\mathbb{Q}}$	0.972
${\theta_{23}}^{\mathbb{Q}}$	0
${\theta_{31}}^{\mathbb{Q}}$	0
${\theta_{32}}^{\mathbb{Q}}$	0
${\theta_{33}}^{\mathbb{Q}}$	0.954
${\theta_{11}}^{\mathbb{P}}$	0.370
${\theta_{12}}^{\mathbb{P}}$	0.174
${\theta_{13}}^{\mathbb{P}}$	0.145
${\theta_{21}}^{\mathbb{P}}$	0.387
${\theta_{22}}^{\mathbb{P}}$	0.847
${\theta_{23}}^{\mathbb{P}}$	-0.131
${\theta_{31}}^{\mathbb{P}}$	0.006
${\theta_{32}}^{\mathbb{P}}$	0.020
${\theta_{33}}^{\mathbb{P}}$	1.018
$\sigma_{\varepsilon 11}$	-0.055
$\sigma_{\varepsilon 12}$	0
$\sigma_{\varepsilon 13}$	0
$\sigma_{\varepsilon 21}$	0.181
$\sigma_{\varepsilon 22}$	0.647
$\sigma_{\varepsilon 23}$	0
$\sigma_{\varepsilon 31}$	-0.161
$\sigma_{\varepsilon 32}$	-0.679
$\sigma_{\varepsilon 33}$	0.183
σ_{ς}	0.115
δ_0	0
δ_{11}	1
δ_{12}	1
δ_{13}	1
Log-Likelihood	**1194.24**
RMSE	**0.217**

Figure 17: Impulse Response Functions - Identification scheme III

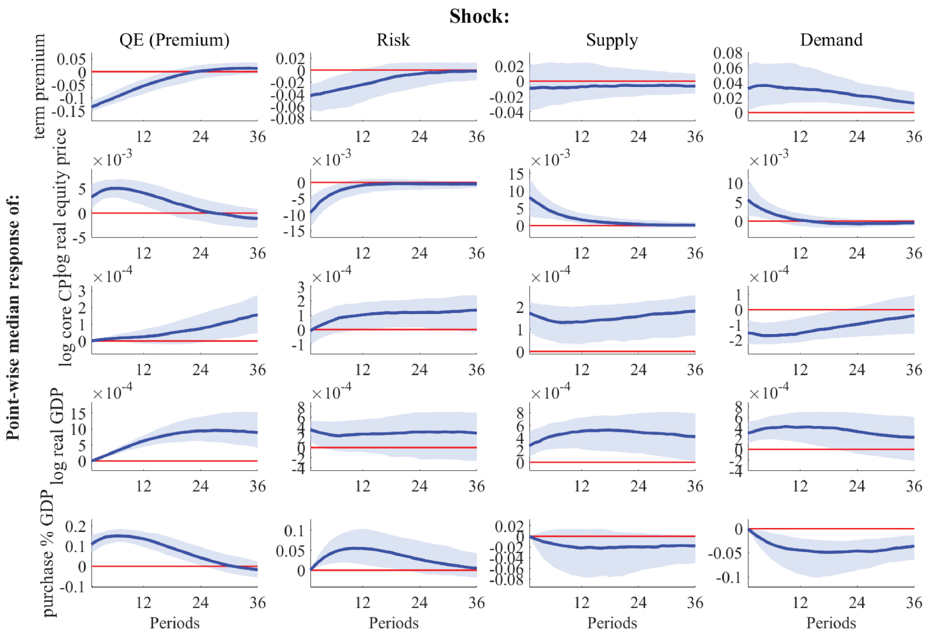
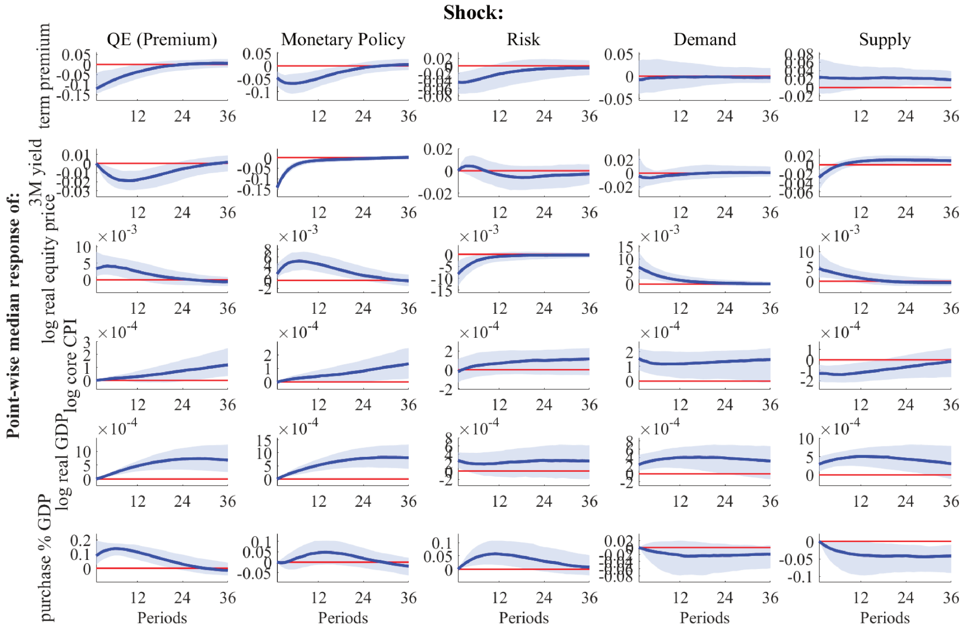


Figure 18: Impulse Response Functions - Identification scheme IV



D Data

- Euro Area government bond yield curve is a nominal synthetic curve available at http://sdw.ecb.europa.eu
- Economic activity is the *log*-real GDP interpolated at a monthly frequency using a *Denton* approach and the OECD Composite leading indicator (CLI) as an index.
  Data are available at http://fred.stlouisfed.org/series/CLVMEURSCAB1GQEA19 and http://data.oecd.org/leadind/composite-leading-indicator-cli.htm
- Consumer price index excluding food and energy is the core Harmonized Index of Consumer Prices (core-HICP).
  Data are available at: http://sdw.ecb.europa.eu
- Real Equity prices is the *log* Euro Stoxx 50 deflated by the HICP.
  Data are available at: http://sdw.ecb.europa.eu

CHAPTER II

Potential growth and natural yield curve in Japan*

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Abstract

The yield curve gap in Japan is estimated to examine whether it has contributed to the sustained low growth and low inflation rates observed in this country since the 1990s. First, real yield curve factors are extracted from a dynamic term structure model. Second, a semi-structural macroeconomic model generalizing the concept of short-term natural rate of interest to the entire range of maturities is estimated. Natural yield factors and potential variables are jointly estimated and analyzed through the lens of the different monetary policy regimes implemented by the Bank of Japan.

JEL classification: C32; E43; E52
Keywords: Yield curve; Potential growth; State-space model; Japan

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1 Introduction

The natural rate is the real interest rate at which output is at its potential and inflation remains stable. First introduced by Wicksell (1898), this neutral rate provides an investment-saving equilibrium. Since the seminal paper of Laubach and Williams (2003), the natural rate of interest has gained in attention. Tracked by central bankers, it is an unobserved target for the policy rate, that enables to close the output gap and reach the inflation objective.

Central banks in advanced economies have recently lowered their policy rates to the zero-lower bound in response to the Great Financial Crisis (GFC). The Bank of Japan (BoJ) is a precursor that initiated such a monetary policy easing to fight deflationary pressures and boost an anemic GDP growth during the *Lost Decade* (1991-2001). Constrained by the zero-lower bound on short-term nominal interest rates, the BoJ is unable to track the natural rate deep into negative territory. In this context, it introduced unconventional monetary policy measures through different monetary regimes, so as to act on medium- to long-term yields and provide further easing. In this paper, we propose to extend the concept of short-term natural rate of interest to all horizons, by estimating the Japanese natural yield curve.

The natural yield curve generalizes the notion of equilibrium rate on the market of investment and savings to all horizons. This extension is based on the two following assessments: first, investments are mostly made at medium- to long-term maturities and are therefore sensitive to long-term rates. Second, consumption-savings decisions are not only driven by the current short-term rate, but also by expectations of future short-term rates via consumption smoothing. The natural yield curve is therefore built from real rate expectations, that provide an equilibrium between investment and savings at all horizons considered by economic agents.

In an environment where nominal short-term rates are constrained by the zero-lower bound, real long-term rates can deviate from natural long-term rates. The same way the real short-term rate may differ from its natural level due to frictions (price stickiness, wage rigidity), real interest rate expectations can diverge from their flexible price equivalent level. Such a gap at short-term and long-term horizons prevents the investment-saving equilibrium to be reached and output to run at potential. As a consequence, inflation either accelerates or decelerates, depending on the sign of the output gap.[1]

There are two opposite views in the literature about whether neutral rates should be interpreted using a short- or a long-run perspective. On the one hand, the long-run interpretation relies on the following assumption: neutral rates are driven by potential output growth and structural factors (total factor productivity growth, the growth rate of population, savings resulting from individual's preferences, etc). This approach is adopted in papers based on Laubach and Williams' methodology. On the other hand, a short-term interpretation relies on the assumption that most of the changes in the actual real rates come from changes in the interest rates gap rather than in the neutral rates. In this case, neutral rates do not depend upon unobserved structural factors and the interest rates gap is correlated with the business cycle. Choosing between these two alternative approaches is usually an empirical issue and the literature is still inconclusive (Garnier and Wilhelmsen (2009)). Here, we adopt the short-run perspective for two reasons. First, the aim of this paper is to analyze the yield

[1] The output gap is defined as the difference between real GDP and potential GDP.

curve gap[2] in the perspective of the different monetary regimes implemented by the BoJ. Second, we consider a maximum maturity horizon of 10 years, which corresponds to the typical duration of a business cycle (Juglar cycles).

In this paper, we estimate the natural yield curve, and some potential variables of the Japanese economy within a semi-structural macroeconomic model. In line with Brzoza-Brzezina and Kotłowski (2014) and Imakubo et al. (2017), we first extract three real yield curve factors from a Nelson-Siegel term structure model. We then plug these three factors representing the real yield curve in a macroeconomic model, where they are treated as observable. In the spirit of Laubach and Williams (2003), we define a set of equations inspired from the New-Keynesian theory and results found in the empirical literature. For instance, we estimate an aggregate demand relation in which the output gap is not only driven by the (short-term) interest rate gap, but by the whole yield curve gap, in line with Imakubo et al. (2017). Thus, the investment-saving equilibrium at all horizons is considered. Besides, we define a backward-looking type of Phillips curve relation, in which inflation is governed by global factors and a measure of capacity utilization gap. It has indeed been found in the literature that inflation has become quite sensitive to international factors (Auer et al. (2017), Leduc and Wilso (2017), Blanchard (2018)), but now responds moderately to the output or unemployment gap (Belz et al. (2020)). Finally, we express the natural yield curve and the yield curve gap as a function of potential output growth and observable macroeconomic and financial variables that we believe drive the yield curve gap. Indeed, some papers suggest that macro-financial fundamentals contain valuable information that account for the impact of the term structure on future output (Bernard and Gerlach (1998)), Rouwenhorst and Plosser (1994), Smets (1997)). We therefore include two policy variables, indicators of the financial cycle and of the fiscal stance, the effective exchange rate, and the inflation rate as explanatory variables of the yield curve gap.

Our results are threefold. First, we find that both real and natural yields have globally decreased since the 1990s. The real and natural yield curves substantially flattened over the last thirty years, so that real yields now converge towards -1% as of 2019:Q4, while natural yields reach -1.30%. Second, the relatively small to negative yield curve gap observed during most of the sample contributed positively to real output, but on average not enough to bring it above potential. Indeed, the elasticity of the output gap with respect to the yield curve gap is found to be positive, indicating that the monetary easing ensuing a negative yield curve gap failed to generate a positive output gap. The yield curve gap nonetheless explains the output gap more than the short-term interest rate gap, in an environment where the nominal short-term rate in constrained by the zero-lower bound. Third, this persisting negative output gap, despite the easing monetary conditions, can explain why inflation has been so muted in Japan over the recent years. We find a strong sensitivity of inflation to the gap between capacity utilization and its potential, estimated to be negative during most of the sample. This indicates that the economy has mostly been running below capacity, which exerted downward pressure on consumer prices. However, it is noteworthy that few estimated coefficients are found to be statistically significant in the model. This weak identification of the parameters reflects the large uncertainty surrounding estimations of the natural rates usually found in the literature.

This paper builds on the rapidly growing literature on the estimation of the natural rate of interest, initiated by the seminal papers of Laubach and Williams (2003) and Holston et al. (2016). However, rather than focusing on the short-term natural rate as in most models

[2] The yield curve gap is defined as the difference between the real yield curve and the natural yield curve.

(see Brand et al. (2020)), we extend the estimation to the entire range of the yield curve and consistently estimate potential variables of the Japanese economy. To our knowledge, very few papers have gone in that direction. Three exceptions are Brzoza-Brzezina and Kotłowski (2014) for the U.S., Imakubo et al. (2017) for Japan, and Brand et al. (2020) for the Euro-zone. The first paper only estimates one factor of the neutral yield curve and therefore fails to estimate the whole yield curve gap. The second is a pure econometric estimation of the natural yield curve alone, and the yield curve gap is not analyzed in the perspective of the performance of the Japanese economy. Finally, the third paper nests the estimation of the natural yield curve in a model à la Laubach and Williams. It falls in the long-run view of the natural rate of interest, where only structural factors are assumed to drive the neutral rate. In contrast to these papers, we propose a semi-structural macroeconomic model that is general enough to capture the interactions between the yield curve gap, the output gap, and inflation, in an economy experiencing a prolonged period of recession and low levels of inflation. We also make several new contributions to the modeling of the neutral yield curve components, by providing a short-run view of a yield curve gap driven by macroeconomic fundamentals.

The rest of the paper is organized as follows. In **Section 2**, we present a brief historical overview of monetary policy in Japan since 1990s. In **Section 3**, we propose an estimation of the Japanese real yield curve using a dynamic Nelson-Siegel model. **Section 4** presents the semi-structural macroeconomic model and the joint estimation of the natural yield curve and potential variables. **Section 5** explains the results and **Section 6** concludes.

2 A brief history of monetary policy in Japan

Since the late 1990s, five monetary easing programs have been implemented in Japan: the Zero Interest Rate Policy (ZIRP), the Quantitative Easing policy (QE), the Comprehensive Monetary Easing policy (CME), the Quantitative and Qualitative Easing policy (QQE), and the Quantitative and the Qualitative Easing policy with yield curve. In each program, the Bank of Japan chose different targets as shown in **Table 1**.

Table 1: Monetary policy regimes in Japan

Period	Monetary policy	Target
1999/02 - 2000/06	ZIRP	Overnight call rate
2001/03 - 2006/02	QE	current account balances of BOJ
2010/10 - 2013/03	CME	Overnight call rate
2013/04 - 2016/09	QQE	monetary base
2016/09 - onward	QQE w. yc control	short-term and long-term rates

"ZIRP" stands for Zero Interest Rate Policy
"QE" stands for Quantitative Easing
"CME" stands for Comprehensive Monetary Easing
"QQE" stands for Quantitative and Qualitative Easing
"QQE w. yc control" stands QQE with yield curve control

In the 1980s, Japan experienced what was called an economic miracle. However, the appreciation of the domestic currency, coupled with higher inflationary pressures and uprising financial asset prices, led the BoJ to adopt a restrictive monetary policy. This resulted into a financial bubble burst. The central bank then turned to easing its policy by cutting its policy rate. Unfortunately and perhaps because this decision was taken lately, the economy sank into depression. This paved the way to the so-called *Lost Decade* (1991-2001), which was characterized by a low potential growth, low interest rates, and low inflation.

In this context, the Bank of Japan implemented the ZIRP in February 1999. This policy consisted in making the overnight call rate move at very low levels through the provision of higher amounts of liquidity injected in the financial markets. In the meantime, forward guidance intended to lower long-term interest rates. Note that if the role of forward guidance in driving long-term yields is not questioned in this paper. A focus is deliberately made on policies leading to a change in the policy rate and or in the monetary base because it is very difficult to capture the impact of central bank's communication. Such a task is beyond the scope of this paper.

The decline in the nominal interest rates engendered by the ZIRP was followed by a QE policy in March 2001, and the start of the purchase of medium- to long-term government bonds. The economy then rebounded during the 2000s until the rise of the 2008 financial crisis. From that moment, monetary authorities introduced several unconventional monetary easing measures known as CME, (that included the purchase of both sovereign bonds and risky assets from the private sector, such as corporate bonds and real estate investment trusts). In addition, the central bank engaged into a forward guidance policy and set an inflation target of 2% to anchor inflation expectations.

These measures were not enough to boost real activity and to raise inflation. The BoJ accordingly moved to a new policy called QQE in April 2013. The central bank pursued a large-scale 10-year Japanese government bond purchase in order to decrease long-term interest rates. It also continued to purchase risky assets in an effort to reduce risk premia.

In spite of the positive effects on real activity, inflation expectations remained low (below the 2% target). This led to a new policy know as QQE with yield curve control in September 2016. This new policy was coupled with price level targeting and an inflation overshooting commitment. Plus, under QQE the operating target are interest rates, which enables the BoJ to determine the amount of government bond to purchase in a flexible manner. No central bank has ever made such a commitment.

3 Estimation of the Japanese real yield curve

This section presents the term structure model used to decompose the Japanese real yield curve into latent factors, and analyzes them through the spectrum of the different monetary policy regimes implemented by the Bank of Japan since the 1990s.

3.1 Dynamic Nelson-Siegel model

The first step consists in estimating three real yield curve factors that will then be considered as observed variables in the semi-structural macroeconomic model of section **Section 4.1**. Especially, the Level L_t, the Slope S_t, and the Curvature C_t of the Japanese real yield curve are obtained using a Nelson-Siegel decomposition. The Level can be thought of describing the long-end of the curve, the Slope the short-end and the Curvature the medium part of the curve. Quite standard in the literature, such approach enables to prices yields of any maturity thanks to three factors and a scale parameter λ. It has been shown in the empirical literature that this decomposition can replicate most of the variations in the shape of the yield curves (see, among others, Diebold and Li (2006), Diebold and Rudebusch

(2013)), Joslin et al. (2014)).

The Dynamic Nelson-Siegel representation of an n-period zero-coupon bond yield is:

$$r_t^{(n)} = L_t + S_t \frac{1-e^{-n/\lambda}}{n/\lambda} + C_t \left(\frac{1-e^{-n/\lambda}}{n/\lambda} - e^{-n/\lambda} \right) + \epsilon_t^{(n)} \tag{1}$$

where $\epsilon_t^{(n)} \sim \mathcal{N}(0, \sigma^2)$, $t \in [\![1, T]\!]$, $n \in \mathbb{N}$ with n the maturity of the bond and $\lambda \in \mathbb{R}$ a scale parameter that determines where the "bow center" (i.e the maximal Curvature interest rate) is located.

Model-implied yields can be gathered in measurement equation:

$$Y_t = \Lambda X_t + \varepsilon_t, \tag{2}$$

where $Y_t = (r_t^{(n_1)}, r_t^{(n_2)}, ..., r_t^{(n_N)})'$ with N the number of yields observed, $X_t = (L_t, S_t, C_t)'$, $\varepsilon_t \sim \mathcal{MVN}(0, \Sigma_\varepsilon)$ with $\varepsilon_t = (\epsilon_t^{(n_1)}, \epsilon_t^{(n_2)}, ..., \epsilon_t^{(n_N)})'$ the vector of observation errors, and $\Lambda \in \mathcal{M}_{N\times 3}(\mathbb{R})$ the matrix of loadings filled with Equation (1).

Standard in the state-space modeling literature, the state equation defining the process of latent vector X_t is defined as a first-order autoregressive process:

$$X_t = \mu + \Theta X_{t-1} + \zeta_t, \tag{3}$$

where $\mu = (\mu_L, \mu_S, \mu_C)'$ is the vector of intercept, $\zeta_t \sim \mathcal{MVN}(0, \Sigma_\zeta)$ with $\zeta_t = (\xi_t^L, \xi_t^S, \xi_t^C)'$ and $\Theta \in \mathcal{M}_{N\times 3}(\mathbb{R})$ an unrestricted feedback matrix.

As is common practice (Diebold and Li (2006) and Imakubo et al. (2017)), we assume that the deviations of the observed real yields from the model-implied yields at various maturities are uncorrelated (i.e variance-covariance matrix $\Sigma_\varepsilon \in \mathcal{M}_{N\times N}(\mathbb{R})$ from Equation (2) is diagonal), and that ξ_t^L, ξ_t^S and ξ_t^C can be correlated (i.e $\Sigma_\zeta \in \mathcal{M}_{3\times 3}(\mathbb{R})$ from Equation (3) is left unrestricted). Details of the model are provided in **Appendix A.1**.

Besides, data are quarterly average of Japanese government zero-coupon bond yields of maturity 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 years from 1989:Q4 to 2019:Q4. Observed nominal yields are deflated with consumer price inflation expectations from surveys. In this framework, we obtain the real factors L_t, S_t, and C_t using a maximum likelihood estimation of the parameters through the Kalman filter. The filter is initialized with estimates from Imakubo et al. (2017).

3.2 Fitting the yield curve

The Nelson-Siegel approach provides a good fit of the term structure of interest rates, with a root-mean squared error of 0.22, as reported in **Table 7 - Appendix B.1** and observable on **Figure 8 - Appendix B.1**. In this section, **Figure 1** shows the yield surface over 1989:Q4-2019:Q4, while the yield curve factors under the different monetary policy regimes can be observed on **Figure 2**.

Strikingly, one can notice a global drop in real rates since the 1990s on **Figures 1**. This is confirmed by the downward trend in the Level factor of the yield curve, observable on **Figure 2**. Besides, the negativity of the Level since 2014:Q2 highlights the low level of long-term yields in Japan. It is consistent with the introduction of the QQE programme in

2013:Q3, aimed at lowering the long-end of the curve.

Moreover, the Japanese real yield curves were relatively flat, with an average Slope of -0.89% since the introduction of the ZIRP in 1999:Q2. Over the whole period studied, the average real interest rate is only at 0.31% at the shortest maturity and at 0.63% at the longest maturity, as shown in **Table 2**. The gap between the short and long-term rates has also been narrowing rapidly since the beginning of the 1990s, which reflects a flattening of the yield curve (also observable in **Figures 10** to **16** in **Appendix B.1** through the different monetary regimes). The negative Slope[3] of the curve presented on **Figure 2** still suggests an upward slopping real term structure during most of the sample, with the exception of a few quarters: 1989:Q4-1990:Q1, 1990:Q4-1991:Q3, 2017:Q3-2017:Q4, and 2019:Q3-2019:Q4. This is explained by the presence of a temporarily inverted nominal yield curve in 1989:Q4-1990:Q1, 1990:Q4-1991:Q3, 2019:Q3-2019:Q4, and by relatively low inflation expectations in 2017:Q3-2017:Q4.

The Curvature factor displays a higher variations than the other latent factors, which is what is usually found in the empirical literature. It is mostly negative throughout the sample, indicating a negative convexity of the yield curve. Besides, the estimated scale parameter λ corresponds to a maximum of the Curvature at 4 years (see **Figure 9 - Appendix B.1**), which is usual.

Finally, comparing the difference between the 1Y nominal and real interest rates on **Table 2** gives a view of inflation expectations in Japan. It is noteworthy that these were on average negative during the ZIRP, QE and CME regimes, thus reflecting the deflationary spiral that the Bank of Japan experienced during this period (average consumer price inflation of -0.29% annualized QoQ over 1999:Q1-2013:Q1). However, the pre-regimes, QQE and YCC periods saw positive inflation expectations, although still not reaching the inflation target of two percent for most horizons (average of 1.27% 1Y ahead, 1.24% 5Y ahead and 1.12% 10Y ahead over the QQE and YCC periods).

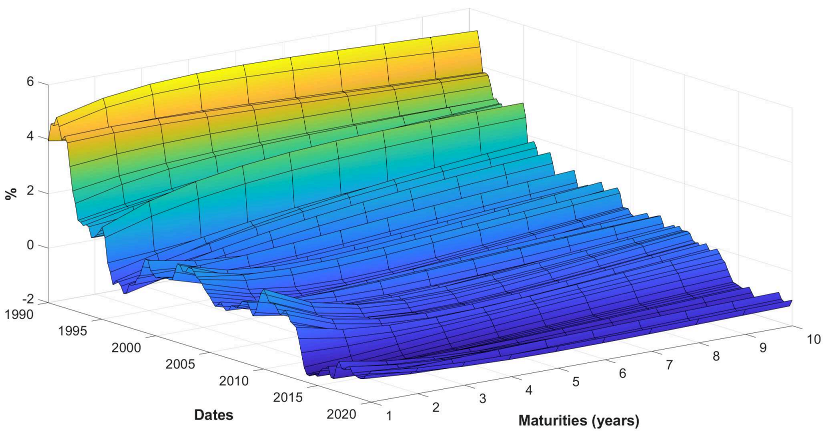
Table 2: Nominal and real levels of yields

	Nominal yield curve		**Real yield curve**	
Years	**1Y**	**10Y**	**1Y**	**10Y**
Pre-regimes (1989-1999)	2.82	4.20	1.39	2.40
ZIRP (1999-2000)	0.21	1.74	0.29	0.75
QE (2001-2006)	0.04	1.31	0.38	0.44
CME (2010-2013)	0.12	0.97	0.15	-0.12
QQE (2013-2016)	-0.01	0.40	-1.44	-1.02
YCC (2016-2020)	-0.18	0.01	-1.08	-1.19
Whole sample (1989-2020)	0.93	1.96	0.31	0.63

Average rates (%) during selected periods

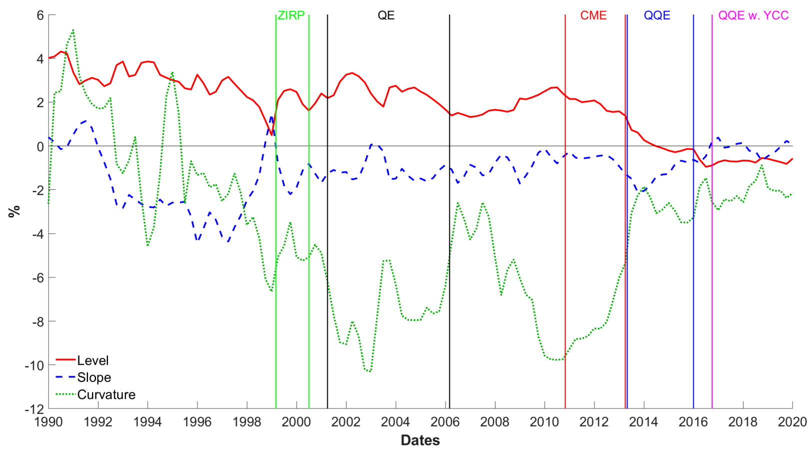
[3]By convention, the Slope is the difference between the short-end and the long-end of the curve.

Figure 1: Real government bond yield surface



Period 1989:Q4-2019:Q4

Figure 2: Nelson-Siegel factors under different monetary policy regimes in Japan



Period 1989:Q4-2019:Q4

4 A semi-structural macroeconomic model

Our approach to estimate the neutral yield curve draws on an empirical literature on the natural rate of interest, where time-varying neutral interest rates are obtained by estimating the investment-saving equilibrium in line with Wicksell (1898). Based on the seminal paper of Laubach and Williams (2003), our paper builds a semi-structural macroeconomic model with filtering techniques to recover the natural yield curve and potential variables. However, we depart from Laubach and Williams (2003) long-term view of the natural rate of interest.

4.1 Model

Most semi-structural macroeconomic models à la Laubach and Williams (2003) include an aggregate demand curve, in which the output gap is related to the interest rate gap, a Phillips curve, in which inflation depends on the output-gap, and an equation relating the neutral rate to potential growth and an unobserved factor. This unobserved factor is supposed to capture shifts in preferences, demographic determinants, changes in productivity, and global savings. This is the long-run view of the natural rate of interest.

In this paper, we take a more short-term view of the natural rate. We do not consider the natural rate of interest as the trend of the real rate, but we assume that the interest rates gap is correlated with the business cycle. This gap is carefully tracked by the central banker in order to close the output gap. As a result, our natural rates are not driven by an unobserved factor of global trends, but we specify observable policy and macroeconomic variables driving the interest rates gap.

In addition to the original framework of Laubach and Williams (2003), we innovate in several manners. First, we extend the concept of short-term natural rate of interest to all maturities. As in Imakubo et al. (2017), the yield curve gap therefore enters the aggregate demand curve, in replacement of the interest rate gap. However, we depart from this latter paper by nesting the estimation of the natural yield curve within a semi-structural macroeconomic model. Thus, our yield curve gap is consistent with the macroeconomic fundamentals of the Japanese economy. Second, we consider global factors as drivers of the Japanese consumer price inflation, as a response to the empirical evidence in the literature on the weakening of the Phillips curve relation. Third, we estimate the potential capacity utilization of the Japanese economy, as a measure of tensions of the market of goods and services from which upward pressure on prices appear.

4.1.1 State equations

Our semi-structural macroeconomic model contains six observed variables and six unobserved variables of interest. Standard in the literature, we nest our model in a state-space framework and we estimate the unobserved variables using the Kalman filter.

We start by specifying the process of the natural yield curve factors, as counterparts of the real yield curve factors estimated in the previous section. The natural Level $L_t^\star$, natural Slope $S_t^\star$, and natural Curvature $C_t^\star$ are defined as:

$$L_t^\star = \beta L_{t-1}^\star + c^L g_{t-1} + u_t^{L^\star} \tag{4}$$

$$S_t^\star = \delta S_{t-1}^\star + c^S g_{t-1} + u_t^{S^\star} \tag{5}$$

$$C_t^\star = \eta C_{t-1}^\star + c^C g_{t-1} + u_t^{C^\star} \tag{6}$$

where $u_t^{L^\star} \sim \mathcal{N}(0, u_{L^\star}^2)$, $u_t^{S^\star} \sim \mathcal{N}(0, u_{S^\star}^2)$, and $u_t^{C^\star} \sim \mathcal{N}(0, u_{C^\star}^2)$.

That is, the natural yield factors are driven by an autoregressive process and by potential growth. As opposed to Laubach and Williams (2003), we omit an unobserved factor z_t and we will instead specify observed macroeconomic variables driving the yield curve gap.

Potential output $y^\star$ is represented as the result of a unit root process with a time-varying drift:

$$y_t^\star = y_{t-1}^\star + g_{t-1} + u_t^{y^\star} \tag{7}$$

where $u_t^{y^\star} \sim \mathcal{N}(0, u_{y^\star}^2)$.

Potential growth g_t is assumed to follow an AR(1) process with a constant drift (possibly with a unit root, if $\phi_{g_1} = 1$):

$$g_t = \phi_{g_0} + \phi_{g_1} g_{t-1} + u_t^g \tag{8}$$

where $u_t^{g^\star} \sim \mathcal{N}(0, u_g^2)$.

We also innovate by estimating the potential capacity utilization, as a measure of saturation of the economy when the real yield curve matches its natural counterpart:

$$cap_t^\star = \chi_0 + \chi_1 cap_{t-1}^\star + u_t^{cap^\star} \tag{9}$$

where $u_t^{cap^\star} \sim \mathcal{N}(0, u_{cap^\star}^2)$.

4.1.2 Measurement equations

As a counterpart of our six state equations, we define six measurement equations to complete our state-space model.

The yield curve factor gaps are determined by autoregressive processes and key observable macroeconomic variables:

$$L_t - L_t^\star = \phi_L(L_{t-1} - L_{t-1}^\star) + (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6)Macro_{t-1} + w_t^L \tag{10}$$

$$S_t - S_t^\star = \phi_S(S_{t-1} - S_{t-1}^\star) + (\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6)Macro_{t-1} + w_t^S \tag{11}$$

$$C_t - C_t^\star = \phi_C(C_{t-1} - C_{t-1}^\star) + (\kappa_1, \kappa_2, \kappa_3, \kappa_4, \kappa_5, \kappa_6)Macro_{t-1} + w_t^C \tag{12}$$

where $w_t^L \sim \mathcal{N}(0, w_L)$, $w_t^S \sim \mathcal{N}(0, w_S)$, $w_t^C \sim \mathcal{N}(0, w_C)$, and
$Macro_{t-1} = (Policy_{t-1}, \Delta BOJbase_t, \Delta pb_{t-1}, \Delta Financial_t, \Delta REER_t, \pi_t)'$

The definition of the macroeconomic variables entering $Macro_{t-1}$ are the following: $Policy_{t-1}$ is the BoJ policy rate, $\Delta BOJbase_t$ is the change in the BoJ's monetary base (M3), Δpb_{t-1} is the lagged change in the primary fiscal balance, $\Delta Financial_t$ is the change in the current financial environment, $\Delta REER_t$ is the change in the real effective exchange rate, and π_t is the inflation rate. Our financial cycle index is the first principal component of stock prices growth, house prices growth, and credit to non-financial corporation growth.

The inclusion of the previous macroeconomic variables as drivers of the yield curve gap is motivated by the vast literature on the macroeconomic determinants of the yield curve (Bikbov and Chernov (2010), Evans and Marshall (2007), Rudebusch and Wu (2008), Chen and Tu (2018)). Indeed, not only their shocks, but also the variables themselves, affect the components of the yield curve. For instance, inflation is known to drive the Level of the real yield curve by changing long-run inflation expectations, while the Slope is quite sensitive to changes in the monetary policy. The literature also emphasizes the role of macro-financial fundamentals in carrying information that account for the impact of the term structure on future output: the exchange rate (Bernard and Gerlach (1998)), the inflation rate (Rouwenhorst and Plosser (1994)) and asset prices (Smets (1997)). The inclusion of a proxy for financial conditions is otherwise justified by our will to better capture the business cycle dynamics, as well as other aspects of monetary transmission not contained in our first two policy variables. Finally, fiscal policy is also believed to be an important determinant of the yield curve gap in Japan, in a environment where the Abenomics policy-mix was a major part of the strategy of the government to revive output growth and inflation.

Furthermore, the IS curve - written in terms of log-deviations of real GDP per capita y_t from its potential $y_t^\star$ - relates the output gap to the real yield curve gap, and not only the short-term interest rate gap (Imakubo et al. (2017)). It is also driven by changes in the government's primary fiscal balance (in percentage of nominal GDP), Δpb_{t-1}, to assess the stance of the government's fiscal policy. It includes an autoregressive term to capture the persistence of the output gap. It is defined by the following equation:

$$y_t - y_t^\star = \phi_1(y_{t-1} - y_{t-1}^\star) + \eta_{pb}\Delta pb_{t-1} + \mu \int_n \phi_n(r_{t-1}^{(n)} - r_{t-1}^{\star(n)})dn + w_t^y \tag{13}$$

where $w_t^y \sim \mathcal{N}(0, w_y^2)$, and $r_t^{(n)}$, $r_t^{\star(n)}$ are respectively the real and natural yields of maturity n.

A quick observation of this equation makes it clear that any policy impacting interest rate expectations, such as forward guidance, would drive the output gap $y_t - y_t^\star$ through a change in real long-term yields r_t^n. However, this is not in contradiction with the New-Keynesian theory. One can, for instance, think of a forward-looking IS curve in a simple linearized DSGE model, in with real output depends on the expected real short-term rates.

pb is the ratio of the primary fiscal balance to GDP. $\Delta pb > 0$ indicates a contractionary fiscal stance, while $\Delta pb < 0$ means that the fiscal stance is expansionary. We do not impose the constraint that $\eta_{pb} < 0$ to let the data speak. That is, fiscal policy may have Ricardian effects, where people anticipate higher taxes and postpone consumption following an increase in the deficit of the government primary balance. In addition, we use Δpb_{t-1} and not Δpb_t as the primary balance is usually computed on a yearly basis, so a fiscal impulse would be reported with at least a lag of a quarter.

μ is a parameter that describes the sensitivity of the output gap to the real yield curve gap and $\phi_n()$ is a weighting function of the interest rates gap of different maturities. For simplicity, we retain one lag to capture the influence of the yield curve. For purpose of simplicity we also assume that this function is described by a Uniform law. The uniform weighting amounts to assigning the same weight to the different interest rates so that no particular maturity has more or less influence on the output gap than another. While Brzoza-Brzezina and Kotłowski (2014) assume a uniform law, Imakubo et al. (2017) consider various types of distribution: a uniform, a step, and a mix of two betas distributions. They conclude that although a decreasing function, the sensitivity of the output gap to the interest rate gap is only lower than in the case of a uniform function for yields with a maturity higher than 10 years.[4] As the maximum maturity considered in this paper is 10 years, we stick to the uniform distribution for tractability.

Using the Nelson-Siegel decomposition of **Section 3.1** and the assumption that the real and natural yield curves have the same scale parameter λ, we can rewrite Equation (13) as:

$$\begin{aligned} y_t - y_t^\star = {} & \phi_1(y_{t-1} - y_{t-1}^\star) + \eta_{pb}\Delta pb_{t-1} \\ & + \mu_L(L_{t-1} - L_{t-1}^\star) + \mu_S(S_{t-1} - S_{t-1}^\star) + \mu_C(C_{t-1} - C_{t-1}^\star) + w_t^y \end{aligned} \tag{14}$$

where it can be shown that, under a uniform distribution, function (Imakubo et al. (2017)):

$$\begin{cases} \mu_L = \mu \int_n dn = \mu \times \phi_{n,L} = \mu \\ \mu_S = \mu \int_n \frac{1-e^{-n/\lambda}}{n/\lambda} dn = \mu \times \phi_{n,S} \\ \mu_C = \mu \int_n \left(\frac{1-e^{-n/\lambda}}{n/\lambda} - e^{-n/\lambda}\right) dn = \mu \times \phi_{n,C} \end{cases}$$

[4] See for example Galí and Gertler (2007) for a micro-founded analysis of the role of interest rate expectations in driving real activity.

where $\phi_{n,L}, \phi_{n,S}$ and $\phi_{n,C}$ come from function ϕ_n in Equation (13), and N is the maximum maturity (10 years here).

Moreover, our equation governing consumer price inflation can be described as a backward-looking type of Phillips curve relation:

$$\pi_t = d_\pi \pi_{t-1} + \eta_1(cap_{t-1} - cap^\star_{t-1}) + \eta_2 \pi^{wages}_{t-1} + \eta_3 \pi^{US,PPI}_{t-1} + \eta_4 TOT^{emerg}_{t-1} + \eta_5 \pi^{energy}_{t-1} + w^\pi_t \quad (15)$$

where $w^\pi_t \sim \mathcal{N}(0, w_\pi)$.

In this relation, inflation π_t is explained by lagged inflation π_{t-1}, lagged capacity utilization gap $cap_{t-1} - cap^\star_{t-1}$ and some variables that reflect the key determinants of the decline in historical inflation found in the literature: lagged wage inflation π^{wages}_{t-1}, lagged U.S. producer price inflation $\pi^{US,PPI}_{t-1}$, lagged emerging countries' terms of trade TOT^{emerg}_{t-1}, and lagged energy inflation π^{energy}_t.

This formulation broadly reflects the main findings of the empirical literature on Phillips curves. Firstly, a consensus seems to emerge in the literature about a weakening of the relationship between inflation and unemployment gap in the U.S. (Belz et al. (2020)), thereby implying that the latter is not necessarily a good predictor of the former. As early as 2004, some economists pointed out the same phenomenon observable in Japan (Mourougane and Ibaragi (2004)). One implication is that demand-pull inflation is better described by an equation relating price changes to an indicator of tensions in the market of goods and services. Capacity utilization is a reliable indicator of such inflationary pressures (Garner (1994)). In our Phillips curve type of relation, we therefore input the capacity utilization gap instead of the output gap.

Secondly, wage inflation is taken as a proxy of the role of unionization in the bargaining process in the workers-employers relations. The bargaining power of employees has decreased over time and this provided more leverage to firms to hold down wages (Kato (2016)). Such decrease in downward wage rigidity may be part of the explanation for the muted inflation observed in Japan over the past thirty years.

Thirdly, there are some empirical evidence that domestic inflation rates in open economies are determined by global factors (Auer et al. (2017), Leduc and Wilso (2017), and Blanchard (2018) among others). Such factors are, for instance, international prices -captured here by the producer price index in the US, $\pi^{US,PPI}$-, the emerging countries' terms of trade -here TOT^{emerg}-, and energy prices -here π^{energy}.

Finally, the formalization of the capacity utilization gap present in Equation (15) is standard. It has an autoregressive component $cap_{t-1} - cap^\star_{t-1}$ with a drift and depends on the output gap $y_{t-1} - y^\star_{t-1}$:

$$cap_t - cap^\star_t = \theta_0 + \theta_1(cap_{t-1} - cap^\star_{t-1}) + \theta_2(y_{t-1} - y^\star_{t-1}) + w^{cap}_t \quad (16)$$

where $w^{cap}_t \sim \mathcal{N}(0, w^2_{cap})$.

The variable *cap* refers to capacity utilization and $cap^\star$ to the long-term capacity utilization, which can be thought of as a non-accelerating inflation rate capacity utilization (NAICU).

4.2 Estimation

State equations (4) to (9) and measurement equations (10), (11), (12), (14), (15), and (16) are gathered in a state-space framework for estimation.

The state equation is written in compact form as:

$$X_t = \alpha + AX_{t-1} + U_t \tag{17}$$

where $X_t = (L_t^\star, S_t^\star, C_t^\star, y_t^\star, g_t, cap_t^\star)$, $A \in \mathcal{M}_{6\times6}(\mathbb{R})$, and $U_t \sim \mathcal{MVN}(0, \Omega_1)$ with $\Omega_1 \in \mathcal{M}_{6\times6}(\mathbb{R})$ a diagonal variance-covariance matrix. Details of matrices α, A and Ω_1 are presented in **Appendix A.2**.

The measurement equation is written as:

$$Y_t = \beta + BY_{t-1} + CZ_{t-1} + DX_t + EX_{t-1} + W_t \tag{18}$$

where $Y_t = (L_t, S_t, C_t, y_t, \pi_t, cap_t)'$, $Z_{t-1} = (Policy_{t-1}, \Delta BOJbase_t, \Delta pb_{t-1}, \Delta Financial_t, \Delta REER_t, \pi_{t-1}^{wages}, \pi_{t-1}^{US,PPI}+, TOT_{t-1}^{emerg}, \pi_{t-1}^{energy})'$, $B \in \mathcal{M}_{6\times6}(\mathbb{R})$, $C \in \mathcal{M}_{6\times9}(\mathbb{R})$, $D \in \mathcal{M}_{6\times6}(\mathbb{R})$, $E \in \mathcal{M}_{6\times6}(\mathbb{R})$, and $W_t = (W_t^L, u_t^S, u_t^C, u_t^y, u_t^\pi, u_t^{cap})'$ with $U_t \sim \mathcal{MVN}(0, \Omega_2)$ and $\Omega_2 \in \mathcal{M}_{6\times6}(\mathbb{R})$ a diagonal variance-covariance matrix. Details of matrices β, B, C, D, E and Ω_2 are presented in **Appendix A.2**.

In this framework, we estimate $L_t^\star$, $S_t^\star$, $C_t^\star$, $y_t^\star$, g_t, and $cap_t^\star$ through the Kalman filter using a maximum a-posteriori estimation of the parameters of the model.[5] Priors as well as the initialization of the Kalman Filter come from OLS regressions based on the trend of L_t, S_t, C_t, y_t and cap_t extracted from an HP-filter. No sign restriction is imposed on any parameter in order to let the data speak. Besides, as measurement equation (18) contains both X_t and X_{t-1}, we use Qian (2014) derivation of the Kalman gain. Data sources are presented in **Appendix C**.

5 Results

In this section, we analyze the results from the semi-structural macroeconomic model introduced in **Section 4**. We here summarize our main findings but complementary results can be found in **Appendix B.2**.

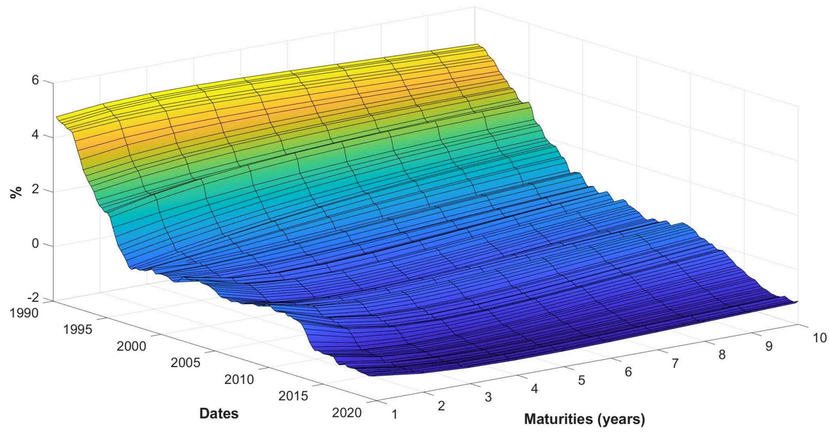
5.1 The natural yield curve

Figure 3 shows the natural yield surface throughout the sample period (1990:Q1-2019:Q4). Similarly to real rates, we observe a downward trend in the natural rates at all maturities. This finding suggests that results in the literature on the historical decline of the neutral short-term interest rates (see, for instance, Holston et al. (2017), and Fujiwara et al. (2016)) also apply to medium- and long-term yields. This is also consistent with Imakubo et al. (2017) that find a decrease in the Level and a flattening of the neutral curve through an increase in its Slope (see **Figure 4**). Despite its flattening, it is noteworthy that the natural yield curve has always been upward slopping throughout the sample, except before 1992:Q2.

[5]A maximum a-posteriori estimation is a maximum likelihood estimation that is penalized by prior beliefs of the econometrician. It enables to let the data speak as much as in a maximum likelihood estimation, while still being possible to input priors as in a full Bayesian method. The reader can refer to Särkkä (2013) for technical details.

Overall, our natural yield factors have the same dynamics as in Imakubo et al. (2017), but display a bit more of variation, especially when comparing the natural curvatures.

Figure 3: Natural yield surface



Period 1990:Q1-2019:Q4

Comparing **Table 3** with **Table 2 - Section 3.2** gives an indication as to whether monetary conditions were loose or tight during the different monetary regimes. On average, the real short-term rate was above the natural rate during the ZIRP (1999:Q1-2000:Q2), QE (2001:Q1-2006:Q1), and CME (2010:Q4-2013:Q1) periods, implying tight short-term monetary conditions. It was below its natural counterpart during the pre-regime (1990:Q1-1998:Q4) and QQE (2013:Q2-2016:Q3) periods, and roughly at the natural level during the QQE with yield curve control period (2016:Q3-2019:Q4). However, a quick look at the same tables for the 10Y rates indicates that long-term monetary conditions were not necessarily matching short-term ones. This is confirmed by comparing the short-term interest rate gap with the yield curve gap on **Figure 30 - Appendix B.2**. The short-term interest rate gap, alone, is thus not an indicator sufficient enough to evaluate monetary conditions.

Table 3: Natural levels of the 1Y and 10Y yields

	Neutral yield curve	
Years	**1Y**	**10Y**
Pre-regimes (1989-1999)	2.09	2.94
ZIRP (1999-2000)	0.20	0.94
QE (2001-2006)	0.24	0.44
CME (2010-2013)	0.08	-0.07
QQE (2013-2016)	-0.84	-0.71
YCC (2016-2019)	-1.10	-1.18
Whole sample (1989-2019)	0.49	0.84

Average rates (%) during selected periods

Indeed, the real level, Slope, and Curvature fluctuated around their natural levels throughout the sample (see **Figures 23**, **24** and **25** in **Appendix B.2**). A closer look at the yield curve gap across the different monetary policy regimes (**Figures 17** to **22** in **Appendix B.2**) is therefore required to assert that monetary conditions were loose or tight overall according to our model.

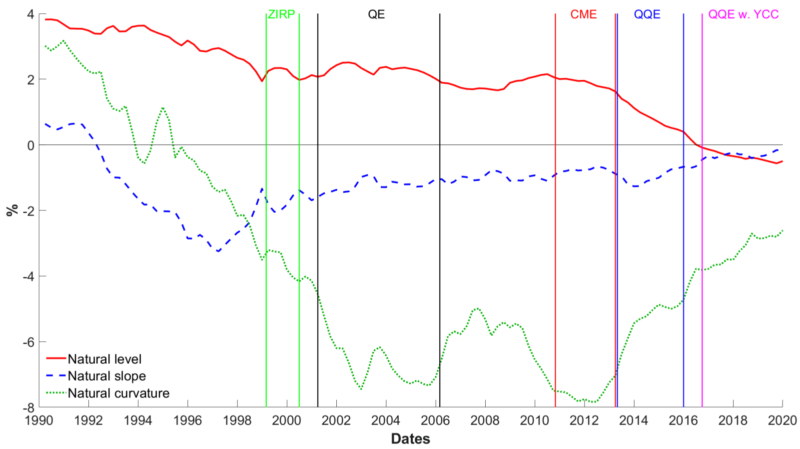
If the short-term real rate was on average above its natural counterpart during the ZIRP period (1999:Q1-2000:Q2), the interest rate gap was substantially negative for all other maturities. Looking at **Figure 29**, we can notice that the negative output gap has been drastically narrowing during the ZIRP period, increasing from -8.47% in 1999:Q1 to +0.44% in 2000:Q2. The positive short-term interest rate gap alone thus contradicts the improving economic activity during this period, and the whole yield curve gap is a better indicator of the monetary conditions.

A similar analysis can be made for the following QE (2001:Q1-2006:Q1) and CME (2010:Q4-2013:Q1) periods: the interest rate gap was positive for short to medium yields, and negative or null for medium to long-term maturities. During the first part of the QE period (2001:Q1-2003:Q4), the yield curve gap switched sign, leading the output gap to alternatively narrow or widen. The yield curve gap then turned negative in 2004:Q1, leading the output gap to reach a high point of +2.86% in 2004:Q3 thanks to easing monetary conditions. During the CME period, a comparable change in the sign of the yield curve gap lead the output gap to fluctuate from -3.75% in 2011:Q2 to +2.32% in 2012:Q1.

Interestingly, the massive drop of -14.73% in the output gap during the Great Financial Crisis was not coupled with a large positive interest rate or yield curve gap. Illustrating that the sharp decline in economic activity did not come from tight monetary conditions but rather from an exogenous shock, the recession lead to a big deterioration in the financial cycle (**Figure 27**) and a strong deflation (**Figure 28**).

Furthermore, the negative yield curve gap of the QQE period (2013:Q2-2016:Q3) saw an average output gap of +1.03%, coupled with improving financial conditions and an annualized QoQ inflation peaking at +3.60% in 2014:Q2. The real yield curve was indeed much lower than its natural counterpart (**Figure 21**). As Imakubo et al. (2017) found, it seems that this monetary regime was the one that implied the loosest monetary conditions. Output was also mostly above potential during the QQE with yield curve control period of (2016:Q3-2019:Q4). **Figure 22 - Appendix B.2** tends to show that this latter monetary regime was quite successful at tracking the natural yield curve. Indeed, the gap is negative or null at the short and long-end of the curve. Nevertheless, the positive yield gap at the medium-end of the curve can be attributed to more muted inflation expectations at a medium-term horizon.

Figure 4: Natural factors under different monetary policy regimes in Japan



Period 1990:Q1-2019:Q4

Unfortunately, the present semi-structural model and its estimation via maximum a-posteriori makes it hard to generate confidence intervals for the obtained natural yield curve factors. On the one hand, uncertainty surrounding natural rate estimates coming from models estimated in a Bayesian way are usually very high, but this approach leaves some flexibility regarding the values of the parameters. On the other hand, calibrated models usually produce no uncertainty around the estimates, but results largely rely on the economist's priors regarding the values of the parameters. The maximum a-posteriori estimation employed here lies in between: it is unable to produce any uncertainty regarding the results, but these are mostly driven by the data.

5.2 Determinants of the yield curve gap

Table 4 presents the estimated parameters of the macroeconomic determinants of the yield curve gap, while the full estimation of the parameters is shown in **Table 8** and **9** in **Appendix B.2**. This table provides information about some structural factors that have contributed to the decline in the natural rates. Some factors are Japan-specific (monetary and fiscal policies, inflation), others are global factors (real effective exchange rate and financial cycle). To interpret the coefficients, we remind that the endogenous variables are different components of the yield curve gap. When the real and neutral interest rates match, output converges to its natural level. A positive yield curve gap may signal a forthcoming recession, while in the case of a negative gap, the output-gap is likely to become positive.

It is noteworthy that very few coefficients of the observable macroeconomic variables driving the yield curve factors gap seem to be statistically significant in **Table 4**. This is due to the large uncertainty surrounding the estimates of the natural rates. Indeed, the big standard errors associated to the estimated parameters lead to high p-values. This phenomenon is usually hidden by nature in full Bayesian estimations and little mentioned in the literature.[6] We will therefore take the statistical significance of the parameters with careful consideration.

Regarding the role of monetary policy, **Table 4** suggests that the use of the policy rate

[6] A complementary explanation can come from the difficulty of Matlab's optimization function to numerically approximate the big hessian matrix (55×55) of the model. Uncertainty in the inversion of the hessian matrix then translates into large standard errors.

did not make monetary policy accommodative enough in the long-term. Indeed, the coefficients of the policy rate variable is negative for the Level gap. This means that a fall in the policy rate has, on average, led to a rise in long-term real rates above their neutral rates, which has a contractionnary effect on output. However, monetary policy did manage to make short to medium-term monetary condition more accomodative on average, as seen by the positive coefficients of the policy rate for the Slope and Curvature gaps.

The estimates also suggest that the BoJ's balance sheet expansion policy has not had a beneficial effect on economic activity when considering the yield curve gap. Indeed, the coefficient is positive in the level, Slope, and Curvature equations, indicating that an increase in the monetary base translates into a rise in the real yield curve above its natural level. It is important to note that, unlike the U.S. Federal Reserve (Fed) or the European Central Bank (ECB), the expansion of the BoJ's balance sheet did not just occur at the time of the QE policies, but had begun as early as 1997 to mitigate the effects of the Asian financial crisis. The average growth in M3 between 1989:Q4 and 1998:Q4 is indeed of 3.49%, whereas it only reaches 1.85% between 1999:Q1 and 2019:Q4. Most of the balance sheet expansion of the BoJ therefore happened outside of the monetary policy regimes studied here, which renders difficult the interpretation of the sign of the coefficients.

Since fiscal policy is a component of global demand, a tightening of the fiscal policy lowers the equilibrium interest rates. How the yield factors gap react to this change in the fiscal stance depends on the type of the fiscal retrenchment. If a cut in public spending is decided, then a drop in investment is likely to affect the long-end of the yield curve gap. The natural Level should therefore go down and the Level gap should increase. If a tax raise is decided, then the short-end of the yield curve gap is likely to be more impacted because of the increase in global savings due to Ricardian behaviors (the short-term natural rate goes down). As a consequence, the natural Slope should increase and the Slope gap should decrease. In the estimation, a fiscal retrenchment is measured by an increase in primary fiscal balance. Consistent with this interpretation, we see that the sign of the coefficient for the change in the primary balance is positive for the Level (a cut in public spending leads to a increase in the Level gap), but negative for the Slope (a tax increase leads to a reduction in the Slope gap).

Regarding the role of the financial cycle, a bubble burst, (or a negative change in the financial index), implies a lower return of investments in stocks, and housing, as well as less availability of credit. This can lower the investment-saving equilibrium rates because of the impact of a lower investment and a higher saving. The negative estimates of the coefficient of the change in the financial cycle in the Level and Slope equations suggest that such effect was on average at stake during our sample.

The ZIRP and QE policies had two effects on the nominal and real exchange rates in Japan. During the ZIRP policy (1999:Q1-2000:Q2), the nominal and real exchange rate of the Yen fell sharply because of the BoJ's objective of targeting the level of foreign exchange reserves. However, exchange rate developments changed after the adoption of the QE policies (2001:Q1-2006:Q1), because the ECB and the Fed initiated massive asset repurchases in much larger proportions than the BoJ, leading to a real and nominal appreciation of the Yen. This can explain why an appreciation of the real effective exchange rate (i.e. a decrease in REER) has a stimulating effect on activity, by pushing the real Level and Slope below their natural counterparts.

Finally, inflation has a negative effect on the Level and Slope gaps: an increase in consumer price growth leads to an increase in inflation expectations and a decrease in real yields. This reduces the gap between the real yield curve and the natural yield curve.

Table 4: Determinants of the yield curve gap

Level gap	**Coefficients**
Policy rate	$\alpha_1 = -0.008$
Change in BOJ base money	$\alpha_2 = 0.013$
Change in overall budget balance	$\alpha_3 = 0.035$
Change in Financial cycle	$\alpha_4 = -0.005$
Change in REER	$\alpha_5 = 0.018$
Inflation	$\alpha_6 = -0.029$
Slope gap	**Coefficients**
Policy rate	$\gamma_1 = 0.008^{***}$
Change in BOJ base money	$\gamma_2 = 0.008$
Change in overall budget balance	$\gamma_3 = -0.157^{***}$
Change in Financial cycle	$\gamma_4 = -0.028$
Change in REER	$\gamma_5 = 0.027$
Inflation	$\gamma_6 = -0.019$
Curvature gap	**Coefficients**
Policy rate	$\kappa_1 = 0.027$
Change in BOJ base money	$\kappa_2 = 0.028$
Change in overall budget balance	$\kappa_3 = 0.235^{***}$
Change in Financial cycle	$\kappa_4 = 0.033$
Change in REER	$\kappa_5 = -0.132^{*}$
Inflation	$\kappa_6 = 0.049$

Period 1990:Q1-2019:Q4

*** means that the coefficient is statistically significant at a 1% level

* means that the coefficient is statistically significant at a 10% level

5.3 Influence of the yield curve gap on the macroeconomic variables

Tables 5 and 6 present the estimates of the coefficient entering the aggregate demand and aggregate supply Equations (14) and (15). To save space, we present a selection of charts: potential output (**Figure** 5), potential growth (**Figure** 6), and potential capacity utilization (**Figure** 7). Besides, **Figures** 26, 27, 28 and 29 in **Appendix** B.2 show the potential output growth, the financial cycle, inflation and the output gap with the yield curve gap.

The yield curve gap coefficient $\mu = \mu_L$, enters the aggregate demand Equation (14) with a positive sign. This indicates that a reduction in the yield curve gap has had positive effect on the output gap by increasing real GDP, but most of the time still below its potential. Why this phenomenon is found to be happening on average during the period studied remains an open question. However, negative demand shocks and a strong price rigidity can be thought of potential suspects. Besides, the Level gap seems to matter more than the Slope gap, which matters more than the Curvature gap for economic activity ($\mu_L > \mu_S > \mu_C$). This means that the output gap is more driven by long-end of the curve than by the short-end, in an environment where the volatility of nominal short-term yields is low due to the zero-lower bound constraint.

The coefficient of changes in fiscal balance η_{pb} captures the behavior of governments when they commit to maintain the same orientation of their fiscal policy over several consecutive periods. According to our estimates, if fiscal balance is initially in surplus and the government decides to increase this surplus further by 1%, this makes real GDP increase from above its potential level by 0.1%. Symmetrically, an additional increase of the deficit by 1% reduces output by 0.1% below its potential. There are two alternative explanations to this finding.

The first interpretation is that, over the whole period, fiscal policies have had non-Keynesian effects. This issue was one of the main topics of fiscal policy debated in Japan during the 1990s, and it has regained in popularity in recent years. The usual hypothesis is that consumption has remained stagnant due to Barro-Ricardo effects, because people had concerns over fiscal sustainability. To avoid an explosion of future debt, fiscal policy has then been restrictive over several periods.

An alternative explanation is that expansionary fiscal policies have had Keynesian effects, by increasing the output, but mostly below potential GDP. **Figure 5** shows the level of real GDP, the estimated potential GDP and, for comparative purpose, the trend computed from an HP-filter. The historical real GDP is very often below its potential. This is a typical characteristic of the Japanese economy during the *Lost Decade* and the Great Recession: as seen on **Figure 29**, the output-gap has remained negative from 1992:Q2 to 2003:Q3 and from 2008:Q3 to 2011:Q2, with an average of -0.24% over 1990:Q1-2019:Q4. Hence, this reflects an economy has been operating under potential capacity utilization. This is confirmed by analyzing **Figure 7**.

Figure 6 shows the estimated potential growth. Outside of the GFC period, is has mostly been fluctuating between +0.20% and +1.5%, with an average of +0.84% over 1990:Q1-2019:Q4. This low level reflects the situation of the Japanese economy: potential growth is not pushed up by an increase in hours worked in a society were the population is aging, the birth rate is low, and there is no immigration. It is also neither boosted by a vigorous rise in capital stock, nor by total factor productivity, whose positive contribution has been declining over the years (Kawamoto et al. (2017)).

Furthermore, coefficients of inflation in Equation (15) are all positive, with the exception of the one for the emerging countries' terms of trade. For instance, cost-push inflation is caused by a rise in U.S. producer prices ($\eta_3 > 0$), energy prices ($\eta_5 > 0$), and a decrease in the terms of trade caused by a rise in import prices ($\eta_4 < 0$). Demand-pull inflation stems from wage inflation ($\eta_2 > 0$) and production over-capacities. The strong link between the potential capacity utilization gap and inflation ($\eta_1 = 1.311$) may explain why inflation has been so muted in Japan over the sample. According to our estimates, a 1% increase in capacity utilization above its potential leads to a $1.31/100 = 0.0131\%$ increase in inflation.[7] As we find an average capacity utilization gap of -4.91% over 1990:Q1-2019:Q4 (**Figure 7**), this indicates that the economy was running below capacity on average and that inflation was therefore decelerating. Furthermore, we calculate an average non-accelerating inflation rate capacity utilization at 94%, higher than the 82% usually found for the U.S. (Emery and Chang (1997)). This high number reinforces the idea that there has been low to negative pressures on consumer prices and an absence of tensions on the market of goods and services.

[7] Our measure of capacity utilization is initially divided by 100 and centered around zero instead of 100.

Table 5: Determinant of the output gap

Variables	Coefficients
Lagged output gap	$\phi_1 = 0.716^{***}$
Change in fiscal balance	$\eta_{pb} = 0.001$
Level gap	$\mu_L = 0.004$
Slope gap	$\mu_S = 0.002$
Curvature gap	$\mu_C = 0.001^{*}$

Period 1990:Q1-2019:Q4

*** means that the coefficient is statistically significant at a 1% level

* means that the coefficient is statistically significant at a 10% level

Table 6: Determinant of inflation

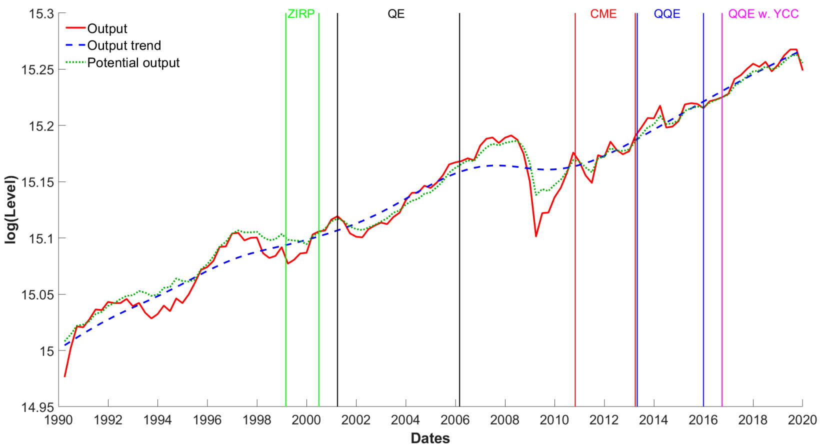
Variables	Coefficients
Lagged inflation	$\mathrm{d}_\pi = 0.825^{***}$
Capacity utilization gap	$\eta_1 = 1.311^{***}$
Wage inflation	$\eta_2 = 0.013$
US PPI inflation	$\eta_3 = 0.024^{*}$
Terms of trade emerging countries	$\eta_4 = -0.010$
Energy prices	$\eta_5 = 0.011$

Period 1990:Q1-2019:Q4

*** means that the coefficient is statistically significant at a 1% level

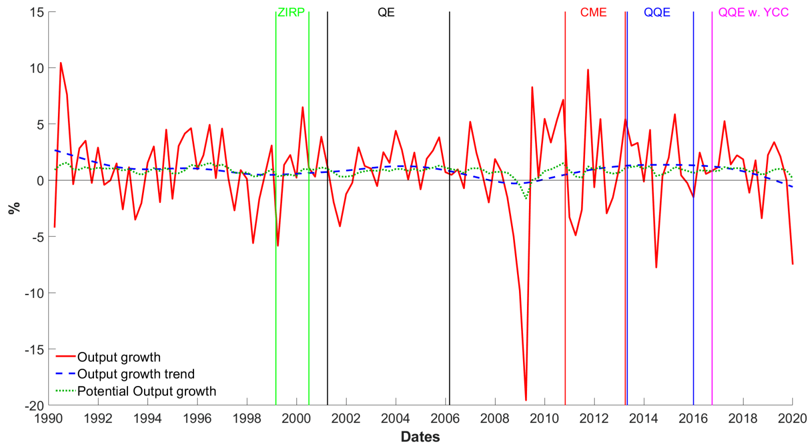
* means that the coefficient is statistically significant at a 10% level

Figure 5: Potential Output



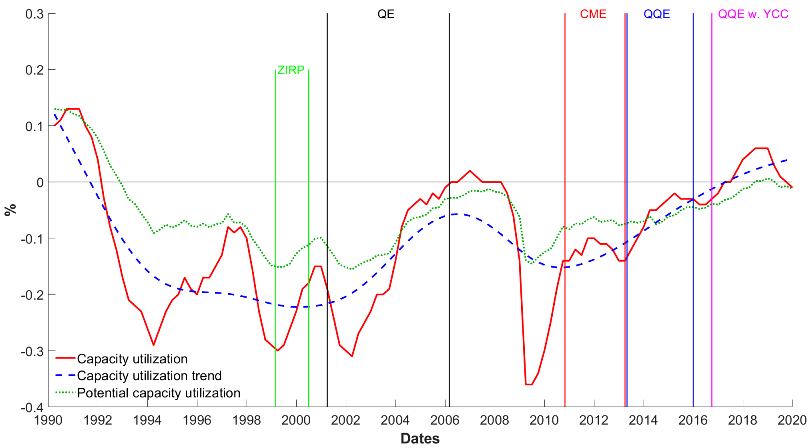
Period 1990:Q1-2019:Q4

Figure 6: Potential growth



Period 1990:Q1-2019:Q4

Figure 7: Potential capacity utilization



Period 1990:Q1-2019:Q4

6 Conclusion

This paper extends the concept of short-term natural rate of interest to all maturities, and consistently estimates the natural yield curve and potential variables in Japan. Three real yield curve factors are first extracted from a Dynamic Nelson-Siegel model. These factors are then plugged in a semi-structural model with some observed macroeconomic variables of interest. Using filtering techniques, natural yield curve factors, potential output, potential growth, and potential capacity utilization of the Japanese economy are jointly estimated for the period 1990-2020.

The main findings are the following: firstly, we find that both real and natural yields have globally decreased since the 1990s. The real and natural yield curves substantially flattened over the last thirty years and they are now in negative territory. Secondly, the relatively small to negative yield curve gap observed during most of the sample contributed positively to output, but on average not enough to bring it above its potential. The yield curve gap nonetheless explains the output gap more than the short-term interest rate gap, in an environment where the nominal short-term rate in constrained by the zero-lower bound. Thirdly, the persisting negative output gap, despite the easing monetary conditions, can explain why inflation has been so muted in Japan over the recent years.

The next step to this paper is to do a similar exercise for the other industrialized countries for purpose of comparison. We leave that for future research.

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Appendices

A Models

A.1 Dynamic Nelson-Siegel model

Here, we present the state-space model for the Dynamic Nelson-Siegel model estimated with the Kalman Filter:

State Equation:

$$X_t = \mu + \Theta X_{t-1} + \zeta_t \quad \text{with} \quad \zeta_t \sim \mathcal{MVN}(0, \Sigma_\zeta) \tag{A.1}$$

Measurement Equation:

$$Y_t = \Lambda X_t + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim \mathcal{MVN}(0, \Sigma_\varepsilon) \tag{A.2}$$

where

$$\mu = \begin{pmatrix} \mu_L \\ \mu_S \\ \mu_C \end{pmatrix} \quad \Theta = \begin{pmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{pmatrix} \quad \Lambda = \begin{pmatrix} 1 & \frac{1-e^{-n_1/\lambda}}{n_1/\lambda} & \frac{1-e^{-n_1/\lambda}}{n_1/\lambda} - e^{-n_1/\lambda} \\ 1 & \frac{1-e^{-n_2/\lambda}}{n_2/\lambda} & \frac{1-e^{-n_2/\lambda}}{n_2/\lambda} - e^{-n_2/\lambda} \\ \vdots & \vdots & \vdots \\ 1 & \frac{1-e^{-n_N/\lambda}}{n_N/\lambda} & \frac{1-e^{-n_N/\lambda}}{n_N/\lambda} - e^{-n_N/\lambda} \end{pmatrix}$$

$$\Sigma_\zeta = \begin{pmatrix} \sigma_{\zeta,11} & \sigma_{\zeta,12} & \sigma_{\zeta,13} \\ \sigma_{\zeta,21} & \sigma_{\zeta,22} & \sigma_{\zeta,23} \\ \sigma_{\zeta,3} & \sigma_{\zeta,32} & \sigma_{\zeta,33} \end{pmatrix} \quad \Sigma_\varepsilon = \begin{pmatrix} \sigma_{\varepsilon,11} & 0 & \dots & 0 \\ 0 & \sigma_{\varepsilon,22} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & \sigma_{\varepsilon,NN} \end{pmatrix} \tag{A.3}$$

A.2 Semi-structural macroeconomic model

The following presents the semi-structural macro model nested in a state-space model:

State equation:

$$X_t = \alpha + AX_{t-1} + V_t \quad \text{with} \quad U_t \sim \mathcal{MVN}(0, \Omega_1) \tag{A.4}$$

Measurement equation:

$$Y_t = \beta + BY_{t-1} + CZ_{t-1} + DX_t + EX_{t-1} + W_t \quad \text{with} \quad W_t \sim \mathcal{MVN}(0, \Omega_2) \tag{A.5}$$

where: $\alpha = (0, 0, 0, 0, \phi_{g_0}, \chi_0)'$, $\quad A = \begin{pmatrix} \beta & 0 & 0 & 0 & c^L & 0 \\ 0 & \delta & 0 & 0 & c^S & 0 \\ 0 & 0 & \eta & 0 & c^C & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & \phi_{g_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \chi_1 \end{pmatrix}$

$\Omega_1 = diag(u_{L^\star}, u_{S^\star}, u_{C^\star}, u_{y^\star}, u_g, u_{cap^\star})$

$\beta = (0, 0, 0, 0, 0, \theta_0)'$, $\quad B = \begin{pmatrix} \phi_L & 0 & 0 & 0 & \alpha_6 & 0 \\ 0 & \phi_S & 0 & 0 & \gamma_6 & 0 \\ 0 & 0 & \phi_C & 0 & \kappa_6 & 0 \\ \mu_L & \mu_S & \mu_C & \phi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_\pi & \eta_1 \\ 0 & 0 & 0 & \theta_2 & 0 & \theta_1 \end{pmatrix}$,

$$C = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 & 0 & 0 & 0 & 0 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & 0 & 0 & 0 & 0 \\ \kappa_1 & \kappa_2 & \kappa_3 & \kappa_4 & \kappa_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & \eta_{pb} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$E = \begin{pmatrix} -\phi_L & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_S & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_C & 0 & 0 & 0 \\ -\mu_L & -\mu_S & -\mu_C & -\phi_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\eta_1 \\ 0 & 0 & 0 & -\theta_2 & 0 & -\theta_1 \end{pmatrix}, \quad \Omega_2 = diag(w_L, w_S, u_C, w_y, w_\pi, u_{cap})$$

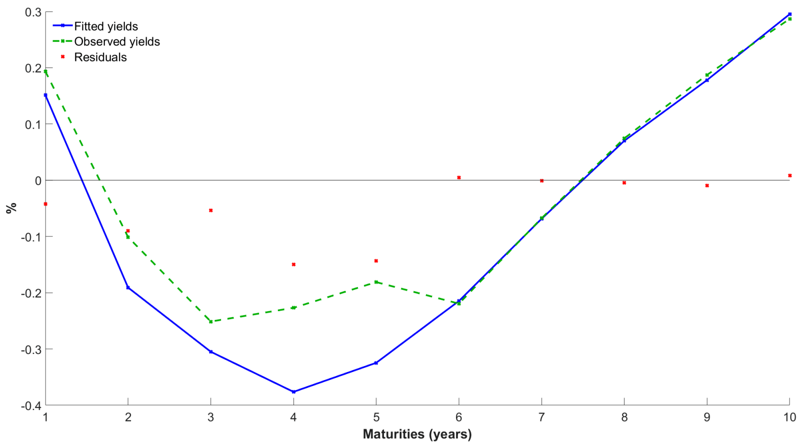
B Additional results

B.1 Dynamic Nelson-Siegel model

Table 7: Estimates - Dynamic Nelson-Siegel model

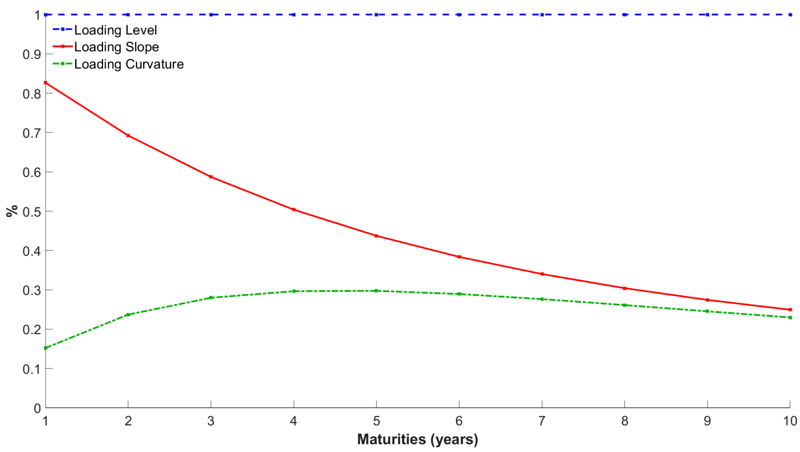
RMSE	**0.216**
Log-Likelihood	**576**
μ_L	0.044
μ_S	-0.128
μ_C	-0.127
θ_{11}	0.985
θ_{12}	-0.045
θ_{13}	0.083
θ_{21}	0.043
θ_{22}	0.857
θ_{23}	0.218
θ_{31}	0.001
θ_{32}	-0.010
θ_{33}	0.940
$\sigma_{\zeta_{11}}$	0.112
$\sigma_{\zeta_{12}}$	-0.118
$\sigma_{\zeta_{13}}$	-0.081
$\sigma_{\zeta_{22}}$	0.298
$\sigma_{\zeta_{23}}$	-0.239
$\sigma_{\zeta_{33}}$	1.734
$\sigma_{\epsilon_{11}}$	0.089
$\sigma_{\epsilon_{22}}$	0.100
$\sigma_{\epsilon_{33}}$	0.148
$\sigma_{\epsilon_{44}}$	0.099
$\sigma_{\epsilon_{55}}$	0.066
$\sigma_{\epsilon_{66}}$	0.003
$\sigma_{\epsilon_{77}}$	0.000
$\sigma_{\epsilon_{88}}$	0.001
$\sigma_{\epsilon_{99}}$	0.000
$\sigma_{\epsilon_{1010}}$	0.002
λ	2.542

Figure 8: Real government bond yield curve



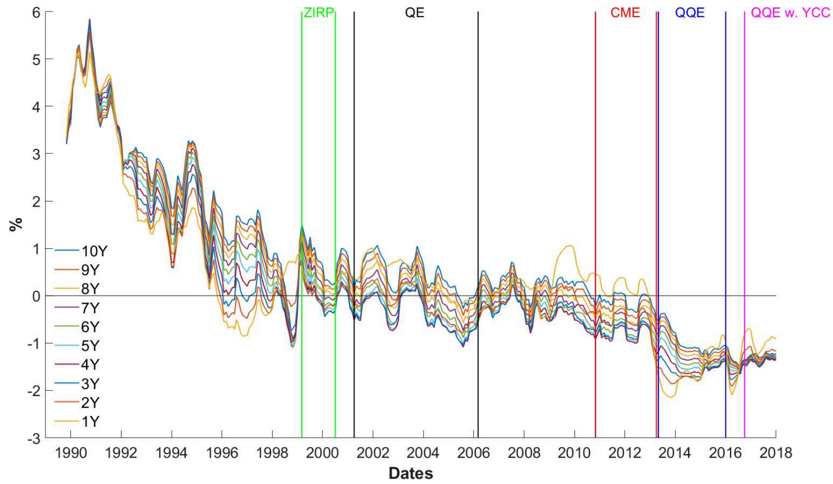
Median over 1989:Q4-2019:Q4

Figure 9: Loadings of the yield curve



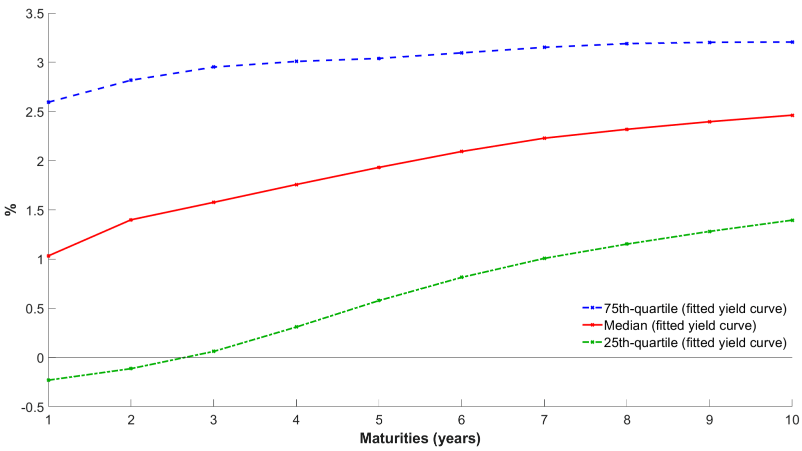
Period 1989:Q4-2019:Q4

Figure 10: Real government bond yields



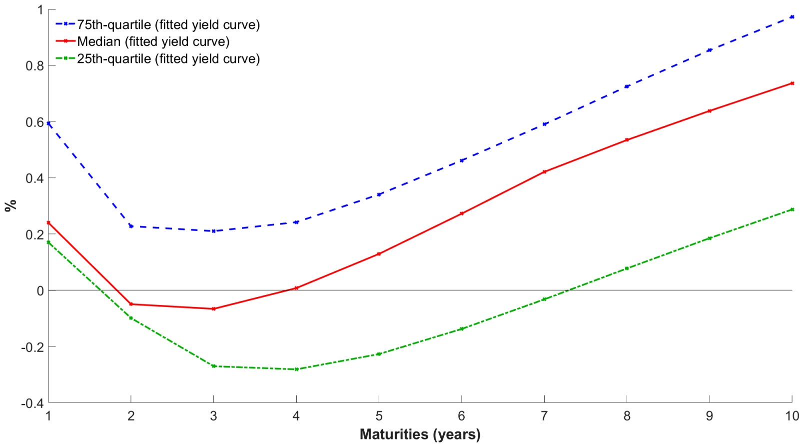
Period 1989:Q4-2019:Q4

Figure 11: Yield curve during the pre-regime period



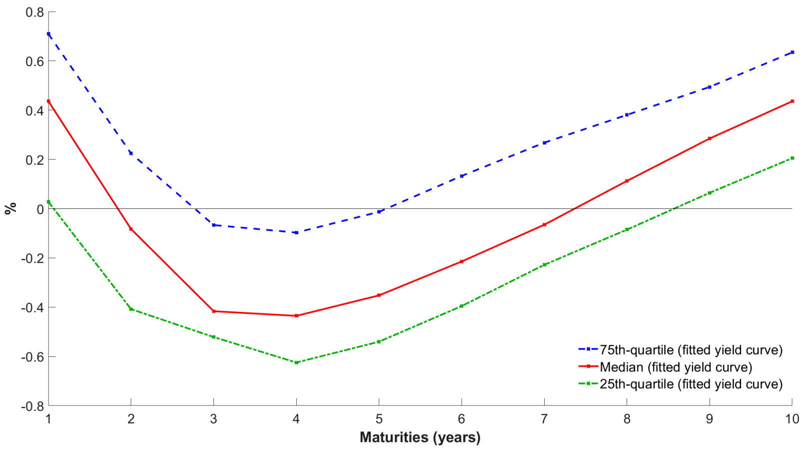
Quartiles over 1989:Q4-1998:Q4

Figure 12: Yield curve during the ZIRP regime period



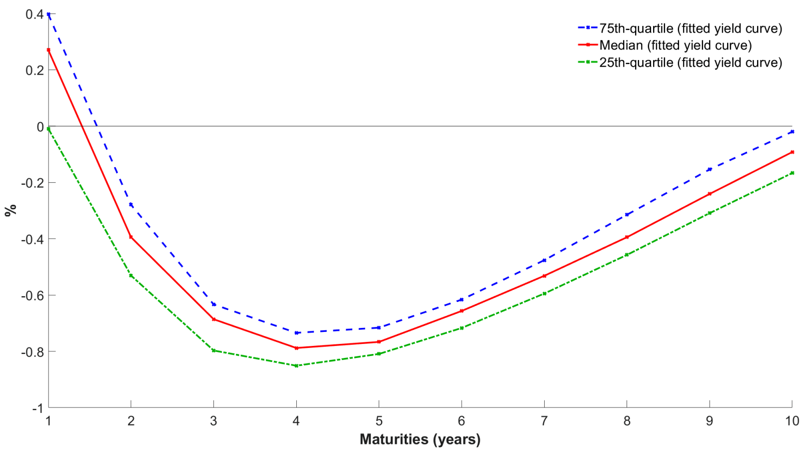
Quartiles over 1999:Q1-2000:Q1

Figure 13: Yield curve during the QE regime period



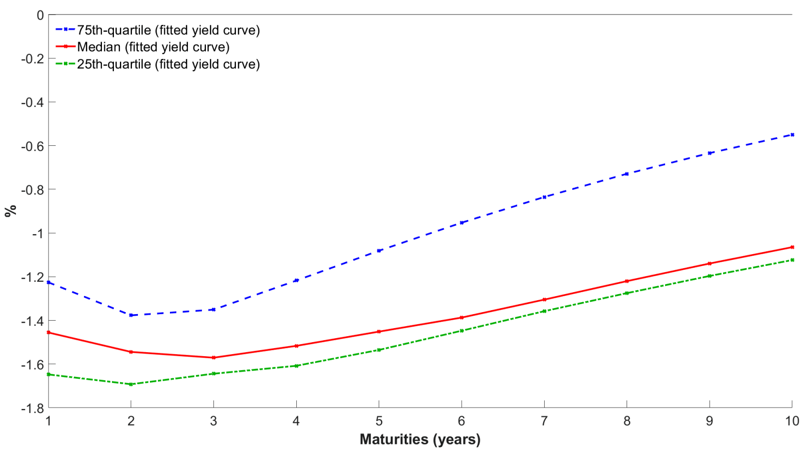
Quartiles over 2001:Q1-2006:Q1

Figure 14: Yield curve during the CME regime period



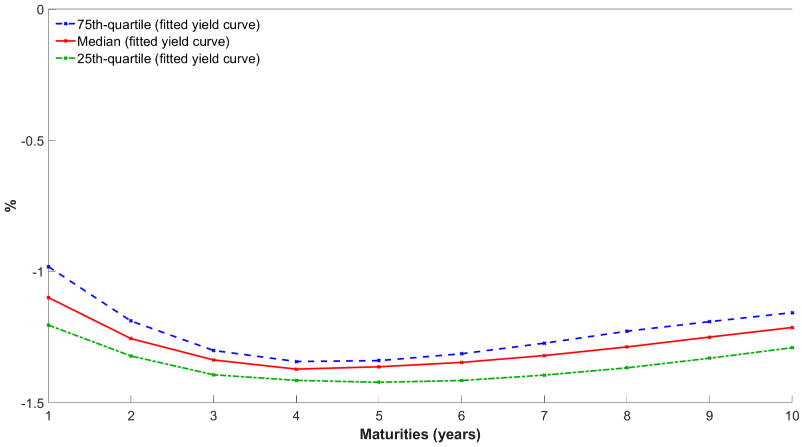
Quartiles over 2010:Q4-2013:Q1

Figure 15: Yield curve during the QQE regime period



Quartiles over 2013:Q2-2016:Q3

Figure 16: Yield curve during the QQE with yield curve control regime period



Quartiles over 2016:Q4-2019:Q4

B.2 Semi-structural macroeconomic model

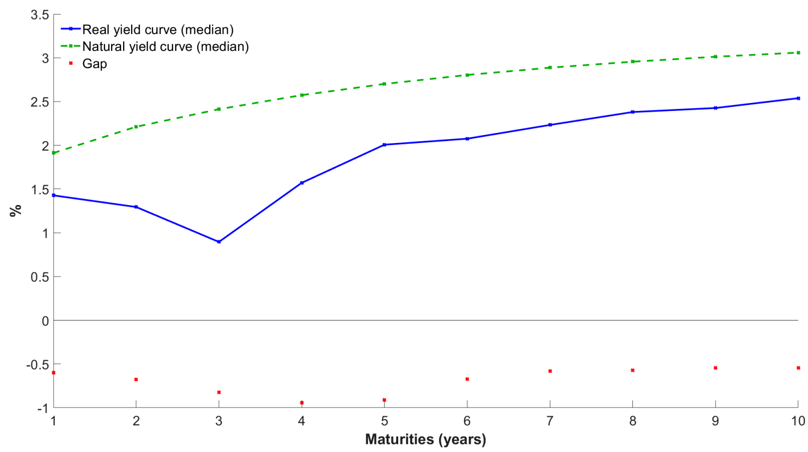
Table 8: Estimates - semi-structural macroeconomic model

Estimation	**OLS**	**MLE**			
RMSE **Log-Likelihood**	**0.467** **182**	**0.464** **250**	**Standard errors**	**Z-ratio**	**P-values**
β	0.980	0.996	0.003	344.56	$< 10^{-4}$
δ	0.988	0.988	0.011	86.46	$< 10^{-4}$
η	0.997	0.998	0.007	154.58	$< 10^{-4}$
c_L	-1.875	-1.875	0.100	-18.74	$< 10^{-4}$
c_S	-1.905	-1.905	0.100	-19.05	$< 10^{-4}$
c_C	1.260	1.260	0.100	12.60	$< 10^{-4}$
ϕ_{g0}	0.002	0.001	0.000	2.22	2.630E-02
ϕ_{g1}	0.576	0.575	0.102	5.67	$< 10^{-4}$
χ_0	-0.004	-0.003	0.002	-1.39	1.633E-01
χ_1	0.973	0.966	0.014	71.21	$< 10^{-4}$
θ_0	0.000	-0.006	0.005	-1.20	2.284E-01
θ_1	0.900	0.904	0.000	65535	$< 10^{-4}$
θ_2	-0.081	-0.081	0.101	-0.80	4.243E-01
ϕ_1	0.716	0.717	0.082	8.69	$< 10^{-4}$
ϕ_L	0.678	0.680	0.000	65535	$< 10^{-4}$
ϕ_S	0.680	0.681	0.000	65535	$< 10^{-4}$
ϕ_C	0.645	0.646	0.069	9.39	$< 10^{-4}$
μ_L	0.002	0.004	0.004	0.93	3.514E-01
μ_S	0.001	0.002	0.002	0.96	3.370E-01
μ_C	0.001	0.002	0.001	1.89	5.830E-02
α_1	-0.009	-0.008	0.026	-0.32	7.524E-01
α_2	0.013	0.013	0.011	1.22	2.231E-01
α_3	0.036	0.035	0.035	1.01	3.129E-01
α_4	-0.005	-0.006	0.030	-0.18	8.550E-01
α_5	0.018	0.018	0.030	0.62	5.378E-01
α_6	-0.028	-0.029	0.033	-0.90	3.656E-01
γ_1	0.009	0.008	0.000	65535	$< 10^{-4}$
γ_2	0.009	0.008	0.021	0.39	6.961E-01
γ_3	-0.158	-0.158	0.046	-3.44	6.000E-04
γ_4	-0.028	-0.028	0.043	-0.65	5.181E-01
γ_5	0.027	0.027	0.042	0.65	5.166E-01
γ_6	-0.018	-0.019	0.044	-0.44	6.582E-01

Table 9: Estimates - semi-structural macroeconomic model

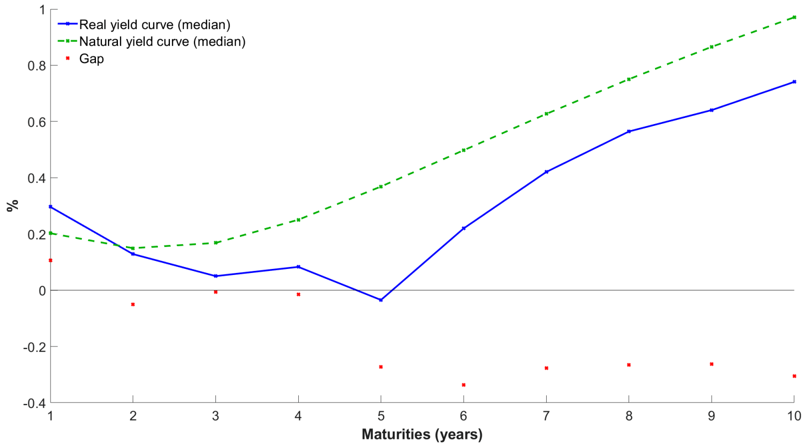
Estimation	**OLS**	**MLE**			
RMSE **Log-Likelihood**	**0.467** **182**	**0.464** **250**	**Standard errors**	**Z-ratio**	**P-values**
κ_1	0.027	0.027	0.072	0.38	7.070E-01
κ_2	0.028	0.028	0.045	0.63	5.308E-01
κ_3	0.235	0.235	0.082	2.85	4.400E-03
κ_4	0.033	0.034	0.074	0.45	6.515E-01
κ_5	-0.132	-0.132	0.074	-1.80	7.240E-02
κ_6	0.049	0.049	0.075	0.65	5.135E-01
d_π	0.825	0.825	0.047	17.40	$< 10^{-4}$
η_1	1.311	1.311	0.101	13.00	$< 10^{-4}$
η_2	0.012	0.013	0.017	0.74	4.611E-01
η_3	0.023	0.024	0.010	2.53	1.130E-02
η_4	-0.010	-0.010	0.023	-0.42	6.757E-01
η_5	0.012	0.011	0.012	0.96	3.378E-01
η_{pb}	0.002	0.001	0.002	0.95	3.428E-01
$u_{L^\star}$	0.090	0.095	0.000	65535	$< 10^{-4}$
$u_{S^\star}$	0.168	0.172	0.000	65535	$< 10^{-4}$
$u_{C^\star}$	0.336	0.338	0.055	6.17	$< 10^{-4}$
$u_{y^\star}$	0.010	0.000	0.000	65535	$< 10^{-4}$
u_g	0.010	0.003	0.002	1.63	1.031E-01
$u_{cap^\star}$	0.009	0.018	0.000	65535	$< 10^{-4}$
w_L	0.297	0.297	0.016	18.24	$< 10^{-4}$
w_S	0.448	0.448	0.000	65535	$< 10^{-4}$
w_C	1.082	1.082	0.085	12.70	$< 10^{-4}$
w_y	0.009	0.009	0.001	7.16	$< 10^{-4}$
w_{pi}	0.543	0.543	0.036	15.28	$< 10^{-4}$
w_{cap}	0.031	0.030	0.000	65535	$< 10^{-4}$

Figure 17: Yield curve gap during the pre-regimes period



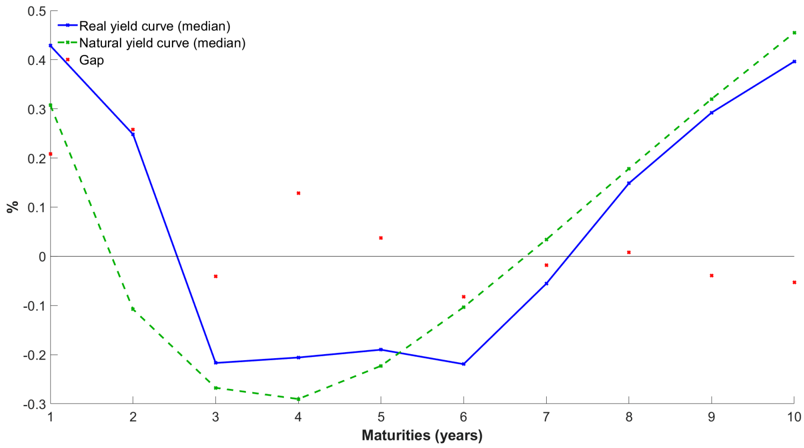
Median over 1989:Q4-1998:Q4

Figure 18: Yield curve gap during the ZIRP regime period



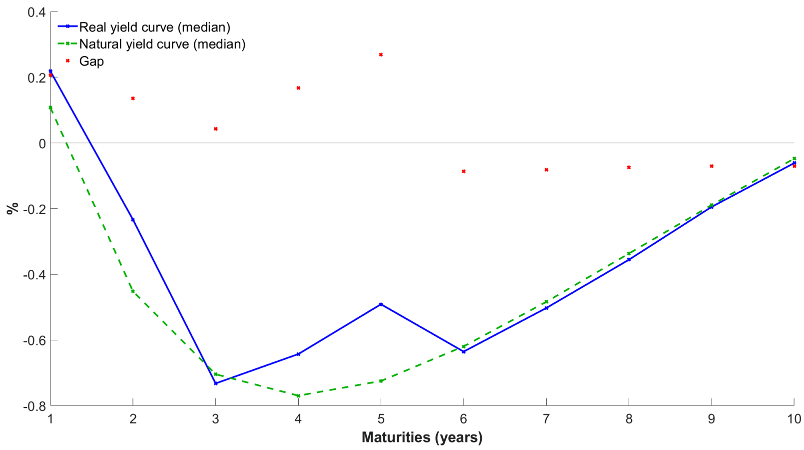
Median over 1999:Q1-2000:Q1

Figure 19: Yield curve gap during the QE regime period



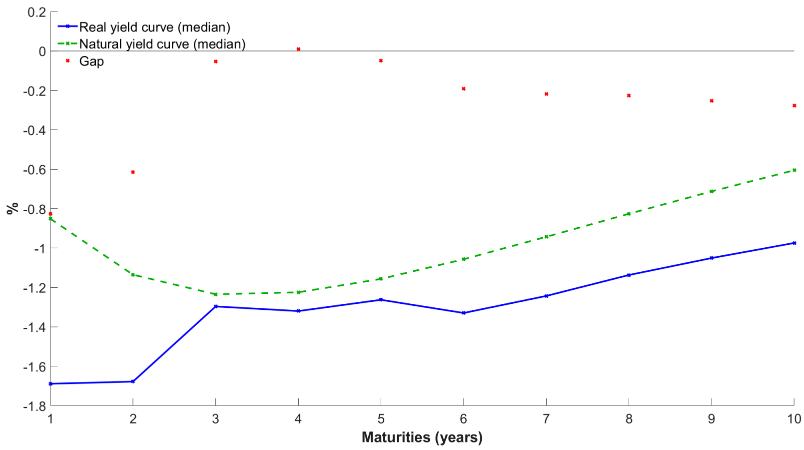
Median over 2001:Q1-2006:Q1

Figure 20: Yield curve gap during the CME regime period



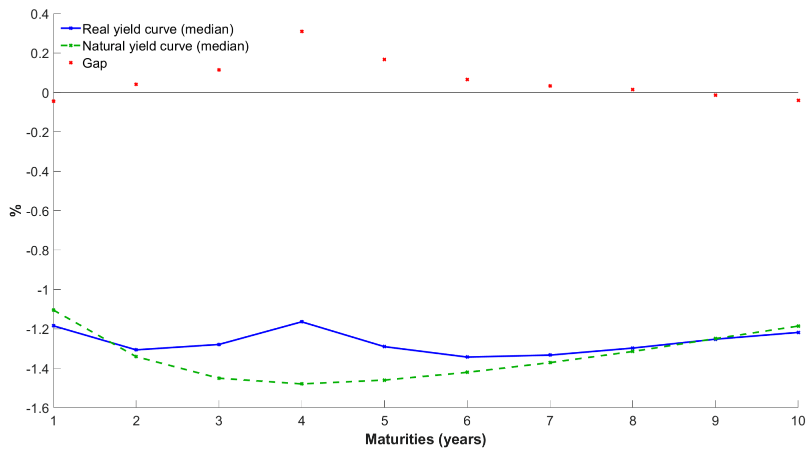
Median over 2010:Q4-2013:Q1

Figure 21: Yield curve gap during the QQE regime period



Median over 2013:Q2-2016:Q3

Figure 22: Yield curve gap during the QQE with yield curve control regime period

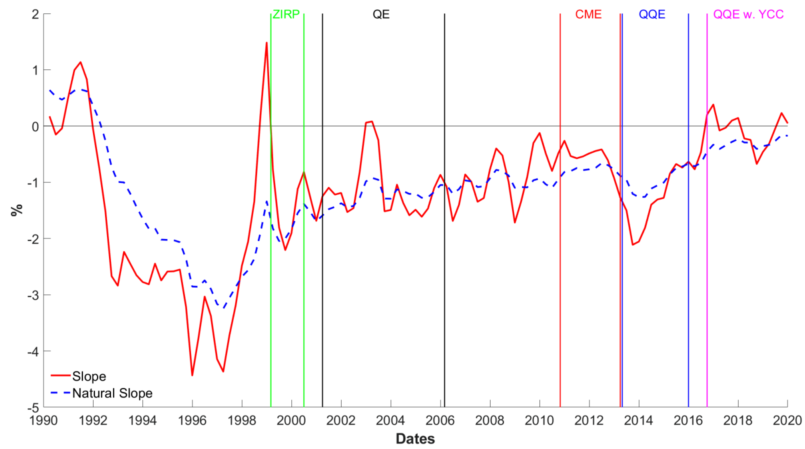


Median over 2016:Q4-2019:Q4

Figure 23: Level gap under different monetary policy regimes

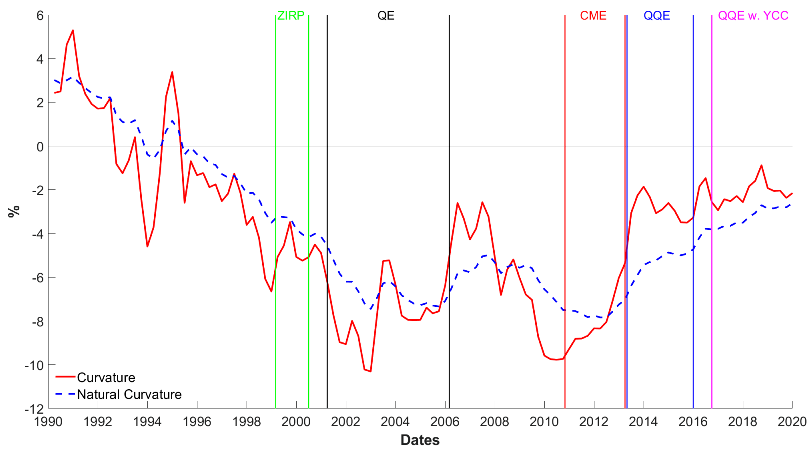
Period 1990:Q1-2019:Q4

Figure 24: Slope gap under different monetary policy regimes



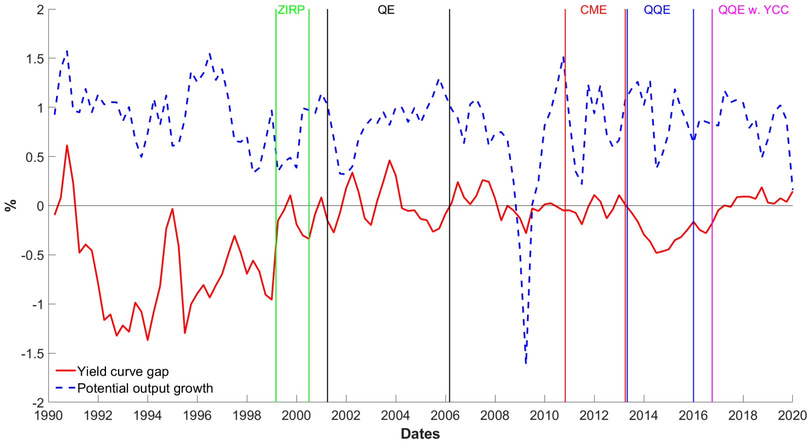
Period 1990:Q1-2019:Q4

Figure 25: Curvature gap under different monetary policy regimes



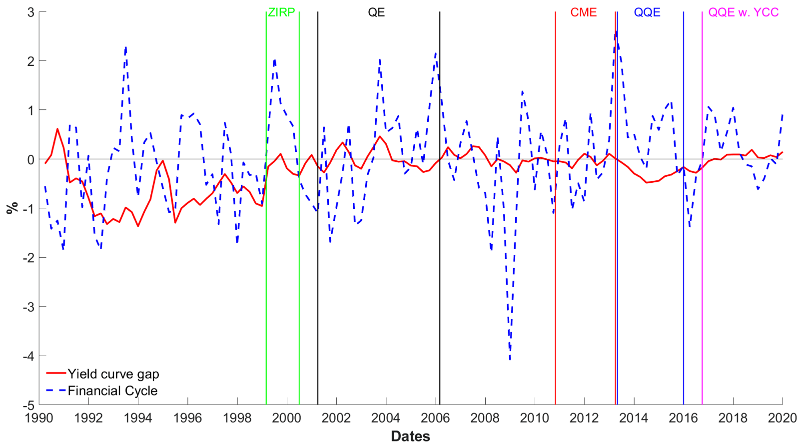
Period 1990:Q1-2019:Q4

Figure 26: Potential output growth and yield curve gap



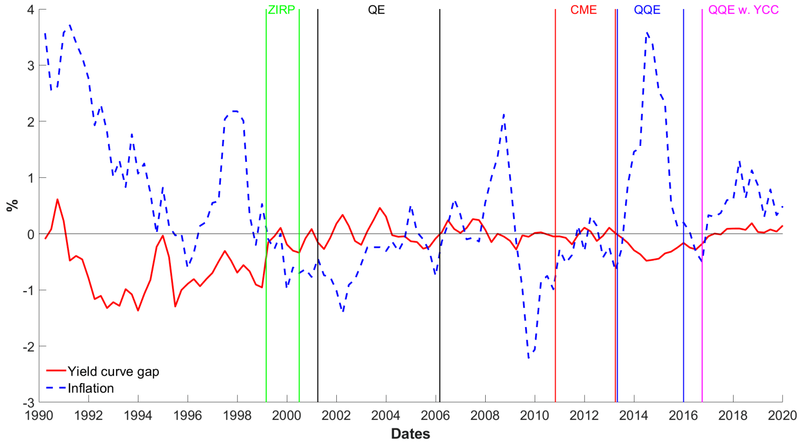
Period 1990:Q1-2019:Q4

Figure 27: Financial cycle and yield curve gap



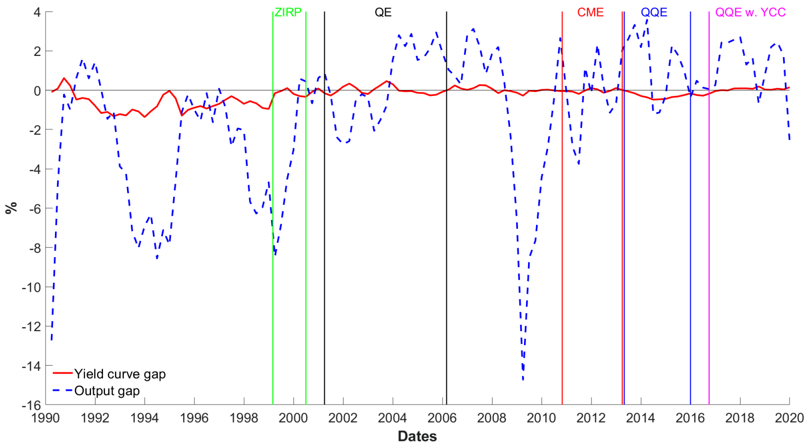
Period 1990:Q1-2019:Q4

Figure 28: Inflation and yield curve gap



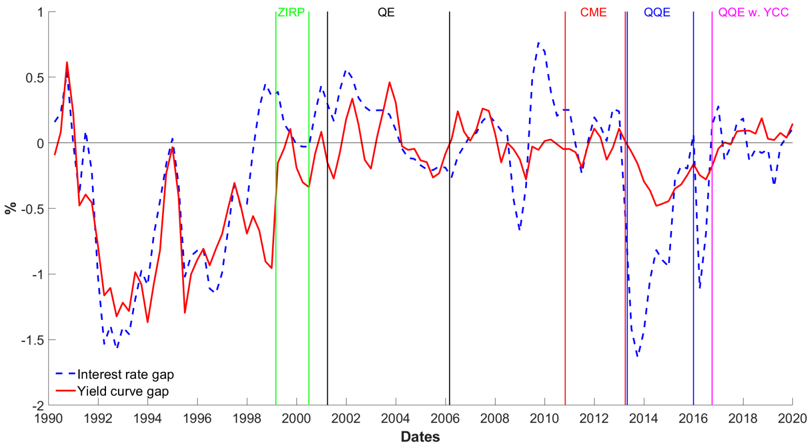
Period 1990:Q1-2019:Q4

Figure 29: Output gap and yield curve gap



Period 1990:Q1-2019:Q4

Figure 30: Yield curve gap and interest rate gap



Period 1990:Q1-2019:Q4

C Data

Table 10: Data sources

Variable name	Data	Source
$y^{(n)}$	nominal yields	Ministry of Finance
REER	Real Effective Exchange Rate	Bank of Japan**
Policy	Call Rate, Uncollateralized Overnight/Average (% per annum)	Bank of Japan
BOJbase	M3	FRED database
π	Consumer Price inflation	Statistics Bureau/Datastream
π^{energy}	Energy Price	OECD
$\pi^{US,PPI}$	Producer Price Index (Total Manu., US)	Fred
cap	Rate of Capacity Utilisation	Fred
y (real GDP per capita)	Quarterly real GDP (AR,SA), level	Cabinet Office/Datastream
y (real GDP per capita)	population level	Fred
financial	housing prices	OECD**
financial	stock prices	OECD**
financial	Credit to Private Non-Financial Sector, Adjusted For Breaks	Fred**
π^{wages}	Hourly earnings (MEI)	OECD
TOT^{emerg}	Terms of trade emerging and developing countries (goods and services)	IMF/Datastream*
pb	Government Primary Balance (% GDP)	Oxford Economics/Datastream

* Interpolated from yearly to quarterly data using a cubic approach
** Data are divided by their standard deviation for homogeneity with other series

CHAPTER III

Inflation-output gap correlation and the slope of the New Keynesian Phillips curve*

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Abstract

Price inflation has been remarkably stable over the past 25 years, in spite of large fluctuations in real economic activity. This observation has led some to believe that the Phillips curve has flattened. We argue that it is crucial to control for all supply shocks (and not only cost-push shocks) when evaluating alternative explanations for the puzzling behavior of inflation. Using a combination of New Keynesian theory and SVAR models, we find limited evidence for a flattening of the Phillips curve once supply shocks are properly taken into account. Our results are more consistent with explanations based on a stricter monetary policy responses to inflation fluctuations, or based on a more important role of supply shocks.

JEL classification: C3; E3; E5
Keywords: Inflation; the Phillips curve; monetary policy; structural VAR models

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1 Introduction

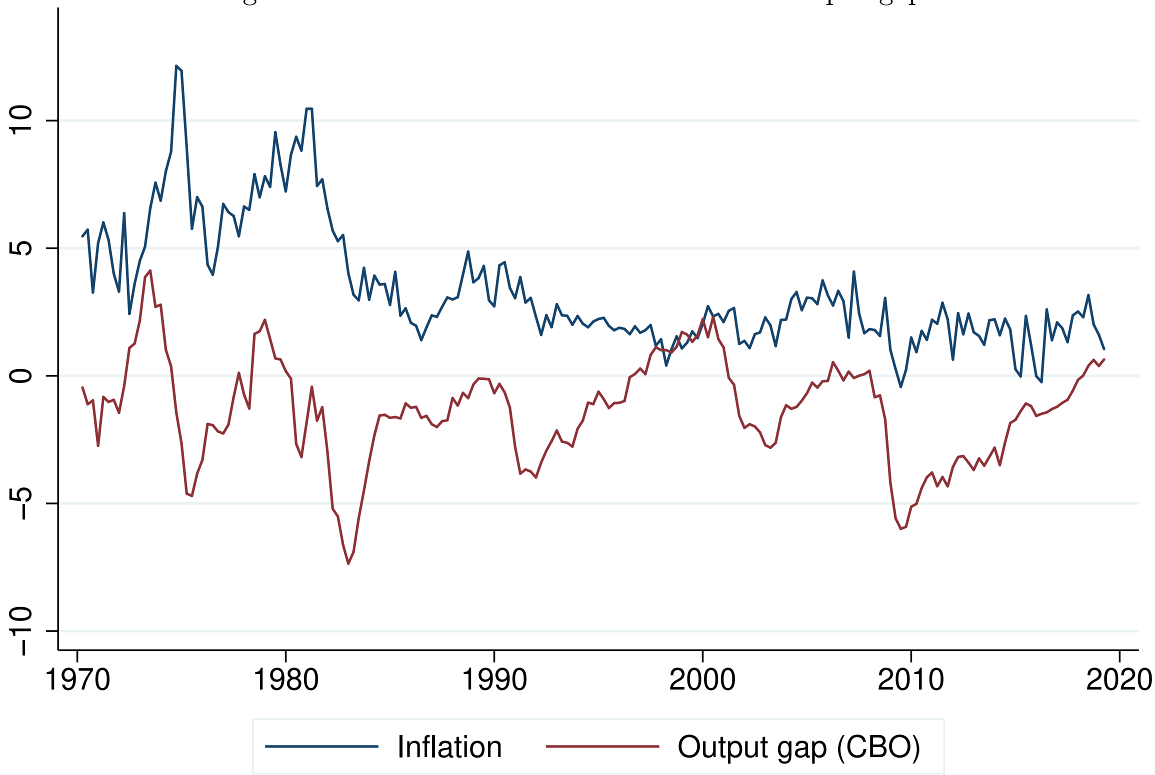
Since the mid 1990s price inflation has remained remarkably stable in the United States, even in presence of large cycles in economic activity, as shown in Figure 1. This tendency has been even more pronounced in recent years. During the pre Great Recession boom, inflation was stubbornly stable, barely above 2 percent. During the Great Recession, in face of the largest decline in real economic activity since the Great Depression, inflation declined by only one percent. In the aftermath of the Great Recession, real economic activity recovered (albeit slowly), unemployment reached a 50-year low slightly under 4 percent but inflation remained consistently below 2 percent. Why has inflation been so stable? At least three explanations have been proposed to solve the puzzle, or at least part of it. The first (and most widely accepted) explanation points towards a decline in the slope of the Phillips curve, possibly driven also by global factors (cf. Del Negro et al. (2020), Forbes (2019), Rubbo (2020)). In such a scenario, demand shocks have large effects on real economic activity but barely affect inflation. A second explanation, possibly complementary to the first, highlights the role of monetary policy that may have become more aggressive over time with respect to achieving inflation stability (McLeay and Tenreyro (2020)). According to this view, the Phillips curve is alive and stable but demand shocks leave no footprint and generate limited fluctuations in inflation (but also in measures of slack in the economy). A third possibility is that the correlation between inflation and real economic activity declines because the relative importance of demand and supply shocks has changed (Gordon (2013)). One could imagine that supply shocks become more important over time or more concentrated in specific periods, as it was the case for oil shocks in the aftermath of the Great Recession (Coibion and Gorodnichenko (2015)). Under that view, inflation may co-move less with real economic activity even if both the Phillips curve and the behavior of monetary policy are perfectly stable. We refer to these three broad explanations for the stability of inflation as the *slope hypothesis*, the *policy hypothesis* and the *shocks hypothesis*.[1]

In this paper we re-evaluate the debate on "What's (not) up with Inflation?", using the words of Yellen (2019). We argue that a careful identification of supply shocks is a crucial step to evaluate the different explanations. In order to achieve our goal, we estimate simple Structural Vector Autoregression (SVAR) models identified with sign restrictions and interpret the results through the lenses of standard New Keynesian theory. While SVAR models have been used in the previous literature on the inflation puzzle (Del Negro et al. (2020) among others), a strong focus on the role of supply shocks constitutes the key contribution of our paper. Supply shocks, defined as shocks that move inflation and measures of real economic activity in opposite directions, play an important role in shaping business cycle fluctuations in the US economy (cf. Smets and Wouters (2007)). We argue that controlling carefully for supply shocks is of paramount importance to evaluate all the three main explanations for the puzzling behavior of inflation during the last 25 years.

We build our argument on a simple observation. Suppose we consider as a measure of real economic activity the output gap as estimated by the CBO. Coibion et al. (2018) show that the CBO estimate (as the ones other international organizations like IMF and OECD) responds in the same way to demand and supply shocks, at least in the short run. These statistical measures are consistent with a smooth measure of potential output, responding very gradually to shocks. Therefore, output and the statistical measure of the output gap

[1] A fourth explanation advocates that some indicators of real economic activity, like the output gap or the unemployment rate, might have become less representative of the state of the economy in recent years. As discussed in Coibion et al. (2018), the measurement of the output gap is subject to large uncertainty. In addition, record low unemployment in recent years might not have been a good indicator of overall health of the labor market in presence of a large decline in the labor force participation rate reflecting at least in part cyclical factors, as shown in Erceg and Levin (2014).

Figure 1: The evolution of inflation and the output gap



co-move in response to any shock and all supply shocks generate a negative correlation between a statistical measure of the output gap and inflation.[2] We argue that this simple observation on the measurement of the output gap has important implications to evaluate different explanations of the inflation puzzle.

First and foremost, estimates of the slope of the Phillips curve based on statistical measures of the output gap need to control for all supply shocks and not only for cost push shocks as suggested by standard New Keynesian theory. Put differently, single equation estimates of the slope based on aggregate data will be heavily biased if supply shocks are important. Using a multivariate approach and conditioning on demand shocks, our SVAR analysis allows us to obtain estimates of the conditional correlation between output and inflation, thus, of the Phillips curve slope, that do not suffer from the bias induced by supply shocks. Second, for the sake of the same argument, a monetary policy responding aggressively to inflation will achieve a good stabilization of inflation as well as statistical measures of the output gap in response to demand shocks, but not in response to supply shocks. In fact, such a policy will stabilize inflation but will generate substantial volatility in statistical measures of the output gap in response to supply shocks. Therefore, disentangling demand from supply shocks is crucial to evaluate this explanation of the inflation puzzle. Finally, and somewhat obviously, any explanation based on shifts in the relative importance of shocks can be evaluated only in a framework where demand and supply shocks can be set apart.

In a nutshell, our results find limited support for the *slope hypothesis* and some support for the *policy hypothesis* or the *shocks hypothesis* depending on the specification. As a first

[2]This is not the case if one considers the model based concept of output gap in New Keynesian models where potential output is defined as the counterfactual level of output in absence of nominal rigidities and cost push shocks. In the New Keynesian model potential output does not necessarily fluctuate little in response to shocks. Technology and labor supply shocks, for example, move potential output more than actual output and generate a negative conditional correlation between model based output gap and output.

experiment, we estimate our SVAR model over the sample period 1969Q4-2019Q4 using data on inflation, the CBO estimate of the output gap, and inflation expectations as measured by the Survey of Professional Forecasters. Notably, the use of data on inflation expectations allows us to contribute to the debate on the role of their stability (cf. Bernanke (2007), Jorgensen and Lansing (2019)) or instability (Coibion and Gorodnichenko (2015)) in driving inflation dynamics. We use the model to purge the data from the influence of supply shocks and obtain an estimate of the correlation between inflation and output gap conditional on demand shocks. In a second step, we impose the straight-jacket of the Phillips curve and we estimate its slope as in Coibion and Gorodnichenko (2015), except that we use the data purified from the effects of supply shocks rather than the unconditional data. Interestingly, while the slope of the Phillips curve has flattened substantially in the unconditional data, we find a remarkably stable slope conditional on demand shocks only. In a second experiment we split the full sample in two and estimate a separate SVAR model for each sub-sample. The second sample isolates the period in which the inflation puzzle has been more pronounced, as shown in Stock and Watson (2019). Results based on the two-sample estimates suggest some flattening of the Phillips curve, but far from the magnitude found in unconditional data.

Our paper contributes to the empirical literature studying the drivers of the connection between inflation and real economic activity. Most papers discuss approaches and challenges to the estimation of the New Keynesian Phillips curve in a single equation framework (cf. Galí and Gertler (1999), Sbordone (2002), Kleibergen and Mavroeidis (2009), Imbs et al. (2011), Mavroeidis et al. (2014) Barnichon and Mesters (2019) and Barnichon and Mesters (2020)). A few papers use SVAR models to study inflation dynamics, mostly focusing on the long-run trade-off between inflation and unemployment (cf. King and Watson (1994), Cecchetti and Rich (2001) and Benati (2015)). Del Negro et al. (2020) find evidence in favor of the slope hypothesis using both SVAR and DSGE models. The paper closest to us is perhaps Galí and Gambetti (2019) who use a SVAR to purge the data from the variation induced by wage mark-up shock to estimate the slope of the Phillips curve. Based on our argument on the measurement of the output gap, we argue that it is important to control for the variation induced by *all supply shocks*. A benefit of our set-up, compared with previous literature, is that we can evaluate the three main explanations for the inflation puzzle in a unified framework.

The rest of the paper is organized as follows: Section 2 discusses the structural relationship between output and inflation using a textbook, New Keynesian model. Section 3 describes our methodological approach, Section 4 documents the main empirical results. Section 5 provides robustness tests, while Section 6 concludes.

2 Theoretical discussion

We start with a log-linearized, textbook New Keynesian model (see Woodford (2003) and Galí (2008) for further details) and then briefly look at some extensions. The model is summarized below:

$$y_t = \mathbb{E}_t y_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) + m_t \tag{1}$$

$$y_t = a_t + n_t \tag{2}$$

$$w_t = \psi_t + \sigma y_t + \varphi n_t \tag{3}$$

$$mc_t = w_t - a_t \tag{4}$$

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda mc_t + z_{p,t} \tag{5}$$

Conditional on monetary policy and exogenous disturbances, these five equations characterize the dynamics of five endogenous variables, all defined in log deviations from their respective steady state values: the output gap y_t, hours worked n_t, the real wage w_t, real marginal costs mc_t, and price inflation π_t. The variables a_t, ψ_t, and $z_{p,t}$ are interpreted as a productivity shock, a labor supply shock, and a cost-push shock respectively. m_t is a demand shock. All parameters have the usual interpretation, including $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, with θ being the Calvo probability in any given period of not being able to adjust the price. The model is closed with a specification of monetary policy. As a baseline, we assume that the nominal interest rate i_t is determined by a simple Taylor rule:

$$i_t = \phi_\pi \pi_t + \phi_y y_t + z_t \tag{6}$$

z_t is interpreted as a monetary policy shock. One can simplify the model by inserting for equations (2)-(4) into (5) and arrive at the canonical New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + s_t, \tag{7}$$

where $s_t = z_{p,t} + \lambda\psi_t - \lambda(1+\varphi)a_t$ collects the three supply shocks. All three supply shocks enter the Phillips curve because we consider output in deviation from steady state (or equivalently, from the trend), rather than in deviation from output with flexible prices.[3] While the latter is relevant for welfare policies, only the former is consistent with the output gap measures used, and the data provided, by statistical agencies. Note that the two demand shocks m_t and z_t do not enter the Phillips curve. Our object of interest, the slope of the Phillips curve, is given by $\kappa = \lambda(\sigma + \varphi)$. A flattening of the Phillips curve amounts to a decline in κ.

In order to discuss the changing output gap-inflation relationship in data, as well as challenges associated with estimation of κ in equation (7), we find it instructive to work with the model's solution. To this end we collect the two demand shocks in $d_t = m_t - \sigma^{-1} z_t$ and impose the heroic assumption that d_t and s_t are independently and identically distributed with variances σ_d^2 and σ_s^2, respectively.[4] Analytical solutions for output and inflation follow:

$$\begin{aligned} y_t &= \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\left(\sigma d_t - \phi_\pi s_t\right) \\ \pi_t &= \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}\left[\sigma\kappa d_t + \left(\sigma + \phi_y\right) s_t\right] \end{aligned}$$

Consider, first, the scope for *unconditional* inference. The unconditional variance of inflation relative to output follows:

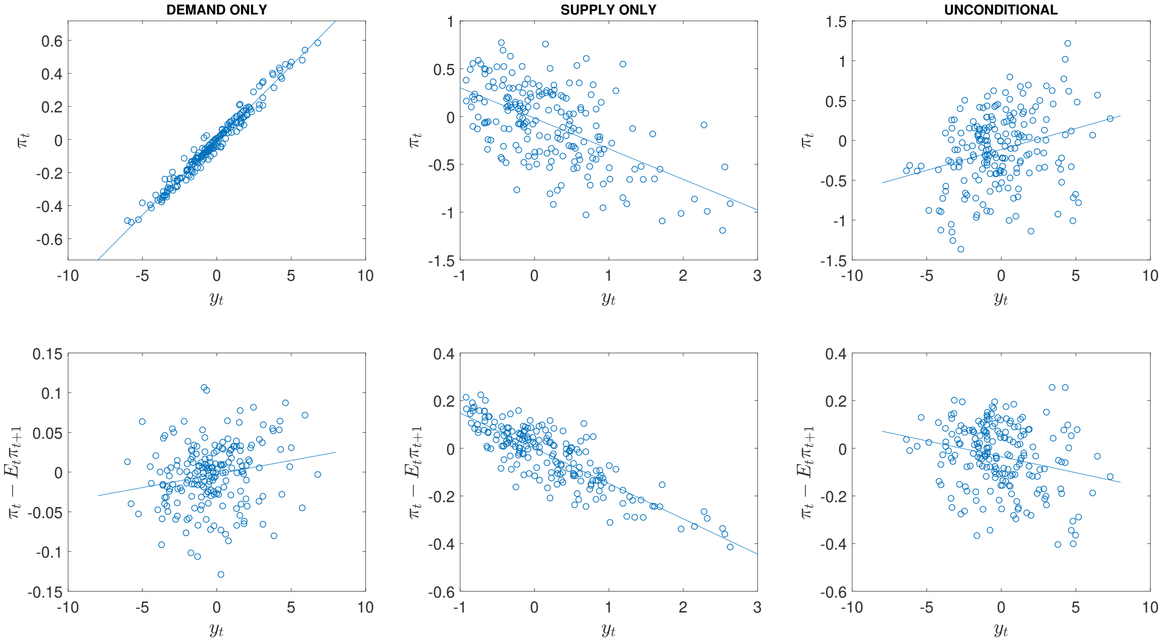
$$\frac{var(\pi_t)}{var(y_t)} = \frac{\sigma^2\kappa^2 + (\sigma + \phi_y)^2 \frac{\sigma_s^2}{\sigma_d^2}}{\sigma^2 + \phi_\pi^2 \frac{\sigma_s^2}{\sigma_d^2}}$$

This expression suggests that a flattening of the Phillips curve ($\kappa \downarrow$) reduces the relative volatility of inflation. Indeed, a quick glance at data seems to provide evidence consistent with such a flattening: the relative volatility of inflation was 0.87 over the time period 1970Q1-1985Q1, 0.58 in 1985Q2-2007Q3, and 0.50 in 2007Q4-2019Q1. However, the expression above makes it clear that also other changes would lead to a similar decline in the relative inflation

[3] Only the cost-push shock enters the Phillips curve if we consider output in deviation from its counterfactual when prices are flexible. The other supply shocks are part of flex-price output in this case.

[4] The i.i.d. assumption is relaxed in the appendix, where we instead consider the more common assumption that shocks follow separate AR(1) processes. None of our conclusions are altered in this case.

Figure 2: Scatter plots from the extended, New Keynesian model



volatility, including stricter focus from monetary policy authorities on inflation targeting ($\phi_\pi \uparrow$), as well as a rise in the relative importance of supply shocks ($\frac{\sigma_s^2}{\sigma_d^2} \uparrow$). The observation that inflation volatility has declined relative to output volatility is not, therefore, evidence of a Phillips curve flattening.

At least two challenges emerge when estimating equation (7) by means of regression methods: first, one must account for inflation expectations in the regression, as failure to do so will lead to biased estimates of κ. Second, even if the role of expectations has been properly addressed, it is also necessary to control for the vector of *unobservable* supply shocks embedded in s_t. And herein lies a major identification problem: it is not sufficient to control for e.g. cost-push disturbances only—all supply shocks enter s_t in equation (7). Any supply shock ignored in the empirical specification will effectively enter the residual term and lead to an omitted variable bias. Given the model solution above, the OLS estimator for κ is equal to

$$\kappa^{OLS} = \frac{cov\left(\pi_t, y_t\right)}{var\left(y_t\right)} = \frac{\sigma^2\kappa - \phi_\pi\left(\sigma + \phi_y\right)\frac{\sigma_s^2}{\sigma_d^2}}{\sigma^2 + \phi_\pi^2\frac{\sigma_s^2}{\sigma_d^2}} \leq \kappa.$$

Note that $\kappa^{OLS} = \kappa$ only if $\sigma_s^2 = 0$, i.e. if *all* variation in the data due to supply shocks is filtered out. Incomplete filtering implies a downward bias in the estimator. The bias stems from two supply driven sources of variation: (i) the variance in y_t given by $\phi_\pi^2\sigma_s^2$, and (ii) the negative covariance between π_t and y_t, given by $-\phi_\pi\left(\sigma + \phi_y\right)\sigma_s^2$. Moreover, the bias might evolve through time. Suppose one decides to estimate the Phillips curve over different sub-samples, but uses unconditional data which are partly explained by supply shocks. Then, one may erroneously conclude that the Phillips curve has flattened if the stance of monetary policy has changed ($\phi_\pi \uparrow\downarrow$, $\phi_y \uparrow$), or if supply shocks have become more volatile relative to demand shocks ($(\sigma_s^2/\sigma_d^2) \uparrow$). It is, therefore, crucial to filter out any variation due to supply, so that the data used for inference is conditional on demand only.

This observation carries over to larger DSGE models as well, including those used by central banks for monetary policy analysis. As a simple illustration, we may add some standard "bells and whistles" to the textbook model and simulate artificial data. This is done in Figure 2. There we report scatter plots based on artificial data from a model with bells and whistles.[5] The two sub-plots on the left show artificial data points conditional on demand only. Importantly, while the OLS line connecting $\pi_t - \mathbb{E}\pi_{t+1}$ to y_t is closely connected to the slope of the Phillips curve (see equation (5)), the OLS line connecting π_t to y_t overstates the slope. Data points conditional on supply shocks are shown in the middle, while the unconditional data (i.e. the combined contributions from demand and supply) are reported in the two sub-plots on the right. Our calibration is such that the correlation between $\pi_t - \mathbb{E}\pi_{t+1}$ and y_t is negative in the unconditional data, even when the true slope of the Phillips curve is positive. Thus, naive regressions which fail to control for supply shocks could grossly underestimate of the true slope of the Phillips curve.

3 Empirical approach

The empirical approach we pursue in this paper is essentially a two-step procedure: in the first step, referred to as the filtering step, we decompose the data into the components coming from demand and supply, respectively. This is done with a SVAR model estimated on data using Bayesian techniques. The second step, referred to as the regression step, is to run Phillips curve regressions on the filtered data in order to make inference about the conditional relationship between output and inflation. This is needed in order to get unbiased estimates of the slope of the Phillips curve. In the following we describe the SVAR methodology used here in more detail.

3.1 The SVAR model

The structural representation of a VAR model can be written as follows:

$$B^{-1}Y_t = \sum_{k=1}^{p} Z_k Y_{t-k} + \varepsilon_t$$

Y is a vector of size n, $B \in \mathcal{M}_{n\times n}(\mathbb{R})$, and $Z_k \in \mathcal{M}_{n\times n}(\mathbb{R})$ are the matrices of structural parameters, and $\varepsilon_t \in \mathbb{R}^n$ is the vector of structural shocks with $\varepsilon_t \sim \mathcal{MVN}(0, I_n)$. We assume that all the roots of the model's characteristic polynomial lie outside the unit disk, so that the VAR model is stationary. Our baseline VAR contains three variables:

$$Y_t = (\pi_t, \pi_t^e, y_t)'$$

where π_t is inflation, π_t^e are inflation expectations, and y_t is the output gap.

We estimate the SVAR model with four lags and a constant on quarterly data. It is estimated using Bayesian methods with standard natural conjugate (Normal-Wishart) priors. Moreover, we specify flat priors for the reduced form parameters in order to remain agnostic about the data generating processes. We also impose sign restrictions on impact using the QR decomposition algorithm proposed by Arias and Waggoner (2018) to identify the structural shocks. This algorithm enables to draw from a conjugate uniform-normal-inverse-Wishart posterior distribution over the orthogonal reduced form parameterization and then to transform the draws into the structural parameterization.

[5]For the illustration in Figure 2 we add nominal wage stickiness, (external) habit formation in demand, partial inflation indexation in wages and prices, as well as monetary policy inertia. These features are typically included in order to get a better model fit to macroeconomic data. We also depart from the assumption that shocks are white noise and instead model them as AR(1) processes. The resulting model equations are summarized in the appendix. The calibration is available upon request.

Table 1: Sign restrictions baseline VAR

	Demand	Supply	Residual
Inflation	+	+	+
Inflation expectations	+	+	-
Output gap	+	-	$\star$

Note: Restrictions are imposed on impact. $\star$ means unrestricted.

An obvious criticism of constant parameter SVARs is that they do not allow for a possible change in the underlying structural parameters. Yet, time-varying parameter SVARs are known for their instability and they have recently been criticized for their inconsistency when identified with sign restrictions under Bayesian inference (Bognanni (2018)). For these reasons, we employ a constant parameter SVAR that we will estimate over 2 samples (1968:Q4-1994:Q4 and 1995:Q1-2019:Q4) as well as on rolling windows, to capture the possibility of a structural change in the parameters.

As shown in Table 1, our baseline VAR only identifies two structural shocks with an economic foundation: a *demand* and a *supply* shock. The third shock, labelled as *residual*, is deliberately built from an unusual combination of restrictions so as do minimize its impact. One would indeed struggle to give an interpretation to a shock that raises inflation while decreasing inflation expectations. Such an unusual shock may only rarely happen in the economy and we thus hope to minimize its importance. Besides, to ensure that the residual shock is indeed a *residual*, we try different non-economically-founded combinations of restrictions that validates the robustness of our results.

Once the model is identified, it is easy to derive the impulse response functions, historical decompositions, and so on. However, Bayesian inference implies the use of a descriptive statistics to visually exploit the results. If the use of the mode or of the point-wise median has been common in the literature, such a statistics suffers from Fry and Pagan (2011) criticism: results may be built from many different point-wise estimated models and may therefore not be consistent with the estimated structural relationships between the variables. To prevent from such a caveat, we follow Fry and Pagan (2011) by basing our results on a median-target estimate that minimizes the squared distance between the point-wise median and every draws, optimized on all variables and all shocks in the impulse response function dimension.

Figure 3: Historical decomposition – inflation

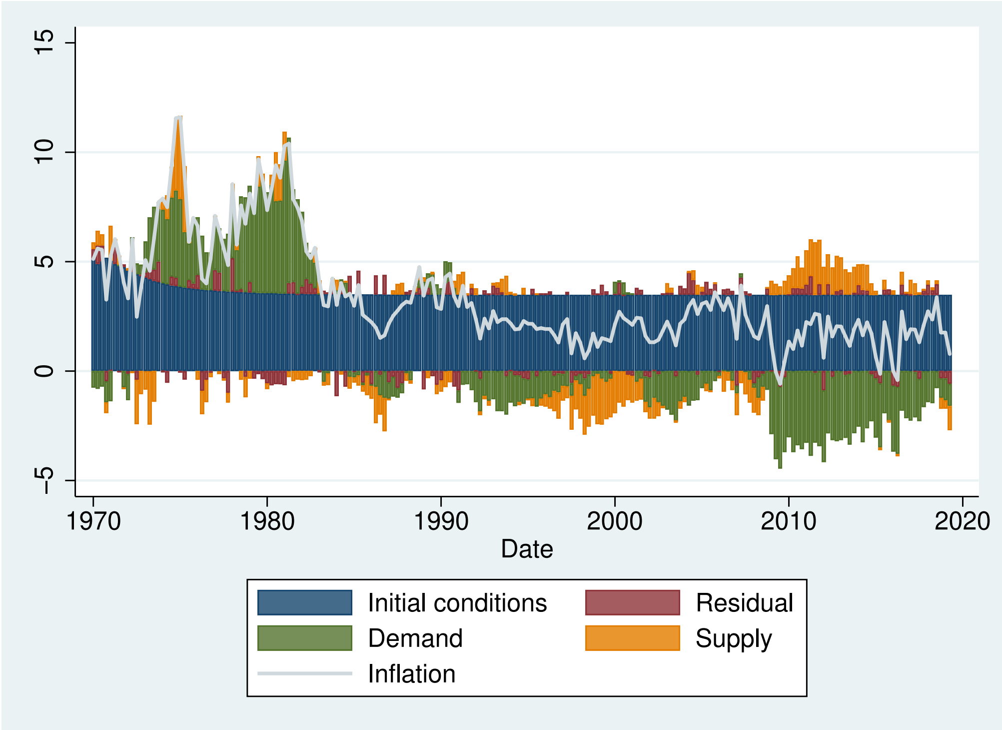


Figure 4: Historical decomposition – inflation expectations

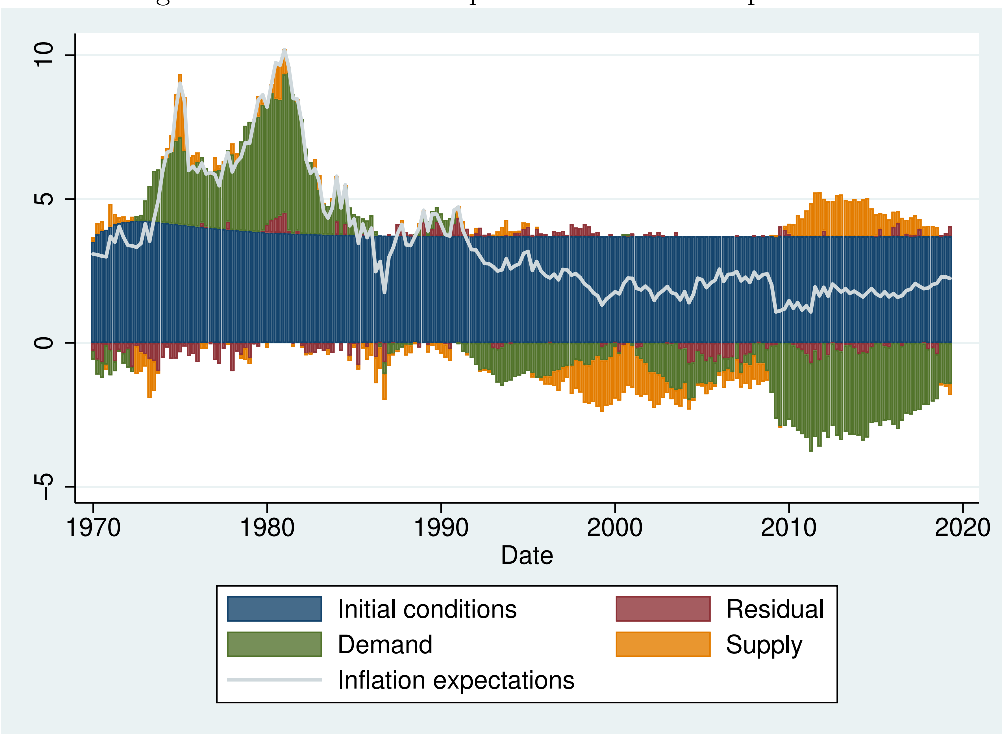
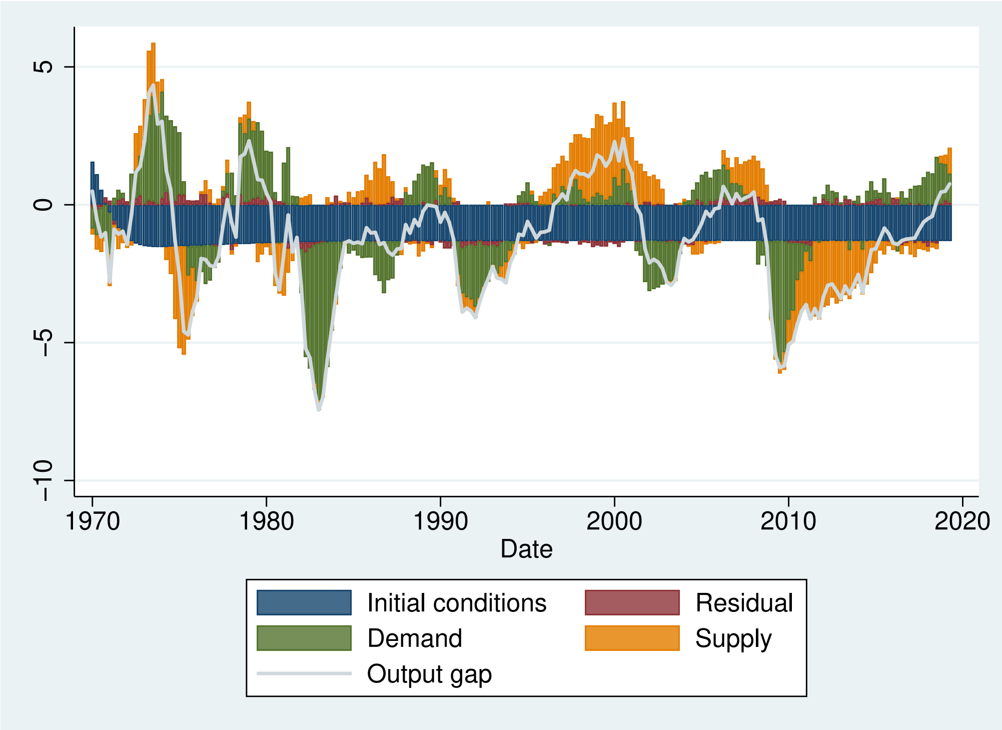


Figure 5: Historical decomposition – output gap



3.2 A decomposition of historical data

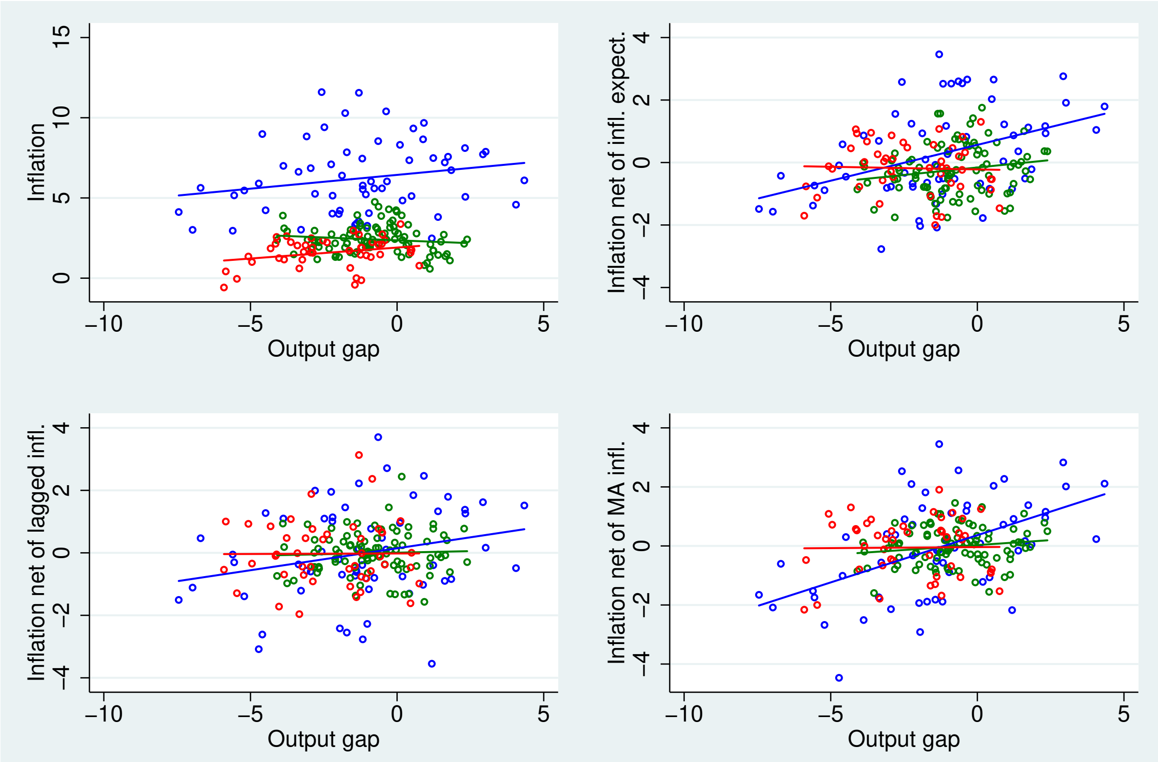
The estimated SVAR model then serves as a filtering device which allows us to decompose data in to variation conditional on demand and supply, respectively. The historical decomposition is given by:

$$H_{j,\varepsilon^i,t} = e_j' \sum_{q=0}^{t-1} C_q b_i \varepsilon_{t-q}^i,$$

where $H_{j,w^i,t}$ is the contribution of shock ε^i to variable j at time t.

In Figure 3-Figure 5 we present historical decompositions for the variables included in the VAR. Fluctuations in historical data are decomposed into a deterministic component (the blue area) and a stochastic component driven by the three shocks. We remark that demand and supply shocks are of comparable importance while the residual shock is totally marginal. Notably, the role of supply shocks is not negligible throughout the entire sample but seems particularly relevant in the second part of the sample. In fact, they seem to be concentrated in the second half of the 1990s and in the aftermath of the Great Recession. According to the model, demand shocks explain the large drop in output during the Great Recession. However, demand shocks are not sufficiently persistent to explain the dynamics of the slow recovery phase. In addition, demand shocks lead to a large decline of inflation, in contrast with the unconditional data. Therefore, the model needs negative supply shocks to match the data. The large drop in the output gap and the limited response of inflation are seen as a combination of negative demand and negative supply shocks. More generally, supply shocks contribute to the stability of inflation throughout the sample by keeping inflation low after 1995, while driving the boom in the output gap in a period of high productivity growth. Overall, the VAR assigns an important role to supply shocks, in particular in the second part of the sample, and seems therefore consistent with the shocks hypothesis detailed in the Introduction.

Figure 6: Unconditional scatter plots



Note: The y-axis shows inflation minus (a) constant expectations $\pi^e_{t+1} = \pi^e$ in top left, (b) Survey of Professional Forecasters $\pi^e_{t+1} = \pi^{spf}_{t+1}$ in top right, (c) lagged inflation $\pi^e_{t+1} = \pi_{t-1}$ in bottom left, and (d) a 4-quarter moving average $\pi^e_{t+1} = \sum_{j=1}^{4} \pi_{t-j}$ in bottom right. The three sample periods are $1969Q4$-$1984Q4$, $1985Q1$-$2007Q3$, and $2007Q4$-$2019Q1$.

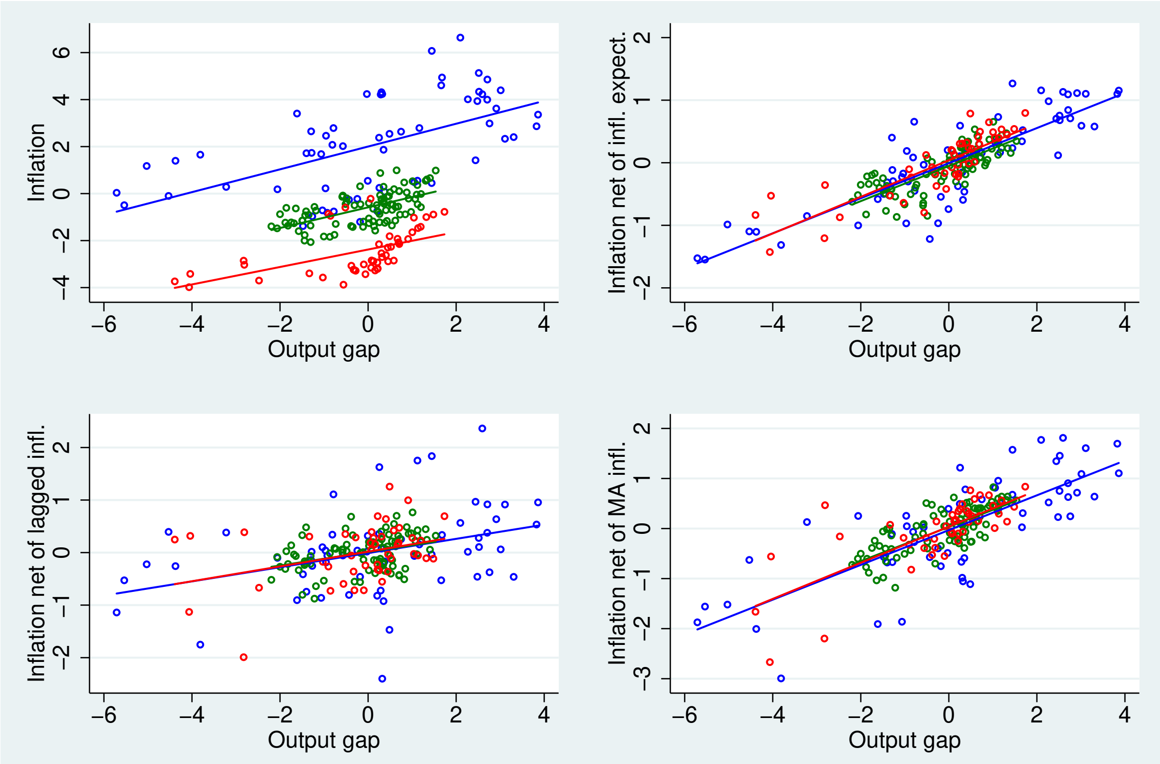
4 Empirical results

This section presents estimates of the relationship between output and inflation measures when the data are purged of supply shocks. Consider a Friedman-type, expectations augmented Phillips curve:

$$\pi_t - \pi^e_{t+1} = \kappa y_t + u_t$$

Note that this Phillips curve is identical to the one in equation (7) if $\beta = 1$ and inflation expectations π^e_{t+1} are rational. However, while the disturbance s_t in equation (7) is purely structural, the residual u_t above is best understood as a statistical error term. Importantly, the slope κ can be recovered by ordinary least squares only if $cov\,(y_t, u_t) = 0$. We consider four cases: first, for comparison, we keep inflation expectations fixed and equal to a constant π^e. This effectively leaves us with the naive correlation between inflation and the output gap in levels. Second, as a baseline proxy for inflation expectations π^e_{t+1}, we consider the Survey of Professional Forecasters. This proxy can be viewed as a benchmark because the survey expectations, at least in principle, should capture the forward looking nature of the New Keynesian Phillips curve. Third, our benchmark is compared with two backward looking measures: lagged inflation π_{t-1}, and the 4-quarter moving average $\sum_{j=1}^{4} \pi_{t-j}$. We follow Coibion and Gorodnichenko (2015) and split the full sample into the following sub-samples: $1969Q4$-$1984Q4$, $1985Q1$-$2007Q3$, and $2007Q4$-$2019Q1$. Broadly speaking, one can interpret these sub-samples as the pre-Great Moderation period, the Great Moderation, and the Post Financial crisis period. For each of the sub-samples we plot the selected measures of inflation

Figure 7: Conditional on demand



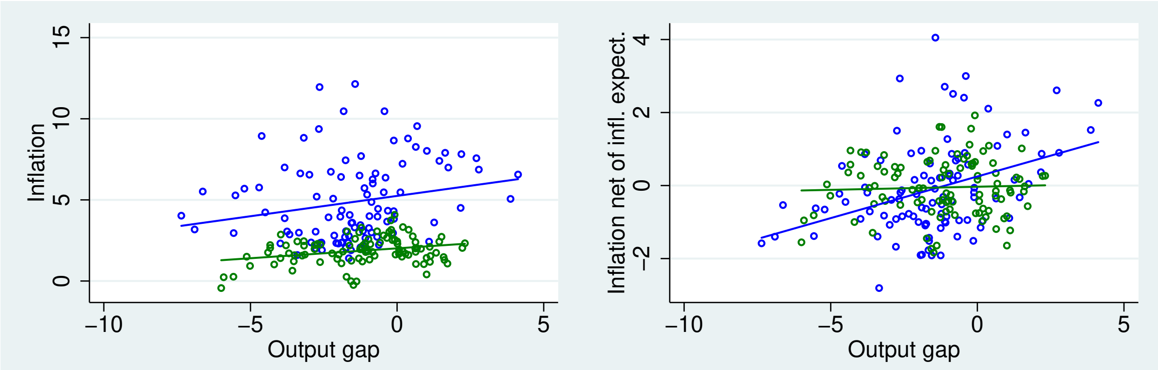
Note: The three sample periods are 1969Q4-1984Q4, 1985Q1-2007Q3, and 2007Q4-2019Q1. See Figure 6 for further details.

(net of inflation expectations) against the output gap. A least squares line is also included for each of the sub-samples.

As a reference, we start with the unconditional data. The results are shown in Figure 6. The relationship between various inflation measures and the CBO output gap was clearly positive in the 70's and the first half of the 80's. However, from the beginning of the Great Moderation and onwards this relationship seems to have broken down, and there is no sign of a return to the Phillips curve in recent years either. This is true both when we ignore inflation expectations (top left) and when expectations from the Survey of Professional Forecasters are taken into account (top right). The bottom two sub-plots, where backward looking measures are subtracted from current inflation, show a similar picture: inflation and output tended to co-move before the Great Moderation, but inflation fluctuations have since become disconnected from movements in the output gap. Thus, all of the scatter plots in Figure 6 confirm previous findings in the literature—the inflation-output gap relationship in unconditional data has weakened substantially in the last decades.

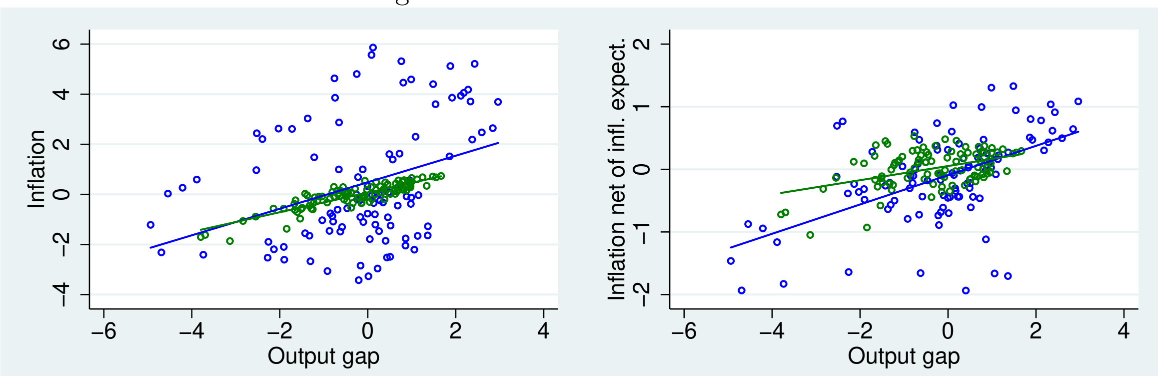
Given that the regression lines in Figure 6 are likely to be influenced by supply side factors, we next turn to the data conditional only on demand shocks. These shocks, which are identified with the SVAR model described in the previous section, are arguably much more informative about the slope of the Phillips curve. Results are shown in Figure 7. Consider first the top left plot, which just reports the conditional inflation-output gap relationship. Now the regression lines are remarkably similar across the different sub-samples. There is a downward shift in average inflation in later sub-samples, but the slope remains very similar. Note that regression lines in the top left plot are likely to be biased upwards compared with κ, because they ignore the role of inflation expectations. Those expectations should, conditional on demand shocks, be positively correlated with the output gap. By ignoring this

Figure 8: Unconditional scatter plots



Note: The two sample periods are 1969Q4-1994Q4 and 1995Q1-2019Q1. See Figure 6 for further details.

Figure 9: Conditional on demand



Note: The two sample periods are 1969Q4-1994Q4 and 1995Q1-2019Q1. See Figure 6 for further details.

conditional correlation, the expectations term end up in the error u_t and makes it correlated with the output gap as well (see the appendix for further discussion). The scatter plots and regression lines in the top right sub-plot are arguably more closely connected with the New Keynesian Phillips curve and its slope κ. And the regression slopes remain remarkably stable here as well. This is also true when we consider the backward looking measures in the bottom sub-plots. Taken together, the scatter plots in Equation 6 are far from consistent with a flattening of the Phillips curve.

A limitation with results in Equation 6 is our use of a constant parameter SVAR, estimated over the full sample, to produce the conditional data. Next we re-estimate the SVAR over the sub-samples as well. This allows for shifting parameters across the sub-samples in the filtering step, both in parameters that determine dynamics and those governing the volatility of structural shocks. Because of limited sample sizes, we split the full sample in two sub-samples of equal size. The first sample ends after 1994Q4. Figure 8 reports unconditional scatter plots in this case. The inflation-output gap relationship has weakened when we ignore inflation expectations (left sub-plot), and disappeared all together when expectations are taken into account. Turning to the data purged of supply shocks in Figure 9, we find some flattening, but far from enough to reject a positive (conditional) relationship between inflation and output. The somewhat weaker relationship obtained in the second sub-sample may be due to a true Phillips curve flattening, a stricter focus on inflation stability among policy makers, a relatively greater role for supply shocks, or to a combination of these explanations. From a statistical point of view, the somewhat flatter relationship implies less of a need to explain the Great Recession with offsetting shocks compared with the baseline.

Instead, the two-sample SVAR model assigns more of the missing disinflation during the financial crisis to weaker pass-through of real activity to inflation. This is a tougher sell in the one-sample SVAR model, which naturally assigns less weight to the financial crisis. Taking stock, while the statistical relationship between inflation and output has weakened substantially in recent decades, we find limited evidence of a flattening of the Phillips curve in US data.

5 Robustness

This section sheds light on the robustness of our results to various alternative specifications. First, we document the statistical significance of changes in Phillips curve slopes using a regression approach on the data purged of supply shocks. Second, we investigate the possibility for parameter instability by means of rolling window regressions.

5.1 Regression analysis

Our econometric strategy follows closely that of Coibion and Gorodnichenko (2015). The equation to estimate is

$$\pi_t - \pi^e_{t+1} = c_1 + D_{2,t}c_2 + D_{3,t}c_3 + (\kappa_1 + D_{2,t}\delta_2 + D_{3,t}\delta_3)\, y_t + u_t.$$

Parameter shifts are modeled by means of the dummy variables $D_{2,t}$ and $D_{3,t}$, which are equal to one in the second (1985Q1-2007Q3) and third (2007Q4-2019Q1) sample, respectively (and zero otherwise). The slope of the Phillips curve in the first sample is κ_1, $\kappa_2 = \kappa_1 + \delta_2$ in the second, and $\kappa_3 = \kappa_1 + \delta_3$ in the third. Thus, the slopes have changed if δ_2 and δ_3 are significantly different from zero. As an additional robustness exercise, we also run all regressions on the purged data produced with the SVAR model over two sub-samples. In this case the first sample spans the period 1969Q4-1994Q4.

The results for various econometric specifications are documented in Table 2. Part A of the table shows the results using data from the 1-sample SVAR model. Part B does the same for the 2-sample SVAR, given that the scatter plots with these data were less clear. We focus first on the results in Part A. Column (1a) reports the estimate of κ if we perform a naive OLS regression on unconditional data. As a starter, we consider one, common slope for all sub-samples. This slope is estimated to be .13, significant at the 95% level. However, the model explains only a minor share of inflation variability, with (adjusted) R^2 being equal to .07. One obvious limitation with OLS in our context is the exposure to omitted variables which might create movements both in inflation and the output gap. To partially address this concern, we re-estimate the same specification in column (1b), but now with lagged output as an instrument for the current output gap. The point estimate increases slightly in this case.

Columns (2a) and (2b) report results from similar regressions, but now with the dummies that allow for breaks in the slope parameter. The OLS estimate of κ falls from .23 in the first sub-sample to 0.10 in the second, and it is essentially zero after the financial crisis. We perform χ^2-tests to check whether κ_2 and κ_3 are significantly different from zero. The test evaluates the joint hypothesis that $\kappa_1 + \delta_2$ (and $\kappa_1 + \delta_3$) is zero. p-values are reported in the table. We can barely not reject the null at the 10% level for κ_2, while the null for κ_3 kept at all reasonable significance values. The IV estimates in column (2b) tell a similar story: when looking at the unconditional data, the slope of the Phillips curve was positive and highly significant in the first sub-sample, but has since declined to zero.

Column (3a) documents results from the same regression as in (2a), but now with data conditional on demand shocks only. The slope in the first sample is positive and highly

Table 2: Regression results

	Unconditional data				Conditional data							
	(1a) OLS	(1b) IV	(2a) OLS	(2b) IV	(3a) OLS	(3b) IV	(4a) OLS	(4b) IV	(5a) OLS	(5b) IV	(6a) OLS	(6b) IV
					PART A: DATA FROM 1-SAMPLE SVAR							
κ_1	.13***	.16***	.23***	.29***	.28***	.26***	.27***	.26***	.28***	.12**	.28***	.27***
δ_2			$-.13^{*}$	$-.19^{**}$	.00	$-.00$	.02	.02	$-.02$	$-.02$	.03	.02
δ_3			$-.25^{**}$	$-.34^{***}$	.01	$-.00$	.01	$-.00$	.03	$-.03$	.00	$-.00$
Controls												
Exp. rhs.							Y	Y	Y	Y	Y	Y
Infl. lag									Y	Y		
Infl. MA											Y	Y
H_0: $\kappa_2 = 0$ (χ^2-*test*)			.11	.10	.00	.00	.00	.00	.00	.01	.00	.00
H_0: $\kappa_3 = 0$ (χ^2-*test*)			.87	.60	.00	.00	.00	.00	.00	.20	.00	.00
R^2	.07	.06	.15	.13	.71	.70	.98	.98	.98	.96	.98	.98
N	198	197	198	197	198	197	198	196	197	196	194	194
					PART B: DATA FROM 2-SAMPLE SVAR							
κ_1	.13***	.16***	.23***	.27***	.23***	.28***	.19***	.24***	.21***	.14***	.36***	.43***
δ_2			$-.21^{***}$	$-.26^{***}$	$-.12^{*}$	$-.23^{***}$	.40**	.34	.34**	.03	.24	.12
Controls												
Exp. rhs.							Y	Y	Y	Y	Y	Y
Infl. lag									Y	Y		
Infl. MA											Y	Y
H_0: $\kappa_2 = 0$ (χ^2-*test*)			.77	.89	.01	.22	.00	.01	.00	.30	.00	.01
R^2	.07	.06	.10	.10	.29	.27	.94	.94	.96	.92	.96	.95
N	198	197	198	197	198	197	198	196	197	196	194	194

Note: HAC robust standard errors. Significant at the 90% level (*), 95% level (**), and 99% level (***).

significant. More importantly, there is no statistical evidence of a decline in κ in later sub-samples, and the null that κ_2 or κ_3 are zero is rejected at all relevant levels. Also, the explanatory power of the regression equation improves substantially when data are purged of supply shocks, with an R^2 that increases from .15 in (2a) to .71 in (3a). A similar story holds true when we instead instrument the output gap in column (3b).

Next, we relax the restriction that inflation expectations enter with a unit coefficient in the Phillips curve. To this end we estimate the following regression equation:

$$\pi_t = c_1 + D_{2,t}c_2 + D_{3,t}c_3 + (\beta_1 + D_{2,t}\beta_2 + D_{3,t}\beta_3)\,\pi^e_{t+1} + (\kappa_1 + D_{2,t}\delta_2 + D_{3,t}\delta_3)\,y_t + u_t$$

The results are reported in column (4a) and (4b).[6] The inclusion of shifts in the role of inflation expectations has very little impact on the point estimates. Our estimate of κ is still relatively stable across sub-samples.

In the last four columns, we also control for historical inflation rates directly, either by including lagged inflation π_{t-1} in the regression (columns (5a) and (5b)) or the 4-quarter moving average $\frac{1}{4}\sum_{j=1}^{4}\pi_{t-j}$ (columns (6a) and (6b)). Once more we find, consistent with the scatter plots for these data, a remarkably stable slope of the Phillips curve. The only exception is when we use instruments for the specification with lagged inflation as a control variable. In this case the slope is only about .12. Although this point estimate remains fairly stable across sub-samples, we cannot reject that it is different from zero after 2007.

Finally, we compare the results discussed above with those in Part B of the table, where data have been estimated over two sub-samples with the SVAR model as well. Even though we are now only comparing two samples, the conditional data suggest a complete disappearance of the Phillips curve in the latter sample (Part B, columns (2a) and (2b)). Also, consistent with the scatter plots shown earlier, columns (3a) and (3b) suggest a flattening even when we consider data conditional on demand. This result, however, does not seem to be robust to more flexible specifications of how inflation expectations enter the Phillips curve. If anything, the results in columns (4a)-(6b) suggest that κ has increased somewhat in the last sub-sample, although this increase is only significant in a couple of specifications. Taken together, we find it hard to conclude that our regression results document strong evidence in favor of a decline in κ, the slope of the New Keynesian Phillips curve.

5.2 Rolling window estimates

Following the bulk of existing literature, our analysis so far is subject to the specified ex ante the periods in which samples are split. As a final exercise, we attempt to account for the timing of shifts in κ in a more flexible manner. We do this by means of rolling window regressions, both when filtering the data and when estimating the Phillips curve on a given dataset. The goal is to shed some light on the evolution of possibly time varying parameters, without having to take a stand ex ante on when these parameters may change.

To this end we estimate the following regression equation:

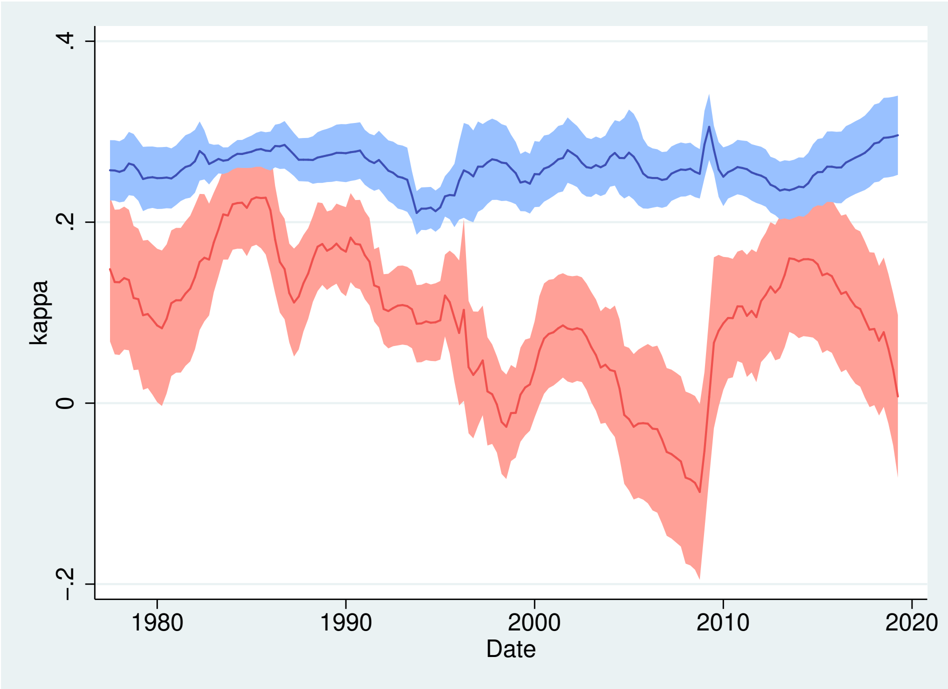
$$\pi_t - \pi^e_{t+1} = c_t + \kappa_t y_t + u_t$$

For each quarter t, the estimates of κ_t and other parameters is obtained using observations from $t-k$ to t (each sample includes k observations). The result is a time series of estimated values κ_t. We use two types of data in these rolling window regressions. First, we consider the conditional data generated by the SVAR model estimated over the entire sample. Second, as an alternative we also re-estimate the SVAR model over each sub-sample period used later on. This leaves us with estimates of π_t, π^e_{t+1} and y_t which come from an SVAR model unique

[6] We add two lags of inflation expectations to the instrument vector when IV methods are used.

for period t. The benefit with the first approach may be that estimates are more precise and stable, since all the conditional data come from the same model. The benefit with the second approach is that it identifies parameter changes in a much more flexible manner, at the expense of considerable noise in the generated data. In each case we choosing a rolling window of fifty observations.

Figure 10: Rolling window estimates based on data from a one-sample SVAR model



Note: Estimates of κ_t from rolling window regressions of $\pi_t - \pi^e_{t+1} = c_t + \kappa_t y_t + u_t$. Estimates based on unconditional data in red, and on conditional data in blue.

Figure 10 plots the time series of realized Phillips curve slopes when data come from the SVAR model estimated one sample. The window size of each sub-sample is 50 periods. We report point estimates as well as 68% confidence bands. Two observations stand out: first, the time varying slope obtained from unconditional data (in red) is consistently smaller than the counterpart obtained from data purged of supply shocks (in blue). Second, the two time series evolves quite different over time. The slope obtained from unconditional data has a downward trend beginning in the early 80's, followed by a temporary rise during the financial crisis. The latest estimate is close to zero, suggesting a statistical disconnect between inflation and output. This observation contrasts with the evolution of κ_t estimated from data conditional on demand. The latter has remained fairly stable over time, with only transitory fluctuations around .25. Again, we find a decline in the statistical inflation-output gap relationship which largely disappears once the data are purged of supply shocks.

Results based on the data from rolling window SVAR models are reported in Figure 11, still with a window size of 50 periods. Now the first observation starts in the early 90's, because the first observations from the SVAR models are used as training samples. Again we find relatively stable estimates of κ_t in the data purged of supply shocks, although these estimates are lower on average compared with those in Figure 10. The point estimates rise at the onset of the financial crisis, but then quickly return to pre-crisis levels. However, there is no apparent downward trend in the conditional estimates.

Figure 11: Rolling window estimates based on data from rolling window SVAR models

Note: Estimates of κ_t from rolling window regressions of $\pi_t - \pi^e_{t+1} = c_t + \kappa_t y_t + u_t$. Estimates based on unconditional data in red, and on conditional data in blue.

6 Conclusions

In this paper we have reconsidered the puzzling stability of inflation in spite or large fluctuations in real economic activity over the last couple of decades. Using a combination of New Keynesian theory and estimated SVAR models, we argue that controlling for the effects of all supply shocks (and not only for cost-push shocks) is of paramount importance to evaluate alternative explanations of the inflation puzzle. While we reconfirm that the unconditional correlation between output and inflation and the slope of Phillips curve estimated on unconditional data have declined, we find that the evidence of a decline of the slope is much weaker when conditioning on demand shocks. We find more support for alternative explanations based on a more aggressive response of monetary policy against inflation or on a more important role of supply shocks in the second part of the sample.

We are considering several robustness checks and extensions. We plan to use other measures of the output gap and inflation expectations. In addition, we are going to disentangle the supply shocks into different components to further evaluate the policy hypothesis and the shocks hypothesis.

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Appendix

A Additional details on the theoretical model

A.1 The textbook model with autoregressive shocks

Suppose that instead of being i.i.d. as in the main text, the structural shocks follow separate AR(1) processes:

$$\begin{aligned} d_t &= \rho_d d_{t-1} + \varepsilon_{d,t} \qquad \varepsilon_{d,t} \sim N\left(0, \sigma^2_{\varepsilon,d}\right) \\ s_t &= \rho_s s_{t-1} + \varepsilon_{s,t} \qquad \varepsilon_{s,t} \sim N\left(0, \sigma^2_{\varepsilon,s}\right) \end{aligned}$$

Closed form solutions for output and inflation follow:

$$\begin{aligned} y_t &= \frac{\sigma\left(1-\beta\rho_d\right)}{\Psi_d} d_t - \frac{\left(\phi_\pi - \rho_s\right)}{\Psi_s} s_t \\ \pi_t &= \frac{\sigma\kappa}{\Psi_d} d_t + \frac{\left[\sigma\left(1-\rho_s\right)+\phi_y\right]}{\Psi_s} s_t \end{aligned}$$

We have defined two auxiliary functions:

$$\begin{aligned} \Psi_d &= \left[\sigma\left(1-\rho_d\right)+\phi_y\right]\left(1-\beta\rho_d\right) + \left(\phi_\pi - \rho_d\right)\kappa > 0 \\ \Psi_s &= \left[\sigma\left(1-\rho_s\right)+\phi_y\right]\left(1-\beta\rho_s\right) + \left(\phi_\pi - \rho_s\right)\kappa > 0 \end{aligned}$$

OLS estimators under different assumptions follow readily:

(a) Unconditional data:

$$\begin{aligned} \kappa^{OLS} &= \frac{cov\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}, y_t\right)}{var\left(y_t\right)} \\ &= \frac{\left(\frac{\sigma(1-\beta\rho_d)}{\Psi_d}\right)^2\kappa - \left(\frac{1}{\Psi_z}\right)^2\left(\phi_\pi-\rho_z\right)\left[\sigma\left(1-\rho_z\right)+\phi_y\right]\left(1-\beta\rho_z\right)\frac{\sigma_z^2}{\sigma_d^2}}{\left(\frac{\sigma(1-\beta\rho_d)}{\Psi_d}\right)^2 + \left(\frac{\phi_\pi-\rho_z}{\Psi_z}\right)^2\frac{\sigma_z^2}{\sigma_d^2}} \le \kappa \end{aligned}$$

where $\sigma_d^2 = \frac{\sigma^2_{\varepsilon,d}}{1-\rho_d^2}$ and $\sigma_z^2 = \frac{\sigma^2_{\varepsilon,z}}{1-\rho_z^2}$.

(b) Purged of supply shocks, but ignoring expectations:

$$\kappa^{OLS} = \frac{cov\left(\pi_t, y_t\right)}{var\left(y_t\right)} = \frac{\kappa}{1-\beta\rho_d} \ge \kappa$$

(c) Purged of supply shocks and accounting for expectations:

$$\kappa^{OLS} = \frac{cov\left(\pi_t - \beta\mathbb{E}_t\pi_{t+1}, y_t\right)}{var\left(y_t\right)} = \kappa$$

A.2 The model with various bells and whistles

Finally, we add nominal wage rigidities, partial price and wage indexation, habit formation, and interest rate inertia. These features tend to make model dynamics more similar to those in estimated SVARs (hump-shaped impulse responses). Importantly, natural equilibrium dynamics are affected by habit formation since the source of habits is preferences. We state the natural equilibrium dynamics consistent with habits below:

$$\begin{aligned} w_t^n &= a_t \\ y_t^n &= \psi_y^h y_{t-1}^n + \psi_a^h a_t - (\sigma_h + \varphi)^{-1} \psi_t \\ r_t^n &= \sigma_h \left(\mathbb{E}_t \Delta y_{t+1}^n - h \Delta y_t^n + \mathbb{E}_t \Delta z_{d,t+1} \right) \end{aligned}$$

We have defined $\sigma_h = \frac{\sigma}{1-h}$, $\psi_y^h = \frac{\sigma_h h}{\sigma_h + \varphi}$ and $\psi_a^h = \frac{1+\varphi}{\sigma_h+\varphi}$ to ease the notation. Importantly, habit formation adds inertia to natural output beyond that implied by productivity a_t. The resulting model in terms of gaps from the natural equilibrium follows:

$$\begin{aligned} \tilde{y}_t - h\tilde{y}_{t-1} &= \mathbb{E}_t \left(\tilde{y}_{t+1} - h\tilde{y}_t \right) - \frac{1}{\sigma_h} \left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n \right) \\ \pi_t - \gamma_p \pi_{t-1} &= \beta \mathbb{E}_t \left(\pi_{t+1} - \gamma_p \pi_t \right) + \lambda \tilde{w}_t + \lambda z_{p,t} \\ \pi_{w,t} - \gamma_w \pi_{t-1} &= \beta \mathbb{E}_t \left(\pi_{w,t+1} - \gamma_w \pi_t \right) + \kappa_w \left(\tilde{y}_t - \psi_y^h \tilde{y}_{t-1} \right) - \lambda_w \tilde{w}_t + \lambda_w z_{w,t} \\ \tilde{w}_t &= \tilde{w}_{t-1} + \pi_{w,t} - \pi_t - \Delta w_t^n \\ y_t &= \tilde{y}_t + y_t^n \\ w_t &= \tilde{w}_t + w_t^n \\ i_t &= \phi_i i_{t-1} + (1 - \phi_i) \left(\phi_\pi \pi_t + \phi_y y_t \right) + z_t \end{aligned}$$

Finally, we assume AR(1) processes for all shocks:

$$\begin{aligned} z_{d,t} &= \rho_d z_{d,t-1} + \sigma_d \varepsilon_{d,t} && \text{(demand shock)} \\ z_{i,t} &= \rho_i z_{i,t-1} + \sigma_i \varepsilon_{i,t} && \text{(monetary policy shock)} \\ a_t &= \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t} && \text{(productivity shock)} \\ z_{p,t} &= \rho_p z_{p,t-1} + \sigma_p \varepsilon_{p,t} && \text{(price markup shock)} \\ z_{w,t} &= \rho_w z_{w,t-1} + \sigma_w \varepsilon_{w,t} && \text{(wage markup shock)} \\ \psi_t &= \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t} && \text{(labor supply shock)} \end{aligned}$$

B Additional results

Historical decompositions of data when the SVAR model is estimated over two sub-samples:

Figure B.1: Historical decomposition inflation

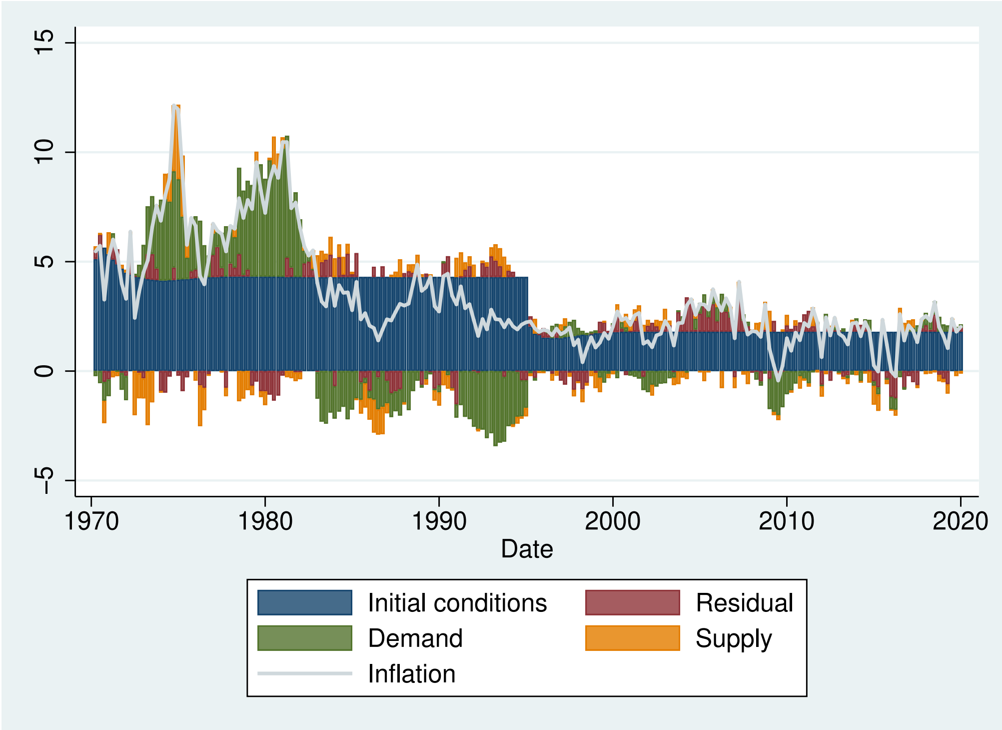


Figure B.2: Historical decomposition inflation expectations

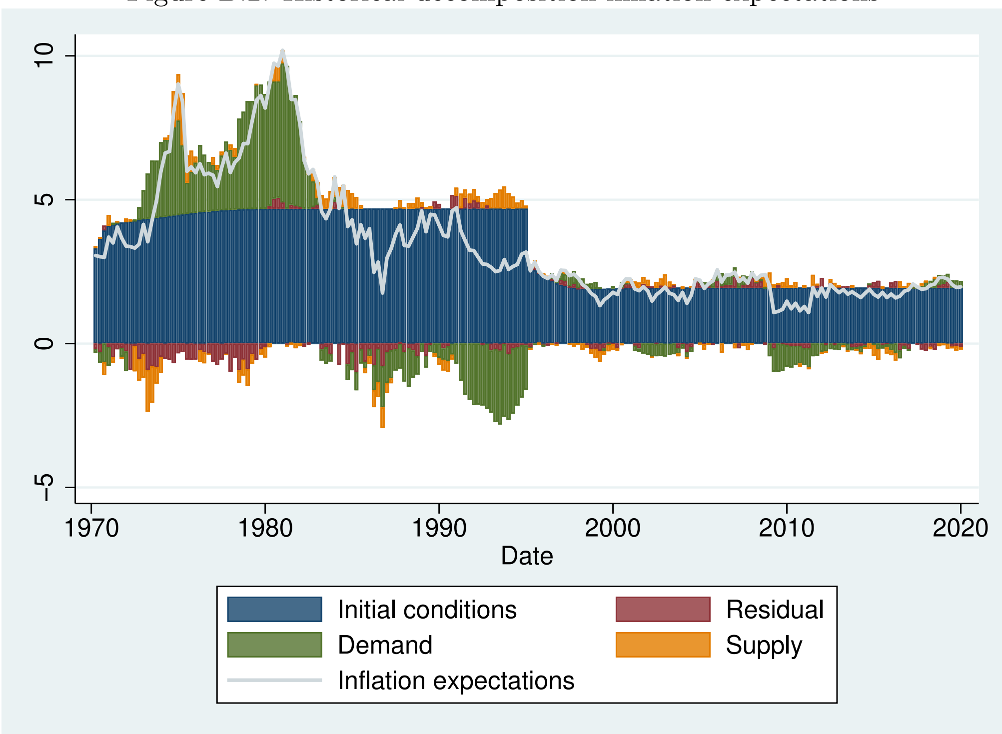
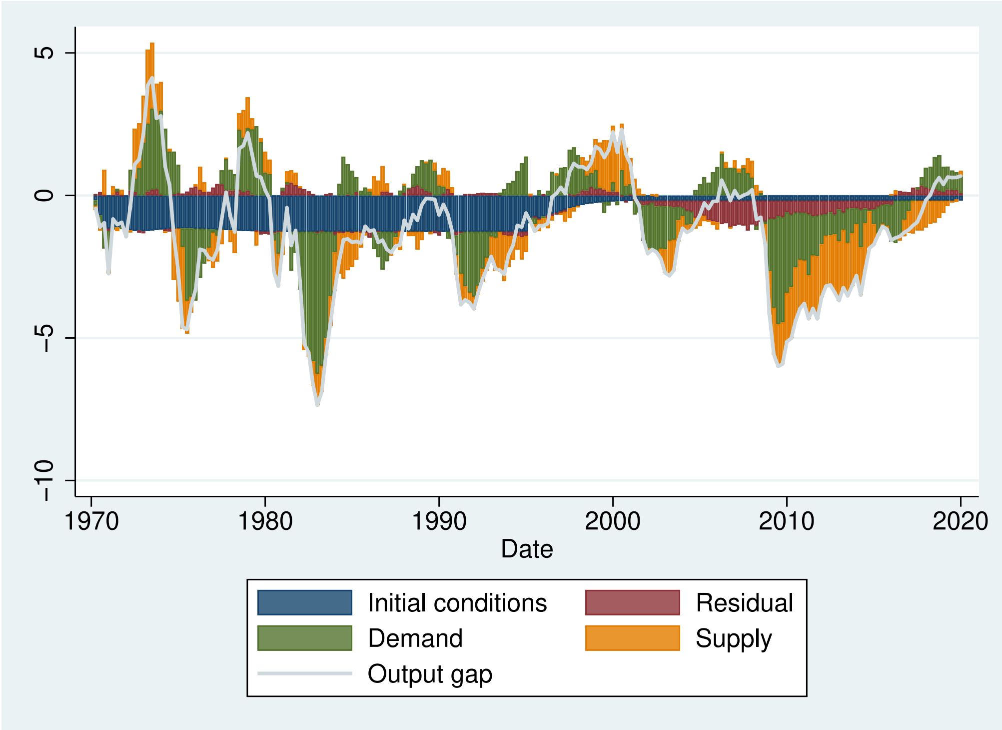


Figure B.3: Historical decomposition output gap



C Data

Variable	Data	Source	Transformation
y	Output gap	CBO	None
π	GDP deflator index	CBO	Annu. % change QoQ
π^e	Inflation expect. (one-q. ahead GDP defl.)	SPF	Median
p	Corporate profits	Fred	Deflated per capita annu. % change QoQ
w	Average hourly earnings	Fred	Deflated annu. % change QoQ
-	U.S. population	Fred	None

Sources: http://www.cbo.gov/about/products/budget-economic-data#11
http://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files/pgdp
http://fred.stlouisfed.org/series/CPATAX
http://fred.stlouisfed.org/series/AHETPI
http://fred.stlouisfed.org/series/POPTHM

General Conclusion

This Ph.D. thesis was written with the ambition to better understand the role of interest rates as an instrument influencing the economy for the central bank. Three main research questions were raised in an attempt to illustrate this role.

How did the term premium transmission channel of Quantitative Easing affect Euro Area macroeconomic variables ? Is the Japanese yield curve gap responsible for the low growth - low inflation environment present in Japan since the 1990s? Is the seemingly dead U.S. price Phillips curve really buried?

To the first question, the first chapter finds that the term premium channel of QE essentially works like the overall yield channel: it is expansionary for real GDP and inflation. A reduction in the reward for risk following a bonds purchase by the central bank stimulates the economy. Yet, using term structure modeling and time series analysis, the chapter also finds evidence that the loosening of monetary conditions engendered by this term premium channel was not enough to contribute positively to consumer prices in the aftermath of the Great Financial Crisis.

The second question is addressed with a semi-structural macroeconomic model in a second chapter. Modeling the natural yield curve and potential macroeconomic variables, this second part of the thesis finds that the different monetary regimes implemented by the Bank of Japan did not have an homogeneous impact on interest rates at all maturities. Besides, the monetary easing generated by the negative yield curve gap was not sufficient to significantly revive the Japanese economy.

Finally, the third question is the topic of a third chapter combining New Keynesian theory with time series analysis. It is found in this third part that the flattening of the slope of the structural relationship between price inflation and measures of real economic activity is only apparent. As suggested by New Keynesian theory, one needs to control for all supply shocks to prevent from a downward bias when estimating the slope of the Phillips curve. A cleaned estimation of the Phillips curve, conditional on demand, shows that the relation has not flattened much through time. More than that, the apparent flattening is actually explained by a more aggressive U.S. Federal Reserve and a change in the relative importance of demand and supply shocks.

Not claiming to resolve all the questions regarding the role of interest rates for the economy, I hope that this thesis will have contributed to extend our knowledge in monetary policy. The underlying research questions of the three chapters contained in this thesis will hopefully be of interest of the central bankers and relevant for policy purposes.