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Deontic Logic
a tool for law formalization ?

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Foreword

The choice of this subject for my PhD thesis is the natural culmination of my academic journey.

I studied law and mathematics simultaneously for many years until the Masters degree, and in mathematics I specialized in formal logic where I discovered the existence of Deontic Logic, and had a glimpse of its state of affairs.

I immediately knew that I wanted it to be my thesis subject, as I saw a big potential in it for bringing more correctness to the law, but I could not guarantee this potential before doing the work ; neither could I affirm that it would be relevant, nor successful, nor sufficient as a way to tackle legal inconsistencies.

I had been committed in entrepreneurship in the legal design domain for many years, and even though I saw the benefits of schematizing law, it was way too insufficient to get any global impact or result on the legal system, I needed to find out if this new method could fulfill its potential.

That is why there is a question mark at the end of my thesis title : I pursued the Deontic Logic path and did not know if it would work out until the very end of my thesis, and hopefully it did.

Introduction

1 Nemo censetur ignorare legem

No one is considered to be ignorant of the law.

But is this saying still relevant today ? Is it possible to "know the law" ?

Not when the law lacks clarity, predictability, consistency.

Imagine if you had two bosses you are supposed to listen to, with consequences if you do not follow what they order you to do, and they gave you two contradictory instructions.

Imagine if you were instructed to clean a locked room that you have never been given the key to.

Imagine if you were asked to "bring that item from the other room" without further explanation on what it is or where it was.

You do not need to imagine much longer, as it is the state of contemporary law, which can be quite absurd : for example, in Tampa, California, "it is illegal to eat cottage cheese after 6:00pm on Sunday"[1] ; and in France, until an old law was repealed in 2013[2], it was illegal for women to wear pants[3].

Contemporary law is unclear, unpredictable, potentially inconsistent ; and this must change.

Indeed, it was established many times by French legal doctrine (for example, but it is true for most legal orders[4][5]) that contemporary laws are more and more numerous and unintelligible, which entails a lack of clarity[6], predictability[7] and consistency[8].

Moreover, the laws change frequently and end up creating a situation of over-complexity and legal insecurity, as the French "Conseil d'Etat" notes in

[1]https://www.mallardperez.com/blog/unusual-laws-in-florida.cfm

[2]https://www.senat.fr/questions/base/2012/qSEQ120700692.html

[3]BARD, Christine. Le droit au pantalon. 2013.

[4]TURK, Matthew C. Overlapping legal rules in financial regulation and the administrative state. Ga. L. Rev., 2019, vol. 54, p. 791.

[5]KATZ, Leo. A theory of loopholes. The Journal of Legal Studies, 2010, vol. 39, no 1, p. 1-31.

[6]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, no 4, p. 905-917.

[7]MACKAAY, Ejan. L'inflation normative. 2018. p. 14-15.

[8]GALLARDO, Eudoxie. Les incohérences du droit pénal des mineurs contemporain. Revue de science criminelle et de droit pénal comparé, 2017, vol. 4, no 4, p. 713-724.

its successive reports[9] of 1991[10] and 2005[11].

One way that many scholars thought relevant to tackle this problem is to correct the law through logical formalization[12].

Indeed, formalization is supposed to bring back correctness into the law, but to this day the resulting impact in the daily life of legal professionals is rather limited except for some simplified diagrams, and one may wonder if this strategy is actually suitable for its purpose, or what we can expect from it.

Therefore, we must address what exactly a good logical formalization of the law would be, how it could benefit concretely the law and the legal professionals, and find out if such a formalization is actually achievable.

These challenges lead us to question formalization methodology, norm theory, and to formally build a system enabling the formalization of the law : this is why the keystone of our dissertation will be Deontic Logic (also known as "logic of norms").

In order to properly tackle it, we will thus thoroughly study Deontic Logic, as a potential tool for legal formalization.

To that end, we must address this question not only through a legal lens, but through a multidisciplinary research.

2 Building bridges

I realized early on that my thesis subject was not an independent one, as it deserves, to be fully addressed, to not be content with a monodisciplinary study.

Indeed, even if legal formalization, and more specifically Deontic Logic, were relatively recent topics, what could I possibly bring to the table if they had been already studied by leading researchers in their respective domains ?

There was my answer : there were numerous domains that often evolved in parallel but without any bridges bringing them into the same direction.

[9]https://www.conseil-etat.fr/publications-colloques/etudes/etudes-annuelles/securite-juridique-et-complexite-du-droit

[10]D'ÉTAT, Conseil. De la sécurité juridique. Rapport public, 1991, p. 15-47.

[11]CONSEIL, d'État. Sécurité juridique et complexité du droit. Rapport public 2006. Jurisprudence et avis de 2005, 2007.

[12]GRIMAUD, Lin. Pourquoi formaliser en travail social? Évolution de la supervision d'équipe depuis la loi 2002. Empan, 2009, no 3, p. 134-145.

Not only the researchers in these different domains did not bring a definitive answer to the legal formalization question, but they were often answering it separately.

Since I had the opportunity to have a background in both law and formal logic, I had to address this question panoramically, and bring these disciplines together in the same direction, while still bringing a novel point of view, which will be nourished by this double heritage.

I am aware that this objective does have consequences on my PhD thesis : making it very unorthodox, and quite ambitious.

This ambition is necessary in order to reconcile the various separate answers to legal formalization that can be found in philosophy of law, norms theory, formal (deontic) logic and legal informatics.

Indeed, I believe none of these answers can be sufficient on its own to grasp the complexity of the subject. Since there is not one single state of the art to assess, but one for each discipline/chapter, the states of the art will be done within each corresponding chapter, but I will however give some idea of it now :

- some authors focused their research on the feasibility of formalizing law, engaging in theoretical debates or dilemmas like Jorgensen's one[13] ; while totally obscuring the question of the appeal : why formalize in the first place ? even if we can, should we ? is it beneficial for the law, and if it is, under what conditions ?

The appeal and the feasibility of a potential formalization of the law must be addressed together, while considering the technical and theoretical interests and limitations.

- some concentrated on norms theory and their conception like Guastini[14] without linking it to formalization, while on the other hand most researchers on Deontic Logic did not question the definition of norms before proposing a logical system, like Jones and Pörn[15].

There should be, prior to any attempt at creating a Deontic Logic, a thorough study of norms definition, conception and properties, that could lead to understanding how to link their nature to some standards for a future formalization.

[13]COYLE, Sean. Facing Jorgensen's dilemma. N. Ir. Legal Q., 2004, vol. 55, p. 341.

[14]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

[15]JONES, Andrew JI et PÖRN, Ingmar. Ideality, sub-ideality and Deontic Logic. Synthese, 1985, p. 275-290.

- some constructed systems and axioms of deontic logics to fight paradoxes, like Prakken and Sergot[16], but these systems were disconnected from the real legal world and tacitly considered that legal systems were perfectly coherent, which they are not.

A Deontic Logic system should be aware of its object : the law is not necessarily coherent, and we should take this into account when building an axiom system.

- some, finally, paved the way for concrete tools and languages to support applications for law like the Akoma-Ntoso project[17], but it often lacks either some accessibility for lawyers, or some way to evaluate the method properly[18].

A language or tool for legal practice should be both accessible and assessable, lawyers should be able to both use it and verify it easily.

Moreover, all these different answers are complementary in order to address the question, being each at a different step of the process of formalizing law : that is why this thesis will build bridges between disciplines, and put these bridges together to close the huge gap separating the law from its computability.

However, the ambition of this thesis is not unbounded, we had to make some epistemological choices. In particular, when discussing law formalization, we will more precisely be focused on normative systems and the study of norms, and not include legal argumentation, for several reasons :

- we will formalize norms as entities of static normative systems, argumentation is essentially dynamic and dialogical, presupposing established norms as a basis, which is why I believe that the systematic study of the structure of normative systems must precede any formal treatment of argumentation ;

- we could not claim before doing this thesis that formalization of legal systems was possible with satisfying results, our first concern was to assess whether or not is was possible, and therefore it would have been methodologically presumptuous and unwarranted to include the study or formalized legal argumentation ;

- argumentation is sidelined from this thesis but not from my global re-

[16]PRAKKEN, Henry et SERGOT, Marek. Dyadic Deontic Logic and contrary-to-duty obligations. In : Defeasible Deontic Logic. Dordrecht : Springer Netherlands, 1997. p. 223-262.

[17]PALMIRANI, Monica et VITALI, Fabio. Akoma-Ntoso for legal documents. In : Legislative XML for the semantic web: Principles, models, standards for document management. Dordrecht : Springer Netherlands, 2011. p. 75-100.

[18]NAKAMURA, Makoto. Development of applications for open data for Japanese laws and regulations. J. Open Access L., 2022, vol. 10, p. 1.

search framework, on the contrary, having a solid theoretical basis of structural formalization is necessary before even thinking of this second step, which will be an interesting follow-up subject to research.

The ultimate goal of this thesis is to make law formal and computable, and in order to achieve that, creating a step-by-step method to make it possible.

To make things clearer, here is how I will structure my answer :

- if I want to tackle legal formalization, I must begin by challenging its importance and relevance (first chapter) ;
- before creating a logic of norms, I must discuss norms, address their definitions and properties (second chapter) ;
- I then have to build a formal Deontic Logic system while taking into account the reality of legal systems, previously addressed (third chapter) ;
- once the norms and formal ground are both known, I must find a way to bring them together, and I will do this by creating an intermediate step between both (fourth chapter) ;
- finally, when all the elements are present, only the final stage remains, i.e. presenting which concrete applications could come from this legal formalization method.

As a consequence of the ambition and the unorthodoxy of this multidisciplinary thesis, one of its main challenges was making it panoramic enough while still mostly readable by scholars regardless of their background, without losing any depth in researching each question, but this was particularly difficult to calibrate, so some parts will still probably be too technical (especially the end of the third chapter).

In order to face this issue, each chapter has been divided so as to have one main discipline, in the most logical order possible (a progression from the most theoretical chapter to the most practical one) ; within each chapter, the difficulty is progressive, with the most technical elements often in the end ; illustrations are often given, and there is a recapitulation at the end of each chapter.

Chapter I

The need for a logical formalization of the law

The first question that spontaneously arises when it comes to legal formalization is not "what for ?", but it is either "what is formalization ?"[19] or "why formalize ?"[20].

It is very intriguing to notice that law (in a broad sense) is sometimes seen as a whole impermeable entity that would naturally escape attempts to rationalize it, or worse, as an irrational entity in its very core, making it idealistic and unrealistic to even try to combine law and rationality.[21]

However, we will argue in this thesis that law can be formalized, and even needs to.

If the concrete ability to formalize it will be tackled later, this first chapter will address the need of law for formalization, which comes from the multiple shortcomings of contemporary legal systems.

We will first discuss what the actual needs of law have become over the years, then check if a formalization could indeed solve them, we will finally address the methodological dilemma we must face in order to achieve such formalization.

[19]PEREGRIN, Jaroslav et SVOBODA, Vladimír. Logical Formalization and the Formation of logic (s). Logique et Analyse, 2016, vol. 233, p. 55-80.

[20]GRIMAUD, Lin. Pourquoi formaliser en travail social? Évolution de la supervision d'équipe depuis la loi 2002. Empan, 2009, no 3, p. 134-145.

[21]BEAL, James Hartley. law making, rational and irrational. Journal of the American Pharmaceutical Association, 1921, vol. 10, no 8, p. 574-581.

1 The contemporary deficiencies of law

Nowadays, the law is not a smooth plane where every issue has a corresponding legal solution[22]. Not that it has ever been the case, it would be an illusion to think that these deficiencies are specific to contemporary legal systems, but they are definitely amplified today[23].

Indeed, norms are both too numerous[24] and too absent[25], as some situations are subject to overlapping norms[26] that may contradict each other, and others do not find a single applicable or clear norm[27].

1.1 Completeness

1.1.1 A dual definition

Defining completeness is relatively straightforward, but there are two ways of doing so, and they are both actually complementary as two sides of the same coin, and many authors refer to both of them.

A first way to define completeness is negatively, by simply saying that the law is complete if it has no (legal) gaps. For example, Michel Troper says that a legal system is complete if it "does not contain any gap"[28]. A legal gap, as Kelsen describes it[29], is the absence of a general norm governing a behavior (we will come back to discussing and classifying gaps in more detail later).

The second way to define completeness is positively, by saying that for every behavior we can find a norm in the legal system that governs it.

"For instance, Carnelutti says : "By the term 'completeness' we understand the demand that in the legal order there should be a mandate which would apply to or provide a solution of any conflict of interest that might

[22]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, no 4, p. 905-917.

[23]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi : retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, no 3, p. 541-563.

[24]MACKAAY, Ejan. L'inflation normative. 2018.

[25]MOREHOUSE, Libby. The Kids Are Not Alright: A Look into the Absence of laws Protecting Children in Social Media. Loy. LA Ent. L. Rev., 2023, vol. 44, p. 75.

[26]TURK, Matthew C. Overlapping legal rules in financial regulation and the administrative state. Ga. L. Rev., 2019, vol. 54, p. 791.

[27]KATZ, Leo. A theory of loopholes. The Journal of Legal Studies, 2010, vol. 39, no 1, p. 1-31.

[28]TROPER, Michel. Chapitre II. Le «dogme» de la complétude. Léviathan, 2011, p. 19-29.

[29]KELSEN, Hans. Théorie pure du droit. (Charles Eisenmann trans., 1962). p.245.

arise in any society regulated by law"[30] [Alchourrón and Bulygin] quite agree with Carnelutti"[31].

This is also the way completeness is defined by María José Falcon y Tella in her book "A Three-Dimensional Theory of law"[32] : "The expression " completeness of the legal system" refers to the need for the whole legal system to be complete, i.e., that it contain the legal norms which are necessary so that any case which may arise in real life can be resolved pursuant to law."

Therefore, for a system to be complete, it must contain enough norms to cover all potential cases, but a mere accumulation of them is not enough, as it is not about quantity[33], but it is their range that matters the most and that must be total (we will ignore for now the potential default norms that could help bring closure, and address them later).

Thus, a completeness issue comes from a lack of norms, not by quantity but by situations coverage ; as for my own definition, I will consider that a legal system is complete if and only if every legal issue does have at least one applicable norm to solve it, i.e. if every contextualized behavior can be deontically characterized (classified as obligatory, forbidden, permitted or optional) within the system.

Some might argue that all issues/behaviors are not legally relevant, we will tackle this lawlessness objection in the subsection "The reachability of completeness".

1.1.2 The "dogma" of completeness

In the literature, the completeness of legal systems is often presented firstly as a dogma, as in the Encyclopedia of the philosophy of law and social philosophy[34].

Indeed, for completeness, it is not the definition that sparks the most debates, but its modalization vis-à-vis the legal system (i.e. its necessity).

Alchourrón and Bulygin, in "Normative Systems", expose the various forms the dogma (they call it "postulate") can take[35], which we will briefly

[30]CARNELUTTI, Francesco. Teoria generale del diritto. 1940. p.107

[31]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.128.

[32]FALCON Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.8

[33]MARSCHIK, Axel. Too much order? The impact of special secondary norms on the unity and efficacy of the international legal system. Eur. J. Int'l L., 1998, vol. 9, p. 212.

[34]SELLERS, Mortimer et KIRSTE, Stephan (ed.). Encyclopedia of the philosophy of law and social philosophy. Dordrecht : Springer Netherlands, 2023. p.2062

[35]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.127.

summarize : some authors claim that the dogma is only an ideal and therefore legal systems "should be complete" ; some claim that legal systems may contain gaps but are necessarily complete since they can fill or eliminate the gaps ; and some authors claim that even though "legal rules" might be incomplete, the "law" *lato sensu* necessarily finds a way to be complete. This last claim is the closest to what our conclusion will be at the end of this section.

In order to support this postulate of the necessary completeness of the law, two main (opposed) arguments will be offered. They will both be discussed in our subsection "The reachability of completeness", but let us present them first :

The first argument is what I will call "default completeness", and it states that every behavior that is not governed by specific norms still falls under a more general default norm that will modalize it. This argument called restrictively "the Principle of Prohibition" by Alchourrón and Bulygin[36] (and rejected by them) ties in with the "general exclusive norm" Kelsenian concept[37].

The second argument is what I will call "partial completeness", and it states that every behavior that is not governed by specific norms is therefore outside of any legal scope, making automatically every legal system complete. This argument invokes the mirrored concepts of "empty legal space"[38] and "lawlessness"[39] (non-droit in French).

If I do agree with the first argument, I reject the second one, as I will develop later.

1.1.3 The need of completeness

Before diving deeper in the subject of necessary completeness, let us first discuss why it is an important property for a system to have, and why we need legal systems to be complete.

This need of filling gaps is not only a theoretical question, but is also very actual[40]. Indeed, T.S. Sales states in her article "The gaps and the legal

[36]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.125.

[37]KELSEN, Hans. Pure theory of law. Univ of California Press, 1967.

[38]GUASTINI Riccardo (2004) L'interpretazione dei documenti normativi. Giuffrè, Milano

[39]CARBONNIER, Jean. L'hypothèse du non-droit. (No Title), 1963.

[40]IRTI, Claudia. A Short Introduction to 'The Problem of Legal Gaps. In : Tulane European and Civil law Forum. 2014, vol. 29, p. 157.

system (in) completeness"[41] that gaps will inevitably exist in the law, and may be an issue for the system's completeness : "there are problems without a solution in explicit, specific standards. Given that social relations are dynamic, changes will occur, new necessities will appear and new demands will be made, and existing standards will be incapable of keeping up, with the accuracy necessary, with transformations and regulating all existing conduct. Gaps, therefore, are necessary to ensure that the deficiencies of the normative system are found and due adjustments are made, in order to ensure that law may walk in lock step with the development of society."

Thus, Sales' point of view is nuanced, by considering that completeness is not synonym of the absence of gaps, but more synonym of the absence of unfillable gaps, since many gaps in the law do actually have default solution coming from custom or interpretative tools for example. Thus, a complete system would be a system where no legal question stays unanswered, regardless of the legal (*stricto sensu*) source of the answer.

In order to illustrate this dichotomy, let us take a simple example : if a law lists exhaustively a series of items that are subject to a new tax, but states explicitly that the tax rate will be specified for each item by a decree, and if the decree later lists one by one all items with their corresponding tax rates except one item, then the tax rate of this specific item can be considered as an unfillable gap because it is explicitly both existing due to its mention in the law and not existing due to its absence in the decree that is supposed to specify it. We could even consider that, if the law does not differ its application, there would be a short period of time between the law and the decree where there is an unfillable gap for all these items tax rates.

On the contrary, we could cite the article 1433 of the French civil code[42] that contains fillable gaps, since instead of listing the items that are concerned by it, it gives a "notable" example of a concerned item, implying that all the others will be analogically inferrable. In this instance, the article gives us itself the gap and the way to fill it, by stating roughly "this law applies to all items that are like this specific one".

Whatever nuances we bring to define completeness, it is a very important feature for a legal system to have, because, as we will see below, it is the main thing that protects the people from what may wrongly seem inevitable, i.e. the arbitrariness of all the authorities that apply and interpret the law

[41] SALES, Tainah Simões. The gaps and the legal system (in) completeness. Revista da Faculdade de Direito da UERJ-RFD-Rio de, 2013, vol. 1, no 23.

[42] https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000006439724/

in general, and of the judges in particular[43].

On a similar level, the incompleteness of the ECHR's decisions has been criticized by Jeremy Letwin in his article "Why Completeness and Coherence Matter for the European Court of Human Rights"[44].

If completeness is an important feature for a legal system to have, it is because completeness protects the citizens from the arbitrariness and the potential abuse of judges, or any law interpreting authority (this idea is developed in detail later in this chapter) ; consequently, it will bring legal predictability by reducing the distance between judicial decisions, because no ruling will be made without legal sources anymore, which will push the system towards a more systematically coherent jurisprudence ; thus, the system will be more legitimate.

Indeed, in systems of civil law (the coherence claim does not stand for common law systems), since precedents are not binding, nothing stops several courts (or even judges of the same court) to deal differently with a legal gap, and it can result in different or even contradictory conclusions. So if we cannot link the completeness of norms with the coherence of norms in a legal system, we can however argue that legal incompleteness can entail judicial incoherence in civil law systems.

Therefore, completeness benefits both the citizens and the legal system, respectively bringing more predictability and less arbitrariness leading to more coherence on the one hand, and more legitimacy and stability on the other hand.

This vision is consistent with Weber's idea of a pure legitimate rational authority[45], making the systematic legality a key-factor of it.

Before going further with completeness, even if it is beneficial, we may wonder if it does make sense to actually pursue it.

1.1.4 The reachability of completeness

Partial completeness. Some may argue that law will never be complete, and even that it must stay that way, i.e. some areas of everyday life do not

[43]CHEMERINSKY, Erwin. In Defense of Judicial Supremacy. Wm. & Mary L. Rev., 2016, vol. 58, p. 1459.

[44]LETWIN, Jeremy. Why Completeness and Coherence Matter for the European Court of Human Rights. In : European Convention on Human Rights law Review. Brill Nijhoff, 2021. p. 119-154.

[45]WEBER, Max. Economy and society: An outline of interpretive sociology. University of California press, 1978.

need any legal involvement, which would not be welcome.

This is the "non-droit" hypothesis that was proposed by Jean Carbonnier[46] (it could be translated by lawlessness, or as Marijan Pavčnik says, "legally empty (free) spaces eluding law and lawyers"[47]). In her article "Le «vide juridique» et le «besoin de loi». pour un recours à l'hypothèse du non-droit."[48], the author develops that idea, by stating that legal gaps have been instrumentalized by politicians wanting every aspect of life ruled by law.

Some authors, like Bobbio in "Teoria dell'ordinamento giuridico"[49] argued that on the contrary the lawlessness was irrelevant to the necessary completeness of law : "there cannot be legal gaps, for everything that is not normatively qualified by the law is irrelevant from the legal point of view, and lies within an "empty space" not reached by the law".

Following this hypothesis, in "The Completeness of International law and Hamlet's Dilemma.", researchers of the University of Utrecht analyzed a decision of the International Court of Justice[50], which "asserted that the current state of international law and the facts at its disposal were insufficient to enable it to reach a definitive conclusion".

We could wonder if we are in presence of a genuine *non liquet*, since the court did not recognize any closing-rule that helps compensate the legal void, which is acknowledged. This absence of closing-rule might be a typical example of "non-droit" decided by the court.

However, we must not jump to such conclusions since, firstly, this is but one decision of fallible judges ; secondly, as the researchers develop in the article, we could consider that the absence of decision means the absence of sanction, which entails tacit permission (see next paragraph), so there would be no *non liquet* there ; and thirdly, the discovery of gaps in the law is only an opportunity to fill them, as "the judicial identification of material gaps constitutes the line of development for the law: 'How could law have developed if the assumed absence of gaps expressive of its formal completeness were to

[46]CARBONNIER, Jean. L'hypothèse du non-droit. (No Title), 1963.

[47]PAVCNIK, Marijan. Why Discuss Gaps in the law?. 1996.

[48]DINH, Anne-Marie Ho. Le «vide juridique» et le «besoin de loi». pour un recours à l'hypothèse du non-droit. L'Année sociologique, 2007, vol. 57, no 2, p. 419-453.

[49]BOBBIO, Norberto. Teoria dell'ordinamento giuridico. (No Title), 1960. p.248-249

[50]WERNER. The Completeness of International law and Hamlet's Dilemma. Nordic Journal of International law, 1999, vol. 68, no 3, p. 225-247.

be identified with the absence of gaps pointing to material perfection?'[51]"[52]

Thus, all of the authors defending partial completeness tend to narrate what I believe is a fiction in which the "non-droit" or "empty space" is somewhat or somehow intentional, and therefore there is neither mistakes nor legal gaps in any legal system (in the sense of behaviors not governed by any norm, I acknowledge that they would disagree with the appellation "legal" gap as they claim they are intentionally outside of legal scope).

Indeed, this theoretical postulate is too extreme, as it would be absurd to claim that the limits of the "empty space" are intentional, since legal gaps can take many forms among which we must recognize that some happen definitely by mistake. Therefore, I reject the argument that completeness is necessary because it is partial.

Default completeness. On the other hand, we have a totally opposed hypothesis that some authors like Troper[53] or Bulygin[54] defend, also stating that the law is always complete by default, but it is the case because general default tacit norms do exist systematically.

The clearest example of such theorists is actually Hans Kelsen in "Pure theory of law"[55], with his "general exclusive norm" theory :

According to Kelsen, the "general exclusive norm" of every legal system states that the people's behaviour is negatively ruled by the legal order, i.e. if a behavior is not legally prohibited, it is therefore in a sense permitted[56].

I consider this theory to be relevant to some extent. Indeed, if we look at the explicit-implicit norms dichotomy[57], there is a deontic debate[58] whether the absence of interdiction (respectively obligation) and the presence of an explicit permission (respectively option) are deontically equivalent. We will

[51]LAUTERPACHT, Hersch. The function of law in the international community. OUP Oxford, 2011.

[52]SCOBBIE, Iain GM. The Theorist as Judge: Hersch Lauterpacht's Concept of the International Judicial Function. Eur. J. Int'l L., 1997, vol. 8, p. 264.

[53]TROPER, Michel. Le droit et la nécessité. Chapitre II. Le «dogme» de la complétude. Léviathan, 2011, p. 19-29.

[54]E. Bulygin et al., Lagunas del derecho y positivismo jurídico.

[55]KELSEN, Hans. Pure theory of law. Univ of California Press, 1967.

[56]KELSEN, Hans. Pure theory of law. Univ of California Press, 1967.

[57]POSTEMA, Gerald J. Implicit law. law & Phil., 1994, vol. 13, p. 361.

[58]MAKINSON, David. On a fundamental problem of Deontic Logic. Norms, logics and information systems. New studies on Deontic Logic and computer science, 1999, vol. 49, p. 29-54.

see in the Chapter III that they are, and therefore every non-forbidden behavior is implicitly permitted.

This explicit-implicit norms debate mirrors the academic dispute that happened between Alchourrón and Bulygin[59] on one side, and Opalek and Wolenski[6061] on the other side (see references for the details of the feud).

We can note as a remark that many academic disputes are parallel to this one, for example the one between Gardiès on one side[62], defending the weak/strong permission distinction, and Bailhache[63] and Kalinowski[64] on the other side, criticizing this distinction, and I will join them (see references for the details of the feud).

Indeed, I would take mostly the side of Opalek and Wolenski as I consider that the deontic status of permission is unique (and equivalent to the non-prohibition), whether the permission is "strong" or "weak". Indeed, and this is consistent with the way I will focus on norms hierarchy in Chapter III, there are generally three main cases where we find strong explicit permission norms :

- it could be to create an exception to a prohibition, but that could be seen as a redefinition of the context of the in principle prohibition, therefore making the permission equivalent to the removal (i.e. absence) of prohibition ;

- it could be to confer exclusive power, for example stating "only A is permitted to do B", but this means that everyone else is prohibited to do B, so this norm is equivalent to "everyone that is not A is prohibited to do B", therefore joining the previous case of permissions ;

- the last case is negligible, and occurs when the legislator wants to clarify the obscure status of a behavior, and states that it is permitted, but this clarification does not create a different permission, it only makes explicit an already existing implicit norm.

Thus, implicit and explicit permissions share the same deontic status, which means that a default norm permitting what is not yet governed by

[59]ALCHOURRÓN, Carlos E. et BULYGIN, Eugenio. Permission and permissive norms. Theorie der Normen, 1984, p. 349-371.

[60]OPALEK, Kazimierz et WOLENSKI, Jan. On weak and strong permissions. Rechtstheorie, 1973, vol. 4, p. 169.

[61]OPALEK, K. et WOLENSKI, J. On weak and strong permissions once more. Rechtstheorie, 1986, vol. 17, p. 83.

[62]GARDIES, Jean-Louis. Essai sur la logique des modalités. FeniXX, 1979. p.97

[63]BAILHACHE, Patrice. Sémantiques pour des systèmes déontiques intégrant permission faible et permission forte. Logique et analyse, 1977, vol. 20, no 79, p. 286-316.

[64]KALINOWSKI, Georges. Sur un nouveau système de logique deontique. Logique et Analyse, 1974, vol. 17, no 67/68, p. 343-387.

other norms can be well integrated to legal systems without creating deontic conflict or problems.

However, we must nuance this immediately, by saying that an implicit default permission/option does not mean that it is automatic in every legal system or for every context.

A first limit to it is explicit default norms, that overrule (because they have priority) implicit default norms ; and some domains are restricted, making default permission insufficient against a general interdiction, for example in norms creation or the exercise of public power, where a special habilitation[65] is necessary. Another example of an explicit default norm is the principle of legality in criminal law[66], where no sanction can happen unless a clear and precise explicit law provides it.

As a remark, we could consider default norms separately as meta-norms, but this will not be necessary, since our definition of norms (see next chapter) can include them, because the non-existence of a norm can be seen as a fact, a context.

A second limit concerns legal systems that would integrate stronger implicit default norms, like for example in an authoritarian illiberal state[67], where everything that is not explicitly permitted could be forbidden. Nevertheless, for most behaviors and in most legal systems, the tacit permission stands if no explicit norm says otherwise.

A third limit is pointed out by Guastini in "L'interpretazione dei documenti normativi"[68] : some systems, like the Swiss legal order, can decide to have no default norm at all by allowing judges to create laws as a legislator whenever there is a legla gap. Not only is it dangerous prerogative vis-à-vis the separation of powers, but it still does not rule out the general exclusive norm, it just cancels its usefulness by creating explicit norms wherever it could have been applied.

My conclusive claim regarding default completeness would be that the implicit default permission is the default default norm ; i.e. when there are no alternative default norm, and when it is not addressed or cancelled, the permission will remain implicitly whenever and wherever there is no prohibition.

[65]TUSSEAU, Guillaume. Les normes d'habilitation. Dalloz, 2006.

[66]*Nullum crimen, nulla pœna sine lege.*

[67]TROPER, Michel. Le droit et la nécessité. Chapitre II. Le «dogme» de la complétude. Léviathan, 2011, p. 19-29.

[68]GUASTINI Riccardo (2004) L'interpretazione dei documenti normativi. Giuffrè, Milano. p.230

If the completeness is by default in any legal system, it might be related to the burden of jurisdiction.

1.1.5 The burden of jurisdiction

Literally, "jurisdiction" comes from the latin *juris dictio*, i.e. "saying the law"[69]. The judge is only but the mouth of the law, "la bouche de la loi"[70], and in civil law countries, he is not supposed to create norms, but only to apply them through interpretation.

But there is a significant burden on the judge, who must at all costs judge without the ability[71] to fill himself the gaps of the law[72].

Indeed, on the one hand, the *non liquet* is out of the question, as evidenced for example in France by the fourth article of the civil code "Le juge qui refusera de juger, sous prétexte du silence, de l'obscurité ou de l'insuffisance de la loi, pourra être poursuivi comme coupable de déni de justice."[73], which could be translated like this : "The judge who refuses to rule under the pretext of the silence, obscurity or deficiency of the law, may be prosecuted as guilty of denial of justice". So the judge must make a ruling. Similar dispositions can be also found in many other legal systems, like the Netherlands, Spain, Argentina or Quebec[74] for example.

On the other hand, the judge is not able to fill norm gaps by creating any, like the following articles states immediatly afterwards, in the fifth article of the French civil law code[75].

This can be seen as a distrust of judges, as Julien Jeanneney stated in "Les lacunes constitutionnelles"[76], and a will to protect the citizens from their arbitrariness. The law does not indeed mention "gaps" in the law, but

[69]NURET, Bertrand. Juridiction. In : Dictionnaire d'administration publique. Presses universitaires de Grenoble, 2014. p. 284-285.

[70]DE MONTESQUIEU, Charles Louis de Secondat. Montesquieu: de l'esprit des lois. Flammarion, 1989.

[71]Except in the very rare cases of legal orders allowing judges to create norms to fill gaps, like Swiss judges (see the first article of the Swiss civil code of 1907), but we will not consider this extreme hypothesis here

[72]GULATI, Mitu et MCCAULIFF, Catherine MA. On" Not" Making law. law and Contemporary Problems, 1998, vol. 61, no 3, p. 157-227.

[73]https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000006419283

[74]JEANNENEY, Julien. Les lacunes constitutionnelles. Dalloz; coll. Nouvelle bibliothèque de thèses, 2016. p.289

[75]https://www.legifrance.gouv.fr/codes/article_lc/LEGIARTI000006419284

[76]JEANNENEY, Julien. Les lacunes constitutionnelles. Dalloz; coll. Nouvelle bibliothèque de thèses, 2016. p.288

its "silence"[77].

Therefore how are judges supposed to deal with gaps ? Or as Alchourrón and Bulygin phrased their section : "Can the judge resolve a case where there is a normative gap without modifying the law ?"[78]

The answer is through interpretation[79] in the broadest sense : "the interpreter is valued as he carries out the sleight of hand consisting of filling in and bringing together the legal vacuums, making the legal system appear a whole and complete system"[80]. But the range of interpretation can be quite tricky, and we will come back to it at the end of the section.

However, we shall add that even if default mechanisms can help preserve completeness, we should not be eluding totally but only partially legal gaps thanks to general exclusive norms, as they do not help solve all of them.

Indeed, even if we acknowledge the existence of general default norms, there is only certain types of gaps that they do cover for. For example, if a regulation gives an explicitly exhaustive list of items subject to a new tax, it is easy to assume that every item that is not cited is not subject to this tax. However, if it also gives precisely a different and exclusive tax percentage for every item except one, no default reasoning can help close this gap here, because "X item is subject to a new specific tax" without defining its amount leaves unanswered and unanswerable the question "how much should we pay for it ?".

Thus, we must distinguish between different types of legal gaps, let us say a few words on their typology, and their consequences.

1.1.6 A typology of gaps

Some authors, like Conte[81], classify legal gaps according to the aspect of the law that they are having an impact on : either the gaps are impacting what

[77]JEANNENEY, Julien. Les lacunes constitutionnelles. Dalloz; coll. Nouvelle bibliothèque de thèses, 2016. p.17

[78]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.158.

[79]EASTERBROOK, Frank H. Legal Interpretation and the Power of the Judiciary. Harv. JL & Pub. Pol'y, 1984, vol. 7, p. 87.

[80]VICTORIA, ITURRALDE SESMA M. Significado e individualización de los enunciados normativos en la aplicación de la ley: lenguaje legislativo, antinomias y lagunas. 1988. Thèse de doctorat. Universidad del País Vasco-Euskal Herriko Unibertsitatea., p. 354-395

[81]CONTE, Amedeo G. DÉCISION, COMPLÉTUDE, CLÔTURE: A propos des lacunes en droit. Logique et analyse, 1966, vol. 9, no 33, p. 1-18.

the law is supposed to be, or they are impacting how the law is supposed to work ; i.e. on the one hand the gap can be "inadequate vis-à-vis the Sollen[82]", the ideal and the goals of the legal system, like for example justice or equality (some laws can create inequality gaps[83] while still being consistent and functional) ; and on the other hand the gap can be "inadequate vis-à-vis the Sein[84]", the material system itself, making it impossible technically to rule or decide legally the status of the situations that are covered by said gaps.

However, from a logical point of view, it is more relevant for us to focus exclusively on material gaps, those that disrupt the actual integrity of the law as we will see below. Indeed, inequality gaps do not disrupt the law in any way other than having unwanted consequences after being applied, but this is to the legislator to correct and adapt ; inequality gaps do not threaten the law itself since they can be applied, the logical danger comes from material gaps which can create inconsistencies and loopholes within the law.

I thus find interesting the classification that María José Falcon y Tella offered in her book[85] (which is close to the classification of "Normative gaps" we can find in the "Encyclopedia of the Philosophy of Law and Social Philosophy"[86] while refining its presentation a little) :

- a first type of gap threatens the completeness of the legal systems, the "normative"/"proper" gap, where a norm lacks although is needed to tackle some issues and situations that need to be dealt with.

- a second type of gap threatens the clarity of legal systems, the "technical"/"irregular" gap, where a norm does cover a subject but with too few material to make any decision/ruling possible without stretching its interpretation too arbitrarily.

- a last type of gap threatens the coherence of the legal systems, the "logic"/"conflict" gap, where norms overlap and contradict each other vis-à-vis the outcome to give a legal situation/issue.

(- a fourth type of gap is presented in the book as the "unfairness" gap, but it shall not be included among material gaps according to Conte's dis-

[82]INCAMPO, Antonio, et al. Sein und Sollen. From Kelsen to Schambeck. In : The 80th Anniversary of Professor Schambeck. Gdansk University Press, 2016. p. 21-30.

[83]DE OLIVEIRA CARVALHO, Catarina. The great inequality gaps: a brief overview1. The quest for labour rights and social justice, 2024.

[84]INCAMPO, Antonio, et al. Sein und Sollen. From Kelsen to Schambeck. In : The 80th Anniversary of Professor Schambeck. Gdansk University Press, 2016. p. 21-30.

[85]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.8

[86]SELLERS, Mortimer et KIRSTE, Stephan (ed.). Encyclopedia of the philosophy of law and social philosophy. Dordrecht : Springer Netherlands, 2023. p.1077

tinction cited earlier[87], since it is either no more than a conflict between a normative element and a higher standard/principle of the normative system or a non-material gap).

This classification is interesting because it really offers a unified and elegant way to partition the deficiencies of legal systems we are addressing in this chapter. It is also logically relevant to distinguish gaps vis-à-vis the structural source of their inadequacy : the norms are alternatively too silent, too vague or too diverse.

I shall nevertheless nuance the distinction by refining the third type mentioned, as we cannot consider as a gap a mere contradiction between norms that do exist and cover the legal issue ; however, there can be incoherence related material gaps : when there is no norm clearly governing a case, but two relatively close norms could be stretched to fill the gap, with incompatible consequences for the case, leading to a potentially inconsistent jurisprudence.

We will deal in the following sections with the clarity and coherence issues in legal systems, but let us first dive deeper in understanding the meaning of completeness, or more precisely what we would ideally want completeness to entail.

1.1.7 Pragmatic completeness

We discussed already the theoretical conception of completeness for a legal system, but we still have to explore how it translates into practice.

When is a legal system complete ? Actually when all the necessary tools are available : either the system already has a complete set of laws and therefore does not need anything, or it must find alternative tools to maintain completeness :

- on the one hand, it can be via default and dense principles that cover for the "gaps" of the law, which can manifest in the form of default or very general laws, or superior standards/principles[88].

[87]CONTE, Amedeo G. DÉCISION, COMPLÉTUDE, CLÔTURE: A propos des lacunes en droit. Logique et analyse, 1966, vol. 9, no 33, p. 1-18.

[88]TROPER, Michel. Chapitre II. Le «dogme» de la complétude. Léviathan, 2011, p. 19-29.

- on the other hand, it can be via interpretation, whether the interpreter (often the judge) needs to "find" the completeness in the law, or outside of it ; i.e. sometimes it will be sufficient to interpret widely a keyword of a law, and sometimes the interpretation needs to go even further and fill the gap with sources that are external to the law itself (for example, in Québec, the interpreters of the law once had to find "external completeness" by comparative reasoning with foreign legal systems[89]).

The arbitrariness of the interpretation depends on the needs and quality of the system, but there seems to be no limit to the interpretation's range since the judge is often forced by law to make a ruling as if it was complete[90]. This obligation for the judge to make a ruling regardless of the state of the law is present in many contemporary legal systems, as illustrated by Jeanneney in "Les lacunes constitutionnelles"[91], where he cites, among others, Argentina, Quebec, Rwanda and Albania where, like in the French civil code, judges are prescribed to judge any case submitted to them.

To circle back to the "dogma" of completeness, it is neither that legal systems are all necessarily complete, nor that they cannot be : the dogma is a self-fulfilling prophecy that legal systems adopt in order to become complete regardless of their quality (i.e. the level of incompleteness of the legal rules, in oposition to the whole legal order which will have to be complete in the end).

Therefore, the law is complete when it "decides" that it must be complete, but at what cost ?

The definitive cost of the completeness, for any system whose law is not complete enough on its own, is the arbitrariness imposed on its judges and interpreters, as Coval and Smith say in "The completeness of rules"[92]. They also consider that it is too much discretion to give the judges, who were supposed to be the mouth of the law, not its hands.

That is why it is so important that the law *stricto sensu* becomes complete on its own, and fills its gaps to avoid discretion and arbitrariness.

But legal systems must not fall into the other trap : creating too much norms to make sure that gaps are filled, risking therefore the coherence of

[89]JUTRAS, Daniel. Cartographie de la mixité: La common law et la complétude du droit civil au Québec. Can. B. Rev., 2009, vol. 88, p. 247.

[90]MONNIER, François. Déni de justice. Droits, 2001, vol. 34, no 2, p. 91-94.

[91]JEANNENEY, Julien. Les lacunes constitutionnelles. Dalloz; coll. Nouvelle bibliothèque de thèses, 2016. p.288

[92]COVAL, S. C. et SMITH, J. C. The Completeness of Rules. The Cambridge law Journal, 1977, vol. 36, no 2, p. 364-368.

the whole system.

"In addition to completeness, the legal system features coherence or cohesion."[93]

1.2 Coherence

1.2.1 A divisive definition

The coherence issue comes from a surplus of norms, not by pure quantity but by situations coverage ; i.e. we will consider that a legal system is coherent if and only if every legal conflict does have at most one normative issue (potentially from several texts if they come to the same conclusion) to solve it.

We choose this definition, which is not a consensual one[94], among many others for several reasons :

- Firstly, this definition of coherence mirrors how we defined completeness earlier, which has the benefit of being immediately practical for further work on norms, since its combination with the completeness criteria would expect from an ideal legal system to have one unique normative solution for every legal issue (simultaneously "at least one" and "at most one" solution per issue). No more, no less, exactly one : "the entirety of the precepts which form a specific legal system is not a mere superimposed addition, but a regulated total, wherein each part acquires meaning in relation to the others, forming a kind of legal puzzle where each piece fits in perfectly with the others in establishing a system."[95]

- Secondly, we must distinguish it from mere consistency, and according to Wintgens[96] this distinction is not always considered by scholars because of the proximity of the two notions, but consistency is the absence of contradictions. Since consistency is a necessary condition for coher-

[93]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

[94]DICKSON, Julie. Interpretation and coherence in legal reasoning. 2001.

[95]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

[96]WINTGENS, Luc J. Coherence of the law. ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of law and Social Philosophy, 1993, vol. 79, no 4, p. 483-519.

ence[97], it seems logical to me (and to other authors like Wintgens[98] or Raz[99]) that a convenient definition for coherence would be the "unity of principle", which I rephrased into "at most one normative issue".

We also had to make a choice since there is no consensual definitions of coherence or consistency in law, as Stephen Pethick states in "Solving the impossible: The puzzle of coherence, consistency and law." : "Though considerable claims are made for the use of coherence in law, its meaning is routinely taken to be elusive, controversial or even mystical. Some argue that defining coherence is logically impossible"[100].

As a remark, we shall point out that we will only consider coherence as a close notion to consistency and as a structural property of normative systems (systemic perspective), not through the coherentist lens of epistemic justification (argumentative perspective), for two reasons :

- firstly, coherence as a systems property is exclusionary, i.e. it excludes norms responsible for contradictions, without challenging the rest of them ; coherence from an argumentative lens risks shifting the burden by requiring every norm to be justified prior to any potential contradiction. My aim is more modest than answering globally both questions, as I will not question the justification of any norm that is not responsible for an inconsistency in the system.

- secondly, since the main goal of the thesis is achieving legal formalization, it is more relevant to focus on this structure perspective rather than the argumentative one, considering that we will not include legal interpretation in our formalization method. Also, a switch of the burden on norms from the default absence of contradiction to the needed justification would be very detrimental from a logical point of view (see the similar quandary concerning the negation as failure (NAF) in Chapter III).

For discussions on the argumentative perspective of coherence, we refer to the works of Hage[101] and Amaya[102]

[97]BALKIN, Jack M. Understanding legal understanding: The legal subject and the problem of legal coherence. Yale law Journal, 1993, p. 105-176.

[98]WINTGENS, Luc J. Coherence of the law. ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of law and Social Philosophy, 1993, vol. 79, no 4, p. 483-519.

[99]RAZ, Joseph. The relevance of coherence. BUL Rev., 1992, vol. 72, p. 273.

[100]PETHICK, Stephen. Solving the impossible: The puzzle of coherence, consistency and law. N. Ir. Legal Q., 2008, vol. 59, p. 395.

[101]HAGE, Jaap. Law and coherence. Ratio Juris, 2004, vol. 17, no 1, p. 87-105.

[102]AMAYA, Amalia. Legal justification by optimal coherence. Ratio Juris, 2011, vol. 24, no 3, p. 304-329.

As we will see in this section, coherence is one of the most important features for legal systems to have, but it is often threatened by the accumulation of legal norms, which nowadays are more than ever piling up :

"The term norm inflation appears to be a generalization, towards all sources of law, of legislative inflation, defined as a disproportionate increase in the volume and length of legislation."[103]

For example, in Canada, the number of pages concerning legislation on intellectual property alone went from 691 in 1995 to 1200 in 2016[104] ; as for France, as Sassier and Lansov pointed out in "Ubu loi"[105] : in France (in 2008) we have "10,500 laws, 120,000 decrees, 7,400 treaties, 17,000 European Union texts, tens of thousands of pages of 62 different codes!".

The more norms[106] you have, the more often you might find inconsistencies. This assumption would be insignificant if the legislators were thorough and enacted only laws written meticulously and qualitatively, but it is not the case, as we will see in the next section.

Another problem, often recurring, is that when a new norm is enacted, all the linked existing norms are not systematically checked or revoked, which leads to contradictions.

However, we must be careful when calling out contradictions, or shall we say "antinomies" more specifically, as María José Falcon y Tella develops in her book[107] :

Indeed, we are not in the presence of an antinomy between norms unless "it is necessary for the norms to be absolutely incompatible or, in other words, that their simultaneous application be impossible."[108]

"According to García Maynez, who, in turn, echoes the Kelsenian doctrine of the four fields of validity, “antinomy exists when two legal norms of one same system are mutually opposed in a contradictory manner, provided they have identical material, spatial and temporal scopes, but one permits

[103]MACKAAY, Ejan. L'inflation normative. 2018.

[104]MACKAAY, Ejan. Législation canadienne en propriété intellectuelle. Scarborough, Ont.: Carswell, 1994.

[105]SASSIER, Philippe et LANSOY, Dominique. Ubu loi. Fayard, 2008. introduction.

[106]see definition in Chapter II

[107]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

[108]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

and regulates and the other prohibits the same subjects from the same conduct."[109][110]

For example, if a law condemns a behavior with a fine, and if another law condemns the same behavior with a jail sentence, then there is no antinomy and these two laws are compatible because we can apply both : the normative answer to this behavior can therefore be unique (in the legal systems' perspective), and it is ((fine) **and** (jail sentence)). However, if a law condemns a behavior with a fine, and if another law permits the same behavior, then we do have an antinomy here, because there are two different and incompatible issues, and only one of the two laws can be applied here.

The same would be true of power-conferring norms, where an antinomy would only exists if two norms respectively permit and prohibit the same person or entity from exerting said power (power is indeed only an explicit option, i.e. an explicit non-prohibition).

This way of defining antinomies may seem restrictive, as its strict interpretation does not include contraries but only contradictions ; however, if we aim at achieving coherence, we shall consider both in a more broad sense of what antinomies are.

1.2.2 The self-awareness of the legal world

Legal actors themselves seem to be more and more aware of the inherent need of coherence of the law, and start to mark out to protect their legal system against itself. One of the driving forces of this phenomenon is the standard of legal certainty[111]/security[112] ("sécurité juridique" in French legal doctrine[113]).

Speaking of "sécurité juridique", many important cases in both public and private law have been establishing it as an important principle, like the "KPMG" case in front of the Conseil d'Etat in 2006[114], or the 1997 case in

[109]GARCÍA MÁYNEZ, Eduardo, et al. Introducción al estudio del derecho. México: Porrúa, 2002, vol. 54.

[110]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

[111]BRAITHWAITE, John. Rules and principles: A theory of legal certainty. Australasian Journal of Legal Philosophy, 2002, vol. 27, no 2002, p. 47-82.

[112]PIAZZON, Thomas. La sécurité juridique. 2006. Thèse de doctorat. Paris 2.

[113]DELLAUX, Julien. Le principe de sécurité juridique en droit constitutionnel: signes et espoirs d'une consolidation de l'ordre juridique interne et de l'État de droit. Revue française de droit constitutionnel, 2019, vol. 119, no 3, p. 665-696.

[114]Conseil d'État 288460, lecture du 24 mars 2006,

front of the Cour de Cassation[115] where the judges insisted on the fact that no one is supposed to anticipate legal changes but only taking into account the positive law.

The Court of Justice of the European Union is also very abundant about this principle, dedicating to it "thousands of decisions"[116].

This is part of the thesis of Delnoy in his article "The legal obligations of consistency of the law or how the law protects itself against itself."[117] : Delnoy states that many legal principles, standards and requirements (even at a high priority normative level, within the Constitution principles *lato sensu* for example) all converge in establishing a coherence constraint that can be imposed on the legislator.

Guiorguieff confirms this point of view, as he states that the French "Conseil d'Etat" itself is the guarantor of the coherence and the predictability of laws,[118] supporting the argument that coherence is both necessary and at risk for maintaining legal certainty.

Therefore, more coherence is beneficial to both the legal system itself which keeps improving, and to the people and citizens, as "little by little, private individuals are better equipped, from a legal point of view, to fight against legal inconsistencies."[119]

ECLI:FR:CEASS:2006:288460.20060324

[115]Civ. 1re, 25 nov. 1997, Bull. civ. I, n° 328, p. 222 ; RTD civ. 1998 (1), p. 210, obs. N. Molfessis

[116]VAN MEERBEECK, Jérémie. De la certitude à la confiance: le principe de sécurité juridique dans la jurisprudence de la Cour de justice de l'Union européenne. Presses universitaires Saint-Louis Bruxelles, 2019. p.33

[117]DELNOY, Michel (2005). Les obligations juridiques de cohérence du droit ou comment le droit se protège contre lui-même. In C. Biquet, A. Deliège, P. Lecocq, Y.-H. Leleu, ... M. Vanwijck (Eds.), Liber Amicorum Paul Delnoy (pp. 923-936). Bruxelles, Belgium: Larcier Intersentia.

[118]GUIORGUIEFF, Johann. Le Conseil d'Etat garantit la prévisibilité de la loi, pas celle de la jurisprudence judiciaire. Principes de sécurité juridique et de confiance légitime (Responsabilité du fait des lois). La Revue des droits de l'homme. Revue du Centre de recherches et d'études sur les droits fondamentaux, 2014.

[119]DELNOY, Michel (2005). Les obligations juridiques de cohérence du droit ou comment le droit se protège contre lui-même. In C. Biquet, A. Deliège, P. Lecocq, Y.-H. Leleu, ... M. Vanwijck (Eds.), Liber Amicorum Paul Delnoy (pp. 923-936). Bruxelles, Belgium: Larcier Intersentia.

1.2.3 Fighting arbitrariness

One of the end goals of making coherence a requirement for legal systems is to neutralize the potential arbitrariness of all law-applying authorities.

Indeed, if in an incomplete system's judges (among others) have an arbitrary margin in "interpreting" norms widely to fill gaps, because of the lack of norms for many legal issues ; in an incoherent system, they have an arbitrary decision to make in considering which norm to apply, since several contradicting norms may all overlap and be individually applicable to the same legal issue, but only one can remain.

The main problem of arbitrariness is that it is a direct threat to the principles of equality and to the due process of law[120]. As Karen Steyn develops in her article "Consistency – A Principle of Public law?"[121] : "The rule of law requires that laws be applied equally, without unjustifiable differentiation." Many other authors also describe arbitrary power as a danger for due process or for human rights, like Breakey[122], Skinner[123] or Ribeiro[124].

This requirement also ties in with the principle of "quality legislation", or "good legislation", which notably "requires that the law produced be consistent within itself, be legally sound (that is, it complies with the constitution and is compatible with existing laws and the legal system generally)"[125] ; this principle has been considered a requirement by many constitutional courts around the world, for example in Latvia[126].

Therefore, if we want a legal system to be arbitrariness-free (as much as possible) - and we do want it that way - we must also pursue coherence.

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[120]BARNETT, Randy E. et BERNICK, Evan D. No Arbitrary Power: An Originalist Theory of the Due Process of law. Wm. & Mary L. Rev., 2018, vol. 60, p. 1599.

[121]STEYN, Karen. Consistency–A Principle of Public law?. Judicial Review, 1997, vol. 2, no 1, p. 22-26.

[122]BREAKEY, Hugh. Arbitrary power, arbitrary interference and the abuse of power: Corruption, natural rights and human rights. In : Responsible leadership and ethical decision-making. Emerald Publishing Limited, 2017. p. 125-145.

[123]SKINNER, Quentin. Freedom as the absence of arbitrary power. Republicanism and political theory, 2008, vol. 3, no 4, p. 83-101.

[124]RIBEIRO, Marc. Limiting arbitrary power: the vagueness doctrine in Canadian constitutional law. UBC Press, 2004.

[125]AITKEN, Victoria E. An exposition of legislative quality and its relevance for effective development. ProLaw Student Journal, 2013, vol. 2, p. 1-43. p.5

[126]Satversmes tiesas 2003. gada 25. marta spriedums lietā Nr. 2002-12-01. Latvijas Vēstnesis, 2003. gada 26. marts, Nr. 47. Sprieduma secinājumu daļas 6.punkts [Judgement of the Constitutional Court of 25 March 2003 in case No. 2002-12-01]; Satversmes tiesas 2005. gada 6. aprīļa spriedums lietā Nr. 2004-21-01. Latvijas Vēstnesis, 2005. gada 7. aprīlis, Nr. 55. Sprieduma 9.3.1. punkts. Judgement of the Constitutional Court of 6 April 2005 in case No. 2004-21-01.

1.2.4 A rationality guarantee

As Ratti and Rodriguez said in their article on Legal systems[127] : "The law should be, but not necessarily is, the product of a rational normative authority."

This statement is part of the presentation of the "dogma of consistency", which presents it as a necessary property for legal systems to have.

One can wonder why consistency would be such an important property of legal systems. We already saw that it ensures legal certainty/predictability for the people, and that it neutralizes excessive arbitrariness among the judges, but there is more to it, "because an inconsistent system cannot fulfill its central function properly, that is, to guide human behavior in a rational way"[128].

We cannot expect people to obey a system that prescribes them to have incompatible behaviors simultaneously, or else, the law would have no rationality anymore : from the individuals perspective[129], if the law can prescribe two incompatible and contradictory behaviors, and if both those behaviors are sanctioned if not adopted, we could have absurd situations where there is no legal move for you to make, like being stalemated in chess.

What rationality could have a system where some situations would have no legal solution, and where sanction becomes inevitable, whatever you do ?

In order to avoid antinomies between norms, we need to establish ""resolution criteria" for the antinomies"[130]. Indeed, norms conflicts would be harmless if the conflicts are all solvable, but we will come back to this in detail in our third chapter when exposing our strategy for this. And as Raz claimed in "The identity of legal systems"[131] : "legal systems (empirically or logically) always provide the criteria for resolving any possible inconsistency"[132].

[127]SELLERS, Mortimer et KIRSTE, Stephan (ed.). Encyclopedia of the philosophy of law and social philosophy. Dordrecht : Springer Netherlands, 2023. p.2063

[128]SPAAK, Torben. Consistency in law. Faculty of law, Stockholm University Research Paper, 2023, no 132.

[129]from the perspective of the legal authorities, we saw earlier that inconsistencies lead to arbitrariness. We here tackle the people's view.

[130]Y TELLA, María José Falcón. A three-dimensional theory of law. Martinus Nijhoff Publishers, 2010. Ch.9

[131]RAZ, Joseph. The identity of legal systems. Calif. L. Rev., 1971, vol. 59, p. 795.

[132]SELLERS, Mortimer et KIRSTE, Stephan (ed.). Encyclopedia of the philosophy of

As for the needs of law, it would be theoretically satisfying to be just complete and coherent, but we cannot ignore that from a pragmatic point of view, it must also be clear.

1.3 Clarity

1.3.1 Interpretability and understandability

The clarity issue comes from the source of norms[133] : the legislator, and the quality of their writing ; i.e. I consider that a legal system is clear if and only if every legal norm does not have any way of being understood other than the way the legislator intended it to be.

I would even add that a norm is clear if it can be understood only in one way. Not interpreted, but understood, and we do distinguish between the two concepts, as Marcelo Dascal did in his book "Interpretation and understanding"[134] : thus, if we consider different definitions and apply them to norms, we distinguish between the understanding which concerns the essence of the obligation, and the interpretation which concerns the context of the obligation ; the understanding is related to the structure of the norm, the interpretation is related to its wording.

Our distinction is relatively close to Marmor's distinction between ambiguity and vagueness in "Varieties of Vagueness in the Law"[135] : when we face an ambiguous expression, it can be understood in several alternative ways, but there is necessarily a unique understanding that is the correct one (the one intended by the author) ; when there is a vague expression, the interpretations can vary, but this is normal practice.

As Asgeirsson stated in "The nature and value of vagueness in the law"[136], "Legal indeterminacy [...], I argue, is sometimes a good thing [...] to let interpreters handle borderline cases if and when they arise."

Let us have an example to make our distinction clear : we consider a norm stating that "every motorized vehicle ought to have insurance"[137]. The mo-

law and social philosophy. Dordrecht : Springer Netherlands, 2023. p.2063

[133]FRANCUS, Michael A. The law of Clarity and the Clarity of law. U. Chi. L. Rev. Online, 2020, p. 1.

[134]DASCAL, Marcelo. Interpretation and understanding. 2003.

[135]MARMOR, Andrei. Varieties of Vagueness in the Law. In : Handbook of Legal Reasoning and Argumentation. Dordrecht : Springer Netherlands, 2018. p. 561-580.

[136]ASGEIRSSON, Hrafn. The nature and value of vagueness in the law. 2020. p.3

[137]fictional norm, inspired by the French car insurance legislation (article L.211-1, Code

torized vehicles are many, and we can have a certain range of interpretation about what is or is not a motorized vehicle (the french jurisprudence is abundant on this matter, like the famous ride-on lawnmower case[138]), but there is only one way to understand the normative statement of this text, which is the obligation to have an insurance for it, which is very clear.

On the other hand, we consider another example : the fictional norm "every person must behave in a friendly manner" (which is not as far-fetched as it seems, considering the legal standards of "reasonable person[139]", or the no so old "bonus pater familias[140][141]"). The interpretation of the word "person" will be very negligible and straightforward, but there is no unique or obvious way to understand what is expected deontically from these people, what does it mean to behave in a friendly manner ? It is not clear.

We thus establish a difference between norms we cannot understand, as we cannot know what behavior is deontically expected, and norms we can understand but still have to interpret, i.e. we know what is expected but the context *lato sensu* is debatable.

1.3.2 The importance of clarity

Before diving into the details of why clarity is so important, we can already notice that, firstly, it is one of the eight standards of the internal morality of law that Fuller considers necessary for a principle to be acceptable as a law [142].

This importance of clarity even seems to become more and more crucial over time. Indeed, as Benezech discusses in his article "L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence."[143], the French Constitutional judge (the "Conseil Constitutionnel") did enshrine

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des Assurances).

[138]Cass. 2e Civ., 24 juin 2004, no 02-20.208, Bull. civ. II, no 308 ; D. 2004. 2197, et les obs. ; ibid. 2005. 1317, obs. H. Groutel.

[139]DIMATTEO, Larry A. Counterpoise of Contracts: The Reasonable Person Standard and the Subjectivity of Judgment, The. SCL Rev., 1996, vol. 48, p. 293.

[140]CONVERT, Laurent. Feu le «bon père de famille» et l'avènement du standard «raisonnable»: un pas vers la «reasonable person» de common law?. Tribonien, 2018, no 01, p. 166-183.

[141]OUDIN, Martin. L'execution du contrat en bon pere de famille (l'obligation de comportement en droit civil compare). 2000. Thèse de doctorat. Paris 11.

[142]TUCKER, Edwin W. The morality of law, by Lon L. Fuller. Indiana law Journal, 1965, vol. 40, no 2, p. 5.

[143]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, no 3, p. 541-563.

as a constitutional requirement for the law to be both accessible and intelligible[144] (which implies clarity and understandability). The same goes for Canada[145], where the jurisprudence insists on the requirement of clear and precise laws, and where vagueness of the law can invalidate it due to its unconstitutionality.

Moreover, the need of clarity in law is more and more crucial as time goes by. As too many authors already state it : not only the quantity of texts is growing exponentially and is overwhelming as we already said[146], but also and especially the quality of the law (i.e. its writing by the legislator) has greatly deteriorated over the past few decades as pointed out by Benezech in "L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence."[147] and by Andone and Leone in "Talking law. Clarity, transparency and legitimacy in rule-making."[148].

So, the law is today getting both more abundant and less clear.

The two main reasons for the importance of clarity for law are the fight against the arbitrariness of the judges (or any law-applying authority), and the legitimacy of the law itself.

Indeed, the more a law is unclear, the more the judge can interpret it widely, but a law too wide is pointless as it gives the judges the power to interpret it as they want. The less a law will be clear, the wider the interpretation window gets. And, as Endicott stated in "Vagueness and law"[149] : "Official discretion is contrary to the rule of law only if it is arbitrary in the pejorative sense".

This is why there could be no law that prescribes to behave according to morality, because this would make the judge both the source and the hammer of new norms that can be very far from the legislator's original intention.

This ties up to the "légisprudence" theory of Wintgens[150], incorporat-

[144]Décision n° 99-421 DC du 16 décembre 1999

[145]RIBEIRO, Marc. Limiting arbitrary power: the vagueness doctrine in Canadian constitutional law. UBC Press, 2004.

[146]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, no 4, p. 905-917.

[147]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, no 3, p. 541-563.

[148]ANDONE, Corina et LEONE, Candida. Talking law. Clarity, transparency and legitimacy in rule-making. The Theory and Practice of Legislation, 2022, vol. 10, no 1, p. 1-4.

[149]ENDICOTT, Timothy. Vagueness and law. In : Vagueness: A guide. Dordrecht : Springer Netherlands, 2011. p. 171-191.

[150]WINTGENS, Luc J. La légisprudence: étude pour une nouvelle théorie de la législa-

ing several principles in a rational theory of legislation, among which is the principle of coherence, where the author underlines the importance of taking into account the interactions between the legislator's norms and the judges decisions in order to maintain coherence within the legal system.

As for the legitimacy, as Andone and Leone state in their article "Talking law. Clarity, transparency and legitimacy in rule-making. The Theory and Practice of Legislation"[151], we cannot expect any legitimacy from a decision based on a text that is at best "merely clear". Citizens must be able to expect what the law dictates to them, or else it would not make any sense to punish behavior that does not comply with an obscure obligation.

If the legal system fails to satisfy the clarity requirement, it could even threaten the rule of law ; as Endicott concluded in "Vagueness and law"[152] : "There is a failure in the rule of law when (because of vagueness or for any other reason) the law does not communicate to the subjects of the law, and to the institutions of the legal system, standards that are sufficiently determinate for the actual purposes of legal regulation."

Therefore, in order to ensure quality legislation, many legistics-based initiatives have been taken ; we could cite for example the "guide de légistique"[153], a french institutional guide to help legislators in drafting normative texts.

Moreover, additionally to clarity issue, if we circle back to legal certainty/predictability issues that come with incomplete or incoherent legal systems, it all results in a huge lack of legitimacy of the law in the words of Fuller's theory[154][155], according to whom the predictability of the law is necessary for a legal system to be regarded as legitimate.

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tion. Archives de philosophie du droit, 2005, vol. 49, p. 93-113.

[151]ANDONE, Corina et LEONE, Candida. Talking law. Clarity, transparency and legitimacy in rule-making. The Theory and Practice of Legislation, 2022, vol. 10, no 1, p. 1-4.

[152]ENDICOTT, Timothy. Vagueness and law. In : Vagueness: A guide. Dordrecht : Springer Netherlands, 2011. p. 171-191.

[153]D'ÉTAT, Conseil et MINISTRE, PREMIER. uide de légistique [en ligne]. 2007.

[154]SCHNEIDERMAN, David. Weber, Fuller, and Belief in the Legitimacy of law. Max Weber Studies, 2024, vol. 24, no 2, p. 183-216.

[155]FELDMAN, Daniel L. law's Legitimacy: Lon Fuller in a Consequentialist Frame. 2024. Thèse de doctorat. City University of New York.

To conclude this chapter, if we look back at the section's title "The contemporary deficiencies of law", we addressed three of them which we consider are the most important to take into account for a formal perspective of the law. However, the literature does not limit legal problems to these three elements.

For example, Frédéric Rouvière, in his article "Typologie des problèmes juridiques et argumentation"[156], exposes a typology of legal problems and considers (exhaustively) four types :

- completeness, which is linked, as in our developments, to the question of legal gaps ;

- harmonization, which can be transposed to our developments on coherence, as it is linked to the overlap of concurring texts ;

- interpretation, which can be transposed to our developments on clarity as the author explicitly links it to the question of clarity and vagueness problems ;

- the fourth type that is missing in our discussion (according to Rouvière's typology) is derogation, which he links to the questions of justice, fairness and equity. If we do not include this discussion, it is for the same reason that we did not include the argumentative perspective of coherence : we do not aim nor claim to question meta-legal standards through logical formalization, and we do not interfere with the interpretation of applied norms by lawyers ; our work is structural and systemic, and it is both outside our scope and outside our reach to address such a subject.

Thus, it seems essential that the law becomes more complete, coherent, and clear. However, it would be a Herculean task for us, humans, to solve this issue since, as we said, the norms are actually way too numerous.

This is why, in our thesis, we will look into logical formalization, its potential, and what it can bring to the legal domain in general, and to the legal systems in particular.

We will now see if and how legal formalization can fix the deficiencies of legal systems we just mentioned.

[156]ROUVIÈRE, Frédéric. Typologie des problèmes juridiques et argumentation. RTD-Civ. Revue trimestrielle de droit civil, 2022, no 03, p. 559.

2 The potential of formalization

The law lacks completeness, coherence and clarity ; we need to find a way to systematically fix it, and in this section, we will elaborate on the conjecture that formalization is a potential method for that.

Of course, we need to first address what formalization is, and why we think it could be a solution, before challenging to what extent it could be beneficial for legal systems.

Formalizing something, according to the Cambridge dictionary, means deciding "to arrange it according to a fixed structure"[157].

I find it interesting here to consider the polysemy of "fixed", and to claim that we will fix the structure of law ; i.e. we will both establish the structure of normative systems and of norms interacting with each other[158], while at the same time repairing the faulty structures that we find nowadays in the law[159].

Thus, we will discuss, respectively, in this section, why we will resort to logical formalization, then how we will make use of it, and finally which strategy we will adopt in order to overcome its potential limits or criticisms.

2.1 Nature

"What exactly is it that we do when we do logical formalization? What do we achieve when we “translate” a natural language sentence into a formula of a formal language?"[160]

In their article "Logical Formalization and the Formation of logic(s)", Peregrin and Svoboda question and address the process of formalization itself, along with its role and relevance.

Formalizing a norm is, actually, capturing its essence, its signifier content, what it abstractly means, its true purpose ; and keeping only this quintessential information, "the logical form of the sentence"[161], and integrating it into

[157]https://dictionary.cambridge.org/fr/dictionnaire/anglais/formalize

[158]KERR, James M. Fixing the law and Finding the law. law. & Banker & S. Bench & B. Rev., 1922, vol. 15, p. 275.

[159]CLAUSING, Kimberly A. Fixing Five Flaws of the Tax Cuts and Jobs Act. Colum. J. Tax L., 2019, vol. 11, p. 31.

[160]PEREGRIN, Jaroslav et SVOBODA, Vladimír. Logical Formalization and the Formation of logic(s). Logique et Analyse, 2016, vol. 233, p. 55-80.

[161]PEREGRIN, Jaroslav et SVOBODA, Vladimír. Logical Formalization and the Formation of logic(s). Logique et Analyse, 2016, vol. 233, p. 55-80.

a formal "language", able to standardize the formalized norms and structure them in relation to each other.

Indeed, the true meaning of a sentence in "natural languages" does not lie within the words themselves (if the legal language is in fact a language with scientific vocabulary[162] with its technical lexicon, and its specificities in regard to the common tongue, it is still structurally a "natural language" [163]).

On the contrary, words and "synonyms" (if ever there were "true" synonyms[164]) can often be misleading. For example, every time the verb "shall" is decisive for the understanding of a legal norm, a new hermeneutical problem arises : according to the Cambridge dictionary[165], this verb can alternatively mean a prediction, a suggestion, or a prescription (the future, the contingency, or the must/the duty).

In a formal language, this division would not exist, since there is a strict bijection between words and meanings (i.e. every word has one and only corresponding meaning).

Moreover, as Jakimenko developed in the article "Logical Validity and Linguistic Correctness: Impact on Changes in Thinking."[166], a change in language always induces a change in thinking. So formalizing law will condition lawyers, by changing the way they think the law, to better conceive norms, and therefore gain clarity and efficiency.

As an example, we could cite how Benjamin Moron-Puech did undertake a "mathematical formalization" of his theoretical definition of a legal act, making it a formal definition, in order to capture its essence and its true meaning, in his PhD thesis "L'acte juridique, une réponse à la crise du contrat."[167]

A more concrete illustration of how formalization can reshape norms can

[162]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

[163]GIBBONS, John. Language and the law. Annual review of applied linguistics, 1999, vol. 19, p. 156-173.

[164]BOON, Andy. "The Illusion of Synonyms: Investigating the Similarities and Differences between Bias and Prejudice." Proceedings of Temple University Japan Applied Linguistics Colloquium , 2005.

[165]https://dictionary.cambridge.org/dictionary/english/shall

[166]JAKIMENKO, Virginija. Logical Validity and Linguistic Correctness: Impact on Changes in Thinking. Societal Studies, 2013, vol. 5, no 4, p. 1029-1039.

[167]MORON-PUECH, Benjamin, FENOUILLET, Dominique, et COMBACAU, Jean. L'acte juridique, une réponse à la crise du contrat. (No Title), 2020. p.261

be found in "The British Nationality Act as a logic program"[168], where legal sentences like "A person born in the United Kingdom after commencement shall be a British citizen if at the time of birth his father or mother is (a) a British citizen; or (b) settled in the United Kingdom." could be represented by the rule "x is a British citizen, if x was born in the U.K., and x was born on date y, and y is after or on commencement, and z is a parent of x, and z is a British citizen on date y"[169].

Formalizing that way can also lead to much simpler processing of rules in a more algorithmic and computable way, but we will come back to this specific point in our last chapter.

Now that the concept of formalization is introduced, we can ask ourselves : why exactly would we use formalization on legal norms ? and how would it help fulfilling lawyers' needs ?

2.2 Correctness

The necessity of legal correctness has already been pointed out by Robert Alexy in "law and correctness"[170], where he argues that correctness is not systematic in legal systems, but I will not be using this term exactly in the same sense.

What I mean by correctness of the law is the absence of problematic deficiencies, which is equivalent to the satisfaction of the properties described in the beginning of this chapter (completeness, coherence and clarity).

If we decide to formalize the norms of a system, then we would be able to check many properties of the said system[171]. We will return in detail in our final chapter to the many possibilities and tools entailed by a successful formalization of the law, but let us just say a few words of how formalizing norms could correct them from being incomplete, incoherent, or unclear.

[168]SERGOT, Marek J. , SADRI, Fariba, KOWALSKI, Robert A. , et al. The British Nationality Act as a logic program. Communications of the ACM, 1986, vol. 29, no 5, p. 370-386.

[169]SERGOT, Marek J.. , SADRI, Fariba, KOWALSKI, Robert A.. , et al. The British Nationality Act as a logic program. Communications of the ACM, 1986, vol. 29, no 5, p. 370-386. | we note that this is not the final formalization of this law by the article, it is criticizable as it stands, but it is a convenient and more simple way to show its process. Please read the whole reference for more details and a more satisfying formalization

[170]ALEXY, Robert. law and correctness. Current Legal Problems, 1998, vol. 51, no 1, p. 205.

[171]NOVOTNÁ, Tereza et LIBAL, Tomer. Towards Automating inconsistency Checking of Legal Texts. Jusletter IT. Die Zeitschrift für IT und Recht., 2020.

In a formal language, it is very easy to check the consistency of a group of assertions, and in many formal languages, there even are proof-assistants softwares that can help doing so, some of which are automated (we could cite as examples SPINdle[172], Coq[173], Prolog[174] or Isabelle/HOL[175]).

As for the completeness of the legal systems, we must check the properties of the logic we build our formalization on (see in Chapter III where we check if our logic is correct, coherent and complete), but a good logic satisfying these properties could solve completeness interrogations with ease[176].

Finally, for clarity issues, the bijection between elements of a formal language and their meanings may solve some of the challenges, but it can only be partially the case since, as we will see now, most of the interpretation will remain (and must absolutely remain) in the hands of humans.

2.3 Limits

Even if a good logical formalization might help fixing legal systems' contemporary flaws, it will only be partially. A formalization will not come without certain limits that it cannot, or sometimes should not, overcome.

2.3.1 The method's relevance

Firstly, as Smith develops in "The limits of correctness"[177], a model we build is not necessarily correct.

Certainly we can argue back, saying that we can prove that a logic we build is complete, or consistent, etc. But even if it is the case, we cannot prove that it is the most faithful model possible, or the most relevant way to

[172]Lam, H. and Governatori, G. The Making of SPINdle. Proceedings of International Symposium on Rule Interchange and Applications (RuleML 2009), Springer-Verlag. 2009.

[173]CHLIPALA, Adam. Certified programming with dependent types: a pragmatic introduction to the Coq proof assistant. MIT Press, 2013.

[174]COLMERAUER, Alain et ROUSSEL, Philippe. The birth of Prolog. In : History of programming languages—II. 1996. p. 331-367.

[175]NIPKOW, Tobias, WENZEL, Markus, et PAULSON, Lawrence C. (ed.). Isabelle/HOL: a proof assistant for higher-order logic. Berlin, Heidelberg : Springer Berlin Heidelberg, 2002.

[176]PEREGRIN, Jaroslav et SVOBODA, Vladimír. Criteria for logical formalization. Synthese, 2013, vol. 190, p. 2897-2924.

[177]SMITH, Brian Cantwell. The limits of correctness. Acm Sigcas Computers and Society, 1985, vol. 14, no 1, 2, 3, 4, p. 18-26.

model or formalize things.

So, the first potential weakness of a logical formalization of the law comes from the potential lack of trust we might have in the whole process itself, and how we could question the relevance of a formalization based on a hierarchized succession of priority criteria. Another subsequent flaw may come from the actor who implements the formalization. Even if we trust the process, we may not trust the processor.

For example, we could have a debate about who should be the actor who will formalize the law (once we have the tools for it) : should it be a lawyer, a logician, or a couple of both expertise ? As for the process, some could argue that instead of a hierarchy we should put all laws on the same level, and that any contradiction is a problem in a legal system.

Hopefully, the specificity of our thesis is that we will not propose a unique formal logic to implement law with, but more of a method, a family of logics, and a scheme of systems that will depend on the choice of priority criteria, and we will justify the choice of a hierarchical structure for legal systems (see Chapter III) ; as for the choice of the processor, as we will see in Chapter IV, if we insert enough high-level interfaces, "simple" lawyers with no background in logic or informatics would be perfectly able to formalize law, and it is what we want in the end.

The specificity of our method will make its relevance for formalization a bit more dependent on its operators, since they are able to make their own choice of criteria, which may be an inconvenient ; therefore, since we do not lock any arbitrary criteria upstream, we have nullified the chances of imposing the wrong ones on those who will be more legitimate to choose them. It will also help adapting easily the method to different legal orders with different systems, structures and priorities.

2.3.2 The restricted use

Along with the limits that are imposed on us, we also have limits that we must impose on ourselves ; we must not formalize law in any way, without having first some preliminary considerations.

The most important one is that we do not want to substitute the human (lawyer) interpretation of the law ; if we formalize norms, it will only be deontically within the norms, and structurally between them. This means that we will not pretend that logic can capture the meaning and the essence of the legal vocabulary, but only the way they are related to each other and

entail each other on a macro level, and the way they prescribe on a micro level. To rephrase, we will link logically all the deontic obligations with each other, and link every one to atomic legal concepts that will not and must not be interpreted by the logic.

To illustrate this, we can take the example of a law stating "it is prohibited for motorized vehicles to park at location X". If there is not a second law giving an exhaustive definition of what "motorized vehicles" are, then we would need interpretation in order to know what it encompasses, but if there is, then we can logically establish the legality of any car, bike of bicycle parked at the location. So the formalization of this text would be respectively partial and dependent on human interpretation or total. But as we develop in the fourth chapter, we do not want any non-human interpretation to take place if interpretation is needed. That is why the legal formalization as we propose it can only be partial.

Indeed, as we will see in the next chapter, we cannot rely (yet ?) on any built informatics ontology for law ; *id est* we cannot capture and compute formally the meaning, nuances and implications of legal vocabulary, nor can we delegate this task to Artificial Intelligence, as it both does not really "understand" it and does not reason transparently on it. We still need human interpretation/intervention in any process attempting to formalize law.

As Leone concluded in " Legal knowledge extraction in the data protection domain based on Ontology Design Patterns", after analyzing all legal ontologies options and attempts : "acknowledging that approaches of NLP on legal documents can not fully address some crucial aspects of the legal domain, such as legal interpretation, and that the formalisation of legal knowledge in highly-structured ontologies necessarily asks for human intervention"[178]

Speaking about Artificial intelligence, we shall be careful and not involve it in our formalization process, since implementing a formalization must be done by a (human) lawyer for two reasons : the operator must both be able to understand and interpret law (only lawyers have both the knowledge[179] and the legitimacy to do so[180], we will though address this question in more details in our fourth chapter), and be able to take responsibility for how it was done. We will come back to Artificial Intelligence matters in the next section in more details.

[178]LEONE, Valentina. Legal knowledge extraction in the data protection domain based on Ontology Design Patterns. 2021.

[179]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

[180]LÉVY-BRUHL, Henri. La Science du Droit ou" Juristique". Cahiers internationaux de sociologie, 1950, vol. 8, p. 123-133.

2.3.3 The approach's legitimacy

As for critiques that could come our way, there is one that we should address even though it is a dismissive one : some might question our will to make law more correct (more coherent, complete and clear), by saying that law is supposed to be incoherent, or incomplete, and that it is in the nature of legal systems to be incorrect.

To illustrate this debate : John Finnis, for instance, did argue the fact that legal positivism is "an incoherent intellectual enterprise"[181] in his article "On the incoherence of legal positivism." But this "dismissive"[182] consideration is thoroughly demystified by Brian Tamanaha in "The contemporary relevance of legal positivism", where he develops that it is but a debate that is no longer relevant on the law and morality confusion, and that the claim of incoherence of the law is not pertinent.

Indeed, legal positivism has evolved beyond this opposition between law and morality, and considers now more complex intricacies ; according to what Raz develops in "About morality and the nature of law"[183], I consider that the law is non-morally binding, regardless of the morality or non-morality of the law.

But more importantly, beyond this debate, there are several arguments that we could put forward in order to defend our will to bring more correctness to the law.

What is crucial, is that the coherence of the law is an important criteria for its legitimacy (according to the theory of Fuller we saw earlier[184][185]). Indeed, how could we impose on citizens to comply with rules if there is no way for them to easily understand what is precisely asked from them ?

This is also why the standard of legal certainty or legal predictability is

[181]FINNIS, John. On the incoherence of legal positivism. Notre Dame L. Rev., 1999, vol. 75, p. 1597.

[182]TAMANAHA, Brian Z. The contemporary relevance of legal positivism. Austl. J. Leg. Phil., 2007, vol. 32, p. 1. p.49

[183]RAZ, Joseph. About morality and the nature of law. Am. J. Juris., 2003, vol. 48, p. 1.

[184]SCHNEIDERMAN, David. Weber, Fuller, and Belief in the Legitimacy of law. Max Weber Studies, 2024, vol. 24, no 2, p. 183-216.

[185]FELDMAN, Daniel L. law's Legitimacy: Lon Fuller in a Consequentialist Frame. 2024. Thèse de doctorat. City University of New York.

so important[186][187]. We cannot expect behavior that is barely foreseeable. *Nemo censetur ignorare legem*[188], but the law should not ignore them either, and therefore should be correct in order to be legitimate.

Furthermore, consistency is structurally needed in law, whether it is in common law systems (where the precedent system specifically guarantees consistency over time) or in civil law systems. As Dworkin and Bentham respectively argue in "law's Empire"[189] and "The theory of legislation"[190] , the ultimate goal of legal consistency is both reaching fairness and avoiding arbitrariness : "law as integrity asks judges to assume, so far as this is possible, that the law is structured by a coherent set of principles about justice and fairness and procedural due process, and it asks them to enforce these in the fresh cases that come before them, so that each person's situation is fair and just according to the same standards"[191] ; "The third condition is, that the laws should be consistent [...] When new laws are made in opposition to a principle established by the old ones, the stronger that principle is, the more odious will the inconsistency appear. A contradiction of sentiment results from it, and disappointed expectations accuse the legislator of tyranny."[192]

[186]PIAZZON, Thomas. La sécurité juridique. 2006. Thèse de doctorat. Paris 2.

[187]DELLAUX, Julien. Le principe de sécurité juridique en droit constitutionnel: signes et espoirs d'une consolidation de l'ordre juridique interne et de l'État de droit. Revue française de droit constitutionnel, 2019, vol. 119, no 3, p. 665-696.

[188]"No one is supposed to ignore the law", latin saying.

[189]DWORKIN, Ronald. law's empire. Harvard university press, 1988.

[190]BENTHAM, Jeremy. The theory of legislation. In : Readings from Liberal Writers. Routledge, 1950. p. 182-186.

[191]DWORKIN, Ronald. law's empire. Harvard university press, 1988.

[192]BENTHAM, Jeremy. The theory of legislation. In : Readings from Liberal Writers. Routledge, 1950. p. 182-186.

3 The methodology dilemma

In this thesis, in order to achieve legal formalization, we could have gone two ways : the first one would have been to take advantage of AI in our approach, but this is not what we chose, since the second one seems more promising through a logic-based approach.

3.1 The machine-based AI approach

The Artificial Intelligence approach is very tempting. Indeed, these last few years, AI developed at a very high rate, and seems to have few to no limit to its potential.

The AI&law field of research is also quite rich[193], and many scholars are very enthusiasts with the rapid growth we are witnessing currently in the AI field[194], and with how it could revolutionize law[195].

However, even if the Artificial Intelligence does have some potential for which it could be integrated in law, we must stay very cautious[196] with how we integrate it, and for what function (our critique shall be limited to stochastic AI systems).

3.1.1 The inescapable bias

Humans are biased creatures, one of the hopes we first had with Artificial Intelligence is that it might be more objective than us, and that it would get rid of every human biased reasoning. Alas, it is the opposite : AI, due to the way it is learning, incorporates the human bias[197], and even accentuates them[198].

[193]BENCH-CAPON, Trevor, ARASZKIEWICZ, Michał, ASHLEY, Kevin, et al. A history of AI and law in 50 papers: 25 years of the international conference on AI and law. Artificial Intelligence and law, 2012, vol. 20, p. 215-319.

[194]COOPER, A. Feder, LEE, Katherine, GRIMMELMANN, James, et al. Report of the 1st Workshop on Generative AI and law. arXiv preprint arXiv:2311.06477, 2023.

[195]SHAHID, Anum, QURESHI, Gohar Masood, et CHAUDHARY, Faiza. Transforming legal practice: The role of AI in modern law. Journal of Strategic Policy and Global Affairs, 2023, vol. 4, no 01, p. 36-42.

[196]GORDON, John-Stewart. AI and law: ethical, legal, and socio-political implications. Ai & Society, 2021, vol. 36, p. 403-404.

[197]FERRYMAN, Kadija, MACKINTOSH, Maxine, et GHASSEMI, Marzyeh. Considering biased data as informative artifacts in AI-assisted health care. New England Journal of Medicine, 2023, vol. 389, no 9, p. 833-838.

[198]COWGILL, Bo, DELL'ACQUA, Fabrizio, DENG, Samuel, et al. Biased programmers? Or biased data? A field experiment in operationalizing AI ethics. In : Proceedings

Indeed, as Rejmaniak develops in "Bias in artificial intelligence systems"[199], the AI does reproduce human bias at larger scales, and even systematizes them ; a human will be biased from time to time, but the problem is that the AI will integrate this bias as data, and consider it among core rules it will use for decision making.

Therefore, what could have been a source of objectivity and correctness against the bias of human reasoning is but a way to generalize and amplify them, as Ganel et al. argue in "Bias in human perception of facial age are present and more exaggerated in current AI technology"[200].

Thus, it may be dangerous to trust AI with the task of formalizing law, the most concerning factor being, as literature tends to prove, that AI will be biased against the categories of people that are already struggling for their rights : whether it is biased against gender[201], or against the poor[202] and the minorities in general[203].

3.1.2 The backlash of using AI

What is even more dangerous forward, is that research tends to prove that the AI bias (inherited from humans then amplified[204]) are inherited back by more humans in return when they use the AI, creating an inescapable vicious circle ; this is well documented by literature since 2022, and we refer to it for more details, for example in "Biased AI systems produce biased humans"[205], "Humans absorb bias from AI—and keep it after they stop using the algo-

of the 21st ACM Conference on Economics and Computation. 2020. p. 679-681.

[199]REJMANIAK, Rafał. Bias in artificial intelligence systems. Białostockie Studia Prawnicze, 2021, vol. 3, no 26, p. 25-42.

[200]GANEL, Tzvi, SOFER, Carmel, et GOODALE, Melvyn A. Bias in human perception of facial age are present and more exaggerated in current AI technology. Scientific Reports, 2022, vol. 12, no 1, p. 22519.

[201]WELLNER, Galit P. When AI Is Gender-biased. HUMANA. MENTE Journal of Philosophical Studies, 2020, vol. 13, no 37, p. 127-150.

[202]CURTO, Georgina, JOJOA ACOSTA, Mario Fernando, COMIM, Flavio, et al. Are AI systems biased against the poor? A machine learning analysis using Word2Vec and GloVe embeddings. AI & society, 2024, vol. 39, no 2, p. 617-632.

[203]ZOU, James et SCHIEBINGER, Londa. AI can be sexist and racist—it's time to make it fair. 2018.

[204]GANEL, Tzvi, SOFER, Carmel, et GOODALE, Melvyn A. Bias in human perception of facial age are present and more exaggerated in current AI technology. Scientific Reports, 2022, vol. 12, no 1, p. 22519.

[205]GLICKMAN, Moshe et SHAROT, Tali. Biased AI systems produce biased humans. OSF Prepr, 2022, vol. 10.

rithm"[206], or the work of Vicente and Matute in "Humans inherit artificial intelligence bias"[207] and "The Inherited Bias Effect: the propagation of artificial intelligence bias to human decisions"[208].

Last but not least, another pitfall that arises when using AI, is trust issues with the machine. In fact, some researchers found out that AI could learn to "lie"[209] about results, and more specifically in order to please its operator[210].

All of these problems would be manageable if AI had a transparent computing process, but many authors like Carabantes in "Why artificial intelligence is not transparent: a critical analysis of its three opacity layers"[211] showed that is was not. Instead, AI suffers from a "black-box" effect, making its reasonings and automations neither traceable nor understandable on how the conclusions were reached, and therefore not trustworthy[212], since AI, as said before, can also lie, bias and fabricate its results without us being able to analyze the reasoning steps.

For all these reasons, we will not count on AI for formalizing the law. However, once the logical theory is thoroughly developed, and the formalization done by lawyers, AI may become a precious tool since it will have a concrete and safer framework.

3.2 The logic-based DL approach

The logical approach to legal formalization might seem a bit old-fashioned today, but I believe it is the best way to proceed for many reasons we will develop below.

[206]LEFFER, Lauren. Humans absorb bias from AI—and keep it after they stop using the algorithm. Scientific American, 2023.

[207]VICENTE, Lucía et MATUTE, Helena. Humans inherit artificial intelligence bias. Scientific reports, 2023, vol. 13, no 1, p. 15737.

[208]VICENTE, Lucia et MATUTE, Helena. The Inherited Bias Effect: the propagation of artificial intelligence bias to human decisions. 2023.

[209]WITELL, Lars et SNYDER, Hannah. Dishonesty Through AI: Can Robots Engage in Lying Behavior?. In : Humane Autonomous Technology: Re-thinking Experience with and in Intelligent Systems. Cham : Springer International Publishing, 2024. p. 233-246.

[210]STRAY, Jonathan. The AI learns to lie to please you: preventing biased feedback loops in machine-assisted intelligence analysis. Analytics, 2023, vol. 2, no 2, p. 350-358.

[211]CARABANTES, Manuel. Why artificial intelligence is not transparent: a critical analysis of its three opacity layers. In : Handbook of Critical Studies of Artificial Intelligence. Edward Elgar Publishing, 2023. p. 424-434.

[212]VON ESCHENBACH, Warren J. Transparency and the black box problem: Why we do not trust AI. Philosophy & technology, 2021, vol. 34, no 4, p. 1607-1622.

As for the choice of Deontic logic, among the various logical options to approach legal formalization, it is the most canonical one ; we will mention some other options in Chapter III as a comparison (paraconsistency, defeasibility, preference logic etc.), and some other non-standard logics might be helpful for future work (for example fuzzy logic to include the formalization of jurisprudence or to start tackling legal argumentation), but our choice is to rely first on Deontic Logic as a simple but solid starting point, which we will refine by developing our own variant of it.

3.2.1 The algorithmic nature of law

Law has a huge algorithmic potential, i.e. the way the legislation is written often enables us to understand it as "a sequence of steps used to obtain a result from input elements"[213].

As Kowalski wrote in "Legislation as logic Programs"[214] : "The linguistic style in which legislation is normally written has many similarities with the language of logic programming [...] the law has other important similarities with computing".

Indeed, most legal texts are contextual obligations, i.e. the prescription of a behavior A in a context B ; we therefore have to successively check if all the conditions of the law are met by the case facts, and if it is the case, conclude that the obligation stands. That is why many templates-based logics or methods are suitable to capture the essence of many norms.

This resulted in the development in the last decades of " legal informatics" a growing field that took interest in these similarities between legislation and programming.[215].

We can witness this in the work of many scholars, among whom R. Kowalski, who developed "Logical English" for legal formalization since 1986, and

[213]https://www.cnil.fr/fr/definition/algorithme

[214]KOWALSKI, Robert A. Legislation as logic programs. In : logic Programming in Action: Second International logic Programming Summer School, LPSS'92 Zurich, Switzerland, September 7–11, 1992 Proceedings 2. Springer Berlin Heidelberg, 1992. p. 203-230.

[215]KATZ, Daniel Martin, Dolin, Ron, et BOMMARITO, Michael J. (ed.). Legal informatics. Cambridge University Press, 2021.

continues today[216][217][218][219].

All these works tend to prove that not only is it possible to formalize law, but it even is a quite natural process to undertake.

Moreover, unlike AI, logic (and informatics theory) guarantees the transparency[220] of the method and of the reasoning, often leaving the legal or litigious interpretations to the human operators.

And even if most law-oriented logics would need a double expertise to operate, with the right method (see Chapter IV), we could superimpose interfaces (like the Legai tool for example[221]) so that the lawyers could operate themselves the legal formalization with little to no knowledge in logic or informatics.

3.2.2 A path from law to code

As for our approach and methodology in this thesis, we intend to develop a step-by-step pathway from A to Z, or in our case from law to code, i.e. a method to go from any group of legal texts to its formalized computer code equivalent, thus achieving what Hildebrandt calls : "code-driven law"[222].

By combining works in legal (norms) theory (see Chapter II) ; in mathematical (deontic) logic, i.e. a non-standard mathematical logic that studies norms (see Chapter III), and in informatics (grammars) theory, i.e. the creation of a language that will serve as an intermediate between the legal text

[216]SERGOT, Marek J.. , SADRI, Fariba, KOWALSKI, Robert A.. , et al. The British Nationality Act as a logic program. Communications of the ACM, 1986, vol. 29, no 5, p. 370-386.

[217]KOWALSKI, Robert et DATOO, Akber. Logical English meets legal English for swaps and derivatives. Artificial Intelligence and law, 2022, vol. 30, no 2, p. 163-197.

[218]KOWALSKI, Robert, DÁVILA, Jacinto, SARTOR, Galileo, et al. Logical English for law. on Methodologies for Translating Legal Norms into Formal Representations (LN2FR 2022), 2022, p. 57.

[219]KOWALSKI, Robert, DÁVILA, Jacinto, SARTOR, Galileo, et al. Logical English for law and education. In : Prolog: The Next 50 Years. Cham : Springer Nature Switzerland, 2023. p. 287-299.

[220]LIBAL, Tomer et NOVOTNÁ, Tereza. Towards transparent legal formalization. In : Explainable and Transparent AI and Multi-Agent Systems: Third International Workshop, EXTRAAMAS 2021, Virtual Event, May 3–7, 2021, Revised Selected Papers 3. Springer International Publishing, 2021. p. 296-313.

[221]LIBAL, Tomer. The legai editor: a tool for the construction of legal knowledge bases. In : Legal Knowledge and Information Systems. IOS Press, 2022. p. 286-289.

[222]HILDEBRANDT, Mireille. Code-driven law: freezing the future and scaling the past. Is law computable, 2020, p. 67-84.

and its formalized version (see Chapter IV), we will be able to go as lawyers from the legal texts themselves, extract the norm, obtain its formal translation, and then obtain a ready and computable corresponding code, in that order.

Our thesis thus brings all the methods that are necessary to achieve ideal legal formalization, which will be transparent and will not need multiple experts to be operated.

Chapter II
Norms in the legal system

Deontic Logic, our main subject in this PhD, is often defined as a "Branch of modal logic that studies the permitted, the obligatory, and the forbidden, which are characterized as deontic modalities (Greek, deontos: “of that which is binding”)"[223].

But if we look beyond this definition, most researchers since 1951 and Von Wright's founding paper on Deontic Logic[224] associate it with normative logic, and consider that Deontic Logic is a logic of norms as Kalinowski explains in the introduction of his book[225]. Kalinowski even titled his book about Deontic Logic "La logique des normes"[226], or in English, "The logic of norms".

So norms are central in Deontic Logic, since norms' deontic modalities are its object of study, as listed by the Stanford Encylopedia of Philosophy[227], making Deontic Logic more specifically a subset of logics of norms ; it is therefore only natural that we first address these norms in a chapter, before discussing Deontic Logic.

But how should we address norms ? The task seems complex, since "norms" is at the same time referring to the concept of norms[228], to the norms themselves as singular items[229], and to the system that integrates them[230].

Indeed, if many authors did address the question of Deontic Logic in the past, not all of them took the time to first discuss apart norms, their definition, their nature. But we will not skip that here, we will take the time to discuss every aspect of what norms are, in order to support the correctness of our following study of the logic of norms, i.e. Deontic Logic.

Studying norms also implies studying their structure, of several kinds : on an absolute level, studying the structure of the norm itself, as a single item ; and on a relative level, studying the structure of groups of norms, in what

[223]The Editors of Encyclopaedia Britannica.

[224]VON WRIGHT, Georg Henrik. Deontic Logic. Mind, 1951, vol. 60, no 237, p. 1-15.

[225]KALINOWSKI, Georges. La logique des normes. FeniXX, 1972. p. 13-14.

[226]id.

[227]MCNAMARA, Paul and VAN DE PUTTE, Frederik. "Deontic Logic", The Stanford Encyclopedia of Philosophy (Fall 2022 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/fall2022/entries/logic-deontic/>.

[228]KELSEN, Hans. On the concept of norm. In : Essays in Legal and Moral Philosophy. Dordrecht : Springer Netherlands, 1973. p. 216-227.

[229]HART, Herbert Lionel Adolphus. Bentham's of laws in general. Rechtstheorie, 1971, vol. 2, p. 55.

[230]DWORKIN, Ronald, et al. Is law a system of rules?. The philosophy of law, 1977, vol. 52.

we shall call systems of norms. As Alchourrón and Bulygin, among others, stated in their book "Normative Systems"[231], whose title is enlightening : it is more than relevant to not only study norms on their own, but to consider them as part of larger systems. Indeed, as we will see in this chapter, the legal systems do exist beyond norms themselves and are not just a mere accumulation of them.

We will be formalizing norms in this thesis, but formalization's utility is very limited if done on single norms and not integrated in a wider logical system, enabling us to work on their structures and connections, and challenge them.

In this chapter, we will first concentrate on norms and their properties, then we will discuss legal systems structures.

1 Norms and their properties

In order to comprehensively study norms, we must start by defining them, so we will review existing literature on the matter.

After that, we will establish our stipulation, and what kind of norms we include or not in our definition.

We will also address the norms' creation, and decide which conception of norms to retain.

We will then discuss a few properties of norms that will be useful to us later on.

Finally, we will focus on legal ontologies, the challenge they represent, and how we may overcome it.

1.1 Defining norms

Let us start by a small state of the art regarding how norms are usually defined, before doing so ourselves, in abstraction and in legal context.

1.1.1 Classically

There are several ways and methods of defining norms that we can refer to, and they all bring something interesting regarding what norms are.

[231] ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.3.

Common parlance. Firstly, let's discuss classical definitions that we can find in dictionaries or encyclopedias. Even if this step might appear too conventional, some decisions of the Supreme Court Of The United States taught us that we shall not underestimate "common parlance" definitions.

We can cite as an example the Nix v. Hedden case[232], where the court decided that tomatoes should be considered as vegetables regarding tax law, in the context of "the Tariff Act of March 3, 1883, c. 121, imposing a duty on "Vegetables..."[233]. The final and decisive argument of the court was that "Botanically speaking, tomatoes are the fruit of a vine, just as are cucumbers, squashes, beans and peas. But in the common language of the people, whether sellers or consumers of provisions, all these are vegetables"[234].

Some might find this decision amusing, and its stakes quite low, but it is very interesting how it makes us question the definitions in legal contexts (and how it is even more challenging to establish a legal ontology).

Indeed, this debate brings us back to more philosophical interrogations about the role of law in a society, and if the legal definitions shouldn't be closer to the people's expectations, since these people are supposed not to ignore the law, and more specifically the legal norms they are bound to respect.

But let us get back to defining norms. We will start with the Collins Dictionary, which defines norms as "ways of behaving that are considered normal in a particular society."[235].

We are facing a problem with this definition, since it is a circular one. But the same dictionary also defines "normal" as something "that is usual and ordinary, and is what people expect."[236]

The Cambridge Dictionary defines norms as "an accepted standard or a way of behaving or doing things that most people agree with".[237].

Finally, here is the french definition of the CNRTL[238] of what a norm is "Type, état, comportement qui peut être pris pour référence ; modèle, principe directeur qu'on tire de l'observation du plus grand nombre."[239], which would translate as "Type, state, behavior that can be taken as a

[232]Nix vs Hedden, 149 US 304 (1893)
[233]id.
[234]id.
[235]Collins English Dictionary
[236]id.
[237]Cambridge Advanced Learner's Dictionary
[238]Centre National de Ressources Textuelles et Lexicales
[239]id.

reference ; pattern, guiding principle that is derived from the observation of the greatest number."

What we can infer from these definitions, taken collectively, is that norms are ways of behaving which are both expected to be followed and accepted as such by most people in a certain society.

This definition is somewhat empirical. Let us check if researchers did define norms in a more absolute and autonomous way.

Scientific literature. The diversity of definitions in the academic field is no different from the one we observe in common parlance.

Indeed, as it is presented in the article "Norms: The Problem of Definition and Classification" by its author Gibbs[240], "literature reveals [...] a lack of agreement in generic definitions"[241]. The author then lists several diverse definitions among which can we find the definitions of Bierstedt " It is a standard to which we are expected to conform"[242], and Morris "Norms are generally accepted, sanctioned prescriptions for, or prohibitions against, others' behavior, belief, or feeling, i.e. what others ought to do, believe, feel or else. Norms must be shared prescriptions [...] Norms always include sanctions."[243].

As for Kalinowski, in his book "the logic of norms", he defines norms in a very empirical way[244]. His very first chapter opens on a list of random examples of norms[245], and he then induces that norms do have common structures that he studies through the lens of semiotics.

Hans Kelsen, in his book "Pure Theory of law", starts defining norms by stating that the word "norm" per se implies that something or some behaviour ought to be[246].

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[240]GIBBS, J P. « Norms: The Problem of Definition and Classification », The American journal of sociology. 1965, vol.70 no 5. p. 586-594.

[241]id.

[242]BIERSTEDT, Robert. The social order. (2d ed.; New York: McGraw-Hill Book Co., 1963), p. 222.

[243]MORRIS, Richard T. A typology of norms. American Sociological Review, 1956, vol. 21, no 5, p. 610-613.

[244]KALINOWSKI, Georges. La logique des normes. FeniXX, 1972. PUF, 8-10.

[245]id.

[246]KELSEN, Hans. Théorie pure du droit. (Charles Eisenmann trans., 1962). p.13.

Regarding the diverse definitions of norms found in the literature, from which we only showed a portion, it may appear difficult to try and find a unique one ; but this is exactly what we will try to do ; and we do not need to develop more with a systematic review since all definitions of norms in literature("see some examples from the extensive literature: Kelsen, 1967; Coleman, 1994; Pound, 1996; Ehrlich, 2001; Ross, 2002; Lewis, 2002; Ajzen, 2005; Bicchieri, 2005; Sugden, 2005; Hechter and Opp, 2005; Sumner, 2007; Posner, 2007; Fishbein and Ajzen, 2009")[247] fall in three categories, already covered by the previous examples, that we expose in the next paragraph.

1.1.2 Abstractly

Different types of norms definitions. Among the many ways that we saw being used to define norms, we can establish one main distinction and three non-exclusive categories :

- some, like Kalinowski[248], will define norms vis-à-vis their common structure, through semiotics.
- some, like Bierstedt[249] or most dictionaries, define norms relatively to the acceptance and expectations of the public.
- others, like Morris[250] or Kelsen[251], do define norms absolutely through a deontic lens, considering that norms are prescriptions that imply behaviours that ought to be.

This last category seems the most interesting for us, because we ultimately want a definition that can help us formalize law withing a logical scope, so both the facts that the definition is absolute and that it is based on deontic considerations are elements that make us think this may be the best way to define norms.

Naive definition. If we stick to a minimalist definition of what norms are, we would say that they are contextual rules creating deontic modalities.

[247]SVENSSON, Måns. Norms in law and society: towards a definition of the socio- legal concept of norms. In : Social and legal norms. Routledge, 2016. p. 39-52.
[248]KALINOWSKI, Georges. La logique des normes. FeniXX, PUF, 8-10.
[249]BIERSTEDT, Robert. The social order. (2d ed.; New York: McGraw-Hill Book Co., 1963), p. 222.
[250]MORRIS, Richard T. A typology of norms. American Sociological Review, 1956, vol. 21, no 5, p. 610-613.
[251]KELSEN, Hans. Pure theory of law. (Charles Eisenmann trans., 1962,). p.13

A sufficient definition ? As we will develop below, in the rest of this section, many things that we might be willing to add to norms' definition could be considered just as properties or characteristics of norms.

For example, if we consider the acceptance of norms by the people, we do not have to include it in the definition itself because it is not a necessary attribute of norms : indeed, it would be hard to deny that if a dictatorship rises in a state, and if the people in majority would prefer to overthrow it, the laws that this government would make are norms.

Some could argue that the people still accept tacitly by not taking the power back, or accept unfair laws because they believe somehow they would benefit the common good... This will not be part of our study, and we will thus assume that norms are not necessarily (by definition) accepted by the people.

Furthermore, the acceptance criteria would have no interest in the scope of our research, since we focus on the norms and their systems, but do not address in any way their public perception.

Another element that we cannot consider as part of the definition of norms is the expectation of the public.

A consensual observation made by the doctrine these last few decades, at least in France, is the constant and ongoing loss of quality and readability of norms ; their quantity, on the contrary, is greatly rising every year[252][253].

France's "Conseil Constitutionnel" even did state in 1999[254] the constitutional value of laws being intelligible and accessible to the public.

This will not be either a focus of ours, we do not include the expectation of the public in the definition of norms, since we cannot guarantee that the said public is aware of the existing norms, nor that it is even humanly possible to have a global knowledge of (for example) the whole French legal system ; so we cannot reasonably expect from the public to expect what they cannot possibly conceive.

Furthermore, many contemporary laws are very precise and technical[255],

[252]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, vol. 123, no 3, p. 541-563.

[253]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, vol. 116, no 4, p. 905-917.

[254]CONSTITUTIONNEL, Conseil. Décision n 99-421 DC du 16 décembre 1999, Loi portant habilitation du Gouvernement à procéder, par ordonnances, à l'adoption de la partie législative de certains codes. 1999.

[255]CRAWFORD, Lisa Burton. The problem of complex legislation. Legal Theory, 2024, vol. 30, no 1, p. 1-21.

making it impossible for the laymen to expect them.

As for the other norms' properties that we did implicitly exclude from the definition, we will come back to them later in the chapter.

1.1.3 Legally

The specificity of legal norms. Among the norms, there is a specific subset that interests us particularly : legal norms.

It may be relevant to give them a more specific and narrow definition in order to distinguish them.

One of the most central points in law, like any other norms system, is obligations : "The obligatory character of law is central"[256]. This may sound tautological, but it is a very polysemous word. In law, obligations also refer to the first half of a couple (obligation, right). So we could start naively by stating that laws do create rights and obligations between individuals.

Refining the definition. However, by pairing rights with obligations, not only would it be quite redundant but we are also limited to cover less potential laws, let us explain why.

It is true that all deontic modalities (obligation, prohibition, permission, option) could be covered by a single one, for example obligation (see Chapter III), but this is not true for rights, which are deontically a restraint, because if it is widely admitted in literature that any "rights are obligations"[257] , we must point out that "not all obligations are rights-based"[258].

Indeed, as Robert Audi develops in the ninth chapter of "Reasons, rights, and values.", it is not a necessity to violate a right to do a wrong, i.e. it is possible to violate an obligation without violating a right.

However, some could retort that the argumentation of Audi is too moral-centered, and could be less relevant for a system of positive law ; this is why I will support his argument by also connecting the example (actually the counter-example) of the abuse of rights[259]. With the abuse of rights, we can have a wrongful action and a prejudice, i.e. the breach of a prohibition,

[256]GREEN, Leslie. law and obligations. 2004.

[257]NICKEL, James W. How human rights generate duties to protect and provide. Hum. Rts. Q., 1993, vol. 15, p. 77.

[258]AUDI, Robert. Reasons, rights, and values. Cambridge University Press, 2015.

[259]GUTTERRIDGE, H. C. Abuse of rights. The Cambridge law Journal, 1933, vol. 5, no 1, p. 22-45.

without the violation of any rights, but only the abusive use of one.

Thus, for our definition, we should stick to the concept of obligation which has more semantic potential than rights, and we would be better stating that a law is a contextual rule creating obligations between individuals.

As for the context of the law, it depends what scope we will consider ; it would not be the same to deal with the context of the written law of a country, or with the custom's context of any geographical region.

1.1.4 The scope of legal norms

Abstractly speaking, our definition of norms is very broad. We could include in it, as examples, any game or sport rules, morality(ies?), religious prescriptions, etc.

As for legal norms, it is much harder to determine what we consider part of a certain legal order.

One of the oldest debates concerning the norms' scope is the natural law against positive law debate ; which is at least as old as Aristotle's theories[260][261].

But the real question is not what we can consider as a legal norm, but what should we consider as a legal norm for our project ? In fact, being potential norms may not be enough to enter our scope of legal norms that we want to formalize.

The uncertainty of natural law. The debate around natural law has been omnipresent in the history of legal thinkers. But here we do not claim to solve it, we will just check the hypothetical possibility of including natural laws - if they do exist - under our scope among other legal norms.

Phrased like this, the answer becomes immediately no. Because how could we consider them as rules within a system if we cannot even assert their existence. Montaigne did criticize this uncertainty of their existence[262], because even without denying the existence of natural laws, if we cannot enunciate what they say, we cannot consider them as binding.

[260]ARISTOTE: Ethique a Nicomaque.

[261]DESTRÉE, Pierre. Aristote et la question du droit naturel (" Eth. Nic.", V, 10, 1134 b 18-1135 a 5). Phronesis, 2000, p. 220-239.

[262]MONTAIGNE, Michel de et SACY, Samuel Sylvestre de. Apologie de Raimond Sebond. (No Title), 1937. Ch 12.

Just to be clear, we do not deny the fact that natural laws could be norms (see infra for abstract norms), but since we cannot enunciate for sure what they stipulate, we will not include them in the legal norms we consider for formalization.

The incompleteness of positive law. Positive laws, on the contrary, are easy to consider, since their existence is obvious. But there is a catch : as Dworkin criticizes in "The Model of Rules"[263], the clear set of written laws is incomplete, as lawyers also use non-written "principles", "standards".

These principles should, in my opinion, have the same status as natural laws until the moment they are brought to existence, for example if they are "discovered" miraculously by a judge that needed a standard for his interpretation of a certain law.

We will therefore consider the written positive laws along with written enunciations of "preexisting" unwritten positive principles.

Now that we discussed what norms are, we must bring more details and answers to "how norms are".

1.2 Norms properties

In this section, we will consider what we will call a "dismemberment" of norms properties from its definition. I name it this way in reference to the French concept of "dismemberment of property rights"[264] which is a way to distinctively consider some elements of the American concept of bundle of rights[265] : *usus*, *fructus* and *abusus*.

That means as we said earlier that all the following concepts will not be considered as elements of definition for norms but only as properties that norms can have or not have, and not having them does not question their normative essence.

The parallel is not utter, as for property the dismemberment consists in considering separately three elements that constituted it, and for norms, it would consist in considering separately the concept and the properties that we exclude from its redefinition.

[263]DWORKIN, Ronald M. The model of rules. The University of Chicago law Review, 1967, vol. 35, no 1, p. 14-46.

[264]PRIGENT, Stephane. Le démembrement de propriété, Revue Française de Comptabilité. 2020 no 546. p. 18-21.

[265]WALL, Jesse. Taking the bundle of rights seriously. Victoria University of Wellington law Review, 2019, vol. 50, no 4, p. 733-752.

However, semantically, the goal is still to manifest the fact that what was considered a whole should not be anymore. Thus, we must distinguish one by one many elements and explicitly not address them together.

This is why I nevertheless consider this new appellation relevant and interesting.

1.2.1 Creation

Abstract norms. It may appear paradoxical to consider that a norm does not need to be created in order to be, but they can satisfy our earlier definition while still being pure abstraction.

For example, a fictional norm that would prohibit wearing blue hats on Mondays is a norm, as it is a contextual rule implying deontic modalities, it just hasn't been created yet. In this case, the context is temporal : Mondays ; and the deontic modality is the prohibition of wearing blue hats. This is sufficient to satisfy our criteria for an abstract norm.

Then many questions immediately arise : why consider any abstract modality as a norm ? Are they all mandatory ? How can anyone respect norms that only exist in pure abstraction ? Isn't that the very reason we decided to exclude natural laws from our scope ? Does that mean that any norm and its contradiction exist in abstraction ?

This is where the dismemberment theory becomes interesting : stating that all abstract norms are norms does not at all imply that these norms have any consequences or any of the other properties a norm can have. So for the great majority of them, they are not mandatory, they have no sanction, no applicability, etc. Most importantly, they mostly are not a member of any system or group of norms (we distinguish between absolute existence in abstraction, and relative existence in a system of norms, as we discuss it later in this chapter) ; so regardless of whether any norm and its contradiction abstractly exists, what matters is which ones we include in our systems and allow them to have consequences (other properties).

And regarding natural laws, if there are any, it would be difficult to argue that they are not norms, we just question their existence, not their essence. Yes, we do question their knowability, and for that reason, we cannot assume that they are absolutely norms (neither that they just are) ; but if they did exist (which we cannot know), they would be a subset of abstract norms.

Discovered norms. Nevertheless, there is a certain amount of these abstract norms that does have a real legal impact. As we said earlier, positive

law is incomplete without some "standards" or "principles"[266] that may be unwritten.

These unwritten applicable principles are often, according to doctrine, "discovered" by the judge and not created by him, like the French "PFRLR" (principes fondamentaux reconnus par les lois de la République), which are unwritten principles "discovered" by the constitutional judge[267] since 1971[268].

So here we do have a relevant example of abstract norms that were not created before being "discovered" and applied, as the judge only changes their status from unwritten to written legal principle but not their essence as norms that preexisted their discovery.

But if norms do not need to be created in order to be, we still have to find out how norms are brought to "existence" from this abstraction ; and the answer depends on the chosen conception of norms.

1.2.2 Conception

The main alternative : expressive conception v. hyletic conception. One of the main issues we must address is about the nature of norms, but this debate can only be answered by tackling first the two main clashing conceptions of norms : the hyletic conception and the expressive conception.

These two conceptions are presented by Alchourrón and Bulygin[269] as incompatible.

On the one hand, the hyletic conception focuses on semantics, considering norms as purely semantic and abstract entities. "While propositions are the meaning-contents of cognitive or descriptive sentences, norms are the meaning-contents of prescriptive (normative, directive) sentences."[270]

On the other hand, the expressive conception sees norms through a pragmatic lens : "norms are essentially commands, the result of the prescriptive use of language"[271].

[266]DWORKIN, Ronald M. The model of rules. The University of Chicago law Review, 1967, vol. 35, no 1, p. 14-46.

[267]SÉNAC, Charles-Édouard. Y a-t-il encore place pour la découverte de nouveaux principes fondamentaux reconnus par les lois de la République?. Titre VII, 2022, vol. 8, no 1, p. 27-34.

[268]Décision n° 71-44 DC du 16 juillet 1971

[269]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975.

[270]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

[271]WEINBERGER, Ota. The expressive conception of norms-an impasse for the logic of norms. law & Phil., 1985, vol. 4, p. 165.

We could summarize this dichotomy by saying that the debate is whether the language is the cause or the consequence of norms.

According to the expressive conception, the norms result from deontic prescriptions that are expressed, generally by language, but not only. According to Hans Kelsen in his "Pure theory of law"[272], the expression that causes the norms can take many forms : a gesture, a sign, a symbol, or language whether it is written or oral. Kelsen gives the example of a circulation agent that orders a car to stop by rising his hand, the obligation to stop would here be a norm resulting from the gesture ; it would be the same with a stop sign or a red light, both being symbols creating a norm by expressing it symbolically.

I do disagree with this conception, because that would mean that every single road sign when seen by an individual would entail a specific command resulting in a new norm ; on the contrary I believe that the obligation to stop at a stop sign is a norm that do preexist every single material stop sign that was put on the roads as "if you arrive at a stop sign, you must stop", and the stop signs are symbols informing us of where the prescription is applicable.

It would be redundant to consider that there is a distinction between a general norm asking drivers to stop at stop signs, and specific norms created for each driver passing by, because it will both create a confusion regarding which of the general or specific norm would be breached if the driver does not stop, and because it might subordinate the specific norms to the actual information or the driver and not just the presence of the stop sign, i.e. if the stop sign is somewhat hidden by a wrongly-parked truck, the general norm would exist but we would debate about the relevance of a specific norm concerning a particular driver that could not know he had to stop. This confusing redundancy is conflicting with the fact that we want norms to be applicable without having to question their existence at all, this is why I argue that there is a unique norm, and the signs are only informative symbols.

More generally, every norm does necessarily preexist to its enunciation, whether it is by a symbol or a sentence. Indeed, when enacting a norm, the authority enunciates it after having willed it ; the legislator chooses the words for his law after having decided to pass it and therefore after having decided on a new deontic modality to pass ; any road sign or written regulation is fabricated and printed after deciding that a norm will need to be applied ; a policeman will order a car to stop after deciding that he wanted this car to

[272]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.15

stop ; etc.

Any rules sheet in a chess set does not create specific commands for the players to respect, there is no norm created there, but the sheet simply informs the players that there are preexisting common norms they shall follow in order to play the game. This is true for any sheet, and therefore also for the very first sheet to ever get printed. These preexisting norms do prescribe behavior to the players, not the paper sheet which is only an information.

According to the hyletic conception, norms do not need to be brought to existence per se, as they are abstract entities ; however, I believe we could go even further.

We distinguish between abstract (absolute) existence and actual (relative) existence : this means, as stated before, that all possible norms already exist in absolute as abstract entities, but they are not part of any group or system of specific norms ; we would then call relative existence the membership to a legal system (our relative existence corresponds to what the literature simply calls existence or specific existence[273]). We could also refer to this relative existence as membership to a system of norms.

I thus believe that norms do actually need to be brought to (relative) existence in order to have properties other than just existing in the void of abstraction ; so I consider norms to have a hyletic essence and expressive properties.

Let us now tackle the claim that "all possible norms already exist in abstraction", which goes even further on norms' essence than the hyletic conception, and that I will call "norm-platonism[274]" (see the article of the Stanford Encyclopedia of Philosophy for more details about Platonism[275]).

Thus, without going too deep in the metaphysical part of the debate, I will argue against the expressive conception of norms (regarding their essence), and conclude further than the hyletic conception. To warrant this conceptual broadening, I would like to present an additional argument by reductio ad absurdum : let us suppose that norms only are and exist specifically and

[273]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.18

[274]I am not the first to use this concept, as authors like Nola already evoked "norm-platonism", but only from an epistemological perspective and not a legal theory one, like in : NOLA, Robert. Naturalism and Norms of Reason and Method. In : Rescuing Reason: A Critique of Anti-Rationalist Views of Science and Knowledge. Dordrecht : Springer Netherlands, 2003. p. 117-172.

[275]BALAGUER, Mark, "Platonism in Metaphysics", The Stanford Encyclopedia of Philosophy (Spring 2025 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/spr2025/entries/platonism/>.

relatively to a normative system, and let there be A and B two distinct and independant legal systems, if we consider a norm that specifically exists in the legal system B, according to the legal system A this norm does not exist and is not a norm. But even if it is obvious that the norm is a member of system B and not a member of system A, i.e. if the norm only specifically "exists" in the system B and not in the system A, we end up with an entity that would be both a norm and not a norm (i.e. a norm of B, and not a norm of A).

Additionally, the status of this norm and the status of all possible norms is the same from the perspective of the system A ; since A does not necessarily recognize B as a normative system, there is no reason to consider that the status of the norms of B is different than the status of all possible norms according to the system A, and symmetrically. The irrelevancy of the non-existence claim extends then to all possible norms, because as their status states, every possible norm could exist specifically in a possible world, and the hypothetical systems of possible worlds would have no reason to be treated differently from the system B by the system A according to our premises.

Therefore, the status of all possible norms is the same : abstract existence regardless of the systems they belong to.

Our conception of norms. It is only logical for us, regarding the previous section, to be closer to the hyletic conception. Indeed, we stated that norms were abstract entities, so we do believe in all norms' absolute existence ; but in my opinion we do need to look at other norms properties through the lens of the expressive conception.

This is where the dismemberment of norms properties become interesting, because we can now distinguish properties and look at them through different conceptions depending on what seems more relevant. And even if some would disagree with the way I distribute the properties between both conceptions, you could still find the idea of distributing them relevant and do it otherwise.

Our statement here is not so far from the conclusion of Guastini in his paper "Two conceptions of norms"[276]. Indeed, Guastini tells us that : "(a) norms are the meanings of sentences used to prescribe (semantic conception), but at the same time (b) no norm exists without a sentence expressing it (pragmatic conception)".

Our statement is slightly different regarding the second part of his, due to

[276]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

our dismemberment theory : if norms are indeed of a semantic nature, none of them does have any property other than simply existing without being expressed. So norms do have a hyletic nature, but expressive properties.

If we come back to Kelsen's example[277], the police agent raising his hand to make vehicles stop does not create any special norm, but brings applicability (notably) to a preexisting norm by expressing it. Road signs are neither norms nor norm creators, but they make some norms applicable by informing people symbolically, which is another way of expressing them.

Norms and sentences. In his article[278], Guastini states two questions that one must address while tackling the conception of norms :

Firstly, the way norms acquire existence ; and I believe that norms do exist independently of other factors, in accordance with the hyletic conception.

"The second question too seems to admit two alternative answers, prima facie independent of the preceding ones: (a) norms are sentences; (b) norms are the meaning-contents of sentences."[279]. As for this second question, I do believe that norms are not sentences, according to their semantic nature, but also for other reasons.

Indeed, if norms were sentences, a first difficulty that we would face building a Deontic Logic would be having to distinguish between internal and external negations ; because in a semantic conception, the negation of a norm is always a norm, for example the negation of "A is prohibited" would be "A is permitted" ; but if we consider norms as sentences, it is "more problematic. "In normative system S, act p is prohibited" is a complex metalinguistic statement that asserts that a certain norm belongs to a normative system. Therefore, its negation can be understood in two different ways: "In system S, act p is not prohibited" may be read as expressing that S does not contain a norm prohibiting p, in which case the negation operates over the membership of the norm in the system (i.e., the norm-proposition is negated); or it may be read as expressing that S does contain a norm not-prohibiting p (i.e., a norm permitting p), in which case the negation affects the norm referred to in the statement."[280]

[277]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.15

[278]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

[279]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

[280]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.80

Guastini also states that "norms, just like propositions, should not be confused with the sentences that express them. A norm is nothing other than the meaning of a sentence, therefore an interpreted sentence"[281]. Even though I totally agree with the first part of his statement, i.e. that norms are not sentences but only their meaning-content ; I wouldn't say that norms are interpreted sentences but quite on the opposite, in my opinion sentences are interpreted norms. I mean by "sentences" every enunciation of a norm in a system, for example written laws in the French civil code are sentences by nature ; but since norms do preexist in our theory to any enunciation of them, they cannot be the result of the interpretation of something posterior to it.

Some could argue that it would be more convenient to consider that norms are interpreted sentences, because in the end sentences always end up being interpreted in a normative process. I would disagree with such a claim because even if sentences are interpreted in a normative process, it is not to result in norms ; sentences are interpreted when it is time to apply them (or not), so in my opinion, if sanctions are a possible positive result of a sentence's interpretation, norms are not.

1.2.3 Prescription

The prescriptive/descriptive distinction. If norms are contextual rules creating deontic modalities, it means that they do have a prescriptive nature.

Indeed, many authors, such as those cited earlier, consider that according to the hyletic conception, norms are the meaning-contents of prescriptive sentences, and are by nature "proposition-like entities"[282] ; which makes them either a subclass or an analogous counterpart to propositions, which are the meaning-contents of descriptive sentences.

I do believe that this distinction is insufficient, as well as irrelevant according to Navarro and Rodríguez : "Now, the explanation of the difference between both types of meanings cannot be that non-modalized propositions are the meaning of descriptive sentences and norms are the meaning of prescriptive sentences, because in that case we would be ultimately locating the difference at the syntactic level [...] Nor is it possible to differentiate both types of meanings as differences in the uses of language, as it would be tanta-

[281]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

[282]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.69

mount to collapsing this conception of norms into the alternative pragmatic conception"[283].

Nuancing the distinction. There is also another thing to take into consideration : the fact that some "descriptive" sentences can become prescriptive when integrated to a system.

Some of the written laws in civil law countries are nothing but descriptive sentences that entail no deontic modality by themselves ; a good example being definition articles, like the fourth article of the GDPR entitled "Definitions"[284], and giving the definitions of twenty-six terms encountered later in the whole regulation.

If definitions are not prescriptive on their own, they can become indirectly prescriptive when seen as part of the whole legal system, i.e. they can indirectly imply deontic modalities.

Indeed, if for example a law states that small vehicles are prohibited in a certain spot, and if another law defines small vehicles and includes electric scooters, then this definition (which is descriptive) does have prescriptive implications, not by itself but through its membership to a system ; as two different laws can be part of the same norm, here the second is just adding substance the the "context" of the first one. But we will come back later on to what a system is.

So here, with the dismemberment of norms properties, we are able to nuance the prescriptive/descriptive and the norms/propositions distinctions which are considered insufficient, and we can consider propositions as entities that are not norms by essence while still being able to share properties with them like prescription. This avoids having to consider definitions (or other kinds of propositions) as fully norms and still pragmatically be able to consider them as prescriptive when included in a system.

1.2.4 Truth

The nonsense of a truth property. Among the properties norms can have, there is a debate whether it even makes sense at all for norms to have some, like truth. Should we consider truth among norms properties, or does

[283]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.70

[284]REGULATION (EU) 2016/679 OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL of 27 April 2016. Art. 4

it even make sense for a norm to be true ?

Unlike most logics that deal with "non-modalized propositions", the logics of norms deal with prescriptive meanings, and I believe like many authors that it doesn't make sense to say that a norm is true or false.

There is no consensus on this, but most authors agree on the fact that prescriptive norms cannot have truth values, they are valid or invalid, not true or false ; among them, some argue that no norm can have truth value[285], some that only the prescriptive ones can[286], some nuance by saying that statements about the validity of norms can be true or false[287].

As Guastini says : "On the one hand, we did not exclude the existence of prescriptive meanings (i.e., norms, assumedly neither true nor false) side by side with descriptive meanings (i.e., true or false propositions)"[288].

And I would add that even for descriptive propositions (like definitions) in systems of norms, truth is not a relevant property to consider, as it is more important to consider them through the lens of other norms properties like validity or applicability directly.

Truth might thus be more relevant if we consider sentences enunciating them with a pragmatic conception (or metalinguistic statements describing the deontic condition of a behavior) ; "A is prohibited" would be a true sentence if a norm prohibits A, and false if not ; but this is too indirect, and we already stated that we would consider norms semantically.

Jørgensen's dilemma. But admitting that truth-values do not make sense for norms raises an issue, presented by Jørgensen in the form of a dilemma : "under the assumption that norms are neither true nor false, there are only two possible alternatives regarding the applicability of logic to normative discourse. Either the notion of logical consequence, and logical connectives, are defined in terms of truth, in which case there is no possibility for a logic of norms; or a logic of norms is possible, but then the scope of logic has to be wider than descriptive discourse, and the notion of logical consequence as well as logical connectives should not be defined in terms of truth, which

[285]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.18
[286]HART, Herbert Lionel Adolphus. The concept of law. OUP Oxford, 2012.
[287]KALINOWSKI, Georges. La logique des normes. FeniXX, 1972.
[288]GUASTINI, Riccardo. Two conceptions of norms. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2018, no 35.

contradicts a firmly established tradition."[289]

Regarding our discourse until now, it is only natural for us to consider the second option of this dilemma, as we believe in a logic of norms, which are also abstract and cannot be subject to truth-values. But it also means that we will face some challenges in the next chapter regarding the creation of our logic of norms.

1.2.5 Validity

A surrogate of truth. "Unfortunately, in legal theory this concept of validity is often confused with a couple of different ideas, such as the existence of norms and the legality in the creation of norms."[290]

In classical logic, validity is a concept strongly linked to truth : logically, we consider that "an inference (or argument) is valid iff it preserves truth by logical necessity"[291], i.e. an inference is not valid if its premises are true but its conclusion is false.

But as we just saw, truth is not a concept we can use easily in a normative context, making such a definition of validity irrelevant for us.

As Navarro and Rodríguez state in their book, validity "seems to operate as a surrogate of truth in normative discourse"[292]. Moreover, we can find many definitions of validity, some of which are independant from the concept of truth : "On the one hand, to say that a norm is valid is sometimes a way to assign binding force to it [...]. On the other hand, validity is also used as a synonym of membership in a normative system."[293]

Specific existence. I believe that binding force should be distinguished as a property of norms, so I disagree with the first option. Validity should therefore be a substitute to truth, and the membership to a legal system, according to the system's criteria, is kind of a relative truth ; i.e. a norm

[289]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.38

[290]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.118

[291]FIELD, Hartry. What is logical validity. Foundations of logical consequence, 2015, p. 33-70.

[292]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.55

[293]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.56

is valid in a normative system if and only if it is true that the norm is a member of said normative system.

Furthermore, according to Navarro and Rodríguez, Weinberger seems to argue[294] that this last option - validity defined as membership in a normative system - is suitable for laying the groundwork for a logic of norms.

In his "pure theory of norms"[295], Hans Kelsen seems to also support this theory that validity is relative and defined membership to a member system with his formulation "specific existence" of norms, a concept close to what we called earlier relative existence : "l'on dénomme "validité" l'existence spécifique des normes".

Therefore, taking into account what we already said about norms, we consider that validity, relatively to a system (absolute validity makes no sense), is a property that only the norms that are members of that system have.

This also gives us a concrete scope for consistency, as it would make no sense to consider contradiction between a norm in a system with a norm that is outside of it ; so validity is the needed common ground for discussing norms consistency vis-à-vis each other.

1.2.6 Applicability

The acceptance of bindingness. We will consider that a norm is applicable if, according to the institutions of a system, this norm ought to actually be obeyed, and therefore potential sanctions against those who do not act accordingly would be justified.

This means that the applicability of a norm is a sine qua non condition for its sanction (if any) to come into effect. Also, let us note that applicability can be restricted spatially or temporally ; for example, it often happens when a law is voted by a parliament that its effective date will be delayed for months or years, this effective date outlines the applicability context of this law.

I would also argue that it is not the same if a contextual restriction exists on the normative level and on the applicability level : we defined norms as contextual obligations, and applicability can also be contextual, but both

[294]WEINBERGER, Ota. Normentheorie als Grundlage der Jurisprudenz und Ethik.: Eine Auseinandersetzung mit Hans Kelsens Theorie der Normen. 2019.

[295]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.18

these contexts do not interact with each other, they are cumulative but of different natures.

For example : a norm can state that parking in a certain street is forbidden every Sunday, and be applicable everyday ; it would be different to have a general prohibition of parking in this street but only applicable on Sundays. Even if it does not change much deontically for the people, if you would like to park in said street on a Monday, in one case the norm would not concern you, and in the second case the norm would concern you but simply not apply to you.

Navarro and Rodríguez did summarize applicability by saying that "applicable norms are those norms accepted as binding"[296].

"Applicability, like validity, [...] both are independent properties of legal norms. However, legal philosophers often overlook such conceptual independence."[297]

Applicability $\neq$ validity Indeed, I do agree with the independence of both properties (as we defined them), as we could find both valid norms that are not applicable, and applicable norms that are invalid.

For example, a norm can be valid since its promulgation, but cannot be applicable yet if its entry into force was delayed to a later date. On the other hand, many laws are applicable for pragmatic reasons despite being invalid, this can happen if the law is necessary for practical reasons until a new valid one takes its place, or in a system with a posteriori control of validity if a norm is considered applicable by the authorities until it is declared invalid.

Vilajosana also develops examples to distinguish applicability from [validity] (pertaining) : "for example, a rule in a period of vacatio legis may be conceived as a rule pertaining to a legal system, but which is not applicable in accordance with the system itself. Likewise, a foreign rule, which by definition does not pertain to a given legal system, may be applicable under such system if there is a rule within it that provides so (i.e., that compels or authorizes the judges of said system to use it to resolve a case)."[298]

[296]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.132

[297]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.132

[298]VILAJOSANA, Josep M. On the Concept of Constitutional Liquidity. Problema. Anuario de Filosofia y Teoria del Derecho, 2024, vol. 18.

1.2.7 Binding force

The theory of binding force. In theory, all applicable norms are binding, but this is only on the deontic level, this is what ought to be. But since we do not live in an ideal world, it often happens that a binding norm is not followed.

In order to enforce compliance with the law, a system of norms can establish sanctions, i.e. consequences to non-compliance with its norms.

These sanctions can be enforced by the authorities of this system. We will not enter in the details of enforcement or its legitimacy, which we will assume here.

The nature of sanctions. So sanctions could be another optional property of norms, but we can ask ourselves if distinguishing sanctions from norms is really relevant, since a sanction is nothing more than another norm : "if the norm A is not obeyed, then consequence B ought to happen".

The true difference between sanctions and other norms is, in my opinion, on the pragmatic/real level, because only sanctions can lead the authorities to physically force someone to comply.

For norms that are not sanctions (or for sanctions that are not enforced), anyone has the choice to comply or not, which is not the case with enforced sanctions that can lead to material constraints.

Thus, we can conclude that the binding force is a more relevant property for norms than having or not a sanction, since all sanctions are theoretical consequences of some norms unless a contingent enforcement happens ; we will say that a norm does have binding force if and only if their breach is able to entail material enforcement by the authorities of the normative system, i.e. if sanctions can become real consequences and not only theoretical threats.

But for a norm to be enforced, it is only fair that the concerned people are first aware of it. As we said earlier, if we consider the nature of norms as semantic, we pragmatically cannot consider its properties (like enforcement) unless the norm is expressed publicly ; it is what Fuller calls "promulgation", considering it as one of the eight standards of the internal morality of law, in "The morality of law"[299] : "The mandates of the law must be communicated

[299]FULLER, Lon Luvois. The morality of law. 1969.

to the people to whom they are directed."[300]

1.2.8 Publicity

The necessity of publicity. "Nemo censetur ignorare legem"[301] is a famous latin saying among lawyers. It implies that we suppose that all laws (positive laws actually) are known by the people, making them applicable to every person in a corresponding situation.

This latin saying is only relevant if the authorities guarantee the publicity of the said laws.

We do consider publicity (i.e. being expressed by means that are sufficient to reach the concerned people) as another property a norm can have, and a property that all valid norms should have ; and unless a norm is public, still in accordance with Fuller's theory of law's morality[302], I do consider that it should not have other properties as applicability or binding force, as it would be absurd to force people to obey injunctions they are not even aware of (unless, of course, in a totalitarian perspective).

The decreasing clarity of the law. But, as seen in our first chapter, some issues do rise when considering publicity nowadays. Indeed, and as some authors did already state regarding the French legal order, the laws are considerably increasing every year in numbers[303], but also noticeably decreasing in clarity[304]. The combination of these two evolutions makes the saying "nemo censetur ignorare legem" less and less relevant over time.

Since publicity justifies other norms properties, the legislator must be very cautious in order to maintain the justification of the bindingness of the laws he passes, i.e. the legislator must continuously ensure that the publicity of the law is effective.

[300]TUCKER, Edwin W. The morality of law, by Lon L. Fuller. Indiana law Journal, 1965, vol. 40, no 2, p. 5.

[301]"No one is supposed to ignore the law"

[302]FULLER, Lon Luvois. The morality of law. 1969.

[303]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, vol. 116, no 4, p. 905-917.

[304]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, vol. 123, no 3, p. 541-563.

1.2.9 Context

Earlier in this chapter, we defined norms as "contextual rules creating deontic modalities". Therefore, the context is already included in the definition, so why consider it as a property ?

What we call context is any material, temporal or spatial element constituting the premise of the norm, and necessary for it to entail its deontic modalities. The material context could be any specific condition that a norm would state as a premise, it could be the existence of a fact, or even an effective deontic modality ; for example, a material context could be "every citizen", "in every contract", "every employer who has the obligation to A", "every person who breached the obligation X", "every parked car", etc.

Moreover, a norm does not necessarily have a specific context in order to be a norm, as there is always a by default implicit context i.e. "whenever, wherever, whoever".

So the actual property, as we consider it, is for a norm to have a specified context. It can be a time period, a limited space, or material conditions for the norm to have effects.

For example, "nobody shall commit murder" is a norm without any context, and "every car owner shall have a car insurance" is a norm with "owning a war" being its material context.

As a remark, we can add that in contemporary legal systems, almost all norms are contextual, being at least temporally contextual, since norms have promulgation dates and cannot (unless rare exceptions) be retroactive.

1.2.10 Efficiency

We discussed earlier the property of applicability, i.e. for a norm to be potentially applicable to the situations it governs.

In his "Pure theory of law", Kelsen discusses the efficiency property[305]. According to Kelsen, who defines efficiency negatively, a norm is not efficient if it is never applied in practice (i.e. obeyed when it is applicable).

We can ask ourselves if efficiency is a relevant property to consider, since it does not concern norms inherently, but how they are reacted to. We could likewise have the same debate with the acceptance of the public, which can also be a property for a norm, but not very relevant for what we want to do in this thesis.

[305]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.19

These properties, like many others, are a posteriori properties that can be interesting to consider in other contexts, like sociology of law, or for other research ; we will however not develop them in our thesis, since we focus on the formalization of norms, on their nature and structures per se, but not on what happens to them once in place.

2 Legal systems and their properties

Once we defined what and how norms were, we still have to discuss how they interact with each other.

Our goal is not to do a deep study of what normative systems are, as many authors already did, like Carlos E. Alchourrón and Eugenio Bulygin in "Normative systems"[306]. However, we will establish some definitions and some standards that will be helpful to us for later chapters.

We will first define legal systems, then address their diversity, before finally discussing their properties.

2.1 The definition of a legal system

The notion(s) of system. Before defining what a legal system is, we must first consider systems and systems of norms, since we defined earlier laws as a specific sub-set of norms.

The concept of system, and its broad perspective, is thoroughly explored by Losano in "The Legal System from Theology to Technology"[307].

Thus, Losano distinguishes between multiple ways to define systems : originally, and generally speaking, "the system is seen as the coordination of a multiform reality under a single principle" by German philosophers ; as for the way the legal tradition defined systems, it differs between the abstract definition in continental law, and the empirical way in common law, i.e. the "system" is either an abstract concept/principle unifying all its elements, or simply a concrete organization of them. "One arrives at this conclusion: in European continental law the concept of system is so abstract as to be irrelevant in practice; in Anglo-Saxon law instead, the concept of system is used for the solution of practical Problems only"[308].

Nevertheless, we can argue that all of these definitions share the fact that a "system" is an entity that encompasses and coordinates all the elements related to it.

[306]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975.

[307]LOSANO, Mario G. The Legal System from Theology to Technology. ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of law and Social Philosophy, 1971, p. 485-502.

[308]LOSANO, Mario G. The Legal System from Theology to Technology. ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of law and Social Philosophy, 1971, p. 485-502.

Normative systems and legal systems. " Legal systems are mainly sets of norms specifically related by certain criteria."[309]

Ultimately, a system of norms is a set of norms (or indirectly prescriptive propositions) structured in a way that sets a framework for norms to acquire their properties in a certain context.

"In contemporary legal systems these properties of norms refer to a highly heterogeneous family of facts, such as promulgation, judicial recognition, ratification by supra-national institutions, specific commercial customs, and so on. These properties are often called "criteria of validity" and, in this sense, a criterion of validity refers to a legal source as a justification of the membership of a certain norm in a legal system."[310]

Indeed, if norms do exist in abstraction, their validity is relative to the system they are in, as are the modalities of enforcement or publicity. But what exactly is a normative system ?

Refining A&B's definition. I do find the definition of Alchourrón and Bulygin in "Normative systems" quite relevant, as they consider that "A normative set is a set of sentences such that among its consequences there are some sentences which correlate cases with solutions. Every normative set which contains all its consequences will accordingly be called a normative system."[311]

However, we must bring some precisions to this definition. First of all, we do not consider elements of normative sets as "sentences", but either norms or (indirectly prescriptive) propositions, as we developed earlier. Secondly, this definition seems insufficient to characterize normative systems unless provided with some internal rules/axioms that will allow the system to be dynamic, as most legal systems are dynamic systems of norms, i.e. considering a fixed set of sentences and its consequences cannot be sufficient to describe systems that create new laws everyday as most systems are precisely not fixed except when at a time t we want to make a ruling on some specific behavior, then we would freeze the norms and consider them as a fixed set.

I acknowledge that, as Bulygin clarified[312], the "legal system" defined by

[309]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.118

[310]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.118

[311]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.54

[312]BULYGIN, Eugenio. Carlos E. Alchourrón and the Philosophy of Law". South American Journal of Logic, 2015, vol. 1, no 2, p. 345-360.

them is intended as "a micro-system, not the whole legal order" (the latter being a sort of maximal legal system, in the sense of set theory) ; nevertheless, the critique still stands as any subset of laws cannot qualify as a legal system per se, and therefore it still has to be normatively autonomous and fulfill the requirements we just listed above.

We will come back to the static/dynamic question later while discussing systems' properties.

Raz supports this idea in "the concept of a legal system", and states that, in a legal system, all laws are not norms : "some laws are not norms; or, more precisely, that, given the minimum content and minimum complexity of legal systems, and given an acceptable set of principles of individuation, there are in any legal system some laws which are not norms."[313]

What he means is that, considering the dynamicity of legal systems, there are some laws that exist solely for the purpose of regulating it, and not to be normative (at least not directly). For example, "De-obligative laws are not norms"[314], i.e. a new law that states "The law X is abrogated" is not a norm, as it does not add any deontic modality but removes one.

However, we could argue that it still is a norm as it states the opposite modality of the law X, and has now priority over it, but this necessitates a clear temporal priority rule in the system.

Speaking of legal systems, they are actually a specific type of normative systems. We can say that a legal system is a dynamic set of laws (as defined earlier) and potentially other (indirectly prescriptive) propositions that contains all its consequences at any time t.

2.2 The diversity of legal systems

Static systems versus dynamic systems. A first standard often used to differentiate legal systems is the basis for the norms validity.

Kelsen, in his pure theory of law[315], distinguishes static and dynamic systems. In the first ones, norms are valid if they can be deducted from valid norms, in the other ones, norms are valid if they are created with conformity

[313]RAZ, Joseph. The concept of a legal system: an introduction to the theory of a legal system. Oxford University Press, 1980. p.125

[314]RAZ, Joseph. The concept of a legal system: an introduction to the theory of a legal system. Oxford University Press, 1980. p.125

[315]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.195-197

to due procedure and higher norms. These two criteria are also called the "deductibility" and the "legality" criteria.[316]

However, I do believe that this distinction will not be relevant within the framework of our thesis, for many reasons.

Since we will focus our work on the consistency of norms, and will mostly assume the legitimacy of the whole systems, we pragmatically can ignore the static/dynamic distinction.

Indeed, as Kelsen said himself [317], a single system of norms can be both static and dynamic, I would even add that all contemporary legal systems are this way, having both at least one valid authority creating norms and deductibility when seen at a time t.

Most of our work will be aimed at formalizing law, and we will only do it at a fixed time t ; so even if the static/dynamic distinction is relevant in theory, we will not make much use of it.

Static = dynamic ? Furthermore, Navarro and Rodríguez do consider that the best model for normative systems including the deductibility and legality criteria was the one combining both[318], i.e. the hybrid model that Kelsen mentioned.

I would go even further, and say that unless for legitimacy purposes for the whole system, this distinction is not quite relevant, and static and dynamic systems are one and the same but quantitatively nuanced. In every legal system, so also in dynamic ones, deductibility creates norms (in the way we defined them), because any general norm does have specific ones as consequences, unless other norms say otherwise. For example, if a law in a system does forbid using vehicles on Sundays, then their use is forbidden on Sunday afternoons. This last proposition may not be a law, but it is a valid norm, and though it may sound a futile example, a legal system still cannot be totally exhaustive (which is not true for normative systems in general, like in a game where simple rules can be exhaustive). Also, having a dynamic system questions the identity of the valid authorities, and it is a much more complex question that it seems as Raz develops well in his critics of Austin's Theory of legal systems[319], but isn't that also a question we can address static systems ?

[316]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.144

[317]KELSEN, Hans. Pure theory of law., 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.197

[318]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.146

[319]RAZ, Joseph. The concept of a legal system: an introduction to the theory of a legal system. Oxford University Press, 1980. p.7-31

In static systems, we cannot have every norm deducted, "Thus, if systematic relations among norms are not circular, then in every legal system there must be at least one norm whose membership does not depend on the satisfaction of any relations with other norms in the system"[320], so if we assume that the system is valid, then this one norm must have been created by a valid authority (this is the case if, of course, we exclude working with circular sets of laws, which we do).

This makes every static system partially dynamic, and every dynamic system partially static, which means they are all hybrid (if we do not consider legitimacy questions and according to our definition of norms). This does not totally deprive the distinction static-dynamic of its relevance, but this criterion is not as distinctive as it seems to be.

2.3 The properties of legal systems

As we just saw, and as Navarro and Rodríguez state it, "in every legal system [...] there must be at least one independent norm"[321].

These independent norms would be the structuring norms of the system, i.e. the norms that give the systems its properties, for example for other norms to have clear membership criteria.

As for the general properties of legal systems, that they may have or not have, Alchourrón and Bulygin state three of them[322] :[323]

- completeness : as discussed in the first chapter, a normative system would be complete if every potential case does have a normative solution, i.e. if any potential legal issue is included in the premises of a norm of the system.

This property is an important one, and a desirable one, particularly in civil law systems, because judges are supposed to apply the law and not create it (even if they indirectly do). That is why, for example in the French legal

[320]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.146

[321]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge university press, 2014. p.146

[322]ALCHOURRÓN, C. E. y Bulygin, E.(1971). Normative Systems. 1975. p.61-64

[323]Note : we will not be exhaustive here regarding the properties of legal systems, and for example will not discuss legitimacy (KELSEN, Hans. Pure theory of law, 2e éd., trad. C. Eisenmann, Paris, Dalloz. p.209), which we will assume.

system, the article 5 of the civil code forbids general rules creation by judges, stating that "Il est défendu aux juges de prononcer par voie de disposition générale et réglementaire sur les causes qui leur sont soumises."[324]

On the other hand, the article 4 of the same code forbids judges from not judging cases they would consider that no norm apply to : "Le juge qui refusera de juger, sous prétexte du silence, de l'obscurité ou de l'insuffisance de la loi, pourra être poursuivi comme coupable de déni de justice."[325]

So we could imagine, by combining these two articles, that the Civil code writers did expect the French normative system to be complete, in order for judges to be able to judge any case without creating norms, that is why this property is so important for a system to have (see chapter I for further development on completeness).

- independence : a normative system would be independent if, for every norm linking a specific case to a specific solution, there is no other norm linking the same case to the same solution.

This property is, in my opinion, unnecessary, as I do not see the problem from a pragmatic point of view of having multiple norms giving the same solution. On the contrary, it helps coherence, and gives people many ways to obtain their rights (as long as the redundant norms are not cumulative, but this is generally implied). Even the authors of "Metanormative principles and norm governed social interaction"[326] seem to consider the independent property as a derogatory "perfection-property".

- consistency (coherence) : a normative system would be consistent (or coherent, if we stick to the terminology we fixed in our first chapter, but the distinction does not seem to be considered here by Alchourrón and Bulygin) if, for every norm linking a specific case to a specific solution, there is no other norm linking the same case to any other solution.

This property is really important for a legal system to have as we saw in the first chapter, but I believe there is an alternative property that could, if met by a system, remove the necessity of consistency. We would then have

[324]Article 5 - Code civil Français

[325]Article 4 - Code civil Français

[326]ŽARNIĆ, Berislav et BAŠIĆ, Gabriela. Metanormative principles and norm governed social interaction. Revus. Journal for Constitutional Theory and Philosophy of law/Revija za ustavno teorijo in filozofijo prava, 2014, no 22, p. 105–120.

two properties, and systems should necessarily have at least one of them, which is an easier requirement to meet instead of having a necessary coherence property.

Well-order : a normative system is well-ordered if, for every sub-set of norms, there is a norm that has priority.

This property I am introducing can replace coherence, making it unnecessary if a system is well-ordered, because inconsistent norms are not a problem anymore if we can definitely decide which one to prioritize.

Well-order is originally a mathematical concept, and a property from set theory, but I believe that we could use it in normative systems, and especially legal systems.

Indeed, not only does it prevent inconsistencies from being a problem, but also we would be able to either automatically abolish laws that do not have priorities in the set of laws that govern a specific case, or not have to worry anymore about abolishing laws if for example we decide to give priority to new laws over old laws (if laws share the same "value").

But we will talk more about priorities in chapter III.

Chapter III

Deontic logics and their "paradoxes"

It it now time for us to discuss Deontic Logic, or shall I say deontic logics.

Indeed, our main theme in the thesis is not an uncountable singular entity, since many attempts already exist, as we will see in the first section ; for example : SDL[327], DL[328], $2SDL$[329], DDL[330][331] and $DL*$[332].

We can therefore induce that it is not an easy task to create a Deontic Logic system, because each one did come with its flaws and challenges as we will see in more details.

But before we deal with this difficulty, we must first address the state of the art, and discuss the many existing deontic logics ; indeed, we must both justify our contribution vis-à-vis what already exists, and show how it fits in the continuity of the research to this day.

1 The limits of Deontic Logic ?

To begin this chapter, we need to first address the genesis of Deontic Logic.

Indeed, SDL served as a base for most deontic logics, and it is still the case for ours. Discussing it now will therefore allow us to faster dive in the novelties of our logic when we get to it later.

We will therefore discuss the first logic of norms, SDL (for Standard Deontic Logic), and then we will go through the multiple other deontic logics that have followed, and how they each tried to complete and improve the previous ones.

It will thus be interesting to analyze what strategies have been attempted, so that we can adapt our strategy thanks to what our predecessors have found

[327]VON WRIGHT, Georg Henrik. Deontic Logic. Mind, 1951, vol. 60, no 237, p. 1-15.

[328]JONES, Andrew JI et PÖRN, Ingmar. Ideality, sub-ideality and Deontic Logic. Synthese, 1985, p. 275-290.

[329]DE BOER, Mathijs, GABBAY, Dov M., PARENT, Xavier, et al. Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn Deontic Logic system]. Synthese, 2012, vol. 187, p. 623-660.

[330]BENZMÜLLER, Christoph, FARJAMI, Ali, et PARENT, Xavier. A dyadic Deontic Logic in HOL. In : 14th International Conference on Deontic Logic and Normative Systems (DEON 2018). College Publications, 2018.

[331]CARMO, José et JONES, Andrew JI. Deontic Logic and contrary-to-duties. In : Handbook of Philosophical logic: Volume 8. Dordrecht : Springer Netherlands, 2002. p. 265-343.

[332]LIBAL, Tomer et PASCUCCI, Matteo. Automated reasoning in normative detachment structures with ideal conditions. In : Proceedings of the seventeenth international conference on artificial intelligence and law. 2019. p. 63-72.

out.

1.1 SDL, the pioneer

We will begin by describing what is SDL, how it was designed, then discuss about the issues that rose against it.

1.1.1 The heir of modal logic

It all began with Von Wright's paper "Deontic Logic"[333] published in 1951.

Von Wright built a logic that is derived directly from a Modal logic, but with some modifications to its operators.

KD, the modal. This Modal logic, noted KD[334], is an extension of Propositional Calculus (PC), and it brings in two new operators and two additional axioms, K and D, resulting in four modalities :

$(K) : \Box(A \to B) \to (\Box(A) \to \Box(B))$
$(D) : \Box(A) \to \neg\Box(\neg A)$

$\Box A :=$ A is necessary (A cannot not be true)
$\Diamond A :=$ A is possible (A can be true)
$\neg\Box A :=$ A is contingent (A can be not true)
$\neg\Diamond A :=$ A is impossible (A cannot be true)

As Jean-Louis Gardiès develops in "Modalités et normes"[335], or in the introduction of "Essai sur la logique des modalités"[336], we quickly see that these four modalities can be described only with the use of one of them (whichever it is, here we choose the square) and negation :

$\Box A :=$ A is necessary
$\neg\Box\neg A :=$ A is possible
$\neg\Box A :=$ A is contingent

[333] VON WRIGHT, Georg Henrik. Deontic Logic. Mind, 1951, vol. 60, no 237, p. 1-15.
[334] CHAGROV, Alexander et ZAKHARYASCHEV, Michael. Modal logic. Oxford University Press, 1997.
[335] GARDIÈS, Jean-Louis. Modalités et normes. ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of Law and Social Philosophy, 1976, p. 465-474.
[336] GARDIES, Jean-Louis. Essai sur la logique des modalités. FeniXX, 1979.

$\Box\neg A :=$ A is impossible

SDL, the deontic. But as we saw in the last chapter, this Modal logic cannot be adequate for norms, since norms are neither true nor false, and Modal logic only deals with concepts capable of truth values[337].

This is why Von Wright thought about modifying the modalities to give them a better grasp over the reality of norms : it does not make sense for a law to be a necessity, or to be true or false, but a law can definitely be an obligation, and be fulfilled or not fulfilled.

In his "Deontic Logic", Von Wright will therefore replace the Necessity operator $\Box$ by an Obligation operator O, and the Possibility operator $\Diamond$ by a Permission operator P.

We can note, nevertheless, that from a strict logical point of view, KD and SDL are identical, it is the interpretation of the modalities that really changes everything, going from $\Box$ to O.

Here too, we easily realize than one modality will be enough to describe norms[338], let it be O :

$O(A) :=$ A is obligatory
$\neg O(\neg A) :=$ A is permitted
$\neg O(A) :=$ A is optional
$O(\neg A) :=$ A is forbidden

The axioms system of SDL will hence be the same one than KD's, after of course substituting O to $\Box$:

PC : the axioms inherited from Propositional Calculus
$(K) : O(A \to B) \to (O(A) \to O(B))$
$(D) : O(A) \to \neg O(\neg A)$
(Modus Ponens) : $\frac{A, A \to B}{B}$
$(N) : \frac{A}{\Box(A)}$

The Modus Ponens and the Necessitation N are not exactly axioms, but inference rules (i.e. meta-axioms), but it is important to note them as part

[337]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge University Press, 2014. Chapters 1-2

[338]MORETTI, Alessio. The geometry of standard Deontic Logic. Logica Universalis, 2009, vol. 3, p. 19-57.

of the formal definition of SDL (we will discuss the Necessitation rule more in detail in our second section).

1.1.2 The rise of the paradoxes

If KD and SDL share a very similar structure, they are very different in practice, since we cannot put on the same level an absolute necessity on the one hand, and a normative obligation on the second hand.

Indeed, if something is necessary, then it will be ; however, if something is obligatory, there is no way to guarantee that it will be fulfilled and realized. And this difference in interpretation between these two logics will generate what the literature calls "paradoxes" among which we have the "contrary-to-duty paradoxes"[339].

We need to address these paradoxes because they are the main reason that has prevented Deontic Logic from becoming a widely used tool for legal formalization. This is because there would be no interest in formalizing law if we could not trust the formalization, and these paradoxes showed us that an SDL-based formalization would have absurd consequences, like entailing new contradictory obligations.

We will therefore indubitably need to find a solution to these paradoxes if we want to create a Deontic Logic suitable for formalization, but let us first assess them, before thinking of solving them.

The Ross paradox. The Ross paradox[340], for example, comes from the demonstrability in SDL of the formula $O(A) \rightarrow O(A \vee B)$.

If we are in possession of a letter, and if A means posting it and B means burning it, then we can conclude that if it is obligatory to post this letter, then it is obligatory to "post it or burn it".

This conclusion is very unsatisfying, since we are left with a new obligation that can be fulfilled by burning the letter, making impossible to fulfill the

[339]CARMO, José et JONES, Andrew JI. Deontic Logic and contrary-to-duties. In : Handbook of Philosophical logic: Volume 8. Dordrecht : Springer Netherlands, 2002. p. 265-343.

[340]ROSS, Alf. Imperatives and logic. Philosophy of Science, 1944, vol. 11, no 1, p. 30-46.

first obligation to post it.

And this is only with a simple example, but the consequences could be way more unpleasant with more extreme cases.

The Good Samaritan paradox. Another paradox worth mentioning is the Good Samaritan paradox[341], which comes from the demonstrability in SDL of another formula : $O(A \wedge B) \rightarrow O(B)$.

The name of this paradox comes from the most frequent example used in literature : if A is "helping your neighbor" and B is "your neighbor is a burglary victim", then since you have (morally) the obligation to help your burglary victim neighbor, then it is obligatory that your neighbor is a burglary victim.

This all leads to an absurd consequence, making an obligation for your neighbor to be a victim of burglary, when the initial obligation was intended to protect him by making you help him.

The Chisholm paradox. Finally, our last example of a paradox of Standard Deontic Logic is the paradox of Chisholm[342], let us consider the following four sentences :

- It ought to be that Jones goes to assist his neighbors.
- It ought to be that if Jones goes, then he tells them he is coming.
- If Jones doesn't go, then he ought not tell them he is coming.
- Jones doesn't go.

If A means "Jones goes to assist his neighbors", and if B means "Jones tells his neighbor that he is coming", then we would have :

(1) $O(A)$
(2) $O(A \rightarrow B)$
(3) $\neg A \rightarrow \neg O(B)$

[341]PRIOR, Arthur N. The paradoxes of derived obligation. Mind, 1954, vol. 63, no 249, p. 64-65.

[342]CHISHOLM, Roderick M. Contrary-to-duty imperatives and Deontic Logic. Analysis, 1963, vol. 24, no 2, p. 33-36.

(4) $\neg A$

From (2) and the axiom (K), we can infer that $O(A) \rightarrow O(B)$, and by Modus Ponens with (1), we can conclude that $O(B)$.

By Modus Ponens with (3) and (4), we infer $\neg O(B)$, and if we apply the (D) axiom, we obtain $O(\neg B)$.

So we arrive at a contradiction, where both $O(B)$ and $O(\neg B)$ are provable, i.e. it is both obligatory and forbidden for Jones to tell his neighbor he is coming, hence the paradox.

Let us add that having $O(B) \wedge O(\neg B)$ would be what we will call an internal contradiction, where something is both obligatory and forbidden.

However, there is a lighter level contradiction, what we would call an external contradiction, where we would have something both obligatory and optional (i.e. not obligatory), and it is the case here in Chisholm's paradox, since, thanks to the O-necessitation rule of SDL, we can have $O(A) \wedge \neg O(A)$.

1.2 Addressing the paradoxes

Now that we have a more comprehensive view of the first Deontic Logic, SDL, and of the first difficulties it encountered, let us examine how the following literature dealt with it.

1.2.1 The relevance of paradoxes

The pseudo-paradoxes. "The expression "paradox" is used in this context in a rather loose sense. Some of the so-called paradoxes of Deontic Logic are no more than counterintuitive results emerging from possible interpretations of valid formulas of deontic systems. [...] By contrast, others are rather genuine paradoxes in a strict sense – that is, they generate logical contradictions from sets of seemingly consistent premises, such as Chisholm's paradox"[343].

As Navarro and Rodríguez developed above, the word "paradox" is actually polysemous.

Indeed, some of them cannot be considered logically as paradoxes, like the first example we had : the Ross paradox.

[343]NAVARRO, Pablo E. et RODRÍGUEZ, Jorge L. Deontic Logic and legal systems. Cambridge University Press, 2014. p.39

The Ross paradox[344] just adds a new obligation that is confusing from a human-reading perspective, but logically, it is not a problem, since the creation of this new obligation to "post or burn" the letter does not make the first obligation to "post" it obsolete, and thus burning the letter does satisfy only one of two obligations whereas posting it is the only action that fulfills both obligations. So it stays the action commanded by the norms, even if the new norm is counter-intuitive from a human perspective.

The real paradoxes. As for the other type of paradoxes, they are real and problematic, like the Chisholm paradox for example.

Indeed, since SDL includes the *ex falso quodlibet*, it is a problem to be able to deduce both $O(A)$ and $O(\neg A)$ in certain situations that could happen and that we could totally encounter in real life.

If we want to create a Deontic Logic that can help us with actual legal systems of norms, we need to either solve the paradoxes, or cancel the consequences of said paradoxes.

These two strategies have both been attempted in the literature, let us see how now in the rest of this section, because before developing our proposition for a Deontic Logic, it is necessary that we review some state of the art, for several reasons :

- firstly, before proposing a strategy for building a Deontic Logic and dealing with the paradoxes, it is important to be aware of the diversity of strategies that already exist, and assess why we still need to innovate in that matter ;

- secondly, some of these strategies will have key elements that will be useful to us later, and it is important to understand where everything comes from ;

- finally, in the field of Deontic Logic more specifically, the logics are often built on top of each other in a kind of lineage, so it is only natural that we pay tribute to the very diverse work that has been done until today.

1.2.2 The solving attempt

The first main strategy that has been attempted since Von Wright's Deontic Logic is trying to solve the paradoxes by creating an improved or alternative Deontic Logic system, hereunder are some examples.

[344]VAN BENDEGEM, Jean Paul. Ross' Paradox is an impossible super-task. The British Journal for the Philosophy of Science, 1994, vol. 45, no 2, p. 743-748.

$DL/DL*$ **: the new operators.** DL and $DL*$ are both deontic systems that were created one after the other in order to solve the paradoxes problem of SDL. They both share the same strategy for reaching this goal : changing and improving the deontic operator O.

DL**.** In their paper "Ideality, sub-ideality and Deontic Logic."[345], Jones and Pörn build DL, a Deontic Logic system based on SDL, but with an improved deontic operator.

Before we dive into it, we shall say a few words about the theory of possible worlds :

it is about considering all the events that can or could come true, every potential possible combination of such events characterizes one possible world. For example, there exists (at least) one possible world in which France wins the next three football world cups and becomes a monarchy within that time.

By applying the theory of possible worlds to Deontic Logic, two categories of possible worlds can be distinguished : the ones within which every obligation is fulfilled, and the others within which at least one obligation is unmet. These categories are called respectively "ideal worlds" and "non-ideal worlds".

Thus, Jones and Pörn came up with an updated definition for the obligation operator O (inherited from SDL), and also add two other complementary operators :

$O(A) := A$ is true in every ideal world.

$O'(A) := A$ is true in every non-ideal world

$Ought(A) := O(A) \wedge \neg O'(A) := A$ is true in all ideal worlds but is not necessarily true (i.e. A is indeed obligatory but is susceptible of unfullfillment, since it would be irrelevant to consider absolute necessities as obligatory entities). We can note in passing that $\vdash_{DL} O(A) \wedge O'(A) \rightarrow A$.

This new language, DL, is characterized by a class of structures $(W, R_O, R_{O'})$, where W is a set of possible worlds, and R_O, $R_{O'}$, are binary relations on W.

We note wR_Ow' the fact that w is in relation with w' through R_O ; and we note $R_O(w)$ the set of W's elements in relation with w through R_O.

[345]JONES, Andrew JI et PÖRN, Ingmar. Ideality, sub-ideality and Deontic Logic. Synthese, 1985, p. 275-290.

These structures satisfy the following properties :

$$\begin{array}{ll} \forall w \in W, & \exists v, u \in W \text{ such as } wR_O v \text{ and } wR_{O'} u \\ \forall w \in W, & R_O(w) \cap R_{O'}(w) = \emptyset \\ \forall w \in W, & \text{either } w \in R_O(w) \text{ or } w \in R_{O'}(w) \end{array}$$

The authors claim that, thanks to these new operators, the main paradoxes of SDL are resolved. For example, we have that $\nvdash_{DL} Ought(A) \rightarrow Ought(A \vee B)$.

However, there are still some inconsistency concerns. Indeed, an advanced version of the Ross paradox exists in DL :

$\vdash_{DL} (\neg A \wedge \neg B) \rightarrow (Ought(A) \rightarrow Ought(A \vee B))$

An example of this new paradox would be : "If I have neither posted nor burned a letter, then having to post it implies having to post it or burn it".

To remedy this inconvenience, Libal and Pascucci did build an updated version of DL : the logic $DL*$.

DL*. The authors Libal and Pascucci, in their article "Automated reasoning in normative detachment structures with ideal conditions."[346], did redefine new operators from DL in order to improve them :

$Ought^*(A) := O(A) \wedge O'(\neg A)$

However, this new operator alone is too rigid, since $Ought^*(A)$ means "A is true only in ideal worlds". For example, $Ought^*$("every obligation is fulfilled").

That's why another operator is redefined to complement $Ought^*()$:

$O'(A) :=$ A is true in every "awful world" (an awful world is a possible world where every obligation is unfulfilled, they are a subset of non-ideal worlds).

For more details about how this logic is formally defined and constructed, we refer to the literature[347].

[346]LIBAL, Tomer et PASCUCCI, Matteo. Automated reasoning in normative detachment structures with ideal conditions. In : Proceedings of the seventeenth international conference on artificial intelligence and law. 2019. p. 63-72.

[347]LIBAL, Tomer et PASCUCCI, Matteo. Automated reasoning in normative detachment structures with ideal conditions. In : Proceedings of the seventeenth international conference on artificial intelligence and law. 2019. p. 63-72.

DDL **: the dyadicity.** Another strategy that was attempted to solve paradoxes is, instead of changing the operator O itself, changing its nature, and making it a dyadic operator instead of a monadic one.

Classically, in SDL, where we are limited to the monadic operator O, there are two different ways to express and formalize a conditional obligation :

$O(A \to B)$; *ie* it is obligatory that (A implies B)
$A \to O(B)$; *ie* A implies that (it is obligatory that B)

But if we introduce a dyadic operator, we reduce the expression of conditional obligations to only one way of doing so :

$O(B/A)$:= we ought to have B if we have A
(we note that $O(A/\top) \leftrightarrow O(A)$ from a monadic point of view, so not only is the dyadic operator broader, but it completely includes the monadic one).

Thus, in their article "Deontic Logic and contrary-to-duties"[348], Carmo and Jones developed a dyadic Deontic Logic, DDL, with this new operator.

More recently, the researchers Benzmüller, Farjami and Parent did present an embedding of DDL in classical higher-order logic, they even prove that this embedding is faithful (i.e. sound and complete)[349].

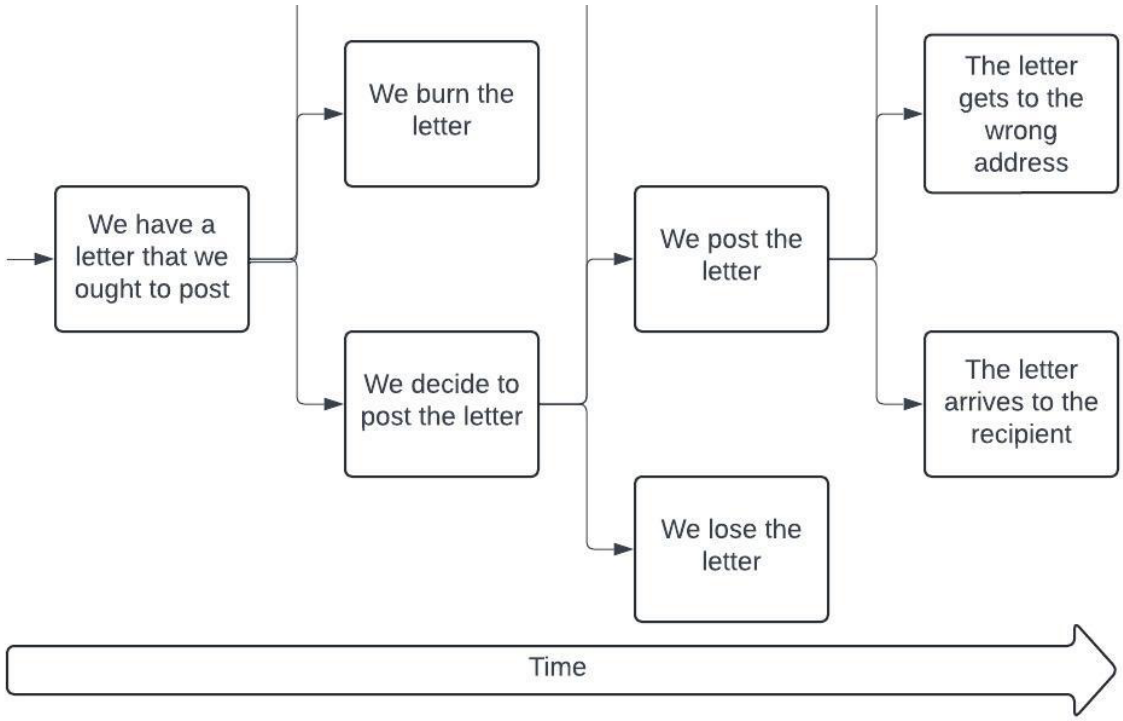
For more details about how this logic is formally defined and constructed, we refer to the literature[350].

$2SDL$ **: the second dimension.** The last strategy of this type consists in taking a second dimension into account : time.

[348]CARMO, José et JONES, Andrew JI. Deontic Logic and contrary-to-duties. In : Handbook of Philosophical logic: Volume 8. Dordrecht : Springer Netherlands, 2002. p. 265-343.
[349]BENZMÜLLER, Christoph, FARJAMI, Ali, et PARENT, Xavier. A dyadic Deontic Logic in HOL. In : 14th International Conference on Deontic Logic and Normative Systems (DEON 2018). College Publications, 2018.
[350]BENZMÜLLER, Christoph, FARJAMI, Ali, et PARENT, Xavier. A dyadic Deontic Logic in HOL. In : 14th International Conference on Deontic Logic and Normative Systems (DEON 2018). College Publications, 2018.

In 2012, De Boer, Gabbay, Parent and Slavkovic introduced $2SDL$[351], the bidimensional version of SDL. They undertook this because they considered that "the letter ought to be posted", "the letter will have to be posted" and "the letter should have been posted" are too different, and therefore the time parameter should be taken into account (as a second dimension) in the logic's construction.

$2SDL$ is based on a tree-like timeline of possible events, where every node is an event from which many possible sub-timelines are issued. Every node however has a unique antecedent temporally PRIOR to it. Here is a visual example below :



Thus, when we want to evaluate propositions in $2SDL$, we do not do it at a time t, but on a time couple (t, s), with t and s being nodes of the tree-timeline.

"We say s is the (real) evaluation point and t is a reference point".

[351]DE BOER, Mathijs, GABBAY, Dov M., PARENT, Xavier, et al. Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn Deontic Logic system]. Synthese, 2012, vol. 187, p. 623-660.

Example : when we say "the letter will have to be posted", the reference point is the time t where the proposition is stated (i.e. now in our example), and we evaluate the proposition at another time s : the proposition guarantees that in every possible world $\exists s$, such that $t < s$ and the letter is indeed posted at the time s.

However, when we say "the letter ought to be posted", the reference point is again t, the time when the proposition was stated, and this proposition guarantees that in at least one possible world, $\exists s$, such that $t < s$ and the letter is indeed posted at the time s.

Finally, when we say "the letter should have been posted", the reference point is still t, when the proposition was stated, and this proposition guarantees that in at least one possible world, $\exists s$, such that $s < t$ and the letter was indeed posted at the time s.

As a matter of fact, $2SDL$ is not the first temporal deontic logic system attempted, it is a more ancient research question. For example, we could cite Patrice Bailhache who, since 1985[352], started addressing the construction of a temporal and deontic logic, which will be completed in 1995[353] with the logic "$R5 - D5$" and a proof of its completeness.

Bailhache also views temporal logic as a tree-like structure, with a "modal ramification" growing towards the future.

For more details about how these logics is formally defined and constructed, we refer to the literature[354][355].

Nevertheless, we note that the authors demonstrated that $2SDL$ was decidable. They later looked into a variant that includes the theory behind DL of Jones and Pörn[356], and created $2DL$, which they proved was complete.

Summary diagram. Here is below a little summary diagram concerning some of the different improvements attempted on SDL by the authors we

[352]BAILHACHE, Patrice. Logique déontique et temporelle. 1985.

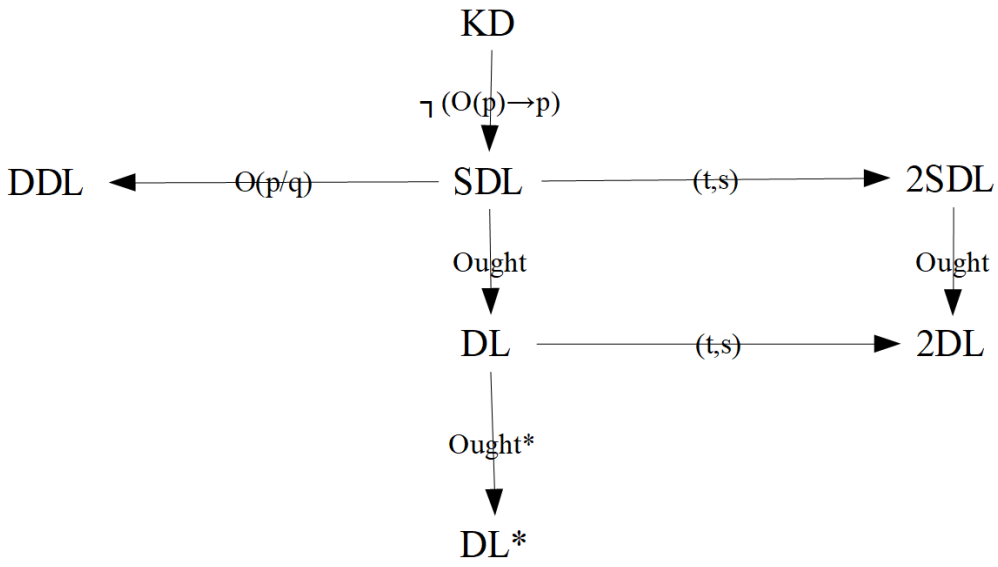
[353]BAILHACHE, Patrice. Canonical models for temporal deontic logic. Logique et analyse, 1995, vol. 38, no 149, p. 3-21.

[354]DE BOER, Mathijs, GABBAY, Dov M., PARENT, Xavier, et al. Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn Deontic Logic system]. Synthese, 2012, vol. 187, p. 623-660.

[355]BAILHACHE, Patrice. Canonical models for temporal deontic logic. Logique et analyse, 1995, vol. 38, no 149, p. 3-21.

[356]JONES, Andrew JI et PÖRN, Ingmar. Ideality, sub-ideality and Deontic Logic. Synthese, 1985, p. 275-290.

discussed until now :



1.2.3 The overriding method

The other main strategy was to get rid of the consequences of paradoxes, like for example getting rid of the principle of explosion.

Paraconsistency. While facing the challenge of building a consistent Deontic Logic, a question rose : why does it have to be consistent ? Does Deontic Logic have to be classically coherent like other logics, or can it inherently tolerate some of laws numerous inconsistencies ? Why do we have to build a consistent logic of norms, if the norms are not consistent themselves ? Is the purpose of Deontic Logic only to formalize norms, or can it aspire to bring them more coherence ?

All these questions led some authors to take an interest in paraconsistent logics[357].

A paraconsistent logic is a construction specifically designed to tolerate inconsistencies, and therefore does not admit the principle of explosion (the "ex contradictione sequitur quodlibet", which states that if any contradiction

[357]TANAKA, Koji, BERTO, Francesco, MARES, Edwin, et al. (ed.). Paraconsistency: logic and applications. Springer Science & Business Media, 2012.

arises in a logic system, then every proposition becomes provable in said system, therefore it "explodes" and loses its relevance).

Strong relevant logics. In their paper "Deontic Relevant logic: A Strong Relevant logic Approach to Removing Paradoxes from Deontic Logic"[358], Takahiro Tagawa and Jingde Cheng justify the use of paraconsistency in order to formalize systems of norms.

Relevant logics[359] are a sub-class of paraconsistent logics, focusing on a more intuitive and human-reasoning way to perceive the entailment, to make it more "natural".

Thus, relevant logics have a specific "natural" implication $\Rightarrow$, provided with "the relevance principle".

According to this principle, for every theorem of "$A \Rightarrow B$"'s type, A and B must share a variable.

Indeed, they consider the "classical" implication to be too unintuitive. For example, $A \wedge \neg A \Rightarrow A$ is a theorem of classical FOL (First Order logic), but is totally irrelevant and unnecessary in terms of human reasoning.

So then the relevant logics were built in response to this issue, with a more restrictive implication that is more faithful to the human way of reasoning.

Nevertheless, "Cheng pointed out that, though the traditional relevant logics have rejected the material-implicational paradoxes, there still are some logical axioms and theorems which are not natural in the sense of entailment".

For example, $A \wedge B \Rightarrow A$ is valid in relevant logics, but is not actually relevant in a "natural" reasoning. It is what Cheng calls "conjunction-implicational paradoxes" and "disjunction-implicational paradoxes"[360] (the word paradox is of course here considered in a very large sense).

To address this inconvenience, the authors Tagawa and Cheng proposed a "strong relevance principle"[361], and start by inductively defining the "an-

[358]TAGAWA, Takahiro et CHENG, Jingde. Deontic relevant logic: A strong relevant logic approach to removing paradoxes from Deontic Logic. In : Pacific Rim International Conference on Artificial Intelligence. Berlin, Heidelberg : Springer Berlin Heidelberg, 2002. p. 39-48.

[359]READ, Stephen. Relevant logic. Oxford: Blackwell, 1988.

[360]CHENG, Jingde. Relevance logic and Entailment logic. In : Software Science And Engineering: Selected Papers from the Kyoto Symposia. 1991. p. 189-211.

[361]TAGAWA, Takahiro et CHENG, Jingde. Deontic relevant logic: A strong relevant logic approach to removing paradoxes from Deontic Logic. In : Pacific Rim International

tecedent part" and "consequent part" of a formula A :

•A is a consequent part of A itself.

•If $\neg B$ is a consequent {antecedent} part of A, then B is an antecedent {consequent} part of A.

•If $B \Rightarrow C$ is a consequent {antecedent} part of A, then B is an antecedent consequent part of A and C is a consequent {antecedent} part of A.

•If $B \vee C$ or $B \wedge C$ is an antecedent {consequent} part of A, then both B and C are antecedent {consequent} part of A.

The authors have therefore proposed the following "strong relevance principle" : "for relevant logics to be free of not only material-implicational but also conjunction-implicational and disjunction-implicational paradoxes, it is a necessary condition that, in all axioms and logical theorems, every variable occurring in the formula should occur at least once as an antecedent part and at least once as a consequent part".

Thus, if we take into consideration both the relevance principle and the strong relevance principle, the logics are cleansed from any paradox or inconsistency in the broadest sense : they are strong relevant logics.

An overly radical method. Inspired by these (strong) relevant logics, Tagawa and Cheng created the deontic relevant logics, whose theory in strictly included in SDL's.

For more details about how it is formally defined and constructed, we refer to the literature[362].

We would have two major benefits by using relevant deontic logics :

- firstly, from a theoretical point of view, all the "paradoxes" of SDL would disappear. Indeed, either the problematic statements would not be provable (like the Ross paradox), or the potential inconsistencies would have no harm vis-a-vis the rest of the logic (like the contrary-to-duty paradoxes).

Conference on Artificial Intelligence. Berlin, Heidelberg : Springer Berlin Heidelberg, 2002. p. 39-48.

[362]TAGAWA, Takahiro et CHENG, Jingde. Deontic relevant logic: A strong relevant logic approach to removing paradoxes from Deontic Logic. In : Pacific Rim International Conference on Artificial Intelligence. Berlin, Heidelberg : Springer Berlin Heidelberg, 2002. p. 39-48.

- secondly, from a practical point of view, it might be fortunate to formalize law with a paraconsistent logic, i.e. a logic that tolerate inconsistencies, since the law is often inconsistent itself. Therefore, getting rid of the principle of explosion might be beneficial to us.

The paradoxes, in Deontic Logic, constitute the biggest obstacle to a formalization of the law ; since the relevant deontic logics offer a paradoxes-free system, we could think that they would be a perfect fit.

However, we must underline that, in order to achieve this paradoxes' cleansing, the paraconsistency did deplete a lot out of Deontic Logic (the theory of relevant Deontic Logic is strictly included in SDL's). So admittedly, we got rid of paradoxes, but at what cost ? Is the logic still "relevant" now that we got rid of a huge portion of its theorems ?

We lost abstraction vis-a-vis the initial issue, and I believe that we can address the paradoxes specifically without ignoring them and considering them forcibly harmless.

The authors Tagawa and Cheng themselves admitted that "the contradiction in contrary-to-duty obligation situation is avoided [in DDL], while the contradiction is regarded as harmless in itself there in our deontic relevant logics"[363].

Thus, it is not necessary to come to such extreme strategies to avoid the paradoxes of SDL, we should on the contrary keep the principle of explosion, since it can be a great tool in order to fight inconsistencies in law (which are not inevitable).

2 The Layered Deontic Logic (LDL)

Now that we have assessed the many ways by which scholars have tried to beat deontic logics' paradoxes to this day, it is time to exhibit the method we believe should be pursued.

In this section, we will expose our own deontic system, that we will call LDL (for Layered Deontic Logic)[364].

[363]TAGAWA, Takahiro et CHENG, Jingde. Deontic relevant logic: A strong relevant logic approach to removing paradoxes from Deontic Logic. In : Pacific Rim International Conference on Artificial Intelligence. Berlin, Heidelberg : Springer Berlin Heidelberg, 2002. p. 39-48.

[364]The formal creation of LDL, and its properties demonstrations, were made thanks to the help of the researcher Vincent Blazy from the LIRMM, with whom we will be

We will start by making a series of choices and trade-offs, and explaining firstly informally how I want LDL to be, and why it would be relevant to give it certain properties.

After this, I will bring more formal and technical details to how the logic should be constructed exactly.

I will then show how it effectively beats the paradoxes, of all kinds.

Finally, we will explore the properties of this new system, and challenge it.

2.1 The need for a cascading hierarchy

The first idea behind LDL is that paradoxes and contradictions come from a conflict between (at least) two deontic modalities that prescribe incompatible behaviors.

In classical logic, it is understandable that contradictions are conflicting, since propositions are either true or false (considering the excluded middle), two true propositions have the same truth value, so there is a real problem by having one contradicting the other, because there is no proposition among them that is true "more" than the other.

However, in systems of norms, we saw already that we could not consider that norms have truth values like classical logical propositions.

2.1.1 The hierarchy advantage

Establishing criteria. Indeed, unlike logical propositions, norms have many different sources. They can be issued from different authorities, at different times, with different purposes, and through different procedures ; all this giving norms various natures, making it possible for us to hierarchize them.

This hierarchy can be established in many ways or forms, the idea for LDL is that we will set up a series of criteria, every criteria being able to rank norms in relation with each other in a certain way, and then we will prioritize and rank the criteria themselves, creating "layers" of priorities on top of each other (the criteria are discussed later with more detail).

The end goal is, like said in Chapter II, to reach a "well-order" thanks to these cascading hierarchies : "a normative system is well-ordered if, for every sub-set of norms, there is a norm that has priority."

publishing these results soon

This is, we believe, the right way to reach consistency without facing any contradiction, since there will always be a "prevailing" solution.

The criteria specification. As for the detail of these criteria, we will make, in this chapter, a proposition ; but let us point out that, even if the number or the order we will choose for the criteria as illustration will certainly be debatable, our main claim in this chapter is that the method (of layered criteria of priority) is adequate.

Another point to make is that the hierarchy's details are necessarily specific to a unique norms system, since we must have a common and shared original source (of legitimacy and normativity) for these norms to be able to compare them with each other. It would not make much sense to hierarchize between conflicting norms if they are issued from different sovereign states. This source is often the Constitution for legal systems, but regardless of the source, we would still need a common ground for the formalization. Indeed, if we deal with different legal systems, then their priorities will be different, and we cannot guarantee the well-order anymore.

The question would though be relevant for a supra-national legal order that would be incorporated in the national one, like, for example, the European Union law in the member states orders.

As a matter of fact, even if we cannot formalize together different national legal systems because their respective priorities would compete and conflict, the case is different with international extensions of legal orders, because, as the monism theory defended by authors like Kelsen[365] suggests, international treaties get incorporated to the national legal order, even if said treaties create an extended supra- legal order ; but since the EU law for example can be seen as an extension of the national legal order and not a simple juxtaposition to it, it would be interesting to incorporate it in the formalization of the national legal systems of member states.

This question will however not be answered here, but would be relevant for future work.

2.1.2 *LDL*'s modalities

Let us develop in more detail how *LDL* would work and what the idea is behind this new Deontic Logic.

[365]KELSEN, Hans. General theory of law and state. Routledge, 2017.

The new operators and subscripts. Once we have established a cascading hierarchy, we assign to each obligation ϕ a decimal subscript $k \in \mathbb{D}$ corresponding to its degree of priority in the system, and we create the operator $\tilde{O}$ that takes as argument both the subscript and the obligation. The higher k is, the more prioritary $\tilde{O}(k, \phi)$ will be). For convenience purposes, when it is not ambiguous, we will note $\tilde{O}(k, \phi)$ as $O_k(\phi)$.

Then, we will consider as actual (active) obligations only the ones that do not contradict higher priority norms, neither through internal nor external contradiction (internal contradiction being a legal contradiction as $OA \wedge O\neg A$, and external contradiction being a direct logical contradiction as $OA \wedge \neg OA$), with the following axiom :

$$O(\phi) \Leftrightarrow (\exists k \in \mathbb{D} \mid O_k(\phi)) \wedge (\nexists l \in \mathbb{D} \mid l \geq k \mid O_l(\neg\phi)) \wedge ((\nexists l \in \mathbb{D} \mid l \geq k \mid \neg O_l(\phi))$$

We will develop later the complete axiom system for LDL, but let us just note here that with this rule, no contradiction could be problematic anymore (if we achieve a well-order thanks to our criteria). Of course, this means that we consider that any obligation that contradicts a higher priority norm is not actually valid (i.e. does not belong to the system of norms in terms of specific existence), but this is a stand I am willing to take in order to pursue norms consistency.

Also, if we have put "$\geq$" instead of "$>$" in the axiom, it is not necessarily that we expect some norms to be of equal value, but we keep our options broader for the priorities order, and keep this a possibility ; and if two norms of same value contradict themselves, then none of their obligations should be "active".

Typically, this could concern two paragraphs of the same law, even if this hypothesis could be called extreme, we could decide that if the same text has two contradicting prescriptions, why would we have to choose between the prescriptions, or give credit to any of them ?

Alchourrón and Bulygin might disagree here[366], and consider that this singular text would still be the expression of two distinct norms whose contradiction should be looked at like any norms antinomies. However, not only did they get criticized by Andrej Kristan[367] for their excessive semantization,

[366]ALCHOURRÓN, Carlos E. et BULYGIN, Eugenio. The expressive conception of norms. In : New studies in deontic logic: Norms, actions, and the foundations of ethics. Dordrecht : Springer Netherlands, 1981. p. 95-124.

[367]KRISTAN, Andrej. In Defence of the Expressive Conception of Norms. Revus. Journal for Constitutional Theory and Philosophy of Law/Revija za ustavno teorijo in filozofijo

and there is no necessity to "choose" between contradictory prescriptions of the same text ; but also more pragmatically, on the human level, if a legislator manages to get two contradicting norms on the same sentence (which is already an extreme hypothetical case), then I consider that there is no way to hierarchize them and that this act of legislation should be invalidated and redone.

The arbitrariness balance. Indeed, the more exhaustive and total the subscripts order system will be, the more arbitrary the lower priority criteria will have to be (exhaustiveness being here obtaining effectively a well-order among all deontic modalities). So when building the system, we must find a good balance between ranking every possible obligation source priority, and limiting arbitrariness even if it means risking to have sometimes same-level norms potentially able to cancel each other. But on another hand, we could judge that it is only fair that if the same norm prescribes contradicting obligations, it gets deactivated and has to be rewritten by the responsible authority.

That is why we may add in our priorities system, for guidance purposes only, some optional criteria. As we said earlier in this chapter, we claim that the method (LDL) is right, but its realization may vary, and the criteria system I will later establish will only be a proposal.

For the rest of the chapter, since we are able to potentially be exhaustive with the criteria (as we will see later), we can assume the following property for the system :

"Two norms of the same priority level do not contradict each other."

Indeed, if we have no faith in norms consistency, we could always create a level of priority per law, or even per article or paragraph, so we can always assume it, the question is how to balance the exhaustiveness of subscripts in order to have as many as needed for the property above to hold, while having as little arbitrariness as possible. But this question will be for the lawyers to answer, not for us.

The needlessness of "completive functions". Setting aside the subscripts, our operators are monadic, i.e. they only take one argument, this argument being the behavior in question.

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prava, 2014, no 22, p. 151-172.

In his papers about "completive functions theory"[368][369], Jean-Louis Gardiès advocates for a framework in which the obligation operator O should take a lot more arguments that would help contextualize the obligation : who obligated whom to do what ?

Gardiès considers that "A is obligatory", "x is obligated to do A", and "y obliges x to do A" are different grammatical constructions that should entail different formulations once formalized by deontic logic. He also advocates for including a time consideration, joining Bailhache[370] in this demand.

I strongly disagree with these statements, at least for the purpose of our research, which is formalizing law deontically.

Indeed, our objective here is not philosophical, we do not need to know what is the duty of citizens x or y, or who should do A or B ; the main goal of our research is practical : determining the deontic status of a behavior, given the context. This context enters the premises of norms we will formalize.

Therefore, we do not need to include any context as operator arguments. And that is exactly what judges do in legal systems : they (almost) never answer questions that were not asked to them, they do not judge about what the duty of x was, they judge if the behavior A complies deontically with the rules that did apply to it.

The same goes for time, we do not have the ambition (yet) to create a dynamic deontic system that evolves with the law ; we will, for a fixed state of the law, at a certain time, be able to formalize it and challenge behaviors vis-à-vis its rules. We do not need time arguments in our system LDL

Let us take an illustration example : in May 2025, x signed a contract with y, who lent him some money for a year ; in June 2026, x still did not pay y back.

In a theoretical system Gardiès-compatible, we would formalize this situation by saying that, O(x,y,give some money, signed contract, June 2026) ; thus including all the context in the obligation operator, making it unwieldy and overloaded.

In LDL, we would distinguish between the norm created by the contract that contains context, the facts of the case, and the deontic status of the behavior. The norm would be "if $date > May2026$ and if no anterior reim-

[368]GARDIES, Jean-Louis. Une particularité du raisonnement juridique: la présence de fonctions complétives. Logique et Analyse, 1971, vol. 14, no 53/54, p. 63-69.

[369]GARDIES, Jean-Louis. Logique déontique et théorie générale des fonctions complétives. Logique et analyse, 1973, vol. 16, no 61/62, p. 143-220.

[370]BAILHACHE, Patrice. Logique déontique et temporelle (Logiki deontyczne i logiki czasowe). 1983.

bursement exists, then O(x pays y back)", the facts of the case in June 2026 (when we fix the law and look at it) do fulfill the premises, therefore it is obligatory that x pays y back.

Thus, in LDL, we get rid of the potential complexity of unnecessary "completive functions" by, as it was defined in chapter II, making the context the premises of the norm, and keeping the obligation operator O light with a single argument.

2.1.3 The negation as failure (NAF)

The principle-exception dilemma. Speaking of priority criteria, there is but one conflict source that we cannot shut down with a criterion in LDL, and it is the principle/exception duality.

Indeed, there is technically always a latent contradiction between a norm that says "if C, then A is obligatory" ($C \rightarrow O(A)$), and another norm that says "A is not obligatory in the context B" ($B \rightarrow \neg O(A)$). Unfortunately, this is not the kind of contradiction that we can resolve by shutting down or deactivating the normativity of one of the norms, both obligations need to stay active in LDL, so we only have two options :

- The first option would be to go back about what we said regarding norms of same value, and keeping both valid while making sure that principles and exceptions always have same priority values, but this also means that we would have to be exhaustive and arbitrary in our criteria choices to be sure that no possible contradicting norms have the same value. We do not want this option for LDL since we will lose the fallback option to deactivate same-level inconsistent norms and therefore struggle more to maintain the consistency guarantee, and we want to keep it.

- The second option consists in formalizing the exceptions together with their respective principles in the same proposition/sentence, when implementing norms into LDL. But, in order to do that, we cannot use simple negations, as we would be left with a too rigid system.

For example, if we merge $C \rightarrow O(A)$ (the principle) and $B \rightarrow \neg O(A)$ (the exception) into the same norm, we would only have $(C \wedge \neg B) \rightarrow O(A)$. But the latter formula implies having to demonstrate $\neg B$ every time we have C before implying $O(A)$, despite the fact that in the context C, $O(A)$ is the principle, and $\neg O(A)$ is only the exception ; so we should not have to demonstrate $\neg B$ by default every time, but we should only consider the exception when it is already present/demonstrated in order to reverse correctly the burden of the demonstration between the principle and its exception (i.e.

we do not want to systematically demonstrate $\neg B$ beforehand, but we want to only check on the contrary if B is demonstrated).

Hopefully for us, such an inference rule does exist : it is the "Negation As Failure" (hereunder abbreviated NAF).

The quandary of the NAF. The NAF is an inference rule which enables to derive "$notA$" if A could not be demonstrated.

Thanks to the NAF, if we now replace the merged norm "$(C \wedge \neg B) \rightarrow O(A)$" by its lighter counterpart "$(C \wedge notB) \rightarrow O(A)$, we obtain what we want, i.e. one single norm and obligation that solves the potential technical conflicts between principles and exceptions, without having to demonstrate the non exceptionality every time we need to apply the principle, we only take the exception into account when its context has already been demonstrated.

However, one problem that may arise is that the NAF is a non-monotonic inference, and it could entail LDL being a non-monotonic logic.

We could of course decide to accept the non-monotonicity disadvantage in order to allow our logic to be dynamic (as we will see below, LDL can either be dynamic and non-monotonic, or static and monotonic), since it will allow us for example to add new norms continuously in real time into the system. Even if this dynamic variant could be appealing somehow, and if it may be the subject of future work, we hereby make the choice of focusing on monotonicity and thus computability and effectiveness of LDL. But we still have an issue before us : how to achieve monotonicity while having the NAF in the same system ?

Let us go back a bit, what exactly does the monotonicity of a logical system consists in ? And why is it important ?

A logical system is said to be monotonic if, whenever a formula is deductively valid, it stays valid in the system regardless of any additional premises. Monotonicity is moreover a key property of logic systems when it comes to computability later on, since most programming tools cannot guarantee the consistency of computation unless the logic is monotonic (like Prolog for example).

Thus, we must proof-check LDL to determine if or how it can be indeed a monotonic logic system. The main issue might come from our most important

rule :

$O(\phi) \Leftrightarrow (\exists k \in \mathbb{D} \mid O_k(\phi)) \wedge (\nexists l \in \mathbb{D} \mid l \geq k \mid O_l(\neg\phi)) \wedge ((\nexists l \in \mathbb{D} \mid l \geq k \mid \neg O_l(\phi))$

In fact, the potential source of defeasibility of formulas we should check first is if some obligations are activated under certain conditions, and are then contradicted by new higher priority obligations, which would imply the non-monotonicity of LDL since the first obligation would not be active anymore (i.e. a valid formula is not valid anymore).

There are only two ways a new obligation can emerge and cause problems : it can be deductively inferred from other norms, or it can come from a new added axiom.

As for the deducted norms, the solution could be to automatize the assignment of subscripts, such that they would always get attributed a priority lower than the axiomatic norms ; but this would not be necessary if our criteria is extensive enough, since obligations with different subscripts cannot interact with each other, and since we do not keep the O-necessitation rule (see later in this chapter).

As for the potential problematic added axioms, we must resign to not take them into account unless "resetting" every active obligation and computing them again afterwards. Which would guarantee the monotonicity only when the system is static, i.e. only concerning the norms at a certain time t without taking into account any continuously added norms.

2.1.4 The novelty of LDL's hierarchical system

With LDL, it is not the first time a Deontic Logic is proposed with priorities or default mechanisms. However, it still stands out by its novelty and innovation. Let us say a few words about what LDL is not : neither a preference-based Deontic Logic, nor a prioritized Deontic Logic, nor a defeasible Deontic Logic.

With its system mixing subscript obligations and active (effective) obligations, LDL stands out from previous attempts of preference-based deontic logics[371]. The norms of a legal order are all distributed, each getting assigned a subscript to its obligations, and we only activate (create non-subscript obligations mirroring) the ones that do not contradict (internally or externally) any higher priority norm.

[371]HANSSON, Sven Ove. Preference-based Deontic Logic (PDL). Journal of Philosophical logic, 1990, vol. 19, p. 75-93.

In fact, and it is a criticism we can also make to the deontic logics I cited in "the solving attempt" section, they are deontic logics created by logicians that did not consider enough the potential of inconsistency of the law. With LDL, and the new axiom we bring, we cancel any potential inconsistency ; and priorities-based logics could have been as far as us, which they came close to.

Let me explain : in priorities-based deontic logics, or in DDL, the obligations are contextual, and this allows us to "cancel" their modality in certain contexts. If the chosen context had been dependent on supra-norms consistency, then it would have been very close to our LDL ; unfortunately, it was only used to implement (cleverly though) principles-exceptions dynamics, which has already been stated by the legislator. But with LDL we will go further..

As for prioritized deontic logics[372], the consistency problem disappears, but another one takes its place : the priorities between norms are too rigid and linear, using 1 for the subscript of the top priority norm, and going up with natural numbers for less and less prioritized norms. It would create lots of problems to update the system and its formalization, as norms are created every day. That is why, with our decimal system and layered criteria of priorities, LDL stands out.

Two other problems of this logic, that are also shared by many logics cited in "the solving attempt" section, are the presence of the O-necessitation rule, and the logic not being intuitionistic ; respectively because as we will discuss hereunder the O-necessitation must not hold in a Deontic Logic, and because it could create unforeseen explicit/implicit norms dilemmas (see hereunder for discussion).

Furthermore, LDL is not either similar to existing defeasible non-monotonic logics by construction, as we focus on static norms systems, where there is no chance for propositions to be defeasible since they cannot contradict each other (the only theoretical contradictions that can happen would be between norms that do not share the same operator thanks to subscripts, so it would be irrelevant).

The only main counterpart would be the lack of dynamism of the logic, but in the short term we do not need to consider legal dynamic systems. Though it can still be an interesting future scope of research.

[372]VAN DE PUTTE, Frederik et STRAßER, Christian. A logic for prioritized normative reasoning. Journal of logic and Computation, 2012, vol. 23, no 3, p. 563-583.

2.2 The creation of LDL

2.2.1 The prerequisites of LDL

The LDL family. Before we dive into the formalism of creating LDL, let us stress that there isn't one single way of building it, as LDL is more accurately a family or a group of logics than a single system, as there are many choices and trade-offs to make regarding the subscripts constructions.

Therefore, we will hereunder make a proposition of an LDL system for the French legal order, while precisely citing optional criteria that you may use or find interesting.

Building the criteria. As for our LDL proposition, our first criterion would be the nature of the norm, which for the subscript would give a single digit. As an example we could have : 5 for constitutional law, 4 for organic law, 3 for ordinary law, 2 for national regulation and 1 for local regulation . We can always choose to add or reduce the number of layers and distinctions (like for local regulations, we could choose to distinguish between regional and municipal regulations).

As a second and also important criteria, we have temporality, which can use up to 8 digits in a YYYYMMDD disposition.

As for optional criteria, if we really want to be exhaustive, we could add the order of publication (in the "Journal Officiel" for national norms, for example) as a criteria to distinguish texts published at the same date (the higher digits go for the first texts to be announced).

We could also distinguish, in the same law, between its articles if we think they could contradict each other, with a higher digit for the first articles. The same goes for the paragraphs of an article.

We could also add many other criteria and change their order, or classify domains of law by priority for example. But this is just a basic example, an illustration, of what we could do to obtain a well-order on norms.

The unnecessary necessitation. Another key-choice we make for LDL is that we will not keep the "O-Necessitation" axiom present in SDL ($A \rightarrow O(A)$). We actually do not want a necessitation rule for any of our modalities.

The same choice was made by the researchers that developed the logic

$2SDL$[373], where the necessitation does not hold, and where the authors proved the completeness of their logic.

Indeed, from many points of view, it does not seem justified to consider every fact as an obligation :

- philosophically, as Jean-Louis Gardiès stated in "Essai sur la logique des modalités"[374], it would not make any sense to have $OO(A)$ as a deontic expression (or $O_kO_k(A)$ for a same k) ; we can still tolerate, nevertheless, to have $O_kO_l(A)$ if $k \neq l$.

- pragmatically, since we know that every obligation is not necessarily satisfied and fulfilled, this means that some obligations will be infringed, and therefore, we would automatically have potential contradictions between O-necessitated obligations (from unlawful facts/actions) and the infringed obligation. And even if we could solve this type of contradiction with our LDL subscript system, we prefer to not deal with it at all since we still need stronger justification to declare unlawful behavior (among others) obligatory.

- realistically, if we have a fact, we do not need for it to become an obligation to be considered by the system, and it is really strange to *a posteriori* declare a behavior obligatory simply because it has been pursued, because this would imply that it should have been like that, and it definitely is not the case.

2.2.2 The early difficulties encountered

The unnecessitated Kripke paradox. Since the researchers that managed to prove the completeness of their Deontic Logic system without necessitation ($2SDL$) did it using Kripke semantics[375], we started by giving some formal semantics to LDL :

Definition 1. *Kripke attempt.*

Consider a first-order multi-modal language with two modalities : the dyadic $\tilde{O}$ taking subscripts as a first argument, and the monadic O.

[373]DE BOER, Mathijs, GABBAY, Dov M., PARENT, Xavier, et al. Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn Deontic Logic system]. Synthese, 2012, vol. 187, p. 623-660.

[374]GARDIES, Jean-Louis. Essai sur la logique des modalités. FeniXX, 1979.p.91

[375]DE BOER, Mathijs, GABBAY, Dov M., PARENT, Xavier, et al. Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn Deontic Logic system]. Synthese, 2012, vol. 187, p. 623-660.

1. A model for the language has the form $(W, \{R_k, R\}, s)$, where W is a non-empty set of worlds, $\{R_k, R\}$ are a set of binary relations on W satisfying the conditions listed below, and s is a function assigning a set of worlds to each propositional variable f^j for $j \in \mathbb{N}$, thus $s(f^j) \subseteq W$. The conditions on $\{R_k, R\}$ are the following :

1.1. $\forall i, \forall x, \exists y, xR_ky$

1.2. $\forall x, \exists y, xRy$

2. Satisfiability of a formula ϕ for a model $M = (W, R, (R_k)_{k\in\mathbb{D}}, s)$ and a world $w \in W$ is expressed by writing $M, w \vDash \phi$, and we define $s^M(\phi) = \{w \in W \mid M, w \vDash \phi\}$. In order to simplify the presentation, whenever the model M is obvious from context, we write $s(\phi)$ instead of $s^M(\phi)$. Moreover, we often use "iff" as shorthand for "if and only if". For all $j \in \mathbb{N}$ and $k \in \mathbb{D}$:

$M, w \vDash \top$	
$M, w \nvDash \bot$	
$M, w \vDash f^j$	iff $w \in s(f^j)$
$M, w \vDash \neg\phi$	iff $M, w \nvDash \phi$ (i.e. $w \notin s(\phi)$)
$M, w \vDash \phi \vee \psi$	iff $M, w \vDash \phi$ or $M, w \vDash \psi$
$\forall i, M, w \vDash O_k(\phi)$	iff $\forall w'$ such that wR_kw', $M, w' \vDash \phi$
$M, w \vDash O(\phi)$	iff $\forall w'$ such that wRw', $M, w' \vDash \phi$

3. ϕ holds in a model M iff $\forall w \in W, M, w \vDash \phi$

4. ϕ is a theorem of LDL iff ϕ holds for every model M.

But as soon as we wrapped up this definition, we realized that without the necessitation rules we could not prove the completeness of this semantics, and that we even could prove the opposite.

Indeed, every usual Kripke relational model M satisfies them by construction: every world satisfies any theorem A, hence in particular every world for instance R-accessible from any fixed one w, so that $M, w \vDash OA$ and the necessitation rule for O automatically holds, which we do not want.

Excluding the necessitation rules then makes it impossible for us to create a complete Kripke semantics for LDL. It was possible for $2SDL$ with their reactive Kripke semantics because of the specificity of the two-dimensionality, of the temporal modality and the two variables evaluation, making the upper reasoning irrelevant.

The explicit/implicit norms dilemma. Another difficulty encountered concerns the main axiom of LDL :

$O(\phi) \Leftrightarrow (\exists k \in \mathbb{D} \mid O_k(\phi)) \wedge (\nexists l \in \mathbb{D} \mid l \geq k \mid O_l(\neg\phi)) \wedge ((\nexists l \in \mathbb{D} \mid l \geq k \mid \neg O_l(\phi))$

Indeed, the main idea behind this axiom was that an obligation can become active if and only if it does not contradict (internally or externally) any obligation that would have priority. But option/contingency can be of two sorts in law, explicit or implicit : for example, when we say $(\nexists l \in \mathbb{D} \mid l \geq k \mid \neg O_l(\phi))$, it could mean either that no option of $\neg\phi$ exists in higher levels of priority, or that at every higher level an explicit norm states that ϕ is an obligation.

But this inconvenience only happens in classical first order logic, where $(\nexists l \in \mathbb{D} \mid l \geq k \mid \neg O_l(\phi)) \Leftrightarrow (\forall l \in \mathbb{D} \mid l \geq k \mid O_l(\phi))$.

We will therefore set LDL_J as intuitionistic in order to maintain normative consistency.

2.2.3 The formal definitions

The semantics of LDL_J.

Definition 2. *LDL_J language.*

We will consider a two-sorted language L, which is bounded first-order on the first sort – which will invariably stand for the ordered decimal subscripts – with infinitely many variable symbols x_j for $j \in \mathbb{N}$ and one constant symbol for each decimal number (say, their decimal expansion), without equality, and multimodal intuitionistic propositional on the second one with countably infinitely many propositional formulae f^j for $j \in \mathbb{N}$ and modalities : one for each decimal subscript of the first sort given by the dyadic $\tilde{O}$ taking a subscript as its first argument, and the monadic one O. Both sorts will further be linked, giving mixed formulae *in addition of purely* modal *ones, by an existential quantifier on modal formulae bounded below by decimals. To sum up, formulae will be generated as the following B.N.F. grammar prescribes:*

$$\begin{array}{lll} \phi, \psi & ::= & \bot \mid f^j \mid \phi \rightarrow \psi \mid \tilde{O}k\phi \mid O\phi \\ \textit{(modal formulae)} & & \\ \alpha, \beta & ::= & \phi \mid \alpha \rightarrow \beta \mid \exists x_j \leq k \mid \phi \\ \textit{(mixed formulae)} & & \end{array}$$

We will also use as abbreviations, the symbols for negation $\neg$, equivalence $\leftrightarrow$ and inexistential quantification $\nexists$, standing as usual and respectively for

the implication of $\bot$, for a conjunction of inter-reciprocal implications and for negations of existential quantifiers.

The axioms of LDL_J.

Definition 3. *LDL_J axiom system.*
The axiom system of LDL_J contains:

- All axiom schemes of some Hilbert system for propositional intuitionistic logic on mixed formulae, plus
- Introduction and elimination axiom schemes for the bounded existential quantifier:
  $\phi[k/x_0] \to \exists x_0 \geq l \mid \phi$, for all $k, l \in \mathbb{D}$ such that $k \geq l$ and
  $(\phi \to \alpha) \to (\exists x_0 \geq k \mid \phi) \to \alpha$ for all $k \in \mathbb{D}$, when x_0 does not occur freely in α,
- J a set of legal norms in force in some legal order, in the form of $\tilde{O}$-modalized formulae whose index is above or equal to min_J,
- It also includes the following modal schemes (hence on the modal formulae only) and the axiom I previously mentioned, defining O in terms of $\tilde{O}$:

$$
\begin{array}{rcl}
\mathrm{K} & : & \mathrm{O}(\phi \to \psi) \to (O\phi \to O\psi) \\
(\mathrm{K}_k)_{k\in\mathbb{D}} & : & \mathrm{O}_k(\phi \to \psi) \to (O_k\phi \to O_k\psi) \\
\mathrm{D} & : & \mathrm{O}\phi \to \neg O\neg\phi \\
(\mathrm{D}_k)_{k\in\mathbb{D}} & : & \mathrm{O}_k\phi \to \neg O_k\neg\phi \\
4 & : & \mathrm{O}\phi \to OO\phi \\
(4_k)_{k\in\mathbb{D}} & : & \mathrm{O}_k\phi \to O_kO_k\phi \\
\mathrm{I} & : & \mathrm{O}\phi \leftrightarrow \exists x_0 \geq min_J \mid (O_{x_0}\phi \wedge (\nexists x_1 \geq x_0 \mid O_{x_1}\neg\phi) \wedge (\nexists x_1 \geq x_0 \mid \neg O_{x_1}\phi))
\end{array}
$$

As an usual abnormal modal Hilbert system, it will have the Modus Ponens as its only inference rule with premises.

Definition 4. *LDL_J semantics.*
We will mix Kripke relational semantics for the intuitionistic aspect and Scott-Montague neighborhood semantics for the modal one, in a classical set

theory with the axiom of choice.

1. A L-structure M will have the form $M = (W, N, (N_k)_{k\in\mathbb{D}}, R^i, V)$, where W is a non-empty set of worlds, $N, (N_k)_{k\in\mathbb{D}}$ are neighborhood functions $W \longmapsto \mathcal{P}(\mathcal{P}(W))$, R^i is an intuitionistic accessibility relation (that is, a preorder on worlds), and V is a intuitionistic valuation function for propositional variables f^j giving the set of worlds where they hold, that is $V(f^j) \subseteq W$ for any $j \in \mathbb{N}$ and V is persistent along R^i:

$$\forall j \in \mathbb{N}, \forall w, w' \mid wR^iw', w \in V(f^j) \text{ implies } w' \in V(f^j).$$

2. Satisfiability of a L-formula α in a L-structure $M = (W, N, (N_k)_{k\in\mathbb{D}}, R^i, V)$ at a world $w \in W$, expressed by writing $M, w \vDash \alpha$, is defined as follows, using when α is a modal one ϕ the notation $(\phi)^M := \{w \in W \mid M, w \vDash \phi\}$ (and the usual shorthand "iff" for "if and only if"). For all $j \in \mathbb{N}$ and $k, l \in \mathbb{D}$, we have:

$M, w \nvDash \bot$	(that is, $w \notin (\bot)^M$, so finally $(\bot)^M = \varnothing$)
$M, w \vDash f^j$	iff $w \in V(f^j)$
$M, w \vDash \alpha \wedge \beta$	iff $M, w \vDash \alpha$ and $M, w \vDash \beta$
$M, w \vDash \alpha \to \beta$	iff $\forall w' \in W \mid wR^iw', M, w' \vDash \alpha$ implies $M, w' \vDash \beta$
$M, w \vDash O_k\phi$	iff $(\phi)^M \in N_k(w)$
$M, w \vDash O\phi$	iff $(\phi)^M \in N(w)$
$M, w \vDash \exists x_j \geq l \mid \phi$	iff $\exists k \in \mathbb{D} \mid k \geq l$ and $M, w \vDash \phi[k/x_j]$

3. α holds in a L-structure M iff $\forall w \in W, M, w \vDash \alpha$.

4. A L-structure M is a model of LDL_J when all of its axioms hold in it.

5. α is a tautology of LDL_J for this semantics iff ϕ holds in every model M of LDL_J.

The class of models of LDL.

Theorem 1. *The models of LDL_J in our semantics are exactly the L-structures M satisfying the following properties, written axiom scheme-wise in the same order as thereabove:*

1. $\forall w, w' \mid wR^iw', (\phi \to \psi)^M \in N(w')$ *implies* $\forall w'' \mid w'R^iw'', (\phi)^M \in N(w'')$ *implies* $(\psi)^M \in N(w'')$

2. $\forall k \in \mathbb{D}, \forall w, w' \mid wR^i w', (\phi \to \psi)^M \in N_k(w')$ *implies* $\forall w'' \mid w'R^i w'', (\phi)^M \in N_k(w'')$ *implies* $(\psi)^M \in N_k(w'')$

3. $\forall w \in W, (\phi)^M \in N(w)$ *iff* $\forall w' \mid wR^i w', (\neg\phi)^M \notin N(w')$

4. $\forall w \in W, (\phi)^M \in N_k(w)$ *iff* $\forall w' \mid wR^i w', (\neg\phi)^M \notin N_k(w')$

5. $\forall w \in W, (\phi)^M \in N(w)$ *implies* $\{w' \in W \mid (\phi)^M \in N(w')\} \in N(w)$

6. $\forall w \in W, (\phi)^M \in N_k(w)$ *implies* $\{w' \in W \mid (\phi)^M \in N_k(w')\} \in N_k(w)$

7. $\forall w, w' \mid wR^i w'$,

$$(\phi)^M \in N(w') \quad \leftrightarrow \quad \left(\begin{array}{ll} & \exists k \in \mathbb{D} \mid M, w \vDash (\phi)^M \in N_k(w') \\ \wedge & \nexists l \in \mathbb{D} \mid l \geq k \wedge \forall w'' \mid w'R^i w'', (\phi)^M \in N_l(w'') \\ \wedge & \nexists l \in \mathbb{D} \mid l \geq k \wedge (\neg\phi)^M \in N_l(w') \end{array} \right)$$

Proof. It amounts to a mere unveiling of the definition of the satisfiability of a formulae in a model at a world.

The soundness of LDL_J.

Theorem 2. *For any J, our semantics for LDL_J is sound.*

Proof. Since R^i is a preorder and hence reflexive in any L-structure M, the Modus Ponens inference rule is valid in all of them at any world $w \in W$: for any L-formulae α and β, $M, w \vDash \alpha \to \beta$ means that $\forall w' \in W \mid wR^i w', M, w' \vDash \alpha$ implies $M, w' \vDash \beta$, so since $wR^i w$ we get $M, w \vDash \beta$. This means, all axioms of LDL_J holding in anyone of its models by definition, that furthermore all its theorems hold in it, hence the soundness of the semantics.

The consistency of LDL_J.

Theorem 3. *For any consistent J, LDL_J is consistent.*

Proof. The theory consists of:

- some fragment of intuitionistic logic with one quantifier that is bounded by decimals,
- the set J of $\tilde{O}$-modalized formulae that is consistent by hypothesis,
- and of the axioms of some multimodal logic $D4$,
- plus the additional axiom I.

Those three first fragments are then notoriously consistent, and their interaction too since this multimodal intuitionnistic logic is (because larger classical $D4$ is, regardless of how many times its modality is multiplied, as modelizable by multi-relational Kripke frames with transitive - for 4 - and serial - for D relations), and does not prove or disprove any modalized formula win the absence of necessitation. Hence, $\tilde{O}$-modalized theorems only come from J through proofs that only may use (Modus Ponens and) axioms for O_k modalities, to derive only formulae not involving the O modality. Now no negation of instance of the last axiom I is provable anymore by the rest of the theory, since it mixes several modalities. So finally even its adjunction does not endanger consistency.

A canonical model for LDL_J.

Definition 5. *Canonical model of LDL_J. Let MCS_J be the set of maximally consistent sets of L-formulae that contain the axioms of LDL_J. The canonical model of LDL_J will be the L-structure*

$$CM_J := \left(MCS_J, N^{CM_J}, \left(N_k^{CM_J}\right)_{k\in\mathbb{D}}, R^{i,CM_J}, V^{CM_J}\right)$$

with the following other components:

- $N^{CM_J} : w \mapsto \{\{w' \in MCS_J \mid \phi \in w'\} \mid O\phi \in w\}$
- *For all* $k \in \mathbb{D}$, $N_k^{CM_J} : w \mapsto \{\{w' \in MCS_J \mid \phi \in w'\} \mid \tilde{O}k\phi \in w\}$
- $R^{i,CM_J} := \{(w, w') \in MCS_J^2 \mid \forall \alpha, \beta, (\alpha \to \beta \in w \wedge \alpha \in w') \to \beta \in w'\}$, *and*
- $V^{CM_J} : f^j \mapsto \{w \in MCS_J \mid f^j \in w\}$

Theorem 4. *For any consistent J, CM_J models LDL_J, because for any L-formula α and $w \in MCS_J$,*

$$CM_J, w \vDash \alpha \text{ iff } \alpha \in w.$$

Proof. By structural induction on α. The base cases are trivial since $\bot$ belongs to no maximally consistent set, and the f^j case is by definitions of their satisfiability by worlds in CM_J and V^{CM_J}.

- If $\alpha = \beta \to \gamma$, then $CM_J, w \vDash \alpha$ iff:

$$\begin{array}{ll} & \forall w' \mid wR^{i,CM_J}w', CM_J, w' \vDash \beta \to CM_J, w' \vDash \gamma, \\ \text{iff} & \forall w' \mid wR^{i,CM_J}w', \beta \in w' \to \gamma \in w' \text{ by induction}, \\ \text{iff} & \forall w' \mid (\forall \delta, \epsilon, (\delta \to \epsilon \in w \wedge \delta \in w') \to \epsilon \in w'), \beta \in w' \to \gamma \in w', \\ \text{iff} & \forall w', \exists \delta, \epsilon, ((\delta \to \epsilon \in w \wedge \delta \in w') \to \epsilon \in w') \to (\beta \in w' \to \gamma \in w'). \end{array}$$

  But for any $w' \in MCS_J$, $\alpha = \beta \to \gamma \in w$ gives us relevant δ and ϵ so that we get $CM_J, w \vDash \alpha$. And in the other direction, either $\beta \in w' \to \gamma \in w'$, so that any δ and ϵ works, or $\beta \in w'$ and $\gamma \notin w'$ so that $\delta, \epsilon := \beta, \gamma$ work iff $\beta \to \gamma \in w$.

- Now if $\alpha = O\phi$, then $CM_J, w \vDash \alpha$ iff $(\phi)^{CM_J} \in N^{CM_J}(w)$,

$$\begin{array}{ll} \text{iff} & \{w' \in MCS_J \mid MCS_J, w' \vDash \phi\} \in \{\{w' \in MCS_J \mid \phi \in w'\} \mid O\phi \in w\}, \\ \text{iff} & \{w' \in MCS_J \mid \phi \in w'\} \in \{\{w' \in MCS_J \mid \phi \in w'\} \mid O\phi \in w\}, \\ \text{iff} & O\phi \in w. \end{array}$$

- And the $\tilde{O}$ case is exactly symmetrical for any $k \in \mathbb{D}$.

Remark: For any consistent J, in CM_J indeed hold all LDL_J theorems, as the soundness of the semantics globally shows, since every maximally consistent set containing its axioms in fact contains all of them, because they all are consistent with its axioms.

The completeness of LDL_J.

Theorem 5. *For any J, our semantics for LDL_J is complete, because in the consistent case, in CM_J hold exactly the LDL_J-theorems.*

Proof. If J is inconsistent, then LDL_J is too so that it does not have a model, and we are vacuously done. Otherwise, again LDL_J is consistent too, so that it has models, in particular a canonical model CM_J. Hence any formula α holding in all of its models in our semantics is by this one. But if α is unprovable by LDL_J, then LDL_J axioms and $\neg\alpha$ are mutually consistent so that by Zorn's lemma, there exists a maximally consistent set w of L-formulae containing both, hence not containing α, by consistency. Now by contraposition of one direction of a previous theorem, $\alpha \notin w$ implies $CM_J, w \nvDash \alpha$, so α does not hold in the canonical model.

Conclusion. Thus, we have introduced LDL_J, a new Deontic Logic system, multimodal, layered and hierarchized, intuitionistic and with an innovative axiom for determining which norms should become active/applicable, indexed by any body J of axiomatic norms that is bounded below. We also developed a set-theoretic semantics that mixes up relational accessibility for the intuitionistic aspect and neighbourhoods for the modal one. We have proved that LDL_J is consistent provided J already is, and that its semantics is adequate, that is sound and complete.

Chapter IV

The necessary intermediate level between logic and law

One of the main challenges that has always risen in the attempts of legal formalization is making the legal-experts (or commonly, the lawyers) participate actively in the process.

Speaking of the process, the aim of the legal formalization is to close the gap between the law and formal logic. But if so, then before we address this question, first we must establish and assess the (huge) existing gap between the law and the logic. It is only natural, before thinking about closing a gap, to start by evaluating it.

Another key-question we must answer in this chapter is who should close this gap. Not only is it necessary for lawyers to participate in the process of formalizing law, but it would be even more valuable if we could make their participation sufficient.

As we will also develop later in this chapter, we often find attempts of formalizing law that make it indispensable to have also logic experts in order to complete the process.

This need of lawyers comes from the fact that interpretation is inevitable at some point in the formalization process, and also both the public and the lawyers deserve a transparent formalization that could be explained, justified or corrected step by step.

Therefore, we will discuss the role of lawyers in the formalization process.

Finally, in our last section, we will develop a way of crossing this long bridge separating law from logic by splitting it into two connected bridges, via a chain of isomorphisms. I.e. we will address how we propose to close the gap in a way that takes into account our assessment and our will to include the lawyers in it.

Before starting this chapter, let us stress that we will not consider the understanding or translation of natural or legal languages by artificial intelligence or its derivatives. As we will develop later, we want this formalization to stay human-based, and the fact that we will automate certain specific translations in the last section does not make this formalization machine-based at all.

1 The gap between the text and the logic

As we just said, since we wish to close the gap between law and formal logic, it is necessary that we start by assessing this gap. Indeed, we must evaluate the reality and the extent of this gap, and this means addressing directly the two concurring elements that makes legal texts syntactically very far from logical propositions : the fact that the human language is itself quite distant from formal languages, and the fact that there are also specificities of the legal language itself.

Furthermore, if we take some time discussing this gap, it is to justify why we decided, in this chapter's last section, to establish a double-bridge strategy with an intermediate step to the formalization, made necessary by the reality of this huge gap. We thus are justifying an important part of our step-by-step formalization that is the core of our thesis, by taking the time to assess this gap that we need to close between the text and the logic.

In order to do so, we will first start by discussing the existing gap between logic and natural language, and we will then focus more specifically on legal language for comparison.

1.1 The specificities of human language(s)

Let us start by establishing the challenges of formalizing natural language into logic, before tackling some other languages obstacles we must face in our process.

1.1.1 From natural language to logic

Some starting definitions. Before we dive into the main topic of this section, let us start by defining some terms.

By "language", we hereby mean "the system of words or signs that people use to express thoughts and feelings to each other"[376].

This definition of language as a countable entity could be a bit too narrow, this is why we will later talk about languageS[377].

By languages, we mean any one of the many specific systems of signs and symbols used in a particular context (the context for human languages often being a group of people).

[376]The Britannica Dictionary ; 1.a.

[377]LEWIS, David. Languages and language. 1975.

These two perspectives of language as a countable entity are complementary, so there is no need of refining our definitions of language or languages for now.

As for "natural" language, it refers to the way by which we, humans, communicate in everyday life, orally or in writing, and without any specific or technical vocabulary inherent to a domain or a language in particular.

Finally, "logic" does refer to formal and computable systems/languages in general, not to a specific logic or program. The objective here is actually to evaluate to what extent is natural language computable, and then later focus more specifically on law.

natural language processing ? "Developing a program that understands natural language is a difficult problem."[378] But why is it so hard ?

Trying to compute natural language is not a new phenomenon. For more than fifty years, logicians tried to develop a logic that understands natural language, one of the most significant works of that time is from 1970 : "Linguistics and natural logic" from Lakoff[379].

But as the years go by, with the development of informatics, it has become both more important and more conceivable for scholars to find a way to compute natural language, helping machines understand it.

Among the disciplines that aim to study language, this is the role of computational linguists, as Allen describes in "natural language understanding" in 1995[380] : "The goal of the computational linguist is to develop a computational theory of language, using the notions of algorithms and data structures [...] must take advantage of what is known from all the other disciplines[381]."[382]

If this goal is completely met, then it would allow computers and hu-

[378]CHOWDHARY, K. R. natural language processing. Fundamentals of artificial intelligence, 2020, p. 603-649. p.3

[379]LAKOFF, George. Linguistics and natural logic. Synthese, 1970, vol. 22, no 1-2, p. 151-271.

[380]ALLEN, James. natural language understanding. Benjamin-Cummings Publishing Co., Inc., 1995.

[381]i.e. linguists, psycholinguists and philosophers

[382]ALLEN, James. natural language understanding. Benjamin-Cummings Publishing Co., Inc., 1995. p.1-2

mankind to communicate with a shared and unique language, making it easier for both to share information and knowledge, and for the other party to process it. But natural language Processing is still a challenge for many reasons[383].

Natural challenges. The difficulty of computing/processing natural language could be summarized in saying that it does depend too much on both how the sentence is phrased, and how it is perceived.

It "depends, amongst other things, on just how that natural language sentence is phrased. [...] the difficulty [...] in translating natural language statements into formal logic will, at least in part, be due to characteristics of the natural language statements themselves."[384]

Indeed, whereas many sentences could be to a certain extent transparent for logical translation when a connective like "and" or "if-then" is used, most of complex sentences of a natural language need some arbitration regarding the structure of the sentence.

For example, we could cite the sybillin sentence given by Chowdary : "I saw the Golden gate bridge flying into San Francisco."[385] In this sentence, who or what is flying ? It seems obvious for a human to find out that "I" is, but it isn't for a machine unless it could interpret it. Technically, grammatically, it could be a sentence where "the Golden gate bridge" is the flying subject, and we easily know that it isn't because we understand the sentence.

This brings us to another challenge, which regards interpretation. In fact, if we were able to compute natural language, we would anyway need to tackle interpreting the sentences of said language at some point in the process.

As Pereira develops in "logic for natural language analysis" : "To find a logical form for a sentence, we need to know how the meanings of words are bound together into the nucleus[386] of the sentence."[387]

[383]For a recent state of the art/progress in NLP, see : CHOWDHARY, KR1442 et CHOWDHARY, K. R. natural language processing. Fundamentals of artificial intelligence, 2020, p. 603-649. p.4

[384]BARKER-PLUMMER, Dave, COX, Richard, DALE, Robert, et al. An empirical study of errors in translating natural language into logic. In : Proceedings of the Annual Meeting of the Cognitive Science Society. 2008.

[385]CHOWDHARY, KR1442 et CHOWDHARY, K. R. natural language processing. Fundamentals of artificial intelligence, 2020, p. 603-649. p.3

[386]SEUREN, Pieter AM. Operators and nucleus: a contribution to the theory of grammar. Cambridge University Press, 2010.

[387]PEREIRA, Fernando Carlos Neves. logic for natural language analysis. 1982. p.120

To back up his point of view, the author gives us a telling example with the sentence "I saw a man in the park with a telescope."[388] The telescope here could potentially belong either to the subject or to the man in the park. Formalizing such a sentence into logic needs either to implement a way to decide how to solve such syntactical dilemmas, or to leave the interpretation out of the scope of logic and let humans take charge (which would be respectively too arbitrary or too subjective), but we will come back to interpretation issues later in the chapter.

This fact, that it would involve too much arbitrariness to totally compute natural language into logic, is supported by Lakoff in his book "Linguistics and natural logic" : "the theory given in Chomsky's Syntactic Structures[389] leaves open the question as to whether [...] any correspondence between grammatical form and logical form is accidental."[390] This means that the logic behind natural language cannot be totally captured by a computable logic since there is too much arbitrariness, which ends up giving us often syntactical dilemmas whose resolution we can never automate.

For all these reasons, translating natural language into logic is not an easy task ; but it is even more complex if we consider languageS and their variety.

1.1.2 The forever obstacles of languages

The impossible common ground. The first obstacle of languages is their plurality. There is indeed a wide variety of languages, and each one has its very own system.

Without entering into the linguistic details, even among human languages, the grammars are very diverse and do not obey at all to the same logic.

Before considering computing any human languages, we can start by looking at what are the difficulties humans themselves face in translating them into one another.

In the Journal of Language and Linguistic Studies, there is an article[391]

[388]PEREIRA, Fernando Carlos Neves. logic for natural language analysis. 1982. p.120

[389]CHOMSKY, Noam. Syntactic structures. Mouton de Gruyter, 2002.

[390]LAKOFF, George. Linguistics and natural logic. Synthese, 1970, vol. 22, no 1-2, p. 151-271. p.2-3

[391]PHAM, Anh Tuan, NGUYEN, Linh Thi Diem, et PHAM, Vi Thi Tuong. English language students' perspectives on the difficulties in translation: Implications for language

where they studied and ranked the difficulties faced in the translation process between Vietnamese and English languages, and especially regarding "four factors namely lexis, grammar, culture, and existing knowledge"[392].

We therefore have a dilemma if we ever want to build a natural logic for these languages :

- either we build one logic per language, which means that we will "only" face grammatical difficulty, since there is no need to translate singular words which will be atoms[393].

- or we try to build one logic that computes all human languages, but this sounds impossible, since not only the grammar will also be an issue, but we have to deal with a whole common ontology, including "untranslatable" words[394]. It is hard enough for humans to translate languages, and for computer scientists to logically process one natural language at a time, so this second option seems like a dead end.

The specific obstacles. We already talked about the difficulty of formalizing a human grammar into logic because of the arbitrariness[395] and the constant need of (human ?) interpretation[396], but other problems arise even when dealing with a single natural language.

Indeed, beyond the complex syntactic level, there are also the semantics of the language that can be difficult, since the words of a language and their meaning are not bijective sets. Semantics are already tricky for casual language translations from one to another, so they will be too for any attempt of formalization :

A first problem can arise when a single word does have several corresponding meanings. Polysemy is already tricky when it comes to translating human languages into each other, as Essam Mohammed develops in"Polysemy as a

education. Journal of Language and Linguistic Studies, 2022, vol. 18, no 1, p. 180-189.

[392]PHAM, Anh Tuan, NGUYEN, Linh Thi Diem, et PHAM, Vi Thi Tuong. English language students' perspectives on the difficulties in translation: Implications for language education. Journal of Language and Linguistic Studies, 2022, vol. 18, no 1, p. 180-189.

[393]an atom is the simplest formula that can exist in logic, with no logical operator and no other purpose than stating itself

[394]MUKHTAROVA, Sandugash Sabyrzhanovna, KARAGULOVA, Balgenzhe, SIDESHOVA, Zarina, et al. Translating the untranslatable: challenges, sticking points and struggles. Space and Culture, India, 2019, vol. 7, no 1, p. 250-263.

[395]LAKOFF, George. Linguistics and natural logic. Synthese, 1970, vol. 22, no 1-2, p. 151-271.

[396]PEREIRA, Fernando Carlos Neves. logic for natural language analysis. 1982.

Lexical Problem in Translation"[397].

To illustrate this issue, Moreso stated for example that "Generic terms like 'woods', 'bald', or 'heap' pose a problem when it comes to determining their referent in many doubtful cases."[398]

Coming to logic, it will also be an additional obstacle : since words are bound to become atoms, we would have identical words but with different meanings depending on the context, and "The translator then has to resort to the context of situation to eliminate the ambiguity"[399], and unless involving human interpretation (or AI), it is a dead end for machines.

As a conclusion for this part, I will cite one of Lakoff's concluding remarks in "Linguistics and natural logic" : "Recall that natural logic is a theory, a theory about the logical structure of natural language sentences and the regularities governing the notion of a valid argument for reasoning in natural language."[400]

1.2 The even more specific legal language

As we have just seen above, translating natural language into logic is a hard undertaking ; but our job would be to do so with legal language, which is a more specific subset of languages, this might make it even harder to formalize, unless it doesn't.

1.2.1 From legal to natural language

"The indeterminacy of the law is connected with the problem of the vagueness of linguistic expressions, and especially the vagueness of the generic terms"[401], but beyond the semantics of natural languages, the legal world does have a specific language of its own.

[397]MOHAMMED, Essam T. Polysemy as a Lexical Problem in Translation. Online Submission, 2009, vol. 55, p. 1-19.

[398]MORESO, José Juan et MORESO, José Juan. A Logical Analysis of Legal Propositions. Legal Indeterminacy and Constitutional Interpretation, 1998, p. 47-100.

[399]MOHAMMED, Essam T. Polysemy as a Lexical Problem in Translation. Online Submission, 2009, vol. 55, p. 1-19.

[400]LAKOFF, George. Linguistics and natural logic. Synthese, 1970, vol. 22, no 1-2, p. 151-271. p.105

[401]MORESO, José Juan et MORESO, José Juan. A Logical Analysis of Legal Propositions. Legal Indeterminacy and Constitutional Interpretation, 1998, p. 47-100.

The legal "language". "Many legal theorists and linguists have addressed the notion of legal language from different perspectives. Despite that, the definitions of legal language vary."[402]

We can address the definition of legal language through many approaches :

- we could focus on the specific lexicon that is used by lawyers in order to discuss legal matters
- we could focus on the finality of a discourse in order to determine if it contains legal language
- we could focus on the source of the discourse to see if it issued by a law worker

But if we were to define precisely the contours of what is legal language, it would be a long way through an old doctrinal debate that is not our main focus here, since we will only assume the existence of a legal language ("law has its own language"[403]).

As for the definition, since the legal language's reach is way beyond norms, and only the latter is our subject in this thesis, we do not need to precisely define what the legal language is, but only will we say that it definitely includes the language norms are written with.

The special semantics. Even if we do not define its boundaries, we can still say a few words about the characteristics of legal language.

Semantically, legal language does have a specific lexicon, a vocabulary that, even if the words are often the same as in natural languages, they have a specific meaning ; the existence of many dictionaries (like the French famous "Vocabulaire juridique"[404]) specific to legal meaning of (often common) words proves it.

" Legal linguistics is a science"[405], and even its use, for example in legistics when writing laws, is a "technique, a science"[406].

[402]GLOGAR, Ondřej. The Concept of legal language: What Makes legal language ‘ Legal ‘?. International Journal for the Semiotics of law-Revue internationale de Sémiotique juridique, 2023, p. 1-27.

[403]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

[404]CORNU, Gérard, et al. Vocabulaire juridique. (No Title), 2016.

[405]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

[406]BÉNABENT, Alain et GAUDEMET, Yves. Dictionnaire juridique: tous les mots du droit. (No Title), 2022.

Indeed, a specific language does exist, as show the numerous specific definitions (or redefinitions) of common terms in written laws. "Polysemy and ambiguity are irreducible phenomena of language, incidents or accidents of linguistic communication. [...] legal definition could at least have the function - and justification - of dispelling equivocation and obscurity. [...] it is a matter of choice and clarification. Selective, it brings out the meaning it selects, at least for the legal effect it determines. It favors one meaning among others."[407]

So this need of giving a new or specific definitions to polysemous common terms attests of the existence of a specific legal language needing a precise lexicon, and the legal language does certainly have a semantic singularity.

As for its use, we could certainly discuss the finality of it, its source, or even the public to whom it is destined, but we will not solve this general debate here. Whether it is destined for lawyers or for the general public to understand, the legal language is specific and technical. What does concern us is finding out if it makes it harder to formalize than natural language or not.

1.2.2 The benefits of legal language

The system's strength. "The advantage of a technical language is that it avoids contradiction/misinterpretation"[408]

It might seem a little counter-intuitive, but having a legal language far more specific than natural ones only makes its formalization easier, and that for many reasons.

Thanks to the (theoretical) precision and technicality of the legal lexicon, a lot of natural language translation problems are avoided like synonymy or polysemy, since we (the lawyers) know exactly the semantics of this technical language (this does not apply to translating a legal language to another since legal languages are too dependant on the natural language they are based upon[409]), "Each word receives one definition"[410].

[407]CORNU, Gérard. Les définitions dans la loi. Mélanges dédiés à Jean VINCENT, 1981, p. 77.

[408]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

[409]CHROMÁ, Marta. Synonymy and polysemy in legal terminology and their applications to bilingual and bijural translation. Research in language, 2011, vol. 9, no 1, p. 31-50.

[410]GAUTIER, Pierre-Yves. Quelle langue parle le droit?. Pouvoirs, 2023, no 3, p. 99-110.

Another point going our way is that legistics are very controlled, i.e. the way norms are written is such that it has to obey certain standards, like clarity or intelligibility. In France for example, norms' intelligibility is even a constitutional objective[411]. So this makes it a lot less likely to find untranslatable sentences like in natural languages.

The legislator's weaknesses. However, we must balance our discourse with a pragmatic observation : norms are not always written in a clear and intelligible way. Indeed, with the passing of time, they are written less and less clearly, and the number of norms becomes higher every year, as many observers underlined[412][413][414].

After all, laws are written one after another by different fallible humans, and when it comes to writing laws, "the art of writing rules does not follow any rules"[415]

So this loss in quality and gain in quantity does not ease the translation of norms, but it is not a question of translatability anymore, it becomes a matter of interpretation.

[411]MALAURIE, Philippe. L'intelligibilité des lois. Pouvoirs, 2005, no 3, p. 131-137.

[412]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, no 3, p. 541-563.

[413]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, no 4, p. 905-917.

[414]MOREL, Benjamin. Les fins de la loi, la fin de la loi. Droit et société, 2020, no 1, p. 235-247.

[415]CORNU Gérard, « L'art d'écrire la loi », Pouvoirs, 2003/4 (n° 107), p. 5-10. DOI : 10.3917/pouv.107.0005. URL : https://www.cairn.info/revue-pouvoirs-2003-4-page-5.htm

2 The necessity of a lawyer-based formalization

Now that we assessed the gap, we need to choose an operator. Because if we wish to propose a method for formalization that will help close this gap, we also need to arbitrate who should do it.

Therefore, we need to address, in this second section, the role of lawyers who, as we will develop, must be these operators for many reasons ; because, and this is also an important element, operating formalization will entail automatically interpreting the law.

Moreover, as we will see, this formalization by lawyers must be provided with clarity and transparency, which are elements supporting and justifying our choice of a method based on an intermediate formal grammar.

In order to assess the necessity of basing the formalization of the law on lawyers, we will first discuss how we cannot escape the interpretation of the law, then how the formalization needs to be clear and transparent.

2.1 The (ir)reducible interpretation ?

When formalizing law, the interpretation step is inescapable, and that is why we need lawyers to do it.

2.1.1 The necessary interpretation

The broad interpretation process. We discussed in first section how the languages themselves can be sometimes tough to interpret semantically, but there is a new difficulty that arises when lawyers have to interpret law : linguistics is not the only complex parameter they must take into account in their interpretation of the law.

"The crucial question for legal interpreters isn't "what do these words mean," but something broader: What law did this instrument make? How does it fit into the rest of the corpus juris? What do "the legal sources and authorities, taken all together, establish"?[416] Questions like these presuppose some particular system of law, and their answers depend on the other legal rules in place. Language will of course be an input to the process, but law begins and ends the inquiry."[417]

[416]FINNIS, John. Philosophy of law: Introduction. John M. Finnis, PHILOSOPHY OF law: COLLECTED ESSAYS, 2011, vol. 4, p. 11-32.

[417]BAUDE, William et SACHS, Stephen E. The law of interpretation. Harv. L. Rev., 2016, vol. 130, p. 1079.

So, according to Baude and Sachs in "The law of interpretation", there are many factors to take into account while interpreting the law, and besides the language itself, most of these factors are law-related : among those we would have the *ratio legis*, the reason of the law, i.e. the intention of who wrote that law ; we would have the context of the other legal texts, and the relative consistency of those with each other ; or we would have the exact entailments that the text legally does or dos not create.

Therefore, the interpretation of the law is a much broader process than a "simple" linguistic interpretation or understanding.

The double distinction. Kelsen, in his "Pure theory of law"[418], did establish the famous distinction between the "authentic" interpretation of the organs that apply the law, and the common interpretation of other actors.

I believe, as does Patterson in "Interpretation in law"[419], that another distinction should prevail : the one between understanding and interpreting. I also think that what we abusively call interpretation is often just the understanding of what the law is stating. Finding out what a norm is should not be qualified as interpretation unless there are conflicting multiple outcomes legally understandable.

"The need for interpretation arises when our conventional ways of understanding break down. This occurs, in law, when our use of the forms of legal argument is in some way rendered problematic and thrown in doubt."[420]

Therefore, a legal interpretation is only a way to arbitrate between various and legally valid understandings that are in dispute.

The ineluctable interpretation. We saw earlier that interpretation cannot be avoided sometimes on a linguistic level, it is even more true in law where the levels are overlapping. "law so conceived is deeply and thoroughly political. Lawyers and judges cannot avoid politics in the broad sense of political theory"[421].

In "law as interpretation", Dworkin exhibits that interpretation in law is general, and unavoidable in its practice. Even if our thesis focuses mainly

[418]KELSEN, Hans. Pure theory of law.

[419]PATTERSON, Dennis. Interpretation in law. San Diego L. Rev., 2005, vol. 42, p. 685.

[420]PATTERSON, Dennis. Interpretation in law. San Diego L. Rev., 2005, vol. 42, p. 685.

[421]DWORKIN, Ronald. law as interpretation. Critical Inquiry, 1982, vol. 9, no 1, p. 179-200.

on civil law systems, our discourse stays true for common law ones : "Interpretation as a technique of legal analysis is less familiar in the case of the common law, but not unfamiliar"[422].

But not only is interpretation necessary for lawyers, it might even be necessary for law itself. As some scholars argue, the interpretation is what activates the effectiveness of norms.

Grau, for example, in his book[423], considers that judges, even if they are authentic interprets of the law (in the Kelsenian sense[424]), do not create the norm but produce it through interpretation ; i.e. they transform the source of the norm, the legal text, into a new text, which would be the norm they use.

I wouldn't go this far, as I consider that norms already exist (see chapter II), but I agree that being applied (and interpreted) by a judge makes them effective.

But if the interpretation does have this much entailment legally, then we must be careful about who interprets the law.

2.1.2 The necessary lawyer-based interpretation

The legitimate interpreters. "Interpretation is a parasitic activity in legal practice"[425], but nonetheless a necessary and unavoidable one ; and since it is an important part of legal activity, we must ensure that it is done the best way possible.

The classical Kelsenian distinction between authentic and non-authentic interpreters of the law[426] seems too restrictive to me, as it is too focused on the consequences of the interpretation, not on the interpretation itself.

I would argue on that subject like Chevallier did in "Les interprètes du droit"[427] : we must differentiate according to the legitimacy of the interpreter. This legitimacy should be based on knowledge, science and expertise

[422]DWORKIN, Ronald. law as interpretation. Critical Inquiry, 1982, vol. 9, no 1, p. 179-200.

[423]EROS, GRAU. Pourquoi j'ai peur des juges: l'interprétation du droit et les principes juridiques. Editions Kimé, 2014.

[424]KELSEN, Hans. Pure theory of law. Univ of California Press, 1967.

[425]PATTERSON, Dennis. Interpretation in law. San Diego L. Rev., 2005, vol. 42, p. 685.

[426]KELSEN, Hans. Pure theory of law. Univ of California Press, 1967.

[427]CHEVALLIER, Jacques. Les interprètes du droit. La doctrine juridique, 1993, p. pp. 259-282.

in the legal domain. Chevallier does thus define the legitimacy negatively, by excluding non-lawyers, the administration, and "simple" legal practitioners.

Even though I think this last exclusion should be at least partially challenged depending on the law there is to interpret, I also argue for a legitimacy-based distinction, because the answer to "who should be trusted with interpreting the law ?" should be the same to "who understands law better ?", since we want the formalization to be faithful to the original text. Therefore, lawyers are the best suited for a law interpreting task, and consequently for operating a legal formalization.

Furthermore, if law is "deeply and thoroughly political"[428], then the legitimacy criteria arises even more as necessary, and we cannot let any common interpreter have an impact on such important matters.

Beyond the legitimacy need before interpreting, since some could argue that lawyers may not be the only legitimate interpreters when it comes to formalizing law, let us discuss another criteria that we consider indispensable.

The expertise criteria. As Patterson, Grau, Baude and Sachs argue in their respective works[429][430][431], interpretation consists most of the time in understanding, and this understanding does necessitate some level of legal expertise because of the multiple factors involved in the process.

Guidère adds in his article[432] that the interpretation is totally dependant on both the interpreter and his understanding("understanding" here is not intended by the cited authors as in our earlier distinction from interpretation).

It is therefore necessary for an interpreter to know the law, and to know it really well, before interpreting it. This criteria, added to the legitimacy one, allows us to define a framework in which we can safely interpret the law at the end of the formalizing process.

[428]DWORKIN, Ronald. law as interpretation. Critical Inquiry, 1982, vol. 9, no 1, p. 179-200.

[429]PATTERSON, Dennis. Interpretation in law. San Diego L. Rev., 2005, vol. 42, p. 685.

[430]EROS, GRAU. Pourquoi j'ai peur des juges: l'interprétation du droit et les principes juridiques. Editions Kimé, 2014.

[431]BAUDE, William et SACHS, Stephen E. The law of interpretation. Harv. L. Rev., 2016, vol. 130, p. 1079.

[432]GUIDÈRE, Mathieu. Introduction à la théorie analytique de la traduction et de l'interprétation. Babel, 2010, vol. 56, no 4, p. 299-312.

Indeed, since multiple norms could result from various interpretations[433] when necessary, this entails two things : we must trust the interpreter, and we must not automate the interpretation task but leave it to humans in complement the formalization process.

2.2 The need for a transparent formalization

When formalizing law, the process must be transparent, i.e. the process must be loyal, readable and explainable even after being done, in order to ensure its trustworthiness on many levels.

The authors of "Transparence et loyauté, deux motifs de la régulation des algorithmes"[434] indeed noted that the notion of algorithmic transparency must not be defined independently from the explainability of its process, and the loyalty of its results.

Let us now address in more detail why we need our formalization to meet this requirement.

2.2.1 The excluded "black box" reasoning

The black box[435] phenomenon is told of "a system that produces results without the user being able to see or understand how it works"[436].

We must avoid any process of legal formalization becoming a black box, i.e. making it unreadable by those who did not participate in it, and that for many reasons.

The understandability and clarity of legal norms is a necessity in order to guarantee the fairness of a justice system vis-à-vis its subjects.

Nemo censetur ignorare legem[437] is an old latin saying that is only relevant if the people have a chance at being able to comprehend the norms they are supposed to obey and know about.

[433]EROS, GRAU. Pourquoi j'ai peur des juges: l'interprétation du droit et les principes juridiques. Editions Kimé, 2014. p.44

[434]CHEVRET-CASTELLANI, Christine et LABELLE, Sarah. Transparence et loyauté, deux motifs de la régulation des algorithmes. Terminal. Technologie de l'information, culture & société, 2019, no 124.

[435]ROUSE, William B. et MORRIS, Nancy M. On looking into the black box: Prospects and limits in the search for mental models. Psychological bulletin, 1986, vol. 100, no 3, p. 349.

[436]Cambridge Advanced Learner's Dictionary & Thesaurus

[437]"Nobody is supposed to ignore the law" in latin

In France, for example, the "Conseil Constitutionnel" did rule in 1999[438] that pursuing the accessibility and the understandability of the law is an objective with constitutional value.

"This objective's ambition is precisely to impose a minimum level of understandability upon the legislator"[439]. I believe, since the formalization is a process that aims at more clarity and consistency in the law, that it should have the same requirement, and hence every formalization process should be accessible, transparent and understandable.

Another important element we must consider for the subjects of a legal system is the predictability criterion. Indeed, predictability is often a requirement for legislation, even sometimes a source of legislation ; to illustrate, we can cite Gaudemet-Tallon according to whom permissive rules' main purpose is the predictability of the legal regime[440].

Beauvais, in his article "Le droit à la prévisibilité en matière pénale dans la jurisprudence des cours européennes"[441], completes the requirements cycle by considering that the needed predictability of the law implies necessarily that the subjects of the law know it, and to be able to know it they must first have complete access to it, which implies physical access and understandibility of said law.

Overall, we see that the clarity of the law is a central requirement in a justice system. But if a formalization process is supposed to bring more clarity to it, the process itself must stay accessible and readable.

2.2.2 The benefits of a readable formalization

The formalization process itself must stay readable and transparent for many reasons that concerns three categories of people.

Lawyers. As we saw above, lawyers must be in charge of interpretation during the formalization process, and they preferably should be able to do the whole formalization without the help of logic experts (and our next section will tackle this problematic) ; but even if they do have the legitimacy to

[438]https://www.legifrance.gouv.fr/cons/id/CONSTEXT000017667970

[439]BENEZECH, Ludovic. L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence. Revue française de droit constitutionnel, 2020, no 3, p. 541-563.

[440]GAUDEMET-TALLON, Hélène. Le pluralisme en droit international privé: richesses et faiblesses (le funambule et l'arc-en-ciel): cours général. Martinus Nijhoff, 2006.

[441]BEAUVAIS, Pascal. Le droit à la prévisibilité en matière pénale dans la jurisprudence des cours européennes. Archives de politique criminelle, 2007, no 1, p. 3-18.

interpret law, they must be controlled in doing so by the other lawyers, who should be able thanks to a transparent process to discuss and challenge any interpretation given.

Even if a formalization process gets agreed upon, this does not mean that we should not control the applications of it by lawyers or practitioners. On the contrary, any application must be challengeable, defensible or controllable at any step of the process.

Legislators. As Beauvais states in his article "Le droit à la prévisibilité en matière pénale dans la jurisprudence des cours européennes" : "The predictability requirement would not be complete unless it also applied to temporal modifications of the law."[442].

We must therefore take temporality into account, and when we do, we realize that since laws are not eternal, we certainly have to make any process of formalization reversible or checkable by legislators, either to make their work finer or easier (since law quantitatively boomed in the last decade, with an equivalent loss of quality, making legislators and lawyers works even harder[443])

Citizens. Transparency is a key-element for good governance in a democracy.

According to Johnston in "Good governance: Rule of law, transparency, and accountability"[444], explains how transparency is a goal and a key-challenge for good governance as it is allows accountability to be brought easily on the table.

This idea is also supported by the authors of "Democracy and transparency"[445] and "What is transparency?"[446].

As a conclusion, we could say that having a transparent formalization

[442]BEAUVAIS, Pascal. Le droit à la prévisibilité en matière pénale dans la jurisprudence des cours européennes. Archives de politique criminelle, 2007, no 1, p. 3-18.

[443]URVOAS, Jean-Jacques. Quantité, qualité, efficacité des lois, quel bilan?. Revue française de droit constitutionnel, 2018, no 4, p. 905-917.

[444]JOHNSTON, Michael. Good governance: Rule of law, transparency, and accountability. New York: United Nations Public Administration Network, 2006, p. 1-32.

[445]HOLLYER, James R., ROSENDORFF, B. Peter, et VREELAND, James Raymond. Democracy and transparency. The Journal of Politics, 2011, vol. 73, no 4, p. 1191-1205.

[446]BALL, Carolyn. What is transparency?. Public Integrity, 2009, vol. 11, no 4, p. 293-308.

process done by expert lawyers can act as a guarantee of both efficiency and democracy within a legal system.

3 An intermediary grammar : Logical Legal Templates (LLT)

The formalization of legal texts is an important step in order to be able to improve law's consistency, or to automate several legal tasks such as reasoning or answering legal questions.

In an early published article[447], T. Libal and I present a new approach for legal texts' formalization, based on a two-step method.

The latter meets several requirements that existing approaches could not satisfy.

We will hereby use an excerpt of the GDPR as an example of this method, translate it into a logical formula, and then translate it back in order to evaluate, successfully, the accuracy of the process.

3.1 A need for a new approach

We will first see what challenge we are facing, then describe what approaches already exist, before explaining why we think it is necessary to have a new one.

3.1.1 The main challenge

A main aspect of the law is to govern the normative behavior of the population. As such, it is of high interest that the understanding of the law is not only available to a selected number of people, such as lawyers, but is widely accessible. At the same time, we would like its application to be fair and objective.

An interesting approach for attaining both these goals is by making the law understandable to computers. The ability of a computer to reason over the law will allow it both to explain it to people, via for example expert systems, and to reason over it in an objective way.

One way of achieving the above task is by making sure legislatures are creating human understandable legislations. There is some active work on this

[447] ABIDI, Abdelhamid et LIBAL, Tomer. A validation process for a legal formalization method. on Methodologies for Translating Legal Norms into Formal Representations (LN2FR 2022), 2022, p. 36.

Some parts of this last section may reuse elements of this article

direction [448] but various aspects seem to prevent this from actually happening. One of the main ones is the fact that legislatures are not logicians and therefore would find it hard to give a formal and unambiguous interpretation of their laws.

Another way of achieving the task is by translating existing laws into a formal representation, a process which is called legal formalization. More specifically, a legal formalization is the process of translating legal texts into a formal representation, while preserving their legal meaning.

Still, one of the biggest problems of legal formalization is the ability of legal experts to participate in the process and their ability to confirm the quality and faithfulness of the formalization.

3.1.2 The state of the art

The need for legal experts when formalizing legal text stems from the need to have a translation which preserves the legal meaning of the text. Since the process requires two different formats - the legal text on the one side and the formal representation on the other - it traditionally required two domain experts, one for each format. Among the various approaches to this problem, one should mention Bartolini et al approach [449], where an agile methodology is introduced which involves both domain experts via short sprints. The goal of this approach is to allow iterations between the two experts and in this way, to increase the confidence in the translation.

Due to the importance of the legal formalization phase, there are various tools and formats designed to facilitate it. These existing tools fall within two categories. The first category contains those approaches which aim to be as close as possible to the legal text, the language used there is normally very general and less structured, which results in translations which are not easily computable. Since the main goal of legal formalization is to allow computers to reason over the text, this is undesirable. There is the advantage though that the correctness and faithfulness of the translation are easier to validate. The second category contains those approaches which aim to be computable. As we will see, these approaches require a translation which is much farther than the original text and therefore, its correctness is harder to validate.

[448] Publications Office of the European Union (2021). AKN4EU Guidelines and modeling. https://op.europa.eu/en/web/eu-vocabularies/akn4eu.

[449] BARTOLINI, Cesare, LENZINI, Gabriele, et SANTOS, Cristiana. An agile approach to validate a formal representation of the GDPR. In : JSAI International Symposium on Artificial Intelligence. Cham : Springer International Publishing, 2018. p. 160-176.

Indeed, and as mentioned earlier, we could classify them in two categories : syntax-based languages and semantics-based languages. In the first class we would have Akoma-Ntoso and DAPRECO, in the second we would have Logical English, PENG^{ASP}, Catala...

The problem here is that all these approaches are either too close to the legal texts, or too close to the machine, with insufficient bridges during the formalization process.

If we take Akoma-Ntoso for example, even if it aims at direct capture of legal information, it is not fit for direct use due to its XML format, and lawyers would struggle to use it.

We could cite the paper "Applying the Akoma Ntoso XML Schema to Japanese Legislation"[450], which enlights some restrictions inherent to Akoma-Ntoso, resulting in "unnatural annotations". In the conclusion of this paper, we can read that "an underlying problem of the structural ambiguity of Akoma Ntoso, [is] that the flexibility of annotations allows the schema to validate unintended structuralisation to the same document.".

So Akoma-Ntoso suffers from a lack of logical structure through the process, and a lack of direct usability by lawyers who would not understand XML easily. There is no clear way of how to computize it, since its format is too general.

Regarding DAPRECO, this language is trying to be both close to the law and formal, but is actually hardly accessible for lawyers, as it carries the XML format weaknesses, and is not really computable. This last issue is rather serious when it comes to formalization languages as computation can be used as a form to validate the correctness of the formalization. As argued in "A meta-level annotation language for legal texts" [451], DAPRECO contains various errors, both syntactical and semantical, which could be avoided if the knowledge base was computable.

As for the rest of the languages, they focus on formal semantics and computability, but by doing so they require a direct translation from the legal texts to the language, which is often close to programming languages. It is also here a problem since it is difficult to maintain and validate, or even

[450]GEN, Kawachi, AKIRA, Nagai, MAKOTO, Nakamura, et al. Applying the Akoma Ntoso XML schema to Japanese legislation. Journal of law, Information and Science, 2016, vol. 24, no 2, p. 49-70.

[451]LIBAL, Tomer. A meta-level annotation language for legal texts. In : International Conference on logic and Argumentation. Cham : Springer International Publishing, 2020. p. 131-150.

to have faith in the translation as it would need at least experts that are competent in both programming and law.

If we take Catala as an example, it is locally not a problem since it focuses on tax law, which is among the few branches of law that can be more easily translated into a programming language (with implications and arithmetic), even if sometimes there is a need of legal interpretation by professionals ; but the problem is that we would struggle to find anyone who can do it alone without both backgrounds, since lawyers would hardly understand programming semantics, and computer scientists would lack legitimacy in interpreting themselves the law in order to directly translate it into code.

A last approach that seems interesting for us is to formalize through annotation. This approach can be found in Tomer Libal's paper "A meta-level annotation language for legal texts"[452], and is based on the idea that direct annotation of legal texts using deontic operators can be too complex, so we should use "macros" to annotate legal texts on a meta-level.

3.1.3 Our contribution

Before talking about what we will do, let us say a few words about some of the guidelines we will base our approach on :

As we just saw, many attempts at formalizing law already exist, ("Logical english"[453], Akoma-Ntoso [454], DAPRECO [455], PENG^{ASP} [456], Catala [457] etc.), but none is actually fitting our expectations, as we believe that a program-

[452]LIBAL, Tomer. A meta-level annotation language for legal texts. In : logic and Argumentation: Third International Conference, CLAR 2020, Hangzhou, China, April 6–9, 2020, Proceedings 3. Springer International Publishing, 2020. p. 131-150.

[453]KOWALSKI, Robert. Logical english. Proceedings of logic and Practice of Programming (LPOP), 2020.

[454]PALMIRANI, Monica et VITALI, Fabio. Akoma-Ntoso for legal documents. In : Legislative XML for the semantic web: Principles, models, standards for document management. Dordrecht : Springer Netherlands, 2011. p. 75-100.

[455]ROBALDO, Livio, BARTOLINI, Cesare, PALMIRANI, Monica, et al. Formalizing GDPR provisions in reified I/O logic: the DAPRECO knowledge base. Journal of logic, Language and Information, 2020, vol. 29, no 4, p. 401-449.

[456]SCHWITTER, Rolf. The Grammar of PENG ASP Explained. In : Proceedings of the Seventh International Workshop on Controlled natural language (CNL 2020/21). 2021.

[457]MERIGOUX, Denis, CHATAING, Nicolas, et PROTZENKO, Jonathan. Catala: a programming language for the law. Proceedings of the ACM on Programming Languages, 2021, vol. 5, no ICFP, p. 1-29.

ming language would not do as much as a formal annotation language[458].

We want our formalization to be both high-level and formal, in two-steps, compatible and faithful, easily understandable and computable.

Indeed, we need it high-level enough for lawyers to use it without any scientific background, we need it also bidirectionally isomorphic so we can both easily check it and update it. And these previous formalizations do not really meet all the requirements we want them to.

Among the requirements we think are essential, most of them are not new :

In "The Grammar of PENG^{ASP} Explained"[459], the author comes up with requirements for a controlled language : "a controlled language specification should be translatable into an executable answer set program", "the grammar for this language should be highly configurable for different application scenarios", "the grammar should be bi-directional".

We could also cite the overlapping criteria of Routen and Bench-Capon in "Hierarchical formalizations"[460] : "a formalization should not only (1) be a faithful representation of what is expressed by the legislation; (2) be computationally adequate, i.e. should permit us to make all relevant derivations by machine; but also (3) be easy to validate and maintain".

We claim that these guidelines are a good start to challenge a formalization methodology, and it appears existing approaches do not meeting all of them.

If, separately, all of our criteria already exist in the literature, our contribution is to show a concrete example of how to create a methodology that fullfills the conjunction of these requirements.

Indeed, let us now develop why these criteria are important, and how our approach is relevant in regard to them.

Our approach satisfies the previous demands as we will see below, since it is usable directly by the lawyers without any programming background, it is

[458]LIBAL, Tomer. A meta-level annotation language for legal texts. In : logic and Argumentation: Third International Conference, CLAR 2020, Hangzhou, China, April 6–9, 2020, Proceedings 3. Springer International Publishing, 2020. p. 131-150.

[459]SCHWITTER, Rolf. The Grammar of PENG ASP Explained. In : Proceedings of the Seventh International Workshop on Controlled natural language (CNL 2020/21). 2021.

[460]ROUTEN, Tom et BENCH-CAPON, Trevor. Hierarchical formalizations. International Journal of Man-Machine Studies, 1991, vol. 35, no 1, p. 69-93.

based on an isomorphic (and bi-directional) methodology, it is computable, and allows a faithful translation of the legal texts' structure and content, finally the isomorphic characteristic makes it easy to validate and maintain.

Firstly, we want law formalization to become an accessible tool for lawyers, and we cannot expect from them to be experts in pure logic, so we must not rely on direct translation from legal text to logical formula as a methodology.

Secondly, it allows us to be more accurate through this intermediate process. Since there is a big gap between a legal text and its computable formalization, it would be too ambitious to do it all in one go, like many other attempts. The smaller the steps are, the easier it is to check and evaluate them, so with the two-steps translation, we can check the correctness of each one and then compose them at the end.

An analogy to this technique is functions composition in mathematics where instead of directly creating a unique function f , you create two functions g and h , and compose them afterwards, so that $h \circ g = f$. Or, for a more simple analogy, it would be like translating French sign language into French then French into English instead of translating directly French sign language into English.

Another key-point will be to create an isomorphism between the legal text and its logical formalization. An isomorphism in mathematics a bijective function that preserves structure from a set to another, back and forth ; by saying we want an isomorphism for our formalization, we mean that we want to be able to translate legal texts into a formal language while preserving the logical structure of it, and being able to go back and forth between both languages without losing any meaning or information in the process.

The relevance of using and creating isomorphisms for knowledge based systems has already been highlighted in the past, for example by T.J.M.Bench-Capon & F.P.Coenen in 1992, in their paper "Isomorphism and Legal Knowledge Based Systems"[461]. Their relevance still holds today as it has been reconfirmed almost 20 years later by Trevor J. M. Bench-Capon and Thomas F. Gordon. in "Isomorphism and argumentation"[462], in 2009.

Isomorphisms are all about creating "a well defined correspondence of the knowledge base to the source texts". But the authors then said that the

[461]BENCH-CAPON, Trevor JM et COENEN, Frans P. Isomorphism and legal knowledge based systems. Artificial Intelligence and law, 1992, vol. 1, no 1, p. 65-86.

[462]BENCH-CAPON, Trevor et GORDON, Thomas F. Isomorphism and argumentation. In : Proceedings of the 12th international conference on artificial intelligence and law. 2009. p. 11-20.

correspondence could not be total since "Ideally there would be a one to one correspondence between the knowledge base items and the source material items, but practical reasons may necessitate deviation from this."

We are trying, with our approach, to go the closest possible to an ideal isomorphism by separating the process into two steps : creating LLTs from the legal texts, then creating logical formulae from the LLTs. Indeed, with an intermediate step, it will be easier to make the correspondence between the items.

The need for an intermediate step for a legal formalization has already been considered, for example by the authors of "An Interdisciplinary Methodology to Validate Formal Representations of Legal Text Applied to the GDPR"[463], saying that "A key element in the methodology is an intermediate representation".

They will then introduce the requirement that this intermediate representation has to be "supposedly human-readable". But this would no longer be necessary if the several levels were isomorphic to each other, the requirement would then be transferable to the higher existing level, ie the legal text itself (to go on with the function metaphor, if $f = h \circ g$ is isomorphic, you no longer need to check manually if $Im(g) = D_h$).

We aim at bridging the gap between the two categories of languages already existing (as seen in the state of the art), and proposing, as a first contribution to have a language close to the original text, therefore allowing an easier way to validate. At the same time, the chosen language enjoys properties which make it easily computable (second contribution). This is achieved by defining two target languages, one close to the original text and one denoted in first-order logic, which is a highly computable format. We then define a translation between the two levels which allows the automatic translation from the high level to the technical one. Moreover, we require that this translation is isomorphic, which allows the translation back from the formal to the legal levels (third contribution), thus allowing the legal expert to validate the result. Our claim is that this approach enjoys the good properties of both categories, without any of the undesired ones.

[463]BARTOLINI, Cesare, LENZINI, Gabriele, et SANTOS, Cristiana. An interdisciplinary methodology to validate formal representations of legal text applied to the GDPR. In : Twelfth International Workshop on Juris-informatics (JURISIN 2018). 2018.

3.1.4 The illustration examples

In order to make our following demonstration more accessible, we will be using two alternative examples to show how our method applies : one simple fictional norm, and the more complex article 13 (1.a, 1.b and 4) of the GDPR.

Example 1. *Let us take for example a simple legal regulation (fictional but close enough to what a real text could be, as it is inspired by french car lights regulation) :*

"It is obligatory to turn on a car's headlights while driving unless it is daytime and the visibility is sufficient".

As for the GDPR[464], its article 13 is the main legal reference with regard to the transparency of data collection, but it is also a complex text regarding its structure and the way it describes this obligation. It is, therefore, a perfect use case for our methodology : both a useful and challenging article for a formalization.

The article, though, has quite a lot of repetition between and within its paragraphs. We will, therefore, focus on the paragraphs 1 and 4. Since paragraph 1 also contains an enumeration, we only consider the first two segments a and b.

Article 13. 1. Where personal data relating to a data subject are collected from the data subject, the controller shall, at the time when personal data are obtained, provide the data subject with all of the following information:

(a) the identity and the contact details of the controller and, where applicable, of the controller's representative;

(b) the contact details of the data protection officer, where applicable;

4. Paragraphs 1, 2 and 3 shall not apply where and insofar as the data subject already has the information.

3.2 A high-level formal language : LLTs

LLT stands for "Legal Logical Template". It is a formal grammar, that helps describing the structure of a legal text.

Informally, LLTs are a formal representation of legal concepts and capture the way a legal expert interprets the law.

[464]European Parliament regulation 2016/679 (2016). https://eur-lex.europa.eu/eli/reg/2016/679/oj

They allow us to dodge the necessity to translate directly legal texts into pure logical formulae, and give us an intermediate step in the process ; which is fortunate as we saw earlier.

Let us first note that the LLTs we are presenting were created for a specific legal context, and are not necessarily universal, it is the method that matters more. And we do not claim that we objectify legal interpretation, since the interpretation of a legal text cannot be absolute, but we do give lawyers all the necessary tools to formalize the law themselves, the way they think is the best.

3.2.1 The syntax

In order to formally denote the LLTs, we will use a Context Free Grammar [465] (CFG). We chose to use CFG for our formalization for its many advantages : it goes without saying, but it is a grammar, which is close enough to "natural" languages ; but it moreover is a formal grammar, and a popular one that can be easily linked to a more formal step with precise semantics. It is, therefore, a great candidate for the intermediate step we needed, between the natural language of the law, and the computable language of logical formulae.

Using such grammar is also very useful here because it allows us to describe law in a very clear and structured way. Indeed, we can capture the very structure of the texts and keep an equivalent structure during the translation, which is not the case of many previous attempts that formalize law part by part.

Let us just point out as a remark that we use some symbols from formal regular grammars, like "?" to indicate that a part is optional in a sentence.

Definition 6. *CFG Formally, a CFG G is a 4-tuple (V, Σ, R, S), where V is a set of N-terminal symbols, Σ is a set of terminal symbols, R is a set of production rules over $V \times (V \cup \Sigma)$ and S is the start variable.*

Our goal here will therefore be to create two isomorphic functions, one between the grammar and the law, and another between the grammar and the logic.

To reach it, we need the grammar's syntax to be both abstract enough to include as much legal situations as possible, and precise enough to avoid

[465] AHO, Alfred V. et ULLMAN, Jeffrey D. Translations on a context free grammar. In : Proceedings of the first annual ACM symposium on Theory of computing. 1969. p. 93-112.

any confusion regarding its relevance when used.

We focus on a formal language close to that of legal texts, as well as on various desired properties of such a language. We claim that, since the language is close to the original legal text, lawyers would find it more intuitive and easy to work with. Nevertheless, allowing a lawyer to actually work with this language would require a user-friendly interface, which is not our focus. An example of an interface for such a language is described in [466].

Furthermore, in order to help our purpose, we claim that we can always add even higher level lawyer-friendly interfaces supporting both the accessibility, the annotations and the validation of our method : the LegAi tool, presented in "The LegAi Editor: An Annotation Tool for the Construction of Legal Knowledge Bases",[467] is a good example of what higher lever interfaces can be.

Definition 7. *LLTs The following is the CFG of the LLTs. The terminals set Σ contains ATOM and all lower case words. The starting symbol S is LLT.*

LLT :=	*EXCEPTION | REF | IF-THEN | OBLIGATION |*
EXCEPTION :=	*{LLT | REF} + {NEG-CONTEXT | CONTEXT} + LLT?*
REF :=	*LABEL + LLT*
IF-THEN :=	*CONTEXT + LLT*
CONTEXT :=	*CUMUL-LIST | N-CUMUL-LIST | ATOM_OR_OBLIGATION*
ATOM_OR_OBLIGATION :=	*ATOM | OBLIGATION*
CUMUL-LIST :=	*{ATOM_OR_OBLIGATION + ATOM_OR_OBLIGATION} | {ATOM_OR_OBLIGATION + CUMUL-LIST}*
N-CUMUL-LIST :=	*{ATOM_OR_OBLIGATION + ATOM_OR_OBLIGATION} | {ATOM_OR_OBLIGATION + N-CUMUL-LIST}*
LABEL :=	*ATOM*
OBLIGATION :=	*LLT*
NEG-CONTEXT :=	*CONTEXT*

Let us explain how we decided to build the syntax :

ATOM := group of words

Let's note that we call an ATOM the smallest portion of a legal text that cannot be finer formalized without interfering with its legal meaning.

LABEL := ATOM

[466]LIBAL, Tomer et STEEN, Alexander. The NAI suite–drafting and reasoning over legal texts. In : Legal Knowledge and Information Systems. IOS Press, 2019. p. 243-246.

[467]LIBAL, Tomer. The legai editor: a tool for the construction of legal knowledge bases. In : Legal Knowledge and Information Systems. IOS Press, 2022. p. 286-289.

REF := (LABEL ; LLT)

A LABEL is a numerotation that refers to a portion of the legal text, for example “paragraph (1)”or “point (f)”. We need labels to capture texts that influence each other, and REF to refer to the corresponding LLT, like pointers ; for example in the article 13.4 of the GDPR, establishing exceptions for article 13’s previous paragraphs.

OBLIGATION:= LLT is obligatory

A general obligation that could concern either a precise, conditional, multiple or even abstract obligation.

As a remark, we can say that in a future work we could add to the obligation some parameters, like for whom or when is it obligatory.

N-CUMUL-LIST := {ATOM_OR_OBLIGATION or ATOM_OR_OBLIGATION} | {ATOM_OR_OBLIGATION or N-CUMUL-LIST}

CUMUL-LIST := {ATOM_OR_OBLIGATION and ATOM_OR_OBLIGATION} | {ATOM_OR_OBLIGATION and CUMUL-LIST}

These correspond to the classic disjunction and conjunction that allow us to group the legal elements we want to address.

They are necessary to capture precisely for example all the different kinds of multiple conditions we can find in law.

CONTEXT := CUMUL-LIST | N-CUMUL-LIST | ATOM_OR_OBLIGATION

A CONTEXT can be any list of elements that can become for example a group of disjunctive conditions or a list of cumulative obligations when combined with other LLTs.

IF-THEN := if CONTEXT then LLT

This is just the usual implication, that is often used to link a cause to its legal consequence.

EXCEPTION := do not LLT/REF if CONTEXT | do LLT if NEG-CONTEXT | do LLT/REF and LLT except if CONTEXT then do only LLT/REF

Finally, an EXCEPTION is a specific sub-case in which a certain norm does not apply, although it should in principle. The excluded-part (the LLT?) is optional as sometimes only a part of the legal text is left apart when the exception context arises.

3.2.2 The semantics

Now that we have the syntax of the grammar, we need matching semantics to create the corresponding logical formulae afterwards.

Indeed, our initial goal was to establish an isomorphic correspondence between legal texts and logical formulae, in two steps ; now that we have LLTs that are the intermediate step, let's build some corresponding formal semantics.

Since both the LLTs and the formulae are formal languages, the isomorphism will be obtained easily by construction. As for the isomorphism between the LLTs and the legal texts, we will see in a coming section that it is bi-directional.

We need a well understood formal language with clear understanding in order to be able to reason over it afterwards, and let's not forget that computability was an important criterion for us. That's why we need to go further by building this.

These semantics are based on the First Order logic, to which we add the classic deontic Obligation operator, and a second negation "$\sim$ "the negation as failure (the first order negation is noted "-").

We could also use a simple arrow "$\rightarrow$ "to show a meta relation between two legal texts that influence each other (for example when the key-point of a text is redefined by another one).

The adding of the obligation operator is essential to capture every deontic prescription. And it is sufficient for that because every modality can be attained by adding a negation or two : if A is obligatory, then A is not optional, (not A) is prohibited and (not A) is not permitted.

It is not the first time a deontic operator is added to a FOL language as can be seen in [468].

[468]LOKHORST, Gert-Jan C. Reasoning about actions and obligations in first-order logic.

As for the new negation added, the negation as failure [469], it is very useful for the exceptions or the conditioned modalities, that are frequent in law. Indeed, the negation as failure (NAF) consists in deriving not A if you cannot derive A, and it is important to consider that by default, exceptions do not apply in law, unless derivable/provable. The most general case in law is generally the default one.

For example, if A is obligatory, and B is a special context where A is exceptionally optional, then we can say that ∼B ⇒ Ob(A).

Let's note, though it is not our scope here, that our semantics are computable, thanks to existing methods that already handle First Order logic, First Order Modal logics and the addition of Negation-as-failure ; various theorem provers like SPINdle[470] can handle our semantics, finalizing the step-by-step path from law to code. We can already automate trees creation as we will see hereunder.

Definition 8. *Semantics We define* $\phi : \{LLTs\} \rightarrow \{FOL \vee \{\sim\} \vee \{Ob\}\}$ *, associating to each LLT its semantics :*

ϕ *(EXCEPTION) :=*	$\sim\phi$*(NEG-CONTEXT)* $\Rightarrow$ ϕ*(LLT/REF) ;*
	$\sim\phi$*(CONTEXT)* $\Rightarrow$ ϕ*(LLT/REF)* $\wedge$ $\neg\phi$*(LLT)*
ϕ*(REF) :=*	ϕ*(LABEL | LLT)*
ϕ*(IF-THEN) :=*	ϕ*(CONTEXT)* $\Rightarrow$ ϕ*(LLT)*
ϕ*(CONTEXT) :=*	ϕ*(CUMUL-LIST) ;* ϕ*(N-CUMUL-LIST) ;* ϕ*(ATOM_OR_OBLIGATION)*
ϕ*(ATOM_OR_OBLIGATION) :=*	ϕ*(ATOM) ;* ϕ*(OBLIGATION)*
ϕ*(CUMUL-LIST) :=*	ϕ*(ATOM_OR_OBLIGATION)* $\wedge$ ϕ*(ATOM_OR_OBLIGATION) ;*
	ϕ*(ATOM_OR_OBLIGATION)* $\wedge$ ϕ*(CUMUL-LIST)*
ϕ*(N-CUMUL-LIST) :=*	ϕ*(ATOM_OR_OBLIGATION)* $\vee$ ϕ*(ATOM_OR_OBLIGATION) ;*
	ϕ*(ATOM_OR_OBLIGATION)* $\vee$ ϕ*(N-CUMUL-LIST)*
ϕ*(LABEL) :=*	ϕ*(ATOM)*
ϕ*(OBLIGATION) :=*	*Ob*(ϕ*(LLT))*
ϕ*(NEG-CONTEXT) :=*	-ϕ*(CONTEXT)*

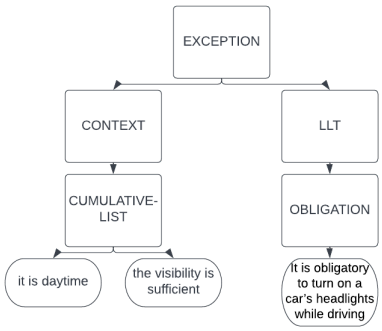
3.2.3 The trees

The methodology to apply the LLTs to the legal texts is to build a syntax tree, level by level, beginning with the treetop as the main LLT of the text, and then recursively replace the leaves by their LLTs if they exist, until there are only ATOMs left.

Studia Logica, 1996, vol. 57, no 1, p. 221-237.

[469]CLARK, Keith L. Negation as failure. logic and Databases. logic and Databases, Plenum, 1978.

[470]LAM, Ho-Pun et GOVERNATORI, Guido. The making of SPINdle. In : International Workshop on Rules and Rule Markup Languages for the Semantic Web. Berlin, Heidelberg : Springer Berlin Heidelberg, 2009. p. 315-322.



Actually, Rolf Schwitter, in the paper "The Grammar of PENG^{ASP} Explained"[471], recommends the use of syntax trees as both a strategy, and a way of checking if the requirements are met.

Here is the result if we apply the previous LLTs to both our simple fictional example and to the articles 13.1.a&b and 13.4 of the GDPR : (see figures below).

Let us note that we do not claim it is the only way to formalize this example or this article, but we do claim it is one of the ways lawyers would, since the interpretation of a legal text is not absolute. The objective is to give lawyers all the necessary tools to, in the end, formalize the law themselves the way they think is the best.

3.2.4 The formulae

Now that we have both the syntax trees and the semantics corresponding to each syntax, we can obtain the logical formulae of the legal texts.

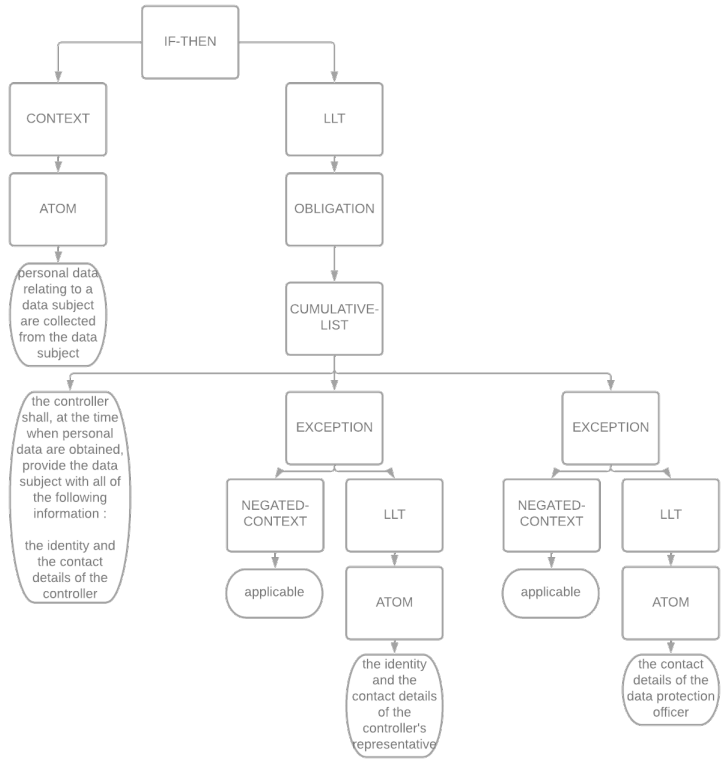
In order to do this, we just have to translate the syntax tree level by level, starting from the top.

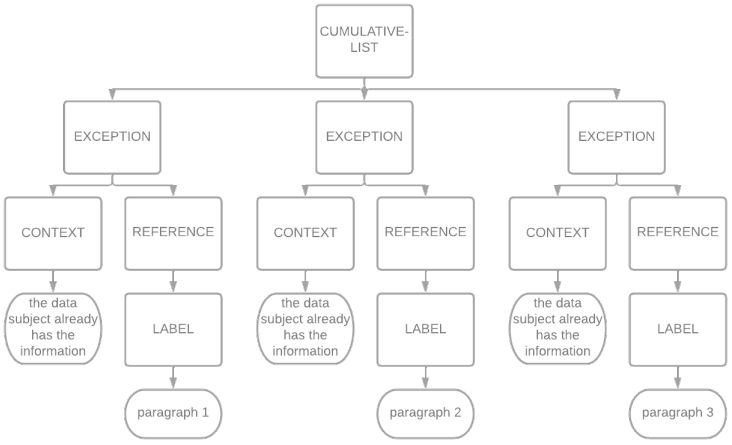
Let's see what we obtain first on our simple example :

Example 2. *($\sim$(it is daytime$\wedge$the visibility is sufficient) $\Rightarrow$ Ob(it is obligatory to turn on a car's headlights while driving)).*

And now here is the result for the GDPR article :

[471]SCHWITTER, Rolf. The Grammar of PENG ASP Explained. In : Proceedings of the Seventh International Workshop on Controlled natural language (CNL 2020/21). 2021.


IF-THEN
CONTEXT
ATOM
personal data relating to a data subject are collected from the data subject
LLT
OBLIGATION
CUMULATIVE-LIST
the controller shall, at the time when personal data are obtained, provide the data subject with all of the following information :
the identity and the contact details of the controller
EXCEPTION
NEGATED-CONTEXT
applicable
LLT
ATOM
the identity and the contact details of the controller's representative
EXCEPTION
NEGATED-CONTEXT
applicable
LLT
ATOM
the contact details of the data protection officer


CUMULATIVE-LIST
EXCEPTION
CONTEXT
the data subject already has the information
REFERENCE
LABEL
paragraph 1
EXCEPTION
CONTEXT
the data subject already has the information
REFERENCE
LABEL
paragraph 2
EXCEPTION
CONTEXT
the data subject already has the information
REFERENCE
LABEL
paragraph 3

13.1.a&b : (personal data relating to a data subject are collected from the data subject) ⇒ Ob((the controller shall, at the time when personal data are obtained, provide the data subject with all of the following information : the identity and the contact details of the controller) ∧ (∼-(applicable) ⇒ (the identity and the contact details of the controller's representative)) ∧ (∼-(applicable) ⇒ (the contact details of the data protection officer)))

13.4 : (∼(the data subject already has the information) ⇒ (paragraph 1)) ∧ (∼(the data subject already has the information) ⇒ (paragraph 2)) ∧ (∼(the data subject already has the information) ⇒ (paragraph 3))

3.3 A double isomorphic bridge

An important property of legal formalization is the ability of the legal expert to validate its quality [472].

Let us demonstrate how this can be obtained when using LLTs, thanks to the bidirectionality of the bridge we are building.

3.3.1 The functional evaluation

We said earlier that one of the interests of LLTs is that we can evaluate them more accurately thanks to the intermediate step. The evaluation we use is a functional one, consisting of a reverse translation of LLTs. If we automate this reverse translation, we just have to check then if the new legal text has the same meaning and the same legal substance. And the closer the new text is, the finer is the LLTs system.

The goal, as mentioned before, is to create isomorphisms, but because of many languages' synonyms, it cannot be a perfect one. We are trying to reach an isomorphism modulo synonyms.

We could cite T.J.M.Bench-Capon & F.P.Coenen's paper [473], in which they mention several advantages of isomorphism-based knowledge representations, among which is "verification and validation". Indeed, the ability to validate a formalization is precious, since it allows us to check not only the result, but also the process itself. And it definitely is an advantage of

[472]LIBAL, Tomer et NOVOTNÁ, Tereza. Towards transparent legal formalization. In : International Workshop on Explainable, Transparent Autonomous Agents and Multi-Agent Systems. Cham : Springer International Publishing, 2021. p. 296-313.

[473]BENCH-CAPON, Trevor JM et COENEN, Frans P. Isomorphism and legal knowledge based systems. Artificial Intelligence and law, 1992, vol. 1, no 1, p. 65-86.

isomorphisms because we can therefore reverse the process at any moment for verification.

With the reverse translations, we are thus going even closer to the creation of an "ideal" isomorphism, by making it bijective.

Indeed, in the paper "The Grammar of PENG^{ASP} Explained"[474], Schwitter supports this idea, and uses the bidirectionality of the grammar as an argument of its executability, unlike the other attempts of controlled languages back then. Bidirectionality is even one of the requirements the author claims to be unavoidable for a controlled language to meet.

3.3.2 The method

Definition 9. *Reverse translation In order to translate the LLTs back, we must think of a standard translation regardless of the LLT's terminals. Here is an attempt of doing so :*

EXCEPTION :=	*LLT/REF and LLT except if NEG-CONTEXT then only LLT/REF applies | LLT/REF does not apply except if CONTEXT*
IF-THEN :=	*if CONTEXT then LLT*
CUMUL-LIST :=	*ATOM_ OR_ OBLIGATION and ATOM_ OR_ OBLIGATION | ATOM_ OR_ OBLIGATION and CUMUL-LIST*
N-CUMUL-LIST :=	*ATOM_ OR_ OBLIGATION or ATOM_ OR_ OBLIGATION | ATOM_ OR_ OBLIGATION or N-CUMUL-LIST*
OBLIGATION :=	*it is obligatory that LLT*
NEG-CONTEXT :=	*not CONTEXT*

Let us note as a remark that we consider that "not not A" is equivalent to A, so we can shorten heavy sentences, for example when we negate a NEG-CONTEXT in an EXCEPTION rule. We can also point out that the combinations of the EXCEPTION are not exhaustive to lighten the definition, since we can for example have a NEG-CONTEXT without any optional excluded part. As for the first translation to LLTs, the reverse one will be recursive and begin at the treetop. We take the syntax tree of the legal text, and replace the main LLT by it's translation, and then recursively replace the sub-LLTs if found by their translations, and finish with the atoms.

3.3.3 The result

Now that we have a method for validation via reverse translation, let's quickly see how it applies to our simple example before we check the syntax tree we created for the articles 13.1.a&b and 13.4 of the GDPR :

Example 3. *:*

[474]SCHWITTER, Rolf. The Grammar of PENG ASP Explained. In : Proceedings of the Seventh International Workshop on Controlled natural language (CNL 2020/21). 2021.

"Is is obligatory to turn on a car's headlights while driving except if it is daytime and the visibility is sufficient."

Now let's compare it with the original text :

"It is obligatory to turn on a car's headlights while driving unless it is daytime and the visibility is sufficient."

We can observe that the reverse translation is effective (of course modulo synonyms). Now, we shall present the reverse translation of the articles 13.1.a&b and 13.4 of the GDPR, which also seems effective to us :

"If personal data relating to a data subject are collected from the data subject,

then it is obligatory that the controller shall, at the time when personal data are obtained, provide the data subject with all of the following information : the identity and the contact details of the controller

and the identity and the contact details of the controller's representative except if not applicable

and the contact details of the data protection officer except if not applicable.

Paragraph 1 does not apply if the data subject already has the information

and paragraph 2 does not apply if the data subject already has the information

and paragraph 3 does not apply if the data subject already has the information."

To conclude this section, we can say that we have introduced two languages. One of them is novel and can be used by legal experts. The second can be used for computation.

We have introduced an algorithm to automatically translate between the two languages. In addition, we have given an algorithm for the automatic translation of one language back into legal text. The resulted approach (helped by higher-level interfaces like the LegAi tool[475]) allows a legal expert to use the methodology and validate the quality of the translation without the need for a second domain expert (a programmer or a logician).

The formalization can be used for computation, and we argue that our multi-layered approach solves some of the issues with the state-of-the-art.

[475]LIBAL, Tomer. The legai editor: a tool for the construction of legal knowledge bases. In : Legal Knowledge and Information Systems. IOS Press, 2022. p. 286-289.

Chapter V

The development of tools for legal practice

1 Annotation tools for automated reasoning

In our last chapter, we advocated for a legal formalization strategy based on annotation.

Indeed, as we saw, it allows lawyers to do the work themselves without a logical background, provided that there is a high-level interface for them to operate. Here come the annotation tools.

1.1 Instruments for a better formalization

The potential for formalization of legal automation is widely recognized in contemporary literature[476], and to attain it, the tools are as diverse as the strategies.

If some plead for approaches based on programming languages, like Catala[477] or ProLeg[478], implementing the law as code to formalize it ; there is another way, as Tomer Libal develops in " Legal linguistic templates and the tension between legal knowledge representation and reasoning."[479].

Indeed, in line with our last chapter's contribution, and as we will develop hereunder, we believe that annotation tools are the best way to achieve legal formalization, like many authors who advocate for it : whether it is Nazarenko et al. with a LegalRuleML tool[480], Soavi et al. with the ContrattoA tool[481], or Libal with the LegAI tool[482] that we already mentioned in

[476]NOVOTNA, Tereza et LIBAL, Tomer. An evaluation of methodologies for legal formalization. In : International Workshop on Explainable, Transparent Autonomous Agents and Multi-Agent Systems. Springer, Cham, 2022. p. 189-203.

[477]HUTTNER, Liane et MERIGOUX, Denis. Catala: moving towards the future of legal expert systems. Artificial intelligence and law, 2022, p. 1-24.

[478]SATOH, Ken, ASAI, Kento, KOGAWA, Takamune, et al. PROLEG: an implementation of the presupposed ultimate fact theory of Japanese civil code by PROLOG technology. In : JSAI international symposium on artificial intelligence. Berlin, Heidelberg : Springer Berlin Heidelberg, 2010. p. 153-164.

[479]LIBAL, Tomer. Legal linguistic templates and the tension between legal knowledge representation and reasoning. Frontiers in Artificial Intelligence, 2023, vol. 6, p. 1136263.

[480]NAZARENKO, Adeline, LEVY, François, et WYNER, Adam. An annotation language for semantic search of legal sources. In : Proceedings of the Eleventh International Conference on Language Resources and Evaluation (LREC 2018). 2018.

[481]SOAVI, Michele, ZENI, Nicola, MYLOPOULOS, John, et al. Semantic annotation of legal contracts with ContrattoA. In : Informatics. MDPI, 2022. p. 72.

[482]LIBAL, Tomer. The legai editor: a tool for the construction of legal knowledge bases. In : Legal Knowledge and Information Systems. IOS Press, 2022. p. 286-289.

our previous chapter.

The reason of this choice is linked to our idea of what the criteria of a good formalization are, which is a topic we discussed many times in this PhD, but it was also summarized specifically for legal formalization methods by Novotna and Libal in "An evaluation of methodologies for legal formalization"[483] ; a good formalization should be :

- correct, i.e. the representation must be the closest possible to the legal text, and "faithful"[484] to it. This correctness, as we developed in our last chapter, can be evaluated thanks to the ability to create isomorphisms[485] through the formalization process ;

- transparent, i.e. legal experts should be able to operate the formalization tool themselves, unlike many methods "usually require at least two experts - a logician (or a computer scientist) and a legal expert [..and..] none of the experts understands perfectly both sides of the process"[486];

- comprehensible, i.e. the ability to understand the formalization method and read its results easily for legal experts;

- supporting multiple interpretations, i.e. leaving a margin for (human) interpretation, preferably by lawyers, who should be able to formalize texts in a way they choose to, without being forced by restrictive semantics.

We believe that, taking these criteria into account, and for reasons we already developed, they totally align with our last chapter's method which, if mixed with a clear high-level interface/tool (like LegAI for example[487]), can be a good method, suitable for legal formalization.

Indeed, with our method, we can verify the correctness at any time, with the ability to reverse the LLTs translation and compare it to the original text ; the formalization is transparent as we can trace every step or the annotation process ; it is comprehensible if provided with a high-level interface that is

[483]NOVOTNA, Tereza et LIBAL, Tomer. An evaluation of methodologies for legal formalization. In : International Workshop on Explainable, Transparent Autonomous Agents and Multi-Agent Systems. Springer, Cham, 2022. p. 189-203.

[484]BARTOLINI, Cesare, LENZINI, Gabriele, et SANTOS, Cristiana. An agile approach to validate a formal representation of the GDPR. In : JSAI International Symposium on Artificial Intelligence. Cham : Springer International Publishing, 2018. p. 160-176.

[485]BENCH-CAPON, Trevor JM et COENEN, Frans P. Isomorphism and legal knowledge based systems. Artificial Intelligence and law, 1992, vol. 1, no 1, p. 65-86.

[486]NOVOTNA, Tereza et LIBAL, Tomer. An evaluation of methodologies for legal formalization. In : International Workshop on Explainable, Transparent Autonomous Agents and Multi-Agent Systems. Springer, Cham, 2022. p. 189-203.

[487]LIBAL, Tomer. Legal linguistic templates and the tension between legal knowledge representation and reasoning. Frontiers in Artificial Intelligence, 2023, vol. 6, p. 1136263.

lawyer-friendly ; and it supports multiple interpretations since the operator can choose how to annotate a legal text following his own interpretation of it.

Therefore, annotation tools really stand out as the greatest option for legal formalization.

1.2 The automation of law computing

The automation of legal reasoning has been a fantasy for decades[488], it is now a concrete application of annotation tools we can create.

However we must nuance by saying that we can only partially automate reasoning, as we are limited to non-interpretative inferences, and only to some extent, as we are limited to some types of norms-related reasonings.

As said in the first chapter, this automation will not be total, but only partial, as we still want the operating lawyers to do the interpretation themselves, so they will have to manually formalize the legal norms into templates/macros. But where does the automation process lie ? It only appears after this human-based first step of the formalization was done.

Thus, once a group of legal texts, a subset of the law, is formalized in an annotation tool, we can automatically and isomorphically convert them to computable code, which can be used as the basis for actually automating legal reasoning in a second phase, involving a second type of instruments : formal verification tools.

2 Theorem provers for verified reasoning

We shall now say a few words about formal verification tools and how they are useful to concluding our approach.

2.1 The automation of legal reasoning

Once we have a coded and computable law, the best way to make use of it is to implement it next in an automated theorem prover.

[488]SHANNON, David T. et GOLSHANI, Forouzan. Sur l'automatisation du raisonnement juridique. Jurimetrics J. , 1987, vol. 28, p. 305.

An automated theorem prover[489] (or ATP) is a software often used by logicians to "reason" over an implemented formal logic. If the formal logic is a representation of the law, this opens up many concrete applications ; let us discuss what the most interesting are.

- Firstly, proving "theorems" means proving or disproving (through proving the negation) statements. This means, in layman's terms, that we could have Q&As with the software regarding the law we previously implemented in it.

For example, once we implement traffic law, we can try proving that it is obligatory to stop in situation X, i.e. we ask the software "is it obligatory to stop if you are in the situation X ?", and the software will answer automatically and do the formal proof itself.

- Secondly, we can verify the consistency of legal reasoning based on legal texts, for example, the justification of judicial decisions. We could therefore check if the reasoning of a judge is composed of valid inferences, and does not contradict any prioritary norms that should have been applied to the case instead.

It is important to note that we do not aim at challenging any interpretation made by the judge, but only assure that his inferences are not fallacies regarding the state of the law. For example, if the judge interprets the facts of a case by stating that tomatoes should be considered vegetables regarding their taxation[490], the ATP would not argue on this interpretation or their fruit/vegetable nature, but only verify if this vegetable nature would indeed entail taxation or not.

Thirdly, we can correct the law itself, as we can set up consistency checks for formal systems in theorem provers. We can therefore more easily identify problematic or inconsistent laws.

The law, thanks to the previous use of the annotation tool, does indeed become a formal system, suitable for ATPs.

This means that, for any group of formalized legal norms, for example all of contract law related texts, we can use the ATP to check its global consistency, and tell us if the group is coherent or not.

We note that some Deontic Logic systems have already been implemented

[489]BIBEL, Wolfgang. Automated theorem proving. Springer Science & Business Media, 2013.

[490]GRAY, Horace. Supreme Court Of The United States. US Reports: Nix v. Hedden, 1892.

successfully in formal verification tools, like the DDL logic in Isabelle/HOL by Benzmuller et al. in "A dyadic Deontic Logic in HOL"[491].

Our long term goal would be to implement our logic LDL in one too, by mixing it with the formal level of our annotation formalization process, but this will be part of our future work (see conclusion).

2.2 Instruments for a better law

Thanks to this double level of practical tools, i.e. annotation and formal verification ones, we will be able to concretely formalize law and help it become more correct, looping back to the requirements of our first chapter, and responding to them.

Indeed, provided some additional future work is done (see conclusion), we now have all the necessary theory allowing lawyers to transparently annotate the law by interpreting it themselves, creating thus a formalization.

This formalization of legal texts will be transformed into code, which will be computed by automatic provers (ATPs) according to an implementation of the LDL logic.

All of this results in a tool that can prove any "theorem", i.e. ask the machine any question about a deontic statement, among our group of legal norms (which can be as big as we need).

As applications, legal practitioners would be able to anticipate the legality of behaviors by checking them beforehand, detect "hidden" or unforeseeable exceptions or concurrent norms, map graphically legal mechanisms to win time in their workflow, run "what-if" simulations and compare multiple scenarios for their clients, simulate any potential contradictions while drafting new rules prior to their integration to the system, etc.

For example, corporate lawyers could quickly compare the consequences of various legal forms for a company, tax experts could more easily anticipate the consequences of new laws before they are even voted, international lawyers could run at little time and effort multiple scenarios to guarantee to their client the applicable law of the country they prefer, legislators could check which problematic or conflicting existing laws clash with their new draft and act accordingly by abrogating the old laws or amending the new one.

[491]BENZMÜLLER, Christoph, FARJAMI, Ali, et PARENT, Xavier. A dyadic Deontic Logic in HOL. In : 14th International Conference on Deontic Logic and Normative Systems (DEON 2018). College Publications, 2018.

Concretely, we can thus bring more completeness to the law by checking if any question is inferrable and solvable with current legislation, or if there is a gap to fill, entailing then a need for a new law to cover it and avoid judicial arbitrariness.

Also, we can bring more coherence to the law, by consistently verifying it, by checking both the source of the law (the group of texts) and its use (the legal reasonings made with said group of texts).

Finally, we can bring more clarity, as we can obtain a single answer for any legal question (concerning the formalized laws), therefore overriding the complexity of the too numerous laws for the public.

3 Case study

Before ending this chapter, we will, on a small scale, see an of formalization.

However, I must point out two things : firstly, as I did not build a concrete tool or software yet, this will only be a simulation of what benefits we could get from such tools if they were to be developed ; secondly, the scale is limited, as much more time and resources would be needed to do a large scale case study on a legal domain with hundreds or thousands of norms, so the scope of benefits will only be glimpsed at.

Therefore, hereunder is presented an illustration, and not a demonstration.

An EU regulation. The article 13, paragraphs 1 to 4 of the EU regulation 883/2004 was phrased like this in 2014[492] :

[492]https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A02004R0883-20140101

"1. A person who normally pursues an activity as an employed person in two or more Member States shall be subject:

(a) to the legislation of the Member State of residence if he/she pursues a substantial part of his/her activity in that Member State; or

(b) if he/she does not pursue a substantial part of his/her activity in the Member State of residence:

(i) to the legislation of the Member State in which the registered office or place of business of the undertaking or employer is situated if he/she is employed by one undertaking or employer; or

(ii) to the legislation of the Member State in which the registered office or place of business of the undertakings or employers is situated if he/she is employed by two or more undertakings or employers which have their registered office or place of business in only one Member State; or

(iii) to the legislation of the Member State in which the registered office or place of business of the undertaking or employer is situated other than the Member State of residence if he/she is employed by two or more undertakings or employers, which have their registered office or place of business in two Member States, one of which is the Member State of residence; or

(iv) to the legislation of the Member State of residence if he/she is employed by two or more undertakings or employers, at least two of which have their registered office or place of business in different Member States other than the Member State of residence.

2. A person who normally pursues an activity as a self-employed person in two or more Member States shall be subject to:

(a) the legislation of the Member State of residence if he/she pursues a substantial part of his/her activity in that Member State;

or

(b) the legislation of the Member State in which the centre of interest of his/her activities is situated, if he/she does not reside in one of the Member States in which he/she pursues a substantial part of his/her activity.

3. A person who normally pursues an activity as an employed person and an activity as a self-employed person in different Member States shall be subject to the legislation of the Member State in which he/she pursues an activity as an employed person or, if he/she pursues such an activity in two or more Member States, to the legislation determined in accordance with paragraph 1.

4. A person who is employed as a civil servant by one Member State and who pursues an activity as an employed person and/or as a self-employed person in one or more other Member States shall be subject to the legislation of the Member State to which the administration employing him/her is subject."

For obvious readability purposes, for the rest of this case study, we may shorten some atoms, i.e. we may use shorter or less precise formulations as a replacement for big sentences, as we already showed in the last chapter what could a faithful formalization be.

For example, we may say "A normally employed person" instead of "A person who normally pursues an activity as an employed person".

Interpretation. Before formalizing, I must start by clarifying some ambiguities of the text through interpretation (as a lawyer myself).

- The first article is very clear in stating two or more Member States for the normal employed activity, and this is regardless of the potential self-employed activity (as article 3 will confirm).
- The second article implies implicitly that the self-employed activity should be considered as exclusive, with no other employers whose presence would completely nullify the provisions of this article (as article 3 will confirm).
- The third article confirms both our interpretations above by stating a specific regime for the case of one self-employed activity and one employer in a different state, then by stating that regardless of the self-employment activity we should consider article 1 if the employed activity is pursued in more than one Member State.
- The fourth article is an exception to all of the above by fixing a civil servant employment as prioritary for determining the legislation.

Formalization. If we consider that there is no contradictions in this article (and there is not), we can locally ignore subscripts to lighten the writing, as they would be useful on a larger scale. We will also abbreviate MS for Member State(s).

Here is how we could formalize it (we will graphically clarify this formalization below) :

(a normally employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge$(pursuing activity substantially in the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS of residence)

(a normally employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge\neg$(pursuing activity substantially in the MS of residence)$\wedge$(employed by 1 employer)$\rightarrow$ O(the person is subject to the legislation of the MS of the employer)

(a normally employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge\neg$(pursuing activity substantially in the MS of residence)$\wedge$(employed by at least 2 employers of the same MS)$\rightarrow$ O(the person is subject to the legislation of the MS of the employers)

(a normally employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge\neg$(pursuing activity substantially in the MS of residence)$\wedge$(employed by at least 2 employers of different MS)$\wedge$(at least 2 employers are from different MS other than the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS of residence)

(a normally employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge\neg$(pursuing activity substantially in the MS of residence)$\wedge$(employed by at least 2 employers of different MS)$\wedge\neg$(at least 2 employers are from different MS other than the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS other than the MS of residence)

(a normally and exclusively self-employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge$(pursuing activity substantially in the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS of residence)

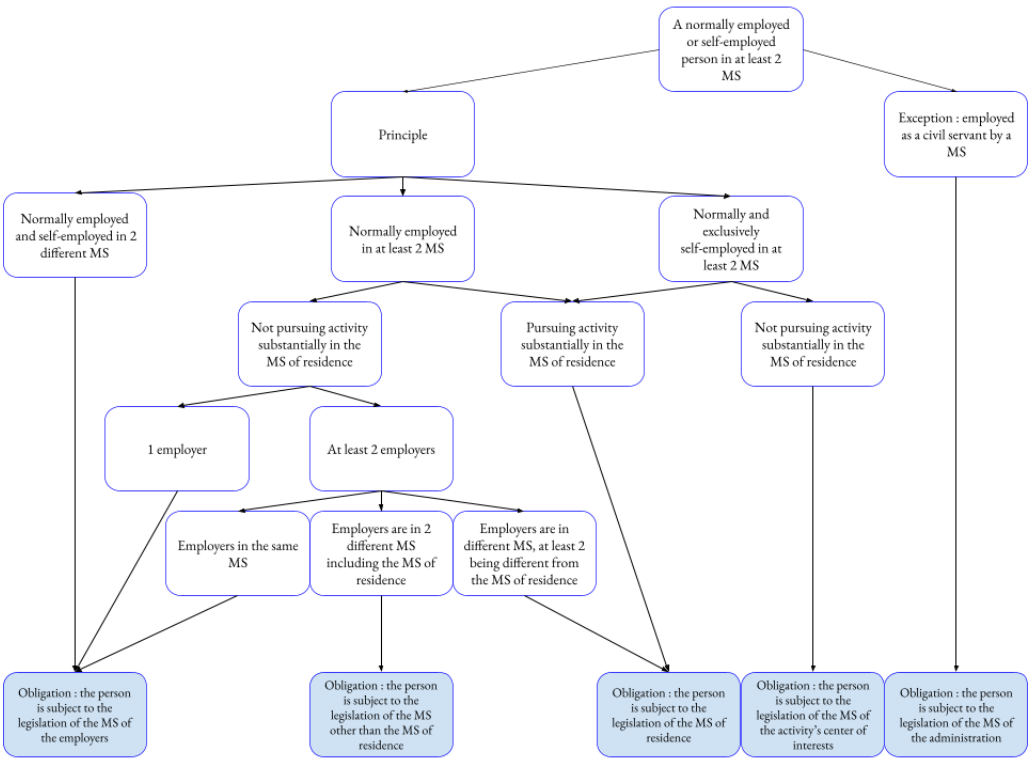
(a normally and exclusively self-employed person in at least 2 MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge\neg$(pursuing activity substantially in the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS of the activity's center of interests)

(a normally employed and self-employed person in 2 different MS)$\wedge\neg$(employed as a civil servant by a MS)$\wedge$(pursuing activity substantially in the MS of residence)$\rightarrow$ O(the person is subject to the legislation of the MS of the employers)

(a person employed as a civil servant by a MS)$\rightarrow$ O(the person is subject to the legislation of the MS of the administration)

Annotation. The annotation can both help non-logicians create indirectly (and isomorphically) the formalization, or help graphically illustrate it if it is already formalized, as it can go both ways.

We will now present a simulation of what a high-level graphic annotation could look like for the article under study :



Automation. Now that the article is both formalized and annotated, we can start concretely discussing the benefits we get, whether as lawyers or clients :

- Being able to visually comprehend the whole regime in a glimpse ;
- Giving the tool factual elements of a case and asking what regime applies ;
- Run simulations for every situation, real or hypothetical, even the most complex ones, with ease ;
- Encoding a questionnaire or a chat-bot that will give the applicable regime in a few steps ;
- Incorporate this formalized article to the whole legal domain (if formalized beforehand) and check for contradictions, redundancies or loopholes ;
- For the legislator, checking every new law's compatibility with this article by formalizing it, and anticipating every conflict of norms before it even happens.

- Being able to identify irrelevant fragments of a law for optimization, by noting a strict logical equivalence between the presence of a disposition and its absence (for example the second part of the point 3. here, or the obviously pointless disposition 7.1.c. of the EU regulation "Bruxelles I Bis"[493]).

Example. For example, a person residing in France is employed as a civil servant by the French administration, and also selling French-learning programs to people residing in mostly Spain and Germany. He wants to quit his civil servant job because he discovers that many people from Italy and Greece want to learn French too (but not more than in Spain). He already received offers from agencies, on operating in Italy alone and based in it, one operating in both Italy and Greece (but mostly in Greece) and based in Greece. He wants to know what is his best option regarding legislations ater quitting :

- He can decide to stay self-employed and sell programs in the four countries, his main center of interest still being Spain ;
- He can decide to stay self-employed for 3 countries (including Greece) and get employed by the Italian agency for the Italian market ;
- He can decide to be employed by the Greek agency for both new markets, in addition to his current self-employment ;
- He can decide to get employed by both the agencies respectively one for Italy and one for Greece, in addition to his current self-employment.

For these four options, we can in a few seconds answer this person's dilemma by asking the tool : the applicable legislation would respectively be Spain, Italy, Greece and France. So the person can "choose" the one they prefer.

[493]https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32012R1215

Conclusion

It is now time to finish off this thesis of ours, which we will do in two stages : we will first bring a conclusion to our work, then we will pave the way for the subsequent questions that we still have to work on in the future.

1 Conclusion

Let us start by recalling what goals we had in the beginning of this thesis, before reviewing our contributions through it.

1.1 Recalling the goals of the thesis

The primary goal of this thesis was to close the gap between law and code through logical formalization.

A subsequent goal of it was to find a method, a process, to make this formalization and computation of the law doable step by step from law to code.

Another big goal was to address the legal formalization question through a disciplinary panoramic lens, from norms theory to formal logic, from informatics theory to philosophy of law ; and find bridges between these disciplines to make them work together towards a more correct law.

Finally, our thesis' title ends with a question mark, inducing the doubt regarding the possibility of reaching or not the goals we had. We wanted to explore and address the questions to first know if they could be answered before doing so.

1.2 Reviewing the contributions of the thesis

Our first chapter did not have a novel contribution per se, but consisted in groundwork we assessed and evaluated the relevance of the goals we had, then justified the purpose of our thesis.

Before diving in any attempt of formalizing law, we assessed the situation, and took a state of affairs of the deficiencies of contemporary law, establishing that it was incorrect, i.e. lacked completeness, coherence and clarity.

We then assessed the potential of formalization which can, through a logic-based approach and to a certain extent, bring more correctness to the law.

In chapter two, we challenged the definition and properties of norms and legal systems.

Since the thesis is about using Deontic Logic to formalize law, and since norms are the main object of study of Deontic Logic, it is only natural that we first established some definitions before dealing directly with said logic.

We thus began by giving our definition of what a norm is, after a brief state of the art. We also specifically addressed legal norms and distinguish them.

We then discussed some properties that are sometimes included in the norms' definition, but we showed how they are not necessary for norms to exist.

We talked about the issue of legal ontology, and how we must keep the formalization partial, in order to avoid objectifying what cannot be : the legal interpretation.

We finally addressed the diverse legal systems, their definitions and properties.

Our main contribution to norm theory in this chapter was to arbitrate the hyletic/expressive debate around the conception of norms through what we called "the dismemberment of norms properties", which totally make the essence of norms independant from its effects and properties, which could then be more easily categorized between hyletic properties and expressive properties, according to which is more relevant for each one.

In our third chapter, we first discussed the basis we need beforehand, theoretically and historically, i.e. exploring what attempts at building a Deontic Logic have been pursued already, and justify why we needed a new one.

We then worked on our own attempt by creating a Deontic Logic system, based on cascading hierarchies, and we developed why we think it is the best way to deal with both paradoxes and priorities among the legal systems.

Our contribution in this chapter was thus the creation of LDL, the Layered Deontic Logic, which was formally defined in collaboration with Vincent Blazy from the LIRMM, with whom we demonstrated that our logic was correct, complete and consistent.

Moreover, unlike most deontic logics, ours does not assume that the law is correct, and therefore is built to make contradictions impossible, both legally and logically, thanks to a subscripts system that is flexible and can be adapted to each legal system by lawyers.

Our fourth chapter focused on the process we propose in order to close the huge gap that we assessed between law and code. We needed to find a

way to connect them, and to go from one to the other, both ways if possible.

We began by drawing up the existing gap between languages and logics, and more specifically the legal language.

We then addressed the need of the formalization to be lawyer-based, i.e. lawyers must be able to close this gap themselves. It is not enough to elaborate a way to formalize law into logic if lawyers cannot benefit directly from it.

We finally discussed a grammar that will be the intermediate step of our formalization method, based on an isomorphism, by creating a double-bridge between logic and law.

Therefore, for our contribution in this chapter, based on existing observations and requirements, we came up, with the help of Tomer Libal from the University of Luxembourg, with a two-steps method involving an intermediate level of templates, based on a formal grammar, and isomorphic to both the code and a potential higher-level interface that can come from tools we implement it in.

In our fifth and last chapter, after having done all this theoretical groundwork in the previous ones, it was only time to say a few words about the potential practical tools we could develop thereafter.

Thus, we presented and categorized several useful strategies and instruments that can serve concrete applications in law.

This last chapter did not really bring a novel contribution but it was important to informatively show some prospects of concrete tools that could be the continuity of our work in this thesis, circling back to necessity for us to make law more correct, as assessed in the first chapter.

Finally, if we read again the question of the title, we can answer it positively : yes, Deontic Logic can be a tool for legal formalization, if it is supported by a cross-disciplinary lens, involving working on norms, formal logic and " legal informatics".

Thus, the goals we had for this thesis have mainly been met, but we still have much work to do to completely attain even more ambitious objectives.

2 Future work

Many of the questions tackled in this thesis deserve further study in the future.

2.1 More ambitious inclusion of norms

In our thesis, we gave a very general definition of norms, but when discussing their integration into LDL's hierarchy, we restricted ourselves to written laws, excluding de facto all the jurisprudence.

Although in theory jurisprudence is not fully considered as a source of law in civil law countries, as they lack the bindingness of precedents ; I believe we could try and find a clever way to incorporate them, maybe though fuzzy logic for example.

Thus, finding a way to let jurisprudence play a role is an important objective for the future.

2.2 More ambitious features for LDL

As we developed in our third chapter, the ideal way to express principle-exception dynamics in law is the negation as failure (NAF), but we did not incorporate it yet in LDL due to lack of time.

This deserves to be addressed next.

Also, if we successfully demonstrated that LDL is correct, complete and consistent, we did not (yet) demonstrate that it is decidable, also due to lack of time, but we think that it may be the case.

So we shall keep our efforts to develop LDL even more in the future.

2.3 More ambitious practical implementations

Finally, we want to make the concrete tools, talked about in our last chapter, exist.

Though an annotation tool, "LegAi"[494], was already developed by Tomer Libal in parallel of our work on the Logical Legal Templates ; this tool does not include any LDL components, as the deontic operator is very basic in it.

[494]LIBAL, Tomer. The legai editor: a tool for the construction of legal knowledge bases. In : Legal Knowledge and Information Systems. IOS Press, 2022. p. 286-289.

That is why a last theoretical work will be to mix up the LDL system with the LLTs method in order to get the ultimate synthesis of this thesis strategies for formalization, and integrate it in an annotation tool.

This means that, instead of just formalizing legal texts with simple obligations, the lawyers would be able to integrate a subscript system and hierarchize these obligations while formalizing them. This would bring even more interest to the annotation/ formalization, while still being accessible to lawyers with no scientific background, since it would already more faithfully transcribe the reality of the group of legal texts they would like to formalize.

As for the formal verification tools, we must plan to implement LDL into a theorem prover, as Benzmuller et al. successfully did for DDL in Isabelle/HOL[495], but with a deontic system that did not satisfy the requirements we exposed in chapter three.

[495]BENZMÜLLER, Christoph, FARJAMI, Ali, et PARENT, Xavier. A dyadic Deontic Logic in HOL. In : 14th International Conference on Deontic Logic and Normative Systems (DEON 2018). College Publications, 2018.

References

Aho, A. V. and J. D. Ullman (1969). "Translations on a context free grammar". en. In: *Proceedings of the first annual ACM symposium on Theory of computing - STOC '69*. Marina del Rey, California, United States: ACM Press, pp. 93–112. DOI: 10.1145/800169.805425. URL: http://portal.acm.org/citation.cfm?doid=800169.805425 (visited on 09/27/2025).

Aitken, Victoria E. (2013). "An exposition of legislative quality and its relevance for effective development". In: *ProLaw Student Journal* 2, pp. 1–43. URL: https://www.luc.edu/media/lucedu/prolaw/documents/AITKEN%20FINAL%20ARTICLE.pdf (visited on 09/27/2025).

Alchourrón, C. E. (1975). *y Bulygin, E.(1971). Normative Systems*.

Alchourrón, Carlos E. and Eugenio Bulygin (1981). "The Expressive Conception of Norms". en. In: *New Studies in Deontic Logic*. Ed. by Risto Hilpinen. Dordrecht: Springer Netherlands, pp. 95–124. ISBN: 978-90-277-1346-9 978-94-009-8484-4. DOI: 10.1007/978-94-009-8484-4_4. URL: http://link.springer.com/10.1007/978-94-009-8484-4_4 (visited on 03/18/2026).

— (1984). "Permission and permissive norms". In: *Theorie der Normen*, pp. 349–371.

Alexy, Robert (1998). "Law and correctness". In: *Current Legal Problems* 51.1, p. 205. URL: https://search.proquest.com/openview/47bb554b7b7a5450798fa7c0101?pq-origsite=gscholar&cbl=2032116 (visited on 09/27/2025).

Allen, James (1995). *Natural language understanding*. Benjamin-Cummings Publishing Co., Inc. URL: https://dl.acm.org/doi/abs/10.5555/199291 (visited on 09/27/2025).

Amaya, Amalia (Sept. 2011). "Legal Justification by Optimal Coherence*". en. In: *Ratio Juris* 24.3, pp. 304–329. ISSN: 0952-1917, 1467-9337. DOI: 10.1111/j.1467-9337.2011.00486.x. URL: https://onlinelibrary.wiley.com/doi/10.1111/j.1467-9337.2011.00486.x (visited on 03/17/2026).

Andone, Corina and Candida Leone (Jan. 2022). "Talking law. Clarity, transparency and legitimacy in rule-making". en. In: *The Theory and Practice of Legislation* 10.1, pp. 1–4. ISSN: 2050-8840, 2050-8859. DOI: 10.1080/20508840.2022.2033941. URL: https://www.tandfonline.com/doi/full/10.1080/20508840.2022.2033941 (visited on 09/27/2025).

Asgeirsson, Hrafn (2020). "The nature and value of vagueness in the law". In: URL: https://www.torrossa.com/it/resources/an/5215200 (visited on 03/18/2026).

Audi, Robert (2015). *Reasons, rights, and values*. Cambridge University Press. URL: https://books.google.com/books?hl=fr&lr=&id=G4mSBwAAQBAJ&oi=fnd&pg=PR7&dq=AUDI,+Robert.+Reasons,+rights,+and+values.+Cambridge+University+Press,+2015&ots=BCocbvmKvP&sig=mRas2Ltp_lNSUXJ3O3OSQmXxitY (visited on 09/27/2025).

Bailhache, Patrice (1977). "Sémantiques pour des systèmes déontiques intégrant permission faible et permission forte". In: *Logique et analyse* 20.79, pp. 286–316. URL: https://www.jstor.org/stable/44083766?casa_token=tDrVxy-ci9YAAAAA:zERPnxUiNBHAxa-BiaFdOj5Rstvlud2Nsd6soi5pPP1G6UW7NMZfjD (visited on 04/02/2026).

— (1985). "Logique déontique et temporelle". In: URL: http://dspace.uni.lodz.pl/handle/11089/8590 (visited on 04/02/2026).

— (1995). "Canonical models for temporal deontic logic". In: *Logique et analyse* 38.149, pp. 3–21. URL: https://www.jstor.org/stable/44084456?casa_token=9mc6z3Q1WEwAAAAA:HQMBpL1C8w0_33RgYFH0i6HnjfxdaETMIgHB4zoxc5zDvt_qDsRYexJ4P6IH4IPJAqFSuwmDBvpPtfqj75kRwiqr-RsT8Uj3z8uFDsNREL7XasUqCw (visited on 04/02/2026).

Balaguer, Mark (2025). *Platonism in Metaphysics*.

Balkin, Jack M. (1993). "Understanding legal understanding: The legal subject and the problem of legal coherence". In: *Yale Law Journal*, pp. 105–176. URL: https://www.jstor.org/stable/797078 (visited on 09/27/2025).

Ball, Carolyn (Sept. 2009). "What Is Transparency?" In: *Public Integrity* 11.4, pp. 293–308. ISSN: 1099-9922. DOI: 10.2753/PIN1099-9922110400. URL: http://www.tandfonline.com/doi/full/10.2753/PIN1099-9922110400 (visited on 09/27/2025).

Bard, Christine (2013). "Le droit au pantalon". In: URL: https://laviedesidees.fr/Le-droit-au-pantalon.html (visited on 09/27/2025).

Barker-Plummer, Dave et al. (2008). "An empirical study of errors in translating natural language into logic". In: *Proceedings of the Annual Meeting of the Cognitive Science Society*. Vol. 30. 30. URL: https://escholarship.org/content/qt66k15611/qt66k15611.pdf (visited on 09/27/2025).

Barnett, Randy E. and Evan D. Bernick (2018). "No Arbitrary Power: An Originalist Theory of the Due Process of Law". In: *Wm. & Mary L. Rev.* 60, p. 1599. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/wmlr60&section=43 (visited on 09/27/2025).

Bartolini, Cesare, Gabriele Lenzini, and Cristiana Santos (2018). "An interdisciplinary methodology to validate formal representations of legal text applied to the GDPR". In: *Twelfth International Workshop on Juris-informatics (JURISIN 2018)*. URL: https://orbilu.uni.lu/handle/10993/39326 (visited on 09/27/2025).

Bartolini, Cesare, Gabriele Lenzini, and Cristiana Santos (2019). "An Agile Approach to Validate a Formal Representation of the GDPR". en. In: *New Frontiers in Artificial Intelligence*. Ed. by Kazuhiro Kojima et al. Vol. 11717. Series Title: Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 160–176. ISBN: 978-3-030-31604-4 978-3-030-31605-1. DOI: 10.1007/978-3-030-31605-1_13. URL: http://link.springer.com/10.1007/978-3-030-31605-1_13 (visited on 09/27/2025).

Baude, William and Stephen E. Sachs (2016). "The law of interpretation". In: *Harv. L. Rev.* 130, p. 1079. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/hlr130&section=63 (visited on 09/27/2025).

Beal, James Hartley (1921). "Law making, rational and irrational". In: *Journal of the American Pharmaceutical Association* 10.8, pp. 574–581. URL: https://scholar.archive.org/work/3m4stitiozajtblnmllms6f2oq/access/ia_file/crossref-pre-1923-scholarly-works/10.1002%252Fjps.3080090604.zip/10.1002%252Fjps.3080100803.pdf (visited on 09/27/2025).

Beauvais, Pascal (2007). "Le droit à la prévisibilité en matière pénale dans la jurisprudence des cours européennes". In: *Archives de politique criminelle* 29.1, pp. 3–18. URL: https://shs.cairn.info/article/APC_029_0003?tab=texte-integral (visited on 09/27/2025).

Bénabent, Alain and Yves Gaudemet (2021). "Dictionnaire juridique: tous les mots du droit". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130290370862226064 (visited on 09/27/2025).

Benacerraf, Paul (1973). "Mathematical truth". In: *The Journal of philosophy* 70.19, pp. 661–679. URL: https://www.jstor.org/stable/2025075?casa_token=k4LPwnIcMGQAAAAA:0RLadEju11i1tDQwrIFZvxGw7rkSUTHLzwIXa82WTp4PlZi44cvuC_eA-NdGXqR2qTtgUGQ (visited on 02/27/2026).

Bench-Capon, T. J. M. and F. P. Coenen (1992). "Isomorphism and legal knowledge based systems". en. In: *Artificial Intelligence and Law* 1.1, pp. 65–86. ISSN: 0924-8463, 1572-8382. DOI: 10.1007/BF00118479. URL: http://link.springer.com/10.1007/BF00118479 (visited on 09/27/2025).

Bench-Capon, Trevor, Michał Araszkiewicz, et al. (Sept. 2012). "A history of AI and Law in 50 papers: 25 years of the international conference on AI and Law". en. In: *Artificial Intelligence and Law* 20.3, pp. 215–319. ISSN: 0924-8463, 1572-8382. DOI: 10.1007/s10506-012-9131-x. URL: http://link.springer.com/10.1007/s10506-012-9131-x (visited on 09/27/2025).

Bench-Capon, Trevor and Thomas F. Gordon (June 2009). "Isomorphism and argumentation". en. In: *Proceedings of the 12th International Conference on Artificial Intelligence and Law*. Barcelona Spain: ACM, pp. 11–20. ISBN: 978-1-60558-597-0. DOI: 10.1145/1568234.1568237. URL: https://dl.acm.org/doi/10.1145/1568234.1568237 (visited on 09/27/2025).

Benezech, Ludovic (2020). "L'exigence d'accessibilité et d'intelligibilité de la loi: retour sur vingt ans d'existence". In: *Revue française de droit constitutionnel* 123.3, pp. 541–563. URL: https://shs.cairn.info/revue-francaise-de-droit-constitutionnel-2020-3-page-541 (visited on 09/27/2025).

Bentham, Jeremy (1950). "The theory of legislation". In: *Readings from Liberal Writers*. Routledge, pp. 182–186. URL: https://www.taylorfrancis.com/chapters/edit/10.4324/9781003589020-32/theory-legislation-jeremy-bentham (visited on 09/27/2025).

Bentham, Jeremy and Herbert Lionel Adolphus Hart (1970). "Of laws in general". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130000797966862976 (visited on 09/27/2025).

Benzmüller, Christoph, Ali Farjami, and Xavier Parent (2018). "A dyadic deontic logic in HOL". In: *14th International Conference on Deontic Logic and Normative Systems (DEON 2018)*. College Publications. URL: https://orbilu.uni.lu/handle/10993/36395 (visited on 09/27/2025).

Bibel, Wolfgang (2013). *Automated theorem proving*. Springer Science & Business Media. URL: https://books.google.com/books?hl=fr&lr=&id=MNcACAAAQBAJ&oi=fnd&pg=PR5&dq=BIBEL,+Wolfgang.+Automated+theorem+proving.+Springer+Science+%5C%26+Business+Media,+2013.&ots=eFyi_ZTj7y&sig=VdBFpmlwpqr0ffVIqYfZMGutih4 (visited on 09/27/2025).

Bierstedt, Robert (1970). "The social order". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130282271127320192 (visited on 09/27/2025).

Bobbio, Norberto (1960). "Teoria dell'ordinamento giuridico". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1970867909775832330 (visited on 03/14/2026).

Boon, Andrew (2025). "The Illusion of Synonyms: Investigating the Similarities and Differences between Bias and Prejudice". In: (). URL: https://www.academia.edu/download/85392477/Boon_2005_Bias_and_Prejudice.pdf (visited on 09/27/2025).

Borges, Georg, Ken Satoh, and Erich Schweighofer (May 2023a). *Proceedings of the International Workshop on Methodologies for Translating Legal Norms into Formal Representations (LN2FR 2022) in association with 35th International Conference on Legal Knowledge and Information Systems (JURIX 2022)*. arXiv:2305.12203 [cs]. DOI: 10.48550/arXiv.

2305.12203. URL: http://arxiv.org/abs/2305.12203 (visited on 09/27/2025).

Borges, Georg, Ken Satoh, and Erich Schweighofer (May 2023b). *Proceedings of the International Workshop on Methodologies for Translating Legal Norms into Formal Representations (LN2FR 2022) in association with 35th International Conference on Legal Knowledge and Information Systems (JURIX 2022)*. arXiv:2305.12203 [cs]. DOI: 10.48550/arXiv.2305.12203. URL: http://arxiv.org/abs/2305.12203 (visited on 09/27/2025).

Braithwaite, John (2002). "Rules and principles: A theory of legal certainty". In: *Australasian Journal of Legal Philosophy* 27.2002, pp. 47–82. URL: https://search.informit.org/doi/pdf/10.3316/ielapa.200206928 (visited on 09/27/2025).

Breakey, Hugh (May 2017). "Arbitrary Power, Arbitrary Interference and the Abuse of Power: Corruption, Natural Rights and Human Rights". en. In: *Research in Ethical Issues in Organizations*. Ed. by Sunil Savur and Sukhbir Sandhu. Vol. 17. Emerald Publishing Limited, pp. 125–145. ISBN: 978-1-78714-416-3 978-1-78714-415-6. DOI: 10.1108/S1529-209620170000017013. URL: http://www.emerald.com/books/edited-volume/11609/chapter/81509815 (visited on 09/27/2025).

Bulygin, Eugenio (2015). "Carlos E. Alchourrón and the Philosophy of Law"". In: *South American Journal of Logic* 1.2, pp. 345–360. URL: http://www.sa-logic.org/sajl-v1-i2/02-Bulygin-SAJL.pdf (visited on 02/28/2026).

Carabantes, Manuel (2023). "Why artificial intelligence is not transparent: a critical analysis of its three opacity layers". In: *Handbook of Critical Studies of Artificial Intelligence*. Edward Elgar Publishing, pp. 424–434. URL: https://www.elgaronline.com/edcollchap/book/9781803928562/book-part-9781803928562-45.xml (visited on 09/27/2025).

Carbonnier, Jean (1963). "L'hypothèse du non-droit". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130282272778728192 (visited on 09/27/2025).

Carmo, José and Andrew J. I. Jones (2002). "Deontic Logic and Contrary-to-Duties". en. In: *Handbook of Philosophical Logic*. Ed. by D. M. Gabbay and F. Guenthner. Dordrecht: Springer Netherlands, pp. 265–343. ISBN: 978-94-010-3919-2 978-94-010-0387-2. DOI: 10.1007/978-94-010-0387-2_4. URL: http://link.springer.com/10.1007/978-94-010-0387-2_4 (visited on 09/27/2025).

Carnelutti, Francesco (1940). "Teoria generale del diritto". In: URL: https://philpapers.org/rec/CARTGD-4 (visited on 03/13/2026).

Chagrov, Alexander and Michael Zakharyaschev (1997). *Modal logic*. Oxford University Press. URL: https://academic.oup.com/book/52910 (visited on 09/27/2025).

Chemerinsky, Erwin (2016). "In Defense of Judicial Supremacy". In: *Wm. & Mary L. Rev.* 58, p. 1459. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/wmlr58&section=39 (visited on 09/27/2025).

Chevret-Castellani, Christine and Sarah Labelle (2019). "Transparence et loyauté, deux motifs de la régulation des algorithmes". In: *Terminal. Technologie de l'information, culture & société* 124. URL: https://journals.openedition.org/terminal/4064 (visited on 09/27/2025).

Chisholm, Roderick M. (1963). "Contrary-to-duty imperatives and deontic logic". In: *Analysis* 24.2, pp. 33–36. URL: https://www.jstor.org/stable/3327064 (visited on 09/27/2025).

Chlipala, Adam (2013). *Certified programming with dependent types: a pragmatic introduction to the Coq proof assistant*. MIT Press. URL: https://books.google.com/books?hl=fr&lr=&id=gevxCwAAQBAJ&oi=fnd&pg=PR5&dq=CHLIPALA,+Adam.+Certified+programming+with+dependent+types:+a+pragmatic+introduction+to+the+Coq+proof+assistant.+MIT+Press,+2013&ots=p0_U3zfBKn&sig=elt44JUAdjuJy8_2pE0CJJlDRlk (visited on 09/27/2025).

Chomsky, Noam (Jan. 2002). *Syntactic Structures*. Berlin, New York: Mouton de Gruyter. ISBN: 978-3-11-021832-9. DOI: 10.1515/9783110218329. URL: http://www.degruyter.com/view/books/9783110218329/9783110218329/9783110218329.xml (visited on 09/27/2025).

Chowdhary, K. R. (2020). "Natural Language Processing". en. In: *Fundamentals of Artificial Intelligence*. New Delhi: Springer India, pp. 603–649. ISBN: 978-81-322-3970-3 978-81-322-3972-7. DOI: 10.1007/978-81-322-3972-7_19. URL: http://link.springer.com/10.1007/978-81-322-3972-7_19 (visited on 09/27/2025).

Chromá, Marta (2011). "Synonymy and polysemy in legal terminology and their applications to bilingual and bijural translation". In: URL: http://dspace.uni.lodz.pl/handle/11089/9580 (visited on 09/27/2025).

Clark, Keith L. (1978). "Negation as failure. Logic and Databases". In: *Logic and Databases, Plenum*.

Clausing, Kimberly A. (2019). "Fixing Five Flaws of the Tax Cuts and Jobs Act". In: *Colum. J. Tax L.* 11, p. 31. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/colujoutl11&section=8 (visited on 09/27/2025).

Colmerauer, Alain and Philippe Roussel (Jan. 1996). "The birth of Prolog". en. In: *History of programming languages—II*. Ed. by Thomas J. Bergin

and Richard G. Gibson. New York, NY, USA: ACM, pp. 331–367. ISBN: 978-0-201-89502-5. DOI: 10.1145/234286.1057820. URL: http://dl.acm.org/doi/10.1145/234286.1057820 (visited on 09/27/2025).

Constitutionnel, Conseil (1999). *Décision n 99-421 DC du 16 décembre 1999, Loi portant habilitation du Gouvernement à procéder, par ordonnances, à l'adoption de la partie législative de certains codes*.

Conte, Amedeo G. (1966). "DÉCISION, COMPLÉTUDE, CLÔTURE: A propos des lacunes en droit". In: *Logique et analyse* 9.33, pp. 1–18. URL: https://www.jstor.org/stable/44083463 (visited on 09/27/2025).

Convert, Laurent (2018). "Feu le «bon père de famille» et l'avènement du standard «raisonnable»: un pas vers la «reasonable person» de common law?" In: *Tribonien* 1.1, pp. 166–183. URL: https://shs.cairn.info/revue-tribonien-2018-1-page-166 (visited on 09/27/2025).

Cooper, A. Feder et al. (Dec. 2023). *Report of the 1st Workshop on Generative AI and Law*. arXiv:2311.06477 [cs]. DOI: 10.48550/arXiv.2311.06477. URL: http://arxiv.org/abs/2311.06477 (visited on 09/27/2025).

Cornu, Gérard (1981). "Les définitions dans la loi". In: *Mélanges dédiés à Jean VINCENT*, p. 77. URL: http://www.initerm.net/public/langues%20de%20sp%C3%A9cialit%C3%A9/langue%20juridique/Les_d_finitions_dans_la_loi.doc (visited on 09/27/2025).

— (2003). "L'art d'écrire la loi". In: *Pouvoirs* 107.4, pp. 5–10. URL: https://droit.cairn.info/article/POUV_107_0005 (visited on 09/27/2025).

— (2024). *Vocabulaire juridique*. Puf. URL: https://books.google.com/books?hl=fr&lr=&id=3tfpEAAAQBAJ&oi=fnd&pg=PT3&dq=CORNU,+G%C3%A9rard,+et+al.+Vocabulaire+juridique.+(No+Title),+2016.&ots=rXqDXUj8mf&sig=b90CP0GXUgAkZjIMikEeyvtMyA8 (visited on 09/27/2025).

Coval, S. C. and J. C. Smith (1977). "The Completeness of Rules". In: *The Cambridge Law Journal* 36.2, pp. 364–368. URL: https://www.cambridge.org/core/journals/cambridge-law-journal/article/completeness-of-rules/B21A62C5DA10A027A1F4310B583163CD (visited on 09/27/2025).

Cowgill, Bo et al. (July 2020). "Biased Programmers? Or Biased Data? A Field Experiment in Operationalizing AI Ethics". en. In: *Proceedings of the 21st ACM Conference on Economics and Computation*. Virtual Event Hungary: ACM, pp. 679–681. ISBN: 978-1-4503-7975-5. DOI: 10.1145/3391403.3399545. URL: https://dl.acm.org/doi/10.1145/3391403.3399545 (visited on 09/27/2025).

Coyle, Sean (2004). "Facing Jorgensen's dilemma". In: *N. Ir. Legal Q.* 55, p. 341. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/nilq55&section=31 (visited on 09/27/2025).

Crawford, Lisa Burton (2024). "The problem of complex legislation". In: *Legal Theory* 30.1, pp. 1–21. URL: https://www.cambridge.org/core/journals/legal-theory/article/problem-of-complex-legislation/670046444760B57B0FAE844D6E6ECFBD (visited on 09/27/2025).

Curto, Georgina et al. (Apr. 2024). "Are AI systems biased against the poor? A machine learning analysis using Word2Vec and GloVe embeddings". en. In: *AI & SOCIETY* 39.2, pp. 617–632. ISSN: 0951-5666, 1435-5655. DOI: 10.1007/s00146-022-01494-z. URL: https://link.springer.com/10.1007/s00146-022-01494-z (visited on 09/27/2025).

d'État, Conseil and PREMIER MINISTRE (2007). *uide de légistique*. URL: https://www.circulaires.gouv.fr/contenu/Media/Files/autour-de-la-loi/guide-de-legistique/guide-de-legistique-edition-2017-format-pdf.pdf (visited on 09/27/2025).

Dascal, Marcelo (Oct. 2003). *Interpretation and Understanding*. en. Amsterdam: John Benjamins Publishing Company. ISBN: 978-90-272-2604-4 978-1-58811-414-3 978-90-272-9589-7. DOI: 10.1075/z.120. URL: http://www.jbe-platform.com/content/books/9789027295897 (visited on 09/27/2025).

De Boer, Mathijs et al. (July 2012). "Two dimensional Standard Deontic Logic [including a detailed analysis of the 1985 Jones–Pörn deontic logic system]". en. In: *Synthese* 187.2, pp. 623–660. ISSN: 0039-7857, 1573-0964. DOI: 10.1007/s11229-010-9866-4. URL: http://link.springer.com/10.1007/s11229-010-9866-4 (visited on 09/27/2025).

Dellaux, Julien (2019). "Le principe de sécurité juridique en droit constitutionnel: signes et espoirs d'une consolidation de l'ordre juridique interne et de l'État de droit". In: *Revue française de droit constitutionnel* 119.3, pp. 665–696. URL: https://shs.cairn.info/article/RFDC_119_0665 (visited on 09/27/2025).

Delnoy, Michel (2005). "Les obligations juridiques de cohérence du droit ou comment le droit se protège contre lui-même". In: URL: https://orbi.uliege.be/bitstream/2268/67799/1/M.%20Delnoy%20-%20Les%20obligations%20juridiques%20de%20coh%C3%A9rence%20du%20droit%20ou%20comment%20le%20droit%20se%20prot%C3%A8ge%20contre%20lui-m%C3%AAme.pdf (visited on 09/27/2025).

Destrée, Pierre (2000). "Aristote et la question du droit naturel (" Eth. Nic.", V, 10, 1134 b 18-1135 a 5)". In: *Phronesis*, pp. 220–239. URL: https://www.jstor.org/stable/4182647 (visited on 09/27/2025).

Dickson, Julie (2001). "Interpretation and coherence in legal reasoning". In: URL: https://plato.sydney.edu.au/entries/legal-reas-interpret/ (visited on 09/27/2025).

DiMatteo, Larry A. (1996). "Counterpoise of Contracts: The Reasonable Person Standard and the Subjectivity of Judgment, The". In: *SCL Rev.* 48, p. 293. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/sclr48&section=26 (visited on 09/27/2025).

Dinh, Anne-Marie Ho (2007). "Le «vide juridique» et le «besoin de loi». pour un recours à l'hypothèse du non-droit". In: *L'Année sociologique* 57.2, pp. 419–453. URL: https://shs.cairn.info/article/ANSO_072_0419?tab=resume (visited on 09/27/2025).

Dworkin, Ronald (1977). "Is law a system of rules?" In: *The philosophy of law* 1.

— (Sept. 1982). "Law as Interpretation". en. In: *Critical Inquiry* 9.1, pp. 179–200. ISSN: 0093-1896, 1539-7858. DOI: 10.1086/448194. URL: https://www.journals.uchicago.edu/doi/10.1086/448194 (visited on 09/27/2025).

— (1988). *Law's empire*. Harvard University Press. URL: https://books.google.com/books?hl=fr&lr=&id=WcfmEAAAQBAJ&oi=fnd&pg=PA1&dq=DWORKIN,+Ronald.+Law%E2%80%99s+empire.+Harvard+university+press,+1988&ots=-ysti8xQmT&sig=y7tRPpGV4Jfwldi35nM2tPCx-uk (visited on 09/27/2025).

Dworkin, Ronald M. (1967). "The model of rules". In: *The University of Chicago Law Review* 35.1, pp. 14–46. URL: https://www.jstor.org/stable/1598947 (visited on 09/27/2025).

Easterbrook, Frank H. (1984). "Legal Interpretation and the Power of the Judiciary". In: *Harv. JL & Pub. Pol'y* 7, p. 87. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/hjlpp7&section=18 (visited on 09/27/2025).

Endicott, Timothy (2011). "Vagueness and Law". en. In: *Vagueness: A Guide*. Ed. by Giuseppina Ronzitti. Dordrecht: Springer Netherlands, pp. 171–191. ISBN: 978-94-007-0374-2 978-94-007-0375-9. DOI: 10.1007/978-94-007-0375-9_7. URL: http://link.springer.com/10.1007/978-94-007-0375-9_7 (visited on 03/18/2026).

EROS, GRAU (2014). *Pourquoi j'ai peur des juges: l'interprétation du droit et les principes juridiques*. Éditions Kimé. URL: https://books.google.com/books?hl=fr&lr=&id=X3c3DwAAQBAJ&oi=fnd&pg=PA7&dq=EROS,+GRAU.+Pourquoi+j%27ai+peur+des+juges:+l%27interpr%C3%A9tation+du+droit+et+les+principes+juridiques.+Editions+Kim%C3%A9,+2014.&ots=YFyMW_jG2D&sig=8_gNZQuFXoTd4ULKk1YM9HCOwjM (visited on 09/27/2025).

Feldman, Daniel L. (2024). "Law's Legitimacy: Lon Fuller in a Consequentialist Frame". PhD Thesis. City University of New York. URL: https://

search.proquest.com/openview/e2205c192e22c299e1ce91c4d349c638/1?pq-origsite=gscholar&cbl=18750&diss=y (visited on 09/27/2025).

Ferryman, Kadija, Maxine Mackintosh, and Marzyeh Ghassemi (Aug. 2023). "Considering Biased Data as Informative Artifacts in AI-Assisted Health Care". en. In: *New England Journal of Medicine* 389.9. Ed. by Jeffrey M. Drazen, pp. 833–838. ISSN: 0028-4793, 1533-4406. DOI: 10.1056/NEJMra2214964. URL: http://www.nejm.org/doi/10.1056/NEJMra2214964 (visited on 09/27/2025).

Field, Hartry (2015). "What is logical validity". In: *Foundations of logical consequence*, pp. 33–70. URL: https://books.google.com/books?hl=fr&lr=&id=f1DCBwAAQBAJ&oi=fnd&pg=PA33&dq=Field,+H.+(2015).+What+is+logical+validity.+Foundations+of+logical+consequence&ots=6iH_oC_khV&sig=o6FXP7XNl8nFHUnyn-GbpnB3fDg (visited on 09/27/2025).

Finnis, John (1999). "On the incoherence of legal positivism". In: *Notre Dame L. Rev.* 75, p. 1597. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/tndl75&section=49 (visited on 09/27/2025).

— (2011). "Philosophy of Law: Introduction". In: *John M. Finnis, PHILOSOPHY OF LAW: COLLECTED ESSAYS* 4, pp. 11–32. URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1850674 (visited on 09/27/2025).

Francus, Michael A. (2020). "The Law of Clarity and the Clarity of Law". In: *U. Chi. L. Rev. Online*, p. 1. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/uchidial88&section=13 (visited on 09/27/2025).

Fuller, Lon Luvois (1969). "The morality of law". In: URL: https://philpapers.org/rec/FULTMO-2 (visited on 09/27/2025).

Gallardo, Eudoxie (2017). "Les incohérences du droit pénal des mineurs contemporain". In: *Revue de science criminelle et de droit pénal comparé* 4.4, pp. 713–724. URL: https://shs.cairn.info/article/RSC_1704_0713 (visited on 09/27/2025).

Ganel, Tzvi, Carmel Sofer, and Melvyn A. Goodale (2022). "Biases in human perception of facial age are present and more exaggerated in current AI technology". In: *Scientific Reports* 12.1, p. 22519. URL: https://www.nature.com/articles/s41598-022-27009-w (visited on 09/27/2025).

García Máynez, Eduardo (2002). "Introducción al estudio del derecho". In: *México: Porrúa* 54. URL: https://www.academia.edu/download/31173020/D1021.pdf (visited on 09/27/2025).

Gardies, Jean-Louis (1971). "Une particularité du raisonnement juridique: la présence de fonctions complétives". In: *Logique et Analyse* 14.53/54,

pp. 63–69. URL: https://www.jstor.org/stable/44074434?casa_token=F2orjtzNzbIAAAAA:Czw0gWWAizIyapCEMjh_-5J6h5DB2Ib2U__P7rjuLseeJ1cxI5M3WqSior4tjWNEQFty3vvDwJUNnxjtnzfvELg (visited on 04/02/2026).

Gardies, Jean-Louis (1973). "Logique déontique et théorie générale des fonctions complétives". In: *Logique et analyse* 16.61/62, pp. 143–220. URL: https://www.jstor.org/stable/44083641?casa_token=9tv7WRe4pO8AAAAA:nT9drk6IkGFLHC83TGMqAUxJvcAM-ta9ctOLLYNPbi1hVWVPlIk2cEG8d66roPPPwjXWNdHjFKYmEMVNA_Skv6X941yCg (visited on 04/02/2026).

— (1979). *Essai sur la logique des modalités*. FeniXX. URL: https://books.google.com/books?hl=fr&lr=&id=BHF6DwAAQBAJ&oi=fnd&pg=PT3&dq=jean+louis+gardies+essai+sur+la+logique+des+modalit%C3%A9s&ots=WZknzqqh_E&sig=3wioRNjPCNxbAUyF7Elm44HFqlI (visited on 04/02/2026).

Gardiès, Jean-Louis (1976). "Modalités et normes". In: *ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of Law and Social Philosophy*, pp. 465–474. URL: https://www.jstor.org/stable/23678969?casa_token=cHbCjzOJfeoAAAAA:sBbZrU2Dtnoj3LLwL5YPikQIQLvntSehoLXb3VYGhewlntZRglsjhVkbCUEb_mSuJGpOke2usef4vbUgRilqPvpdeNPUOly6vTXaMtXMFoSjMs4g (visited on 04/02/2026).

Gaudemet-Tallon, Hélène (2006). *Le pluralisme en droit international privé: richesses et faiblesses (le funambule et l'arc-en-ciel): cours général*. Martinus Nijhoff.

Gautier, Pierre-Yves (2023). "Quelle langue parle le droit?" In: *Pouvoirs* 186.3, pp. 99–110. URL: https://shs.cairn.info/revue-pouvoirs-2023-3-page-99 (visited on 09/27/2025).

Gen, Kawachi et al. (2016). "Applying the Akoma Ntoso XML schema to Japanese legislation". In: *Journal of Law, Information and Science* 24.2, pp. 49–70. URL: https://search.informit.org/doi/abs/10.3316/informit.556807524334387 (visited on 09/27/2025).

Gibbons, John (1999). "Language and the law". In: *Annual Review of Applied Linguistics* 19, pp. 156–173. URL: https://www.cambridge.org/core/journals/annual-review-of-applied-linguistics/article/language-and-the-law/6D78A6A4594CF756FF119A9B84CAA516 (visited on 09/27/2025).

Gibbs, Jack P. (Mar. 1965). "Norms: The Problem of Definition and Classification". en. In: *American Journal of Sociology* 70.5, pp. 586–594. ISSN: 0002-9602, 1537-5390. DOI: 10.1086/223933. URL: https://www.journals.uchicago.edu/doi/10.1086/223933 (visited on 09/27/2025).

Glickman, Moshe and Tali Sharot (2022). "Biased AI systems produce biased humans". In: *OSF Prepr* 10. URL: https://scholar.archive.org/work/nwomk7onovc3vfsufe6fsogtpm/access/wayback/https:

//files.osf.io/v1/resources/c4e7r/providers/osfstorage/637340db0e715d3451a9999b?action=download&direct&version=1 (visited on 09/27/2025).

Glogar, Ondřej (June 2023). "The Concept of Legal Language: What Makes Legal Language 'Legal'?" en. In: *International Journal for the Semiotics of Law - Revue internationale de Sémiotique juridique* 36.3, pp. 1081–1107. ISSN: 0952-8059, 1572-8722. DOI: 10.1007/s11196-023-10010-5. URL: https://link.springer.com/10.1007/s11196-023-10010-5 (visited on 09/27/2025).

"5. The Rules of the Game" (2002). In: *Isabelle/HOL*. Ed. by Gerhard Goos et al. Vol. 2283. Series Title: Lecture Notes in Computer Science. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 67–104. ISBN: 978-3-540-43376-7 978-3-540-45949-1. DOI: 10.1007/3-540-45949-9_5. URL: http://link.springer.com/10.1007/3-540-45949-9_5 (visited on 09/27/2025).

Gordon, John-Stewart (June 2021). "AI and law: ethical, legal, and socio-political implications". en. In: *AI & SOCIETY* 36.2, pp. 403–404. ISSN: 0951-5666, 1435-5655. DOI: 10.1007/s00146-021-01194-0. URL: https://link.springer.com/10.1007/s00146-021-01194-0 (visited on 09/27/2025).

Gray, Horace (1892). "Supreme Court Of The United States". In: *US Reports: Nix v. Hedden*. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/joscus1891&section=7 (visited on 09/27/2025).

Green, Leslie (2004). "Law and obligations". In: URL: https://academic.oup.com/edited-volume/34507/chapter/292816462?searchresult=1&utm_source=TrendMD&utm_medium=cpc&utm_campaign=Oxford_Academic_Books_TrendMD_1 (visited on 09/27/2025).

Grimaud, Lin (2009). "Pourquoi formaliser en travail social?" In: *Empan* 75.3, pp. 134–145. URL: https://shs.cairn.info/article/EMPA_075_0134?tab=resume (visited on 09/27/2025).

Guastini, Riccardo (2018). "Two conceptions of norms". In: *Revus. Journal for Constitutional Theory and Philosophy of Law/Revija za ustavno teorijo in filozofijo prava* 35. URL: https://journals.openedition.org/revus/4849 (visited on 09/27/2025).

Guidere, Mathieu (Dec. 2010). "Introduction à la théorie analytique de la traduction et de l'interprétation". en. In: *Babel. Revue internationale de la traduction / International Journal of Translation* 56.4, pp. 299–312. ISSN: 0521-9744, 1569-9668. DOI: 10.1075/babel.56.4.01gui. URL: http://www.jbe-platform.com/content/journals/10.1075/babel.56.4.01gui (visited on 09/27/2025).

Guiorguieff, Johann (2014). “Le Conseil d’Etat garantit la prévisibilité de la loi, pas celle de la jurisprudence judiciaire. Principes de sécurité juridique et de confiance légitime (Responsabilité du fait des lois)”. In: *La Revue des droits de l’homme. Revue du Centre de recherches et d’études sur les droits fondamentaux*. URL: https://journals.openedition.org/revdh/864 (visited on 09/27/2025).

Gulati, Mitu and Catherine MA McCauliff (1998). “On" Not" Making Law”. In: *Law and Contemporary Problems* 61.3, pp. 157–227. URL: https://www.jstor.org/stable/1192422 (visited on 09/27/2025).

Gutterridge, H. C. (1933). “Abuse of rights”. In: *The Cambridge Law Journal* 5.1, pp. 22–45. URL: https://www.cambridge.org/core/journals/cambridge-law-journal/article/abuse-of-rights/B8E92D6778E35377614E6CE371B60C2 (visited on 09/27/2025).

Hage, Jaap (Mar. 2004). “Law and Coherence”. en. In: *Ratio Juris* 17.1, pp. 87–105. ISSN: 0952-1917, 1467-9337. DOI: 10.1111/j.0952-1917.2004.00257.x. URL: https://onlinelibrary.wiley.com/doi/10.1111/j.0952-1917.2004.00257.x (visited on 03/17/2026).

Hansson, SvenOve (Feb. 1990). “Preference-based deontic logic (PDL)”. en. In: *Journal of Philosophical Logic* 19.1. ISSN: 0022-3611, 1573-0433. DOI: 10.1007/BF00211186. URL: http://link.springer.com/10.1007/BF00211186 (visited on 09/27/2025).

Hart, Herbert Lionel Adolphus (2012). *The concept of law*. OUP Oxford. URL: https://books.google.com/books?hl=fr&lr=&id=YhFVBAAAQBAJ&oi=fnd&pg=PP1&dq=HART,+Herbert+Lionel+Adolphus.+The+concept+of+law.+OUP+Oxford,+2012.&ots=7EfBGDVS_y&sig=Ss0Ml65Z6v8TD3tTbgM6OBrL1Uk (visited on 09/27/2025).

Hildebrandt, Mireille (2020). “Code-driven law: freezing the future and scaling the past”. In: *Is law computable*, pp. 67–84. URL: https://www.torrossa.com/gs/resourceProxy?an=5204787&publisher=FZ0252#page=90 (visited on 03/18/2026).

Hollyer, James R., B. Peter Rosendorff, and James Raymond Vreeland (Oct. 2011). “Democracy and Transparency”. en. In: *The Journal of Politics* 73.4, pp. 1191–1205. ISSN: 0022-3816, 1468-2508. DOI: 10.1017/S0022381611000880. URL: https://www.journals.uchicago.edu/doi/10.1017/S0022381611000880 (visited on 09/27/2025).

Huttner, Liane and Denis Merigoux (Aug. 2022). “Catala: Moving towards the future of legal expert systems”. en. In: *Artificial Intelligence and Law*. ISSN: 0924-8463, 1572-8382. DOI: 10.1007/s10506-022-09328-5. URL: https://link.springer.com/10.1007/s10506-022-09328-5 (visited on 09/27/2025).

Incampo, Antonio (2016). "Sein und Sollen. From Kelsen to Schambeck". In: *The 80th Anniversary of Professor Schambeck*. Gdansk University Press, pp. 21–30. URL: https://ricerca.uniba.it/handle/11586/181107 (visited on 09/27/2025).

Irti, Claudia (2014). "A Short Introduction to 'The Problem of Legal Gaps". In: *Tulane European and Civil Law Forum*. Vol. 29. URL: https://journals.tulane.edu/teclf/article/view/1683/1530 (visited on 09/27/2025).

Jakimenko, Virginija (2013). "Logical Validity and Linguistic Correctness: Impact on Changes in Thinking". In: *Societal Studies* 5.4, pp. 1029–1039. URL: https://www3.mruni.eu/ojs/societal-studies/article/view/1613 (visited on 09/27/2025).

Jeanneney, Julien (2016). *Les lacunes constitutionnelles*. Dalloz; coll. Nouvelle bibliothèque de thèses. URL: https://hal.science/hal-03200864/ (visited on 09/27/2025).

Johnston, Michael (2006). "Good governance: Rule of law, transparency, and accountability". In: *New York: United Nations Public Administration Network* 1, p. 32. URL: https://www.researchgate.net/profile/Michael-Johnston-26/publication/267974525_Good_Governance_Rule_of_Law_Transparency_and_Accountability/links/570d0b1908aed31341cf00c6/Good-Governance-Rule-of-Law-Transparency-and-Accountability.pdf (visited on 09/27/2025).

Jones, Andrew JI and Ingmar Pörn (1985). "Ideality, sub-ideality and deontic logic". In: *Synthese*, pp. 275–290. URL: https://www.jstor.org/stable/20116198 (visited on 09/27/2025).

Jutras, Daniel (2009). "Cartographie de la mixité: La common law et la complétude du droit civil au Québec". In: *Can. B. Rev.* 88, p. 247. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/canbarev88&section=21 (visited on 09/27/2025).

Kalinowski, Georges (1972). *La logique des normes*. FeniXX. URL: https://books.google.com/books?hl=fr&lr=&id=Hs__EAAAQBAJ&oi=fnd&pg=PP1&dq=KALINOWSKI,+Georges.+La+logique+des+normes&ots=GrZwfQ_OZs&sig=DvwR9GJ6CxkiBqChMvcWQd3PHb4 (visited on 09/27/2025).

— (1974). "Sur un nouveau système de logique deontique". In: *Logique et Analyse* 17.67/68, pp. 343–387. URL: https://www.jstor.org/stable/44083887?casa_token=0zigT-_KYwUAAAAA:mzQHbxh3FtjGlnrHxJz_GGYnRV1dkurmd1jNO5Fu4JQPb53ThV9ThpbIDG-DX6SLph17Ry6brFclKK4v1-5Dw-Z_fhCIoug49yLDN2NmxevaWkfpuA (visited on 04/02/2026).

Katz, Daniel Martin, Ron Dolin, and Michael J. Bommarito (2021). *Legal informatics*. Cambridge University Press. URL: https://books.google.com/books?hl=fr&lr=&id=c_wYEAAAQBAJ&oi=fnd&pg=PR9&dq=KATZ,

+Daniel+Martin,+DOLIN,+Ron,+et+BOMMARITO,+Michael+J.+(ed.).+Legal+informatics&ots=Vlt87C-pEF&sig=92cDihKr-5GzJ9C-HadB5qn8qAc (visited on 09/27/2025).

Katz, Leo (Jan. 2010). "A Theory of Loopholes". en. In: *The Journal of Legal Studies* 39.1, pp. 1–31. ISSN: 0047-2530, 1537-5366. DOI: 10.1086/649046. URL: https://www.journals.uchicago.edu/doi/10.1086/649046 (visited on 09/27/2025).

Kelsen, Hans (1967). *Pure theory of law*. Univ of California Press. URL: https://books.google.com/books?hl=fr&lr=&id=DttWqqkBbZ8C&oi=fnd&pg=PA1&dq=kelsen+pure+theory+of+law&ots=SqUYgj8AqB&sig=kqRbqFdB35Bkqdr2ybqjFHxrfF8 (visited on 09/27/2025).

— (1973). "On the Concept of Norm". en. In: *Essays in Legal and Moral Philosophy*. Dordrecht: Springer Netherlands, pp. 216–227. ISBN: 978-94-010-2655-0 978-94-010-2653-6. DOI: 10.1007/978-94-010-2653-6_9. URL: http://link.springer.com/10.1007/978-94-010-2653-6_9 (visited on 09/27/2025).

— (2017). *General theory of law and state*. Routledge. URL: https://www.taylorfrancis.com/books/mono/10.4324/9780203790960/general-theory-law-state-hans-kelsen (visited on 09/27/2025).

Kerr, James M. (1922). "Fixing the Law and Finding the Law". In: *Law. & Banker & S. Bench & B. Rev.* 15, p. 275. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/lbancelj15&section=59 (visited on 09/27/2025).

Kowalski, Robert and Akber Datoo (June 2022). "Logical English meets legal English for swaps and derivatives". en. In: *Artificial Intelligence and Law* 30.2, pp. 163–197. ISSN: 0924-8463, 1572-8382. DOI: 10.1007/s10506-021-09295-3. URL: https://link.springer.com/10.1007/s10506-021-09295-3 (visited on 09/27/2025).

Kowalski, Robert, Jacinto Dávila, et al. (2023). "Logical English for Law and Education". en. In: *Prolog: The Next 50 Years*. Ed. by David S. Warren et al. Vol. 13900. Series Title: Lecture Notes in Computer Science. Cham: Springer Nature Switzerland, pp. 287–299. ISBN: 978-3-031-35253-9 978-3-031-35254-6. DOI: 10.1007/978-3-031-35254-6_24. URL: https://link.springer.com/10.1007/978-3-031-35254-6_24 (visited on 09/27/2025).

Kowalski, Robert A. (1992). "Legislation as logic programs". In: *Logic Programming in Action*. Ed. by J. Siekmann et al. Vol. 636. Series Title: Lecture Notes in Computer Science. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 203–230. ISBN: 978-3-540-55930-6 978-3-540-47312-1. DOI: 10.1007/3-540-55930-2_15. URL: http://link.springer.com/10.1007/3-540-55930-2_15 (visited on 09/27/2025).

Kristan, Andrej (2014). "In Defence of the Expressive Conception of Norms". In: *Revus. Journal for Constitutional Theory and Philosophy of Law/Revija za ustavno teorijo in filozofijo prava* 22, pp. 151–172. URL: https://journals.openedition.org/revus/2883 (visited on 03/18/2026).

Lakoff, George (Dec. 1970). "Linguistics and natural logic". en. In: *Synthese* 22.1-2, pp. 151–271. ISSN: 0039-7857, 1573-0964. DOI: 10.1007/BF00413602. URL: http://link.springer.com/10.1007/BF00413602 (visited on 09/27/2025).

Lam, Ho-Pun and Guido Governatori (2009). "The Making of SPINdle". In: *Rule Interchange and Applications*. Ed. by David Hutchison et al. Vol. 5858. Series Title: Lecture Notes in Computer Science. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 315–322. ISBN: 978-3-642-04984-2 978-3-642-04985-9. DOI: 10.1007/978-3-642-04985-9_29. URL: http://link.springer.com/10.1007/978-3-642-04985-9_29 (visited on 09/27/2025).

Lauterpacht, Hersch (2011). *The function of law in the international community*. OUP Oxford. URL: https://books.google.com/books?hl=fr&lr=&id=P6LFAgAAQBAJ&oi=fnd&pg=PP1&dq=LAUTERPACHT,+Hersch.+The+function+of+law+in+the+international+community.+OUP+Oxford,+2011.&ots=uqwXpSwjfI&sig=MZ8T2Bo1F5RkwHJtenT0o5f7u9Y (visited on 09/27/2025).

Leffer, Lauren (2023). "Humans absorb bias from AI—and keep it after they stop using the algorithm". In: *Scientific American* 26. URL: https://courtleader.net/wp-content/uploads/2024/05/humans-absorb-bias-from-ai-and-keep-it-after-they-stop-using-the-algorithm-_-scientific-american.pdf (visited on 09/27/2025).

Leone, Valentina (2021). "Legal knowledge extraction in the data protection domain based on Ontology Design Patterns". In: URL: https://orbilu.uni.lu/handle/10993/47854 (visited on 09/27/2025).

Letwin, Jeremy (2021). "Why Completeness and Coherence Matter for the European Court of Human Rights". In: *European Convention on Human Rights Law Review, The*. Vol. 2. 1. Brill Nijhoff, pp. 119–154. URL: https://brill.com/view/journals/eclr/2/1/article-p119_119.xml (visited on 09/27/2025).

Lévy-Bruhl, Henri (1950). "La Science du Droit ou" Juristique"". In: *Cahiers internationaux de sociologie* 8, pp. 123–133. URL: https://www.jstor.org/stable/40688734 (visited on 09/27/2025).

Lewis, David (1975). "Languages and language". In: URL: https://conservancy.umn.edu/bitstreams/361dde7c-8e3e-463c-a74b-eb5d79ce7f8a/download (visited on 09/27/2025).

LIBAL, Tomer (2020). "Towards automating inconsistency checking of legal texts". In: *Internationales Rechtsinformatik Symposion*. URL: https://orbilu.uni.lu/handle/10993/45476 (visited on 09/27/2025).

Libal, Tomer (2020). "A Meta-level Annotation Language for Legal Texts". en. In: *Logic and Argumentation*. Ed. by Mehdi Dastani, Huimin Dong, and Leon Van Der Torre. Vol. 12061. Series Title: Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 131–150. ISBN: 978-3-030-44637-6 978-3-030-44638-3. DOI: 10.1007/978-3-030-44638-3_9. URL: http://link.springer.com/10.1007/978-3-030-44638-3_9 (visited on 09/27/2025).

— (Dec. 2022). "The LegAi Editor: A Tool for the Construction of Legal Knowledge Bases". In: *Frontiers in Artificial Intelligence and Applications*. Ed. by Enrico Francesconi, Georg Borges, and Christoph Sorge. IOS Press. ISBN: 978-1-64368-364-5 978-1-64368-365-2. DOI: 10.3233/FAIA220482. URL: https://ebooks.iospress.nl/doi/10.3233/FAIA220482 (visited on 09/27/2025).

— (2023). "Legal linguistic templates and the tension between legal knowledge representation and reasoning". In: *Frontiers in Artificial Intelligence* 6, p. 1136263. URL: https://www.frontiersin.org/articles/10.3389/frai.2023.1136263/full (visited on 09/27/2025).

Libal, Tomer and Tereza Novotná (2021). "Towards Transparent Legal Formalization". en. In: *Explainable and Transparent AI and Multi-Agent Systems*. Ed. by Davide Calvaresi et al. Vol. 12688. Series Title: Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 296–313. ISBN: 978-3-030-82016-9 978-3-030-82017-6. DOI: 10.1007/978-3-030-82017-6_18. URL: https://link.springer.com/10.1007/978-3-030-82017-6_18 (visited on 09/27/2025).

Libal, Tomer and Matteo Pascucci (June 2019). "Automated reasoning in normative detachment structures with ideal conditions". en. In: *Proceedings of the Seventeenth International Conference on Artificial Intelligence and Law*. Montreal QC Canada: ACM, pp. 63–72. ISBN: 978-1-4503-6754-7. DOI: 10.1145/3322640.3326707. URL: https://dl.acm.org/doi/10.1145/3322640.3326707 (visited on 09/27/2025).

Libal, Tomer and Alexander Steen (2019). "The NAI suite–drafting and reasoning over legal texts". In: *Legal Knowledge and Information Systems*. IOS Press, pp. 243–246. URL: https://ebooks.iospress.nl/volumearticle/53680 (visited on 09/27/2025).

Lokhorst, Gert -Jan C. (July 1996). "Reasoning about actions and obligations in first-order logic". en. In: *Studia Logica* 57.1, pp. 221–237. ISSN: 0039-3215, 1572-8730. DOI: 10.1007/BF00370676. URL: http://link.springer.com/10.1007/BF00370676 (visited on 09/27/2025).

Losano, Mario G. (1971). "The Legal System from Theology to Technology". In: *ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of Law and Social Philosophy*, pp. 485–502. URL: https://www.jstor.org/stable/23678518 (visited on 09/27/2025).

Mackaay, Ejan (1994). *Législation canadienne en propriété intellectuelle*. Scarborough, Ont.: Carswell.

— (2018). "L'inflation normative". In: URL: https://lexelectronica.openum.ca/files/sites/103/NE_art3_EM.pdf (visited on 09/27/2025).

Makinson, David (1999). "On a fundamental problem of deontic logic". In: *Norms, logics and information systems. New studies on deontic logic and computer science* 49, pp. 29–54. URL: https://www.researchgate.net/profile/David-Makinson-2/publication/228558710_On_a_fundamental_problem_of_deontic_logic/links/0046353c67786cb950000000/On-a-fundamental-problem-of-deontic-logic.pdf (visited on 09/27/2025).

Malaurie, Philippe (2005). "L'intelligibilité des lois". In: *Pouvoirs* 114.3, pp. 131–137. URL: https://shs.cairn.info/article/POUV_114_0131 (visited on 09/27/2025).

Marmor, Andrei (2018). "Varieties of Vagueness in the Law". en. In: *Handbook of Legal Reasoning and Argumentation*. Ed. by Giorgio Bongiovanni et al. Dordrecht: Springer Netherlands, pp. 561–580. ISBN: 978-90-481-9451-3 978-90-481-9452-0. DOI: 10.1007/978-90-481-9452-0_19. URL: http://link.springer.com/10.1007/978-90-481-9452-0_19 (visited on 03/18/2026).

Marschik, Axel (1998). "Too much order? The impact of special secondary norms on the unity and efficacy of the international legal system". In: *Eur. J. Int'l L.* 9, p. 212. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/eurint9&section=16 (visited on 09/27/2025).

Masbernat, Patricio (2007). "Lagunas en el Derecho". en. In: *Ius et Praxis* 13.1. ISSN: 0718-0012. DOI: 10.4067/S0718-00122007000100018. URL: http://www.scielo.cl/scielo.php?script=sci_arttext&pid=S0718-00122007000100018&lng=en&nrm=iso&tlng=en (visited on 09/27/2025).

McNamara, Paul and Frederik Van de Putte (2022). *Deontic Logic*.

Merigoux, Denis, Nicolas Chataing, and Jonathan Protzenko (Aug. 2021). "Catala: a programming language for the law". en. In: *Proceedings of the ACM on Programming Languages* 5.ICFP, pp. 1–29. ISSN: 2475-1421. DOI: 10.1145/3473582. URL: https://dl.acm.org/doi/10.1145/3473582 (visited on 09/27/2025).

Mohammed, Essam T. (2009). "Polysemy as a Lexical Problem in Translation." In: *Online Submission* 55, pp. 1–19. URL: https://eric.ed.gov/?id=ED601174 (visited on 09/27/2025).

Monnier, François (2001). "Déni de justice". In: *Droits* 34.2, pp. 91–94. URL: https://droit.cairn.info/revue-droits-2001-2-page-91 (visited on 09/27/2025).

Montaigne, Michel de and Samuel Sylvestre de Sacy (1937). "Apologie de Raimond Sebond". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130282269879335040 (visited on 09/27/2025).

Morehouse, Libby (2024). "The Kids Are Not Alright: A Look into the Absence of Laws Protecting Children in Social Media". In: *Loyola of Los Angeles Entertainment Law Review* 44.2, p. 74. URL: https://digitalcommons.lmu.edu/cgi/viewcontent.cgi?article=1663&context=elr (visited on 09/27/2025).

Morel, Benjamin (2020). "Les fins de la loi, la fin de la loi". In: *Droit et société* 104.1, pp. 235–247. URL: https://droit.cairn.info/revue-droit-et-societe-2020-1-page-235?tab=texte-integral (visited on 09/27/2025).

Moreso, José Juan (1998). "A Logical Analysis of Legal Propositions". In: *Legal Indeterminacy and Constitutional Interpretation*. Ed. by Francisco J. Laporta et al. Vol. 37. Series Title: Law and Philosophy Library. Dordrecht: Springer Netherlands, pp. 47–100. ISBN: 978-90-481-5061-8 978-94-015-9123-2. DOI: 10.1007/978-94-015-9123-2_3. URL: http://link.springer.com/10.1007/978-94-015-9123-2_3 (visited on 09/27/2025).

Moretti, Alessio (July 2009). "The Geometry of Standard Deontic Logic". en. In: *Logica Universalis* 3.1, pp. 19–57. ISSN: 1661-8297, 1661-8300. DOI: 10.1007/s11787-009-0003-4. URL: http://link.springer.com/10.1007/s11787-009-0003-4 (visited on 09/27/2025).

Moron-Puech, Benjamin, Dominique Fenouillet, and Jean Combacau (2020). "L'acte juridique, une réponse à la crise du contrat". In: *(No Title)*. URL: https://cir.nii.ac.jp/crid/1130848326094177792 (visited on 09/27/2025).

Morris, Richard T. (1956). "A typology of norms". In: *American Sociological Review* 21.5, pp. 610–613. URL: https://www.jstor.org/stable/2089098 (visited on 09/27/2025).

Mukhtarova, S. S. et al. (2019). "Translating the untranslatable: challenges, sticking points and struggles". In: *Space and Culture, India* 7.1, pp. 250–263. URL: https://elibrary.ru/item.asp?id=41615768 (visited on 09/27/2025).

Nakamura, Makoto (2022). "Development of applications for open data for Japanese laws and regulations". In: *J. Open Access L.* 10, p. 1. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/jopacc10&section=4 (visited on 09/27/2025).

Nakata, I and M Hagiya (Sept. 1991). *Software Science and Engineering: Selected Papers from the Kyoto Symposia*. en. WORLD SCIENTIFIC. ISBN: 978-981-02-0776-2 978-981-4360-12-8. DOI: 10.1142/1496. URL: https://www.worldscientific.com/worldscibooks/10.1142/1496 (visited on 09/27/2025).

Navarro, Pablo E. and Jorge L. Rodríguez (2014). *Deontic logic and legal systems*. Cambridge university press. URL: https://books.google.com/books?hl=fr&lr=&id=SkVpBAAAQBAJ&oi=fnd&pg=PR9&dq=navarro+rodr%C3%ADguez+deontic+logic&ots=lYcVwQBAEg&sig=Zv7BB3m3-MpbZMuIPaxKPQvcnM0 (visited on 09/27/2025).

Nazarenko, Adeline, François Levy, and Adam Wyner (2018). "An annotation language for semantic search of legal sources". In: *Proceedings of the Eleventh International Conference on Language Resources and Evaluation (LREC 2018)*. URL: https://aclanthology.org/L18-1177.pdf (visited on 09/27/2025).

Nickel, James W. (1993). "How human rights generate duties to protect and provide". In: *Hum. Rts. Q.* 15, p. 77. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/hurq15&section=11 (visited on 09/27/2025).

Nola, Robert (2003). "Naturalism and Norms of Reason and Method". In: *Rescuing Reason*. Ed. by Robert S. Cohen, Jürgen Renn, and Kostas Gavroglu. Vol. 230. Series Title: Boston Studies in the Philosophy of Science. Dordrecht: Springer Netherlands, pp. 117–172. ISBN: 978-1-4020-1043-9 978-94-010-0289-9. DOI: 10.1007/978-94-010-0289-9_4. URL: http://link.springer.com/10.1007/978-94-010-0289-9_4 (visited on 03/18/2026).

Novotná, Tereza and Tomer Libal (2022). "An Evaluation of Methodologies for Legal Formalization". en. In: *Explainable and Transparent AI and Multi-Agent Systems*. Ed. by Davide Calvaresi et al. Vol. 13283. Series Title: Lecture Notes in Computer Science. Cham: Springer International Publishing, pp. 189–203. ISBN: 978-3-031-15564-2 978-3-031-15565-9. DOI: 10.1007/978-3-031-15565-9_12. URL: https://link.springer.com/10.1007/978-3-031-15565-9_12 (visited on 09/27/2025).

Nuret, Bertrand (2014). "Juridiction". In: *Dictionnaire d'administration publique*. Presses universitaires de Grenoble, pp. 284–285. URL: https://shs.cairn.info/article/PUG_KADA_2014_01_0284?tab=texte-integral (visited on 09/27/2025).

Oliveira Carvalho, Catarina de (2024). "The great inequality gaps: a brief overview1". In: *The quest for labour rights and social justice*. URL: https://www.torrossa.com/gs/resourceProxy?an=5862414&publisher=FM0520#page=60 (visited on 09/27/2025).

Opalek, K. and J. Wolenski (1986). "On weak and strong permissions once more". In: *Rechtstheorie* 17, p. 83. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/recthori17&section=7 (visited on 03/15/2026).

Opalek, Kazimierz and Jan Wolenski (1973). "On weak and strong permissions". In: *Rechtstheorie* 4, p. 169. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/recthori4&section=19 (visited on 03/15/2026).

Oudin, Martin (2000). "L'execution du contrat en bon pere de famille (l'obligation de comportement en droit civil compare)". PhD Thesis. Paris 11. URL: https://theses.fr/2000PA111016 (visited on 09/27/2025).

Palmirani, Monica and Fabio Vitali (2011). "Akoma-Ntoso for Legal Documents". en. In: *Legislative XML for the Semantic Web*. Ed. by Giovanni Sartor et al. Vol. 4. Series Title: Law, Governance and Technology Series. Dordrecht: Springer Netherlands, pp. 75–100. ISBN: 978-94-007-1886-9 978-94-007-1887-6. DOI: 10.1007/978-94-007-1887-6_6. URL: https://link.springer.com/10.1007/978-94-007-1887-6_6 (visited on 09/27/2025).

Patterson, Dennis (2005). "Interpretation in law". In: *San Diego L. Rev.* 42, p. 685. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/sanlr42&section=39 (visited on 09/27/2025).

Pavcnik, Marijan (1996). *Why Discuss Gaps in the Law?* URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/raju9&section=7 (visited on 09/27/2025).

Peregrin, Jaroslav and Vladimír Svoboda (2016). "Logical formalization and the formation of logic (s)". In: *Logique et Analyse* 233, pp. 55–80. URL: https://poj.peeters-leuven.be/content.php?id=3149531&url=article (visited on 09/27/2025).

Pereira, Fernando Carlos Neves (1982). "Logic for natural language analysis". In: URL: https://era.ed.ac.uk/handle/1842/6646 (visited on 09/27/2025).

Pethick, Stephen (2008). "Solving the impossible: the puzzle of coherence, consistency and law". In: *N. Ir. Legal Q.* 59, p. 395. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/nilq59&section=36 (visited on 09/27/2025).

Pham, Anh Tuan, Linh Thi Diem Nguyen, and Vi Thi Tuong Pham (2022). "English language students' perspectives on the difficulties in translation:

Implications for language education." In: *Journal of Language and Linguistic Studies* 18.1, pp. 180–189. URL: https://eric.ed.gov/?id=EJ1325750 (visited on 09/27/2025).

Piazzon, Thomas (2006). "La sécurité juridique". PhD Thesis. Paris 2. URL: https://theses.fr/2006PA020028 (visited on 09/27/2025).

Postema, Gerald J. (Aug. 1994). "Implicit law". en. In: *Law and Philosophy* 13.3, pp. 361–387. ISSN: 0167-5249, 1573-0522. DOI: 10.1007/BF01001679. URL: http://link.springer.com/10.1007/BF01001679 (visited on 09/27/2025).

Prakken, Henry and Marek Sergot (1997). "Dyadic Deontic Logic and Contrary-to-Duty Obligations". en. In: *Defeasible Deontic Logic*. Ed. by Donald Nute. Dordrecht: Springer Netherlands, pp. 223–262. ISBN: 978-90-481-4874-5 978-94-015-8851-5. DOI: 10.1007/978-94-015-8851-5_10. URL: http://link.springer.com/10.1007/978-94-015-8851-5_10 (visited on 09/27/2025).

Prigent, Stephane (Oct. 2020). "Le démembrement de propriété". French. In: *Revue Française de Comptabilité* 546. Place: Paris, pp. 18–21. ISSN: 04848764. URL: https://scpo.idm.oclc.org/login?url=https://www.proquest.com/trade-journals/le-d%C3%A9membrement-de-propri%C3%A9t%C3%A9/docview/2464651333/se-2?accountid=13739.

Prior, Arthur N. (1954). "The paradoxes of derived obligation". In: *Mind* 63.249, pp. 64–65. URL: https://academic.oup.com/mind/article-abstract/LXIII/249/64/995760 (visited on 09/27/2025).

Raz, Joseph (1980). *The concept of a legal system: an introduction to the theory of a legal system*. Oxford University Press. URL: https://academic.oup.com/book/10825?searchresult=1&utm_source=TrendMD&utm_medium=cpc&utm_campaign=Oxford_Academic_Books_TrendMD_0 (visited on 09/27/2025).

— (1992). "The relevance of coherence". In: *BUL Rev.* 72, p. 273. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/bulr72&section=15 (visited on 09/27/2025).

— (2003). "About morality and the nature of law". In: *Am. J. Juris.* 48, p. 1. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/ajj48&section=4 (visited on 09/27/2025).

Read, Stephen (1988). *Relevant logic*. Oxford: Blackwell. URL: http://www.st-andrews.ac.uk/~slr/Relevant_Logic.pdf (visited on 09/27/2025).

Rejmaniak, Rafał (2021). "Bias in artificial intelligence systems". In: *Białostockie Studia Prawnicze* 3.26, pp. 25–42. URL: https://www.ceeol.com/search/article-detail?id=986514 (visited on 09/27/2025).

Ribeiro, Marc (2004). *Limiting arbitrary power: the vagueness Doctrine in Canadian constitutional law*. UBC Press. URL: https://books.google.com/books?hl=fr&lr=&id=EJ3TDYwq2jEC&oi=fnd&pg=PA1&dq=RIBEIRO,+Marc.+Limiting+arbitrary+power:+the+vagueness+Doctrine+in+Canadian+constitutional+law.+UBC+Press,+2004&ots=l2M7asVR-l&sig=KYOO3ZI5obUeDvL5NZfHmyFTXMQ (visited on 09/27/2025).

Robaldo, Livio et al. (Dec. 2020). "Formalizing GDPR Provisions in Reified I/O Logic: The DAPRECO Knowledge Base". en. In: *Journal of Logic, Language and Information* 29.4, pp. 401–449. ISSN: 0925-8531, 1572-9583. DOI: 10.1007/s10849-019-09309-z. URL: http://link.springer.com/10.1007/s10849-019-09309-z (visited on 09/27/2025).

Ross, Alf (1944). "Imperatives and logic". In: *Philosophy of Science* 11.1, pp. 30–46. URL: https://www.cambridge.org/core/journals/philosophy-of-science/article/imperatives-and-logic/81AE86DF302E06D582DEB6F9D806BDEC (visited on 09/27/2025).

Rouse, William B. and Nancy M. Morris (1986). "On looking into the black box: Prospects and limits in the search for mental models." In: *Psychological bulletin* 100.3, p. 349. URL: https://psycnet.apa.org/record/1987-09358-001 (visited on 09/27/2025).

Routen, Tom and Trevor Bench-Capon (1991). "Hierarchical formalizations". In: *International Journal of Man-Machine Studies* 35.1, pp. 69–93. URL: https://www.sciencedirect.com/science/article/pii/S0020737307800083 (visited on 09/27/2025).

Rouvière, Frédéric (2021). "L'esprit contre la lettre: un fauteuil roulant électrique n'est pas un véhicule terrestre à moteur". In: *RTDCiv. Revue trimestrielle de droit civil* 4, p–975. URL: https://amu.hal.science/hal-03574804/document (visited on 09/27/2025).

— (2022). "Typologie des problèmes juridiques et argumentation". In: *RTDCiv. Revue trimestrielle de droit civil* 03, p. 559. URL: https://shs.hal.science/halshs-03825446/document (visited on 03/17/2026).

Sales, Tainah Simões (2013). "The gaps and the legal system (in) completeness". In: *Revista da Faculdade de Direito da UERJ-RFD-Rio de* 1.23. URL: https://www.academia.edu/download/118199398/4759.pdf (visited on 09/27/2025).

Sassier, Philippe and Dominique Lansoy (2008). *Ubu loi*. Fayard. URL: https://books.google.com/books?hl=fr&lr=&id=xoaNjJDumlAC&oi=fnd&pg=PA1994&dq=SASSIER,+Philippe+et+LANSOY,+Dominique.+Ubu+loi.+Fayard,+2008.+introduction.&ots=Lb2hgm6Co-&sig=PoQL1VQGfxcLWahgf_yb_1hI1Pc (visited on 09/27/2025).

Satoh, Ken et al. (2011). "PROLEG: An Implementation of the Presupposed Ultimate Fact Theory of Japanese Civil Code by PROLOG Tech-

nology". en. In: *New Frontiers in Artificial Intelligence*. Ed. by Takashi Onada, Daisuke Bekki, and Elin McCready. Vol. 6797. Series Title: Lecture Notes in Computer Science. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 153–164. ISBN: 978-3-642-25654-7 978-3-642-25655-4. DOI: 10.1007/978-3-642-25655-4_14. URL: https://link.springer.com/10.1007/978-3-642-25655-4_14 (visited on 09/27/2025).

Schneiderman, David (2024). "Weber, Fuller, and Belief in the Legitimacy of Law". In: *Max Weber Studies* 24.2, pp. 183–216. URL: https://muse.jhu.edu/pub/421/article/938237/summary (visited on 09/27/2025).

Schwitter, Rolf (2021). "The Grammar of PENG ASP Explained". In: *Proceedings of the Seventh International Workshop on Controlled Natural Language (CNL 2020/21)*. URL: https://aclanthology.org/2021.cnl-1.5.pdf (visited on 09/27/2025).

Scobbie, Iain GM (1997). "The Theorist as Judge: Hersch Lauterpacht's Concept of the International Judicial Function". In: *Eur. J. Int'l L.* 8, p. 264. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/eurint8&section=30 (visited on 09/27/2025).

Secondat Montesquieu, Charles-Louis de and Charles de Secondat baron de Montesquieu (2005). *The spirit of laws*. The Lawbook Exchange, Ltd. URL: https://books.google.com/books?hl=fr&lr=&id=ueKGVv-hyEkC&oi=fnd&pg=PR1&dq=MONTESQUIEU+CHARLES-LOUIS+DE+SECONDAT,,+De.+De+l%27esprit+des+lois.+1748&ots=4J42fR-m4G&sig=uiLQVWkRaY1E-PlC4M44SKKdtiA (visited on 09/27/2025).

"Legal Conventionalism" (2023). en. In: *Encyclopedia of the Philosophy of Law and Social Philosophy*. Ed. by Mortimer Sellers and Stephan Kirste. Dordrecht: Springer Netherlands, pp. 1905–1905. ISBN: 978-94-007-6518-4 978-94-007-6519-1. DOI: 10.1007/978-94-007-6519-1_300053. URL: https://link.springer.com/10.1007/978-94-007-6519-1_300053 (visited on 03/14/2026).

Sénac, Charles-Édouard (2022). "Y a-t-il encore place pour la découverte de nouveaux principes fondamentaux reconnus par les lois de la République?" In: *Titre VII* 8.1, pp. 27–34. URL: https://droit.cairn.info/article/TVII_008_0027 (visited on 09/27/2025).

Sergot, M. J. et al. (May 1986). "The British Nationality Act as a logic program". en. In: *Communications of the ACM* 29.5, pp. 370–386. ISSN: 0001-0782, 1557-7317. DOI: 10.1145/5689.5920. URL: https://dl.acm.org/doi/10.1145/5689.5920 (visited on 09/27/2025).

Seuren, Pieter AM (2010). *Operators and nucleus: a contribution to the theory of grammar*. Vol. 2. Cambridge University Press. URL: https://books.google.com/books?hl=fr&lr=&id=MlG01v3WXesC&oi=fnd&pg=PA1&dq=P.+Seuren,+Operators+and+nucleus:+a+contribution+to+the+

theory+of+grammar.+London:+Cambridge+University+Press,+I969.+pp.+X%2B230.&ots=QgN-mY5j_C&sig=SAh-L17-t6PfWv9U3n8tbl08akA (visited on 09/27/2025).

Shahid, Anum, Gohar Masood Qureshi, and Faiza Chaudhary (2023). "Transforming legal practice: The role of AI in modern law". In: *Journal of Strategic Policy and Global Affairs* 4.01, pp. 36–42. URL: https://www.researchgate.net/profile/Gohar-Qureshi/publication/376521706_36_Transforming_Legal_Practice_The_Role_of_AI_in_Modern_Law/links/657c111ffc4b416622c98f28/36-Transforming-Legal-Practice-The-Role-of-AI-in-Modern-Law.pdf (visited on 09/27/2025).

Shannon, David T. and Forouzan Golshani (1987). "On the automation of legal reasoning". In: *Jurimetrics J.* 28, p. 305. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/juraba28&section=35 (visited on 09/27/2025).

Skinner, Quentin (2008). "Freedom as the absence of arbitrary power". In: *Republicanism and political theory* 3.4, pp. 83–101. URL: https://books.google.com/books?hl=fr&lr=&id=zxgnZx0ccuQC&oi=fnd&pg=PA83&dq=SKINNER,+Quentin.+Freedom+as+the+absence+of+arbitrary+power.+Republicanism+and+political+theory,+2008,+vol.+3,+no+4,+p.+83-101.&ots=ItympKjMui&sig=N0gYNIuKBQTivio9IN-pWK2Jgwk (visited on 09/27/2025).

Smith, Brian Cantwell (Jan. 1985). "The limits of correctness". en. In: *ACM SIGCAS Computers and Society* 14,15.1,2,3,4, pp. 18–26. ISSN: 0095-2737. DOI: 10.1145/379486.379512. URL: https://dl.acm.org/doi/10.1145/379486.379512 (visited on 09/27/2025).

Soavi, Michele et al. (2022). "Semantic annotation of legal contracts with ContrattoA". In: *Informatics*. Vol. 9. 4. MDPI, p. 72. URL: https://www.mdpi.com/2227-9709/9/4/72 (visited on 09/27/2025).

Spaak, Torben (2023). "Consistency in Law". In: *Faculty of Law, Stockholm University Research Paper* 132. URL: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4418149 (visited on 09/27/2025).

Steyn, Karen (Mar. 1997). "Consistency – A Principle of Public Law?" en. In: *Judicial Review* 2.1, pp. 22–26. ISSN: 1085-4681, 1757-8434. DOI: 10.1080/10854681.1997.11426924. URL: https://www.tandfonline.com/doi/full/10.1080/10854681.1997.11426924 (visited on 09/27/2025).

Stray, Jonathan (2023). "The AI learns to lie to please you: preventing biased feedback loops in machine-assisted intelligence analysis". In: *Analytics* 2.2, pp. 350–358. URL: https://www.mdpi.com/2813-2203/2/2/20 (visited on 09/27/2025).

Svensson, Måns (2016). "Norms in law and society: towards a definition of the socio-legal concept of norms". In: *Social and legal norms*. Routledge, pp. 39–52. URL: https://www.taylorfrancis.com/chapters/edit/10.4324/9781315609416-4/norms-law-society-towards-definition-socio-legal-concept-norms-m%C3%A5ns-svensson (visited on 09/27/2025).

Tagawa, Takahiro and Jingde Cheng (2002). "Deontic Relevant Logic: A Strong Relevant Logic Approach to Removing Paradoxes from Deontic Logic". In: *PRICAI 2002: Trends in Artificial Intelligence*. Ed. by G. Goos et al. Vol. 2417. Series Title: Lecture Notes in Computer Science. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 39–48. ISBN: 978-3-540-44038-3 978-3-540-45683-4. DOI: 10.1007/3-540-45683-X_7. URL: http://link.springer.com/10.1007/3-540-45683-X_7 (visited on 09/27/2025).

Tamanaha, Brian Z. (2007). "The contemporary relevance of legal positivism". In: *Austl. J. Leg. Phil.* 32, p. 1. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/ajlph32&section=3 (visited on 09/27/2025).

Tanaka, Koji et al. (2012). *Paraconsistency: Logic and applications*. Vol. 26. Springer Science & Business Media. URL: https://books.google.com/books?hl=fr&lr=&id=vbYPVdHBnQoC&oi=fnd&pg=PR5&dq=TANAKA,+Koji,+BERTO,+Francesco,+MARES,+Edwin,+et+al.+(ed.).+Paraconsistency:+Logic+and+applications.+Springer+Science+%5C%26+Business+Media,+2012.&ots=w-IlbAKPY9&sig=q1TXzqkaup7wuuORrhOqxqwfPkI (visited on 09/27/2025).

Tella, María José Falcón y (2010). *A three-dimensional theory of law*. Martinus Nijhoff Publishers. URL: https://brill.com/downloadpdf/display/title/17285.pdf (visited on 09/27/2025).

Troper, Michel (2011). "Chapitre II. Le «dogme» de la complétude". In: *Léviathan*, pp. 19–29. URL: https://shs.cairn.info/article/PUF_TROPE_2011_02_0019?tab=texte-integral (visited on 09/27/2025).

Tucker, Edwin W. (1965). "The morality of law, by Lon L. Fuller". In: *Indiana Law Journal* 40.2, p. 5. URL: https://www.repository.law.indiana.edu/cgi/viewcontent.cgi?article=3571&context=ilj (visited on 09/27/2025).

Turk, Matthew C. (2019). "Overlapping legal rules in financial regulation and the administrative state". In: *Ga. L. Rev.* 54, p. 791. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/geolr54&section=24 (visited on 09/27/2025).

Tusseau, Guillaume (2006). *Les normes d'habilitation*. Dalloz. URL: https://sciencespo.hal.science/hal-03595929/ (visited on 09/27/2025).

Urvoas, Jean-Jacques (2018). "Quantité, qualité, efficacité des lois, quel bilan?" In: *Revue française de droit constitutionnel* 116.4, pp. 905–917. URL: https://shs.cairn.info/article/RFDC_116_0905?tab=texte-integral (visited on 09/27/2025).

Van Bendegem, Jean Paul (1994). "Ross' Paradox is an impossible supertask". In: *The British Journal for the Philosophy of Science* 45.2, pp. 743–748. URL: https://www.jstor.org/stable/687695 (visited on 09/27/2025).

Van De Putte, Frederik and Christian Straßer (2012). "A logic for prioritized normative reasoning". In: *Journal of Logic and Computation* 23.3, pp. 563–583. URL: https://academic.oup.com/logcom/article-abstract/23/3/563/1028847 (visited on 09/27/2025).

Van Meerbeeck, Jérémie (2019). *De la certitude à la confiance: le principe de sécurité juridique dans la jurisprudence de la Cour de justice de l'Union européenne*. Presses universitaires Saint-Louis Bruxelles. URL: https://books.google.com/books?hl=fr&lr=&id=IgqfDwAAQBAJ&oi=fnd&pg=PA7&dq=VAN+MEERBEECK,+J%C3%A9r%C3%A9mie.+De+la+certitude+%C3%A0+la+confiance:+le+principe+de+s%C3%A9curit%C3%A9+juridique+dans+la+jurisprudence+de+la+Cour+de+justice+de+l%27Union+europ%C3%A9enne.+Presses+universitaires+Saint-Louis+Bruxelles,+2019&ots=RH7YiozXT4&sig=eu1-suRVPtCcwKHGJWkwbVul0k8 (visited on 09/27/2025).

Vicente, Lucia and Helena Matute (2023). "The Inherited Bias Effect: the propagation of artificial intelligence biases to human decisions". In: URL: https://osf.io/f2x4n/download (visited on 09/27/2025).

Vicente, Lucía and Helena Matute (2023). "Humans inherit artificial intelligence biases". In: *Scientific reports* 13.1, p. 15737. URL: https://www.nature.com/articles/s41598-023-42384-8 (visited on 09/27/2025).

VICTORIA, ITURRALDE SESMA M. (1988). "Significado e individualización de los enunciados normativos en la aplicación de la ley: lenguaje legislativo, antinomias y lagunas". PhD Thesis. Universidad del País Vasco-Euskal Herriko Unibertsitatea. URL: https://ekoizpen-zientifikoa.ehu.eus/documentos/5ecb7f76299952131520 36d5?lang=en (visited on 09/27/2025).

Vilajosana, Josep M. (2024). "On the Concept of Constitutional Liquidity". In: *Problema anuario de filosofía y teoría del derecho* 18, pp. 97–143. URL: https://www.scielo.org.mx/scielo.php?pid=S2007-43872024000100097&script=sci_arttext&tlng=en (visited on 09/27/2025).

Von Eschenbach, Warren J. (Dec. 2021). "Transparency and the Black Box Problem: Why We Do Not Trust AI". en. In: *Philosophy & Technology* 34.4, pp. 1607–1622. ISSN: 2210-5433, 2210-5441. DOI: 10.1007/s13347-

021-00477-0. URL: https://link.springer.com/10.1007/s13347-021-00477-0 (visited on 09/27/2025).
Von Wright, Georg Henrik (1951). "Deontic logic". In: *Mind* 60.237, pp. 1–15. URL: https://www.jstor.org/stable/2251395 (visited on 09/27/2025).
Wall, Jesse (2019). "Taking the bundle of rights seriously". In: *Victoria University of Wellington Law Review* 50.4, pp. 733–752. URL: https://search.informit.org/doi/abs/10.3316/INFORMIT.926232072624789 (visited on 09/27/2025).
Weber, Max (1978). *Economy and society: An outline of interpretive sociology*. Vol. 2. University of California press. URL: https://books.google.com/books?hl=fr&lr=&id=MILOksrhgrYC&oi=fnd&pg=PR3&dq=WEBER,+Max.+Economy+and+society:+An+outline+of+interpretive+sociology.+University+of+California+press,+1978&ots=oCCAvl6ftG&sig=RQrfdxeuue2HBuRVizwKkQyIfGU (visited on 09/27/2025).
Weinberger, Ota (1985). "The expressive conception of norms-an impasse for the logic of norms". In: *Law & Phil.* 4, p. 165. URL: https://heinonline.org/hol-cgi-bin/get_pdf.cgi?handle=hein.journals/lwphil4&section=16 (visited on 09/27/2025).
— (2019). "Normentheorie als Grundlage der Jurisprudenz und Ethik.: Eine Auseinandersetzung mit Hans Kelsens Theorie der Normen." In: URL: https://www.torrossa.com/gs/resourceProxy?an=5550547&publisher=FZ1820 (visited on 09/27/2025).
Wellner, Galit P. (2020). "When AI Is Gender-biased." In: *HUMANA. MENTE Journal of Philosophical Studies* 13.37, pp. 127–150. URL: https://www.humanamente.eu/index.php/HM/article/download/307/273 (visited on 09/27/2025).
Werner (1999). "The Completeness of International Law and Hamlet's Dilemma". In: *Nordic Journal of International Law* 68.3, pp. 225–247. URL: https://brill.com/view/journals/nord/68/3/article-p225_1.xml (visited on 09/27/2025).
Wintgens, Luc J. (1993). "Coherence of the Law". In: *ARSP: Archiv für Rechts-und Sozialphilosophie/Archives for Philosophy of Law and Social Philosophy* 79.4, pp. 483–519. URL: https://www.jstor.org/stable/23680253 (visited on 09/27/2025).
— (2005). "La légisprudence: étude pour une nouvelle théorie de la législation". In: *Archives de philosophie du droit* 49, pp. 93–113. URL: https://www.academia.edu/download/80696471/350.pdf (visited on 09/27/2025).
Witell, Lars and Hannah Snyder (2024). "Dishonesty Through AI: Can Robots Engage in Lying Behavior?" In: *Humane Autonomous Technology: Rethinking Experience with and in Intelligent Systems*. Springer International Publishing Cham, pp. 233–246. URL: https://library.oapen.

org/bitstream/handle/20.500.12657/94667/1/978-3-031-66528-8.pdf#page=243 (visited on 09/27/2025).

Žarnić, Berislav and Gabriela Bašić (2014). "Metanormative principles and norm governed social interaction". In: *Revus. Journal for Constitutional Theory and Philosophy of Law/Revija za ustavno teorijo in filozofijo prava* 22, pp. 105–120. URL: https://journals.openedition.org/revus/2859 (visited on 09/27/2025).

Zou, James and Londa Schiebinger (2018). "AI can be sexist and racist—it's time to make it fair". In: *Nature* 559.7714, pp. 324–326. URL: https://www.nature.com/articles/d41586-018-05707-8 (visited on 09/27/2025).

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