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under the supervision of **Wilfried Sand-Zantman**

Jury

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Trois Essais en Microéconomie de l'Environnement

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Jury

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Man muss Geduld haben
Mit dem Ungelösten im Herzen,
und versuchen, die Fragen selber lieb zu haben,
wie verschlossene Stuben,
und wie Bücher, die in einer sehr fremden Sprache
geschrieben sind.

Es handelt sich darum, alles zu leben.
Wenn man die Fragen lebt,
lebt man vielleicht allmählich,
ohne es zu merken,
eines fremden Tages
in die Antworten hinein.

Rainer Maria Rilke (1929), *Briefe an einen jungen Dichter*

Be patient toward all that is unsolved in your heart
and try to love the questions themselves,
like locked rooms and like books that are now written
in a very foreign tongue.
Do not now seek the answers, which cannot be given you
because you would not be able to live them.

And the point is, to live everything.
Live the questions now.
Perhaps you will then gradually,
without noticing it,
live along some distant day
into the answer.

Rainer Maria Rilke (1929), *Letters to a Young Poet*

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Summary in English

This thesis explores how incomplete information shapes both environmental policies and market outcomes. Across three chapters, it investigates different situations of informational asymmetries – between regulators, consumers, and firms – in contexts involving pollution, the provision of green products, and climate-related risks.

The first chapter, titled *"Tax the Emissions You Can't See : Designing CBAMs under Incomplete Information"* examines the design of a Carbon Border Adjustment Mechanism when a regulator cannot verify the emissions embedded in imported goods. The chapter builds a Principal-Agent model in which a regulator aims to regulate firms with different emission levels. It shows that when emissions cannot be verified, the optimal CBAM must distort the tax structure relative to the optimal solution : clean firms may face quantity restrictions while polluting firms are rewarded through lower taxes to induce truthful reporting of their emissions. The analysis also considers the role of exogenous constraints such as WTO rules, and the firms' endogenous choice of technology.

The second chapter, *"Green, Greener or Neutral ? Signaling Environmental Quality When Consumers Can't Tell"*, explores how a firm can credibly convey information about the environmental quality of its product when consumers value this attribute but cannot directly observe it, reflecting the credence nature of green products. I develop a signaling model in which a monopoly firm can be either green or neutral, and consumers derive both private and public-good benefits from purchasing the environmentally superior product. The firm chooses observable instruments (price and costly effort) to provide information about its environmental performance. Effort has asymmetric effects : it yields further environmental gains for a green firm but is purely wasteful for a neutral one. I show that a separating equilibrium can arise where the green firm distinguishes

itself by exerting positive effort and charging higher price. As a consequence, incomplete information can partially offset the under-provision of environmental quality by inducing green effort for signaling purposes, which indirectly contributes to the public good. The chapter also analyzes how subsidies for green effort may both restore optimal levels of environmental quality while ensuring proper separation between green and neutral firms.

The third chapter, titled *"The Insurer-Investor Dilemma : When Insurers Fuel the Risks They Cover"* investigates the interaction between insurers' financial decisions and the design of insurance policies, when the nature of investments influence climate-related risks. The model considers two insurers that operate in two indirectly linked activities. On the one hand, they compete to offer insurance policies to agents with heterogeneous probabilities of loss ; on the other, they invest on financial markets, choosing between a risky and a safe strategy. Risky investments (*e.g.* in carbon-intensive assets) yield higher returns but simultaneously increase the likelihood of loss for agents exposed to climate-related hazards. When insurers can perfectly identify agents' risk types, risky investments are always preferred, as they justify higher premiums and increase expected returns. However, when insurers cannot perfectly observe agents' risk types, the coverage distortions necessary to separate low and high-risk agents worsen with risky investments, reducing their attractiveness relative to safe strategies. In this setting, asymmetric information can therefore discipline excessive risk-taking and incentivize safe investments. The chapter further analyzes how the introduction of coverage regulations influences insurers' portfolio choices.

Field : Economics

JEL Classification : D42, D62, D82, Q50, Q58, L12, L15, G22

Keywords : Environmental Economics, Incomplete Information, Adverse Selection, Environmental Policy, Externality

Résumé en français

Cette thèse explore la manière dont l'information incomplète influence à la fois les politiques environnementales et l'équilibre sur le marché. À travers trois chapitres, elle étudie différentes situations d'asymétries informationnelles – entre régulateurs, consommateurs et firmes – dans des contextes impliquant la pollution, la production de biens verts et les risques climatiques.

Le premier chapitre, intitulé *"Tax the Emissions You Can't See : Designing CBAMs under Incomplete Information"*, examine la conception d'un mécanisme d'ajustement carbone aux frontières (MACF) lorsque le régulateur ne peut pas vérifier les émissions liées aux biens importés. Ce chapitre élabore un modèle Principal-Agent dans lequel un régulateur vise à réglementer des firmes ayant des niveaux d'émissions différents. Les résultats montrent que lorsque les émissions ne peuvent pas être vérifiées, le mécanisme optimal d'ajustement carbone aux frontières doit distordre la structure fiscale par rapport à la régulation optimale : les firmes non-polluantes doivent être soumises à des plafonds de production, tandis que les firmes polluantes sont "récompensées" par des taxes moins élevées afin de les inciter à déclarer leurs émissions réelles. L'analyse explore également l'impact des contraintes exogènes telles que les règles de l'OMC, ainsi que la possibilité pour les firmes d'investir dans la technologie non-polluante.

Le second chapitre, *"Green, Greener or Neutral ? Signaling Environmental Quality When Consumers Can't Tell"*, étudie la manière dont une firme peut transmettre de manière crédible des informations sur la qualité environnementale de son bien, lorsque les consommateurs valorisent cette caractéristique, mais ne peuvent pas l'observer directement, reflétant le fait que ces biens soient des biens de croyance. Ce chapitre développe un modèle de jeux de signaux dans lequel une firme en situation de monopole peut être soit verte, soit neutre ; et où les consommateurs tirent à la fois des bénéfices privés de

la consommation de biens verts (*warm glow*), mais aussi des bénéfices collectifs, quel que soit leur niveau de consommation individuelle. La firme peut choisir d'utiliser des instruments observables des consommateurs (prix et niveau d'effort coûteux) pour fournir des informations sur la qualité environnementale de son bien. L'effort a des effets asymétriques : il génère des gains environnementaux supplémentaires pour une firme verte, mais ne produit aucun bénéfice pour une firme neutre. Les résultats montrent qu'un équilibre séparateur peut émerger, dans lequel la firme verte se distingue en fournissant un effort positif et en pratiquant un prix élevé. Par conséquent, la présence d'asymétrie d'information peut compenser en partie la sous-production de qualité environnementale en incitant à fournir un effort, ce qui contribue indirectement au bien public. Le chapitre analyse également comment les subventions à l'effort vert peuvent à la fois restaurer le niveau optimal de qualité environnementale tout en assurant la séparation entre firmes verte et neutre.

Le troisième chapitre, intitulé *"The Insurer-Investor Dilemma : When Insurers Fuel the Risks They Cover"*, examine l'interaction entre les décisions financières des assureurs et la conception des contrats d'assurance lorsque la nature des investissements influence les risques liés au climat. Le modèle considère deux assureurs opérant dans deux activités indirectement liées. D'une part, ils sont en concurrence sur le marché assurantiel, où ils proposent des couvertures et des primes à des agents présentant des risques hétérogènes ; d'autre part, ils investissent sur les marchés financiers, en choisissant entre une stratégie risquée et une stratégie non-risquée. Les investissements risqués (par exemple dans des actifs à haute intensité carbone) offrent des rendements plus élevés mais augmentent simultanément le risque pour les agents exposés à des risques climatiques. Lorsque les assureurs peuvent identifier parfaitement le type de risque des agents, les investissements risqués sont toujours préférés, car ils justifient des primes plus élevées et augmentent les profits. Cependant, lorsque les assureurs ne peuvent pas observer parfaitement le type de risque des agents, les distorsions de couverture nécessaires pour séparer les agents à faible et à haut risques s'aggravent avec les investissements risqués, réduisant leur attractivité par rapport aux stratégies non-risquées. Dans ce contexte, l'information incomplète peut donc discipliner la prise de risque excessive et inciter à des investissements non-risqués. Le chapitre analyse également

comment l'introduction de régulations sur le niveau de couverture influence les choix de portefeuille des assureurs.

Discipline : Sciences économiques

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General Introduction

"Scientific knowledge is as much an understanding of the diversity of situations for which a theory or its models are relevant as an understanding of its limits."
– Elinor Ostrom (1990), *Governing the Commons*

Scientific research has produced robust evidence of climate change and its link to human activities. According to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change, *"Human activities, principally through emissions of greenhouse gases, have unequivocally caused global warming"* (IPCC, 2023). This rise in the concentration of greenhouse gases in the atmosphere since the Industrial Revolution is causing a diversity of losses and damages to nature and people. The observable consequences include rising sea levels, increasing frequency and intensity of extreme weather events, and severe disruptions to ecosystems. These effects carry profound economic implications, ranging from productivity losses and increased inequalities to threats to food and water security (UNEP, 2022).

1 The Rationale for Environmental Economics

There are two connected reasons why economists should care about these problems. First, because human-induced climate change is directly linked to the way we conduct our economic activities, *i.e.* how we produce and consume. As written by Charles Kolstad, *"the essence of the environmental problem is the economy"* (Kolstad, 2011). Second, because climate change represents a perfect case of market failure, causing markets to

not directly yield the optimal amount of emissions. When a firm decides on how much to produce, it maximizes its profits, including the revenues from selling its product and the private costs of production. As a consequence, it does not account for the full social cost of its actions, which involves damages created by emissions or other pollutants (Coase, 1960). Economists have theorized these issues as externalities. Because economic agents do not bear the full cost of their choices, they over-provide emissions and under-provide abatement, compared to the social (welfare-maximizing) optimum. Recognizing the environment as a whole as a global public good makes climate change the largest and most complex market failure to ever exist (Stern, 2006; Nordhaus, 2018). As a result, *"when the 'environmental revolution' arrived in the late 1960s, economists were ready and waiting."* (Cropper & Oates, 1992).

The persistence and magnitude of these market failures have motivated a vast body of research, with both positive and normative perspectives. The positive approach seeks to understand and explain how the world is, while the normative approach focuses on identifying how to best intervene to correct inefficiencies. In the context of environmental issues, the two perspectives are inherently connected: understanding how economic agents' behavior shapes environmental outcomes is essential for designing instruments aimed to effectively improve the situation. Therefore, restoring optimality requires the internalization of externalities, so that private decision-makers account for the full social costs and benefits of their actions (Pigou, 1920).

In the literature, there are two categories of instruments aimed at internalizing the damages caused by pollution (see Baumol and Oates (1988) for a textbook treatment). First, the command-and-control approach involves the direct imposition of constraints, such as technology or performance standards. These instruments offer predictability in achieving environmental targets but may be highly costly due to their inflexibility, which ignores differences in abatement costs across polluters. For example, one of the first car pollution regulations in California required the use of positive crankcase ventilation (PCV) valves on all engines.

The second category consists of incentive-based (or market-based) instruments, such as Pigouvian taxes, tradable emission permits (Dales, 1968; Montgomery, 1972), or subsidies. These tools provide economic incentives for pollution reduction in an efficient

way, minimizing abatement costs. The core intuition is the following: the optimal level of emissions is achieved when the marginal benefit of pollution equals the marginal damage. A regulator can induce this outcome by allocating a total amount of tradable emission permits corresponding to the optimal total quantity of emissions. Regardless of the initial allocation, firms with lower abatement costs will reduce pollution in priority and the optimal level of emissions will be attained. A Pigouvian tax induces this outcome when the per-unit tax is set equal to the marginal damage. These two regulations are theoretically equivalent and equally efficient.[1] An early example of a market for emission permits is the U.S. Acid Rain Program which successfully targeted reductions in sulfur dioxide and nitrogen oxide emissions starting 1995 (Stavins, 1998; Ellerman et al., 2000). In 2005, the European Union launched the largest market for tradable permits: the EU ETS. Over the 2005-2024 period, the EU ETS has contributed to reduce emissions from electricity and heat generation and industrial production by 47% (European Commission, 2024).

Outisde of the regulatory literature, a strand of environmental economics focuses on the private provision of public goods and the role of consumer preferences for green products. Although the core idea that firms may produce public goods is not new (see Cornes and Sandler, 1984; Bergstrom et al., 1986), the environmental applications have shed light on how goods with both private and public benefits are provided in these markets, and how consumer demand shapes the quantity and quality of these products. Because not all benefits are captured privately, the level of supply often falls short of the socially optimal amount (Kotchen, 2006; Besley & Ghatak, 2007).

2 The Role of Information

In practice, however, the situation is often more complex. If the standard theoretical framework held perfectly, climate change would not remain an unsolved global issue. Among other factors, one contributing limitation is that conventional results in the literature rely on the assumption that all relevant information is common knowledge among agents. In reality, this assumption might not hold. For instance, firms may

1. For a comprehensive comparison between command-and-control and market-based instruments, see Tietenberg (1990).

possess private information about their emissions or technologies that is not known by regulators (Chapters 1 and 2) or consumers (Chapter 2). Conversely, consumers may hold information about their own characteristics that is not observed by other agents (Chapter 3). In short, imperfect information is pervasive in environmental contexts, as emphasized by Pindyck (2007). These informational frictions complicate both the analysis of market outcomes and agent behavior (positive analysis), as well as the design of optimal regulatory interventions (normative analysis).

Uncertainty. An early recognition of the importance of imperfect knowledge for environmental regulation is found in Weitzman (1974), who introduced uncertainty into the analysis of regulatory instrument choice (further extended by Roberts and Spence, 1976). This seminal contribution derives crucial policy implications, showing that quantity- and price-based regulations become nonequivalent with uncertainty. However, uncertainty does not change the way firms make decisions: agents have no strategic incentive to misreport their characteristics.

Asymmetric information and incentives. Therefore, and more importantly for the focus of this thesis, information asymmetries between economic agents must be considered, as they modify incentives and, consequently, affect equilibrium outcomes and the very design of regulatory interventions. Information asymmetries create different types of problems, the most fundamental of which are issues of hidden information and hidden action.

Hidden information relates to situations where an agent's decisions depend on her privately known attributes (or type) *"in a manner that adversely affects uninformed participants"* (Mas-Colell et al., 1995). The seminal work of Akerlof (1970) demonstrated how private information regarding the quality of a good can lead to market collapse.

On the other hand, hidden action refers to situations where an agent's unobserved actions (*e.g.* effort or precaution) affect outcomes, leading to inefficient behavior when contracts cannot perfectly align incentives. Holmström (1979) established the foundational framework for analyzing how contracts can address these inefficiencies.

This thesis primarily concentrates on hidden information. Economic theory provides two main approaches to addressing this issue: screening and signaling. The distinction

lies in which agent (informed or uninformed) takes action to address the asymmetry. With screening, the uninformed party designs a contract or a policy to induce the informed party to reveal their private information (Chapters 1 and 3). Rothschild and Stiglitz (1976) formalized this in the context of insurance markets, and Baron and Myerson (1982) in the context of monopoly regulation. Signaling involves the informed agent to voluntarily disclose their type through costly actions, for instance education or certification (Chapter 2). A fundamental example is the job-market signaling model of Spence (1973). These tools are not only theoretically elegant but also offer a coherent framework for understanding a wide range of situations, including environmental issues.

3 Environmental Economics under Information Asymmetries

The integration of information concerns into environmental economics has provided important lessons for positive analysis and environmental policy design.

Early contributions focused on the question of emission control when polluters are privately informed about their own abatement costs or emissions levels.[2] Kwerel (1977), Dasgupta et al. (1980), and Spulber (1988) were among the first to formalize how hidden information distorts regulatory outcomes, and to explore the design of second-best policies. Mainly drawing from the mechanism design literature (*e.g.* Clarke, 1971; Groves, 1973), these contributions develop the idea that well-designed regulations can provide polluters with the proper incentives to reduce emissions and report information truthfully. However, such mechanisms inevitably leave polluters with an information rent, a residual payoff that compensates them for revealing their private knowledge (which translates, in this case, to complying with the environmental policy).

The incentive regulation literature explores the trade-off involved in designing policies that reduce information rents while maintaining truthful reporting (Laffont & Tirole, 1993). For instance, in environmental applications, efforts to minimize the rents of privately informed agents can lead to distortions in pollution abatement away from

2. See Lewis (1996) for an extensive review.

efficient levels, or to the implementation of non-linear taxes targeted to different firms.

Beyond regulation, the study of environmental issues with information asymmetries has also enriched our understanding of the strategic choices of firms and consumers, and their interaction. Part of this literature focuses on the existence of markets for green goods seen as impure public goods, where firms possess private information about the environmental attributes of their products. The main studies in this field are concerned with the way firms disclose this information (or when it leads to greenwashing), but also with the implications of these asymmetries on the level of impure public goods provided, on social welfare, consumer surplus, etc. (see Lyon and Maxwell (2007) for a survey). These considerations opened a vast range of research, notably on labeling and certification, exploring under which conditions these tools increase social welfare and/or the provision of environmental attributes.

4 Thesis Contribution

Across three independent but conceptually connected essays, this thesis aims to (modestly) contribute to this literature. Each chapter explores different aspects of the interplay between environmental and information economics, combining insights from the first with tools from the latter.

More specifically, each chapter examines how hidden information shapes market outcomes and the design of policies in contexts involving pollution (Chapter 1), the provision of green products (Chapter 2), and climate-risk management (Chapter 3). They share a unifying motivation: the recognition that incomplete information is not a marginal feature but a defining characteristic of real-world environmental challenges, and that theoretical models help us understand the underlying mechanisms, and guide the design of more effective regulations.

Collectively, these chapters reflect on a common set of overarching questions: How does incomplete information affect equilibrium allocations and the provision of public goods or negative externalities? How does it alter the effectiveness of existing policy instruments, and how should regulation be designed in such informational environments? How does social welfare respond to the interaction between informational and

environmental inefficiencies?

Limitations and avenues for future research. Like any theoretical work, this thesis relies on simplified models that abstract from several real-world complexities. Each chapter focuses on a specific source of information asymmetry in a static environment, leaving aside (among other things) dynamic considerations and empirical validation. The models mostly rely on specific functional forms chosen for tractability, and the results would benefit from a generalization exercise.

Chapter 1. Tax the Emissions You Can't See: Designing CBAMs under Incomplete Information

The first chapter of this thesis investigates the optimal design of a Carbon Border Adjustment Mechanism (CBAM) when a regulator cannot directly verify the emissions embedded in imported goods.

Context and motivation. CBAMs are relevant in contexts where environmental policies are heterogeneous. When domestic carbon emissions are regulated but foreign regulations are more lenient or nonexistent, firms may decide to relocate outside of the regulated area in order to escape the carbon price. Carbon leakage not only creates an inequity between domestic producers and producers located abroad, but also perpetuates the inefficiency of carbon policies, as emissions are merely displaced rather than reduced. In such situation, one solution is the implementation of a CBAM in the form of a border tax on the carbon content of imports from unregulated countries. As an example, this policy lies at the core of the *Fit for 55* package proposed by the European Commission in 2021.

Main question. Current CBAM implementations have relied on self-reporting mechanisms: firms located abroad declare their carbon emissions to determine the corresponding carbon price to be paid. This approach creates a major limitation: firms using carbon-intensive technologies have strong incentives to under-report their emissions.

When declarations made by firms are unreliable, regulators face a classic problem of asymmetric information where the true technologies of firms remain unknown. Therefore, what is the optimal design of a CBAM when the regulator has no information about firms' emissions?

Model overview. The paper develops a Principal-Agent model in which a regulator (the uninformed Principal) designs a tax structure to incentivize firms located abroad (the informed Agents) to truthfully report their emissions, in order to maximize social welfare. Firms differ in their technology of production: "clean" firms are less polluting but produce at a relatively higher cost, while "dirty" firms produce for cheaper but impose a greater environmental damage on society through their emissions. Thus, the firm's type is its environmental performance, which has opposite effects on private costs compared to social costs: clean technologies have higher private costs but reduce the negative externality imposed on social welfare.

Main results. If emissions are verifiable (complete information), the regulator can directly regulate firms to maximize social welfare. In this case, dirty firms are constrained to produce less than their profit-maximizing quantity, to internalize the environmental damage they generate. However, when the firms' technologies are unobservable, this first-best outcome is not attainable, as dirty firm misrepresent as clean firms to avoid higher taxes. To ensure that firms are willing to report their true technology, the regulator must distort the non-linear tax structure (combining taxes and quantity caps). More specifically, dirty firms are still constrained to produce the first-best quantity but enjoy lower taxes; and clean firms suffer from quantity restrictions although they do not pollute. Overall welfare decreases relative to the first-best situation, due to the reward given to dirty firms in exchange of truthful reporting. Taking into account potential exogenous constraints such as WTO rules further exacerbates these distortions. Finally, when firms can choose their technology, the results showcase a fundamental tension between the provision of different types of incentives: the incentive to invest in the clean technology and the incentive to truthfully report. The paper shows that both objectives cannot be reconciled simultaneously when the clean technology is relatively more expensive.

Broader application. Beyond its modest theoretical contribution, the paper provides insights into the ongoing policy debate on the feasibility of CBAMs. It suggests that truth-telling mechanisms may require rewarding or tolerating some inefficiency, and that policy instruments should account explicitly for informational limitations. The framework can be generalized to other environmental taxes where emissions are self-reported, such as carbon offsets.

Chapter 2. Green, Greener or Neutral? Signaling Environmental Quality When Consumers Can't Tell

The second chapter studies how a firm can credibly signal the environmental quality of its product when consumers value environmental performance but cannot verify it directly.

Context and motivation. Consumer increasingly value products that are environmentally responsible or ethically produced. However, the environmental quality of a product is often a credence attribute: it cannot be verified by consumers even after consumption. Therefore, polluting firms may have incentives to appear greener by engaging in misleading claims and marketing to exploit consumer preferences, charge higher prices, and ultimately undermine welfare.

Main question. When the technology of production is private information of the firm, consumers are therefore uncertain about the environmental quality of the product they purchase. In this context, how can a firm credibly signal its environmental quality to consumers? What combinations of observable instruments such as price and costly effort can separate green and neutral firms in equilibrium? And finally, what are the welfare implications of these signaling strategies when the environmental attribute of the product has a public good component?

Model overview. The paper builds a signaling model in a monopoly setting. The firm (informed party) can be one of two types: a green firm, which produces a non-polluting

product at a higher marginal cost but generates positive externalities ; or a neutral firm, which produces with a cheaper technology without environmental benefits. Consumers (uninformed party) derive extra utility from the green product in two ways: (*i*) through their personal purchase of the product (warm-glow benefit) and (*ii*) through positive externalities regardless of their own consumption of the product (public good benefit). However, they cannot directly observe the firm's true type. The firm can influence consumers' beliefs through two observable decisions: the price it charges and the level of costly effort it exerts. Beyond conveying information, effort has a different impact and cost depending on the firm. For a neutral firm, effort is more costly and purely dissipative (*e.g.* costly marketing to appear greener with no real improvements). For a green firm, effort is relatively cheaper, and genuinely increases the public good benefit above its baseline value.

Main results. The full information monopoly strategies involve both types of firms adopting standard monopoly pricing. Also, when consumers can perfectly assess the environmental quality of the product, firms exert zero effort. This implies an under-provision of environmental quality by the green firm relative to the social optimum. Under incomplete information, however, a green firm must change its strategy to be properly perceived by consumers. A separating equilibrium can emerge in which the green firm credibly distinguishes itself by choosing a higher price and (under some conditions) a positive level of effort. In such equilibrium, signaling restores consumers' beliefs and increases the public good benefit associated with green consumption. The analysis reveals that incomplete information can enhance overall welfare: conveying information through effort contributes to the public good, and this gain may be larger than the losses associated with the signaling cost (*i.e.* higher prices and costly effort). This illustrates the possibility that informational inefficiencies can sometimes partially correct other forms of inefficiencies (here, the under-provision of the public good). The paper also analyzes the effects of regulatory instruments on the equilibrium. It provides insights into the implementation of subsidies for green effort, identifying the conditions that enable both the first-best level of effort and the achievement of separation.

Broader application. The findings suggest that costly but credible actions can substitute for formal certification when such schemes are absent or unreliable. Beyond product markets, the analysis can be extended to other contexts where agents must signal socially desirable but unobservable characteristics, such as corporate social responsibility or sustainable investment practices.

Chapter 3. The Insurer-Investor Dilemma: When Insurers Fuel the Risks They Cover

The third chapter uses the insurance sector to investigate the interaction between financial decisions and the design of insurance policies, when investments influence climate-related risks.

Context and motivation. Insurance companies operate in two activities related to climate risks. On the one hand, they play a crucial role in absorbing climate risks by providing coverage against environmental damages. On the other, they are major institutional investors whose portfolio choices can influence the very risks they insure. For instance, large insurers continue to heavily invest in carbon-intensive industries, thereby contributing to the frequency of climate-related losses. This introduces what might initially seem like a paradox. Additionally, insurance markets are often characterized by incomplete information: agents face different risks, and insurers often lack the precise data needed to assess each agent's exposure to specific events.

Main question. This dual activity raises a fundamental question: how does the interplay between investment and insurance activities affect insurers' strategic decisions? In particular, can the presence of asymmetric information alter insurers' choices? This paper aims to study the strategic choices of insurers in the two activities. The results differ depending on the level of information available to the insurers regarding the probability of loss of the agents.

Model overview. The paper develops a theoretical model featuring two insurers competing in an insurance market, and investing the collected premiums in financial markets. Each insurer can choose between a safe investment, which yields a low return and does not affect insured risks; or a risky investment, which yields a higher return but increases the probability of loss for a proportion of agent who are more exposed to climate-related risks. On the contrary, unexposed agents face a lower probability of loss and remain unaffected by insurers' financial decisions. We are interested in the insurers' choice of insurance policies (premiums and coverages) and investment strategies (safe or risky).

Main results. The analysis proceeds in two stages. First, the case where insurers know the agents' probabilities of loss (exposed or unexposed). Competition on the insurance market leads insurers to offer full insurance at actuarially fair premiums, depending on the agent's exposure. In this case, risky investments are always preferred, because they simultaneously increase expected returns and justify higher premiums for exposed agents whose probability of loss rises. This explains why insurers deliberately contribute to higher risks. When the agents' risk types become private information, insurers are not able to offer the full information insurance policies without inducing exposed agents to mimic unexposed ones. To separate agents, insurers must offer partial insurance to unexposed types, which reduces profits as the volume of collected premiums declines. Risky investments increase the magnitude of these distortions, and therefore become relatively less attractive (despite higher return) as the volume of investable funds decreases. Consequently, safe investments can emerge in equilibrium. Asymmetric information therefore mitigates insurers' incentive to engage in excessive risk-taking. The paper also shows that regulatory policies, such as mandating full coverage for all agents, can sometimes shift investments towards safe options.

Broader application. This model introduces a novel mechanism whereby a (or some) Principal(s) can take a profitable action that alters the type of some Agents, and thus affects the extent of adverse selection. This creates an interesting trade-off, that could potentially be generalized to other situations besides insurance (*e.g.* banking, pharmaceutical markets, etc.).

References

Akerlof, G. A. (1970). The Market for "Lemons": Quality Uncertainty and the Market Mechanism. *The Quarterly Journal of Economics*, *84*(3), 488–500.

Baron, D. P., & Myerson, R. B. (1982). Regulating a Monopolist with Unknown Costs. *Econometrica*, *50*(4), 911–930.

Baumol, W. J., & Oates, W. E. (1988, November). *The Theory of Environmental Policy*. Cambridge University Press.

Bergstrom, T. C., Blume, L., & Varian, H. R. (1986). On the Private Provision of Public Goods. *Journal of Public Economics*, *29*(1), 25–49.

Besley, T., & Ghatak, M. (2007). Retailing Public Goods: The Economics of Corporate Social Responsibility. *Journal of Public Economics*, *91*(9), 1645–1663.

Clarke, E. H. (1971). Multipart Pricing of Public Goods. *Public Choice*, *11*, 17–33.

Coase, R. H. (1960). The Problem of Social Cost. *The Journal of Law & Economics*, *56*(4), 837–877.

Cornes, R., & Sandler, T. (1984). The Theory of Public Goods: Non-Nash Behaviour. *Journal of Public Economics*, *23*(3), 367–379.

Cropper, M. L., & Oates, W. E. (1992). Environmental Economics: A Survey. *Journal of Economic Literature*, *30*(2), 675–740.

Dales, J. H. (1968). Pollution, Property and Prices. *Canadian Journal of Economics*, *2*, 322.

Dasgupta, P., Hammond, P., & Maskin, E. (1980). On Imperfect Information and Optimal Pollution Control. *The Review of Economic Studies*, *47*(5), 857–860.

Ellerman, A. D., Joskow, P. L., Schmalensee, R., Montero, J.-P., & Bailey, E. M. (2000). *Markets for Clean Air: The U.S. Acid Rain Program*. Cambridge University Press.

European Commission. (2024). *Report from the Commission to the European Parliament and the Council on the Functioning of the European Carbon Market in 2023* (tech. rep. No. COM 2024 538). Brussels.

Groves, T. (1973). Incentives in Teams. *Econometrica*, *41*(4), 617–631.

Holmström, B. (1979). Moral Hazard and Observability. *The Bell Journal of Economics*, *10*(1), 74–91.

IPCC. (2023). *Climate Change 2023: Synthesis Report. Contribution of Working Groups I, II and III to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Core Writing Team, H. Lee and J. Romero (eds.)]* (tech. rep.). IPCC, Geneva, Switzerland.

Kolstad, C. D. (2011). *Environmental Economics* (2nd ed.). Oxford University Press.

Kotchen, M. J. (2006). Green Markets and Private Provision of Public Goods. *Journal of Political Economy*, *114*(4), 816–845.

Kwerel, E. (1977). To Tell the Truth: Imperfect Information and Optimal Pollution Control. *The Review of Economic Studies*, *44*(3), 595–601.

Laffont, J.-J., & Tirole, J. (1993). *A Theory of Incentives in Procurement and Regulation*. MIT Press.

Lewis, T. R. (1996). Protecting the Environment When Costs and Benefits are Privately Known. *The RAND Journal of Economics*, *27*(4), 819–847.

Lyon, T., & Maxwell, J. (2007). Corporate Social Responsibility and the Environment: A Theoretical Perspective. *Review of Environmental Economics and Policy*, *2*, 240–260.

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic Theory*. Oxford University Press.

Montgomery, W. (1972). Markets in Licenses and Efficient Pollution Control Programs. *Journal of Economic Theory*, *5*(3), 395–418.

Nordhaus, W. (2018). Projections and Uncertainties about Climate Change in an Era of Minimal Climate Policies. *American Economic Journal: Economic Policy*, *10*(3), 333–60.

Ostrom, E. (1990). *Governing the Commons: The Evolution of Institutions for Collective Action*. Cambridge University Press.

Pigou, A. C. (1920). *The Economics of Welfare*. MacMillan Co.

Pindyck, R. S. (2007). Uncertainty in Environmental Economics. *Review of Environmental Economics and Policy*, *1*(1), 45–65.

Roberts, M. J., & Spence, M. (1976). Effluent Charges and Licenses under Uncertainty. *Journal of Public Economics*, *5*(3-4), 193–208.

Rothschild, M., & Stiglitz, J. (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information [Publisher: Oxford University Press]. *The Quarterly Journal of Economics*, *90*(4), 629–649.

Spence, M. (1973). Job Market Signaling. *The Quarterly Journal of Economics*, *87*(3), 355–374.

Spulber, D. F. (1988). Optimal Environmental Regulation under Asymmetric Information. *Journal of Public Economics*, *35*(2), 163–181.

Stavins, R. N. (1998). What Can We Learn from the Grand Policy Experiment? Lessons from SO2 Allowance Trading. *Journal of Economic Perspectives*, *12*(3), 69–88.

Stern, N. (2006, October). Stern Review: The Economics of Climate Change.

Tietenberg, T. H. (1990). Economic Instruments for Environmental Regulation. *Oxford Review of Economic Policy*, *6*(1), 17–33.

UNEP. (2022). *Emissions Gap Report 2022*. United Nations.

Weitzman, M. (1974). Prices vs. Quantities. *Review of Economic Studies*, *41*, 477–91.

Chapter 1

Tax the Emissions You Can't See: Designing CBAMs under Incomplete Information

Abstract

This paper studies the optimal design of a Carbon Border Adjustment Mechanism to address carbon leakage resulting from the relocation of domestic firms to unregulated countries. As proposed by the European Commission in 2021, such mechanisms usually rely on firms self-reporting their emissions to determine the carbon content of imported goods. When verification is impossible, this induces firms to under-report their emissions to pay lower taxes. Using a theoretical model in which firms can either be clean or dirty and the regulator has incomplete information about firms' emissions, we show that truthful reporting by firms requires the design of a non-linear tax structure, specifying taxes and quantity caps. In this framework, clean firms are less cost-efficient, forcing the regulator to reward dirty firms relative to the social optimum. When firms can invest in the acquisition of the clean technology, the regulator cannot simultaneously incentivize clean investments and maintain a truth-revealing tax structure.

JEL Codes: D86, Q58, D62.

Keywords: Environmental Economics, Carbon Tax, CBAM, Externality.

1 Introduction

Research question. Global warming is induced by the excess of greenhouse gases emissions into the atmosphere from human activities. The environment can be considered as a global public good and therefore suffers from the *"greatest and widest-ranging market failure ever seen"* (Stern, 2007). The optimal solution to regulate carbon emissions would be the implementation of a global carbon price, but efforts towards such regulation have fallen short. The emergence of regional regulations to pursue climate ambitions means that, while regulated domestic firms pay for their carbon emissions, foreign firms located in more lenient countries do not. A widely-discussed solution for regulating these outside firms is the implementation of a Carbon Border Adjustment Mechanism (CBAM), which takes the form of a border tax imposed on the carbon content of imported goods.

In this paper, we are interested in the design of a CBAM when the emissions of firms located abroad are unknown to the regulator. In other words, what is the optimal CBAM regulation under incomplete information about firms' emissions?

Context. Although global negotiations on climate mitigation have led to the adoption of important international environmental agreements such as the Kyoto Protocol (adopted in 1997) and the Paris agreement (2015), these agreements remain non-binding and never led to the creation of a unique carbon price. Therefore, countries face incentives to behave as free-riders (Nordhaus, 2015). As a response to the inability of the global community to decide on a unique regulation, sub-global regulations have emerged. For instance, the European Union created the EU Emissions Trading System (EU ETS) in 2005, the largest market for emission allowances. While these types of regulations enabled the imposition of a domestic carbon price and a reduction in local emissions (Ellerman et al., 2016), they also created a new issue: the issue of carbon leakage. Carbon leakage is defined as an increase in foreign emissions following a reduction in domestic emissions (European Commission, 2015). Therefore, regulating foreign emissions is a crucial challenge for regional regulations as the EU ETS, because not doing so can compromise efforts and even result in increased emissions globally (Monjon & Quirion, 2011). Empirical studies estimate the carbon leakage rates between

10 percent and 30 percent (Bohringer et al., 2018). The creation of a CBAM is intended to tackle this issue. The European Commission voted the creation of such a mechanism in 2021 as part of its *Fit for 55* package[1]. However, the design of a CBAM presents many challenges. Among them, the inability of regulators to access information about the carbon content of imported goods. When the regulation relies on self-reporting to retrieve this information, foreign firms with carbon-intensive technologies face incentives to under-report their emissions in order to pay a lower tax.

Model overview and main results. This paper proposes a simple theoretical model of regulation in which a regulator aims to regulate firms with respect to their emissions. We consider firms with two different technologies: *dirty* firms, which have a lower cost of production but pollute more; and *clean* firms, which do not pollute but are less cost-efficient. The extent of information available to the regulator depends on the location of firms. When firms are located inside the region, the regulator can perfectly monitor their emissions. However, when firms are located abroad, the regulator is unable to access information about firms' emissions.

The benchmark case under complete information gives the welfare-maximizing optimum. At this optimum, the regulation is designed such that the environmental negative externality created by dirty firms is internalized, and tax revenue is maximized. However, this welfare-maximizing regulation is not applicable to firms when they relocate abroad, as information asymmetries leads to under-reporting of emissions from dirty firms.

In this case, the CBAM must impose lower taxes on dirty firms compared to the social optimum, and require lower production from clean firms, in order to ensure that firms honestly report their emissions. This results in a welfare loss.

Additional exogenous constraints such as WTO rules can limit the regulator's ability to tax firms beyond their environmental damage. Account for these restrictions further distorts the CBAM.

Finally, when firms can choose their technology of production, a fundamental tension arises. Because adopting the clean technology is more costly, firms prefer to remain dirty

1. For details on this proposal, see European Commission (2021a) and European Commission (2021b).

and misrepresent themselves as clean. This makes it impossible for the regulator to reconcile the incentive to invest in the environmentally "good" type with the provision of incentives that induce truthful reporting of emissions.

Related literature and contribution. This work relates to two main strands of literature.

First, it aims to contribute to the literature on Carbon Border Adjustment Mechanisms, and more generally on the use of border tax structures to address carbon leakage. One of the first papers to consider tax structures (production, consumption and trade taxation) as a way to correct international externalities (such as pollution) and maximize social welfare is Markusen (1975). Hoel (1996) extends this framework to allow for more than two countries and goods traded, and to consider differentiation across sectors. Both of these papers consider a model with complete information about firms' costs and damages. The use of such tax structures for the purpose of a CBAM has gained prominence in the literature in recent years. Some studies such as Cosbey et al. (2019) provide an overview of the legal and economic issues at stake in order to offer guidelines on its design and implementation. They acknowledge and discuss the difficulties raised by the presence of incomplete information, but do not derive a specific model to deal with this issue. This strand of literature has undertaken a quantification of the amount of carbon leakage in order to demonstrate the importance of this issue. It is estimated to lie between 5 and 30 percent for industrialized countries (Bohringer et al., 2012). It also justifies the adoption of a CBAM to tackle this problem (Monjon & Quirion, 2011). To summarize, there are concerns regarding the assessment of the carbon content of imported products, but most papers rely on the use of default values as a way out. The present paper intends to address this issue by incorporating incomplete information in the design of a CBAM.

This paper also aims to contribute to the theoretical literature on pollution control under incomplete information. In Kwerel (1977), a regulator wants to regulate polluting firms without knowing their characteristics. In their model, firms are heterogeneous in cost of production but they all impose the same environmental damage, which results in an optimal regulation with a unique price for all firms. In the present paper, we allow

for firms with different costs of production to be more or less polluting. Spulber (1988) also considers a model in which a social planner wants to regulate heterogeneous polluting firms which have private information about their costs. While this paper assumes that the regulator assigns firms emission levels and payments for damage after firms send a message (signal), the present paper intends to design a model with one-round communication only (no signaling, the regulator does not have any information about firms when designing the optimal policy). The paper by Dasgupta et al. (1980) improves Kwerel (1977) by designing a model in which truth-telling is dominant strategy for all firms, and not only Nash equilibrium. They consider both one-round communication (the social planner chooses the regulation without any information) and two-round communication (signaling). This paper derives a modified VCG mechanism, which applicability to policy-making remains vague. We enrich this approach by designing a Principal-Agent model in which a regulator designs incentives to regulate firms (Baron & Myerson, 1982; Laffont & Martimort, 2002). We also incorporate the concerns with WTO rules compatibility, and consider endogenous investment in the clean technology by firms. Finally, we assume that clean firms, which are the "good type" from an environmental damage perspective, are the less cost-efficient type. This results in the necessity of rewarding the polluting firms to acquire information. Such assumption differs from traditional models in which the less costly type is always the good type.

Structure of the paper. The structure of this paper is as follows. First, we build a benchmark case in which we consider the choice of a tax structure to regulate domestic firms, whose technologies are known by the regulator (section 2). In section 3, we turn to the problem of interest and derive the CBAM chosen by the regulator for firms located abroad, assuming that he does not have any information about their emissions, and that he needs to induce them to voluntary declare their true technology to him. Then, section 4 explores the compatibility of this regulation with WTO rules, and derives a policy that would be more likely to be in line with these requirements. In section 5, we allow for firms to choose whether to invest in the clean technology, and we discuss the tension between inducing firms to adopt such "good" but costly behavior and the truthful report of technology. We demonstrate that the two dimensions cannot be reconciled.

2 Model and Benchmark

Consider a market with consumers, firms, and a regulator.

Consumers

For simplification, assume that consumers have perfectly elastic demand. As a consequence, at a given price, consumer surplus is zero.

Firms

The market consists of a unit mass of perfectly competitive and profit-maximizing firms. There are two technologies of production: a firm i can either be clean ($i = C$) with probability $\lambda \in [0, 1]$; or dirty ($i = D$) with probability $(1 - \lambda)$. Each firm $i \in \{C, D\}$ produces a quantity $q_i \geq 0$ of the homogeneous good, and sells this good at a fixed price $\overline{p} = 1$. Profits from production depend on: (*i*) the revenue from selling the good, (*ii*) the cost of production, assumed to be increasing and convex in the quantity produced, and (*iii*) the tax payment to the social planner. Thus, assume that profits write

$$\pi_i(q_i, t_i) = q_i - \frac{1}{2\theta_i} q_i^2 - t_i q_i,$$

where $\theta_i \geq 1$ is the firm's type, and $T_i = t_i q_i \geq 0$ is the total tax on production paid to the regulator.

Assumption 1. *Clean firms have a higher marginal cost of production than dirty firms:* $\theta_C < \theta_D$.

Indeed, at least in the short term, clean firms might have higher costs of production arising from acquiring renewable energy and greener raw materials, especially in a framework where they are not able to differentiate their product from "brown" products.[2] Clean firms do not generate carbon emissions, therefore they do not impose any environmental damage on society. However, dirty firms create a damage $\gamma > 0$ per unit produced. The total environmental damage created by a dirty firm is thus equal to γq_D.

2. For insights on the possibility of reducing costs by going green that is ruled out in this paper, refer to the works of Porter and van der Linde (1995).

Regulator

The regulator is a benevolent social planner who may use two types of instruments to maximize social welfare: taxes on production, and quantity caps. Social welfare takes into account firms' profits, environmental damages imposed by dirty firms, and tax revenues with a weight $\beta \in [0,1]$ (Laffont & Tirole, 1996).[3] Formally, the social welfare function can therefore be written as:

$$W = \lambda\Big(\pi_C(q_C, t_C) + (1+\beta)t_C q_C\Big) + (1-\lambda)\Big(\pi_D(q_D, t_D) - \gamma q_D + (1+\beta)t_D q_D\Big). \tag{1}$$

2.1 Unregulated Case

First of all, consider the situation in which the social planner does not intervene: $t_C = t_D = 0$. In this case, each firm chooses to produce the quantity that maximizes its individual profits. Thus, the unregulated quantity produced by each type of firm is $q_i^* = \theta_i \; \forall i \in \{C, D\}$, and the resulting profits are equal to $\pi_i^*(q_i^*) = \frac{1}{2}\theta_i$. Dirty firms produce more than clean firms because they are more cost-efficient, and therefore they make higher profits. They do not account for the negative externality created by their emissions when maximizing their own profits, imposing a total damage on society equal to $\gamma\theta_D$. Total welfare is equal to:

$$W^* = \frac{1}{2}\lambda q_C^* + (1-\lambda)\left(\frac{1}{2} - \gamma\right)q_D^*.$$

2.2 Regulation under Complete Information

Suppose all domestic firms are located inside the region, implying that their technologies are perfectly observable by the regulator. To illustrate this situation, consider how EU authorities can verify European producers' emissions. How can the unregulated situation be improved using regulatory instruments?

The regulator chooses taxes and quantities to produce for each type of firm by max-

3. This is motivated by the double dividend hypothesis: environmental taxes can both reduce environmental damages and finance reductions in other types of taxes which are sources of distortions.

imizing the social welfare (1) while ensuring that firms make non-negative profits.[4] The solution to this problem is the welfare-maximizing regulation.[5] Notice that social welfare (1) can be written in terms of quantities and profits as:

$$W = \lambda\Big((1+\beta)\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big) - \beta\pi_C\Big) + (1-\lambda)\Big((1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta\pi_D\Big). \quad (2)$$

This expression gives some intuition about the way tax revenues are values: gross profits (before taxation) made by firms are valued with a weight superior to one because they represent the amount that may be retrieved by the regulator with a tax. On the other hand, net profits enter negatively in social welfare, showing that society is willing to tax firms as high as possible.

Formally, the regulator thus solves:

$$\begin{aligned} &\max_{q_D, q_C, \pi_C, \pi_D} (2) \\ &\text{s.t.} \quad \pi_C \geq 0 \\ &\text{and} \quad \pi_D \geq 0. \end{aligned}$$

To ensure non-negative production, we impose the following assumption.

Assumption 2. *The environmental marginal damage is low enough to ensure that production from dirty firms is socially desirable:* $\gamma < 1 + \beta$.

The following proposition summarizes the welfare-maximizing regulation under complete information.

Proposition 1 (Welfare-maximizing regulation under complete information)**.** *Suppose all firms are located within the domestic region, implying complete information about their technologies. The regulator chooses a regulation to maximize social welfare in the form of a tax structure* $\{(t_C, q_C); (t_D, q_D)\}$ *specifying taxes and quantities for each type of firm. The results imply the following.*

4. By construction, the maximization program of the regulator is always concave in both q_C and q_D for all acceptable values of the model parameters.

5. We denote the elements of the welfare-maximizing regulation with the superscript *FB*, which stands for "first-best", to follow the usual notations in contract theory.

(i) Dirty firms are required to produce less than in the unregulated case: $q_D^{FB} < q_D^*$*, where* $q_D^{FB} = \theta_D\left(1 - \frac{\gamma}{1+\beta}\right)$.

(ii) Clean firms produce their unregulated quantity: $q_C^{FB} = q_C^* = \theta_C$.

(iii) Firms profits are totally retrieved through taxes such that $\pi_C^{FB} = \pi_D^{FB} = 0$.

More precisely: $\begin{cases} t_D^{FB} = \frac{1}{2}\left(1 + \frac{\gamma}{1+\beta}\right) \\ t_C^{FB} = \frac{1}{2} \end{cases}$.

Proof. See Appendix 1.A. □

To maximize tax revenues, the regulator sets the highest possible tax rate on each type of firm such that firms make non-negative profits. As a consequence, all profits are maintained at zero. The regulator extracts the entire gross profits, with $t_i^{FB} = 1 - \frac{1}{2\theta_i} q_i$.

Clean firms are free to produce their unregulated quantity as they do not create any negative externality. However, dirty firms are required to produce a lower quantity than their profit-maximizing production, in order to internalize the negative externality from emissions. The quantity produced by the dirty firms is increasing in θ_D: a more cost-efficient firm is allowed to produce more, as the regulator values gross profits.

The results show the existence a trade-off between environmental protection and the extraction of profits through taxation. Indeed, the quantity produced by dirty firms is decreasing in the marginal damage of emissions γ because it is optimal to pollute less when the environmental damage is higher. However, it is increasing in the value attributed by society to the extraction of profits β. Indeed, when the social planner highly values tax revenue (high β), he is willing to let dirty firms produce more in order to extract more. Thus, parameter β illustrates the willingness of the regulator to increase dirty firms' production above the level that would be required if we were to mitigate the environmental damage only (in other words, above the "Pigouvian" level of production).

Under the zero-profit condition maintained by the social planner, per-unit taxes and total taxes move in opposite directions when quantity varies. Indeed, per-unit taxes ($t_i = 1 - \frac{1}{2\theta_i} q_i$) are decreasing in the quantity produced q_i; while total taxes ($T_i = q_i - \frac{1}{2\theta_i} q_i^2$) are increasing in q_i.[6] Intuitively, higher production increases firm's gross profits, re-

6. This is true for $q_i < \theta_i$, which is always the case here.

quired a larger total tax to keep net profits at zero. However, because this larger output provides a broader tax base, zero profits can be maintained with a lower tax rate per unit. This relationship captures how taxes on clean and dirty firms adjust when quantities change in response to variations in the model parameters (β, γ, θ_i).

The per-unit tax imposed on dirty firms is greater than the per-unit tax imposed on clean firms. If production did not harm society with the creation of an environmental damage ($\gamma = 0$), per-unit taxes on clean firms and dirty firms would be identical. However, as long as there exists an environmental damage $\gamma > 0$, the regulator requires dirty firms to produce less than their profit-maximizing quantity, to internalize the negative effect. The decrease in quantity translates to an increase in per-unit tax when profits are maintained at zero. A positive value of β attenuates this effect.

Quite directly, the welfare attained under this regulation is higher than in the unregulated case. More precisely, the maximized value of social welfare is equal to

$$W^{FB} = \frac{1}{2}\lambda(1+\beta)q_C^{FB} + \frac{1}{2}(1-\lambda)(1+\beta-\gamma)q_D^{FB}.$$

There is no issue of implementation when considering domestic firms, as the regulator is able to perfectly verify firms' technologies. Thus, firms are directly required, depending on their type θ_i, to produce the welfare-maximizing quantity and pay the corresponding tax to the regulator.

Figure 1.1 illustrates the welfare-maximizing regulation described in proposition 1. The imposition of a quantity to be produced can be regarded as a cap that firms are not allowed to exceed, with a corresponding total tax. In other words, from zero production up to $q_i = q_C^{FB}$, firms are required to pay a total tax equal to $\frac{1}{2}q_i$ (green area on the graph). From $q_i = q_C^{FB}$ up to $q_i = q_D^{FB}$, firms are required to pay a total tax of $\frac{1}{2}(1 + \frac{\gamma}{1+\beta})q_i$ (orange area). This is represented by the non-linear red curve. As firms are profit-maximizing, facing this choice, they will choose to produce a quantity exactly equal to the quantities q_i^{FB}. Firms therefore bunch on the blue dots on the figure.

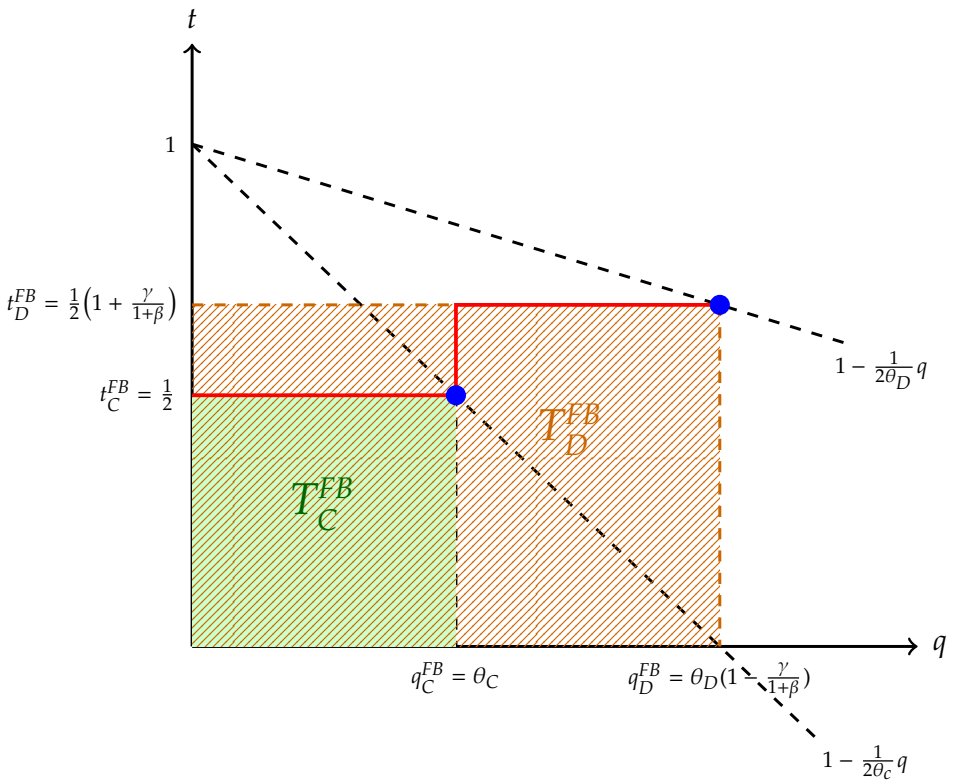


Figure 1.1 – *Optimal regulation of domestic firms under complete information*

3 Regulation of Firms Relocated in Foreign Countries

3.1 CBAM against Carbon Leakage & Asymmetric Information

In the previous section, we derived the optimal regulation chosen by the regulator for domestic firms when they are located inside the country. The aim of the present analysis is to investigate the design of a regulation when firms try to escape taxation by relocating their production facilities outside the regulated region. This situation corresponds to the problem of carbon leakage. A Carbon Border Adjustment Mechanism is a regulation intended to address this issue by imposing a carbon price on emissions generated by the production of imported goods. It aims to maintain a level playing field between firms operating within and outside the regulated area.

However, we assume that the regulator cannot directly observe the emissions of the firms located abroad, that is, each firm's type type θ_i is private information. This is a crucial concern for the design of a CBAM, since obtaining reliable information on the carbon emissions of firms located in foreign countries can be particularly difficult. The

European Commission's proposal relies on a declarative system, in which firms self-report their emissions. Such a system creates incentives for strategic under-reporting of emissions when it is profitable to do so. Although the proposal mentions the potential use of audits to deter misreporting, this option remains remains vaguely specified and would likely entail high administrative costs, if not outright infeasibility. In the absence of declaration, the proposal stipulates that firms be assigned "default values", based on the average emission intensity of the worst 10% performing EU producers. Besides contradicting the very logic of carbon pricing, this approach fails to ensure an accurate representation of the carbon content of imported goods.[7]

These informational and practical constraints motivate the development of a regulation that ensures compliance even when the regulator lacks direct information on firms' emissions. Accordingly, we derive an incentive-compatible tax structure that sustains truthful reporting.

3.2 Optimal CBAM under Asymmetric Information

The problem of incentives. For simplification, assume that all firms (clean and dirty) have relocated their production plants outside of the country, produce and export the entire quantity to the domestic country. The regulator does not know θ_i. He designs a tax structure and lets firms self-select the tax and associated quantity in $\{(q_C, t_C); (q_D, t_D)\}$. In this case, offering the regulation derived in complete information from the previous section (Proposition 1) no longer allows to reach maximum welfare. Indeed, when faced with the first-best regulation, dirty firms prefer not to choose according to their actual technology, but mimic the clean types by choosing (q_C^{FB}, t_C^{FB}) to earn a positive profits: $\pi_D(q_C^{FB}, t_C^{FB}) > 0$. To the uninformed regulator, the two technologies are now indistinguishable, and all firms behave as clean types. Note however that clean firms would choose accordingly, as $\pi_C(q_D^{FB}, t_D^{FB}) < 0$.

Solving the problem. To separate firms with different technologies and avoid mimicking from dirty firms, the regulator needs to design the regulation differently. In

7. Moreover, studies call for a differentiation of benchmark values among exporters but this raises two issues: (*i*) data for country-specific default values may not be available, (*ii*) exporter-specific benchmarks might violate GATT's Most Favoured Nation principle (Cosbey et al., 2019).

particular, to ensure that dirty firms reveal their technology by choosing the correct tax structure, he will need to let them earn some profits.[8]

Formally, the regulator chooses a tax structure $\{(q_C, t_C); (q_D, t_D)\}$ to maximize social welfare (2) subject to

$$q_C - \frac{1}{2\theta_C} q_C^2 - t_C q_C \geq q_D - \frac{1}{2\theta_C} q_D^2 - t_D q_D, \tag{3}$$

$$q_D - \frac{1}{2\theta_D} q_D^2 - t_D q_D \geq q_C - \frac{1}{2\theta_D} q_C^2 - t_C q_C, \tag{4}$$

$$\pi_C(q_C, t_C) \geq 0, \tag{5}$$

$$\pi_D(q_D, t_D) \geq 0. \tag{6}$$

The first two constraints (3) and (4) ensure that each type of firm (clean and dirty respectively) makes higher profits when selecting the tax structure intended for its emissions, relative to the profits it would earn from behaving like the other type.

The latter two constraints (5) and (6) ensure that none of the firms make negative profits, and therefore both types are willing to participate in the market.[9]

The problem can be simplified to maximizing social welfare subject to $\pi_D^{SB} = q_C^{SB^2}\left(\frac{1}{2\theta_C} - \frac{1}{2\theta_D}\right)$ and $\pi_C^{SB} = 0$.[10] The following proposition described the choice of regulation of the regulator under incomplete information. The results denoted with the superscript SB standing for "second-best".

Proposition 2 (Second-best regulation under incomplete information about firms located abroad)**.** *Suppose all firms are located abroad, implying incomplete information about their technologies. The regulator chooses a regulation (CBAM) in the form of a tax structure* $\{(t_C, q_C); (t_D, q_D)\}$ *specifying taxes and quantities to maximize social welfare such that firms reveal their true technology. The results imply the following.*

(i) No distortion of the quantity produced by dirty firms compared to the first-best regulation, but a lower per-unit tax paid by these firms, allowing them to earn positive profits: $\pi_D^{SB} = q_C^2\left(\frac{1}{2\theta_C} - \frac{1}{2\theta_D}\right)$.

8. In contract theory, this is usually called information rent.

9. In other words, constraints (3) and (4) are incentive compatibility constraints; while (5) and (6) are participation constraints.

10. Concavity of the program in q_D is always satisfied. Concavity of the program in q_C is satisfied for $(1-\lambda)\beta\theta_C \leq (\lambda+\beta)\theta_D$. Details on the simplification of the problem can be found in the Appendix.

(ii) A downward distortion of the quantity produced by clean firms compared to the first-best:

$$q_C^{SB} = \varphi q_C^{FB},$$

$$\textit{where } \varphi = \frac{\theta_D \lambda(1+\beta)}{\theta_D(\lambda+\beta) - \theta_C \beta(1-\lambda)} < 1.$$

Clean firms' profits are fully retrieved through taxes ($\pi_C^{SB} = \pi_C^{FB} = 0$).

(iii) Social welfare decreases, but environmental damages are internalized.

Proof. See Appendix 1.B. □

There are two ways in which the regulator ensures that dirty firms do not misreport their emissions. First, the quantity that clean firms are allowed to produce decreases relative to the full information case: $q_C^{SB} < q_C^{FB}$. By reducing clean production, the regulator makes it less attractive for dirty firms to misrepresent themselves as clean. Consequently, the second-best clean quantity is decreasing in θ_D: the more cost-efficient the dirty firms, the stronger its incentive to misreport, and thus the greater the reduction in the clean firms' quantity required to maintain incentive compatibility. The total tax imposed on clean firms is still set such that their profits are equal to zero: $T_C^{SB} = q_C^{SB} - \frac{1}{2\theta_C} q_C^{SB^2}$. Thus, the per-unit tax on the second-best clean quantity is equal to $t_C^{SB} = 1 - \frac{1}{2\theta_C} q_C^{SB}$.

Second, dirty firms get rewarded (*i.e.* earn an information rent) through lower taxation. Indeed, the total tax imposed on dirty firms is equal to $T_D^{SB} = T_D^{FB} - \pi_D^{SB}$, where $\pi_D^{SB} > 0$. Dirty firms quantity is set at the first-best level $q_D^{SB} = q_D^{FB}$. The per-unit tax on this quantity therefore writes $t_D^{SB} = t_D^{FB} - \frac{\pi_D^{SB}}{q_D^{SB}}$, and not only depends on the quantity produced by dirty firms, but also on the quantity produced by clean firms, which affects the profits that dirty firms earn.

Total taxes are non-linear and increasing in quantities.[11] The total tax imposed on clean firms is lower than under the first-best regulation since their profits are maintained at zero but the quantity produced is reduced. Dirty firms also face a lower total tax compared to the first-best, as they make positive profits despite producing the same

11. T_D^{SB} is increasing in q_D as long as $q_D < 2\theta_D$, which holds. T_C^{SB} is increasing in q_C as long as $q_C < 2\theta_C$, which is satisfied as well.

quantity. Because $q_D > q_C$ (as implied by (3) and (4)), the total tax imposed on dirty firms is always greater than the total tax imposed on clean firms: $T_D^{SB} > T_C^{SB}$. Intuitively, the dirty type being more cost-efficient at producing the good produces more and thus pays a higher total tax, but is willing to do so to benefit from its cost advantage.

As discussed in the previous section, total and per-unit taxes respond in opposite direction to changes in quantities, when profits are maintained at zero. The per-unit tax on the second-best clean quantity therefore rises compared to the first-best solution, as the regulator maintains the zero-profit condition but quantity decreases; whereas the per-unit tax on the second-best dirty quantity decreases, to generate the correct incentives.

In this framework, clean firms do not generate environmental damages: they are the "good" type with respect to the public good. However, being less cost-efficient than dirty firms, they are the "bad" from the private perspective. As a consequence, under incomplete information, the regulator is forced to reward dirty firms in order to acquire information and penalize clean firms, by decreasing their production, even though the socially optimal allocation would allow clean firms to produce more. These results show how information asymmetries may undermine the provision of clean goods when being green is more costly.

Social welfare is unambiguously lower under incomplete information than under complete information ($W^{SB} < W^{FB}$). On the one hand, welfare associated with clean firms decreases due to their reduced output. On the other hand, welfare from dirty firms also falls because the tax revenues decrease, while their quantity remains constant. The overall loss in welfare does not however stem from increased environmental damage: the regulator manages to fully internalize the environmental damages generated by dirty firms.

If we relax the simplifying assumption that clean firms cause no environmental damage at all, and instead assume that they generate some emissions at a lower rate than dirty firms (*i.e.* $\gamma_C < \gamma_D$), the model yields an additional insight. In this case, the reduction in the clean firms' quantity for incentive purposes would lead to an over-reduction of their emissions relative to the first-best outcome. Additionally, this "over-preservation" effect would come at the cost of not targeting the most polluting

firms.

Figure 1.2 gives an illustration of the regulation chosen by the regulator for firms located abroad, under incomplete information. Identical to the previous section, the imposition of a quantity to be produced can be considered as a cap on quantity with a corresponding total tax. From $q_i = 0$ up to q_C^{SB}, a firm must pay a total tax equal to T_C^{SB} (green area); and from q_C^{SB} up to q_D^{FB}, a firm must pay a total tax that leaves it with π_D^{SB} (red area). This tax structure is represented by the non-linear red curve. Being profit-maximizing, clean firms will choose exactly q_C^{SB} and dirty firms will produce q_D^{SB}, bunching on the two blue points in the figure.

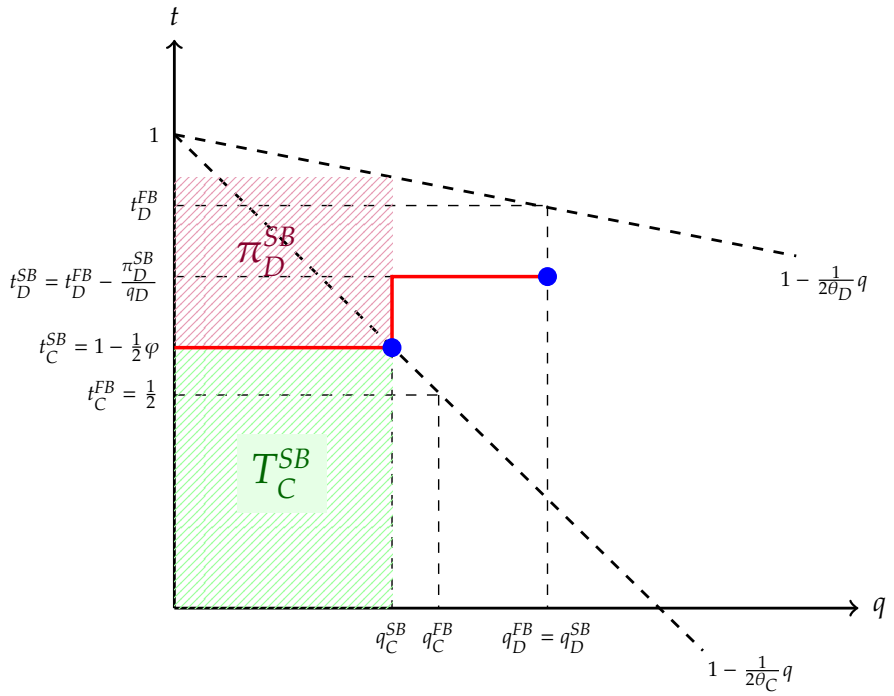


Figure 1.2 – *Regulation of firms located abroad under incomplete information*

4 WTO Compatibility

The primary objective of environmental taxation is to ensure that polluting firms internalize the social cost of their emissions. In the present framework, however, the regulator also seeks to extract profits from firms as he values tax revenues, in addition to correcting environmental externalities. When applies to imported goods, such a

mechanism may conflict with the rules of the World Trade Organization (WTO). To be compatible with WTO requirements, a border adjustment mechanism must be designed to target only emissions and not firm profits (Cosbey et al., 2019; Pauwelyn, 2013).

To incorporate this exogenous constraint into the design of a CBAM, consider that the tax imposed on firms must not exceed the Pigouvian tax level, defined as the per-unit tax rate equal to the marginal environmental damage. Under this rule, the regulator would not be able to tax clean firms, since they generate no environmental damage, implying $t_C^W = 0$. Also, this would constrain the tax imposed on dirty firms such that $t_D^W \leq \gamma$. The regulation developed under incomplete information in section 3 clearly violates at least the first condition, implying that a WTO-compatible regulation must impose this constraint. Since clean firms can no longer be subject to taxation, they will earn positive profits as long as $q_C^W \in [0; 2\theta_C]$. By contrast, the second condition is not necessarily violated: for all values of $\beta \in [0, 1]$, there exists a range of admissible values of γ such that the WTO constraint on t_D is satisfied. [12]

Solving the problem. To design a regulation applicable to firms located abroad under incomplete information about their emissions, and in compliance with WTO rules, the regulator maximizes social welfare (2) subject to

$$\pi_C \geq q_D - \frac{1}{2\theta_C} q_D^2 - t_D q_D, \tag{7}$$

$$\pi_D \geq q_C - \frac{1}{2\theta_D} q_C^2, \tag{8}$$

$$\pi_C \geq 0, \tag{9}$$

$$\pi_D \geq 0, \tag{10}$$

$$t_C = 0, \tag{11}$$

$$t_D \leq \gamma. \tag{12}$$

Similarly to section 3, the first two constraints ensure that both types of firms choose correctly when facing the regulation, and do not misreport their emissions. Clean firms are not likely to behave as dirty firms, so (7) can be ignored and checked ex-post. Inequalities (9) and (10) ensure the participation of both types. Constraints (11) and (12)

12. Refer to the Appendix for an intuition on this matter.

ensure compatibility with the WTO rules, as discussed above.

The problem can be simplified to maximizing the welfare function (2) subject to $\pi_D = q_C\left(1 - \frac{1}{2\theta_D}q_C\right)$ and $\pi_C = q_C\left(1 - \frac{1}{2\theta_C}q_C\right)$.[13]

The following proposition describes the WTO- and incentive-compatible regulation chosen by the regulator under incomplete information about firms' emissions, and to comply with the Pigouvian taxation constraints. The results are denoted with the superscript W.

Proposition 3 (WTO- and incentive-compatible regulation under incomplete information)**.** *Suppose all firms are located abroad, implying incomplete information about their technologies. Also, suppose that taxing firms more than the Pigouvian level violates WTO rules. The regulator chooses a regulation (CBAM) in the form of a tax structure* $\{(t_C, q_C); (t_D, q_D)\}$*, specifying taxes and quantities to maximize social welfare such that firms reveal their true technology, and to ensure compliance with the WTO. The results imply the following.*

(i) No distortion of the quantity produced by dirty firms compared to the first-best and second-best regulations: $q_D^W = q_D^{SB} = q_D^{FB}$*. An increase or a decrease of dirty firms' profits depending on the share of clean firms* λ *and the value attributed to tax revenues* β*. A higher or a lower total tax imposed on dirty firms depending on the change in profits.*

(ii) An additional downward distortion of the quantity produced by clean firms compared to the second-best regulation:

$$q_C^W = \eta q_C^{FB},$$

where $\eta = \dfrac{\theta_D(\lambda - (1-\lambda)\beta)}{\lambda\theta_D - (1-\lambda)\beta\theta_D} < \varphi < 1.$

Positive profits earned by clean firms: $\pi_C^W = q_C^W\left(1 - \dfrac{1}{2\theta_C}q_C^W\right)$.

Proof. See Appendix 1.C. □

First, the imposition of WTO constraints make the design of an incentive-compatible regulation more difficult, as it limits the regulator's ability to tailor taxes to elicit truthful

13. Concavity of the program in q_D is always satisfied. Concavity of the program in q_C is assured for $\lambda\theta_D > (1-\lambda)\beta\theta_C$. Details on the simplification of the problem can be found in the Appendix.

reporting of emissions. Indeed, by preventing the taxation of clean firms ($t_C^W = 0$), dirty firms face stronger incentives to misreport to avoid paying a tax altogether. As a consequence, the regulator must enforce lower quantities to be produced by clean firms, $q_C^W < q_C^{SB}$; while allowing dirty firms to produce the same quantity as under the first- and second-best regulations, $q_D = \theta_D\left(1 - \frac{\gamma}{1+\beta}\right)$.

Second, because clean firms now face no taxation, they necessarily earn positive profits ($\pi_C^W > 0$). The tax on dirty firms is chosen such that they are exactly indifferent between reporting their true emissions and mimicking clean firms. That is, it leaves them just enough profits so that they are willing to reveal their type. The size of this information rent depends on how costly it is for the regulator to distort clean quantities. When there are many clean firms (λ large) and when tax revenues are not highly valued (β small), the regulator prefers to reduce distortions on the clean quantity and instead compensate dirty firms with higher profits. This leads to an increase in dirty profits compared to the second-best. In this case, the total tax charged on dirty firm decreases ($T_D^W < T_D^{SB}$). Conversely, when few firms are clean (λ small) or when revenue extraction is highly valued (β large), the regulator prefers limiting the information rent left to dirty firms as much as possible by severely reducing clean production. In this case, dirty profits decrease with the WTO-compatible regulation compared to the second-best, and the dirty total tax ($T_D^W > T_D^{SB}$) increases.

Intuitively, concavity conditions favor scenarios with high λ and/or low β. In the extreme case where $\lambda = 1$ and $\beta = 0$, clean firms produce the same quantity both under the second-best and WTO-compatible regimes ($q_C^{SB} = q_C^W = \theta_C$), while dirty firms benefit from increased information rents ($\pi_D^W > \pi_D^{SB}$).

Similar to the second-best tax discussion, it is possible to show that, for all values of β, there exists a range of acceptable values of the marginal damage γ such that the WTO-compatible tax satisfies the Pigouvian constraint $t_D^W \leq \gamma$.[14]

Figure 1.3 illustrates the regulation chosen by the regulator for foreign located abroad and to comply with the WTO Pigouvian taxation rule, in the case where $\pi_D^W > \pi_D^{SB}$. For quantities ranging from zero to $q_i = q_C^W$, a firm is subject no taxation. For quantities from $q_i = q_C^W$ to q_D^{FB}, a firm has to pay a total tax equal to T_D^W, as represented by the red curve.

14. Refer to the Appendix for intuition on that matter.

Firms still bunch on the two blue points. Both types of firms make positive profits (green and orange areas), with clean profits unambiguously lower than dirty profits.

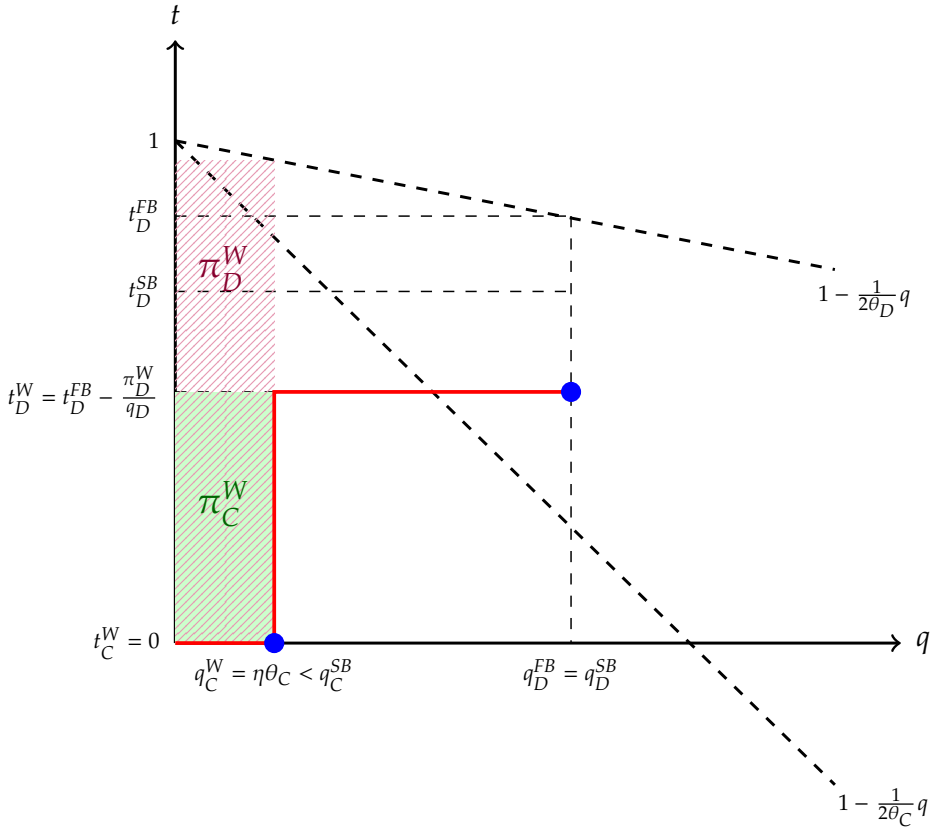


Figure 1.3 – *Regulation of firms located abroad under incomplete information and WTO rules*

5 A Note on Firms' Choice of Technology

This section explores the interaction between incentivizing firms to adopt the clean technology and the feasibility of inducing such an investment through an environmental policy under incomplete information.

We show that, because adopting the clean technology is inherently less cost-efficient than producing with the dirty technology, it is impossible to induce firms to invest in the acquisition of the "good" type. All firms prefer to remain dirty while pretending to have adopted the clean technology. This highlights a contradiction between encouraging investments environmentally superior type and reporting emissions truthfully.

Building on the model developed in previous sections, assume now that all firms lo-

cated abroad initially have the dirty technology, *i.e.* $\theta_i = \theta_D \ \forall i$. Before production, firms may choose to incur a fixed cost $F > 0$ to acquire the clean technology of production, that is to change to $\theta_i = \theta_C$. For simplicity, the probability of success in developing the clean technology when performing the investment is equal to one. [15]

Suppose that it is in the regulator's interest to encourage firms to invest in the clean technology. In this case, firms that invest can be considered to exhibit the "good" behavior, as they choose to acquire in a technology that generates no environmental damage.

Under the WTO-compatible regulation described in section 4, if firms are free to choose their technology ex-ante, none would invest F in the clean technology, as clean firms earn lower profits than dirty firms.

To address this, the regulator could try to adjust the policy to provide sufficient incentives for investment. Under the Pigouvian constraint where $t_C = 0$, the regulator would like to choose q_D, q_C, T_D and a subsidy S to give to clean firms for investing F, in order to maximize social welfare such that firms (*i*) perform the investment in the clean technology, (*ii*) report their investment truthfully to the regulator, and (*iii*) make non-negative profits.

Formally, the regulator must guarantee that the following conditions hold:

$$q_C - \frac{1}{2\theta_C} q_C^2 - F + S \geq q_D - \frac{1}{2\theta_C} q_D^2 - T_D - F, \tag{13}$$

$$q_D - \frac{1}{2\theta_D} q_D^2 - T_D \geq q_C - \frac{1}{2\theta_D} q_C^2 + S, \tag{14}$$

$$q_C - \frac{1}{2\theta_C} q_C^2 - F + S \geq 0, \tag{15}$$

$$q_D - \frac{1}{2\theta_D} q_D^2 - T_D \geq 0, \tag{16}$$

$$q_C - \frac{1}{2\theta_C} q_C^2 - F + S \geq q_D - \frac{1}{2\theta_D} q_D^2 - T_D. \tag{17}$$

The first two constraints are the incentive compatibility constraints, ensuring that a firm that invested in the acquisition of the clean technology does not report that it is dirty (condition (13)), and that a dirty firm does not report that it is has invested

15. Assuming that there exists a probability of failure in acquiring the clean technology only reinforces our point.

in the clean technology (condition (14)). Constraints (15) and (16) are participation constraints, ensuring that none of the firms exit the market. Finally, constraint (17) is the investment-inducing condition. It ensures that firms prefer performing the investment rather than keeping the dirty technology.

However, investing in the clean technology makes the firm less cost-efficient, with $\theta_C < \theta_D$. Therefore, there is a contradiction between conditions (14) and (17).[16]

Therefore, when clean production is relatively more expensive than using a dirty technology, there is an incompatibility between inducing firms to choose the "good" behavior (investment in the clean technology) and inducing firms to report their true technology to the regulator (incentive compatibility constraints).

6 Conclusions

Since some regions regulate carbon emissions more stringently than others, global climate action is undermined by the issue of carbon leakage. A regulator may choose to regulate firms located abroad with a Carbon Border Adjustment Mechanism, aiming to impose a price on the carbon content of imported goods. However, it may be very costly, if not impossible, to acquire information about these firms' emissions. The absence of such information is the particular area of focus of this paper.

The main model studies the design of a CBAM when firms located abroad keep as private information their technology of production. In order to properly distinguish between clean and dirty firms, the regulator must design a non-linear tax structure that differs from the domestic regulation, and thus induces a lower total welfare than under full information. This decrease in total welfare comes from a lower quantity produced by clean firms and a lower tax imposed on dirty firms. The assumption that clean firms have a higher cost of production than dirty firms implies that, under incomplete information, the regulator is forced to choose a tax structure that rewards dirty firms and penalizes clean firms. Including the WTO compliance constraints in a simple manner in the model sheds light on how international agreements may impose further distortions on CBAMs in the presence of information asymmetries. The result highlights a key limitation

16. Adding them together gives $\frac{\theta_C}{\theta_D} \geq 1 + \theta_C F$, which is necessarily violated when $\theta_C < \theta_D$.

of carbon border policies under incomplete information: information asymmetries create inherent conflicts between encouraging green investment and enforcing accurate emissions reporting.

Future research could relax the assumption of perfectly elastic demand in order to include consumer surplus and price in the model.

References

Baron, D. P., & Myerson, R. B. (1982). Regulating a Monopolist with Unknown Costs. *Econometrica*, *50*(4), 911–930.

Bohringer, C., Balistreri, E. J., & Rutherford, T. F. (2012). The Role of Border Carbon Adjustment in Unilateral Climate Policy: overview of an Energy Modeling Forum study (EMF 29). *Energy Economics*, *34*, S97–S110.

Bohringer, C., Carbone, J. C., & Rutherford, T. F. (2018). Embodied Carbon Tariffs. *Scandinavian Journal of Economics*, *120*(1), 183–210.

Cosbey, A., Droege, S., Fischer, C., & Munnings, C. (2019). Developing Guidance for Implementing Border Carbon Adjustments: lessons, cautions, and research needs from the literature. *Review of Environmental Economics and Policy*, *13*(1), 3–22.

Dasgupta, P., Hammond, P., & Maskin, E. (1980). On Imperfect Information and Optimal Pollution Control. *The Review of Economic Studies*, *47*(5), 857–860.

Ellerman, A. D., Marcantonini, C., & Zaklan, A. (2016). The European Union Emissions Trading System: Ten Years and Counting. *Review of Environmental Economics and Policy*, *10*(1), 89–107.

European Commission. (2021a). Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions. 'Fit for 55': Delivering the EU's 2030 Climate Target on the Way to Climate Neutrality.

European Commission. (2021b). Proposal for a Regulation of the European Parliament and of the Council establishing a Carbon Border Adjustment Mechanism [COM(2021) 564 final].

Hoel, M. (1996). Should a Carbon Tax be Differentiated Across Sectors? *Journal of Public Economics*, *59*(1), 17–32.

Kwerel, E. (1977). To Tell the Truth: Imperfect Information and Optimal Pollution Control. *The Review of Economic Studies*, *44*(3), 595–601.

Laffont, J.-J., & Martimort, D. (2002). *The Theory of Incentives*. Princeton University Press.

Laffont, J.-J., & Tirole, J. (1996). Pollution Permits and Environmental Innovation. *Journal of Public Economics*, *62*(1), 127–140.

Markusen, J. R. (1975). International Externalities and Optimal Tax Structures. *Journal of International Economics*, *5*(1), 15–29.

Monjon, S., & Quirion, P. (2011). Addressing Leakage in the EU ETS: Border Adjustment or Output-Based Allocation? *Ecological Economics*, *70*(11), 1957–1971.

Nordhaus, W. (2015). Climate Clubs: overcoming Free-Riding in International Climate Policy. *American Economic Review*, *105*(4), 1339–70.

Pauwelyn, J. (2013). Chapter 15: carbon Leakage Measures and Border Tax Adjustments under WTO Law. In *Research handbook on environment, health and the WTO*. Edward Elgar Publishing.

Porter, M. E., & van der Linde, C. (1995). Toward a New Conception of the Environment-Competitiveness Relationship. *Journal of Economic Perspectives*, *9*(4), 97–118.

Spulber, D. F. (1988). Optimal Environmental Regulation under Asymmetric Information. *Journal of Public Economics*, *35*(2), 163–181.

Stern, N. (2007). *The Economics of Climate Change: the Stern Review*. Cambridge University Press.

Appendix

1.A Proof of Proposition 1

The regulator solves the following problem:

$$\max_{q_D,q_C,T_C,T_D} W = \lambda\Big((1+\beta)\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big) - \beta\pi_C\Big) + (1-\lambda)\Big((1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta\pi_D\Big) \quad (2)$$

subject to: $\pi_C(q_C) \geq 0$ and $\pi_D(q_D) \geq 0$.

Profits enter negatively in the welfare function, so the regulator chooses $\pi_C^{FB} = \pi_D^{FB} = 0$. To maintain these profits at zero, he has to offer, in the regulation, the following total taxes: $T_C^{FB} = \Big(q_C - \frac{1}{2\theta_C}q_C^2\Big)$ and $T_D^{FB} = \Big(q_D - \frac{1}{2\theta_D}q_D^2\Big)$. The solution of this problem with respect to quantities is given by deriving the first-order conditions:

- With respect to q_C: $q_C^{FB} = \theta_C$;
- With respect to q_D: $q_D^{FB} = \theta_D\Big(1 - \frac{\gamma}{1+\beta}\Big)$.

As a consequence; we can rewrite total taxes as their exact expressions:
$T_C^{FB} = \frac{1}{2}\theta_C$ and $T_D^{FB} = \frac{1}{2}\theta_D\Big(1 - \frac{\gamma^2}{(1+\beta)^2}\Big)$.

1.B Proof of Proposition 2

The maximization problem of the regulator writes:

$$\max_{q_D,q_C,T_C,T_D} W = \lambda\Big((1+\beta)\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big) - \beta\pi_C\Big) + (1-\lambda)\Big((1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta\pi_D\Big) \quad (2)$$

subject to:

$$q_C - \frac{1}{2\theta_C}q_C^2 - t_C q_C \geq q_D - \frac{1}{2\theta_C}q_D^2 - t_D q_D, \quad (3)$$

$$q_D - \frac{1}{2\theta_D}q_D^2 - t_D q_D \geq q_C - \frac{1}{2\theta_D}q_C^2 - t_C q_C, \quad (4)$$

$$\pi_C(q_C, t_C) \geq 0, \quad (5)$$

$$\pi_D(q_D, t_D) \geq 0. \quad (6)$$

First, note that we need $q_C < q_D$ for both (3) and (4) to hold simultaneously. As discussed in the previous subsection, the problem of incentives usually arises from dirty technology firms, so we can ignore the constraint ensuring that the clean type behave according to their true technology (3) and check that it is satisfied ex-post. Constraint (4) can be rewritten $\pi_D \geq \pi_C + q_C^2\Big(\frac{1}{2\theta_C} - \frac{1}{2\theta_D}\Big)$. Constraint (5) imposes non-negative profits for clean firms, and the second term on the right-hand side is strictly positive, so the participation constraint of the dirty type (6) is necessarily satisfied and can be ignored. Because π_C and π_D enter negatively in the objective welfare function (1), the regulator would like to set them at their lowest value possible. As a consequence, both remaining constraints hold with equality: the regulator chooses $\pi_C = 0$ and $\pi_D = q_C^2\Big(\frac{1}{2\theta_C} - \frac{1}{2\theta_D}\Big)$. Proceeding by substitution, the simplified maximization problem rewrites:

$$\max_{q_D,q_C \geq 0} \lambda\Bigg((1+\beta)\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big)\Bigg) + (1-\lambda)\Bigg((1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta q_C^2\Big(\frac{1}{2\theta_C} - \frac{1}{2\theta_D}\Big)\Bigg).$$

Computing the first-order conditions and using the constraints give the following solutions:

- $q_D = \theta_D\Big(1 - \frac{\gamma}{1+\beta}\Big)$,

- $q_C^{SB} = \theta_C \dfrac{\theta_D \lambda(1+\beta)}{\theta_D(\lambda+\beta) - \theta_C \beta(1-\lambda)}$,
- $\pi_D^{SB} = q_C^2\Big(\dfrac{1}{2\theta_C} - \dfrac{1}{2\theta_D}\Big)$,
- $\pi_C^{SB} = 0$,
- $T_D^{SB} = T_D^{FB} - \pi_D^{SB}$, so $t_D^{SB} = t_D^{FB} - \frac{\pi_D^{SB}}{q_D}$,
- $T_C^{SB} = q_C^{SB} - \dfrac{1}{2\theta_C} q_C^{SB^2}$, so $t_C^{SB} = 1 - \frac{1}{2\theta_C} q_C^{SB}$.

Showing that $t_D^{SB} < t_D^{FB}$ is straightforward as $\pi_D^{SB} > 0$ and $q_D > 0$. Showing that $T_C^{SB} < T_D^{SB}$ is straightforward as well with a graphical illustration. Indeed, replacing the expressions in T_D^{SB}, we can rewrite:

$$T_D^{SB} = T_C^{SB} + \underbrace{q_D^{SB}\Big(1 - \frac{1}{2\theta_D} q_D^{SB}\Big)}_{(a)} - \underbrace{q_C^{SB}\Big(1 - \frac{1}{2\theta_D} q_C^{SB}\Big)}_{(b)}.$$

Figure 1.4 graphically shows that $(a) > (b)$.

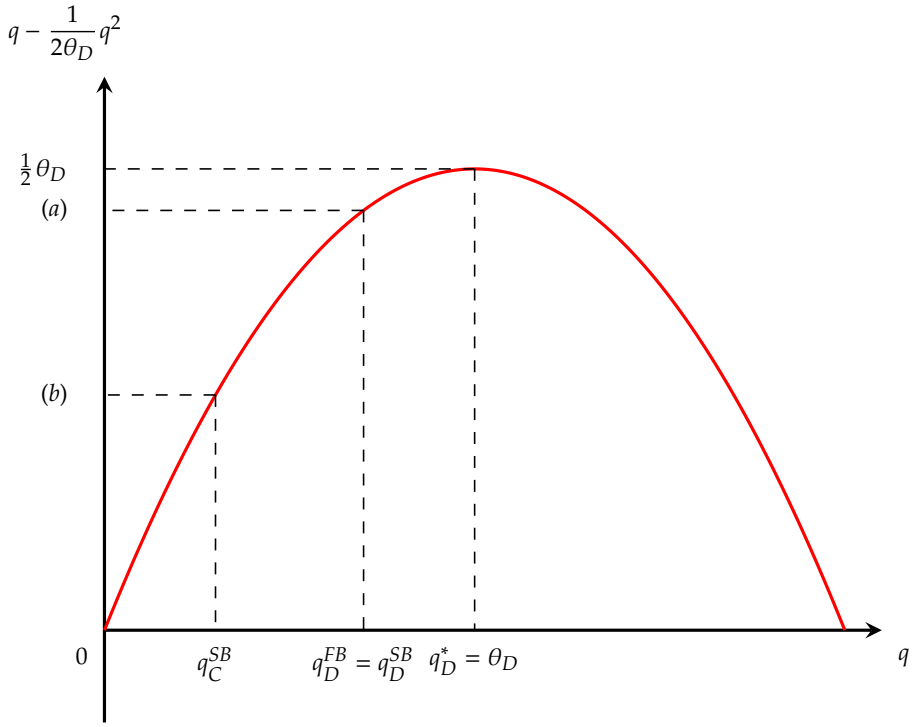


Figure 1.4 – *Graphical comparison of* (a) *and* (b)

1.C Proof of Proposition 3

Sketch of proof – Footnote 14 This reasoning aims to provide intuition for the validity of the following statement: *"For all values of $\beta \in [0,1]$, there exists a range of acceptable values of γ that satisfy the WTO requirement under second-best taxation, i.e. $t_D^{SB} \leq \gamma$"*. Recall, in first-best we have $t_D^{FB} = \frac{1}{2}\left(1 + \frac{\gamma}{1+\beta}\right)$. The first-best per-unit tax would satisfy $t_D^{FB} \leq \gamma$ if and only if $\gamma \geq \frac{1+\beta}{1+2\beta}$, where we also assume $\gamma < 1 + \beta$ for non-negativity of q_D. Therefore, $t_D^{FB} \leq \gamma$ is satisfied for all values of $\gamma \in \left[\frac{1+\beta}{1+2\beta}; (1+\beta)\right]$. Therefore, there exists values of γ inducing the first-best per-unit tax on dirty firms to satisfy the WTO rule. Now, we know that $t_D^{SB} < t_D^{FB}$. As a direct consequence, we can state that there also exists acceptable values of γ induxing the second-best per-unit tax on dirty firms to satisfy the WTO rule, *i.e.* $t_D^{SB} \leq \gamma$.

Simplifying and solving the problem. The maximization problem of the regulator writes:

$$\max_{q_D, q_C, T_C, T_D} W = \lambda\Big((1+\beta)\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big) - \beta\pi_C\Big) + (1-\lambda)\Big((1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta\pi_D\Big) \quad (2)$$

subject to:

$$\pi_C \geq q_D - \frac{1}{2\theta_C}q_D^2 - t_D q_D, \quad (7)$$

$$\pi_D \geq q_C - \frac{1}{2\theta_D}q_C^2, \quad (8)$$

$$\pi_C \geq 0, \quad (9)$$

$$\pi_D \geq 0, \quad (10)$$

$$t_C = 0 \Rightarrow \pi_C = q_C - \frac{1}{2\theta_C}q_C^2, \quad (11)$$

$$t_D \leq \gamma. \quad (12)$$

We ignore (7) and check that it is satisfied ex-post, as the problem of incentives arises from dirty firms and not clean firms. We also ignore (12) and check that there exists values of the marginal damage γ such that it holds ex-post. Given (11), then constraint

(9) is satisfied for any $q_C \in [0, 2\theta_C]$, which we check ex-post as well. Of constraints (8) and (10), only one can be binding. Therefore, one of them is irrelevant in the program. We can show that (10) is irrelevant and (8) is binding at the optimum. Proceed by contradiction. If (10) binds before (8), then $\pi_D = 0$. In this case, solving the program yields $q_C = \theta_C$. But this would lead to dirty firms making positive profits when reporting θ_C (lie), with $\pi_D(q_C, T_C) = \theta_C\big(1 - \frac{\theta_C}{2\theta_D}\big) > 0$. Therefore, the constraint on dirty firms to make sure they tell the truth about their technology (8) is binding at the optimum, and we have $\pi_D = q_C - \frac{1}{2\theta_D}q_C^2$. Proceeding by substitution, the simplified maximization problem rewrites:

$$\max_{q_D, q_C} W = \lambda\Big(q_C - \frac{1}{2\theta_C}q_C^2\Big) + (1-\lambda)\Big[(1+\beta)\Big(q_D - \frac{1}{2\theta_D}q_D^2\Big) - \gamma q_D - \beta\Big(q_C - \frac{1}{2\theta_D}q_C^2\Big)\Big].$$

Computing the first-order conditions and using the constraints give the following solutions:

- $q_D = \theta_D\big(1 - \frac{\gamma}{1+\beta}\big)$,
- $q_C^W = \theta_C \frac{\theta_D(\lambda - (1-\lambda)\beta)}{\lambda\theta_D - (1-\lambda)\beta\theta_C}$,
- $\pi_D^W = q_C^W\big(1 - \frac{1}{2\theta_D}q_C^W\big)$,
- $\pi_C^W = q_C^W\big(1 - \frac{1}{2\theta_C}q_C^W\big)$,
- $T_D^W = T_D^{FB} - \pi_D^W$, so $t_D^W = t_D^{FB} - \frac{\pi_D^W}{q_D}$,
- $t_C^W = 0$.

With these results, we can check ex-post the ignored constraints. Participation constraint for clean firms (9) is satisfied for q_C^W. Intuitively, there exists an interval of acceptable values of γ for which $t_D^W \leq \gamma$, as we have $t_D^W < t_D^{FB}$ (refer to the sketch of proof of footnote 14 for a similar reasoning). Therefore, constraint (12) can be satisfied ex-post.

We want to prove that $q_C^W < q_C^{SB}$. By replacing each quantity by its expression, we get $\beta^2(1-\lambda)(\theta_D - \theta_C) > 0$. This is true as long as $\theta_D > \theta_C$, which holds as an assumption of the model.

Now compare the profits given up to dirty firms in second-best with respect to WTO. Recall, we can write: $q_C^{SB} = \varphi q_C^{FB}$ where $\varphi = \dfrac{\theta_D\lambda(1+\beta)}{\theta_D(\lambda+\beta)-\theta_C\beta(1-\lambda)} < 1$ and $q_C^W = \eta q_C^{FB}$ where $\eta = \dfrac{\theta_D(\lambda-(1-\lambda)\beta)}{\lambda\theta_D-(1-\lambda)\beta\theta_D} < \varphi < 1$. So replacing gives: $\pi_D^{SB} = \theta_C^2\varphi^2\left(\dfrac{1}{2\theta_C}-\dfrac{1}{2\theta_D}\right)$ and $\pi_D^W = \theta_C\eta\left(1-\dfrac{1}{2\theta_D}\theta_C\eta\right)$.

Study the conditions under which we have $\pi_D^W > \pi_D^{SB}$:

$$\theta_C\eta\left(1-\frac{1}{2\theta_D}\theta_C\eta\right) > \theta_C^2\varphi^2\left(\frac{1}{2\theta_C}-\frac{1}{2\theta_D}\right) \Leftrightarrow \eta-\frac{\theta_C}{2\theta_D}\eta^2-\frac{1}{2}\varphi^2+\frac{\theta_C}{2\theta_D}\varphi^2 > 0.$$

First assume that η and φ are close. For simplification, look at the case where $\eta = \varphi$. This happens when β tends towards zero and/or λ tends towards 1. The inequality becomes $\dfrac{-1}{2}\varphi^2+\varphi > 0$ which holds for any $\varphi \in [0,2]$, therefore is always satisfied. Thus, for η and φ close (*i.e.* β small and/or λ high), we have $\pi_D^W > \pi_D^{SB}$. Now consider an increase in β and/or a decrease in λ. Given that $0 < \eta < \varphi < 1$, we can do the latter until $\eta = 0 \Leftrightarrow \lambda-(1-\lambda)\beta = 0$. In this case, the LHS of the inequality becomes: $\dfrac{-1}{2}\varphi^2+\dfrac{\theta_C}{2\theta_D}\varphi^2$. With the assumption that $\theta_D > \theta_C$, this expression is negative. Therefore, when β is high enough and/or λ small enough, the inequality is reversed and $\pi_D^W < \pi_D^{SB}$. This analysis translates directly into comparing the total and per-unit taxes imposed on dirty firms in second-best compared to WTO.

Chapter 2

Green, Greener or Neutral? Signaling Environmental Quality When Consumers Can't Tell

Abstract

How can a firm signal the environmental quality of its product to consumers when its technology is unobservable? This paper analyzes a monopoly market where the firm can be either green, using a costly non-polluting technology that generates positive externalities; or neutral, relying on a standard technology. Consumers cannot directly observe the firm's environmental performance and derive a warm-glow benefit from purchasing green products. Under incomplete information, we derive a separating equilibrium where the green firm uses either one instrument (price) or two instruments (price and effort to improve the greenness of its product) to signal its environmental quality to consumers. As a result, effort can be over- or under-provided relative to the first-best. Signaling can induce an improvement in total welfare by increasing the public good benefit of green consumption. This highlights the role of effort as a potential dual-purpose instrument: mitigating informational asymmetries and improving social surplus. Additionally, we explore how regulations such as green effort subsidies influence this equilibrium.

JEL Codes: D62, D42, D82, Q50, L15, L12.

Keywords: Signaling, Green Products, Incomplete Information, Monopoly.

1 Introduction

Research question. In recent years, consumer preferences have increasingly shifted towards more sustainable or ethical versions of certain products. Numerous surveys show that a significant portion of consumers is willing to pay more for environmentally friendly products. For example, an IPSOS MORI survey reports that 70 % of consumers are willing to pay a premium for products they consider ethically superior (MORI, 2003).

However, the credence nature of a product's environmental quality entails that consumers cannot directly observe the technology used by the firm that they are purchasing from. While consumers may notice observable efforts, such as recyclable packaging or other visible green practices, these signals do not necessarily reflect genuine improvements in the product's environmental impact. For example, a polluting firm might attempt to provide misleading signals (*i.e.* engage in forms of greenwashing) by making the product appear more environmentally friendly without actually reducing the product's environmental footprint. Such strategies may include investing in advertising campaigns that emphasize sustainability claims or designing packaging to incorporate eco-themed symbols. Thus, consumers are left uncertain about whether the effort truly enhances the product's greenness or is merely a costly attempt to appear more ethical.

Therefore, how can a monopolistic firm, whether green or neutral, use price and costly effort to signal its environmental type to consumers?

This paper aims to study the optimal choice of instruments in situations where consumers cannot perfectly observe the type of product they are purchasing, and analyze the welfare implications of such signaling strategies.

Context and model overview. In the context of this paper, the firm's *type* refers to the technology used by the firm, which can be either *green* or *neutral*, and is randomly determined by nature. Specifically, a green firm uses a non-polluting technology that creates a positive externality, but is more costly than the neutral, standard technology of production (following traditional hypothesis in this literature and abstracting from the Porter hypothesis, see Palmer et al. (1995)). These higher costs can arise from factors such as sourcing sustainable raw materials, adopting advanced energy-efficient pro-

cesses, and investing in specialized equipment to enhance environmental performance. Notably, the positive externality generated by green production makes it an impure public good, as its consumption leads to benefits such as improved air quality and better public health, which are enjoyed by everyone regardless of their individual purchase decisions. However, the firm's type is inherently difficult for consumers to observe. Indeed, consumers often have limited or no access to information regarding the specific technologies employed by firms — such as the types of machines, raw materials, or sources of energy used in production. One main interest of this paper therefore lies in the assumption that the technology used is private information of the firm.

However, while the underlying technology is unobservable to consumers, the firm can invest in observable but costly *effort* level. If the firm is green, exerting a positive effort improves further the greenness of the product (hence the size of the positive externality). For instance, a green firm might go the extra mile by adopting fully recyclable packaging or introducing take-back and repair programs. On the other hand, a neutral firm can choose to invest in effort as well to appear greener to consumers, but its effort does not lead to any real change in the environmental impact of the products, as its technology does not generate any positive externality. In this case, effort is a purely dissipative signal. Consumers therefore observe the level of effort provided by a given firm, but not its actual impact on public good.

Because the neutral firm would use effort solely to perform a form of greenwashing [1] (*i.e.* to be perceived as environmentally friendly by consumers), the marginal cost of effort it incurs is higher than for the green type. Indeed, lacking the necessary technology and knowledge to make substantial, genuine improvements, it must rely on costlier, less efficient external adjustments or superficial changes. [2]

For example, Volkswagen's "Clean Diesel" campaign marketed its diesel cars as low emission vehicles. However, it was revealed in 2015 by the EPA that the company had invested in the installation of a software to cheat emissions tests, allowing the vehicles to appear greener than they were, without any reduction in emissions.

1. For papers investigating strategic greenwashing decisions by firms, see notably Lyon and Maxwell (2011); Schmittmann and Gao (2022). For empirical work on greenwashing in the electric utility sector, see Kim and Lyon (2011).

2. We could also assume that higher cost includes reputational threats if the neutral firm is exposed as making false green claims.

One could think of a labels or certification as a solution to incomplete information about the firm's environmental quality. In this paper, we rule out the use of such instruments, and only rely on two instruments to provide information: price and effort. Even when labels do exist to indicate the environmental quality of a product, studies have shown that the large number and vast diversity of labels causes considerable confusion among consumers.[3] In other situations, labels can be manipulated by powerful firms and therefore lose their value.[4]

A crucial aspect of our model is the assumption that consumers derive a *warm glow* benefit from purchasing the green product. The concept of warm glow refers to the intrinsic satisfaction that consumers experience from contributing to the public good, such as purchasing a product that is perceived as green. This idea was first formally introduced by Andreoni (1990). We assume that all consumers experience the same level of warm glow they believe they are consuming the green product, which adds to their utility and is independent of the other product's attributes. This uniform warm-glow benefit captures the idea that green consumption is partly motivated by the consumers' desire to "do their part".[5]

Main results. The results provide three main insights.

First, as a benchmark, we derive the equilibrium monopoly strategies under complete information, as well as the welfare-maximizing solution. Under complete information, firms follow a standard monopoly pricing rule (where marginal revenue equals marginal cost) and the green firm does not take into account the public good aspect of its product, resulting in zero effort — contrary to the positive effort level that a social planner would require as optimal.

Then, we describe how these strategies change when consumers cannot directly ob-

3. See Giovannucci and Koekoek (2003) for a market study of coffee.

4. There are many examples of ecolabel controversies to refer to. Among others, the Marine Stewardship Council, the Forest Stewardship Council, and the Roundtable on Sustainable Palm Oil certification violation scandals.

5. An extensive strand of experimental and psychology literature has investigated the precise factors generating warm-glow benefits. This effect can be linked to feelings of moral satisfaction and alignment with personal values, as well as social recognition (or avoidance of guilt) for making environmentally responsible choices. See Cason and Gangadharan (2002), Elfenbein and McManus (2010), Griskevicius et al. (2010). We model this preference as a vertical attribute of the product, in line with previous theoretical works by Cremer and Thisse (1999), Bansal and Gangopadhyay (2003), and Boyer et al. (2007).

serve the environmental quality of the product (*i.e.* the type of firm they are facing). We examine the conditions for the emergence of a separating perfect Bayesian equilibrium. We show that the green firm may use two instruments to signal itself to consumers: a higher price and a positive effort level to improve the greenness of the product.

We shed light on the implications for overall welfare. We demonstrate that welfare under incomplete information can surprisingly increase towards the first-best outcome, resulting in an improvement over the full information case, when the net public good benefit from green consumption is sufficiently high. As such, this setting highlights a potential benefit of incomplete information in the presence of public goods: the effort exerted by the green firm to mitigate the informational asymmetry is costly but contributes to correcting the under-provision of the public good, leading to an overall improvement of the situation when the gains generated by the public good attribute outweigh the costs of signaling.

Finally, we investigate the effect of regulation by a benevolent social planner. Specifically, we analyze policies such as subsidies for green effort or taxes on neutral firms and assess their impact on welfare and on the structure of a separating equilibrium.

Related literature and contribution. This paper relates to three main strands of literature.

First, this paper contributes to the theoretical literature on signaling games.A classic contribution is Wolinsky (1983), which focuses on the use of prices to signal quality in markets for experience private goods. Like Wolinsky's model, this paper derives a separating perfect Bayesian equilibrium. They share important features such as higher production costs for high quality products (more specifically in our case, green products), and the idea that equilibrium prices include premiums for quality. However, our model differs in several dimensions. Indeed, it allows for the possibility of signaling with two instruments (price and effort), rather than restricting to price only. Also, the type of good we consider is different, as it can be defined as an impure public good: the "quality" of the product has a public good characteristic. Finally, the credence nature of the environmental quality implies that consumers cannot infer the product's true type through experience, which differs from Wolinsky. There are numerous papers

that examine this very issue of quality provision in markets where consumers have incomplete information and are very close to Wolinsky (1983). Among them, we can cite Farrell (1981), Klein and Leffler (1981), Cooper and Ross (1984), and Shapiro (1982). Bagwell and Riordan (1991) extend this line of work by considering a durable good, allowing for the optimal price to vary over time.

A related seminal paper is Milgrom and Roberts (1986), which examines signaling with two instruments: price and advertising. Similarly, our model explores the use of multiple instruments (price and effort) to achieve separation. In both papers, there are situations in which the firm only uses the price to achieve separation, and situations in which both instruments are used. However, as mentioned above, we focus on the environmental quality as a public good attribute, which introduces an additional form of inefficiency, linked to the provision of the public good. This key feature introduces new welfare insights in the presence of incomplete information. Also, this paper differs in the nature of the signaling instruments: in Milgrom and Roberts' model, advertising is a purely dissipative signal, while effort by the green firm here directly enhances the product's environmental quality and contributes to the public good benefit.

Several related papers examine signaling settings in which the signal itself generates externalities (*e.g.* Krasteva and Saboury, 2021; Abou El-Komboz and Goldbeck, 2025). In these frameworks, the signaling action contributes to a pure public good, whereas in this paper, the signal enhances an impure public good traded in a market.

Second, this paper extends the literature on markets for credence goods, in which consumers are unable to assess the true quality of the product, even after consumption (Darby & Karni, 1973; Dulleck & Kerschbamer, 2006). This literature demonstrates that such a feature can lead to inefficient outcomes, such as fraud, in many different sectors (healthcare, car and computer repair, financial and insurance advice, etc.). Building on this foundation, this paper ties the credence attribute of the product with a public good dimension, and specifically analyzes firm strategies that can serve as credible signals for this credence characteristic.

Finally, this paper contributes to the literature on the private provision of public goods. Seminal works in this field include Warr (1983) and Bergstrom et al. (1986), which establish conditions under which private action can lead to the provision of public goods.

More recently, Besley and Ghatak (2007) investigate the feasibility of such provision and its desirability (*e.g.* compared to government provision), and conclude that private provision generally falls short of the first-best outcome. Similar to our paper, they assume that (at least some) consumers derive a personal benefit from contributing to the public good.[6] This assumption is also used by Bagnoli and Watts (2003), who study how the market structure affects the private provision of an impure public good, when consumers enjoy a homogenous warm glow from consuming the "ethical" product. Finally, Kotchen (2006) studies the choice of consumers between consuming an impure public good or consuming the private good separately and making contributions to a pure public good. The main contribution of our paper with respect to this strand of the literature lies in the integration of incomplete information, analyzing how signaling through price and effort affects the provision of a public good.

A more recent body of research started to bridge these previously distinct strands of literature. Closely related to the present paper are Mahenc (2007, 2008) and Sengupta (2012)[7] which investigate firms' signaling strategies when the environmental quality of a product is unobservable to consumers. These papers reach similar conclusions to ours, notably the use of high prices to signal environmental quality when green production entails higher costs. However, their analyses focus exclusively on price as a signaling instrument. In contrast, we extend the framework by incorporating effort. Additionally, we explicitly model the public good nature of environmental quality, analyzing how it affects welfare under incomplete information.

Structure of the paper. This paper is organized as follows: section 2 describes the model; section 3 discusses the monopoly strategies in equilibrium under complete information and the first-best solution; section 4 characterizes and analyzes the separating perfect Bayesian equilibrium; section 5 studies the welfare implications; and section 6 discusses extensions of the model in which a benevolent social planner intervenes.

6. For a related analysis of corporate public good provision without relying on consumer warm glow, and instead focusing on the alignment of managerial incentives with shareholder interests, see Morgan and Tumlinson (2019).

7. Further related work also includes the working paper by Mahenc and Volle (2021) which explores the interaction between price signaling and consumer monitoring.

2 The Model

Consider a market with a monopoly and consumers.

Monopoly

The firm can be either of two types, randomly chosen by nature. It is either a green firm (type $\theta = g$), and produces a green good, or it is a neutral firm ($\theta = n$), and produces a regular neutral good. The type of the firm can be easily interpreted as the technology it uses, which determines the environmental impact of its production process. In other words, we could equivalently assume that with probability λ, the firm is green and uses an environmentally friendly technology to produce. Conversely, with probability $(1 - \lambda)$, the firm is neutral and relies on a conventional technology. Green production creates a positive externality[8] which can be further enhanced if the firm invests in some positive effort to increase the greenness of the product. The firm chooses a price p_θ and an effort level $e_\theta \geq 0$ to maximize its profits, which can be written as: $\pi_\theta(p_\theta, e_\theta) = (p_\theta - c_\theta)q_\theta(p_\theta) - \gamma_\theta e_\theta$ where c_θ is the constant marginal cost of production for type θ, $q_\theta(p_\theta)$ is the demand faced by the firm, and γ_θ is the marginal cost of effort. We assume that the marginal cost of producing the green good is higher than the marginal cost of producing the neutral good, *i.e.* $c_g > c_n$. This assumption can be justified in various ways. Indeed, sourcing sustainable raw materials can be more costly than using conventional alternatives. Moreover, green production may rely on advanced, energy-efficient technologies, which require significant capital investment. We can simplify further by assuming that $c_n = 0$ and thus denote the marginal cost of producing the green good by $c_g = c > 0$. On the contrary, it is less costly for the green firm to exert a certain effort level: $\gamma_n > \gamma_g > 0$. This could arise from the fact that, for the green firm, effort directly enhances the greenness of its product, improving its quality and aligning with the firm's expertise in environmentally friendly practices. On the other hand, for the neutral firm, effort incurs a cost without resulting in any real improvement of the product's environmental attribute. The only incentive for a neutral

8. In this paper, we assume that green production creates a positive externality (public good aspect) while neutral production does not. It would be equivalent to assume that neutral production creates a negative externality (for instance, through pollution), while green production does not.

firm to exert effort could arise when consumers cannot directly observe the firm's type and thus the firm might gain some profit by mimicking the behavior of a green type. This makes the marginal cost of effort heavier for the neutral firm compared to the green firm.

Consumers

Let consumers have unit demand. In this case, they decide whether they want to purchase one unit of the good or not consume. Let v_i be consumer i's valuation of the intrinsic features of the product, and $v_i \sim \mathcal{U}[0,1]$.

If consumer i faces a neutral firm, she gets utility $u_i^n = v_i - p_n$ if she consumes and $u_0^n = 0$ if she does not. However, if the consumer faces a green firm, she gets utility $u_i^g = v_i + b - p_g + B(q_g, e_g)$ if she consumes and $u_0^g = B(q_g, e_g)$ if she does not; where $b > 0$ represents the warm glow enjoyed by the consumer's own purchase of the product, and $B(q_g, e_g)$ is the positive externality created by the existence of the green product, where:

$$B(q_g, e_g) = \begin{cases} 0 & \text{if } q_g = 0, \\ \beta + h(e_g) & \text{if } q_g > 0, \text{ where} \quad h(0) = 0, \quad h'(e_g) > 0, \quad h''(e_g) \leq 0. \end{cases}$$

This public good benefit is enjoyed by consumer i regardless of her own purchasing decision. It exists even when the firm exerts zero effort, as long as the quantity produced is strictly positive. In this case, it is equal to a constant $\beta > 0$, which represents the benefit from having green production on this market compared to neutral production (*e.g.* less waste, less pollution, etc.). In the case where the firm invests in a positive level of effort to increase the greenness of its product, this benefit increases with e_g at a decreasing rate, following function $h(\cdot)$. Assuming that the public good benefit does not increase with the quantity produced captures settings where the existence of green production (rather than its scale) creates value (*e.g.*, reduced baseline pollution). This simplification keeps the analysis simple and focuses attention on the firm's effort choice rather than output level.[9]

9. Appendix 2.G explores an alternative specification where $B(q_g, e_g) = q_g(1 + h(e_g))$, allowing the public good benefit to increase with quantity. The qualitative results and main economic insights remain consistent.

Notice that neutral production and consumption do not create any type of positive externality, and although a neutral monopoly may choose a positive level of costly effort, it does not have any impact on the quality of the product and utility for consumers.

Let us further assume that the warm glow, identical across consumers, exceeds the marginal cost of producing the green product: $b > c > 0$. This assumption ensures that consumers derive greater utility from the personal benefit of purchasing a green product than the cost of producing this product. Therefore, it guarantees the optimality of this production despite the higher cost.

Timing of the Game

Figure 3.1 summarizes the timing of the game, which unfolds as follows. First, the firm's type $\theta \in \{g, n\}$ is randomly drawn by Nature. The firm observes its type, and offers a price and an effort level. Consumers observe the price and effort and formulate their demand. Finally, consumption, profits and utilities are realized.

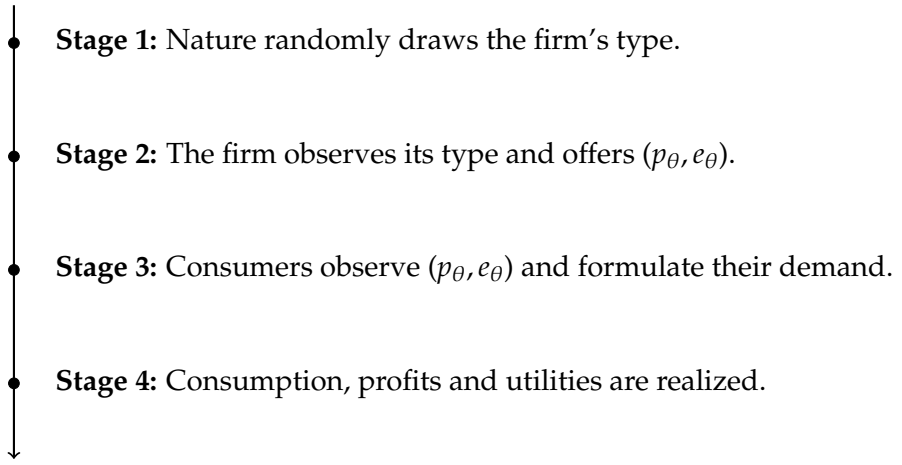


Figure 2.1 – *Timing of the game*

3 Complete Information Benchmark and First-Best

3.1 Monopoly Strategy

First of all, we establish the benchmark under complete information. This will allow us to characterize the monopoly strategy chosen by each type of firm. In this context,

assume that consumers possess complete information about the type of firm they are facing. They know whether the monopoly is a green or neutral producer, and they can base their consumption on this observation. Facing their demand, the firm will choose a price in order to maximize its profits.

Neutral firm. If the firm is neutral, a consumer i chooses to purchase the good if and only if $u_i^n \geq u_0^n$, that is when $v_i \geq p_n$. Given the distribution of the individual valuations, the number of active consumers (*i.e.* total demand for the neutral product) is given by $q_n(p_n) = 1 - p_n$, decreasing in the price charged by the firm. Facing this demand, the neutral monopoly chooses price p_n^m and effort level e_n^m to maximize its profits as follows:

$$\max_{p_n, e_n \geq 0} \ \pi_n(p_n, e_n) = p_n(1 - p_n) - \gamma_n e_n.$$

The solution implies zero effort exerted $e_n^m = 0$ as effort level is costly and does not change anything for consumers; and the price offered is the standard monopoly price: $p_n^m = 1/2$. In this case, the final demand is equal to $q_n(p_n^m) = 1/2$, and profits amount to $\pi_n(p_n^m, 0) = 1/4$.

Green firm. If the firm is green, a consumer i chooses to purchase the good if and only if $u_i^g \geq u_0^g$, that is when $v_i \geq p_g - b$. Given the distribution of the individual valuations, the number of active consumers (*i.e.* total demand for the green product) is given by: $q_g(p_g) = 1 + b - p_g$, decreasing in the price charged by the firm and increasing in the warm-glow benefit. As a consequence, the green monopoly chooses price p_g^m and effort level e_g^m in order to maximize its profits as follows:

$$\max_{p_g, e_g \geq 0} \ \pi_g(p_g, e_g) = (p_g - c)(1 + b - p_g) - \gamma_g e_g.$$

The optimal solution still implies zero effort $e_g^m = 0$ and the price offered by the green monopoly is higher than the neutral price, and is given by $p_g^m = \dfrac{1 + b + c}{2} > p_n^m$. As a consequence, the quantity of green good on the market is given by $q_g(p_g^m) = \dfrac{1 + b - c}{2}$, and monopoly profits are equal to $\pi_g^m(p_g^m, 0) = \dfrac{(1 + b - c)^2}{4}$.

Remark 1. Consumer demand is not influenced by the effort level e_g of the green firm either. Indeed, the costly effort exerted to increase the greenness of the product benefits the consumers regardless of their decision to purchase or not, as it only enters in the public good benefit. Also, the size of the positive externality received by a consumer is not (or to a negligible extent) affected by his decision to purchase or not. As a consequence, the purchase decision of the consumers does not depend on the effort undertaken by the firm, which explains why, under complete information, the firm chooses zero effort. The only influence of effort will be through pre-purchase perceptions of the product's type by consumers, which may only be useful under incomplete information (Milgrom & Roberts, 1986).

Remark 2. In the same spirit, the public good attribute of the green product does not influence the price charged by the firm. Only the individual warm glow enjoyed from the consumers' own purchase has a role in the pricing strategy of the monopoly. Indeed, because the public good benefit is enjoyed by the consumers whether they decide to buy the good or not, it does not have any impact on this decision itself. This implies that the price premium charged by green firms is induced by two dimensions: first, a higher cost of production compared to neutral production, and second, the personal preference of consumers for green consumption through the warm-glow parameter.

3.2 First-Best Solution

To complement the benchmark monopoly strategies, we derive the welfare-maximizing quantities[10] and efforts. A benevolent social planner would maximize the total social welfare by considering the sum of firm profits, consumer surplus, and the potential positive externality. The main insights are summarized in the following proposition.

Proposition 1 (Complete information benchmark and first-best solution)**.** *Under complete information, the first-best solution maximizing social welfare entails the following choices by the benevolent social planner:*

(i) If the firm is neutral, the quantity of the good provided ensures that all consumers are

10. This is equivalent to deriving the prices allowing to implement these optimal quantities.

served. As a consequence, the socially optimal price is zero. The effort level remains zero, identically to the monopoly strategy, as neutral production provides no public good benefit.

(ii) If the firm is green, the socially optimal quantity of the product on the market increases compared to the monopoly strategy, resulting in a lower price equal to the marginal cost. The first-best effort level is strictly positive – exceeding the monopoly level – as it accounts for the public good benefit.

Proof. See Appendix 2.A. □

In the case where the firm is neutral, a benevolent social planner would set the first-best quantity and effort in order to maximize the sum of profits ($\pi_n(p_n, e_n)$) and consumer surplus, given by $S(q_n, p_n) = q_n - \frac{1}{2}q_n^2 - p_n q_n$. Quite obviously, the first-best effort level for the neutral firm is zero, as it implies a cost for the firm without any positive counterpart (*i.e.* no positive externality is generated by neutral consumption).[11] As the marginal cost of production is zero, the first-best price for the neutral product is zero, and in this case, all consumers are served with one unit of the good ($q_n = 1$).[12] Total welfare is equal to $W_n^{FB} = 1/2 > W_n^m = 3/8$.

In the case where the firm is green, the welfare-maximizing solution implies optimizing the sum of profits ($\pi_g(p_g, e_g)$), consumer surplus ($S(q_g, p_g) = q_g(1 + b) - \frac{1}{2}q_g^2 - p_g q_g$), and the positive externality created by the public good aspect of the green product ($B(q_g, e_g) = \beta + h(e_g)$). As the effort to improve the greenness of the product does not only imply a cost but also a benefit, a benevolent social planner therefore chooses a positive effort level indirectly defined by $h'(e_g^{FB}) = \gamma_g$. This optimal effort level is directly influenced by the shape of function $h(\cdot)$, as it is determined by the marginal benefit associated with improved greenness. For a given cost of effort, the greater this marginal benefit, the higher the first-best effort level. Moreover, the quantity chosen by the social planner, given by $q_g^{FB} = 1 + b - c$, is greater than the monopoly quantity as the social planner accounts for consumer surplus. This results in pricing at marginal cost.

As an example, we can take $h(e_g) = \sqrt{e_g}$, which satisfies the assumptions on function

11. With $e_n^{FB} = 0$, concavity of the program in q_n is satisfied, ensuring the existence of a maximum.

12. Note that this is a standard marginal cost pricing rule, under the extreme case where marginal cost is zero. If the neutral firm has a positive marginal cost of production $c_n < c_g$, the first-best price for the neutral product would be $p_n = c_n$. In this case, the quantity served is $q_n^{FB} = 1 - c_n$, ensuring that all consumers with valuations above c_n are served.

$h(\cdot)$. As a result, the first-best effort level is given by $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$. As a result, total welfare amounts to $W_g^{FB} = \frac{1}{2}\left(1+b-c\right)^2 + \frac{1}{4\gamma_g} + \beta > W_g^m = \frac{3}{8}\left(1+b-c\right)^2 + \beta$.

4 Signaling under Incomplete Information

Assume now that consumers do not directly observe the monopoly's type. In other words, the costs $(c_\theta, \gamma_\theta)$ are private information of the firm, and there is no credible direct way for the firm to provide information about these costs. There is only one period. Consumers observe the price and effort level offered by the monopoly, and must try to infer the firm's type from this information. This illustrates the credence nature of the green attribute: consumers base their purchase decision on their belief; and even if they learn the true type of the product ex-post, they are not able to change their decision. In other words, the price and effort level observed by consumers will determine their perception of the firm's type, based on which they formulate their demand. If they believe the firm is green, they enjoy the warm-glow benefit, and their demand will be given by $q_g(p_g)$. If they believe it is neutral instead, their demand will be $q_n(p_n)$. In this context, we are interested in the firm's optimal strategy, and we focus more particularly on separating strategies.[13]

4.1 Conditions for a Separating Equilibrium

First, note that the pair of monopoly strategies under complete information $\{(p_n^m, e_n^m); (p_g^m, e_g^m)\}$ does not yield a separating equilibrium, because it does not provide the necessary conditions for effective signaling.[14] Indeed, the neutral firm can choose p_g^m and zero effort to lead consumers to believe that its product is green, and therefore enjoys higher demand and makes higher profits without incurring the cost of producing the green quality. For a separating perfect Bayesian equilibrium to arise, the following conditions need to be satisfied (Wolinsky, 1983):

13. Here, we disregard the analysis of pooling equilibria.
14. Note that if the warm glow (b) is absent, the complete information monopoly strategies are separating because the neutral firm would have no incentive to mimic the green firm. Indeed, demand would only depend on price, and the neutral firm would not achieve higher demand or profit by pretending to be green.

(i) Type-θ firm maximizes its profits such that profits are non-negative;

(ii) Expectations are fulfilled *i.e.* in equilibrium, the perceived type by consumers given the observed price and effort corresponds to the real type of the firm;

(iii) Consumers make optimal choices by maximizing their utility.

Formally, the monopoly of type $\theta \in \{g, n\}$ maximizes its profits with respect to the price and effort level ensuring that the following necessary equilibrium constraints are simultaneously satisfied:

$$(p_g - c)q_g(p_g) - \gamma_g e_g \geq (p_n - c)q_n(p_n) - \gamma_g e_n, \tag{1}$$

$$p_n q_n(p_n) - \gamma_n e_n \geq p_g q_g(p_g) - \gamma_n e_g, \tag{2}$$

$$\pi_g(p_g, e_g) \geq 0, \tag{3}$$

$$\pi_n(p_n, e_n) \geq 0. \tag{4}$$

Condition (1) ensures that a green firm prefers choosing (p_g, e_g) and be perceived by consumers as green than be perceived as neutral with (p_n, e_n). Similarly, condition (2) ensures that a neutral firm prefers offering (p_n, e_n) and be perceived as neutral than choosing (p_g, e_g) and be perceived as green. Conditions (3) and (4) ensure that profits for each type are non-negative, for the firm to be willing to participate in production. [15] Finally, beliefs off the equilibrium path are required to satisfy the Cho-Kreps Intuitive Criterion (Cho & Kreps, 1987), allowing to focus on the least-cost separating equilibrium.

4.2 Solving for the Least-Cost Separating Equilibrium

In a separating equilibrium, a neutral monopoly always offers the same price and zero-effort package as in full information: $(p_n^m = 1/2, e_n^m = 0)$. Taking this into account, a green monopoly will choose to offer (p_g, e_g) in order to maximize its profits while satisfying the constraints for a separating equilibrium, that is to signal its type to

15. For sufficiency, a single-crossing type of condition needs to be added and verified. See Appendix 2.B for more details on this matter.

consumers. Formally, the program writes:

$$\max_{p_g, e_g \geq 0} \quad \pi_g(\cdot) = (p_g - c)(1 + b - p_g) - \gamma_g e_g \tag{5}$$
$$\text{s.t. } (1) - (4).$$

As a result, two cases can arise. The following proposition summarizes these results under incomplete information about the firm's type.

Proposition 2 (Least-cost separating equilibrium with price and effort)**.** *Suppose that consumers cannot observe the true type of the monopoly. In a separating perfect Bayesian equilibrium satisfying (1)-(4):*

(i) A neutral firm offers its full information strategy, consisting of the monopoly price and zero effort: $p_n^m = 1/2$ *and* $e_n^m = 0$.

(ii) A green firm offers a higher-than-monopoly price $p_g > p_g^m$ *and may offer a positive effort level* $e_g > 0$ *if the warm-glow benefit is large enough compared to the firm's marginal cost of production; and/or the costs of effort among types are not too close. More specifically, compared to the first-best and with* e_g^{FB} *implicitly defined by* $h'(e_g^{FB}) = \gamma_g$*:*

- *If* $c\frac{\gamma_n}{\Delta\gamma} < \sqrt{2b + b^2 - 4\gamma_n e_g^{FB}}$*, a green firm over-provides effort;*
- *If* $c\frac{\gamma_n}{\Delta\gamma} \in \left[\sqrt{2b + b^2 - 4\gamma_n e_g^{FB}},\ \sqrt{2b + b^2}\right]$*, a green firm under-provides effort;*
- *If* $c\frac{\gamma_n}{\Delta\gamma} > \sqrt{2b + b^2}$*, a green firm provides zero effort.*

Proof. See Appendix 2.B. □

Case 1: Positive Effort

First, if $c\frac{\gamma_n}{\Delta\gamma} \leq \sqrt{2b + b^2}$, where $\Delta\gamma = \gamma_n - \gamma_g$, the green firm uses two instruments (price and effort) to signal its type to consumers. More precisely it offers the following price:

$$p_g = \frac{1}{2}\left(1 + b + c\frac{\gamma_n}{\Delta\gamma}\right)$$

and a positive effort to improve the greenness of its product, equal to:

$$e_g = \frac{1}{\gamma_n}\left[\frac{1}{4}b^2 + \frac{1}{2}b - \frac{1}{4}c^2\left(\frac{\gamma_n}{\Delta\gamma}\right)^2\right].$$

This price is higher than the full information monopoly price p_g^m. The cost of separation for the green firm in this case is equal to the loss in profits associated with charging a price above p_g^m plus the total cost of providing effort e_g.

The occurrence of such situation is facilitated by two channels. Firstly, when the warm glow b enjoyed by consumers from purchasing a green product is large enough compared to the marginal cost of producing such product, c. And secondly, when the marginal costs of effort for the green and the neutral types, γ_g and γ_n, are different enough.[16] Intuitively, when the warm-glow benefit b is large, the green product is desirable as it is more strongly valued by consumers. As a consequence, the gain in revenues associated with being perceived as green are higher. For a true green monopoly to distinguish itself from the neutral type using solely the price, that price has to be very high. In this case, the green firm finds it preferable to use a combination of price and positive effort to reduce the distortion on the price. Also, using positive effort as a signaling instruments is clearly advantageous when it is relatively cheap for the green firm to increase effort compared to the neutral firm. Indeed, the neutral monopoly will be deterred from mimicking as exerting the green effort is too costly.

Technically, the green firm prefers using both effort and price to distinguish itself to consumers because, for a fixed price, increasing effort from zero to some positive level relaxes the constraint of interest for a separating equilibrium to arise, constraint (2). As a consequence, the firm is able to charge a price lower than the price required to achieve separation with zero effort (until constraint (2) becomes binding), therefore a price closer to the monopoly price. Moreover, this price p_g is decreasing with the firm's own cost of effort γ_g. Indeed, as the cost of effort shrinks, the firm can achieve separation by exerting greater effort and charging lower price. Similarly, the effort level chosen by the firm is decreasing in the neutral type's cost of effort γ_n: for a given price

16. If $\gamma_g = \gamma_n = \gamma$, or more generally, if the cost of effort functions are identical across types, the green firm never exerts a positive effort to improve the greenness of the product. Refer to Appendix 2.C for more details on this matter.

in equilibrium, separation is achieved with lower effort for the green firm when the cost of effort for the neutral firm grows.

Of course, overall welfare under incomplete information is lower than in the first-best case. However, effort can be under-provided or over-provided relative to the first-best level. More specifically, effort is over-provided ($e_g > e_g^{FB}$) whenever $c\frac{\gamma_n}{\Delta\gamma} < \sqrt{2b + b^2 - 4\gamma_n e_g^{FB}}$, where $e_g^{FB} = h^{(-1)'}(\gamma_g)$. On the other hand, effort is under-provided ($e_g < e_g^{FB}$) when $c\frac{\gamma_n}{\Delta\gamma} \in \left[\sqrt{2b + b^2 - 4\gamma_n e_g^{FB}}, \sqrt{2b + b^2}\right]$.
By using both instruments in this case, the firm gains some profit with respect to using the price only, as long as effort and green production are not too costly.[17]

Case 2: Zero Effort

On the other hand, when $c\frac{\gamma_n}{\Delta\gamma} > \sqrt{2b + b^2}$, the green firm provides zero effort ($e_g = 0$) and only uses the price to signal its type to consumers. As a result, the firm offers the following price:

$$p_g = \frac{1 + b + \sqrt{2b + b^2}}{2}.$$

This price is higher than the full information green monopoly price p_g^m, and the loss in profits for the green firm associated with offering a higher price is the cost of separation. As the firm tries to minimize this cost, the price chosen is the lowest price possible given the constraints for a separating equilibrium. This optimum implies that constraint (2) binds. As a consequence, the marginal cost of producing the green good c does not play a role in this case. Indeed, the separating price here is determined by the constraint on the neutral firm, which does not involve such cost. It is however increasing in the warm glow b, as the firm enjoys higher demand when this benefit increases. Here, even if the firm tried to provide a positive costly effort level, given the constraints for separation, it would not be able to offer a more profitable price (*i.e.* lower) than the one described above.

Figure 2.2 below provides an illustration of the three possible signaling effort levels depending on the parameter values: over-provision, under-provision, and zero effort.

17. See Appendix 2.D.

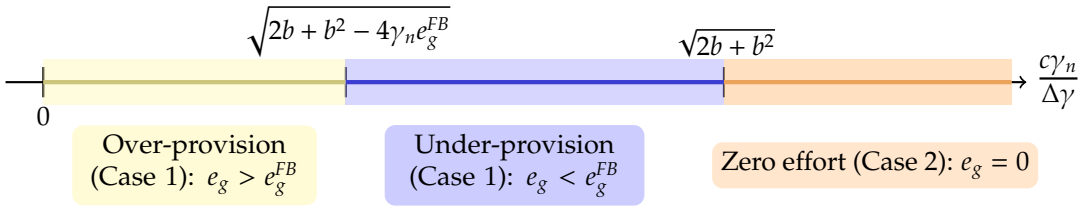


Figure 2.2 – *Signaling effort level depending on parameter values*

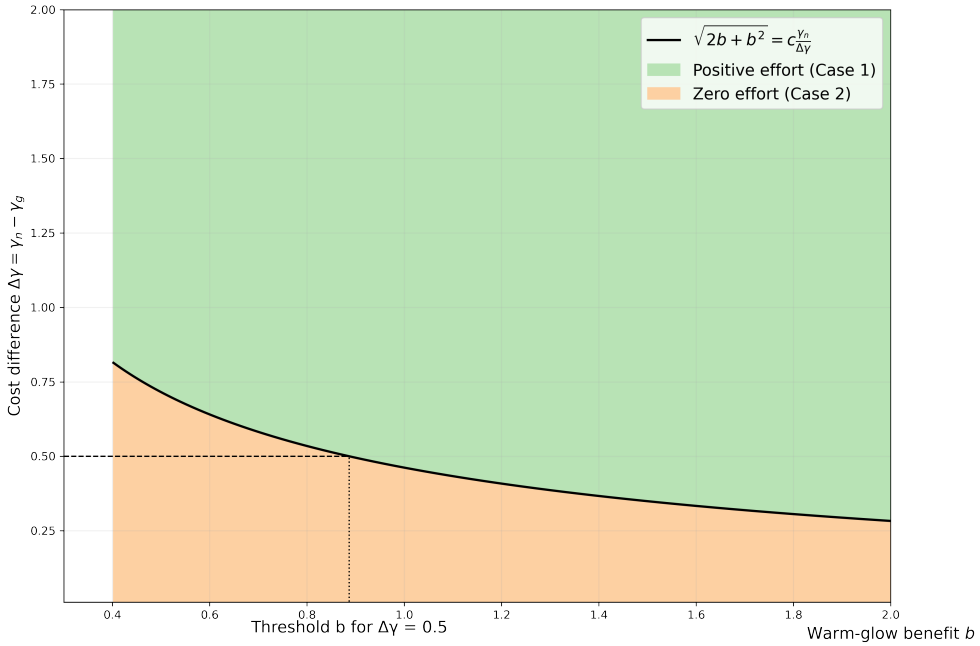
Numerical Example I

Figure 2.3 displays a numerical exercise which gives the signaling strategy chosen by the green monopoly for different values of the warm-glow benefit b and the difference between marginal costs of effort $\Delta\gamma$, when $c = 0.4$ and $\gamma_n = 2$. The black curve corresponds to the threshold at which the monopoly is indifferent between using price and effort or only the price for signaling, *i.e.* $c\frac{\gamma_n}{\Delta\gamma} = \sqrt{2b + b^2}$. Below this curve (orange region), the green monopoly chooses zero effort. Above this curve (green region), it chooses to exert a positive level of effort. As described above, signaling with positive effort is facilitated for higher values of b and $\Delta\gamma$.

As an example, set $\Delta\gamma = 0.5$ (*i.e.* $\gamma_g = 1.5$). A green firm uses only the price as a signal (Case 2) as long as $b < 0.89$ (approximately); and uses both the price and a positive level of effort (Case 1) when $b \geq 0.89$.

To illustrate that a firm exerts positive effort when doing so reduces the signaling cost relative to relying solely on price, keep the previous values of the parameters and set $b = 1.5$, ensuring that the least-cost separating equilibrium involves positive effort. The signaling cost associated with price signaling only is equal to 0.894 which is larger than the signaling cost associated with the strategy described in Case 1, resulting in $e_g = 0.33$ and a signaling cost of $\pi_g^m - \pi_g = 0.864$.

Table 2.1 summarizes two representative examples of over- and under-provision of effort by a green firm when positive effort arises in a separating equilibrium. Keeping $c = 0.4$ and assuming $h(e_g) = \sqrt{e_g}$ and $\gamma_g = 0.5$, the first-best effort level is given by $e_g^{FB} = 1$. Under incomplete information, a green firm over-provides effort when $b = 2$ and $\gamma_n = 1$, exerting $e_g \approx 1.8 > e_g^{FB}$. In contrast, a lower warm glow $b = 1.5$ and higher neutral marginal cost of effort $\gamma_n = 1.3$ leads to under-provision, $e_g \approx 0.9 < e_g^{FB}$.



Simulation for $c = 0.4$ and $\gamma_n = 2$.

Figure 2.3 – *Signaling strategy of a green monopoly across $\Delta\gamma$ and b*

Parameter values			Results		
b	γ_n	$\Delta\gamma$	e_g^{FB}	e_g	$\Delta e_g = e_g^{FB} - e_g$
2.0	1.0	0.5	1.0	1.83	**Over-provision**
1.5	1.3	0.8	1.0	0.93	**Under-provision**

Assumptions: $h(e_g) = \sqrt{e_g}$, $c = 0.4, \gamma_g = 0.5$ for all cases.
Signaling effort levels are approximated.

Table 2.1 – *Numerical examples of over- and under-provision of effort in the separating equilibrium*

To summarize, the green monopoly may exert a positive level of costly effort combined with a higher-than-monopoly price when it helps reducing its signaling cost relative to the situation in which it uses only the price, without violating conditions for separation. In other words, exerting some effort allows to reach separation with a price closer to monopoly price.

5 Welfare Analysis

This section focuses on the welfare implications under incomplete information, in the described equilibrium.

Quite obviously, welfare does not change when the firm is neutral, as the offered price and effort remain identical under complete and incomplete information.

However, when the firm is green, there are conditions under which welfare is higher under incomplete information compared to the full information monopoly strategies. We also investigate the factors that influence this outcome. The main insight is summarized in the following proposition.

Proposition 3 (Welfare implications in separating equilibrium)**.** *When the green firm offers a positive effort level, total welfare is higher under incomplete information than under complete information, provided the net benefit of improved greenness is sufficiently large.*

Proof. See Appendix 2.E. □

Zero effort

First of all, when the green firm provides zero effort in a separating equilibrium to improve the greenness of its product (*i.e.* Case 2), overall welfare under incomplete information is strictly lower than under complete information. This is because, in a separating equilibrium, the green firm's signaling price is higher than the complete information monopoly price, making both the firm and consumers worse off. Put differently, we have $q_g < q_g^m < q_g^{FB}$. In this case, we are facing two types of inefficiencies: a public good inefficiency (zero effort to improve greenness) and an informational inefficiency (higher price for differentiation).

Positive effort

However, when the green firm exerts a positive level of effort to signal itself (*i.e.* Case 1), two opposing effects come into play and may result in higher overall welfare compared to the full information situation.

On the one hand, and similarly to Case 1, the price offered by the green firm in a separating equilibrium is higher than the full information price, which decreases welfare on both the firm and consumers' side, moving further away from the first-best. Note, however, that the welfare loss in this case is smaller compared to the previous scenario, as price does increase but to a lesser extent than when only one signaling

instrument is used.

On the other hand, the green firm's effort is costly but also improves the public good benefit associated with green production, and thus increases this dimension of total welfare.

The net effect on welfare depends on the balance between these two effects. It is possible for the positive effort (and its associated public good benefit) to outweigh the price increase negative effect, leading to an overall welfare improvement: this occurs when the net benefit from improved greenness ($h(e_g) - \gamma_g e_g$) is sufficiently high.

Therefore, the shape of function $h(\cdot)$, which governs the relationship between effort and the incremental positive externality, plays a crucial role in determining whether or not welfare under incomplete information is higher than under complete information. More specifically, if $h(\cdot)$ is sufficiently steep, meaning that the public good benefit increases significantly with effort, then the green firm's effort can generate enough public good benefit to outweigh the costs of effort and increase in price.

To summarize, when the green firm can use price and effort to signal itself to consumers, the presence of the informational inefficiency helps correcting another form of inefficiency: the under-provision of the public good. Moreover, if welfare increases, we conclude that the gains from addressing the public good inefficiency outweigh the losses caused by informational asymmetry. This surprising result leads to an overall improvement in the situation.

Numerical example II

We provide examples of specific parameter values to illustrate the different welfare implications that arise as a result of this model. Suppose $\beta = 2$, $c = 0.4$ and $\gamma_g = 0.5$. Table 2.2 summarizes the examples given thereafter.

First, assume that $h(e_g) = \sqrt{e_g}$, as used previously. Consider the following values for the model parameters: $b = 2$ and $\gamma_n = 1$ (line (a)). In a separating equilibrium, the green firm exerts a positive effort $e_g \approx 1.84$ to increase the greenness of its product. In this case, the benefit from improved greenness is sufficiently large compared to the total cost of effort, and welfare in incomplete information increases compared to the monopoly

case in complete information. More specifically, we have $W_g \approx 4.69 > W_g^m = 1.69$.

As discussed above, it could be the case that the green firm signals itself to consumers using a positive effort in addition to the price, but welfare decreases compared to the monopoly case in complete information. For instance, decrease the cost of effort of the neutral firm closer to the one of the green firm ($\gamma_n = 0.75$), and shrink the gap between the warm-glow benefit and the marginal cost of production by setting $b = 1.6$ (line (b)). In this case, effort to increase the greenness of the product is still positive ($e_g \approx 1.44$), but the total cost of providing such effort becomes too large compared to its positive public good effect, so total welfare drops to $W_g \approx 3.78 < W_g^m \approx 3.82$. With these changes in the parameter values, effort level has increased compared to the previous example, but the signaling cost is too large compared to the public good benefit, inducing a loss in overall welfare.

Second, it is possible to observe the role of the curvature of $h(\cdot)$ by assuming a different functional form. Suppose now that $h(e_g) = e_g^{0.9}$. For a given level of green effort, the associated public good benefit is larger here than in the previous case. Using the same parameter values as the two previous examples, welfare in the first case still rises but even more relative to the complete information benchmark (line (c)). More interestingly, in the second case (line (d)), overall welfare now increases under incomplete information: the stronger public good benefit associated with effort offsets the cost of signaling.

	Parameter values			Results			
	b	γ_n	$\Delta\gamma$	e_g	$W_g^{\text{signaling}}$	W_g^m	ΔW
	Assumption: $h(e_g) = \sqrt{e_g}$						
(a)	2.0	1.0	0.5	1.84	4.69	1.69	+ 0.156
(b)	1.6	0.75	0.25	1.44	3.78	3.82	– 0.04
	Assumption: $h(e_g) = e_g^{0.9}$						
(c)	2.0	1.0	0.5	1.84	5.01	1.69	+ 0.531
(d)	1.6	0.75	0.25	1.44	3.96	3.82	+ 0.148

$\beta = 2, c = 0.4, \gamma_g = 0.5$ *for all cases.*
Effort levels and welfare values are approximated.

Table 2.2 – *Numerical examples of different welfare outcomes under incomplete information with positive effort*

6 Subsidizing First-Best Green Effort

We now introduce the role of regulation in this framework. The objective of this section is to analyze how a benevolent social planner can intervene to influence the firm's behavior and improve overall welfare. More specifically, we explore the implementation of a subsidy for the first-best green effort, when the regulator does not directly observe the type of the monopoly, just like the consumers. Throughout this section, we keep the assumption that $h(e_g) = \sqrt{e_g}$.

Suppose the regulator wants to use the subsidy to implement the first-best effort levels, therefore make the green firm exert $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$. Assume that the regulator does not have any other regulatory instruments (for instance, he cannot set the price or the quantity to be offered by each type of firm). For the green firm to be willing to set e_g^{FB}, the subsidy has to cover the cost of doing so. Therefore, the "full" per-unit subsidy is equal to $s = \gamma_g$.

Under complete information, the regulator is able to condition the allocation of this subsidy on the firm's type: if the firm is green, it receives the subsidy in exchange of setting the first-best green effort level; and if the firm is neutral, it does not receive anything. As a consequence, a green monopoly will optimally choose its monopoly price p_g^m and make profits $\pi_g^m(p_g^m, e_g^{FB}, s) = \pi_g^m(p_g^m, 0)$ as the cost of exerting the first-best effort is totally covered by the subsidy. On the other hand, the neutral firm's strategy is identical as in section 3.1, with zero-effort level and monopoly pricing ($p_n^m = 1/2$) and profits ($\pi_n^m(p_n^m, 0) = 1/4$).

The main issue of interest here is to analyze how the addition of the subsidy in exchange of exerting e_g^{FB} alters the separating equilibrium under incomplete information. When the type is private information of the firm, the regulator will give out the subsidy to a firm offering the green price and first-best green effort level; and will not grant any subsidy to a firm offering any other price and effort pair. In particular, we will demonstrate that under some conditions, the complete information strategies $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ can be directly separating under incomplete information, without the neutral firm facing incentive to mimic the green type. The key insights are summarized in the following proposition.

Proposition 4 (Separation with first-best green effort subsidy)**.** *Suppose there exists incomplete information about the monopoly's type and the regulator offers a per-unit subsidy* $s = \gamma_g$ *to a firm offering the green monopoly price* p_g^m *and exerting the green first-best effort* e_g^{FB}*. The strategies* $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ *can directly induce separation under some conditions:*

(i) When the difference in costs of effort $\Delta\gamma = \gamma_n - \gamma_g$ *is sufficiently large, making the residual cost to provide the first-best green effort level too high for the neutral firm, despite the subsidy.*

(ii) When the warm-glow benefit b *is sufficiently low, inducing the gain in revenues made by the neutral type perceived as green too weak to attenuate the residual cost of effort.*

Proof. See Appendix 2.F. □

The neutral firm chooses its monopoly strategy, allowing for a separating equilibrium to arise directly, if and only if $\pi_n^m(p_n^m, 0) > \pi_n(p_g^m, e_g^{FB}, s)$, where the profits from offering the green type's price and effort and receiving the subsidy are given by $\pi_n(p_g^m, e_g^{FB}, s) = p_g^m(1 + b - p_g^m) - (\gamma_n - s)e_g^{FB}$. As a consequence, we can rewrite this condition as follows:

$$\Delta\gamma > \gamma_g^2(2b + b^2 - c^2). \tag{6}$$

Intuitively, this condition is more easily satisfied when the costs of effort are sufficiently different, and/or, for a given marginal cost of green production, the warm glow is not too large.

First of all, suppose b and c are fixed. Thus, the minimum difference between the costs of efforts necessary for these strategies to be separating is $\widehat{\Delta\gamma} = \alpha$ where $\alpha = \gamma_g^2(2b + b^2 - c^2)$. This is intuitive: when the costs of effort are similar, the neutral firm's incentive to behave like the green type increases, because the subsidy will cover most of the cost of exerting e_g^{FB}, while revenues increase as consumers perceive the firm as green. However, when the difference between the costs of effort exceeds $\Delta\hat{\gamma}$, the residual cost $(\gamma_n - s)$ that the neutral firm has to incur to provide e_g^{FB} remains too high compared to the benefit it retrieves from being perceived as green.

Secondly, the warm glow generates a gain in revenues for the neutral firm perceived as green by consumers. Therefore, a low warm-glow benefit deters the neutral firm from mimicking the green type, and even facilitates satisfying condition 6 for separation

at a lower threshold value $\widehat{\Delta\gamma}$. More specifically, direct separation can still be induced as marginal costs of effort converge, as long as b simultaneously decreases towards c, with the limit of b very close to but still greater than c. In the limiting case where $b \approx c$, direct separation remains feasible only if the difference in marginal costs of effort $\Delta\hat{\gamma}$ exceeds $2c\gamma_g^2$. This cutoff cannot be further reduced.

Figure 2.4 illustrates the extent of the difference between marginal costs of effort required to directly achieve separation with the first-best effort subsidy. The red line gives condition (6) as a function of γ_n for fixed values of the other parameters. Increasing b above c parallel shifts this line to the right. For values of $\Delta\gamma \in [0, \alpha]$, the subsidy does not yield a separating equilibrium (blue segment). For values of $\Delta\gamma > \alpha$, the subsidy directly yields a separating equilibrium (green segment).

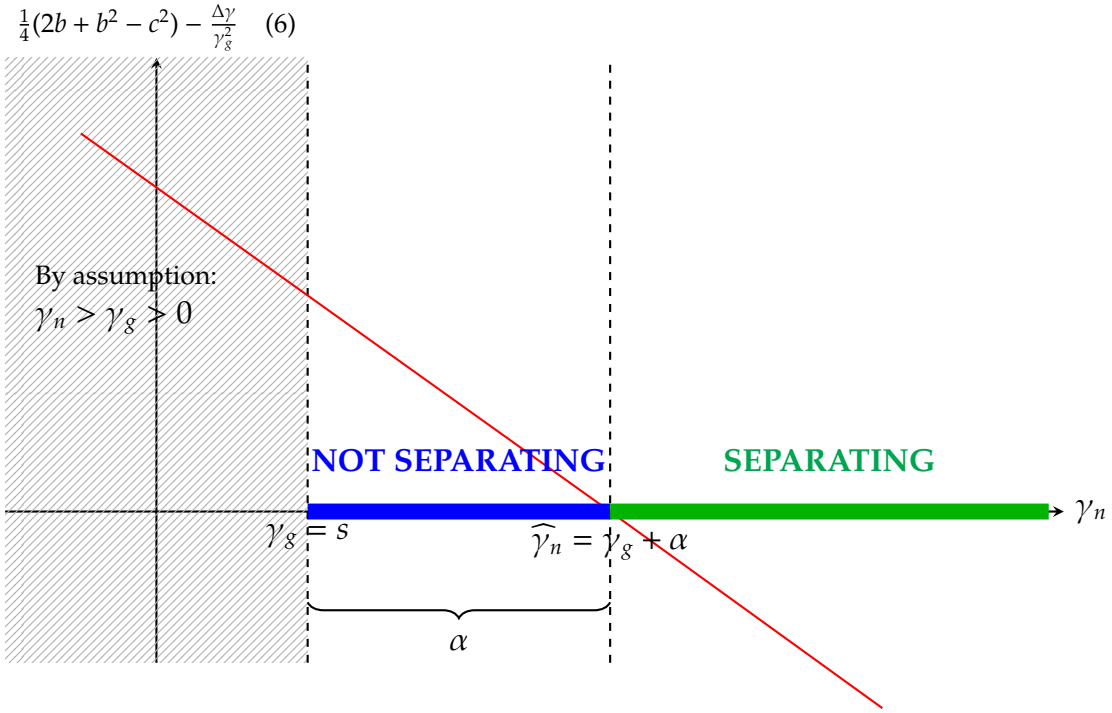


Figure 2.4 – *Graphical illustration of condition (6) as a function of the neutral firm's marginal cost of effort*

This analysis gives insights on the conditions under which a regulator should implement a subsidy to support first-best green effort and expect to tackle two inefficiencies: the informational and the public good provision issues. When the marginal costs of efforts differ sufficiently across types, the social planner should introduce the subsidy

without concern that the neutral firm will mimic the green firm's behavior. Moreover, even when these cost differences narrow, the regulator may still have room to implement the subsidy directly, when the warm-glow benefit is low. The regulator's interest in implementing this policy when the conditions are favorable lies in the fact that total welfare is inevitably higher (or unchanged) when the subsidy leads to the emergence of a separating equilibrium rather than not intervening and letting the firm play its signaling strategy. Indeed, if the firm is green, the quantity (resp. price) offered in this context will be higher (resp. lower) compared to the signaling strategy, both for Case 1 and Case 2, representing a gain for firm profits as well as consumer surplus. Moreover, the level of effort will be the first-best level, maximizing the net public good benefit.

Numerical example III

Below is an illustration of the two cases discussed in this section. Assume $c = 0.4$, $h(e_g) = \sqrt{e_g}$ and $\gamma_g = 0.5$. This means $e_g^{FB} = 1$. If $\gamma_n = 2$ and $b = 1$, offering the first-best green effort subsidy directly yields a separating equilibrium: the difference between the marginal costs of effort is large enough, and the warm glow is low enough. If $\Delta\gamma$ shrinks (take $\gamma_n = 1$ here) and b grows (take $b = 2$), the neutral firm has incentive to claim the subsidy and act as a green producer to make higher profits: $\pi_n(p_g^m, e_g^{FB}, s) = 1.71 > \pi_n^m$. Figure 2.5 summarizes these two examples.

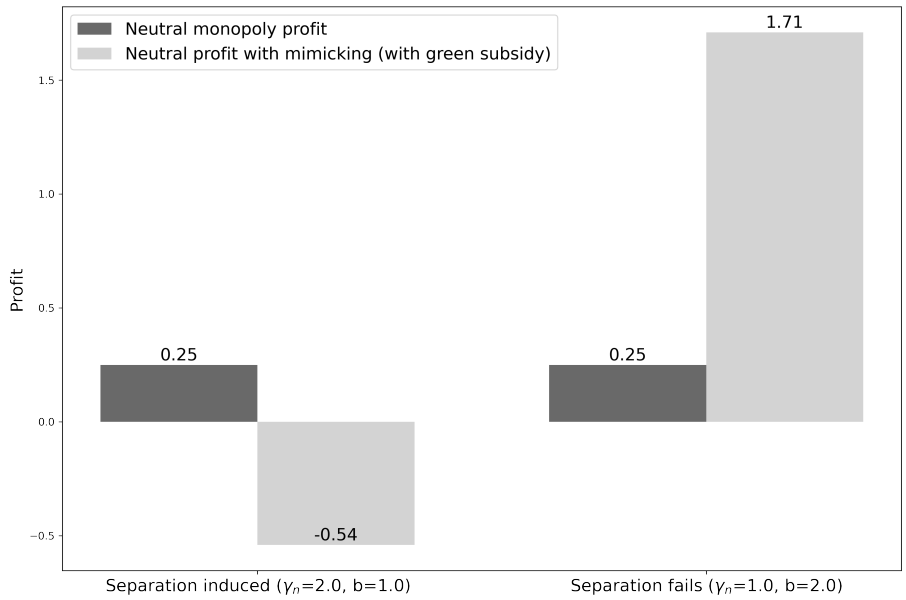


Figure 2.5 – *Numerical example of neutral monopoly profits vs. mimicking (with subsidy) profits*

7 Conclusions

This model highlights how markets for credence impure public goods can achieve effective differentiation even in the absence of third-party certification and labels, using firm-specific signaling instruments of price and costly effort. By combining the warm-glow effect with the impure public good aspect of green products, the analysis offers a novel perspective on signaling. The results discuss conditions under which the signaling strategies chosen in the least-cost separating perfect Bayesian equilibrium improve of overall welfare relative to the complete information case.

Under incomplete information, a higher green price emerge because an environmentally responsible firm charges a premium, reflecting both the higher production costs of the green quality and the need to signal this quality credibly. However, this price increase may be mitigated when a green firm can use costly effort to enhance the environmental quality of its product. Effort thus serves a dual purpose: signaling and improving social outcomes as a byproduct.

The analysis of the implementation of a subsidy for the first-best green effort generates some policy recommendations. Such regulation improves the situation when firm types are sufficiently different, allowing for a separating equilibrium to directly emerge without requiring costly signaling by the green firm.

More broadly, this framework applies to settings in which an observable action that coveys information is privately costly for the sender but simultaneously creates externalities that affect the welfare of others.

Future research could extend the analysis by endogenizing the choice of technology. Another extension to be considered would be the design of optimal regulation when the first-best green effort subsidy fails to induce separation, exploring how a regulator might structure alternative instruments (*e.g.* transfers, quantity caps, etc) to get separation.

References

Abou El-Komboz, L., & Goldbeck, M. (2025). Career Concerns as a Public Good: The Role of Signaling for Open Source Software Development. *Labour Economics*, *97*, 102800.

Andreoni, J. (1990). Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving. *The Economic Journal*, *100*(401), 464–477.

Bagnoli, M., & Watts, S. G. (2003). Selling to Socially Responsible Consumers: Competition and The Private Provision of Public Goods. *Journal of Economics and Management Strategy*, *12*(3), 419–445.

Bagwell, K., & Riordan, M. H. (1991). High and Declining Prices Signal Product Quality. *The American Economic Review*, *81*(1), 224–239.

Bansal, S., & Gangopadhyay, S. (2003). Tax/Subsidy Policies in the Presence of Environmentally Aware Consumers. *Journal of Environmental Economics and Management*, *45*(2, Supplement), 333–355.

Bergstrom, T., Blume, L., & Varian, H. (1986). On the private provision of public goods. *Journal of Public Economics*, *29*(1), 25–49.

Besley, T., & Ghatak, M. (2007). Retailing public goods: The economics of corporate social responsibility. *Journal of Public Economics*, *91*(9), 1645–1663.

Boyer, M., Mahenc, P., & Moreaux, M. (2007). Environmental Protection, Consumer Awareness, Product Characteristics, and Market Power. *Frontiers in the Economics of Environmental Regulation and Liability*.

Cason, T. N., & Gangadharan, L. (2002). Environmental Labeling and Incomplete Consumer Information in Laboratory Markets. *Journal of Environmental Economics and Management*, *43*(1), 113–134.

Cho, I.-K., & Kreps, D. M. (1987). Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics*, *102*(2), 179–221.

Cooper, R., & Ross, T. W. (1984). Prices, Product Qualities and Asymmetric Information: The Competitive Case. *The Review of Economic Studies*, *51*(2), 197–207.

Cremer, H., & Thisse, J.-F. (1999). On the Taxation of Polluting Products in a Differentiated Industry. *European Economic Review*, *43*(3), 575–594.

Darby, M. R., & Karni, E. (1973). Free Competition and the Optimal Amount of Fraud. *The Journal of Law and Economics*, *16*(1), 67–88.

Dulleck, U., & Kerschbamer, R. (2006). On Doctors, Mechanics, and Computer Specialists: The Economics of Credence Goods. *Journal of Economic Literature*, *44*(1), 5–42.

Elfenbein, D. W., & McManus, B. (2010). A Greater Price for a Greater Good? Evidence that Consumers Pay More for Charity-Linked Products [Publisher: American Economic Association]. *American Economic Journal: Economic Policy*, *2*(2), 28–60.

Farrell, J. (1981). *Prices as Signals of Quality* [Doctoral dissertation, University of Oxford].

Giovannucci, D., & Koekoek, F. J. (2003). The State of Sustainable Coffee: A Study of Twelve Major Markets. *IISD, UNCTAD, ICO*.

Griskevicius, V., Tybur, J., & van den bergh, B. (2010). Going Green to Be Seen: Status, Reputation, and Conspicuous Conservation. *Journal of Personality and Social Psychology*, *98*, 392–404.

Kim, E.-H., & Lyon, T. P. (2011). Strategic environmental disclosure: Evidence from the DOE's voluntary greenhouse gas registry. *Journal of Environmental Economics and Management*, *61*(3), 311–326.

Klein, B., & Leffler, K. B. (1981). The Role of Market Forces in Assuring Contractual Performance [Publisher: University of Chicago Press]. *Journal of Political Economy*, *89*(4), 615–641.

Kotchen, M. J. (2006). Green Markets and Private Provision of Public Goods [Publisher: The University of Chicago Press]. *Journal of Political Economy*, *114*(4), 816–834.

Krasteva, S., & Saboury, P. (2021). Informative Fundraising: The Signaling Value of Seed Money and Matching Gifts. *Journal of Public Economics*, *203*(100), 104501.

Lyon, T., & Maxwell, J. (2011). Greenwash: Corporate Environmental Disclosure under Threat of Audit. *Journal of Economics and Management Strategy*, *20*(1), 3–41.

Mahenc, P. (2007). Are Green Products Over-Priced? *Environmental and Resource Economics*, *38*, 461–473.

Mahenc, P. (2008). Signaling the Environmental Performance of Polluting Products to Green Consumers. *International Journal of Industrial Organization*, *26*(1), 59–68.

Mahenc, P., & Volle, A. (2021). Price Signaling and Quality Monitoring in Markets for Credence Goods. *Working Paper hal-03098440, HAL*.

Milgrom, P., & Roberts, J. (1986). Price and Advertising Signals of Product Quality [Publisher: University of Chicago Press]. *Journal of Political Economy*, *94*(4), 796–821.

Morgan, J., & Tumlinson, J. (2019). Corporate Provision of Public Goods. *Management Science*, *65*(10), 4489–4504.

MORI, I. (2003). Ethical Companies. https://www.ipsos.com/en-uk/ethical-companies

Palmer, K., Oates, W. E., & Portney, P. R. (1995). Tightening Environmental Standards: The Benefit-Cost or the No-Cost Paradigm? *Journal of Economic Perspectives*, *9*(4), 119–132.

Schmittmann, J. M., & Gao, Y. (2022). Green Bond Pricing and Greenwashing under Asymmetric Information. *Working Paper No. 2022/246*.

Sengupta, A. (2012). Investment in Cleaner Technology and Signaling Distortions in a Market with Green Consumers. *Journal of Environmental Economics and Management*, *64*(3), 468–480.

Shapiro, C. (1982). Consumer Information, Product Quality, and Seller Reputation [Publisher: [RAND Corporation, Wiley]]. *The Bell Journal of Economics*, *13*(1), 20–35.

Warr, P. G. (1983). The private provision of a public good is independent of the distribution of income. *Economics Letters*, *13*(2), 207–211.

Wolinsky, A. (1983). Prices as Signals of Product Quality. *The Review of Economic Studies*, *50*(4), 647–658.

Appendix

2.A Proof of Proposition 1

Welfare function. The total welfare function includes consumer surplus, firm profits, and possibly the public good benefit. We can either write a welfare function for each situation: the firm is green or the firm is neutral. Alternatively, we can write a single welfare function which takes into account that the firm is green with probability $\lambda \in [0,1]$, and neutral with probability $(1-\lambda)$. In this case, we have:

$$W(q_n, e_n, q_g, e_g) = \lambda\Bigg(\underbrace{q_g(1+b) - \frac{1}{2}q_g^2 - p_g q_g}_{\text{Green Consumer Surplus}} + \underbrace{p_g q_g - c q_g - \gamma_g e_g}_{\text{Green Profits}} + \underbrace{B(q_g, e_g)}_{\text{Public Good Benefit}}\Bigg)$$
$$+ (1-\lambda)\Bigg(\underbrace{q_n - \frac{1}{2}q_n^2 - p_n q_n}_{\text{Neutral Consumer Surplus}} + \underbrace{p_n q_n - \gamma_n e_n}_{\text{Neutral Profits}}\Bigg),$$

where $B(q_g, e_g) = \begin{cases} 0 \text{ if } q_g = 0, \\ \beta + h(e_g) \text{ if } q_g > 0 \text{ where } h(0) = 0,\ h'(e_g) > 0,\ h''(e_g) \leq 0. \end{cases}$

Monopoly strategies under complete information. The derivation of pair of monopoly strategies under complete information $\{(p_n^m, e_n^m); (p_g^m, e_g^m)\}$ is explained in section 3.1 and is fairly straightforward. Based on the definition of welfare above, when the firm is neutral (*i.e.* $\theta = n$ is realized), total welfare in this situation is equal to $W_n^m = 3/8$. On the other hand, when the firm is green ($\theta = g$ is realized), total welfare is equal to $W_g^m = \frac{3}{8}(1+b-c)^2 + \beta$.

Welfare-maximizing solution (first-best). A benevolent social planner chooses maximizes quantities (equivalent to prices) and effort levels in order to maximize total welfare. This optimization problem simplifies to:

$$\max_{q_n,e_n,q_g,e_g\geq 0} \quad W = \lambda\left(q_g(1+b-c) - \frac{1}{2}q_g^2 - \gamma_g e_g + \beta + h(e_g)\right) + (1-\lambda)\left(q_n - \frac{1}{2}q_n^2 - \gamma_n e_n\right).$$

We can consider the situation where the firm is neutral and the situation where the firm is green as separate cases, as (q_g, e_g) does not interact with (q_n, e_n).
For the neutral firm, the solution implies $e_n^{FB} = e_n^m = 0$ as it only enter negatively in the welfare function. As mentioned in Foonote 11, the program is therefore concave in q_n, ensuring the existence of a maximum. As explained in section 3.2, with zero marginal cost, the first-best price is zero and we have $q_n^{FB} = 1$ (all consumers are active). If this state of nature is realized ($\theta = n$), total welfare is equal to $W_n^{FB} = 1/2$, greater than W_n^m.
If the firm is green, we can compute the first order conditions with respect to q_g and e_g:

$$h'(e_g^{FB}) = \gamma_g,$$
$$q_g^{FB} = 1 + b - c \text{ which translates to } p_g^{FB} = c.$$

Using $h(e_g) = \sqrt{e_g}$ as in the discussed examples, we can derive the exact expressions for $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$.
If this state of nature is realized ($\theta = g$), total welfare is equal to:

$$W_g^{FB} = \frac{1}{2}(1+b-c)^2 + \frac{1}{4\gamma_g} + \beta$$

which is greater than W_g^m.

We can write the expected total maximum welfare in first best as:

$$W^{FB} = \lambda\left(\frac{1}{2}(1+b-c)^2 + \frac{1}{4\gamma_g} + \beta\right) + \frac{1-\lambda}{2}.$$

To conclude, we can verify the concavity of the program to ensure the existence of this maximum. The Hessian writes:

$$H_{(q_g,e_g)} = \begin{bmatrix} -1 & 0 \\ 0 & h''(e_g) \end{bmatrix}.$$

For concavity, as the Hessian is a diagonal matrix, we verify the eigenvalues which are the diagonal elements : $-1 < 0$ and $h''(e_g) \leq 0$ as assumed in the model.

2.B Proof of Proposition 2

First of all, we add a single-crossing condition to the set of constraints to ensure necessity and sufficiency. With a fixed price p, we have $\frac{\partial \pi_g(\cdot)}{\partial p} = 1 + b + c - 2p$ and $\frac{\partial \pi_n(\cdot)}{\partial p} = 1 - 2p$. The first condition we verify is: $\frac{\partial \pi_g(\cdot)}{\partial p} > \frac{\partial \pi_n(\cdot)}{\partial p} \Rightarrow b + c > 0$ which holds by assumption. With a fixed effort level e, we compute $\frac{\partial \pi_g(\cdot)}{\partial e} = -\gamma_g$ and $\frac{\partial \pi_n(\cdot)}{\partial e} = -\gamma_n$. We verify the second condition: $\frac{\partial \pi_g(\cdot)}{\partial e} < \frac{\partial \pi_n(\cdot)}{\partial e} \Rightarrow \gamma_n > \gamma_g$ which is also true by assumption.

Secondly, it is straightforward to show that the green monopoly strategy is not separating when there is incomplete information. Indeed, we can compute:
$\pi_n(p_n^m, e_n^m) = 1/4 < \pi_n(p_g^m, e_g^m) = \frac{(1+b-c)(1+b+c)}{4}$.

Now, turn to the derivation of the solution. First, we can prove that, in a separating equilibrium, the neutral monopoly offers the same package as the monopoly strategy in full information, that is $(p_n^m = 1/2, e_n^m = 0)$. Proceed by contradiction: at a separating equilibrium, consumers who observe p_n and e_n believe that the good they purchase is neutral, and thus formulate demand $q_n(p_n)$. If $p_n \neq p_n^m$, the neutral firm is not maximizing its profits given $q_n(p_n)$. In this case, it deviates to offer $p_n^m = 1/2$. Similarly, providing any positive level of effort $e_n > 0$ entails a cost for the neutral firm that does not yield any benefit, so it provides zero effort $e_n^m = 0$. Notice that this ensures that constraint (4) is satisfied, with $\pi_n(p_n^m) = 1/4$.

To solve (5), we can replace (p_n, e_n) by these values in the constraints. The program simplifies to:

$$\max_{p_g, e_g \geq 0} \quad \pi_g(\cdot) = (p_g - c)(1 + b - p_g) - \gamma_g e_g$$

$$\text{s.t.} \quad (1) \quad -p_g^2 + p_g(1 + b + c) - c\left(\frac{1}{2} + b\right) - \frac{1}{4} - \gamma_g e_g \geq 0,$$

$$(2) \quad p_g^2 - p_g(1 + b) + \frac{1}{4} + \gamma_n e_g \geq 0,$$

$$(3), \quad (4).$$

Temporarily ignore constraints (3) and (4) and check that they are satisfied ex-post. These constraints ensure that the firm's profits are non-negative. As mentioned in the first part of the proof, the neutral firm chooses its monopoly strategy, and therefore earns non-negative profits, satisfying constraint (4). For constraint (3), as the firm is a monopoly, it is likely to earn positive profits, so we can ensure that this is the case after we solve for a separating equilibrium.

Constraint (2) is binding at the optimum: separation is costly to the green firm, so at the optimum, it will choose price and effort such that separation is just attained (in other words, to minimize its loss due to the informational inefficiency). As a consequence, we have: $p_g^2 - p_g(1 + b) + \frac{1}{4} + \gamma_n e_g = 0$, which gives an expression for the effort level as a function of the price: $e_g(p) = \frac{1}{\gamma_n}\left(-p_g^2 + p_g(1+b) - \frac{1}{4}\right)$. From this, we can deduce that the effort provided by the green firm to increase the greenness of its product is positive if and only if $p_g \in \left]\frac{1 + b - \sqrt{2b + b^2}}{2}; \frac{1 + b + \sqrt{2b + b^2}}{2}\right[$.

There are two cases to consider:

1. Interior solution for e_g: the green firm uses two instruments (price and positive effort level) to signal itself into a separating equilibrium.

2. Zero-effort equilibrium: the green firm uses only the price to signal itself into a separating equilibrium and provides no effort.

Case 1. Consider an interior solution for $e_g > 0$.

In this case, $e_g(p) = \frac{1}{\gamma_n}\left(- p_g^2 + p_g(1+b) - \frac{1}{4}\right)$. The program writes:

$$\max_{p_g} \pi_g(\cdot) = (p_g - c)(1 + b - p_g) - \frac{\gamma_g}{\gamma_n}\left(- p_g^2 + p_g(1+b) - \frac{1}{4}\right)$$
$$\text{s.t.} \quad (2) - p_g^2\left(1 - \frac{\gamma_g}{\gamma_n}\right) + p_g\left[(1+b)\left(1 - \frac{\gamma_g}{\gamma_n}\right) + c\right] - c\left(\frac{1}{2} + b\right) - \frac{1}{4}\left(1 - \frac{\gamma_g}{\gamma_n}\right) \geq 0.$$

The program is concave in p_g. The first-order condition gives:

$$p_g = \frac{1}{2}\left(1 + b + c\frac{\gamma_n}{\Delta\gamma}\right)$$

where $\Delta\gamma = \gamma_n - \gamma_g > 0$, and

$$e_g = \frac{1}{\gamma_n}\left[\frac{1}{4}b^2 + \frac{1}{2}b - \frac{1}{4}c^2\left(\frac{\gamma_n}{\Delta\gamma}\right)^2\right].$$

Notice that $\frac{\gamma_n}{\Delta\gamma} > 1$ so $p_g > p_g^m$.

We can derive the condition under which we indeed have

$$p_g \in \left]\frac{1 + b - \sqrt{2b + b^2}}{2}; \frac{1 + b + \sqrt{2b + b^2}}{2}\right[$$

as required for $e_g > 0$:

1. $p_g > \frac{1 + b - \sqrt{2b + b^2}}{2} \Leftrightarrow \underbrace{c\frac{\gamma_n}{\Delta\gamma}}_{>0} > \underbrace{- \sqrt{2b + b^2}}_{<0}$, which is always satisfied;
2. $p_g < \frac{1 + b + \sqrt{2b + b^2}}{2} \Leftrightarrow \frac{\sqrt{2b + b^2}}{c} > \frac{\gamma_n}{\Delta\gamma}$.
   This condition needs to be true for the green firm to be willing to choose a positive effort level given by e_g. This is the case when b and c are not too close, and/or when γ_n and γ_g are different enough.

Finally, verify all remaining constraints ((1) and (3)).

- First, verify that constraint (1) holds. Recall the condition:

$$-p_g^2\left(1-\frac{\gamma_g}{\gamma_n}\right)+p_g\left[(1+b)\left(1-\frac{\gamma_g}{\gamma_n}\right)+c\right]-c\left(\frac{1}{2}+b\right)-\frac{1}{4}\left(1-\frac{\gamma_g}{\gamma_n}\right)\geq 0.$$

This is a second-degree equation in p_g. It admits to real roots $\underline{p}$ and $\overline{p}$ given by:

$$\underline{p}=\frac{1}{2}\left(\frac{c+\alpha(1+b)-\sqrt{\delta}}{\alpha}\right) \text{ and}$$

$$\overline{p}=\frac{1}{2}\left(\frac{c+\alpha(1+b)+\sqrt{\delta}}{\alpha}\right),$$

where $\alpha=1-\frac{\gamma_g}{\gamma_n}\in[0,1]$ for simplification and $\delta=(\alpha b-c)^2+2\alpha^2 b>0$ is the discriminant. For constraint (1) to be satisfied, it remains to show that $p_g\in[\underline{p},\overline{p}]$.

- We have $p_g\geq\underline{p}\Leftrightarrow-\sqrt{\delta}\leq 0$, which is true.
- We have $p_g\leq\overline{p}\Leftrightarrow\sqrt{\delta}\geq 0$, which is true as well.

As a result, constraint (1) is ex-post satisfied.

- Now, turn to constraint (3): under these conditions, the profits of the green firm are non-negative: $\pi_g(p_g,e_g)=\frac{1}{4}+\left(1-\frac{\gamma_g}{\gamma_n}\right)\left(\frac{1}{4}b^2+\frac{1}{2}b\right)-\frac{1}{2}c(1+b)+\frac{1}{4}\frac{\gamma_n}{\Delta\gamma}c^2>0.$

Case 2. Assume that the condition for an interior solution for e_g is not satisfied, *i.e.* we have $\frac{\sqrt{2b+b^2}}{c}\leq\frac{\gamma_n}{\Delta\gamma}$. In this case, $e_g=0$ and the green firm uses only the price to signal itself into a separating equilibrium. As mentioned above, ignore constraints (3) and (4) for the moment, and rewrite the objective and constraints (1) and (2) as:

$$\begin{aligned}
&\max_{p_g}\pi_g(\cdot)=(p_g-c)(1+b-p_g)\\
&\text{s.t. (1)}\quad -p_g^2+p_g(1+b+c)-c\left(\frac{1}{2}+b\right)-\frac{1}{4}\geq 0,\\
&\text{(2)}\quad p_g^2-p_g(1+b)+\frac{1}{4}\geq 0.
\end{aligned}$$

(1) and (2) are both second-degree equations in p_g with two real roots, and therefore both give a range of prices for which the constraints are satisfied. Let us characterize these ranges:

- The interval satisfying constraint (1) is given by:
  $p_g\in\left[\underline{p_1}=\frac{1+b+c-\sqrt{2b+(b-c)^2}}{2};\overline{p_1}=\frac{1+b+c+\sqrt{2b+(b-c)^2}}{2}\right].$
- On the other hand, values of p_g which satisfy constraint (2) are:

$$p_g \leq \underline{p_2} = \frac{1 + b - \sqrt{2b + b^2}}{2},$$
$$p_g \geq \overline{p_2} = \frac{1 + b + \sqrt{2b + b^2}}{2}.$$

It is quite straightforward to prove the ordering $\underline{p_2} > \underline{p_1} > \overline{p_2} > \overline{p_1}$ using the expressions for each and under the assumption of the model that $b > c > 0$. From the two ranges of acceptable values for p_g described above and the ordering of the roots, we can demonstrate that the unique range of prices that satisfies both (1) and (2) simultaneously is $[\overline{p_2}; \overline{p_1}]$, as shown in green by the graphical representation below (figure 2.6).

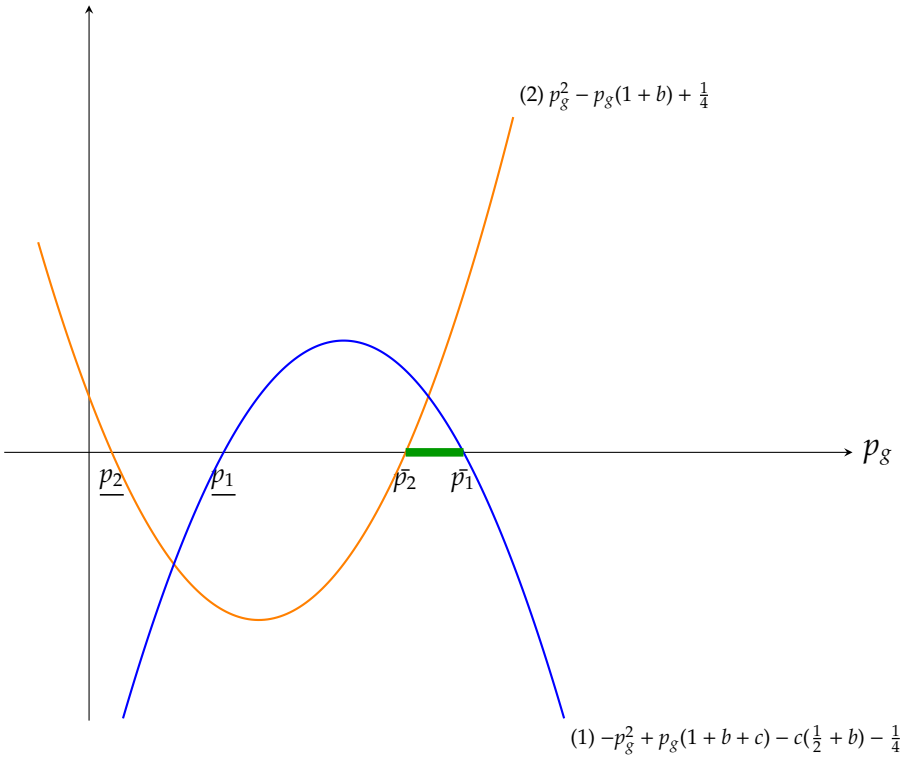


Figure 2.6 – *Graphical illustration of constraints (1) and (2) with zero effort*

The optimization problem therefore boils down to: $\max_{p_g}(p_g - c)(1 + b - p_g)$ subject to $p_g \in [\overline{p_2}; \overline{p_1}]$. Notice that, for the separating equilibrium conditions to be met under zero effort, the price offered by the green firm must be higher than the full information monopoly price $p_g^m = \frac{1+b+c}{2} < \overline{p_2}$. Therefore, the green firm optimally chooses to offer the lowest price possible in the range of acceptable prices for a separating equilibrium, in order to get as close as possible to the full-information monopoly price (consistent with the least-cost equilibrium selection refinement, as imposed by the Intuitive Criterion). Constraint (2) will indeed be binding, and in a separating equilibrium with zero effort,

we have:

$$p_g = \frac{1 + b + \sqrt{2b + b^2}}{2}.$$

As a consequence, notice that constraint (1) is automatically satisfied. We can verify that constraint (3) holds. When effort is zero, green profits are non-negative as long as the price belongs to the interval $[c; (1 + b)]$, which is the case with the price p_g described above.

2.C Footnote 16

When the neutral and green types have identical costs of effort, the green firm will always offer zero effort in a separating equilibrium. In other words, the green firm only uses price as an instrument to signal its type to consumers, and does not find it worth it to provide a costly effort. The intuition behind this is the following: the benefit from choosing a positive effort level for the green monopoly arises from the possibility of offering a lower price than $p = \frac{1+b+\sqrt{2b+b^2}}{2}$, which is the price offered in a separating equilibrium with zero effort. Thus, effort is costly but enables the firm to charge a price closer to its full information monopoly price (which increases revenues). The firm will effectively choose to provide a positive effort if this gain is important enough compared to the loss in profits due to the cost of exerting this effort level. However, when the cost of effort is identical for the green and neutral types, for a positive effort to be worth it for the green firm, the price would need to decrease a lot, to an extent that is not permitted by constraint (2). As a consequence, the green firm cannot remain in a separating equilibrium by choosing a package (p_g, e_g) where $e_g > 0$. To see this formally, suppose that $\gamma_n = \gamma_g = \gamma$. First analyze constraint (2), and determine how p_g decreases when e_g increases to keep the Ccnstraint binding. Recall (2):

$$p_g^2 - p_g(1+b) + \frac{1}{4} + \gamma e_g = 0.$$

Thus, we have:

$$\underbrace{\frac{dp_g}{de_g} = \frac{-\gamma}{2\left(p_g - \frac{1+b}{2}\right)}}_{\star} < 0 \text{ as } p_g > \frac{1+b}{2}.$$

Now turn to the change in profits associated with a change in price and effort:

$$d\pi = \frac{\partial \pi}{\partial p_g} dp_g + \frac{\partial \pi}{\partial e_g} de_g = (1+b+c-2p_g)dp_g - \gamma de_g.$$

To get a gain from simultaneously increasing effort and decreasing the price, we would need:

$$d\pi \geq 0 \Leftrightarrow \frac{dp_g}{de_g} \leq \frac{-\gamma}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

Now we can prove that this cannot be reconciled with condition $\star$ on constraint (2). Indeed, we would need the following to be true:

$$\frac{-\gamma}{2\left(p_g - \frac{1+b}{2}\right)} \leq \frac{-\gamma}{2\left(p_g - \frac{1+b+c}{2}\right)} \Leftrightarrow c \leq 0$$

which is a contradiction. Therefore, increasing effort to a positive level and in exchange lowering the price closer to the full information monopoly price never yields a gain in profits for the green firm when the marginal costs of effort are identical, and this results in zero effort level in separating equilibrium.

2.D Footnote 17

When $\gamma_n > \gamma_g > 0$, we can see how the gain in profits generated by using both the price and effort to achieve separation can be facilitated depending on the parameters of the model, by proceeding in a similar manner. Constraint (2) binding implies:

$$\frac{dp_g}{de_g} = \frac{-\gamma_n}{2\left(p_g - \frac{1+b}{2}\right)}.$$

A gain in profits by increasing e_g and decreasing p_g is possible if:

$$d\pi \geq 0 \Leftrightarrow \frac{dp_g}{de_g} \leq \frac{-\gamma_g}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

Therefore, combining, we need to have the following:

$$\frac{-\gamma_n}{2\left(p_g - \frac{1+b}{2}\right)} \leq \frac{-\gamma_g}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

This condition is more easily satisfied when the difference between the costs of effort, $\Delta\gamma$, is high. Moreover, it is also facilitated by a small marginal cost of production of the green product, c.

2.E Proof of Proposition 3

We provide an intuitive reasoning to support the validity of this argument. When the firm is green, total welfare under complete information monopoly strategies can be written as a function of the monopoly quantity:

$$W_g^m = \underbrace{q_g^m(1+b-c) - \frac{1}{2}q_g^{m^2}}_{(A)} + \beta.$$

When there is incomplete information and parameters of the models allow for Case 1, total welfare in a separating equilibrium can also be written in terms of separating quantities and efforts:

$$W_g = \underbrace{q_g(1+b-c) - \frac{1}{2}q_g^2}_{(B)} + \underbrace{h(e_g) - \gamma_g e_g}_{\text{Net benefit of improved greenness}} + \beta.$$

As $q_g^m > q_g$ (or, equivalently, the separating price is higher than the full information monopoly price), we necessarily have $(A) > (B)$. This illustrates the loss in welfare due to the decrease (resp. increase) in quantity (resp. price) when moving from complete to incomplete information. However, there is an extra component in the welfare function under incomplete information when effort is positive: the net benefit of improved greenness (positive externality minus the private cost of exerting such effort). It could be the case that this dimension gets large enough so that it outweighs the loss from (A) to (B). This is more likely to occur as the cost of effort decreases (low γ_g), and as the public good benefit from improved greenness increases. The latter heavily depends on the shape of function $h(\cdot)$ (a steeper slope facilitates an increase in total welfare when under incomplete information).

2.F Proof of Proposition 4

Suppose that the regulator does not directly observe the type of the monopoly. The regulator gives out a subsidy of $s = \gamma_g$ per unit of effort, to a firm offering $p_g^m = \frac{1+b+c}{2}$ and $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$. It gives out zero subsidy for any level of effort below this one.
The question of interest is the following: does a neutral firm facing this policy offer its usual monopoly strategy (monopoly price and zero effort), or does it face an incentive to behave like a green firm to receive the subsidy?
In other words, we examine whether the menu $\{(p_n^m, 0); (p_g^m, e_g^{FB}, s)\}$ is directly separating or not.
To do so, neutral monopoly profits must be compared to neutral profits when perceived as green. We have:

$$\pi_n(p_n^m, 0) = 1/4,$$
$$\pi_n(p_g^m, e_g^{FB}, s) = \frac{1}{4} + \frac{1}{4}\left(2b + b^2 - c^2 - \frac{\Delta\gamma}{\gamma_g^2}\right).$$

The policy offered directly induces a separating equilibrium where the neutral firm has no incentive to mimic the green firm if and only if $\pi_n(p_n^m, 0) > \pi_n(p_g^m, e_g^{FB}, s)$, *i.e.* when the following condition holds:

$$2b + b^2 - c^2 - \frac{\Delta\gamma}{\gamma_g^2} < 0. \quad (6)$$

For given values of b and c, notice that when $\gamma_g = \gamma_n = s$, condition (6) is necessarily violated as the left-hand side becomes $\frac{1}{4}(2b + b^2 - c^2) > 0$. Therefore, when the marginal costs of effort are identical, the menu cannot be separating in incomplete information, as the neutral firm offers e_g^{FB} at zero cost (fully covered by the subsidy) and enjoys higher profits.
Let us now analyze how the expression changes when γ_n increases above γ_g. The left-hand side of condition (6) is linear and decreasing in γ_n. There exists a cutoff point

$\hat{\gamma_n}$ above which (6) is satisfied, given by:

$$\hat{\gamma_n} = \gamma_g + \underbrace{\gamma_g^2(2b + b^2 - c^2)}_{\alpha}$$

where α is the minimum extent of the difference between γ_g and γ_n required for the policy to directly induce a separating equilibrium, for $b > c > 0$ fixed. In other words, we need $\Delta\gamma > \hat{\Delta\gamma} = \gamma_g^2(2b + b^2 - c^2)$ for (6) to be satisfied. Figure 2.4 in the main part of the paper provides a graphical representation of this cutoff.

We first assumed b and c fixed with $b > c > 0$. Now consider the effect of a variation in b. Recall the minimum difference between γ_g and γ_n required to directly achieve separation: $\hat{\Delta\gamma} = \gamma_g^2(2b + b^2 - c^2)$. Notice that this cutoff can be decreased by a lower value of b. For a fixed γ_g, such variation in b expands the range of values of γ_n for which a separating equilibrium is directly obtained. On figure **??**, this corresponds to moving the point $\hat{\gamma_n}$ to the left, shrinking the extent of α and extending the green interval. However, the warm glow b can take values as low as very close but greater than c, as assumed by the model. In the limiting case, assume that $b = c + \varepsilon$ with $\varepsilon > 0$ very small such that $b \approx c$. Then, the difference between marginal costs of efforts has to be at least $\hat{\Delta\gamma} = 2c\gamma_g^2$ in order to get separation.

The arguments above provide evidence that condition (6) is more easily satisfied when $\Delta\gamma$ is high and b is low.

2.G Footnote 9: Model with Public Good Benefit Increasing in Quantity

This section presents a version of the model where the public good benefit generated by green production increases with the quantity produced. Specifically, assume that the public good benefit is given by:

$$B(q_g, e_g) = q_g \left(1 + h(e_g)\right),$$

where $h(0) = 0$, $h'(e_g) > 0$, and $h''(e_g) \leq 0$. If the green firm does not exert a costly effort to increase the greenness of its product ($e_g = 0$), this "public good" benefit is equal to the total quantity of green goods consumed: $B(q_g) = q_g(p_g)$. In the case where the green firm improves the greenness of its product, this benefit increases with effort at a decreasing rate. All other assumptions and elements of the model remain the same as in the main specification.

Rationale. This alternative captures situations in which a greater scale of green production generates larger aggregate environmental benefits. It reflects the idea that the more consumers a green monopoly serves, the fewer consumers will turn to alternatives (possibly polluting) sources to meet their demand, such as suppliers in adjacent sectors or foreign markets, which are not directly modeled here but could exist. For example, if a small green producer supplies only a few units, the overall environmental benefit may be limited compared to a large firm producing many units, which could meaningfully displace less sustainable options.

Complete information monopoly strategies and first-best solution. The monopoly strategies under complete information are identical to the main model (zero effort, monopoly pricing). The welfare-maximizing solution solves the following problem:

$$\max_{q_n,e_n,q_g,e_g\geq 0} \quad W = \lambda\left(q_g(1+b-c) - \frac{1}{2}q_g^2 - \gamma_g e_g + q_g(1+h(e_g))\right) + (1-\lambda)\left(q_n - \frac{1}{2}q_n^2 - \gamma_n e_n\right)$$

The first-best neutral effort and price are identical to the main model. If the firm is green, we can compute the first order conditions with respect to q_g and e_g:

$$\begin{aligned} q_g^{FB} h'(e_g^{FB}) &= \gamma_g, \\ q_g^{FB} &= 1 + b - c + \left(1 + h(e_g^{FB})\right) \end{aligned}$$

which indirectly define the first-best quantity and effort level for the green firm. Using $h(e_g) = \sqrt{e_g}$ as in the discussed examples, we can derive the exact expressions for q_g^{FB} and e_g^{FB}:

$$q_g^{FB} = \frac{2\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)$$
$$\text{and } e_g^{FB} = \left(\frac{2+b-c}{2\gamma_g - 1}\right)^2$$

where we impose $\gamma_g > 1/2$. If this state of nature is realized ($\theta = g$), total welfare is equal to $W_g^{FB} = \frac{\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)^2$, greater than $W_g^m = \frac{1}{8}\left(1 + b - c\right)\left(7 + 3(b-c)\right)$.

We can write the expected total maximum welfare in first best as:

$$W^{FB} = \lambda\left(\frac{\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)^2\right) + \frac{1-\lambda}{2}.$$

To conclude, we can verify the concavity of the program to ensure the existence of this maximum. The Hessian writes:

$$H_{(q_g,e_g)} = \begin{bmatrix} -1 & h'(e_g) \\ h'(e_g) & q_g h''(e_g) \end{bmatrix}.$$

For concavity, we verify the diagonal elements : $-1 < 0$ and $q_g h''(e_g) \leq 0$, and require $\det\left(H_{(q_g,e_g)}\right) \geq 0 \Leftrightarrow -q_g h''(e_g) \geq (h'(e_g))^2$. Using the example $h(e_g) = \sqrt{e_g}$, we verify: $q_g \geq \sqrt{e_g}$. At the computed solution, this translates to $\gamma_g \geq 1/2$, which is assumed to hold.

Here, the first-best quantity is higher than the green firm's monopoly strategy, for two main reasons:

1. As in the main model, in order to take into account the consumer surplus.
2. To reflect the positive externality generated by the green product, which grows with the size of production (new mechanism compared to the main model).

Separating strategies. As the public good benefit does not play a role in determining the firms' separating strategies in incomplete information, the resulting expressions (and cases) are identical as in the main model.

Welfare implications. Welfare does not change in incomplete information when the firm is neutral, as the price and effort level remain the same as the complete information monopoly strategy. However, similar to the main model, when the firm is green, there are conditions under which welfare is higher under incomplete information. The intuition is identical as before: when the green firm exerts a positive level of effort to signal itself, it is possible that the gain in welfare associated with the public good benefit exceeds the loss associated with charging higher price and costly effort. For this to be the case, the net benefit from improved greenness $h(e_g)q_g - \gamma_g e_g$ needs to be sufficiently large.

Numerical example. We adapt the examples of specific parameter values to illustrate the different scenarios that arise as a result. Keep the assumption that $h(e_g) = \sqrt{e_g}$.

First, consider the following values for the model parameters: $b = 6$, $c = 0.4$, $\gamma_n = 2$, and $\gamma_g = 1$. In a separating equilibrium, the green firm exerts a positive effort $e_g = 5.92$ to increase the greenness of its product. In this case, the benefit from improved greenness is sufficiently large compared to the total cost of effort, and welfare in incomplete information increases compared to the monopoly case in complete information. More

specifically, we have $W_g \approx 20.4 > W_g^m = 19.635$.

As discussed previously, it could be the case that the green firm signals itself to consumers using a positive effort in addition to the price, but welfare decreases compared to the monopoly case in complete information. For instance, take the same parameters but increase the cost of effort for the green firm: $b = 6$, $c = 0.4$, $\gamma_n = 2$, and $\gamma_g = 1.8$. In this case, effort to increase the greenness of the product is still positive ($e_g = 4$), but the total cost of providing such effort becomes too large compared to its positive public good effect, so total welfare drops to $W_g = 6.075 < W_g^m = 19.635$.

There exist parameter values that induce a separating equilibrium where the green firm exerts zero effort and relies solely on the price to signal its type to consumers. As mentioned before, this outcome is more likely to arise when the warm-glow benefit is low, the marginal cost of production is high, and the costs of effort are relatively similar across firm types. For instance, we can take $b = 2$, $c = 0.5$, $\gamma_n = 2$, and $\gamma_g = 1.7$.

Subsidizing the first-best green effort. In a similar fashion as the main model, assume that the objective of the regulator is to use a subsidy $s = \gamma_g$ per unit of effort to implement the first-best green effort level, $e_g^{FB} = \left(\frac{2+b-c}{2\gamma_g - 1}\right)^2$. Under incomplete information, the regulator offers this subsidy to a firm setting e_g^{FB} and p_g^m. In line with the main model's conclusions, under some conditions, the complete information strategies $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ can be directly separating under incomplete information. This happens when the following condition is satisfied:

$$\frac{2b + b^2 - c^2}{4} < \frac{\Delta\gamma}{(2\gamma_g - 1)^2}(2 + b - c)^2. \qquad (6')$$

For this condition to be true, the costs of effort need to be sufficiently different (*i.e.* $\Delta\gamma$ large enough). For given values of b and c, the minimum difference between the costs of efforts necessary for these strategies to be separating is $\Delta\hat{\gamma} = \dfrac{(2b + b^2 - c^2)(2\gamma_g - 1)^2}{4(2 + b - c)^2}$.

The intuition is identical as the main model. The role of the warm-glow benefit in condition (6') is slightly more complex than in the main version of the model. Indeed, the warm-glow creates two opposite effects in the neutral profits from mimicking the green type. First, a positive effect from the gain in revenues from being perceived as green by consumers and getting higher demand and charging a higher price, increasing

in b. Second, a negative effect (which did not exist in the previous version), as the level of first-best green effort required by the regulator increases with b, increasing the cost of providing such effort for the neutral firm. Following the same intuition as above, when the costs of effort across types are sufficiently different ($\Delta\gamma > \frac{1}{4}(2\gamma_g - 1)^2$), regardless of the value of b, the gain in revenues will not be sufficient to offset the residual cost of exerting effort e_g^{FB}, meaning that the complete information strategies will always be separating. However, when $\Delta\gamma \in \left[\frac{c(2\gamma_g-1)^2}{8(1+c^2)}; \frac{1}{4}(2\gamma_g - 1)^2\right]$, separation becomes more difficult to achieve as the subsidy covers most of the neutral's cost of effort, but it can still arise if b is low enough to mitigate the gain in revenues from mimicking green. More specifically, in this case, for the complete information strategies to directly induce separation, we need $b < \frac{2\lambda(2+c)-\frac{1}{2}+\sqrt{\delta}}{\frac{1}{2}-2\lambda}$, where $\lambda = \frac{\Delta\gamma}{(2\gamma_g-1)^2}$, and $\delta = 32c\lambda^2 - 6c\lambda + \frac{1}{4}(1 + c^2)$.

Chapter 3

The Insurer-Investor Dilemma: When Insurers Fuel the Risks They Cover

Abstract

Insurers engage in two core activities: underwriting risks and investing in financial markets. This paper examines how these activities interact, particularly in the context of climate-related risks, where investment decisions can influence the probability of loss. We develop a model of two insurers who offer insurance policies to agents with different probabilities of loss, and invest the collected premiums, following either a risky or a safe strategy. Risky investments yield higher returns but increase the probability of loss for some agents. When insurers know the probability of loss, they always choose risky investments in equilibrium, purposely increasing the risk faced by some agents. When insurers do not know the probability of loss of agents, they must offer distorted insurance policies. The associated loss in profits increases as insurers choose risky investments. As a result, safe investments can emerge in equilibrium. This framework generalizes to settings in which a Principal's profitable action negatively alters the type of some Agents, thereby influencing the severity of adverse selection.

JEL Codes: D82, G22, Q54, D62.

Keywords: Insurance Markets, Investment, Adverse Selection, Externality, Climate Risk.

1 Introduction

Research question. The 2025 Los Angeles wildfires inflicted up to $40 billion in insured losses and renewed a debate about the impact and responsibility of insurers with regard to climate-related risks. As private insurers increasingly withdrew from high-risk areas, citing constraining regulations, millions were left uninsured or reliant on the financially strained FAIR Plan. This episode exposed a striking contradiction: insurers play a crucial role in covering risks and absorbing the costs of future climate damages[1] but continue to heavily invest in fossil fuels and other high-emission sectors, thereby contributing to the increase in frequency and intensity of these events. For instance, the California Department of Insurance's Data Climate Risk Initiative, which requires a large number of insurers to disclose their investments, reported $586 billion of fossil fuel investments in 2017 by the 1,187 insurers surveyed. For some of the largest companies in the dataset, the ratio of fossil fuels investments to their total assets under management can largely exceed 20%.[2] While insurers' investment decisions affect agents' risks, these markets are also often characterized by adverse selection: agents, who face different probabilities of loss, know more about their own risk exposure than insurers. In the context of climate-related risks, inequality across agents regarding the exposure to certain events is particularly pronounced. Asymmetric information regarding exposure may not only affect the insurance policies that insurers are able to offer to agents, but also their choice of investment strategy as it indirectly impacts the likelihood of a loss occurring for (at least) some agents.

Therefore, how does the interplay between insurance and investment activities shape insurers' strategic decisions? And how do these choices change under incomplete information about agents' probability of loss?

This paper aims to study the strategic choices of insurers in the two indirectly-linked activities. We demonstrate that the results differ depending on the level of information available to the insurers regarding the probability of loss of the agents they face.

1. Some studies demonstrate that countries with larger insurance coverage recover faster from climate-related losses (Melecky and Raddatz, 2011; von Peter et al., 2024; OECD, 2018; EIOPA et al., 2021).
2. https://interactive.web.insurance.ca.gov/apex_extprd/f?p=250:1:0::NO:10,25,30,40::

Context and model overview. In this paper, two insurers engage in two separate activities: they provide insurance policies to households and firms on a competitive insurance market, and they invest collected premiums in financial markets. Each insurer chooses between a *safe* investment strategy, which yields lower returns but leaves risk probabilities unchanged, and a *risky* investment strategy, which yields higher returns but increases the probability of loss for some agents in the population. Suppose an insurance company invests heavily in coal mining, oil and gas infrastructure, or other carbon-intensive industries. Those investments contribute to higher global greenhouse gas emissions and thus contribute to the likelihood of a climate-related loss occurring. This creates a link between the insurers' financial decisions and the distribution of risks in the population.

Agents differ in their degree of exposure to climate-related risks (OECD, 2024). A diversity of empirical papers and reports prove that there exists a multitude of factors that induce differences in exposure (even within-country). These of course include geographical factors, but also poverty[3], environmental factors (*e.g.* urbanization, degradation), and political and institutional factors (*e.g.* local regulatory measures). In the model, the agent's *type* captures this dimension. *Unexposed* agents face a constant baseline probability of loss, regardless of the insurers' financial decisions. In contrast, *exposed* agents face a probability of loss that increases if insurers adopt risky strategies.

However, insurers sometimes cannot directly observe whether a given agent is exposed or unexposed. In the model, we first assume that the true probability of loss of an agent is known by the insurers, to then compare with the situation where it is private information of this agent. In practice, such asymmetries arise because households and firms hold private information about micro-level characteristics that affect their risk, related to their specific location, direct environment, or preventive measures. For instance, a household may know whether its home has fire-resistant roofing, or whether it regularly clears flammable vegetation, but these relevant details are costly for insurers to verify. Similarly, firms may hold private information about maintenance history and building materials. It is true that insurers may access some information

3. Low-income agents tend to live in flood-prone areas due to lower prices (Rentschler et al., 2022), are less likely to invest in adaptation measures due to a lack of financial resources (Hallegatte & Rozenberg, 2017), and are more dependent on agriculture income (FAO, 2005).

on agents' locations via coarse indicators of exposures (*e.g.* wildfire-prone zones, designated floodplains). However, it is reasonable to assume that it remains difficult to identify the more granular, site-specific features. Incomplete information implies that insurers must design insurance policies that induce self-selection by agents, rather than pricing directly based on true exposure. These incentive-compatible insurance policies are dependent on the investment strategies chosen by insurers as these choices alter the probability of loss faced by exposed agents.

Main results. In the complete information case, insurers offer full insurance at fair premiums, and prefer risky investment strategies. Risky investments both increase expected returns and allow charging higher premiums on exposed agents. As a result, insurers face perverse incentives to raise the probability of loss for these agents.

These results change depending on the extent of information available to insurers. Indeed, when agents' risk exposure is not known by insurers, they must distort insurance policies compared to the complete information case, to prevent exposed agents from picking policies designed for unexposed agents. More particularly, unexposed agents cannot obtain full insurance and are instead offered policies with partial coverage, making their policy unattractive to exposed agents.

These distortions result in two dimensions of efficiency loss: under-insurance of some types, and a reduction in insurers' premium revenues, *i.e.* the amount that can be invested in financial markets. Insurers therefore face a tradeoff between the profitability of risky investments and the severity of the selection problem. This contributes to making risky investments relatively less attractive than under complete information. As a result, safe investment strategies can, under some conditions, be sustained in equilibrium, since they reduce the distortion required on unexposed policies to achieve incentive compatibility.

Whenever risky investments remain the most profitable choice for insurers even under incomplete information, it is possible for a regulator to intervene with the aim of encouraging safe investments. For instance, making full coverage mandatory for all agents in the population can lead to safe investment strategies in equilibrium.

Related literature and contribution. This paper contributes to the theoretical literature on insurance markets, notably with adverse selection (see Dionne et al., 2025). Rothschild and Stiglitz (1976) is the seminal paper investigating the design of insurance policies in competitive markets where agents are privately informed of their probability of loss (extended to imperfect competition by Stiglitz (1977) and Dahlby (1987)). The present paper adopts a similar framework, in which insurers must distort the offered insurance contracts to incentivize agents to reveal their true risk type. We extend this theoretical framework by introducing insurers' investment decisions as an additional strategic dimension that affects some agents' probability of loss, hence the degree of adverse selection. This feature explains why insurers choose risky investment strategies when they observe the risk types of agents, despite the fact that it increases their probability of loss. It also highlights additional sources of distortions on insurance contracts under incomplete information about agent types. As a consequence, it enables to analyze the extent to which incomplete information in the insurance market affects the investment decisions of insurers, showing that safe investments can emerge in this case.

Many papers have enriched the theoretical foundation by conducting empirical tests of adverse selection (and/or moral hazard) in different markets (see Chiappori and Salanié, 1997, 2000; Chiappori et al., 2006).

This paper is related to different strands of research in which firms, insurers, or financial intermediaries influence the risk or damage ultimately borne by agents. It echoes the literature on insurance markets with moral hazard, in which the probability of loss of agents can increase as a response to contract structure. Foundational contributions include Pauly (1968, 1974), Shavell (1979), Arnott and Stiglitz (1991), Arnott (1992). In contrast, this paper does not consider changes in agents' behaviors in response to coverage, rather how insurers' financial activities affect agents' probability of loss. For standard goods, Spence (1977) and Shavell (1980) investigate how firms' quality or safety choices affect accident probability for consumers. In the systemic risk literature, risky but profitable investments by insurers or bankers increase the probability of a crisis, of default, or insolvency (see Plantin and Rochet (2007) for a textbook treatment, Eling and Pankoke (2014) for a literature review on systemic risk in

the insurance industry, and Cummins and Weiss (2014)). More closely related, Mimra and Wambach (2019) incorporates endogenous insolvency risk into a Rothschild-Stiglitz framework. A common theme across these contributions is the existence of perverse incentives: risk-taking can be privately profitable for firms but increases the probability of costly losses for others. However, this paper focuses on the interaction between insurers' investment choices and the probability of loss faced by agents, rather than on sector-wide risks. This highlights insurers' dual role as both generators and absorbers of risk, linking investment behavior, insurance policy distortions, and the trade-off between profitability and adverse selection.

Finally, this paper is related to the literature that links insurers' financial activities to the design of their insurance contracts. While most recent contributions (*e.g.* Knox and Sørensen, 2024) examine how asset returns influence underwriting decisions, our focus lies on insurers' choice of investment when these strategies directly affect the risk environment and thus the structure of insurance contracts.

Structure of the paper. The remainder of this paper is organized as follows. Section 2 describes the model, section 3 derives the complete information benchmark, and section 4 derives the results in incomplete information with identical insurers. Finally, section 5 discusses two extensions: the case of asymmetric insurers, and the regulatory enforcement of full coverage.

2 The Model

Consider an economy consisting of two sectors: an insurance market with insurers and agents (policyholders); and a financial market in which insurers invest their revenues.

Insurers

Consider a setting in which there are two risk-neutral insurers on the market, indexed by $i = 1, 2$. Insurers have two activities. First, they operate on a perfectly competitive insurance market. They offer insurance policies, specifying premiums and coverage.

Insurers also have a financial activity. They can invest on financial markets to generate interest revenues. Each insurer chooses an investment strategy $a_i \in \{R, S\}$, which may be either a risky investment ($a_i = R$), yielding high return r_H; or a safe investment ($a_i = S$), yielding low return r_L, with $r_H > r_L$. Define $\mathbf{a} = (a_1, a_2)$ as the investment strategy profile of the two insurers. Insurers are limited in the amount they can invest on financial markets: they can only invest up to the volume of collected premiums in the insurance activity. They are not allowed to exceed their insurance capital when investing. This assumption follows the many regulations and directives adopted worldwide. They are typically called Risk-Based Capital requirements, and were adopted to guarantee that insurance companies have enough financial resources to honor insurance claims and other difficulties, and avoid becoming insolvent.[4]

Agents

Consider a unit mass population of risk-averse agents. Agents are endowed with initial wealth $W > 0$ and face a potential loss $L \in]0, W]$ in the event of an accident or a damage. There are two types of agents: exposed and unexposed agents, and we denote their type $\theta \in \{E, U\}$. A part $\alpha \in [0, 1]$ of the population are exposed agents (type $\theta = E$) who face weakly higher probability of loss, and the remaining fraction $(1 - \alpha)$ of the population are unexposed agents (type $\theta = U$). The probability of a loss occurring for an agent of type θ is denoted by $\lambda_\theta(\mathbf{a})$, and it may depend on the investment strategy profile $\mathbf{a}$ chosen by insurers. Preferences over a certain level of wealth in one state of the world (loss or no loss) are represented by a strictly increasing and concave utility function $u(\cdot)$. The expected utility of a type-θ agent over all states of the world (loss and no loss) is denoted $U_\theta(\cdot)$, such that $U_\theta(\cdot) = \lambda_\theta(\mathbf{a})u(\mathcal{W}_{\text{loss}}) + (1 - \lambda_\theta(\mathbf{a}))u(\mathcal{W}_{\text{no loss}})$, where $\mathcal{W}_{\text{loss}}$ and $\mathcal{W}_{\text{no loss}}$ represent the agent's final wealth in the loss and no-loss states, respectively.

4. For instance, the European Union's Solvency II Directive imposes risk-based capital charges that discourage speculative borrowing. Similar regulations exist in other sectors. For instance, in the banking sector (*e.g.* Basel III) and for pension funds (*e.g.* the Pensions Regulator's funding requirements in the UK, the Financieel Toetsingskader (FTK) rules in Netherlands).

Unexposed agents. For unexposed agents, the probability of a loss λ_U is unaffected by investment strategies. These agents face a constant baseline probability of loss, such that

$$\lambda_U(\mathbf{a}) = \lambda \in [0,1] \; \forall \; a_i \in \{R, S\} \; \forall \; i = 1, 2.$$

In our context, examples of unexposed agents include residents located in low-risk areas, such as urban residents in non-fire-prone areas or outside floodplains; and firms operating in climate-resilient infrastructures.

Exposed agents. For exposed agents, the probability of loss $\lambda_E(\mathbf{a})$ depends on the investment decisions $a_i \in \{R, S\} \; \forall i$ made by insurers before contracts are issued. A risky investment increases the likelihood of a loss above the baseline risk, while a safe investment does not. Formally, assume

$$\lambda_E(\mathbf{a}) = \lambda + \beta(a_1) + \beta(a_2) \; \in [0,1],$$

where $\beta(a_i) = \begin{cases} 0 \text{ if } a_i = S \\ \beta_i > 0 \text{ if } a_i = R \end{cases}$.

For instance, these agents can be households located on coastlines, in drought-prone areas or near flammable vegetation, but also populations living in older buildings that lack climate-resilient features such as fire-resistant materials, flood-proofing, etc.

To synthesize, while unexposed agents always face the baseline probability of loss, table 3.1 below summarizes the probability of loss faced by exposed agents, depending on insurers' investment strategies. [5]

5. In reality, it is possible to argue that the likelihood of climate-related losses is determined by a multitude of factors beyond the investment decisions of the insurance sector. The main objective of this paper is to study how the link between investment choices and the probability of loss affects insurer strategic choices, and modeling probability as such does not fundamentally contradict this statement. Thus, consider that β_i can be as small as desired, and that all other determinants of the probability of loss are included in λ.

		Insurer 1's investment strategy a_1	
		Safe	Risky
Insurer 2's investment strategy a_2	Safe	$\lambda_E(S,S) = \lambda$	$\lambda_E(S,R) = \lambda + \beta_1$
	Risky	$\lambda_E(R,S) = \lambda + \beta_2$	$\lambda_E(R,R) = \lambda + \beta_1 + \beta_2$

Table 3.1 – *Exposed agents' probability of loss given insurers' investment strategies*

If insurers are perfectly identical, their impact on the probability of loss from choosing the risky investment strategy will be the same, *i.e.* $\beta_1 = \beta_2 = \beta$. This simplification will be used in some sections of the paper.

Competition

Insurers $i = 1,2$ first commit to an investment strategy $a_i \in \{R,S\}$. Then, agents' types $\theta \in \{E,U\}$ are randomly drawn by Nature. Insurers compete to offer insurance policies consisting of coverage $c_\theta(\mathbf{a})$ and an associated premium $p_\theta(\mathbf{a})$. Facing a policy $(p_\theta(\mathbf{a}), c_\theta(\mathbf{a}))$, type-$\theta$ agent gets insured if and only if his expected utility doing so is weakly higher than his expected utility uninsured, that is when $U_\theta(p_\theta(\mathbf{a}), c_\theta(\mathbf{a}), \mathbf{a}) \geq U(\mathbf{a})$. This is the case when the following condition holds:

$$\lambda_\theta(\mathbf{a})u\Big(W - p_\theta(\mathbf{a}) + c_\theta(\mathbf{a}) - L\Big) + \Big(1 - \lambda_\theta(\mathbf{a})\Big)u\Big(W - p_\theta(\mathbf{a})\Big) \geq \lambda_\theta(\mathbf{a})u\Big(W - L\Big) + \Big(1 - \lambda_\theta(\mathbf{a})\Big)u\Big(W\Big).$$

Once contracts are sold, premiums are collected and insurers earn the financial returns depending on their investment strategy. Finally, insurance claims (if any) are realized. Therefore, an insurer expects to make the following overall profits from insuring one agent of type θ and committing to investment strategy a_i:

$$\pi_i(p_\theta(\mathbf{a}), c_\theta(\mathbf{a}); a_i) = (1 + r_{a_i})p_\theta(\mathbf{a}) - \lambda_\theta(\mathbf{a})c_\theta(\mathbf{a}).$$

This paper focuses on the least-cost separating allocation. Section 5 discusses the market environments that support this type of allocation.

Figure 3.1 summarizes the timing of events.

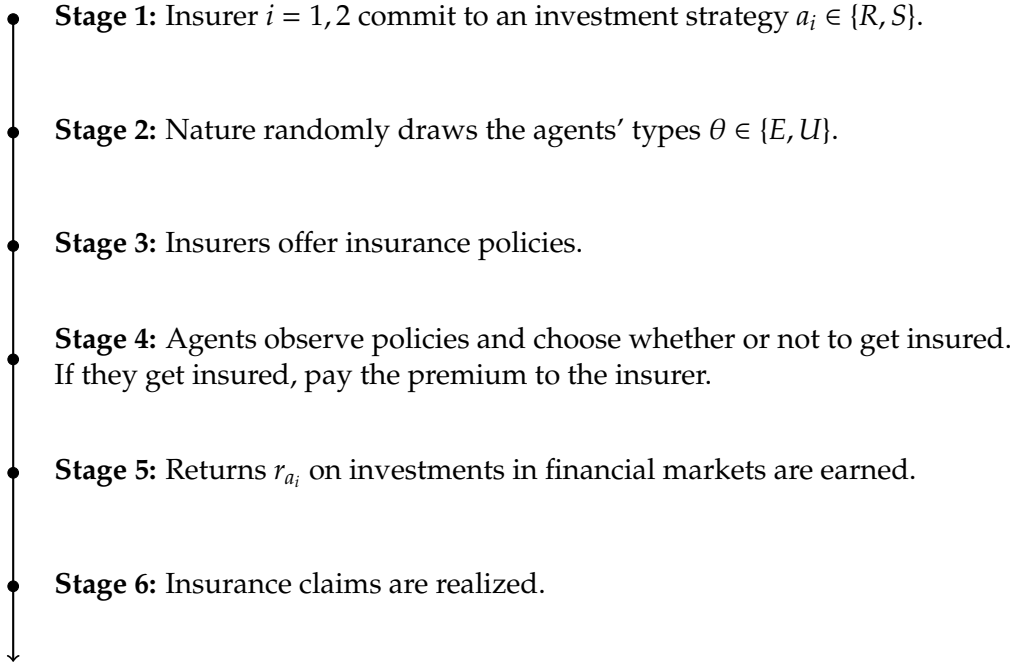


Figure 3.1 – *Timing of the game*

3 Insurers' Strategic Decisions in Complete Information

First of all, we establish the results under complete information. In this context, assume that insurers possess complete information about the types of agents in the population. They directly observe whether an agent is exposed or unexposed to the risk. In other words, they can directly infer the probability of loss faced by a particular agent in the population, and make strategic decisions accordingly. We are interested in the insurers' offer of insurance policies and choices of investment strategies based on this observation. First determine the insurance policies offered to agents and then turn to the investment strategies chosen by insurers.

3.1 Complete Information Insurance Policies

Perfect competition on the insurance market drives expected profits from the insurance activity to zero:

$$p_\theta(\mathbf{a}) - \lambda_\theta(\mathbf{a}) c_\theta(\mathbf{a}) = 0.$$

This leads insurers to offer actuarially fair premiums.[6] Since risk-averse agents prefer full insurance to any level of partial coverage[7], the two competing insurers therefore offer full insurance. All agents in the population are insured. Each insurer serves half of the market. More specifically, they offer the following policies, based on their observation of the types of the agents. We denote these complete information policies as follows: $\{(p_E^*(\mathbf{a}), c_E^*); (p_U^*, c_U^*)\}$.

Unexposed policy. Unexposed agents are offered full coverage. In the event of a loss, the insurer will cover the totality of the loss $c_U^* = L$. They are charged a fair premium, equal to the expected loss: $p_U^* = \lambda L$.

Exposed policy. Similarly, exposed agents receive full coverage, $c_E^* = L$. They are charged a fair premium $p_E^*(\mathbf{a}) = \lambda_E(\mathbf{a})L$, which depends on the investment strategy of the insurers. Note that the premium paid by exposed agents increases when insurers choose risky investment strategies, as these raise the probability of loss.

3.2 Complete Information Investment Strategies

Now that the insurance policies offered under complete information are described, the investment strategies to which insurers commit ex-ante can be determined. The following proposition formalizes the equilibrium outcome.

Proposition 1 (Investment strategies under complete information)**.** *Suppose insurers observe agent types and consequently offer full insurance and fair premiums, as described in section 3.1. In equilibrium, both insurers commit to risky investment strategies, such that:*

$$a_1^* = a_2^* = R.$$

6. Although insurers commit to an investment strategy beforehand, we assume that they do not anticipate future financial returns when setting insurance premiums. This reflects common regulations on these markets. For instance, US state-level rate-filing rules do not permit premiums to be discounted by the full expected return on assets (NAIC, 2024), and Solvency II in the EU requires premiums to be based on expected claims rather than speculative returns (EIOPA, 2015). These constraints rule out systematic under-pricing in anticipation of investment gains.

7. More precisely, the expected utility of a type-θ agent from getting an insurance providing coverage c at a fair premium is strictly increasing in coverage up to the maximum feasible $c = L$: $\frac{\partial \mathcal{U}_\theta(\cdot)}{\partial c} = \lambda_\theta(\cdot)(1 - \lambda_\theta(\cdot))[u'(W - L + (1 - \lambda_\theta(\cdot))c) - u'(W - \lambda_\theta(\cdot)c] > 0 \; \forall \theta = E, U$. Appendix 3.A formally addresses this point.

For each insurer i, given any investment strategy chosen by the other insurer $(-i)$, the expected profits earned with the risky investment strategy are higher than the expected profits with the safe investment strategy, given the complete information insurance policies $\{(p_E^*(\mathbf{a}), c_E^*); (p_U^*, c_U^*)\}$. This translates to

$$\pi_i(p_U^*, p_E^*(\mathbf{a}); R)\Big|_{a_{-i}} > \pi_i(p_U^*, p_E^*(\mathbf{a}); S)\Big|_{a_{-i}},$$

where $\pi_i(p_U^*, p_E^*(\mathbf{a}); R)\Big|_{a_{-i}} = \frac{1}{2} r_H\Big(\alpha L\Big(\lambda + \beta_i + \beta(a_{-i})\Big) + (1-\alpha)\lambda L\Big)$, and $\pi_i(p_U^*, p_E^*(\mathbf{a}); S)\Big|_{a_{-i}} = \frac{1}{2} r_L\Big(\alpha L\Big(\lambda + \beta(a_{-i})\Big) + (1-\alpha)\lambda L\Big)$.

The risky investment strategy is preferred by insurers for two main reasons. First, a risky investment increases the probability of loss faced by exposed agents, which translates to higher premiums charged on these agents, and therefore more revenues available for investments on the financial market. And secondly, the risky investment has a higher return. These two dimensions create perverse incentives for insurers, who are willing to strategically contribute to increasing the risk probability for some agents. As a result of risky investment strategies, the probability of loss faced by exposed agents is the highest and equal to $\lambda_E(R, R) = \lambda + \beta_1 + \beta_2$, resulting in a high premium paid by these agents to get insured, where $p_E^*(R, R) = (\lambda + \beta_1 + \beta_2)L$. Expected profits from returns on investments are equal to $\pi_i(p_U^*, p_E^*(R, R); R) = \frac{1}{2} r_H L\Big(\alpha(\beta_1 + \beta_2) + \lambda\Big)$ for each insurer.

A note on mandatory pooling. Notice that the implementation of mandatory pooling by a regulator does not change the investment strategy choices of insurers in complete information and cannot constitute an improvement. Indeed, if insurers are required to offer the same contract to all agents, then competition on the insurance market leads to offer full insurance at a premium $p^{\text{pooling}}(\mathbf{a}) = \alpha\lambda_E(\mathbf{a})L + (1-\alpha)\lambda L$. For insurers, the total volume of premiums collected is identical to the situation where they can adjust the premium to the agent's type. Therefore, their expected profits in the financial market and the incentives they face when committing to an investment strategy remain the same. Both insurers still commit to the risky investment strategy in equilibrium. On the agents' side, unexposed agents subsidize part of the exposed agents' risk, and are therefore worse-off.

4 Insurers' Strategic Decisions under Incomplete Information about Agent Risk Types

Assume now that insurers do not know whether an agent is exposed or unexposed. Therefore, insurers cannot directly infer the probability of loss faced by this agent. In this case, they must offer a menu of insurance policies and let agents self-select their preferred policy when they get insured. As mentioned in the introduction, there are several reasons for the unobservability of risk, specifically in the climate risk scenario. Indeed, agents can hold private information about micro-level characteristics such as maintenance, building materials and the implementation of protective measures; but also detailed information on their specific location that is inaccessible (or too costly to verify) to insurers. Intuitively, the presence of incomplete information is exacerbated by risky investment strategies, and there is no informational asymmetry when both insurers choose the safe investment strategy. This section therefore investigates how incomplete information changes the design of insurance policies in equilibrium, and how these distortions, in turn, feed back into insurers' incentives to commit to either safe or risky investment strategies.

First of all, note that if an insurer offers the complete information insurance policies described in section 3.1, exposed agents will mimic the behavior of unexposed types by picking the (p_U^*, c_U^*) insurance policy. Indeed, by not revealing their risk type, exposed agents manage to get fully insured for a lower premium, with

$$U_E(p_U^*, c_U^*) = u(W - \lambda L) > U_E(p_E^*(R,R), c_E^*) = u(W - (\lambda + \beta_1 + \beta_2)L).$$

Consequently, an insurer has to design and offer an incentive-compatible menu of insurance policies to distinguish exposed from unexposed agents.

4.1 Solving for the Least-Cost Separating Insurance Policies

The following subsection characterizes the least-cost separating allocation. An insurer must offer insurance policies $\{(\widehat{p_E}(\mathbf{a}), \widehat{c_E}(\mathbf{a})); (\widehat{p_U}(\mathbf{a}), \widehat{c_U}(\mathbf{a}))\}$ while ensuring non-

negative profits and satisfying simultaneously the following constraints:

$$\lambda_E(\mathbf{a})\, u(W - L + \widehat{c_E}(\mathbf{a}) - \widehat{p_E}(\mathbf{a})) + (1 - \lambda_E(\mathbf{a}))\, u(W - \widehat{p_E}(\mathbf{a}))$$
$$\geq \lambda_E(\mathbf{a})\, u(W - L + \widehat{c_U}(\mathbf{a}) - \widehat{p_U}(\mathbf{a})) + (1 - \lambda_E(\mathbf{a}))\, u(W - \widehat{p_U}(\mathbf{a})) \quad (1)$$

$$\lambda u(W - L + \widehat{c_U}(\mathbf{a}) - \widehat{p_U}(\mathbf{a})) + (1 - \lambda) u(W - \widehat{p_U}(\mathbf{a}))$$
$$\geq \lambda u(W - L + \widehat{c_E}(\mathbf{a}) - \widehat{p_E}(\mathbf{a})) + (1 - \lambda) u(W - \widehat{p_E}(\mathbf{a})) \quad (2)$$

$$\lambda_E(\mathbf{a})\, u(W - L + \widehat{c_E}(\mathbf{a}) - \widehat{p_E}(\mathbf{a})) + (1 - \lambda_E(\mathbf{a}))\, u(W - \widehat{p_E}(\mathbf{a}))$$
$$\geq \lambda_E(\mathbf{a})\, u(W - L) + (1 - \lambda_E(\mathbf{a}))\, u(W) \quad (3)$$

$$\lambda u(W - L + \widehat{c_U}(\mathbf{a}) - \widehat{p_U}(\mathbf{a})) + (1 - \lambda) u(W - \widehat{p_U}(\mathbf{a})) \geq \lambda u(W - L) + (1 - \lambda) u(W) \quad (4)$$

Condition 1 ensures that an exposed agent prefers choosing the insurance policy $(\widehat{p_E}(\mathbf{a}), \widehat{c_E}(\mathbf{a}))$ designed for its risk type rather than mimicking the unexposed agents. Condition 2 does the same for unexposed agents. Conditions 3 and 4 ensure that each type of agent prefers getting insured over having no insurance.

As in Rothschild and Stiglitz (1976), the solution to this problem implies no pooling contracts. The following lemma summarizes the menu of insurance policies chosen by an insurer under incomplete information.

Proposition 2 (Least-cost separating insurance policies)**.** *Suppose that insurers cannot observe agent types, and at least one insurer chooses the risky investment strategy (i.e. exposed and unexposed agents face different probabilities of loss). The separating insurance policies, when they exist, are described below.*

- *For exposed types, full insurance at a fair premium:* $\widehat{c_E} = L$, $\widehat{p_E}(\mathbf{a}) = (\lambda + \beta(a_1) + \beta(a_2))L$.
- *For unexposed types, partial insurance at a fair premium:* $\widehat{c_U}(\mathbf{a}) < L$, $\widehat{p_U} = \lambda \widehat{c_U}(\mathbf{a})$, *where the distortion away from full coverage increases with the number of insurers choosing the risky investment strategy.*

When both insurers choose the safe investment strategy, all agents in the population face probability of loss λ*, and insurance policies are identical to the complete information case.*

Proof. See Appendix 3.A. □

In incomplete information, the insurance policy offered to exposed agents remains unchanged compared to the full information case. However, full insurance is no longer attainable for unexposed agents: insurers have to distort coverage downwards for these agents to ensure separation. More specifically, coverage has to be distorted below full coverage to the extent where exposed agents are indifferent between picking the insurance policy designed for their risk type or mimicking unexposed agents (formally, their incentive compatibility constraint 1 binds). Although the premium paid by unexposed agents is lower than the premium offered to exposed agents, the distortion in coverage makes the unexposed policy non-attractive to exposed agents.

Technical details. Different market environments can support this allocation as the unique market equilibrium allocation. For instance, Azevedo and Gottlieb (2017) demonstrate that in a competitive market with adverse selection, the least-cost separating allocation constitutes the unique equilibrium allocation. Alternatively, a signaling game in which consumers offer a contract and insurers accept or reject could be considered. Applying the Intuitive Criterion (Cho & Kreps, 1987) in such framework yields the least-cost separating allocation as the unique equilibrium allocation. Another formulation would consider a game in which insurers simultaneously compete in contracts. However, in this setting, it is well established that a pure-strategy equilibrium fails to exist for some parameter values, and will have to rely on alternative games (Wilson, 1977; Miyazaki, 1977; Riley, 1979; Picard, 2014; Mimra & Wambach, 2014; Dosis, 2018, 2019, 2022).[8]

4.2 Coverage Distortions

The extent of the distortion on unexposed coverage depends on the investment strategies chosen by insurers. The following analysis describes in further details how the separating insurance policies change with different investment strategy profiles. The question of the existence of such profiles in equilibrium will be addressed in the

8. Additional robustness and insights would be added to this paper if other kinds of allocations were considered. One possibility is the Miyazaki-Wilson-Spence allocation, which maximizes the payoff of the low-risk type and breaks even. This coincides with the standard least-cost separating allocation when the low-risk share is small, but may differ if it is large. Another approach would consider Pareto-optimal allocations.

next section.

Risky-risky strategy profile. Coverage offered to unexposed agents is the most significantly distorted (*i.e.* the lowest) when both insurers choose risky investment strategies. Indeed, when this is the case, the probability of loss faced by exposed agents $\lambda_E(R,R)$ is the highest compared to unexposed agents. Therefore, exposed agents' incentives to cheat to be perceived as low risk are strong, and insurers must offer an unexposed insurance policy with lower partial coverage to make it unattractive to exposed agents. Note that, quite clearly, the extent of the distortion on coverage for unexposed agents depends on the size of $\beta_i \ \forall i$, that is on the extent of the impact of an insurer choosing the risky investment strategy on the probability of loss for exposed agents (sometimes referred to as the exposure parameter). The higher the impact, the more distorted the insurance contract must be to satisfy incentive compatibility.

Split-strategy profile. The downward distortion required on unexposed coverage with the Risky-risky profile can be somewhat attenuated when only one insurer chooses the risky investment strategy. This situation where insurers choose different investment strategies will be referred to as split-strategy profiles, covering two distinct cases: $\mathbf{a} = (R,S)$ and $\mathbf{a} = (S,R)$. Under this profile, $\widehat{c_U}(\cdot)$ is less distorted than in the previous case as $\lambda_E(R,R) > \lambda_E(R,S), \lambda_E(S,R)$. If both insurers have the same impact of a risky investment strategy on exposed agents' probability of loss ($\beta_1 = \beta_2 = \beta$), then we will have $\widehat{c_U}(R,S) = \widehat{c_U}(S,R) \in]\widehat{c_U}(R,R), L[$. That is, the partial coverage offered to unexposed agents in incomplete information will be identical regardless of which insurer adopts the risky strategy, and the distortion below full coverage will be less severe than with two risky investments. However, when the impacts of risky investment are heterogeneous among insurers, the distortion is more severe when it is the insurer with the highest β_i that chooses the risky investment strategy. For instance, assuming that $\beta_1 > \beta_2$, then $\widehat{c_U}(R,S) < \widehat{c_U}(S,R)$, with both still within the range $]\widehat{c_U}(R,R), L[$.

Safe-safe strategy profile. When both insurers choose the safe investment strategy, informational asymmetries vanish: all agents in the population face the same probability of loss equal to λ. Therefore, there is no need for insurers to alter the complete

information insurance policies, given by full coverage ($\widehat{c_U}(S,S) = c_U^* = c_E^* = L$) at fair premium ($p_U^* = p_E^* = \lambda L$).

Figure 3.2 provides a simulation that displays the incentive-compatible unexposed partial coverage $\widehat{c_U}(\mathbf{a})$ for each strategy profile $\mathbf{a}$, with $\lambda = 0.4$, $\beta_1 = 0.15$, $\beta_2 = 0.1$, $W = 100$, $L = 50$ and for a log-utility function.

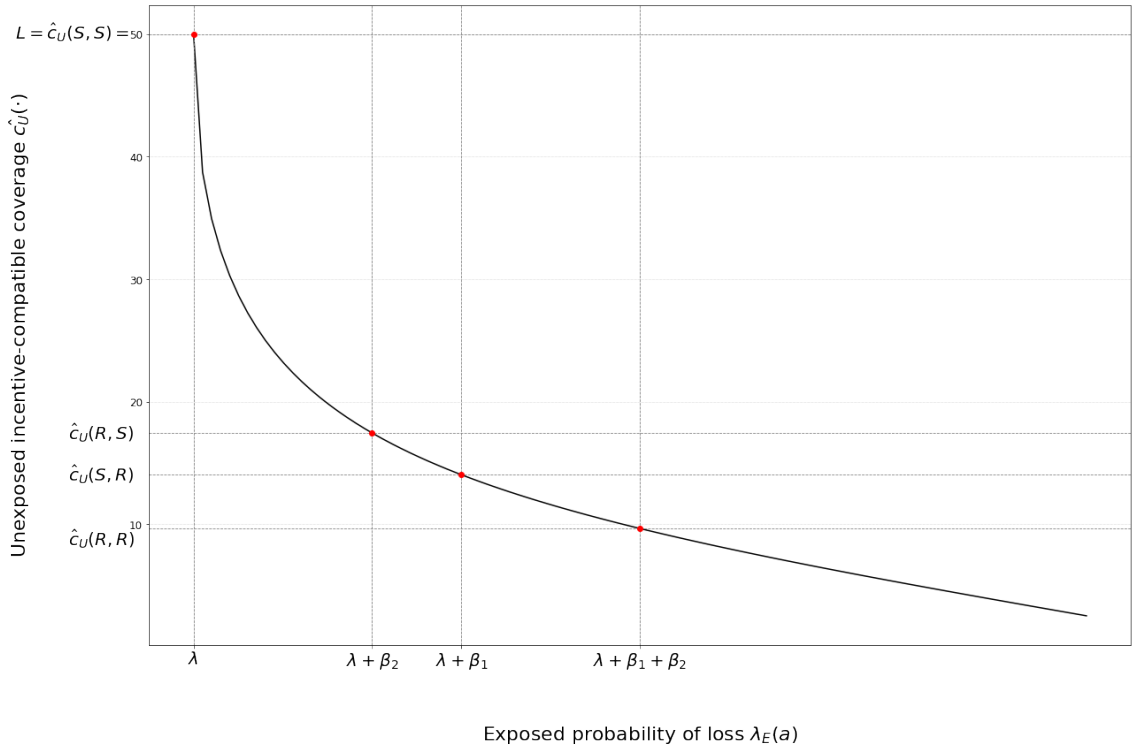


Figure 3.2 – *Simulation of incentive-compatible unexposed coverage for each investment strategy profile*

Distortions on the insurance policy offered to unexposed agents results in efficiency loss, as stated in the following proposition.

Proposition 3. *The equilibrium incentive-compatible insurance policies described in proposition 2 entail a two-fold loss of efficiency compared to the complete information equilibrium: under-insurance of unexposed agents and a loss of expected profits for insurers.*

Indeed, consistent with the conclusions of Rothschild and Stiglitz (1976), unexposed agents are under-insured (and some unexposed agents may choose to remain uninsured altogether), which is sub-optimal considering they are risk-averse while the insurers are risk-neutral.

Second, insurers experience a loss in expected profits from the financial activity: as the coverage offered to unexposed agents decreases, the premiums collected through these contracts are lower than under complete information. This diminishes the volume of investments and therefore lowers financial revenues. More precisely, the expected proifts of an insurer committed to any investment strategy a_i is given by $\pi_i(\widehat{p_U}(\mathbf{a}), \widehat{p_E}, \widehat{c_U}(\mathbf{a}), \widehat{c_E}; a_i) = \frac{1}{2} r_{a_i}[\alpha L(\lambda + \beta(a_1) + \beta(a_2)) + (1-\alpha)\lambda \widehat{c_U}(\mathbf{a})]$. Of course, as $\widehat{c_U}(\mathbf{a}) \leq L$, the expected profits under incomplete information are lower than the complete information expected profits. This dimension of inefficiency highlights a novel perspective, emerging from the interplay between the insurance and financial activities.

Intuitively, the efficiency loss characterized in proposition 3 can be attenuated by choosing safe investment strategies over risky ones, as this reduces the degree of distortion required on coverage for unexposed agents to achieve incentive compatibility. In other words, if this distortion it too severe, insurers may strategically choose the safe investment strategy in incomplete information. Indeed, in this case, keeping the risky investment strategy would result in too little revenues available for investment, while the safe strategy (although less profitable) allows to offer the best possible contracts to all agents. The following section investigates whether and in what cases this situation may arise.

4.3 Investment Strategies with Identical Insurers under Incomplete Information

Under incomplete information, a key question is whether insurers continue committing to the risky investment strategy and offer the corresponding incentive-compatible menu of insurance policies, or instead switch to the safe investment. Intuitively, committing to the safe investment allows an insurer to modify the distribution of risk in the population of agents, increasing the share of unexposed agents relative to exposed agents. This strategic decision affects not only the insurer's financial returns, but also the insurance market outcome – influencing both the policies offered and the realized profits. Table 3.2 summarizes the profits made by insurers for each strategy profile under incomplete information. At Stage 1, each insurer commits to an investment

strategy by comparing the profits of each option, given the strategy chosen by their competitor. That is, $\forall\, i = 1, 2$, the relationships of interest are

$$\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\Big|_{a_{-i}=S} \lesseqgtr \pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\Big|_{a_{-i}=S}$$
$$\text{and} \quad \pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\Big|_{a_{-i}=R} \lesseqgtr \pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\Big|_{a_{-i}=R} \quad .$$

		Insurer 1's investment strategy a_1	
		S	R
a_2	S	$\pi_1(\cdot) = \pi_2(\cdot) = \frac{1}{2} r_L \lambda L$	$\pi_1(\cdot) = \frac{1}{2} r_H \Big[\alpha(\lambda + \beta_1)L + (1-\alpha)\lambda \widehat{c_U}(R, S)\Big]$ $\pi_2(\cdot) = \frac{1}{2} r_L \Big[\lambda + \beta_1)L + (1-\alpha)\lambda \widehat{c_U}(R, S)\Big]$
	R	$\pi_1(\cdot) = \frac{1}{2} r_L \Big[\alpha(\lambda + \beta_2)L + (1-\alpha)\lambda \widehat{c_U}(S, R)\Big]$ $\pi_2(\cdot) = \frac{1}{2} r_H \Big[\alpha(\lambda + \beta_2)L + (1-\alpha)\lambda \widehat{c_U}(S, R)\Big]$	$\pi_1(\cdot) = \pi_2(\cdot) = \frac{1}{2} r_H \Big[\alpha(\lambda + \beta_1 + \beta_2)L + (1-\alpha)\lambda \widehat{c_U}(R, R)\Big]$

Table 3.2 – *Insurers' profits by strategy profile under incomplete information*

The present subsection characterizes the resulting pure strategy Nash equilibrium (or equilibria).[9] In particular, in some cases, the safe-safe strategy profile may be an equilibrium under incomplete information, contrary to complete information. However, in some cases, it may not be the unique equilibrium and coexist with other strategy profiles, or even fail to be an equilibrium altogether.

Assume that insurers have an identical impact on the probability of loss faced by exposed agents when they choose the risky investment strategy, *i.e.* $\beta_1 = \beta_2 = \beta$. This simplification is useful because insurers face the same incentives, and this reduces the set of pure-strategy Nash equilibria while providing the main intuitions for the emergence of certain outcomes. In this case, there are two conditions that govern the strategic decisions of both insurers:

$$r_L \lambda L \geq r_H \Big(\alpha(\lambda + \beta)L + (1-\alpha)\lambda \widehat{c_U}(S, R)\Big), \tag{5}$$
$$\text{and} \quad r_L \Big(\alpha(\lambda + \beta)L + (1-\alpha)\lambda \widehat{c_U}(S, R)\Big) \geq r_H \Big(\alpha(\lambda + 2\beta)L + (1-\alpha)\lambda \widehat{c_U}(R, R)\Big), \tag{6}$$

where $\widehat{c_U}(S, R) = \widehat{c_U}(R, S)$. If condition (5) is satisfied, an insurer prefers to commit to the safe investment strategy if its competitor also commits to the safe investment. If condition (6) is satisfied, an insurer prefers to commit to the safe investment strategy

9. Mixed strategy equilibria are not discussed in this paper.

even if its competitor commits to the risky investment.

Denote by $\widetilde{\alpha_5}$ and $\widetilde{\alpha_6}$ the values of α that make an insurer indifferent between the two investment strategies when its competitor commits to the safe investment, and when its competitor commits to the risky investment, respectively. Equivalently, these are the threshold proportions of exposed agents in the population that satisfy conditions (5) and (6) with equality. They are given by:

$$\widetilde{\alpha_5} = 1 - \frac{L(\Delta r\lambda + r_H\beta)}{r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)}$$

$$\text{and} \quad \widetilde{\alpha_6} = 1 - \frac{L\Big(\Delta r(\lambda+\beta) + r_H\beta\Big)}{\Delta rL(\lambda+\beta) + r_H\beta L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)}.$$

The following proposition describes the four equilibrium outcomes that may exist and characterizes the conditions under which each arises.

Proposition 4 (Investment strategies under incomplete information with identical insurers)**.** *Consider the investment strategy choice of insurers, and suppose that they cannot observe agent types. Let $\beta_1 = \beta_2 = \beta > 0$ and let the incentive-compatible menu of insurance policies be as described by proposition 2. Then, the different possible pure strategy Nash equilibria are as follows.*

1. ***Safe dominance.*** *(S,S) is the unique equilibrium if both (5) and (6) are satisfied. This is the case when $\alpha \in [0, \min\{\widetilde{\alpha_5}, \widetilde{\alpha_6}\}]$.*
2. ***Risky dominance.*** *(R,R) is the unique equilibrium if both (5) and (6) are violated. This occurs when $\alpha > \max\{\widetilde{\alpha_5}, \widetilde{\alpha_6}\}$.*
3. ***Coordination.*** *(S,S) and (R,R) strategy profiles coexist in equilibrium if (5) is satisfied but (6) is violated. This is the case when $\alpha \in]\widetilde{\alpha_6}, \widetilde{\alpha_5}]$.*
4. ***Split-strategy.*** *(S,R) and (R,S) are symmetric equilibria if (5) is violated but (6) is satisfied. This is the case when $\alpha \in]\widetilde{\alpha_5}, \widetilde{\alpha_6}]$.*

Proof. See Appendix 3.B. □

Figure 3.3 provides an illustration of each outcome described by proposition 4. To further understand the intuitions behind each outcome further, note that conditions

(5) and (6) capture the essential trade-off insurers face when choosing their investment strategies. On the one hand, the risky investment strategy is attractive because it yields a higher return, and the premium offered to exposed agents is higher, as their probability of loss is larger. However, it tightens the incentive compatibility constraint of exposed agents, which forces insurers to reduce the coverage offered to unexposed agents $\widehat{c_U}(\mathbf{a})$. This reduction lowers the volume of premiums collected and therefore shrinks the funds available for investment on the financial market. Conversely, a safe investment strategy maintains a larger insurance base, but yields lower return. The equilibrium outcome, among those described in proposition 4, depends on which of these forces dominates. It is notably the values of the model parameters that determine which of these forces dominates.

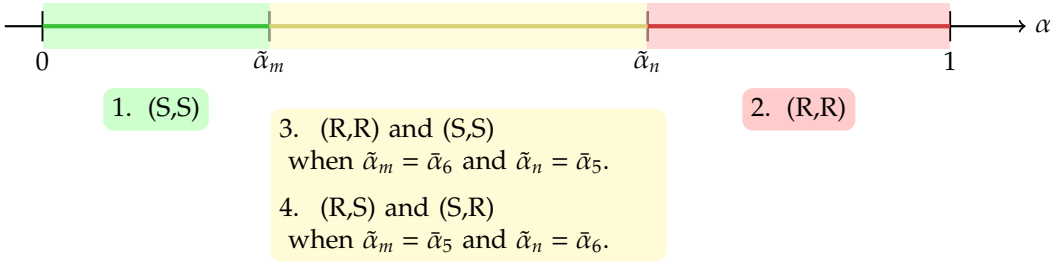


Figure 3.3 – *Illustration of equilibria in terms of exposed agents' proportion α*

Safe dominance. For the (S, S) equilibrium to be unique, the above-mentioned gains from a risky investment strategy must be small enough and/or the distortions on unexposed coverage large enough.

This configuration is more likely to arise when the population consists of a wide unexposed base (small α, as given by proposition 4, provided that the upper bound is positive[10]). In that case, insurers would mainly sell contracts to unexposed types, which under risky investment would be heavily distorted, while only a small fraction of exposed agents would purchase policies at higher premiums.

The existence of such equilibrium is favored when the risky return premium $\Delta r = r_H - r_L$ is low, making the risky investment only marginally more profitable compared to the safe investment.

10. This requires $r_L L > r_H \widehat{c_U}(S, R)$ and $r_L \widehat{c_U}(S, R) > r_H \widehat{c_U}(R, R)$. Only one of these conditions will be relevant, but it is a priori not possible to state which one.

Of course, the safe investment strategies are more attractive to insurers whenever the distortions on insurance policies required to achieve incentive compatibility with risky investment strategies become too severe. The degree of risk aversion plays a role in the magnitude of this distortion. Because more risk-averse agents place a higher value on coverage, insurers must significantly reduce unexposed coverage to separate types in this case. Consequently, higher risk aversion increases the likelihood of a unique (S, S) equilibrium emerging. Another relevant factor is the size of loss L relative to initial wealth W. A larger loss increases the marginal value of insurance coverage, and therefore amplifies the required distortion. This favors the emergence of safe investment strategies in equilibrium.

A variation in the exposure parameter β has different effects on the incentive to choose the safe investment strategy for insurers. First a *premium effect*: as β increases, so does the premium that insurers with risky investment strategies can offer to exposed agents. This raises their insurance revenues, and therefore the volume of investment made in financial markets, making the risky investment strategy more attractive. On the other hand, an increase in β raises the probability of loss for exposed agents and creates a *distortion effect*: their incentive compatibility constraint becomes harder to satisfy, and this requires more severe distortions on unexposed coverage $\widehat{c_U}(\mathbf{a})$. This effect improves the attractiveness of the safe investment strategy, or equivalently, expands the range of values of α that allows for this equilibrium to emerge. Which one of these two effects dominates depends on the sensitivity of unexposed coverage distortions to variations in β.[11]. To summarize, if the separating unexposed coverages $\widehat{c_U}(S, R)$ and $\widehat{c_U}(R, R)$ are very responsive to changes in the exposure parameter β, then the distortion effect dominates and higher values of β facilitate the emergence of the unique (S, S) equilibrium. On the contrary, if $\widehat{c_U}(\mathbf{a})$ is fairly inelastic with respect to changes in β, the premium effect dominates and lower values of β favor the (S, S) equilibrium.

Together, these elements reduce the relative attractiveness of risky investment and make the safe strategy dominant.

11. See Appendix 3.C for a more formal development.

Risky dominance. The (R, R) equilibrium dominates whenever the gains from committing to the risky investment strategy outweigh the losses caused by distortions. The arguments follow the opposite logic to the unique (S, S) case.

Therefore, it is most likely to arise when there is a large proportion of exposed agents in the population (α large enough, as given by proposition 4), to which insurers sell non-distorted policies with high premiums reflecting their risk type.

This is also facilitated by a relatively high return from the risky investment compared to the safe investment (Δr high), making financial returns profitable enough regardless of distortions.

The occurrence of such an equilibrium increases when distortions on insurance policies are mild. This can be the case when agents are not very risk-averse.

In parallel with the previous case, if the distortion effect tends to dominate when β grows, then low values of the exposure parameter β facilitate the emergence of this equilibrium. On the other hand, if the premium effect dominates, then high values of β allow insurers to collect higher premiums from exposed agents and therefore increase the profitability of the risky investment.

Coordination. The (S, S) and (R, R) equilibria coexist when it is in each insurer's interest to imitate their competitor.

This coexistence arises for intermediate values of α (as long as $\widetilde{\alpha_5}$ is strictly positive [12]) and when the split-strategy profile is never profitable for insurers (that is when $\widetilde{\alpha_5} > \widetilde{\alpha_6}$). Intuitively, such coordination happens when the unexposed coverage offered under the split profile is far below full insurance (large gap between L and $\widehat{c_U}(S, R)$), but the additional distortion from moving from the split profile to the risky-risky profile is mild (smaller gap between $\widehat{c_U}(S, R)$ and $\widehat{c_U}(R, R)$). [13] The first dimension implies that aligning on the safe investment strategy allows to avoid excessive coverage distortions, but the second dimension entails that once an insurer chooses the risky investment, it is attractive for the other one to align on risky to reap the higher return without creating too much additional distortions. This is the case under standard assumptions on agent preferences. Indeed, usual curvature conditions involve prudence, that is

12. This requires $r_L L > r_H \widehat{c_U}(S, R)$.
13. See Appendix 3.D.1 for further details.

$u'''(\cdot) > 0$ (Kimball, 1990), which is verified for most CRRA and CARA utility functions. Prudence entails that agents value an additional unit of coverage more and more as the risk increases. Therefore, exposed agents become more sensitive to coverage variations when their probability of loss increases in $[\lambda, \lambda + 2\beta]$. In other words, their incentive to cheat gets weaker as $\lambda_E(\mathbf{a})$ increases. To sum up, in standard cases, unexposed separating coverage decreases at a decreasing rate with the probability of loss of exposed agents, *i.e.* with the number of insurers committing to the risky investment strategy, as illustrated by figure 3.2.

The coordination equilibria arise for intermediate Δr: the risky investment must be sufficiently profitable for insurers to tolerate distortions when both commit to the risky investment strategy; while the safe investment must remain relatively profitable that insurers prefer to coordinate on safe and avoid distortions rather than deviating to risky.

Ultimately, when this outcome arises, the final investment strategies chosen by insurers depend on their expectations. If they expect caution, they will coordinate on the safe investment. If they expect risk-taking, they coordinate on the risky investment.

Split-strategy. The pair of split-strategy equilibria (R, S) and (S, R) arises whenever it is profitable for an insurer to choose the risky investment strategy when its competitor commits to the safe strategy, so that both insurers benefit from moderately high premiums offered to exposed agents and one insurer from higher return. Yet, jointly choosing the risky investment depresses the unexposed policy so severely that the gains from r_H and even higher premiums for exposed agents cannot compensate such a loss, and it is better for one insurer to switch back to the safe investment. In other words, the split-strategy outcome requires enough gain from unilateral risky investment choice but enough penalty from both insurers choosing the risky strategy.

Such equilibria also arise for intermediate values of α (as long as $\widetilde{\alpha_6}$ is strictly positive[14]) and when the coordination strategy is never profitable, contrary to the previous case (that is when $\widetilde{\alpha_6} > \widetilde{\alpha_5}$). This would occur when the unexposed coverage offered under the split profile is close to full (undistorted) insurance offered under

14. This requires $r_L \widehat{c_U}(S, R) > r_H \widehat{c_U}(R, R)$.

the safe-safe strategy profile (small gap between L and $\widehat{c_U}(S,R)$); while the additional distortion required to reach incentive compatibility when both insurers choose risky investments is large (wide gap between $\widehat{c_U}(S,R)$ and $\widehat{c_U}(R,R)$).[15] Therefore, when one insurer chooses the safe investment strategy, it is profitable for the other insurer to switch to the risky investment strategy to enjoy higher returns without creating much distortion on unexposed coverage. However, the first insurer should not coordinate on the risky investment as well because the incremental distortion on coverage is too severe, reducing the volume of investment excessively, while higher return on investment does not outweigh this loss. Under standard assumptions on agents' preferences, such a configuration is generally difficult to obtain. Indeed, usual curvature conditions for $u(\cdot)$ involve prudence. As a consequence, $\widehat{c_U}(\mathbf{a})$ is decreasing and convex in $\lambda_E(\mathbf{a})$, implying that the first distortion from L to $\widehat{c_U}(S,R)$ is large compared to $\widehat{c_U}(S,R) - \widehat{c_U}(R,R)$. The reverse ordering required to obtain a split-strategy profile would call for an anti-prudence assumption.

To summarize, an equilibrium in which both insurers commit to the safe investment strategy is attainable as long as condition (5) holds, but it is not necessarily the only equilibrium. This demonstrates that incentives involved in the investment strategy choice change relative to the complete information situation, due to the interaction between the risky investment strategy and the exposed probability of loss. This emerges from the fact that insurers face a trade-off between investment return and the degree of adverse selection. As they indirectly control the severity of adverse selection via their investment strategy, they might find it optimal to reduce it when it gets too costly.

Numerical simulations. Figure 3.4 displays the equilibrium investment strategies when insurers are identical ($\beta_1 = \beta_2 = 0.1$) across different values of $\alpha \in [0,1]$ and Δr, given $W = 200$, $L = 80$, $\lambda = 0.4$, $r_L = 0.9$, and for a log-utility function. This simulation illustrates the emergence of the safe-safe strategy profile in equilibrium (in green) when the proportion of unexposed agents is large and the difference in financial returns is low; as well as the prevalence of the risky-risky strategy profile (in red) for the opposite. It is

15. See Appendix 3.D.2 for further details.

also possible to see that coordination (in yellow) arises for intermediate values of α and Δr. Fixing $\Delta r = 0.25$ as an example, then $\widetilde{\alpha}_5 \approx 0.51$ and $\widetilde{\alpha}_6 \approx 0.08$, as plotted below. These give the cutoff proportions of exposed agents for each equilibrium outcome. Similarly, it is possible to reverse the exercise by fixing $\alpha = 0.2$ for instance. In this case, Δr must be lower than 0.082 for the (S, S) equilibrium to be unique. On the other hand, if Δr is larger than 0.939, the unique (R, R) equilibrium will arise.

Figures 3.5 and 3.6 illustrate how equilibrium investment strategies change with the size of potential loss L. Using the parameter values $W = 200$, $\lambda = 0.4$, $r_H = 1.1$, $r_L = 1$ and $\beta_1 = \beta_2 = 0.2$, figure 3.5 shows that as long as α is sufficiently low, an increase in L expands the range of α for which the (S, S) equilibrium emerges. For instance, when $\alpha = 0.1$, both insurers strictly prefer committing to the safe investment strategy once L exceeds approximately 55.3. This occurs because the distortions in coverage for unexposed agents intensify as L increases, as illustrated by figure 3.6. When distortions become too severe, the risky investment strategy becomes unattractive. As mentioned before, if α is not sufficiently low such that the condition for safe dominance stated in proposition 4 is not satisfied, then regardless of the size of L, the (S, S) equilibrium is never feasible. In that case, for intermediate values of α, low values of L support the coordination equilibrium, while large L favors risky dominance. For high values of α, the risky-risky equilibrium remains dominant across all L.

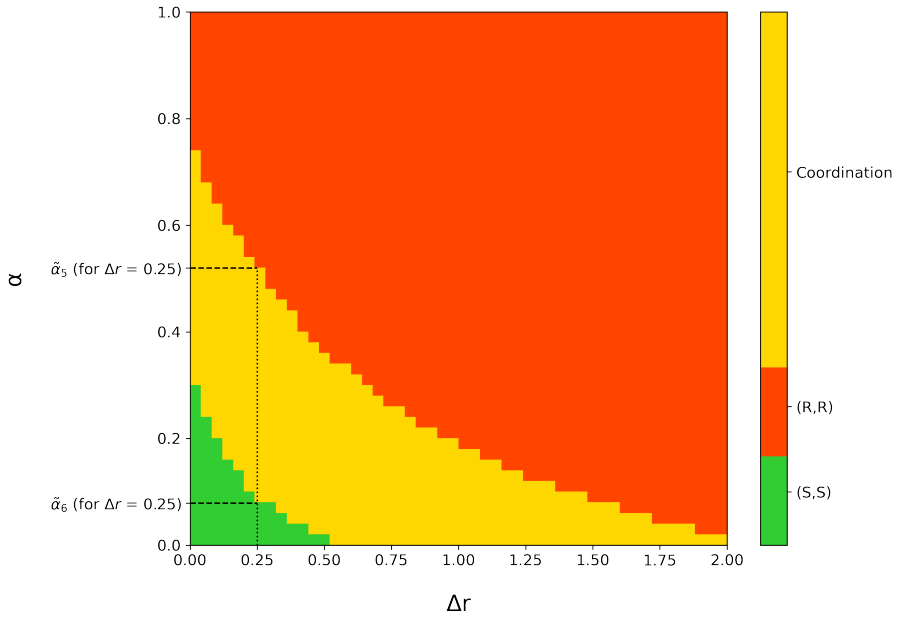


Figure 3.4 – *Equilibrium investment strategies across α and Δr*

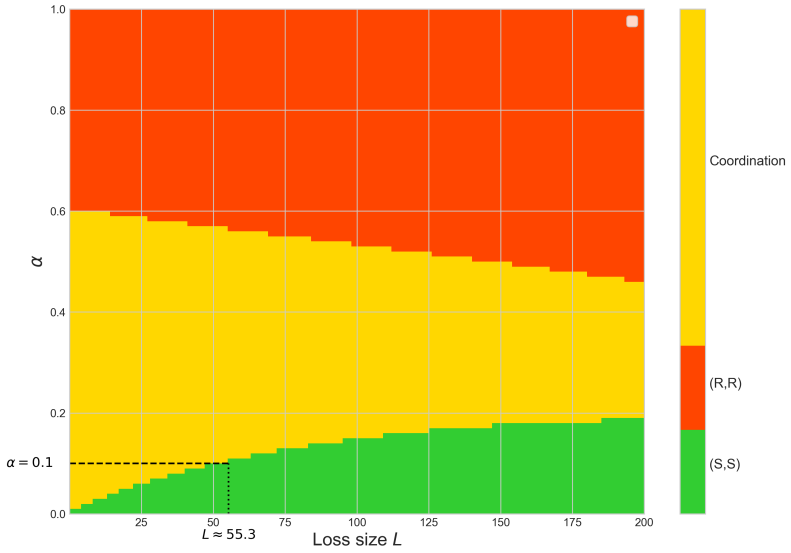


Figure 3.5 – *Equilibrium investment strategies across α and L*

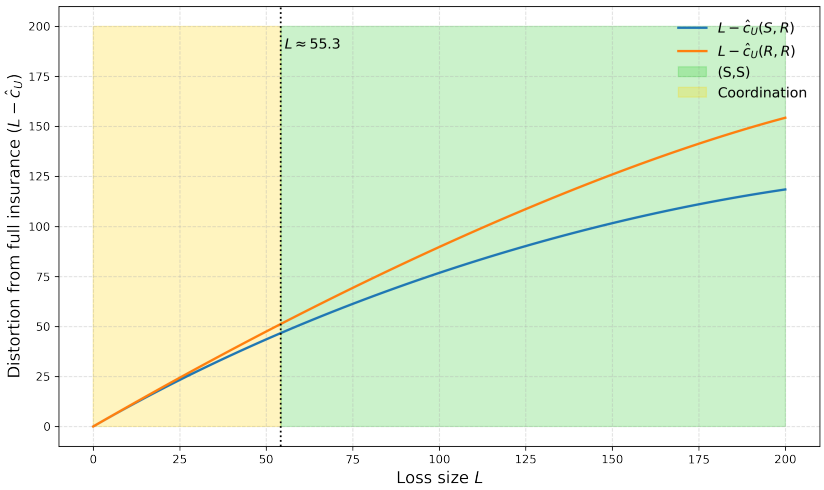


Figure 3.6 – *Distortions in unexposed coverage as a function of loss size L and equilibrium regions ($\alpha = 0.1$)*

5 Extensions

5.1 Investment Strategies with Asymmetric Insurers

The symmetric analysis makes clear the main forces at play. However, when $\beta_1 \neq \beta_2$, the two insurers may face different incentives regarding their choice of investment strategy, as their impact on exposed agents' probability of loss with risky investments

differ. There are four conditions that govern the strategic decisions of insurers:

$$r_L\lambda L \geq r_H\Big(\alpha(\lambda+\beta_1)L + (1-\alpha)\lambda\widehat{c_U}(R,S)\Big), \tag{7}$$

$$r_L\lambda L \geq r_H\Big(\alpha(\lambda+\beta_2)L + (1-\alpha)\lambda\widehat{c_U}(S,R)\Big), \tag{8}$$

$$r_L\Big(\alpha(\lambda+\beta_2)L + (1-\alpha)\lambda\widehat{c_U}(S,R)\Big) \geq r_H\Big(\alpha(\lambda+\beta_1+\beta_2)L + (1-\alpha)\lambda\widehat{c_U}(R,R)\Big), \tag{9}$$

$$\text{and } r_L\Big(\alpha(\lambda+\beta_1)L + (1-\alpha)\lambda\widehat{c_U}(R,S)\Big) \geq r_H\Big(\alpha(\lambda+\beta_1+\beta_2)L + (1-\alpha)\lambda\widehat{c_U}(R,R)\Big), \tag{10}$$

If condition (7) (resp. condition (8)) is satisfied, insurer 1 (resp. insurer 2) prefers to commit to the safe investment strategy if its competitor commits to the safe investment. If condition (9) (resp. condition (10)) is satisfied, insurer 1 (resp. insurer 2) prefers to commit to the safe investment strategy even if its competitor commits to the risky investment.

Denote by $\widetilde{\alpha_7}$, $\widetilde{\alpha_8}$, $\widetilde{\alpha_9}$ and $\widetilde{\alpha_{10}}$ the values of α that satisfy each condition with equality. They are given by:

$$\widetilde{\alpha_7} = 1 - \frac{L(\Delta r\lambda + r_H\beta_1)}{r_H\Big(L(\lambda+\beta_1) - \lambda\widehat{c_U}(R,S)\Big)},$$

$$\widetilde{\alpha_8} = 1 - \frac{L(\Delta r\lambda + r_H\beta_2)}{r_H\Big(L(\lambda+\beta_2) - \lambda\widehat{c_U}(S,R)\Big)},$$

$$\widetilde{\alpha_9} = 1 - \frac{L\Big(\Delta r(\lambda+\beta_2) + r_H\beta_1\Big)}{\Delta rL(\lambda+\beta_2) + r_H\beta_1 L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)},$$

$$\text{and} \quad \widetilde{\alpha_{10}} = 1 - \frac{L\Big(\Delta r(\lambda+\beta_1) + r_H\beta_2\Big)}{\Delta rL(\lambda+\beta_1) + r_H\beta_2 L + \lambda\Big(r_L\widehat{c_U}(R,S) - r_H\widehat{c_U}(R,R)\Big)}.$$

In this more general case, the equilibrium set includes not only the four types of outcomes described in the previous proposition, but also the possibility of a unique asymmetric split-strategy equilibrium where only one insurer invests in the risky strategy. The following proposition summarizes these possibilities.

Proposition 5 (Investment strategies under incomplete information with asymmetric insurers)**.** *Consider the investment strategy choice of insurers, and suppose that they cannot observe agent types. Let* $\beta_1, \beta_2 > 0$ *and let the incentive-compatible menu of insurance policies*

be as described by proposition 2. Then, the different possible pure strategy Nash equilibria are as follows.

1. ***Safe dominance.*** *(S,S) is the unique equilibrium if both (7) and (8) are satisfied, and at least one condition among (9) and (10) is satisfied. This is the case when* $\alpha \leq \min\left\{\min\{\widetilde{\alpha_7}, \widetilde{\alpha_8}\}, \max\{\widetilde{\alpha_9}, \widetilde{\alpha_{10}}\}\right\}$.
2. ***Risky dominance.*** *(R,R) is the unique equilibrium if both (9) and (10) are violated, and at least one condition among (7) and (8) is violated. This is the case when* $\alpha > \max\left\{\min\{\widetilde{\alpha_7}, \widetilde{\alpha_8}\}, \max\{\widetilde{\alpha_9}, \widetilde{\alpha_{10}}\}\right\}$.
3. ***Coordination.*** *(S,S) and (R,R) strategy profiles coexist in equilibrium if (7) and (8) are satisfied and (9) and (10) are violated. This is the case when* $\alpha \in \left]\max\{\widetilde{\alpha_9}, \widetilde{\alpha_{10}}\}, \min\{\widetilde{\alpha_7}, \widetilde{\alpha_8}\}\right]$.
4. ***Symmetric split-strategy.*** *(S,R) and (R,S) are symmetric equilibria if (7) and (8) are violated, but (9) and (10) are satisfied. This is the case when* $\alpha \in]\max\{\widetilde{\alpha_7}, \widetilde{\alpha_8}\}, \min\{\widetilde{\alpha_9}, \widetilde{\alpha_{10}}\}]$.
5. ***Asymmetric split-strategy.***
   - *(R,S) is the unique equilibrium if (7) is violated, (10) is satisfied, and at least one condition for the (S,R) equilibrium is not met: (8) satisfied and/or (9) violated. This is the case when* $\alpha \in]\widetilde{\alpha_7}, \widetilde{\alpha_{10}}]$ *and* $\alpha \leq \widetilde{\alpha_8}$ *and/or* $\alpha > \widetilde{\alpha_9}$.
   - *(S,R) is the unique equilibrium if (9) is satisfied, (8) is violated, and at least one condition for the (R,S) equilibrium is not met: (7) satisfied and/or (10) violated. This is the case when* $\alpha \in]\widetilde{\alpha_8}, \widetilde{\alpha_9}]$ *and* $\alpha \leq \widetilde{\alpha_7}$ *and/or* $\alpha > \widetilde{\alpha_{10}}$.

It is possible that no pure strategy Nash equilibrium exists. This is the case when:

(i) Conditions (7) and (10) are satisfied but conditions (8) and (9) are violated;

(ii) Conditions (7) and (10) are violated but conditions (8) and (9) are satisfied.

Proof. See Appendix 3.E. □

The complete analysis of each outcome would be extensive and would partly replicate the intuitions developed in the symmetric setting. We focus on the main insights underlying the emergence of the asymmetric split-strategy equilibrium, which may arise only when insurers differ.

The asymmetric split-strategy equilibrium arises for intermediate values of α, so that the safe-safe and risky-risky equilibria do not strictly dominate; and moderate values of

Δr.

The insurer whose investment has the larger impact β_i on exposed agents' probability of loss faces a stronger trade-off, both in terms of the premium and the distortion effects. Indeed, for this insurer, choosing the risky investement strategy significantly increases the exposed premium through $\lambda_E(\mathbf{a})$, allowing for the investment of a large volume on financial markets, while benefiting from higher return. However, a high value of β_i also implies that when this insurer chooses the risky investment strategy, the distortion on unexposed coverage must be severe, inducing a loss in profits in this dimension. The insurer with higher β_i is the one who is more likely to choose the safe investment strategy unilaterally if the distortion effect dominates the premium effect. If this is true, greater heterogeneity among insurers makes this outcomes more likely to happen.

5.2 Mandatory Full Coverage and Fair Premiums

Suppose there exists a regulator in this framework, who seeks to enforce a minimum level of coverage to be guaranteed by insurers, at fair premiums. In the extreme case, a regulator would impose the provision of full coverage for all agents. We are interested in the impact of such regulation on the investment strategy choices of insurers in equilibrium, under incomplete information about agents' probabilities of loss.

When coverage is fixed at $c_U^* = c_E^* = L$, insurers who compete in the insurance activity would like to offer fair premiums to each type of agent: $p_U^* = \lambda L$ to unexposed agents, and $p_E^*(\mathbf{a}) = \lambda_E(\mathbf{a})L$ to exposed agents. However, under incomplete information, insurers do not distinguish the different risks in the population, and have to let agents self-select. As a consequence, exposed agents do not reveal their risk type and all agents in the population get fully insured at the lowest premium, p_U^*. Insurers only sell unexposed contracts (even to exposed agents), and are enable to separate risk types. This happens because the regulation eliminates the possibility for insurers to adjust coverage to induce separation.

Because insurers cannot charge exposed agents higher premiums, risky investment strategies become relatively less attractive than under complete information. Indeed, risky investments increase the probability of loss among exposed types, which increases the expected losses for insurers as premium revenues do not reflect the true risk of these

agents. Insurers face a new trade-off: a higher financial return from risky assets versus higher expected claim costs arising from the exposed agents pooled with unexposed agents. When the latter dominates, safe investments become preferable in equilibrium.

Formally, insurers' expected profits under each strategy profile are given by table 3.3.

		Insurer 1's investment strategy a_1	
		Safe	Risky
Insurer 2's investment strategy a_2	Safe	$\pi_1(\cdot) = \pi_2(\cdot) = \frac{1}{2} r_L \lambda L$	$\pi_1(\cdot) = \frac{1}{2}(r_H \lambda L - \alpha \beta_1 L)$ $\pi_2(\cdot) = \frac{1}{2}(r_L \lambda L - \alpha \beta_1 L)$
	Risky	$\pi_1(\cdot) = \frac{1}{2}(r_L \lambda L - \alpha \beta_2 L)$ $\pi_2(\cdot) = \frac{1}{2}(r_H \lambda L - \alpha \beta_2 L)$	$\pi_1(\cdot) = \pi_2(\cdot) = \frac{1}{2}(r_H \lambda L - \alpha(\beta_1 + \beta_2) L)$

Table 3.3 – *Insurers' profits by strategy profile under incomplete information with mandatory full coverage*

Insurers strategically choose their investment strategy by comparing these expected profits. An insurer $i = 1, 2$ prefers the safe investment strategy if the following holds:

$$\alpha \geq \frac{\Delta r \lambda}{\beta_i}. \tag{11}$$

The results are summarized in proposition 6 below.

Proposition 6 (Investment strategies under incomplete information and mandatory full coverage)**.** *Suppose full coverage is mandated by a regulator, and insurers cannot observe agent types. Then, all agents in the population obtain full insurance at premium $p_U^* = \lambda L$. As a consequence, the different possible pure strategy Nash equilibria are as follows.*

1. ***Safe dominance.*** *(S,S) is the unique equilibrium if (11) is satisfied for both $i = 1, 2$. This is the case when $\alpha \geq \dfrac{\Delta r \lambda}{\min\{\beta_1, \beta_2\}}$.*
2. ***Risky dominance.*** *(R,R) is the unique equilibrium if (11) is violated for both $i = 1, 2$. This occurs when $\alpha < \dfrac{\Delta r \lambda}{\max\{\beta_1, \beta_2\}}$.*
3. ***Asymmetric split-strategy.***
   - *(S,R) is the unique equilibrium if (11) is satisfied for $i = 1$ but violated for $i = 2$. This is the case when $\alpha \in \left[\dfrac{\Delta r \lambda}{\beta_1}, \dfrac{\Delta r \lambda}{\beta_2}\right[$.*
   - *(R,S) is the unique equilibrium if (11) is violated for $i = 1$ but satisfied for $i = 2$. This is the case when $\alpha \in \left[\dfrac{\Delta r \lambda}{\beta_2}, \dfrac{\Delta r \lambda}{\beta_1}\right[$.*

Proof. See Appendix 3.F. □

Figure 3.7 provides an illustration of each outcome described in proposition 6. The intuition behind the emergence of a safe-safe equilibrium differs from the situation developed in section 4.3, where insurers face no regulatory constraint and choose their investment strategies while offering least-coast separating insurance policies. In that case, the cost of choosing the risky investment strategy arises from the under-insurance of unexposed agents, whose coverage must be distorted to ensure separation. Under the full-coverage regulation, however, the cost of choosing the risky investment strategy stems from the inability to screen exposed agents who benefit from paying lower unexposed premiums. Therefore, the mechanism discouraging risky investments is fundamentally different, reversing the intuition compared to the unregulated case. The following discussion details each possible outcome.

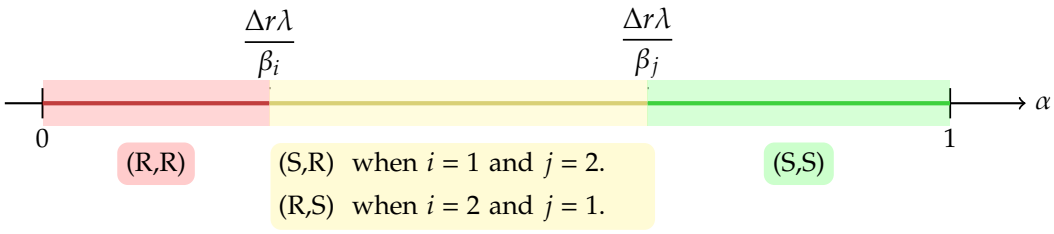


Figure 3.7 – *Illustration of equilibria in terms of α with mandatory full coverage*

Safe dominance. Safe investments dominate in equilibrium when the share of exposed agents α is large enough. When there is a wide exposed base in the population, the number of contracts sold at low premiums to high-risk agents who mimic unexposed types is large. This creates an additional expected claim cost equal to $\alpha(\beta(a_1) + \beta(a_2))L$ as the premium p_U^* does not cover the entirety of the true exposed risk. This expected loss increases with risky investments. Committing to the safe strategy allows insurers to reduce the probability of loss for all agents to λ. In this case, they prefer doing so in order to avoid the expected loss despite the lower financial returns on safe investments.

The existence of such equilibrium is facilitated when the risky return premium Δr is low, making the risky investment not so profitable relative to the safe investment.

Also, it is favored by high values of β_i, meaning that choosing the risky investment

strategy would significantly increase the probability of loss of exposed agents, and therefore the associated expected loss uncovered by premium p_U^*.

Risky dominance. On the contrary, the risky investment strategies dominate in equilibrium when there are few exposed agents in the population (*i.e.* α low). In this case, insurers sell only a small amount of policies to cheating types, and the expected claim costs remain low. This explains their preference for the risky investment strategy, which yield higher return.

Of course, this situation facilitated when the risky investment is relatively highly profitable relative to the safe option, that is when Δr is large.

Finally, it is more likely to arise when the risky investment strategy does not raise the probability of loss of exposed agents too much, *i.e.* when β_i is not too high.

Asymmetric split-strategy. For intermediate values of α, one insurer commits to the safe investment while the other commits to the risky investment strategy. The insurer with the larger exposure parameter β_i has more to lose from choosing the risky investment strategy, as this decision would significantly increase the probability of loss of exposed agents, and therefore the expected residual claim cost to incur. Therefore, this insurer prefers choosing the safe investment strategy to mitigate the effect on $\lambda_E(\mathbf{a})$ despite the lower returns. The insurer with the lower β_i continues to choose risky investments to benefit from higher returns without increasing the expected loss too severely. In this case, the market features one cautious and one aggressive insurer in equilibrium.

Implications. The implementation of mandatory full coverage by a regulator alters insurers' incentives to choose the risky investment strategy, by limiting their ability to screen agents' risk types. If the aim of the regulator is to induce safe investments in equilibrium, the results provide insights regarding the circumstances under which imposing full coverage allows to achieve this goal, or when it is inefficient. In short, the desirability of full-coverage mandates depends on the distribution of risk types in the population.

When exposed agents are numerous in the population, safe investment strategies

are not likely to arise in equilibrium without intervention (see proposition 4). Therefore, mandating full coverage can push insurers towards safe investments.

On the other hand, when the population is largely composed of unexposed agents, such regulation may be unnecessary or even counterproductive. Indeed, insurers are likely to voluntarily choose safe investment strategies to avoid insurance policy distortions. If these costly distortions are not made possible, safe investments are disincentivized. In this case, imposing full coverage creates the reverse effect and motivates risky investments.

6 Conclusions

This paper examines how insurers' choice of investment strategies interact with their core insurance activity, particularly when these strategies affect the underlying risk they insure. This framework illustrates the link between financial choices such as investments in fossil fuels and increases in the probability of climate-related damages for a part of the population.

The analysis demonstrates that the equilibrium investment strategies crucially depend on the informational environment. When agents' risk types are known, insurers always find it optimal to choose risky investments. However, when risk types are private information of agents, there are conditions under which safe investments can emerge in equilibrium, as the distortions on insurance contracts required to separate agents reduce the relative profitability of risky investments. When a market under incomplete information does not directly yield these safe strategies, regulatory interventions – such as the enforcement of full coverage – can effectively steer insurers towards safer choices.

Future research could extend the analysis along several dimensions. A natural extension would be to introduce a richer portfolio choice problem, allowing insurers to diversify across multiple asset classes. It would also be valuable to incorporate consumer preferences for socially responsible insurers, in order to explore how demand-side considerations influence insurers' behavior in equilibrium. Finally, it would be interesting to consider the implications of increasing the number of insurers on the

market, and discuss the impacts on investment strategies.

References

Arnott, R. (1992). Moral Hazard and Competitive Insurance Markets. In G. Dionne (Ed.), *Contributions to Insurance Economics* (pp. 325–358). Springer Netherlands.

Arnott, R., & Stiglitz, J. (1991). Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out of Peer Monitoring? *The American Economic Review*, *81*(1), 179–190.

Azevedo, E. M., & Gottlieb, D. (2017). Perfect Competition in Markets with Adverse Selection. *Econometrica*, *85*(1), 67–105.

Chiappori, P.-A., Jullien, B., Salanié, B., & Salanié, F. (2006). Asymmetric Information in Insurance: General Testable Implications. *The RAND Journal of Economics*, *37*(4), 783–798.

Chiappori, P.-A., & Salanié, B. (1997). Empirical Contract Theory: The case of insurance data. *European Economic Review*, *41*(3-5), 943–950.

Chiappori, P.-A., & Salanié, B. (2000). Testing for Asymmetric Information in Insurance Markets. *Journal of Political Economy*, *108*(1), 56–78.

Cho, I.-K., & Kreps, D. M. (1987). Signaling Games and Stable Equilibria. *The Quarterly Journal of Economics*, *102*(2), 179–221.

Cummins, J. D., & Weiss, M. A. (2014). Systemic Risk and the U.S. Insurance Sector. *The Journal of Risk and Insurance*, *81*(3), 489–527.

Dahlby, B. (1987). Monopoly versus Competition in an Insurance Market with Adverse Selection. *The Journal of Risk and Insurance*, *54*(2), 325–331.

Dionne, G., Nathalie, F., & Mimra, W. (2025, October). Adverse Selection in Insurance. In G. Dionne (Ed.), *Handbook of Insurance* (Third Edition, pp. 165–221, Vol. 2).

Dosis, A. (2018). On Signalling and Screening in Markets with Asymmetric Information. *Journal of Mathematical Economics*, *75*, 140–149.

Dosis, A. (2019). Optimal Ex Post Risk Adjustment in Markets with Adverse Selection. *Journal of Mathematical Economics*, *85*, 52–59.

Dosis, A. (2022). Price Caps and Efficiency in Markets with Adverse Selection. *Journal of Mathematical Economics*, *99*, 102591.

EIOPA. (2015). Guidelines on the Valuation of Technical Provisions under Solvency ii.

EIOPA, Fache Rousovà, L., Giuzio, M., Kapadia, S., Kumar, H., Mazzotta, L., Parker, M., & Zafeiris, D. (2021). *Climate Change, Catastrophes and the Macroeconomic Benefits of Insurance*. Publications Office.

Eling, M., & Pankoke, D. (2014). Systemic Risk and the Insurance Industry: Principal Linkages and Dependencies. In J. Monkiewicz & M. Małecki (Eds.), *Macroprudential Supervision in Insurance: Theoretical and Practical Aspects* (pp. 38–65). Palgrave Macmillan UK.

FAO. (2005). The State of Food and Agriculture 2005.

Hallegatte, S., & Rozenberg, J. (2017). Climate Change through a Poverty Lens. *Nature Climate Change*, *7*, 250–256.

Kimball, M. S. (1990). Precautionary Saving in the Small and in the Large. *Econometrica*, *58*(1), 53–73.

Knox, B., & Sørensen, J. (2024). Insurers' Investments and Insurance Prices. *FEDS Working Paper No. 2024-58*, 1–86.

Melecky, M., & Raddatz, C. (2011). How do Governments Respond After Catastrophes? Natural-Disaster Shocks and the Fiscal Stance.

Mimra, W., & Wambach, A. (2014). New Developments in the Theory of Adverse Selection in Competitive Insurance. *The Geneva Risk and Insurance Review*, *39*(2), 136–152.

Mimra, W., & Wambach, A. (2019). Endogenous Insolvency in the Rothschild-Stiglitz Model. *The Journal of Risk and Insurance*, *86*(1), 165–181.

Miyazaki, H. (1977). The Rat Race and Internal Labor Markets. *The Bell Journal of Economics*, *8*(2), 394–418.

NAIC. (2024). Product Filing Review Handbook (PFR-24).

OECD. (2018). *The Contribution of Reinsurance Markets to Managing Catastrophe Risk* (tech. rep.). OECD Publishing.

OECD. (2024, December). *Accelerating Climate Adaptation: a framework for assessing and addressing adaptation needs and priorities* (OECD Economic Policy Papers No. 35). OECD Publishing.

Pauly, M. V. (1968). The Economics of Moral Hazard: Comment. *The American Economic Review*, *58*(3), 531–537.

Pauly, M. V. (1974). Overinsurance and Public Provision of Insurance: the Roles of Moral Hazard and Adverse Selection. *The Quarterly Journal of Economics*, *88*(1), 44–62.

Picard, P. (2014). Participating Insurance Contracts and the Rothschild-Stiglitz Equilibrium Puzzle. *The Geneva Risk and Insurance Review*, *39*(2), 153–175.

Plantin, G., & Rochet, J.-C. (2007). *When Insurers Go Bust: An Economic Analysis of the Role and Design of Prudential Regulation*. Princeton University Press.

Rentschler, J., Salhab, M., & Jafino, B. (2022). Flood Exposure and Poverty in 188 Countries. *Nature Communications*, *13*.

Riley, J. G. (1979). Informational Equilibrium. *Econometrica*, *47*(2), 331–359.

Rothschild, M., & Stiglitz, J. (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information [Publisher: Oxford University Press]. *The Quarterly Journal of Economics*, *90*(4), 629–649.

Shavell, S. (1979). On Moral Hazard and Insurance. *The Quarterly Journal of Economics*, *93*(4), 541–562.

Shavell, S. (1980). Strict Liability versus Negligence. *The Journal of Legal Studies*, *9*, 1–25.

Spence, M. (1977). Consumer Misperceptions, Product Failure and Producer Liability. *The Review of Economic Studies*, *44*(3), 561–572.

Stiglitz, J. (1977). Monopoly, Non-linear Pricing and Imperfect Information: The Insurance Market. *The Review of Economic Studies*, *44*(3), 407–430.

von Peter, G., von Dahlen, S., & Saxena, S. (2024). Unmitigated Disasters? Risk sharing and macroeconomic recovery in a large international panel. *Journal of International Economics*, *149*, 103920.

Wilson, C. (1977). A Model of Insurance Markets with Incomplete Information. *Journal of Economic Theory*, *16*(2), 167–207.

Appendix

3.A Proof of Proposition 2

Problem of incentives. Recall the complete information menu of insurance policies, where $\mathbf{a}^* = (R, R)$:

$$\begin{cases} p_U^* = \lambda L, \;\; c_U^* = L, \\ p_E^*(\mathbf{a}^*) = (\lambda + \beta_1 + \beta_2)L, \;\; c_E^* = L. \end{cases}$$

An exposed agent's expected utility with the truthful insurance policy $(p_E^*(\mathbf{a}^*), c_E^*)$ is given by:

$$U_E(p_E^*(\mathbf{a}^*), c_E^*, \mathbf{a}^*) = u\Big(W - (\lambda + \beta_1 + \beta_2)L\Big).$$

An exposed agent's expected utility with the (p_U^*, c_U^*) insurance policy is given by:

$$U_E(p_U^*, c_U^*, \mathbf{a}^*) = u\Big(W - \lambda L\Big).$$

With $\beta_1, \beta_2 > 0$ and $u(\cdot)$ strictly increasing and concave, we have $U_E(p_U^*, c_U^*, \mathbf{a}^*) > U_E(p_E^*(\mathbf{a}^*), c_E^*, \mathbf{a}^*)$. Therefore, exposed agents will pick the insurance policy designed for unexposed agents, and there exists a problem of incentives.

Constraints for an incentive-compatible menu of insurance policies. An insurer $i = 1, 2$ must offer insurance policies $\{(\widehat{p_E}(\mathbf{a}), \widehat{c_E}(\mathbf{a})); (\widehat{p_U}(\mathbf{a}), \widehat{c_U}(\mathbf{a}))\}$ satisfying simultaneously the following constraints:

$$\Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u(W-L+\widehat{c_E}(\mathbf{a})-\widehat{p_E}(\mathbf{a}))+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u(W-\widehat{p_E}(\mathbf{a}))$$
$$\geq\Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u(W-L+\widehat{c_U}(\mathbf{a})-\widehat{p_U}(\mathbf{a}))+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u(W-\widehat{p_U}(\mathbf{a})) \quad (1)$$

$$\lambda u(W-L+\widehat{c_U}(\mathbf{a})-\widehat{p_U}(\mathbf{a}))+(1-\lambda)u(W-\widehat{p_U}(\mathbf{a}))$$
$$\geq\lambda u(W-L+\widehat{c_E}(\mathbf{a})-\widehat{p_E}(\mathbf{a}))+(1-\lambda)u(W-\widehat{p_E}(\mathbf{a})) \quad (2)$$

$$\Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u(W-L+\widehat{c_E}(\mathbf{a})-\widehat{p_E}(\mathbf{a}))+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u(W-\widehat{p_E}(\mathbf{a}))$$
$$\geq\Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u(W-L)+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u(W) \quad (3)$$

$$\lambda u(W-L+\widehat{c_U}(\mathbf{a})-\widehat{p_U}(\mathbf{a}))+(1-\lambda)u(W-\widehat{p_U}(\mathbf{a}))\geq\lambda u(W-L)+(1-\lambda)u(W) \quad (4)$$

No pooling contracts in equilibrium. First, it is possible to show that exposed and unexposed agents will never be offered the same contract in equilibrium. Proceed by contradiction. Suppose insurers offer a pooling policy where the unique premium is determined by competition on the insurance market:

$$p^{\text{pooling}}=\alpha\Big(\lambda+\beta(a_1)+\beta(a_2)\Big)L+(1-\alpha)\lambda L.$$

Insurers loose some revenue on exposed types and gain some revenues on unexposed types compared to the full information policies, and still make zero expected profits on the insurance activity. In this case, an insurer could deviate and offer a "cream skimming" policy that would only attract unexposed agents (*e.g.* slightly lower coverage and premium) and thus generate positive profits on the insurance market. This is a profitable deviation that leads to a contradiction. Therefore, a pooling contract cannot be a Nash equilibrium, and in equilibrium, exposed and unexposed agents must not select the same policy. In other words, each type will purchase a separate contract.

Pricing of insurance premiums. Competition between insurers on the insurance activity implies fair premiums in equilibrium. Therefore, we will have

$$\widehat{p_\theta}(\mathbf{a})=\lambda_\theta(\mathbf{a})\widehat{c_\theta}(\mathbf{a}).$$

Single-crossing. Compare how agents value a marginal increase in coverage for the same offered contract $(\overline{p}, c)$. Recall the expected utility of an agent facing a probability of loss $\lambda_\theta(\cdot)$ from getting this insurance contract:

$$\mathcal{U}_\theta(\overline{p}, c) \;=\; \lambda_\theta(\cdot)\, u\Big(W - L + c - \overline{p}\Big) + (1 - \lambda_\theta(\cdot))\, u\Big(W - \overline{p}\Big).$$

Then compute

$$\frac{\partial}{\partial \lambda_\theta(\cdot)}\left(\frac{\partial \mathcal{U}_\theta(\overline{p}, c)}{\partial c}\right) = u'(W - L + c - \overline{p}) > 0 \text{ by assumption on } u(\cdot).$$

The marginal utility of coverage is strictly increasing in the probability of loss. Hence, at any given contract, the exposed agent values an additional unit of coverage more than the unexposed agent. Consequently, to prevent mimicking, the unexposed insurance policy will be distorted.

Least-cost separating insurance policy for exposed types. Building on previous results, exposed types are offered full insurance at a fair premium, with:

$$\begin{cases} \widehat{c_E} = L, \\ \widehat{p_E}(\mathbf{a}) = (\lambda + \beta(a_1) + \beta(a_2))L \end{cases}.$$

Least-cost separating insurance policy for unexposed types. As discussed above, the premium offered to unexposed agents will be a fair premium given their probability of loss and the coverage offered in the policy: $\widehat{p_U}(\mathbf{a}) = \lambda \widehat{c_U}(\mathbf{a})$.
Therefore, given the policy offered to exposed types, an insurer $i = 1, 2$ must choose $\widehat{c_U}(\mathbf{a})$ while simultaneously satisfying the updated set of constraints:

$$u\Big(W-(\lambda+\beta(a_1)+\beta(a_2))L\Big)$$
$$\geq \Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u\Big(W-L+(1-\lambda)\widehat{c_U}(\mathbf{a})\Big)+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u\Big(W-\lambda\widehat{c_U}(\mathbf{a})\Big) \quad (1')$$

$$\lambda u\Big(W-L+(1-\lambda)\widehat{c_U}(\mathbf{a})\Big)+(1-\lambda)u\Big(W-\lambda\widehat{c_U}(\mathbf{a})\Big)\geq u\Big(W-(\lambda+\beta(a_1)+\beta(a_2))L\Big) \quad (2')$$

$$u\Big(W-(\lambda+\beta(a_1)+\beta(a_2))L\Big)\geq \Big(\lambda+\beta(a_1)+\beta(a_2)\Big)u(W-L)+\Big(1-(\lambda+\beta(a_1)+\beta(a_2))\Big)u(W) \quad (3')$$

$$\lambda u\Big(W-L+(1-\lambda)\widehat{c_U}(\mathbf{a})\Big)+(1-\lambda)u\Big(W-\lambda\widehat{c_U}(\mathbf{a})\Big)\geq \lambda u(W-L)+(1-\lambda)u(W) \quad (4')$$

Condition (2′) is necessarily satisfied. Unexposed agents will prefer the insurance policy $(\widehat{p_U}(\mathbf{a}),\widehat{c_U}(\mathbf{a}))$ to an overpriced (relatively to their risk) full insurance. The constraint of interest is (1′). Constraints (3′) and (4′) are also trivial here: as long as coverage is fairly priced, risk-averse agents prefer to smooth their wealth across the two states of nature, rather than buying no insurance.

Given the problem of incentives described above, it is straightforward that full coverage for unexposed agents is not attainable, as it makes their contract attractive to exposed agents *i.e.* it violates (1′). Let us prove that an insurer must decrease the coverage to satisfy this condition. The right-hand side of condition (1′) writes $\text{RHS} = \lambda_E(\mathbf{a})u\Big(W)L+(1-\lambda)\widehat{c_U}(\mathbf{a})\Big)+(1-\lambda_E(\mathbf{a}))u\Big((W-\lambda\widehat{c_U}(\mathbf{a})\Big)$. Differentiating with respect to $\widehat{c_U}(\mathbf{a})$ gives:

$$\frac{\partial \text{RHS}}{\partial \widehat{c_U}(\mathbf{a})}=\lambda_E(\mathbf{a})(1-\lambda)u'\Big(W-L+(1-\lambda)\widehat{c_U}(\mathbf{a})\Big)-\lambda(1-\lambda_E(\mathbf{a}))u'\Big(W-\lambda\widehat{c_U}(\mathbf{a})\Big).$$

By assumption, recall that coverage cannot exceed total loss, $\widehat{c_U}(\mathbf{a})\leq L$. Hence $W-L+(1-\lambda)\widehat{c_U}(\mathbf{a})\leq W-\lambda\widehat{c_U}(\mathbf{a})$. Concavity of $u(\cdot)$ implies

$$u'(W-L+(1-\lambda)\widehat{c_U}(\mathbf{a}))\geq u'(W-\lambda\widehat{c_U}(\mathbf{a})).$$

Moreover, when at least one insurer chooses $a_i = R$, we have $\lambda_E(\mathbf{a}) > \lambda$. This implies

$$\lambda_E(\mathbf{a})(1-\lambda) > \lambda(1-\lambda_E(\mathbf{a})).$$

These arguments imply

$$\frac{\partial \text{RHS}}{\partial \widehat{c_U}(\mathbf{a})} > 0.$$

The right-hand side of (1′) is strictly increasing in $\widehat{c_U}(\mathbf{a})$. To satisfy (1′), an insurer must reduce unexposed coverage below full insurance.

There are many candidates for $\widehat{c_U}(\mathbf{a})$ that satisfy (1′). The problem can be further simplified. As insurers maximize profits in the financial activity, they aim to maximize the volume of premiums collected in the insurance market to invest in the financial market. Equivalently, under incomplete information, insurers minimize the downward distortion on unexposed coverage, and offer a coverage $\widehat{c_U}(\mathbf{a})$ that makes exposed types exactly indifferent between revealing their true risk type and mimicking unexposed agents. In equilibrium, $\widehat{c_U}(\mathbf{a})$ is such that constraint (1′) binds. Formally, $\widehat{c_U}(\mathbf{a})$ is the solution of:

$$u\Big(W - \lambda_E(\mathbf{a})L\Big) = \lambda_E(\mathbf{a})\, u\Big(W - L + (1-\lambda)\widehat{c_U}(\mathbf{a})\Big) + \Big(1-\lambda_E(\mathbf{a})\Big)u\Big(W - \lambda\widehat{c_U}(\mathbf{a})\Big). \quad (1'')$$

For a given $\lambda_E(\mathbf{a})$, this equation implicitly defines $\widehat{c_U}(\mathbf{a})$ and is of the form $F(\widehat{c_U}(\mathbf{a})) = X$. We previously proved that $F(\widehat{c_U}(\mathbf{a}))$ is increasing in $\widehat{c_U}(\mathbf{a}) \in [0, L[$. Therefore, for each $\lambda_E(\mathbf{a})$ and standard function $u(\cdot)$, (1″) has a unique solution for the incentive-compatible unexposed coverage $\widehat{c_U}(\mathbf{a})$.

Connected result on the impact of the strategy profile on $\widehat{c_U}(\mathbf{a})$. Recall that investment choices affect the probability of loss for exposed agents: $\lambda_E(\mathbf{a}) = \lambda + \beta(a_1) + \beta(a_2)$. From equation (1″) and the monotonicity of the RHS in $\widehat{c_U}(\mathbf{a})$, it follows that a higher value of $\lambda_E(\mathbf{a})$ results in a lower value of $\widehat{c_U}(\mathbf{a})$. Intuitively, when the exposed probability of loss rises, the incentive compatibility constraint becomes more stringent, and insurers must

reduce coverage for unexposed agents to prevent mimicking. In particular:

- Coverage is lowest when both insurers commit to risky investment strategies.
- Coverage is intermediate when insurers commit to different investment strategies (split-strategy profile).
- Coverage is full when both insurers commit to safe investment strategies (corner solution).

For convenient comparisons, assume $\beta_1 > \beta_2$. Thus, $\lambda_E(S,S) = \lambda < \lambda_E(S,R) < \lambda_E(R,S) < \lambda_E(R,R)$, and

$$\widehat{c_U}(R,R) < \widehat{c_U}(S,R) < \widehat{c_U}(R,S) < \widehat{c_U}(S,S) = L.$$

Ex-post verification of constraint (2′)**.** Under the described menu of insurance policies, verify that the incentive-compatibility constraint of unexposed agents is satisfied. Recall this condition:

$$\lambda u\Big(W - L + (1-\lambda)\widehat{c_U}(\mathbf{a})\Big) + (1-\lambda)u\Big(W - \lambda\widehat{c_U}(\mathbf{a})\Big) \geq u\Big(W - (\lambda + \beta(a_1) + \beta(a_2))L\Big). \quad (2')$$

First, notice that $W - \lambda\widehat{c_U}(\mathbf{a}) > W - L + (1-\lambda)\widehat{c_U}(\mathbf{a})$ as $\widehat{c_U}(\mathbf{a}) < L$. This implies $u(W - \lambda\widehat{c_U}(\mathbf{a})) > u(W - L + (1-\lambda)\widehat{c_U}(\mathbf{a}))$ with $u(\cdot)$ increasing.
Second, constraint (1′) holding with equality entails

$$u\Big(W - \lambda_E(\mathbf{a})L\Big) = \lambda_E(\mathbf{a})u\Big(W - L + (1-\lambda)\widehat{c_U}(\mathbf{a})\Big) + (1-\lambda_E(\mathbf{a}))u\Big(W - \lambda\widehat{c_U}(\mathbf{a})\Big).$$

Plugging this expression in constraint (2′), we obtain:

$$\underbrace{(\lambda_E(\mathbf{a}) - \lambda)}_{>0 \text{ by definition}}\Big[\underbrace{u\Big(W - \lambda\widehat{c_U}(\mathbf{a})\Big) - u\Big(W - L + (1-\lambda)\widehat{c_U}(\mathbf{a})\Big)}_{>0}\Big] \geq 0.$$

Therefore, with the described menu of insurance policies, constraint (2′) is ex-post satisfied.

Note about participation constraints. Under the incentive-compatible menu of insurance policies, both types of agents purchase (at least some) insurance. It is straightfor-

ward to demonstrates that constraints (3′) and (4′) are satisfied.

- By concavity of $u(\cdot)$ and Jensen's inequality, (3′) necessarily holds.
- Given that expected utility increases with coverage, starting at the no insurance point, (4′) is ex-post satisfied as well.

3.B Proof of Proposition 4

Let each insurer $i \in \{1, 2\}$ choose an investment strategy $a_i \in \{R, S\}$. Overall profits depend on the strategy profile $\mathbf{a} = (a_1, a_2)$, the incentive-compatible menu of insurance policies given in proposition 2, and the various parameters of the model, including the financial returns r_H and r_L, the proportion of exposed agents in the population α, and the exposure parameter β.

To select an investment strategy, an insurer i compared the payoff from the safe investment strategy to the payoff from the risky investment strategy, given its competitor's $(-i)$ strategy. Hence, an insurer compares $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\big|_{a_{-i}=S}$ with $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\big|_{a_{-i}=S}$ and $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\big|_{a_{-i}=R}$ with $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\big|_{a_{-i}=R}$.

Since insurers are identical, this collapses into two conditions governing insurers' investment strategy decisions:

$$r_L \lambda L \geq r_H\Big(\alpha(\lambda + \beta)L + (1 - \alpha)\lambda \widehat{c_U}(S, R)\Big), \quad (5)$$

$$\text{and} \quad r_L\Big(\alpha(\lambda + \beta)L + (1 - \alpha)\lambda \widehat{c_U}(S, R)\Big) \geq r_H\Big(\alpha(\lambda + 2\beta)L + (1 - \alpha)\lambda \widehat{c_U}(R, R)\Big). \quad (6)$$

With identical insurers, $\widehat{c_U}(S, R) = \widehat{c_U}(R, S) > \widehat{c_U}(R, R)$.

Condition (5) implies $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\big|_{a_{-i}=S} \geq \pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\big|_{a_{-i}=S}$. Condition (6) implies $\pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); S)\big|_{a_{-i}=R} \geq \pi_i(\widehat{p_U}, \widehat{p_E}(\mathbf{a}); R)\big|_{a_{-i}=R}$.

Given the model, neither condition (5) nor condition (6) is predetermined to hold or fail. Both inequalities depend on the set of parameters $(r_H, r_L, \lambda, \alpha, \beta)$ and on the incentive-compatible unexposed coverage $\widehat{c_U}(\mathbf{a})$, also dependent on these parameters. These conditions can be rewritten in terms of the proportion of exposed agents in the population, α:

$$\alpha \leq \widetilde{\alpha_5} = 1 - \frac{L(\Delta r\lambda + r_H\beta)}{r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)} \quad (5')$$

$$\text{and} \quad \alpha \leq \widetilde{\alpha_6} = 1 - \frac{L\Big(\Delta r(\lambda+\beta) + r_H\beta\Big)}{\Delta rL(\lambda+\beta) + r_H\beta L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)} \quad (6').$$

There are four cases to investigate. Each table below gives the insurers' investment strategy choices given their competitor's choice. The green cell indicates the Nash equilibrium(a).

- Both conditions hold: (S,S) is the unique Nash equilibrium. An insurer commits to the safe investment strategy when its competitor chooses the safe investment strategy, and keeps the safe investment strategy even when its competitor chooses the risky strategy. Therefore, the safe investment strictly dominates the risky investment for both insurers.

		Insurer 1	
		Safe	Risky
Insurer 2	Safe	$a_1,\ a_2$	a_2
	Risky	a_1	

- Both conditions are violated: (R,R) is the unique Nash equilibrium. The same logic as the previous point applies to the risky investment strategy: this strategy strictly dominates the safe investment strategy for both insurers.

		Insurer 1	
		Safe	Risky
Insurer 2	Safe		a_1
	Risky	a_2	$a_1,\ a_2$

- Condition (5) is satisfied but (6) is violated: (S,S) and (R,R) are both Nash equilibria. Each insurer finds it profitable to choose the safe investment strategy when its competitor chooses the safe investment as well. However, they prefer switching to the risky investment whenever their competitor chooses the risky investment.

		Insurer 1	
		Safe	Risky
Insurer 2	Safe	$a_1,\ a_2$	
	Risky		$a_1,\ a_2$

- Condition (5) is violated but (6) is satisfied: (S, R) and (R, S) are both Nash equilibria. Each insurer finds it profitable to choose the investment strategy not chosen by its competitor.

		Insurer 1	
		Safe	Risky
Insurer 2	Safe		$a_1,\ a_2$
	Risky	$a_1,\ a_2$	

These four cases exhaust all possible configurations of the inequalities.

3.C Footnote 11

The unique equilibrium (S, S) is more easily sustained with high values of β if $\min\{\widetilde{\alpha_5}, \widetilde{\alpha_6}\}$ increases with β.

1. First, consider $\widetilde{\alpha_5} < \widetilde{\alpha_6}$. Compute the derivative of $\widetilde{\alpha_5}$ with respect to β:

$$\frac{\partial \widetilde{\alpha_5}}{\partial \beta} = -\frac{Lr_H^2\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big) - Lr_H(\Delta r\lambda + r_H\beta)\Big(L - \lambda\frac{\partial \widehat{c_U}(S,R)}{\partial \beta}\Big)}{\Big[r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)\Big]^2}$$

   where $\dfrac{\partial \widehat{c_U}(S,R)}{\partial \beta} < 0$. We have $\dfrac{\partial \widetilde{\alpha_5}}{\partial \beta} > 0$ if and only if:

$$\left|\frac{\partial \widehat{c_U}(S,R)}{\partial \beta}\right| > \frac{Lr_L - r_H\widehat{c_U}(S,R)}{\Delta r\lambda + r_H\beta}.$$

   If this condition is verified, the magnitude of the distortion required on $\widehat{c_U}(S,R)$ for incentive compatibility when β grows is large enough so that the distortion effect outweighs the premium effect. Therefore, higher values of β facilitate the emergence of the (S,S) equilibrium (*i.e.* relax condition (5)). If this condition is violated, the premium effect dominates and the (S,S) unique equilibrium is more easily obtained for low values of β.

2. Then, consider $\widetilde{\alpha_5} > \widetilde{\alpha_6}$. Proceeding in the same manner, we have $\dfrac{\partial \widetilde{\alpha_6}}{\partial \beta} > 0$ if and only if:

$$\left|\frac{\partial \widehat{c_U}(R,R)}{\partial \beta}\right| > \frac{r_L}{r_H}\frac{\partial \widehat{c_U}(S,R)}{\partial \beta} + \frac{\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)(\Delta r + r_H)}{r_H\Big(r_H\beta + \Delta r(\lambda+\beta)\Big)}.$$

   If this condition is satisfied, the sensitivity of coverage $\widehat{c_U}(R,R)$ to an increase in β is strong enough compared to the sensitivity of $\widehat{c_U}(S,R)$ and the premium effect so that a higher value of β favors the emergence of the unique (S,S) equilibrium (*i.e.* facilitates condition (6)). If this condition is not verified, the premium effect dominates and low values of β facilitate the emergence of the (S,S) equilibrium.

3.D Footnotes 13 and 15

3.D.1 Footnote 13

Recall the condition under which the coordination equilibria (S,S) and (R,R) arise: $\alpha \in]\widetilde{\alpha_6}, \widetilde{\alpha_5}]$ with $\widetilde{\alpha_5} > \widetilde{\alpha_6}$ and $\widetilde{\alpha_5} > 0$. The aim is to give a bit more subtance to the intuition regarding this interval. To have $\widetilde{\alpha_5} > \widetilde{\alpha_6}$, we need:

$$\frac{\Delta r(\lambda+\beta) + r_H\beta}{\Delta rL(\lambda+\beta) + r_H\beta L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)} > \frac{\Delta r\lambda + r_H\beta}{r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)}.$$

With $\beta > 0$, we have $\Delta r(\lambda+\beta) + r_H\beta > \Delta r\lambda + r_H\beta$. The range for α expands when $\Delta rL(\lambda+\beta) + r_H\beta L + \lambda(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R))$ is low, making the LHS grow. Quite obviously, this term becomes smaller for values of $\widehat{c_U}(R,R)$ close to $\widehat{c_U}(S,R)$.
The interval also expands when $r_H(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R))$ is large, making the RHS small. This term grows when $\widehat{c_U}(S,R)$ is small relative to L.

3.D.2 Footnote 15

Recall the condition under which the symmetric split-strategy equilibria (R,S) and (S,R) arise: $\alpha \in]\widetilde{\alpha_5}, \widetilde{\alpha_6}]$ with $\widetilde{\alpha_6} > \widetilde{\alpha_5}$ and $\widetilde{\alpha_6} > 0$. The ranking $\widetilde{\alpha_6} > \widetilde{\alpha_5}$ implies:

$$\frac{\Delta r\lambda + r_H\beta}{r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)} > \frac{\Delta r(\lambda+\beta) + r_H\beta}{\Delta rL(\lambda+\beta) + r_H\beta L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)}.$$

The LHS term grows when $r_H\Big(L(\lambda+\beta) - \lambda\widehat{c_U}(S,R)\Big)$ decreases, *i.e.* when L and $\widehat{c_U}(S,R)$ are close to each other. On the other hand, the RHS term decreases when $\Delta rL(\lambda+\beta) + r_H\beta L + \lambda\Big(r_L\widehat{c_U}(S,R) - r_H\widehat{c_U}(R,R)\Big)$ increases, *i.e.* when $\widehat{c_U}(R,R)$ is small relative to $\widehat{c_U}(S,R)$.

3.E Proof of Proposition 5

The main logic behind the strategic choice of an investment strategy for an insurer is identical as the previous proof. However, when insurers are not identical in their impact on the exposed agents' probability of loss if they commit to the risky investment strategy, there are four conditions governing their investment strategy decision:

$$r_L \lambda L \geq r_H\Big(\alpha(\lambda+\beta_1)L + (1-\alpha)\lambda \widehat{c_U}(R,S)\Big), \quad (7)$$

$$r_L \lambda L \geq r_H\Big(\alpha(\lambda+\beta_2)L + (1-\alpha)\lambda \widehat{c_U}(S,R)\Big), \quad (8)$$

$$r_L\Big(\alpha(\lambda+\beta_2)L + (1-\alpha)\lambda \widehat{c_U}(S,R)\Big) \geq r_H\Big(\alpha(\lambda+\beta_1+\beta_2)L + (1-\alpha)\lambda \widehat{c_U}(R,R)\Big), \quad (9)$$

$$r_L\Big(\alpha(\lambda+\beta_1)L + (1-\alpha)\lambda \widehat{c_U}(R,S)\Big) \geq r_H\Big(\alpha(\lambda+\beta_1+\beta_2)L + (1-\alpha)\lambda \widehat{c_U}(R,R)\Big). \quad (10)$$

Assuming $\beta_1 > \beta_2$, we have $\widehat{c_U}(R,R) < \widehat{c_U}(R,S) < \widehat{c_U}(S,R) < \widehat{c_U}(S,S) = L$.

Condition (7) implies $\pi_1(\widehat{p_U}, \widehat{p_E}(S,S); S)\big|_{a_2=S} \geq \pi_1(\widehat{p_U}, \widehat{p_E}(R,S); R)\big|_{a_2=S}$.
Condition (8) implies $\pi_2(\widehat{p_U}, \widehat{p_E}(S,S); S)\big|_{a_1=S} \geq \pi_2(\widehat{p_U}, \widehat{p_E}(S,R); R)\big|_{a_1=S}$.
Condition (9) implies $\pi_1(\widehat{p_U}, \widehat{p_E}(S,R); S)\big|_{a_2=R} \geq \pi_1(\widehat{p_U}, \widehat{p_E}(R,R); R)\big|_{a_2=R}$.
Condition (10) implies $\pi_2(\widehat{p_U}, \widehat{p_E}(R,S); S)\big|_{a_1=R} \geq \pi_2(\widehat{p_U}, \widehat{p_E}(R,R); R)\big|_{a_1=R}$.

Below we characterize every possible category of equilibrium outcomes described in the proposition, which exhaust all combinations of the conditions.

- (S,S) unique equilibrium: (7) and (8) hold, and at least one condition among (9) and (10) hold.

		$i=1$	
		S	R
$i=2$	S	a_1, a_2	a_2
	R	a_1	

(9) & (10) satisfied.

		$i=1$	
		S	R
$i=2$	S	a_1, a_2	
	R	a_1	a_2

(9) satisfied, (10) violated.

		$i=1$	
		S	R
$i=2$	S	a_1, a_2	a_2
	R	a_1	

(9) violated, (10) satisfied.

- (R,R) unique equilibrium: (9) and (10) are violated, and at least one condition among (7) and (8) is violated.

		$i=1$	
		S	R
$i=2$	S		a_1
	R	a_2	a_1, a_2

(7) & (8) violated.

		$i=1$	
		S	R
$i=2$	S	a_1	
	R	a_2	a_1, a_2

(7) satisfied, (8) violated.

		$i=1$	
		S	R
$i=2$	S	a_2	a_1
	R		a_1, a_2

(7) violated, (8) satisfied.

- (S, S) and (R, R) are both Nash equilibria: (9) and (10) are satisfied but (7) and (8) are violated.

		$i=1$	
		S	R
$i=2$	S	a_1, a_2	
	R		a_1, a_2

- (S, R) and (R, S) are both Nash equilibria: (9) and (10) are violated but (7) and (8) are satisfied.

		$i=1$	
		S	R
$i=2$	S		a_1, a_2
	R	a_1, a_2	

- (S, R) unique equilibrium: (8) violated, (9) satisfied and at least on condition for the (R, S) equilibrium is not met: (7) satisfied and/or (10) violated.

		$i=1$	
		S	R
$i=2$	S	a_1	
	R	a_1, a_2	a_2

(7) satisfied, (10) violated.

		$i=1$	
		S	R
$i=2$	S		a_1
	R	a_1, a_2	a_2

(7) & (10) violated.

		$i=1$	
		S	R
$i=2$	S	a_1	a_2
	R	a_1, a_2	

(7) & (10) satisfied.

- (R, S) unique equilibrium: (7) violated, (10) satisfied and at least one condition for the (S, R) equilibrium is not met: (8) satisfied and/or (9) violated.

		$i=1$	
		S	R
$i=2$	S	a_2	a_1, a_2
	R		a_1

(8) satisfied, (9) violated.

		$i=1$	
		S	R
$i=2$	S		a_1, a_2
	R	a_2	a_1

(8) & (9) violated.

		$i=1$	
		S	R
$i=2$	S	a_2	a_1, a_2
	R	a_1	

(8) & (9) satisfied.

- No pure strategy Nash equilibrium.

		$i=1$	
		S	R
$i=2$	S	a_1	a_2
	R	a_2	a_1

(7) & (10) satisfied, (8) & (9) violated.

		$i=1$	
		S	R
$i=2$	S	a_2	a_1
	R	a_1	a_2

(7) & (10) violated, (8) & (9) satisfied.

These cases exhaust all possible configurations of the four inequalities.

3.F Proof of Proposition 6

When insurers are mandated to offer full insurance and they compete on premiums, they offer fair premiums but are subject to cheating by exposed agents. As a result, they only sell unexposed policies at low premiums to all agents in the population. The incentive of an insurer i to choose the safe investment strategy does not depend on the investment strategy chosen by its competitor. Indeed, an insurer $i = 1, 2$ commits to the safe investment if

$$\alpha \geq \frac{\Delta r}{\beta_i} \quad (11)$$

Below we characterize the four possible equilibrium outcomes described in the proposition.

- If (11) holds for both $i = 1, 2$: (S, S) is the unique Nash equilibrium. Both insurers commit to the safe investment strategy regardless of their competitor's choice.

		$i = 1$	
		S	R
$i = 2$	S	a_1, a_2	a_2
	R	a_1	

- If (11) is violated for both $i = 1, 2$: (R, R) is the unique Nash equilibrium. Both insurers commit to the risky investment strategy regardless of their competitor's choice.

		$i = 1$	
		S	R
$i = 2$	S		a_1
	R	a_2	a_1, a_2

- If (11) is satisfied for $i = 1$ but violated for $i = 2$: (S, R) is the unique Nash equilibrium. Insurer 1 strictly prefers the safe investment, and insurer 2 strictly prefers the risky investment. This implies $\beta_1 > \beta_2$.

		$i=1$	
		S	R
$i=2$	S	a_1	
	R	a_1, a_2	a_2

- If (11) is violated for $i = 1$ but satisfied for $i = 2$: (R, S) is the unique Nash equilibrium. Insurer 1 strictly prefers the risky investment, and insurer 2 strictly prefers the safe investment. This implies $\beta_2 > \beta_1$.

		$i=1$	
		S	R
$i=2$	S	a_2	a_1, a_2
	R		a_1