École des Hautes Études en Sciences Sociales

Ecole doctorale 465 – Economie Panthéon Sorbonne

Paris-Jourdan Sciences Économiques - Paris School of Economics

Doctorat

Discipline : Sciences Économiques

**Wei Bi**

# A Study on Markets with Noise, Information Asymmetry and Behavioral Agents

**Thèse dirigée par:** Philippe Jehiel, Olivier Compte

**Date de soutenance : le 13 juillet 2023**

Rapporteurs
1. Bruno Jullien, Toulouse School of Economics
2. Régis Renault, CY Cergy Paris Université

Jury
1. Olivier Compte, Paris School of Economics
2. Philippe Jehiel, Paris School of Economics
3. Laurent Mathevet, European University Institute
4. John Wooders, New York University Abu Dhabi

# Acknowledgements

First, I want to express my sincere gratitude and appreciation to my supervisors Philippe Jehiel and Olivier Compte for their guidance and support throughout my thesis. Their expertise, knowledge, and dedication have been invaluable in helping me navigate through the challenges and complexities of my work. Their insightful feedback, constructive criticism, and encouragement have been instrumental in shaping my ideas and enhancing the quality of my work. I am truly grateful for all that they have done for me. I feel extremely fortunate to have had the opportunity to work under their supervision, and I have learned a great deal from you both, both professionally and personally. I hope to emulate the same level of excellence in my future endeavors.

Secondly, I want to express my heartfelt gratitude to my collaborator and soulmate Huiyun Ding for your invaluable contributions. Working with you has been an absolute pleasure. Your creativity, intelligence, and commitment to excellence have been an inspiration to me, and I feel blessed to have been able to work with someone as passionate and driven as you. But more than that, I want to acknowledge the deep and meaningful connection that we have built throughout our collaboration. Your support, encouragement, and understanding have been a constant source of strength and inspiration to me. Our shared vision, dedication, and passion have helped us to create something truly special, and I am proud of what we have accomplished together.

Finally, I want to thank my parents. Their unwavering support and encouragement have given me the strength and confidence to pursue my dreams and overcome the challenges that life has thrown my way. I also want to thank my colleagues in the economics theory research group of PSE and at ENS Paris-Saclay. I want to thank Régis Renault from the THEMA, CY Cergy Paris Université, Bruno Jullien from the TSE, Laurent Mathevet from the EUI, and John Wooders from the NYU Abu Dhabi for being the jury member.

# Abstract

This thesis studies how real-world markets affected by various sources of noise, information asymmetry and bounded rationality. Chapter 1 examines markets in which one party "sells" their talent to the other party, yet the true value of this talent is not easily quantifiable by either side. How to submit a paper to a journal, for instance, becomes a strategic task for authors, and whether to accept a seemingly good paper is equally challenging for editors. In equilibrium, the seller adjusts her strategy by learning (rejection). Recognizing that the "talent" could have been previously rejected by others, the buyer corrects his selection bias by raising the threshold of acceptance. Furthermore, in the induced dynamic game with incumbent and entrant buyers, the competition is unfair for the latter because at the time of entry, the latter will also receive the "talent" previously rejected by the former. This finding brings a new insight into the formation of entry barriers in such markets. Chapter 2 introduces a search model to examine how information asymmetry evolves in a labor market where Graduates search for jobs and Employers make offers. The model shows that while the market can be efficient initially, as low-type Graduates remain in the market, Employers lower their offers to correct for selection bias. However, in large markets, where search costs are low, these effects dissipate, leading to more efficient outcomes. The presence of noise in the market and graduate's belief that she can receive a better offer than her outside option enhance employer competition. The results show that the presence of noise in the market has a significant impact on market efficiency. Chapter 3 explores how bounded rationality leads to suboptimal decision-making by individuals in the market for talent and the Graduate-Employer market. In the market for talent, editors who are subject to cognitive errors may set lower thresholds for paper acceptance, which could lead to lower-quality papers being published. This offers entrants the chance to challenge incumbents' status. In the graduate-employer market, unsophisticated employers who do not consider a graduate's type or the signal provided by an interview may overbid or underbid, respectively, leading to adverse selection effects and decreasing deal prices in the

market. The findings from this chapter could be applied to explain overbidding in corporate acquisitions.

***Keywords:*** Market, information asymmetry, noisy perception, bounded rationality, searching.

# Résumé

Cette ths̀e examine l'impact de diverses sources de bruit, d'asymt́rie d'information et de rationalité limitée sur les marchés du monde réel. Le premier chapitre se concentre sur les marchés où une partie "vend" son talent à une autre partie, mais où la véritable valeur de ce talent n'est pas facilement quantifiable pour les parties impliquées. Par conséquent, soumettre un article á une revue devient une tâche stratégique pour les auteurs, tandis que les éditeurs sont confrontés à la difficulté d'accepter ou de refuser un article apparemment bon. A l'équilibre, le vendeur ajuste sa stratégie par apprentissage (rejet). L'acheteur, reconnaissant que le "talent" aurait pu être précédemment rejeté par d'autres, corrige son biais de sélection en relevant son seuil d'acceptation. De plus, dans le jeu dynamique qui se crée entre les acheteurs historiques et les nouveaux entrants, ces derniers sont désavantagés car, au moment de leur entrée, ils reçoivent également le "talent" précédemment rejeté par les premiers. Cette découverte apporte un nouvel éclairage sur la formation des barrières à l'entrée sur ces marchés. Le deuxième chapitre présente un modèle de recherche visant à examiner comment l'asymétrie d'information évolue sur un marché du travail où les diplômés recherchent des emplois et les employeurs font des offres. Le modèle montre que si le marché peut être efficace au départ, comme les diplômés de faible niveau restent sur le marché, les employeurs réduisent leurs offres pour corriger le biais de sélection. Cependant, sur les grands marchés, où les coûts de recherche sont faibles, ces effets se dissipent, conduisant à des résultats plus efficaces. La présence de bruit sur le marché et la conviction de la diplômée qu'elle peut recevoir une meilleure offre que son option extérieure renforcent la concurrence des employeurs. Les résultats montrent que la présence de bruit sur le marché a un impact significatif sur l'efficacité du marché. Le troisième chapitre explore comment la rationalité limitée entraîne une prise de décision sous-optimale par les individus sur les marchés du talent et des diplômés-employeurs. Sur le marché des talents, les éditeurs sujets à des erreurs cognitives peuvent fixer des seuils inférieurs d'acceptation des articles, ce qui pourrait entraîner la publication d'articles de moindre qualité. Cela offre aux participants la possibilité de contester le statut des titulaires. Sur le marché des diplômés-employeurs, les employeurs non avertis qui ne tiennent pas compte du type de diplômé ou du signal fourni par un entretien peuvent respectivement surenchérir ou sous-enchérir, ce qui entraîne des effets de sélection adverse et une baisse des prix des transactions sur le marché. Les conclusions de ce chapitre pourraient être appliquées pour expliquer la surenchère dans les acquisitions d'entreprises.

***Mots clés:*** Marché, asymétrie de l'information, perception bruyante, rationalité limitée, recherche.

# Contents

# General Introduction

Markets are one of the most important institutions in modern economies, facilitating the exchange of goods, services, and resources. In a perfectly competitive market, buyers and sellers have equal access to information, and prices reflect the true underlying value of the goods and services traded. However, real-world markets are rarely perfectly competitive, and various sources of noise, information asymmetry and bounded rationality can create inefficiencies and distortions that affect market outcomes.

Noise:

One of the key frictions that arises in markets is noise. In many markets, both buyers and sellers may lack a precise understanding of the true value or quality of the product, good, or service being exchanged. For buyers, noisy perception can arise from a variety of factors. For example, a buyer may lack the expertise or experience needed to accurately assess the quality of a product or service. In addition, buyers may also be influenced by various biases or heuristics that can lead them to overestimate or underestimate the value of a product or service. For example, buyers may be more likely to trust the opinions of others, such as friends or online reviews, rather than conducting their own independent research.

On the seller's side, noise can also be a significant challenge. Sellers may lack precise information about the true demand for their product or service, which can make it difficult to sell optimally. In addition, sellers may also struggle to accurately assess the quality of their own product or service, particularly if they lack objective measures or benchmarks to compare against.

Information Asymmetry:

Another key friction that arises in markets is information asymmetry. Information asymmetry refers to situations where one party in a transaction has more information than the other party. This can lead to market inefficiencies, as the party with more information may be able to exploit the party with less information.

There are many factors that can contribute to information asymmetry in markets. One factor is the fact that information is often costly to acquire. For example, conducting research or performing due diligence can be time-consuming and expensive, which can create a barrier to entry for some market participants.

Bounded Rationality:

A third key friction that arises in markets is bounded rationality. Bounded rationality refers to the fact that people's decision-making capabilities are limited by their cognitive abilities and the information available to them. This can lead to suboptimal decisions, even in situations where complete information is available.

There are several factors that can contribute to bounded rationality in markets. One factor is cognitive limitations, which refer to the fact that people's cognitive abilities are finite. For example, people may have limited working memory or may be prone to mental shortcuts or heuristics that can lead to biases and errors in decision-making.

Another factor that can contribute to bounded rationality in markets is the complexity of the information environment. Markets are often complex and dynamic, with a large amount of information available to participants. This can make it difficult for individuals to process all of the available information and make optimal decisions.

This thesis delves into the intricacies of markets, specifically with regard to these three factors. The first chapter focuses on markets without monetary transfer, exploring the impact of noise and information asymmetry on market outcomes. The second chapter delves into markets with monetary transfer. Finally, the third chapter considers the influence of bounded rationality on market outcomes, shedding light on the ways in which human biases and cognitive limitations impact markets.

In the first chapter, entitled "Market for Talent under Asymmetric Information", I aim to contribute to the literature on market by providing a comprehensive understanding of how agents with noisy perception learn and make decisions in markets under information asymmetry, as well as the mechanisms that lead to the formation of entry barriers. The study employs various game-theoretic models and simulation methods to analyze the dynamics of such markets and identify the factors that affect agents' behaviors and market outcomes. By shedding light on the complexities of these markets, this study has potential to contribute to the development of more efficient market designs and policies that can enhance social welfare and promote competition.

The study builds upon existing academic research that examines the dynamics of markets with similar structures. Some studies have explored the search model, where one side has an information disadvantage and receives noisy signals about the state, as in Zhu [2012] and Lauermann and Wolinsky [2016]. They present the search model where the side with information disadvantage is sampled and receives noisy signals about the state. Different from the previous studies, this study focuses on how the agents learn about the state through each interaction with noisy perception. Previous research (Cooper and John [1988] and Milgrom and Roberts [1990]) also highlights the issue of equilibrium multiplicity in markets, where strategic complementarities and agent interactions can result in multiple equilibria. In this study, the equilibrium multiplicity comes from the buyers' ignorance of the seller's history and their noisy perception of quality. Moreover, it shows that in the induced dynamic game, an equilibrium that favors the incumbent will be selected, which provides a cornerstone for the existence of entry barriers.

I study a specific market that considers the behavior of authors and journals in academic publishing. It assumes that an author's submission history is unobservable and that the author's and the journal's perception of quality is noisy. Moreover, the authors are assumed to learn from each interaction and adjust their submission behavior accordingly. This assumption reflects the reality that agents in such markets face uncertainty about the quality of their submissions and must learn from their experiences to improve their chances of success. A key phenomenon in this market is the selection bias effect. It arises from the fact that journal editors cannot observe how many times a manuscript has been rejected by other journals. In the absence of this information, editors only rely on the perceived quality of a manuscript to make a decision on acceptance or rejection.

To correct the selection bias, journals tend to raise their acceptance threshold. This is because they are uncertain about the number of times a paper has been rejected by other journals, so they may mistakenly perceive a paper as of high quality when it has actually been rejected multiple times. By setting a higher acceptance threshold, they aim to ensure that only high-quality papers are accepted. However, this corrective measure also creates inefficiency in the market, especially when the market size is large.

The formation of entry barriers in such markets is a complex process that has received limited attention in the literature. In this study, I analyze the emergence of entry barriers as a consequence of the selection bias effect between authors and journals. Specifically, I examine how journals' inability to perfectly assess the quality of submissions and authors' imperfect learning about the quality of their own work interact to create an environment in which the entrant suffers immediately upon entering the market.

In the second chapter, entitled "Adverse Selection with Dynamic Learning", I explore situations where information asymmetry does not exist at the outset and whether it has any qualitative impact on the outcome in a market setting. The traditional models of adverse selection (Akerlof [1970]) assume that one side has all the information, and the other side learns about the state from history or external signals. However, in many real-life examples, such as in the labor market, a graduate and potential employer are unaware of the graduate's value at the outset, but the graduate can learn about her value through feedback from job interviews. Other real-life examples include high-tech firm acquisitions, startups seeking seed funds, and securities trading in the secondary market.

I use a search model (similar setting as Lauermann and Wolinsky [2016] and Moreno and Wooders [2016]) to analyze a game where a graduate searches for employers sequentially. The graduate's ability can be high or low, and both parties have the same prior belief about the graduate's ability. After each interview, both the graduate and the employer receive a noisy signal of either good or bad. The graduate updates her belief using Bayes' rule based on her past signals, while the employer only knows the signal they receive and is unaware of the graduate's past signals or how many employers she has already interviewed. The sampled employer then makes an offer, and the graduate has three options: accept the offer and end the search, reject the offer and continue sampling, or leave the market and receive an outside option.

I find that when search costs are not extremely high, the market can avoid collapse and allow for efficient trades, as long as there is no initial information asymmetry. In the presence of noise in the market, the high-type graduate is inclined to continue searching after receiving bad signals under the belief that better opportunities may arise, leading to an incentive for employers to compete with each other to offer more attractive deals. However, in small markets with a low number of employers and negligible search costs, the adverse selection effect can lower the average deal price and prolong the search time for a trade, leading to market inefficiency.

This study contributes to the literature on adverse selection by exploring situations where information asymmetry deepens over time and its impact on market outcomes. It highlights the importance of considering noise in the market and how it affects the behavior of both sides. The study also shows that the level of information asymmetry is crucial in determining market efficiency and that search costs play a significant role in determining the outcome. Finally, the study sheds light on the importance of market size in determining market efficiency, with large markets recovering efficiency even in the presence of adverse selection.

In the third chapter, entitled "Markets with Behavioral Agents", I revisit the two markets in the first two chapters to explore how agents with bounded rationality can affect market outcomes. In the first market, the market for talent, where editors make decisions about whether to publish papers based on their quality, editors can make cognitive errors, such as failing to account for selection bias or underestimating the quality of a paper. These errors can reduce the level of selection bias in the market, providing entrants with an opportunity to challenge incumbents.

In the second market, the graduate-employer market, I let employers use previous experiences to make optimal strategies. However, the misuse of information can lead to irrational behavior. I explore how employers who lack sophistication can fail to make optimal decisions by failing to consider the probability of a graduate's acceptance based on their type or the importance of the signal in determining the acceptance probability. These errors can lead to overbidding or underbidding, affecting market outcomes and the willingness of employers to offer high prices due to the adverse selection effect.

This study uses the analogy-class approach (Jehiel [2005]) to characterize the behavior of bounded rational employers, where they bundle the nodes into analogy-classes to form expectations about agents' behavior. By revisiting these markets, we gain insight into the ways in which bounded rationality affects market outcomes, providing a better understanding of the limitations of markets and the challenges they face. Ultimately, this knowledge can help policymakers and market participants design more effective market mechanisms that account for the presence of agents with bounded rationality.

# Chapter 1

# Market for Talent under Asymmetric Information

## 1.1 Introduction

In labor markets, some workers perform routine tasks, such as truck driving, accounting, or programming, while others possess unique talents, such as playing football, writing film scripts, or conducting innovative research projects. However, accurately quantifying these talents remains a challenge. While certificates can be obtained for tasks like accounting or truck driving, there is no clear certification process for subjective talents such as playing football or writing novels. As a result, assessing the true value of one's talent is often noisy and uncertain, representing ***a key feature*** of the talent market that distinguishes it from traditional labor markets. This "seller-talent-buyer" market is illustrated with an example of "author-paper-journal" throughout this chapter. With numerous journals available, authors face a trade-off between the quality of journals and the possibility of acceptance. Journals, in turn, aim to publish high-quality papers but face uncertainty over the paper's submission history. Upon receiving a seemingly strong paper (after observing some good but noisy signals from the referees' reports), editors may worry about overvaluing it due to concerns that the paper may have been rejected by other journals in the past. This information asymmetry between journals and authors constitutes ***the second key feature*** of this market, the information asymmetry. Due to it, for one journal, "receiving a paper" or "being selected" indicates that the paper in hand may not be as good as it seems. This selection bias effect is the main topic of this article. It discovers how journals take it into account when making optimal choices.

The author-paper-journal market is composed of several journals and an author who writes a paper of unknown quality. The quality depends on her type, which is her private information. The higher the type, the more likely the paper is of good quality. Papers are submitted to journals at a cost, and if rejected, the author may try another journal. Journals only publish papers above a certain quality standard, but the editor cannot precisely determine quality and relies on a noisy signal. High-quality papers are more likely to generate good signals.

In a homogeneous journal market, a low-type author knows that her paper is unlikely to be accepted. Therefore, only when the author's type is high enough will she submit her paper to a randomly selected journal. Being rejected indicates that her paper may not be of high quality and it might not be worthwhile to try again, leading only those with relatively higher types to continue the process. The process goes on until either the author finds it better to stop or she tries all journals.

Counterintuitively, as the market size gets larger with more journals, authors' welfare does not necessarily increase though they have more places to try. This is because once an editor receives a paper, it could have been rejected many times before. In other words, the selection bias effect is more significant. To correct it, the journals should raise their threshold, which makes it harder for the author to get a publication.

If journals are heterogeneous, the market splits. A high-type author targets top-class journals publishing high-quality papers because it brings a high payoff. A low-type author has an alternative, the ordinary journals with lower standards, because they have lower thresholds of signals and bring a higher probability of acceptance. Due to this separation, top-class journals receive papers of higher average quality. Thus, compared to the homogeneous case, they set a lower threshold of signals. It lets them publish more good-quality papers. Diversity is beneficial to efficiency.

However, if the difference (the standards of the quality, the payoffs from publication) between a top-class journal and an ordinary one is not that significant, a counterintuitive equilibrium could also exist. The ordinary journal with a lower standard of quality sets a higher threshold than the top-class one. More precisely, in this case, the top-class journal is the author's first option. The ordinary journal receives only the papers having been rejected and perceives that the quality is more likely to be bad, leading to inverted thresholds of acceptance. The major finding of this study is, with multiple equilibria, which one is selected depends largely on which journal exists at first due to the two features discussed initially. This finding brings insights into the formation of entry barriers in such markets, even without tangible costs of entry.

If the top-class journal (incumbent) exists before the ordinary journal (entrant), the entrant is not able to challenge the incumbent by publishing papers of similar quality. Because when entering, it receives papers rejected by the incumbent previously and it can not distinguish between them from the unrejected ones, it must set an unfairly high threshold to select the good-quality papers mixed with bad-quality ones. As a result, both journals publish papers with similar quality but the entrant has a lower possibility of acceptance. This generates the convention among the authors that they should submit to the incumbent first, leading to the entrant receiving rejected papers again in the next period and all following periods. Then, competing with the incumbent leads to either the amount of publications being so small or many bad-quality papers being published. The better choice is to avoid competition and to be the authors' second option.

The selection bias effect is the key factor that generates this vicious cycle in the market, stemming from noisy perceptions of quality and journals' ignorance of authors' submission histories. When journals have perfect information about paper quality, the entry barrier disappears, rendering authors' submission histories irrelevant and thus eliminating information asymmetry. Conversely, when authors have perfect information about journal quality, the entry barrier weakens. For instance, By setting a high threshold, an entrant can become the preferred choice for authors with high-quality papers, as they want to find a place that only publishes top-tier papers and yields a higher payoff.

With the two features, noise and information asymmetry in the market, entry barriers are inevitable to exist. The implications of this study extend beyond the academic market, offering insights into the formation of oligopolies in other markets with these two features, such as venture capital incumbent firms (e.g., Accel, Andreessen Horowitz, Bessemer Venture Partners, Benchmark, and Sequoia Capital) (Hochberg et al. [2010]), and record labels' contracts with musicians (e.g., Universal, Sony, and Warner) (Alexander [1994]).

The rest of the article is organized as follows: Section 1.2 presents the model. Section 1.3 characterizes the equilibrium. Section 1.4 analyzes the case with heterogeneous journals and shows the possibility of multiple equilibria. Section 1.5 discusses the existence of entry barriers. Section 1.6 concludes the study and provides scope for further discussion. All proofs are in Appendix 1.7.4.

### 1.1.1 Related Literature

This study contributes to the literature of the market with noisy perception and information asymmetry: i) the analysis of the agents' learning process under noisy environment, and ii) the existence of an entry barrier in such markets.

Several existing studies of academic publishing have a similar model structure, Cotton [2013], Leslie [2005], Muller-Itten [2017], Ellison [2002] and Azar [2015]. Among them, Cotton [2013], Leslie [2005] and Ellison [2002] focus on the necessity of the submission fee and the lengthy refereeing. A similar result is found in this study that journals have the incentive to set some cost to screen those high-type authors. Muller-Itten [2017] puts more emphasis on the author's behavior. She defines a score system where the author's ranking of submission is based on a score including some factors like the quality of the paper, the difficulty of publication, etc. This idea could be traced back to Oster [1980]. Azar [2015] presents a simple model with one author and one journal. He characterizes the agents' behavior and analyzes how it changes with submission cost, journals' standards, and the noise in the editor's signal. The information structure used in the model is similar to the one in Zhu [2012] and Lauermann and Wolinsky [2016]. They present the search model where the side with information disadvantage is sampled and receives noisy signals about the state. Different from the previous studies, this study focuses on how the agents learn about the state through each interaction.

In regard to the equilibrium multiplicity, many studies (Cooper and John [1988] and Milgrom and Roberts [1990]) attribute it to strategic complementarities. Brock and Durlauf [2001] present a random field model where the agents are influenced by their neighbors' behavior, which follows the same intuition as the above two. In this study, the equilibrium multiplicity comes from the journals' ignorance of the authors' submission order and their noisy perception of quality. Moreover, it shows that in the induced dynamic game, an equilibrium that favors the incumbent will be selected, which provides a cornerstone for the existence of entry barriers. The mutation method introduced in Kandori et al. [1993] and Young [1993] is also used to see which equilibrium is selected in the long run.

There is little literature discussing the formation of entry barriers under information asymmetry. Some discuss the asymmetry between the incumbent and the entrant. Dell'Ariccia et al. [1999], Dell'Ariccia [2001] and Marquez [2002] present a model of the banking industry where the entrants face the adverse selection problem because they can not know whether the borrower has been rejected by the incumbent. Bofondi and Gobbi [2006] find evidence in Italian local markets. Seamans

[2013] studies the U.S. cable TV industry and verifies the incumbent's limit pricing behavior. Langinier [2004] studies how the asymmetry influences the frequency of patent renewal which further changes the likelihood of entry. Some (Aghion and Bolton [1987], Martimort et al. [2021]) consider the asymmetry between the buyer and the seller where the information advantage side gains by contracting. Different from the previous studies, this study gives a simpler insight where the entrant suffers immediately once it enters the market due to the information asymmetry.

## 1.2 The Model

An author writes a paper of quality $q \in \mathbb{R}$, where the exact value of $q$ is unknown. It is contingent on her type $\theta \in \mathbb{R}$, which is her private information. The cumulative distribution function of the quality conditional on the type is a log-concave function $F(q|\theta)$ (corresponding probability density function $f(q|\theta)$). $f(\cdot|\theta)$ satisfies the monotone likelihood ratio property (MLRP). It means that a high-type author is more likely to write a paper of higher quality. Let the continuous function $\mu(\theta)$ be the prior distribution of the author's type.

Assuming there are $m$ class-A journals, each publication in one of these journals yields a payoff of $v > 1$ to the author. The author must pay a submission cost of $c < 1$ for each submission made to a journal. In each round, the author submits their paper to a journal and, upon rejection, chooses another journal in the subsequent round. The process is repeated until the author either succeeds in publishing their paper or decides to stop trying.

The journals only accept papers of sufficiently high quality for publication. Specifically, a journal's payoff for publishing a paper with quality $q$ is given by $q - q_A$. The journal publishes a paper if and only if the quality is higher than $q_A$, where $q_A$ is the minimum quality threshold. The journal can only observe a noisy signal of the paper's quality, denoted by $s = q + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma_s^2)$.

Notably, the author keeps a record of their historical submission failures, while the selected journal remains unaware of this information. The set of all possible histories is represented by $H = \emptyset, (A), (A, A), ...,$ where $h \in H$ represents the author's submission history. For example, $h = (A, A)$ denotes that the author submitted their paper to two class-A journals in the first and second rounds and was rejected on both occasions. For simplicity, I use $A^i$ to represent the history where the author attempts $i$ submissions, all of which fail ($A^0 = \emptyset$).

*Strategy*

The author's strategy is a mapping from her type $\theta$ and her history $h$ to either submit her paper to a class-A journal or stop trying, denoted by $\tau : \mathbb{R} \times H \to \{A, stop\}$. The journal's strategy is a mapping from the noisy signal $s$ to either accept or reject the paper, denoted by $\eta_A : s \to \{Ac, Rj\}$. Journals have no commitment power to their strategies. For example, it can not raise its ranking in authors' submission ordering by claiming that $\eta_A(s) = Ac$ for any signal $s$.

*Belief*

The author, given the history $h$, updates a posterior distribution of the quality $\gamma(q|\theta, h)$ by applying Bayes' rule. For instance, if $h = (A)$, then

$$\gamma(q|\theta, h) = \frac{f(q|\theta) \int_{\eta_A(s)=Rj} \phi(s, q, \sigma_s) ds}{\int dq f(q|\theta) \int_{\eta_A(s)=Rj} \phi(s, q, \sigma_s) ds}$$

where $\phi$ is the probability density function of a normal distribution.

Receiving a paper, the editor of a class-A journal forms a belief $\beta_A$ of the quality $q$.

*Equilibrium*

The perfect Bayesian equilibrium of this game is studied. A tuple $(\gamma, \tau, \eta_A, \beta_A)$ is a perfect Bayesian equilibrium if

1. Given the signal $s$ and belief $\beta_A$, class-A journals accept a paper ($\eta_A(s) = Ac$) if and only if the expected quality is higher than $q_A$,

$$\mathbb{E}_{\beta_A}[q|s] \geq q_A \tag{1.1}$$

2. Given her history $h$ and her belief $\gamma$, the author calculates the expected payoff of submitting her paper to a class-A journal. That is,

$$\pi_A(\theta, h) = v \int \gamma(q|\theta, h) \int_{\eta_A(s)=Ac} \phi(s, q, \sigma_s) ds dq - c \tag{1.2}$$

If $\pi_A(\theta, h) \geq 0$ and the author has not tried all journals, she submits her paper to a journal of class-A she has not tried before ($\tau(\theta, h) = A$). Otherwise, she stops ($\tau(\theta, h) = stop$).

3. Given $\tau$ and $\eta_A$, $\gamma(q|\theta, h)$ and $\beta_A(q)$ are derived by Bayes' rule.

### 1.2.1 Preliminary Observation

**Journal's problem.** Upon receiving a paper, the editor is concerned that it may have already been rejected by other journals, implying that receiving a paper is not necessarily good news. This selection bias effect needs to be taken into account when forming the belief of the quality. I begin by examining the symmetric equilibrium in which the author sets the submission order randomly. Specifically, a journal's position in the order is uniformly distributed. Under this assumption, the likelihood of receiving a paper of quality $q$ from an author with type $\theta$ and history $h$ is given by:

$$L(q,\theta,h=A^i)=\frac{1}{m}\cdot\mu(\theta)\cdot f(q|\theta)\cdot\left(\prod_{j=0}^{i}\mathbb{1}_{\{\tau(\theta,A^j)=A\}}\right)\cdot\left(\int\mathbb{1}_{\{\eta_A(s)=Rj\}}\phi(s,q,\sigma_s)ds\right)^i \tag{1.3}$$

It is the probability that the journal is ranked in the $(i+1)^{th}$ position of the author's submission order, times the prior of $\theta$, times the distribution of the quality conditional on type, and times the probability that the author submits and resubmits before and all get rejected. $\phi$ is the probability density function of a normal distribution.

The editor forms a belief of the quality

$$\beta_A(q)=\sum_{i=0}^{m-1}\int L(q,\theta,h=A^i)d\theta\Bigg/\sum_{i=0}^{m-1}\iint L(q,\theta,h=A^i)d\theta dq \tag{1.4}$$

Given his belief, the editor uses a cutoff strategy because the distribution of signal $s$ satisfies MLRP.

**Lemma 1.** *Given the editor's belief $\beta_A$, there exists a threshold $s_A$ such that the journal accepts a paper if it observes a signal $s\geq s_A$, and otherwise, it rejects.*

**Author's problem.** Before deciding to submit, the author forms a belief of the quality based on her type and her submission history $h$, $\gamma(q|\theta,h)$, and it satisfies the MLRP. Given lemma 1, it can be rewritten as follow

$$\gamma(q|\theta,h)=\frac{f(q|\theta)\Phi^i(s_A,q,\sigma_s)}{\int f(q|\theta)\Phi^i(s_A,q,\sigma_s)dq},\ if\ h=A^i$$

where $\Phi$ is the cumulative distribution function of a normal distribution.

The author faces a trade-off between the benefits of publication and the cost of submission. In order to decide whether to submit or not, she compares the expected gain from publication to the cost of submission. Specifically, she finds it optimal to submit if

$$v \int \gamma(q|\theta, h)[1 - \Phi(s_A, q, \sigma_s)]dq \geq c$$

As the author receives more rejections, her belief in the quality of her paper decreases. Thus, the expected gain from publication decreases as well, and eventually falls below the cost of submission. At this point, the author should stop submitting. Additionally, because of the MLRP, the expected gain increases with the author's type. Thus, there exists a sequence of types, $\theta_0 < \theta_1 < \theta_2 < ... < \theta_{m-1}$, where authors with types $\theta < \theta_0$ never submit, those with types $\theta \in (\theta_0, \theta_1)$ submit only once, those with types $\theta \in (\theta_1, \theta_2)$ submit twice, and so on.

**Lemma 2.** *Given $s_A$, for any history $h$, there exists a unique $\theta^*_A(h) \in (-\infty, +\infty)$ such that $\pi_A(\theta^*_A(h), h) = 0$. The author with history $h$ submits her paper to a class-A journal if her type $\theta \geq \theta^*_A(h)$. She stops if her type $\theta < \theta^*_A(h)$ or $h = A^m$. Moreover, $\theta^*_A(A^{m-1}) > ... > \theta^*_A(A) > \theta^*_A(\emptyset)$.*

## 1.3 Equilibrium Characterization

This section provides an analysis of the equilibrium in the model. Specifically, I demonstrate the existence of a unique symmetric equilibrium and examine how agents' behavior is impacted by the number of journals, the author's benefit-cost ratio, and the noise level in agents' perception.

**Proposition 1.** *A unique symmetric equilibrium exists. The journals' threshold is $s^*_A$. The author with history $A^i$ ($i = 0, 1, ..., m-1$) submits her paper to a journal randomly selected from those she has not tried yet if and only if her type $\theta > \theta^*_A(A^i)$.*

### 1.3.1 Comparative Statics

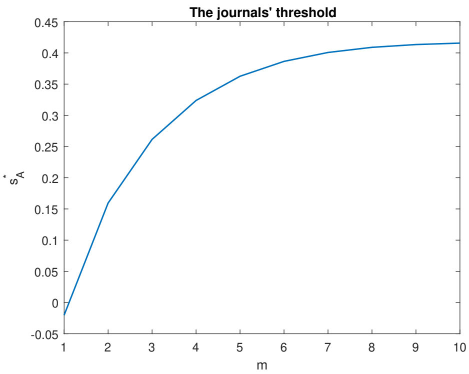
#### The Number of Journals

The behavior of agents is influenced by two effects as the number of journals increases. Firstly, the probability of a journal being at the top of an author's submission order decreases, resulting in a higher chance of the submitted paper being rejected several times, making it less likely to be of high quality. Therefore, journals should increase their threshold to correct this selection bias.

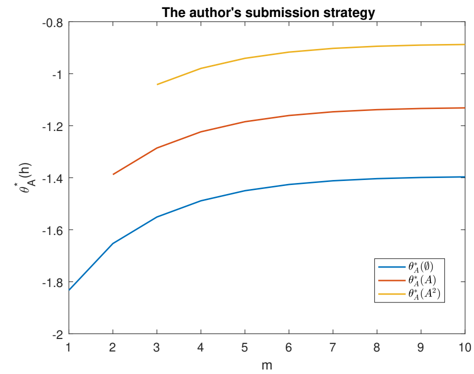
Secondly, authors with higher types are more likely to resubmit after being rejected, leading to an increase in the average quality of submitted papers, resulting in journals setting lower thresholds. However, this effect is only dominant when the submission cost is extremely high.

**Proposition 2.** *There is* $\underline{c} \in (0, v]$ *such that if* $c < \underline{c}$*,* $s_A^*$ *is increasing in* $m$*, and* $\theta_A^*(h)$ *is increasing in* $m$ *for any* $h \in H$*.*

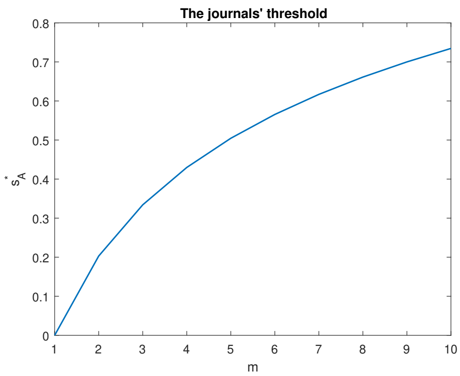
**Example 1.** *The author's type* $\theta$ *follows a normal distribution:* $\mu(\theta) = \phi(\theta, \mu, \sigma_\theta)$*. The quality conditional on the type follows a normal distribution:* $f(q|\theta) = \phi(q, \theta, \sigma_q)$*.*



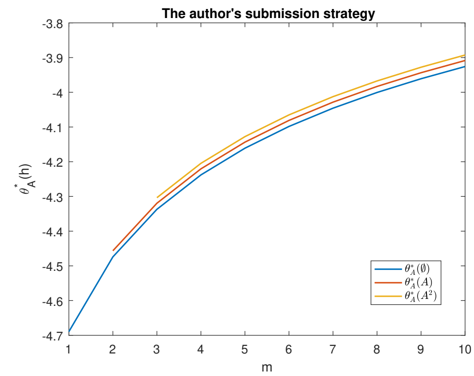

((a))




((b))




((c))




((d))

Figure 1.1: The upper-left graph is journals' strategy $s_A^*$ under different numbers of journals. The upper-right graph is the author's strategy $\theta_A^*(h)$. The parameters are: $\mu = 0$, $\sigma_\theta = \sigma_q = \sigma_s = 1$, $q_A = 0$, $v = 2$ and $c = 0.2$. The bottom two graphs assume $c = 0.001$.

*In Figure 1.1, panels (a) and (b) illustrate that* $s_A^*$ *and* $\theta_A^*$ *increase as the number of journals increases, and eventually converge to some value. This result may seem* *counterintuitive because, as the number of journals becomes extremely large, the probability of being selected by any one journal becomes very small. Consequently, the selected paper could have been rejected numerous times, suggesting lower quality. This intuition implies that journals should set an infinitely high threshold.*

*However, for most authors, their beliefs of quality become passive after multiple rejections, and the expected payoff of resubmitting becomes lower than the submission cost. Therefore, once a journal receives a paper, it can infer that the author has not been rejected an infinite number of times, leading to convergence of* $s_A^*$ *and* $\theta_A^*$ *as the number of journals increases.*

*In contrast, when submission cost approaches zero,* $s_A^*$ *and* $\theta_A^*$ *do not converge. This is illustrated in Panels (c) and (d) of Figure 1.1.* □

Increasing the number of journals in the market provides authors with more opportunities to submit their work, but it does not necessarily lead to an increase in welfare because the selection bias effect can raise the threshold for acceptance. This effect can make it difficult for authors to get published even when the submission cost is negligible and they keep trying until there is no chance left, regardless of their type. I define the market welfare as $TQ(m)$, which represents the sum of the quality of the papers that are ultimately accepted.

$$\begin{aligned} TQ(m) &= \mathbb{E}[q|Accepted] \cdot Pr[Accepted] \\ &= \iint q\mu(\theta) f(q|\theta) \sum_{i=0}^{m-1} \Phi^i(s_A, q, \sigma_s)[1 - \Phi(s_A, q, \sigma_s)] d\theta dq \end{aligned}$$

In Figure 1.2, I compare $TQ(1)$ and $TQ(2)$ under different $\mu$, average authors' type ($\theta \sim \mathcal{N}(\mu, \sigma_\theta)$). I find that welfare is higher with less journals when $\mu$ is high. In this case, the papers in the market are of high quality on average. Then, being rejected means that this paper is really bad, which makes the selection bias effect more significant. Thus, adding another journal raises both thresholds significantly and makes acceptance harder, not only for rejected papers but also for unrejected ones.

### The Ratio $v/c$

As the value from publication increases compared to the cost, it leads the author with lower type to try or try again. Under this case, journals receive low-quality papers more often. Thus, they will increase their thresholds.

**Proposition 3.** *$\theta_A^*(h)$ is decreasing in $v/c$ for any $h \in H$. $s_A^*$ is increasing in $v/c$.*

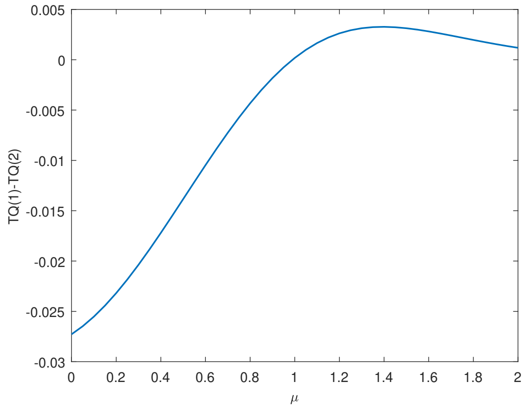



Figure 1.2: Keep the settings of Example 1. The parameters are: $\sigma_\theta = \sigma_q = 1$, $\sigma_s = 2$, $q_A = 0$ and $c = 0$.

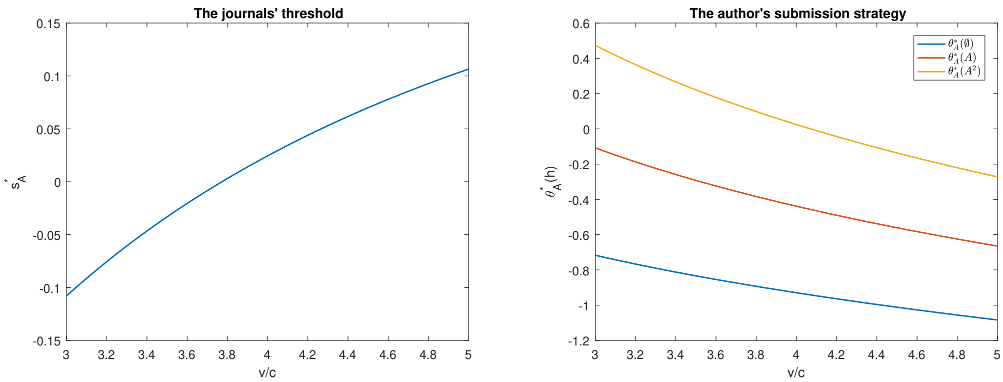



Figure 1.3: The left graph is journals' strategy $s^*_A$ under different $v/c$ ratios. The right graph is the author's strategy $\theta^*_A$. The parameters are: $\sigma_\theta = \sigma_q = \sigma_s = 1$; $q_A = 0$ and $m = 3$.

Keeping the setting of example 1, figure 1.3 shows the trends of $s^*_A$ and $\theta^*_A$ changing with different $v/c$. A higher submission cost discourages submission from low-type authors. It can be used as a tool to filter them. If journals form a coalition and set a positive submission cost cooperatively, they should let the expected quality of the marginal author's paper equal $q_A$.

### 1.3.2 Asymmetric Equilibrium and Stability

In the preceding analysis, the symmetric equilibrium was considered, where the author chooses a journal randomly. However, if the author has a specific submission order, an asymmetric equilibrium may arise. Nevertheless, the existence of such an equilibrium is not guaranteed.

For example, suppose there are two class-A journals, A1 and A2, and the author always tries A1 first, and then A2 after getting rejected. In this case, it is essential that in equilibrium, A2 sets a higher threshold than A1. Otherwise, the author would find it more advantageous to submit to A2 first. A2 has an incentive to do so because it receives papers rejected by A1. However, it has less incentive to do so because those low-type authors getting rejected stop trying. If the second effect dominates, the asymmetric equilibrium may not exist.

**Example 2.** *There are two class-A journals: A1 and A2. The author's type $\theta$ follows a normal distribution: $\mu(\theta) = \phi(\theta, 0, \sigma_\theta)$. The quality conditional on the type follows a normal distribution: $f(q|\theta) = \phi(q, \theta, \sigma_q)$. The parameters are: $\sigma_\theta = 0.5$, $\sigma_q = \sigma_s = 1$; $q_A = 3$; $v = 2$ and $c = 1.2$.*

*I try to find the asymmetric equilibrium where the author does not randomly select the journal. If the author has a specific submission order: 'first A1 then A2', A1 sets a threshold of signal $s_1 = 2.78$, and the author with a type $\theta > 3.14$ submits to A1. Getting rejected, she submits to A2 when her type $\theta > 4.08$, and A2 sets a threshold $s_2 = 2.73$.*

*A2's threshold is lower than A1. The author has no incentive to submit A1 first. Similarly, 'first A2 then A1' can not be an equilibrium.* □

Furthermore, note that as submission costs approach zero, the second effect diminishes, and multiple equilibria may exist. Sections 1.4.2 and 1.5 discuss the general situation in more detail. Moreover, if asymmetric equilibria exist, the symmetric one is not stable. This is because if there is little difference between the thresholds of the journals, the journal with the lower threshold becomes the first option for the author. In other words, there is a specific order of submission that represents the asymmetric equilibrium. However, this order of submission may not be the same for every author in reality.

One way to explain this paradox is to assume that the authors' payoffs are heterogeneous. Specifically, let author $i$'s payoff be $v$ plus some subjective preference $\epsilon_i^j$ for journal $j$.

$$v_i^j = v + \epsilon_i^j,\ \epsilon_i^j \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

Then, if we consider two journals, A and B, and their approximately close thresholds $s_A > s_B$, author $i$ submits to A first if

$$(v + \epsilon_i^A)Pr[Accepted\ by\ A|s_A] > (v + \epsilon_i^B)Pr[Accepted\ by\ B|s_B]$$

$$\epsilon_i^A - \epsilon_i^B > \frac{v\big(Pr[Accepted\ by\ A|s_B] - Pr[Accepted\ by\ B|s_A]\big)}{Pr[Accepted\ by\ A|s_B]} =: k$$

The approximate probability that author $i$ submits to A first is $1 - \Phi(k, 0, 2\sigma_\epsilon^2)$. As long as the variance is sufficiently large to resist the perturbation of thresholds, the symmetric equilibrium is stable.

## 1.4 Heterogeneity

In this section, I consider the scenario where there exist class-B journals, which have a lower standard compared to class-A journals. I analyze the behavior of agents in this new setting and compare it to the previous case. I find that there could be multiple equilibria in this market, which raises the question of potential entry barriers.

### 1.4.1 Class-B Journals Exist

Suppose there are infinitely many class-B journals besides class-A journals. The payoff from publishing in a class-B journal is normalized to 1, and the submission cost is denoted by $c$. The payoff of publishing a paper with quality $q$ for a class-B journals is $q - q_B$, $q_B < q_A$. Similar to class-A journals, the quality of a paper is not precisely known, but an agent can observe a noisy signal $s = q + \epsilon$, where $\epsilon \sim \mathcal{N}(0, \sigma^2)$.

In each round, an author can submit her paper to a journal of either class. If the paper is accepted and published, the author receives the corresponding payoff. If not, her can choose another journal to submit to in the next round. I assume that the author is extremely impatient and the discount rate $\delta = 0$. The general case $\delta > 0$ will be discussed later.

I use $\tilde{h} \in \tilde{H} = \emptyset, (A), (B), (A, A), (A, B), (B, B), ...$ to denote the set of possible histories of an author's submissions. For instance, $\tilde{h} = (A, A, B)$ means that the author submitted their paper to two class-A journals in the first and second rounds and got rejected. In the third round, they submitted it to a class-B journal and got rejected. For simplicity, I denote $A^i B^j$ as the history where the author tries class-A journals $i$ times and class-B journals $j$ times but fails all.

*Strategy*

The strategy of the author is a mapping from her type $\theta$ and her history $\tilde{h}$ to either submitting her paper to $A$, to $B$, or stopping, $\tilde{\tau} : \mathbb{R} \times \tilde{H} \to \{A, B, stop\}$.

The strategy of class-A journal is a mapping from the signal $s$ it receives to either accepting or rejecting, $\tilde{\eta}_A : s \to \{Ac, Rj\}$. Similarly, $\tilde{\eta}_B : s \to \{Ac, Rj\}$.

*Belief*

Given the history $\tilde{h}$, the author forms a posterior distribution of the quality $\gamma(q|\theta, \tilde{h})$ by applying Bayes' rule. Receiving a paper, the editor of a class-A journal forms a belief of the quality $\tilde{\beta}_A$. Similarly, denote $\tilde{\beta}_B$ as class-B journal's belief after receiving a paper.

*Equilibrium*

I use the perfect Bayesian equilibrium concept. A tuple $(\gamma, \tilde{\tau}, \tilde{\eta}_A, \tilde{\eta}_B, \tilde{\beta}_A, \tilde{\beta}_B)$ is a perfect Bayesian equilibrium if

1. Given signal $s$ and belief $\tilde{\beta}_A$, class-A journals accept the paper ($\tilde{\eta}_A(s) = Ac$) if and only if its expected quality is higher than $q_A$,

$$\mathbb{E}_{\tilde{\beta}_A}[q|s] \geq q_A \tag{1.5}$$

Given signal $s$ and belief $\tilde{\beta}_B$, class-B journals accept the paper ($\tilde{\eta}_B(s) = Ac$) if and only if

$$\mathbb{E}_{\tilde{\beta}_B}[q|s] \geq q_B \tag{1.6}$$

2. Given her history $\tilde{h}$, the author compares the expected payoff from submitting her paper to a journal of either class. That is,

$$\pi_A(\theta, \tilde{h}) = v \int \gamma(q|\theta, \tilde{h}) \int \mathbb{1}_{\{\tilde{\eta}_A(s)=Ac\}} \phi(s, q, \sigma_s) ds dq - c \tag{1.7}$$

and

$$\pi_B(\theta, \tilde{h}) = \int \gamma(q|\theta, \tilde{h}) \int \mathbb{1}_{\{\tilde{\eta}_B(s)=Ac\}} \phi(s, q, \sigma_s) ds dq - c \tag{1.8}$$

If $\pi_A(\theta, \tilde{h}) \geq \max\{0, \pi_B(\theta, \tilde{h})\}$, then she submits her paper to a journal of class-A: $\tilde{\tau}(\theta, \tilde{h}) = A$. If $\pi_B(\theta, \tilde{h}) \geq \max\{0, \pi_A(\theta, \tilde{h})\}$, she submits it to one of class-B: $\tilde{\tau}(\theta, \tilde{h}) = B$. Otherwise, she stops: $\tilde{\tau}(\theta, \tilde{h}) = stop$.

3. Given $\tilde{\tau}$, $\tilde{\eta}_A$ and $\tilde{\eta}_B$, $\gamma$, $\tilde{\beta}_A$ and $\tilde{\beta}_B$ are derived by Bayes' rule.

**Journal's problem.** Journals have the same problem as the previous case. Given their belief, class-A journals set a threshold of the signal $s_A$, and class-B journals set $s_B$.

**Lemma 3.** *There exists a $s_A$ ($s_B$) such that the journal in class-A (B) accepts the paper if it observes a signal $s > s_A$ ($s > s_B$), and otherwise, it rejects.*

**Author's problem.** The author's posterior belief can be expressed as follows when the history of submissions is $\tilde{h} = (A^i B^j)$:

$$\gamma(q|\theta, \tilde{h}) = \frac{f(q|\theta)\Phi^i(s_A, q, \sigma_s)\Phi^j(s_B, q, \sigma_s)}{\int dq f(q|\theta)\Phi^i(s_A, q, \sigma_s)\Phi^j(s_B, q, \sigma_s)}, \; if \; \tilde{h} = (A^i B^j)$$

Next, the author must decide whether to submit her paper to a class-A journal, to a class-B journal, or to stop. The expected payoff from submitting her paper to class-A (B) journals can be expressed as:

$$\pi_A(\theta, \tilde{h}) = v \int \gamma(q|\theta, \tilde{h})[1 - \Phi(s_A, q, \sigma_s)]dq - c$$

and

$$\pi_B(\theta, \tilde{h}) = \int \gamma(q|\theta, \tilde{h})[1 - \Phi(s_B, q, \sigma_s)]dq - c$$

If the payoffs from submitting to either type of journal are positive, the author compares them. For a high-type author, submitting to a class-A journal is optimal, as it may lead to a higher payoff from publication. After experiencing several rejections, the author may infer that her paper's quality is less likely to be high, leading her to turn to a class-B journal if $\pi_B > 0$, or to stop if $\pi_B < 0$.

For a medium-type author, submitting to a class-B journal is optimal, as it provides a relatively higher probability of acceptance. The author will continue to submit until $\pi_B < 0$.

Finally, for a low-type author, it is not optimal to submit to either class of journals.

**Lemma 4.** *Given $s_A$ and $s_B$, for any history $\tilde{h}$, there exists a unique $\theta^*_A(\tilde{h}) \in (-\infty, +\infty)$ such that $\pi_A(\theta^*_A(\tilde{h}), \tilde{h}) = 0$; there exists a unique $\theta^*_B(\tilde{h}) \in (-\infty, +\infty)$ such that $\pi_B(\theta^*_B(\tilde{h}), \tilde{h}) = 0$; there exists a unique $\theta^*(\tilde{h}) \in [-\infty, +\infty)$ such that $\pi_A(\theta^*(\tilde{h}), \tilde{h}) = \pi_B(\theta^*(\tilde{h}), \tilde{h})$.*

*If the author has not tried all class-A journals, then*[1]

1. *if $\theta^*(\tilde{h}) > \theta^*_A(\tilde{h}) > \theta^*_B(\tilde{h})$, the author with history $\tilde{h}$ submits her paper to a class-A journal if her type $\theta \geq \theta^*(\tilde{h})$. She submits it to a class-B journal if her type $\theta \in [\theta^*_B(\tilde{h}), \theta^*(\tilde{h}))$. She stops if her type $\theta < \theta^*_B(\tilde{h})$.*

[1]Figure 1.4 helps explaining the relation between $\theta^*$, $\theta^*_A$ and $\theta^*_B$.

2. *if $\theta^*(\tilde{h}) \leq \theta_A^*(\tilde{h}) \leq \theta_B^*(\tilde{h})$, the author with history $\tilde{h}$ submits her paper to a class-A journal if her type $\theta \geq \theta_A^*(\tilde{h})$. She stops if her type $\theta < \theta_A^*(\tilde{h})$.*

*Otherwise, the author submits her paper to a class-B journal if her type $\theta \geq \theta_B^*(\tilde{h})$. She stops if her type $\theta < \theta_B^*(\tilde{h})$.*

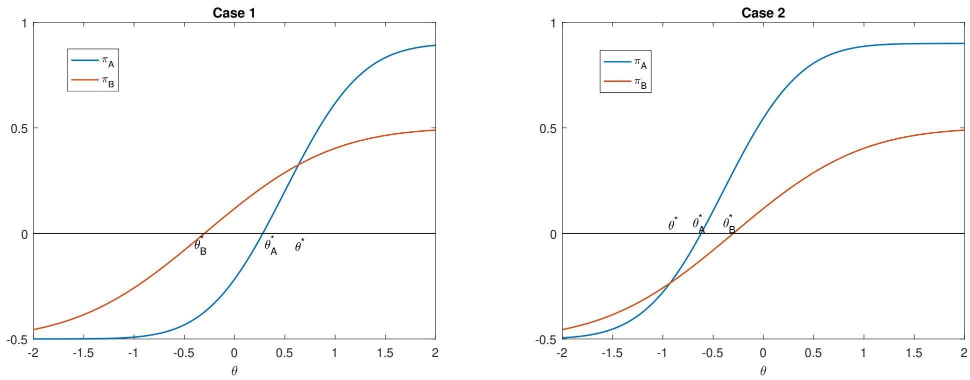



Figure 1.4: Two cases of the order of $\theta^*(\tilde{h})$, $\theta_A^*(\tilde{h})$ and $\theta_B^*(\tilde{h})$.

These two lemmas characterize the agents' best responses. Moreover, one could find that $\theta^*(\tilde{h})$, $\theta_A^*(\tilde{h})$ and $\theta_B^*(\tilde{h})$ are continuous in $s_A$ and $s_B$, and the best responses of journals $s_A^*$ and $s_B^*$ should be bounded. Therefore, an equilibrium exists.

**Proposition 4.** *An equilibrium exists where class-A (B) journals' threshold is $s_A^*$ ($s_B^*$), and the author behaves in the way described in lemma 4.*

*Remark 1: Thresholds of Class-A Journals*

In comparison to the previous section, the availability of an alternative option for low-type authors, i.e., class-B journals, leads to high-type authors submitting their papers to class-A journals. Consequently, class-A journals are more likely to receive high-quality papers, and thus set a lower threshold.

**Example 3.** *Consider a scenario with one class-A journal and other settings identical to Example 1. In the left graph of Figure 1.5, the blue curve represents the threshold of journal $s_A^*$ under different quality standards $q_A$. In the right graph, the blue curve represents the author's strategy $\theta^*$, where she submits her paper to class-A journal if and only if her type $\theta$ is greater than $\theta^*$. With the introduction of class-B journals, the red curve in the left graph represents $s_A^*$, while in the right graph, the author submits her paper to class-A journal only if her type is above the red curve. Otherwise, she chooses a class-B journal or stops.*

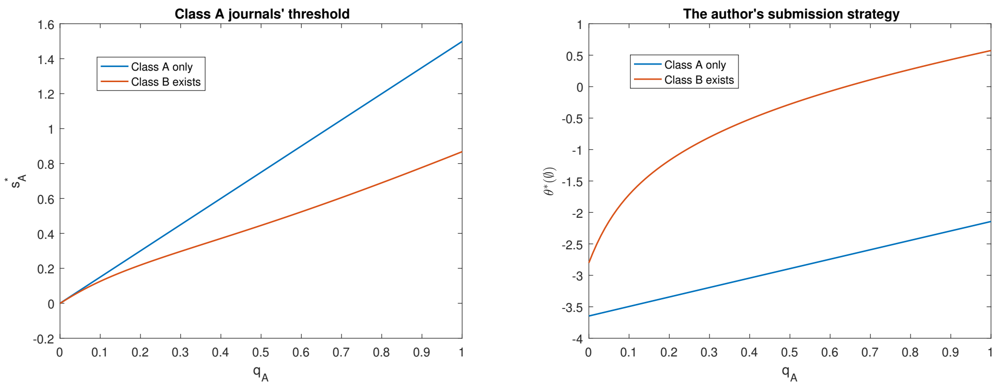



Figure 1.5: The left graph is journals' strategy $s_A^*$. The right graph is the author's strategy $\theta^*(\emptyset)$. If her type $\theta$ is higher than $\theta^*(\emptyset)$, she submits her paper to class-A journal. The parameters are: $\sigma_\theta = \sigma_q = \sigma_s = 1$; $q_B = -1$; $v = 2$ and $c = 0.01$.

*In the right graph, the red curve is above the blue one, indicating that with the availability of class-B journals, authors with lower types will submit their papers to them instead of class-A journals. The left graph shows that the threshold $s_A^*$ of class-A journals is lower because authors' types are higher.* □

The introduction of class-B journals has the effect of making it easier for high-quality papers from high-type authors to be accepted by class-A journals. In Appendix 1.7.2, I provide a detailed analysis of how this diversity impacts market efficiency. Specifically, my findings suggest that the presence of class-B journals leads to an increase in the number of published papers, and high-quality papers are more likely to be published.

*Remark 2: Author's patience*

The above analysis assumes that authors aim to publish their paper as quickly as possible. However, in reality, some authors may choose to be more patient and try submitting to more class-A journals before considering class-B journals. In such cases, let $\delta$ represent the author's discount factor. An author with type $\theta$ and history $\tilde{h}$ will find it optimal to submit their paper to a class-A journal instead of a class-B journal if the following inequality holds:

$$\begin{aligned} &vPr[s > s_A|\theta, \tilde{h}] + \delta u(\tilde{h}A)(1 - Pr[s > s_A|\theta, \tilde{h}]) \\ >&Pr[s > s_B|\theta, \tilde{h}] + \delta u(\tilde{h}B)(1 - Pr[s > s_B|\theta, \tilde{h}]) \end{aligned}$$

Here, $u(\tilde{h}A)$ is the valuation function if the author's history becomes $\tilde{h}A$. It is important to note that $u(\tilde{h}A)$ is greater than $u(\tilde{h}B)$, and $Pr[s > s_A|\theta, \tilde{h}] < Pr[s >$

$s_B|\theta, \tilde{h}]$. Therefore, if an extremely impatient author with type $\theta$ and history $\tilde{h}$ is indifferent between submitting to a class-A or class-B journal ($vPr[s > s_A|\theta, \tilde{h}] = Pr[s > s_B|\theta, \tilde{h}]$), then for a more patient author, submitting to a class-A journal results in a higher payoff than submitting to a class-B journal. As a result, in such situations, class-A journals will raise their thresholds because they receive papers that have likely been rejected more times.

### 1.4.2 Equilibrium Multiplicity

Finite number of journals may result in multiple equilibria. To better understand the intuition, we consider a simplified case of two journals, $A$ and $B$, with different quality standards ($q_A > q_B$). There is no submission fee ($c \to 0$), and publication in either journal yields the same payoff to the author ($v \to 1$). In this case, we do not need to determine the author's cutoff strategy $\theta^*$, but rather consider her submission order. This simplified case can be extended to a more general setting, as discussed in Section 1.5.

In one equilibrium, journal A sets a higher threshold than journal B ($s_A > s_B$) because of its higher quality standard. Consequently, the author submits her paper to journal B first, regardless of her type, because it has a lower threshold and is more likely to accept her paper. After being rejected by journal B, she submits her paper to journal A.

However, when the quality standards of journals A and B are close ($q_B$ is close to $q_A$), another equilibrium arises. In this equilibrium, the author submits her paper to journal A first, regardless of her type. After being rejected by journal A, she submits her paper to journal B. In this case, journal B's threshold $s_B$ is higher than $s_A$ because it receives papers that have been rejected by journal A and are possibly of lower quality. The second equilibrium may seem counterintuitive as the lower-standard journal sets a higher threshold of acceptance.

**Proposition 5.** *There exists $\Delta$ such that when $q_B \in (q_A - \Delta, q_A)$, two asymmetric equilibria exist:*

1. *the author submits her paper to journal B first, and $s_A^* > s_B^*$;*
2. *the author submits her paper to journal A first, and $s_A^* < s_B^*$.*

Which equilibrium is selected depends on which journal exists at first. If journal B is established first, the author submits their paper to it. Then, when journal A is established and receives the rejected paper, it sets a higher threshold not only due to its higher standard but also because it receives papers of potentially lower quality. Conversely, if journal A is established first and then journal B is established, the latter sets a higher threshold despite its lower standard. This insight sheds light on the entry barrier in such markets, where the entrant must set an unfairly high threshold to compete with the incumbent.

## 1.5 Entry Barrier

This section employs a dynamic model to explore the formation of entry barriers. I find that an entrant cannot compete with an incumbent unless it can provide significantly higher value. I then examine how authors' and journals' noisy perceptions of quality further strengthen the entry barrier. Finally, I analyze the robustness of our results and identify the conditions under which the entry barrier exists in the long run (see Appendix 1.7.3).

I consider a market with a continuum of (extremely impatient) authors, each of whom writes one new paper in each period $t = 0, 1, 2, \ldots$ and seeks to publish it. To simplify the model, I assume that there is no submission cost and that authors aim to get published as soon as possible. I also assume that authors have no information about the quality of their own paper other than its prior distribution $f(q)$.

Suppose an incumbent journal exists initially, and its objective is to maximize the sum of the quality of the published papers.[2] The incumbent sets the threshold $s_I^0$ and publishes papers of which signals $s \sim \mathcal{N}(q, \sigma_s^2)$ are higher than the threshold.[3] Denote $Q_I^t$ as the average quality of the papers published in the incumbent's journal in period $t$. $s_I^0$ and $Q_I^t$ are public information. In period 0, the incumbent should publish any paper with the expected quality higher than 0. Thus, it sets $s_I^0$ such that

$$\mathbb{E}[q|s_I^0] = \frac{\int qf(q)\phi(s_I^0, q, \sigma_s)dq}{\int f(q)\phi(s_I^0, q, \sigma_s)dq} = 0$$

The corresponding average quality $Q_I^0$ is given by

$$Q_I^0 = \frac{\int qf(q)[1 - \Phi(s_I^0, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s_I^0, q, \sigma_s)]dq}$$

[2]This preference is a specific case where the minimum standard of journals $q_A$, defined in the preference of journals in section 1.2, is 0.

[3]Sometimes journals do not have this choice but publish the best ones among what they receive under a fixed capacity. However, intuitively, choosing a small capacity is equivalent to setting a high threshold, and vice versa.

Since the incumbent is the only option for the authors for now, they submit their new papers to it.

In the subsequent period $t'$ (without loss of generality, we can say $t' = 1$), an entrant with the same objective as the incumbent issues a new journal and must set its threshold $s_E$ to compete with the incumbent. Journals have no commitment power of their strategies. Even if they announce their threshold, they can change it upon receiving a paper. Let $Q_E^t$ denote the average quality of papers published in the entrant's journal in period $t$. In this period $t'$, authors (probably with two papers in hand: one new and one rejected by the incumbent journal) must simultaneously decide to which journal they should submit new papers. Importantly, authors with papers rejected by the incumbent in the previous period will submit them to the entrant.

In each subsequent period $t \geq t' + 1$, authors simultaneously decide to which journal they should submit their new papers. The journal also receives papers rejected by the other in the previous period.

Let a continuous function $v(Q)$ be the payoff authors receive for publishing in a journal with average quality $Q$ of published papers. $v(Q)$ is assumed to be increasing in $Q$. Without loss of generality, I normalize $v(Q_I^0)$ to be 1.

The question then arises: under which conditions will the incumbent remain the first option for authors, even after the entrant enters the market? To find the answer, I consider the worst (best) case for the incumbent (the entrant): in period $t'$, almost all the authors submit their new papers to the entrant, resulting in the highest average quality $Q_E$ and value $v(Q_E)$. Note that the entrant also receives papers rejected by the incumbent in the previous period $t' - 1$. Thus,

$$Q_E(s_E) = \frac{\int q f(q)[1 + \Phi(s_I^0, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)]dq}{\int f(q)[1 + \Phi(s_I^0, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)]dq}$$

The expected payoff of submitting to the incumbent is

$$v(Q_I^0) \cdot \int f(q)[1 - \Phi(s_I^0, q, \sigma_s)]dq = \int f(q)[1 - \Phi(s_I^0, q, \sigma_s)]dq$$

and to the entrant

$$v(Q_E) \cdot \int f(q)[1 - \Phi(s_E, q, \sigma_s)]dq$$

Then, I define

$$\tilde{v}(Q_E) := \frac{\int f(q)[1 - \Phi(s_I^0, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s_E, q, \sigma_s)]dq}$$

such that if $v < \tilde{v}$, even in the worst case for the incumbent, it is still better to choose it first and we can say the entrant will never become the authors' first option and always receive rejected papers by the incumbent.

**Lemma 5.** $\tilde{v}(Q_I^0) > v(Q_I^0) = 1$.

*Mimicking the incumbent is not optimal*

Lemma 5 shows that $v(Q_E)$ must be lower than $\tilde{v}(Q_E)$ locally around $Q_E = Q_I^0$. The left graph in Figure 1.6 illustrates this point. The blue curve represents the upper bound $\tilde{v}(Q_E)$. Even when $Q_E$ is less than $Q_I^0$, $\tilde{v}(Q_E)$ can be greater than 1. This indicates that if $Q_E$ is close to $Q_I^0$, or if the entrant sets a threshold $s_E$ that leads to an average quality close to the incumbent's, the entrant will never be the authors' first option. The reason to cause that is similar to the formation of the counter-intuitive equilibrium in which the thresholds are inverted. Because the entrant receives rejected papers by the incumbent previously and it can not distinguish between them, it must set an unfairly high threshold to select the good-quality papers mixed with bad-quality ones (as shown in the right graph of Figure 1.6). Consequently, the entrant brings a similar payoff but has a much higher threshold (lower possibility of acceptance), which generates the convention among the authors that they should submit to the incumbent first. This convention leads the entrant to receive rejected papers again in the next period and all subsequent periods.

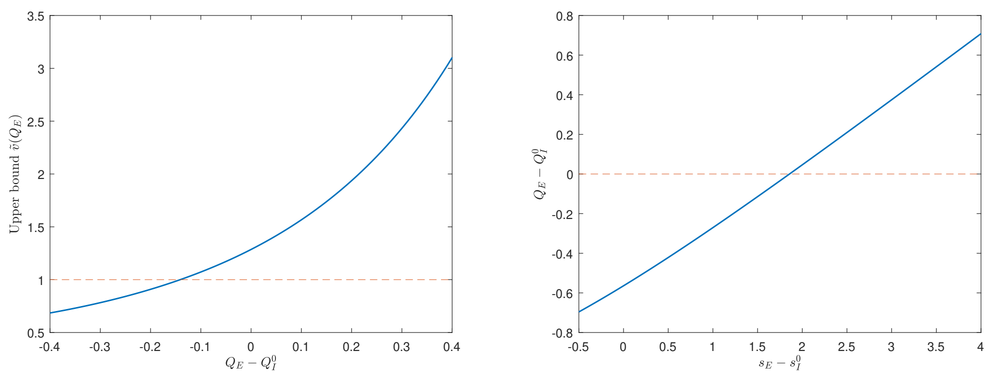



Figure 1.6: The left graph is the upper bound $\tilde{v}$ of the entrant. The right graph is the entrant's average quality corresponding to its threshold. The quality follows a normal distribution: $f(q) = \phi(q, 0, \sigma_q)$. The parameters are: $\sigma_q = \sigma_s = 1$.

**Corollary 1.** *For any value function $v$, there exists $\bar{s}_E > s_I^0$ such that when $s_E \in (s_I^0, \bar{s}_E)$, authors always submit their papers to the incumbent first, where $\bar{s}_E$ induces $Q_E = Q_I^0$.*

Then, the entrant's only option is to differentiate from the incumbent by setting a threshold that leads to an average quality either much higher or much lower than $Q_I^0$. In these cases, $v(Q_E)$ can be higher than $\tilde{v}(Q_E)$, and the entrant can receive not only rejected papers. However, if $Q_E > Q_I^0$ and $v$ is not steep enough that the entrant needs to set an extremely high threshold to have $v(Q_E) > \tilde{v}(Q_E)$, the amount of publication will be minimal. On the other hand, if $Q_E \ll Q_I^0$ and $v$ is not flat enough, the entrant needs to set an extremely low threshold, and many of the papers published will be of poor quality. In either case, although the entrant may become the authors' first option, its utility (sum of the quality of the papers published) will be even worse than being the second option and publishing relatively good papers among those rejected by the incumbent. It is indeed true when the value function is linear: $\bar{v}(Q) = \frac{\max\{0,Q\}}{Q_I^0}$.

**Lemma 6.** $\tilde{v}(Q) > \bar{v}(Q), \forall Q.$

In this linear case, the objectives of the incumbent journal and the authors of new papers are aligned. The journal aims to maximize the sum of quality of the papers published, which is the average quality $Q$, times the probability of acceptance, and times the volume of papers received (if there is only the incumbent, it is 1). Similarly, authors with new papers aim to maximize the expected payoff from publication, which is the value from the journal $\bar{v}(Q)$ multiplied by the acceptance rate. In the absence of the entrant, the incumbent and the authors have already reached mutual optimization. The entrant suffering from rejected papers in the candidate pool can not strictly improve the situation. Therefore, in this specific case, the authors with new papers have no incentive to switch to the entrant's journal, regardless of the threshold set by the entrant.

To quantify the steepness of the value function $v$, I introduce a parameter $\alpha \geq 0$ as follows:

$$v(Q) = \left[\frac{\max\{0, Q\}}{Q_I^0}\right]^\alpha$$

If $v$ is neither flat nor steep (i.e., $\alpha$ is neither too low nor too high), the entrant would instead choose to be the authors' second option and set $s_E^1$ such that:

$$\mathbb{E}[q|s_1 < s_I^0, s_2 = s_E^1] = \frac{\int qf(q)\Phi(s_I^0, q, \sigma_s)\phi(s_E^1, q, \sigma_s)dq}{\int f(q)\Phi(s_I^0, q, \sigma_s)\phi(s_E^1, q, \sigma_s)dq} = 0$$

**Proposition 6.** *There exists* $\underline{\alpha} \in [0, 1)$ *and* $\bar{\alpha} \in (1, +\infty)$ *such that when* $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, *in the equilibrium, the incumbent sets the threshold* $s_I^0$ *and the entrant sets* $s_E^1$*; in* *each period, the authors with new papers always submit to the incumbent first and then to the entrant after rejection.*

*Remark:* In the case where the steepness or flatness of $v$ falls outside the interval $[\underline{\alpha}, \bar{\alpha}]$, multiple equilibria may exist from the authors' perspective, rendering the result unavailable. To overcome this issue, I assume that the equilibrium that the last mover favors will prevail (i.e., all authors with new papers will submit to the entrant after it enters if multiple equilibria exist; after that, if the incumbent changes its threshold, all authors with new papers will submit to it if multiple equilibria exist, and so on).

Suppose $v$ is steep, and both journals want to set a higher threshold than the other. This process continues until one finds that the amount of publication is too low and it is better to set a low threshold. The other then finds it better to lower its threshold as well, and the process begins anew. As a result, there is no pure-strategy equilibrium; only a mixed-strategy equilibrium exists.[4]

### 1.5.1 Noisy Perception

The generation of entry barriers is driven by two key factors: the noisy perception of quality and the ignorance of journals about authors' submission histories. In this section, I examine how perfect perception from either side reduces the barriers to entry.

#### Impact of Journals' Perception of Quality

As the signal of journals becomes less noisy, it becomes harder for the incumbent to maintain its advantage. As shown in the left graph of Figure 1.7, the upper bound $\tilde{v}$ moves downward and becomes closer to the point $(Q_I^0, 1)$. If I simulate the range $[\underline{\alpha}, \bar{\alpha}]$, it shrinks as $\sigma_s$ decreases, but it does not converge to a singleton as $\sigma_s \to 0$. When the signal is perfect ($\sigma_s = 0$), $\tilde{v}$ crosses $(Q_I^0, 1)$.

**Lemma 7.** *If $\sigma_s = 0$ and $s_E = s_I$, then $Q_E = Q_I$.*

This lemma implies that when the signal is perfect and the entrant sets the same threshold as the incumbent, the entrant can always mimic the incumbent. Even though the entrant still receives rejected papers, it knows they will eventually be eliminated because the signal is perfect and both journals have the same threshold.

[4] Similar result can be found in Varian [1980].

The average quality of the two journals will be the same, and the authors with new papers randomly submit to either journal first.

**Proposition 7.** *Let* $\underline{\alpha}^0 = \lim_{\sigma_s \to 0} \underline{\alpha}$ *and* $\bar{\alpha}^0 = \lim_{\sigma_s \to 0} \bar{\alpha}$*. When* $\sigma_s = 0$ *and* $\alpha \in [\underline{\alpha}^0, \bar{\alpha}^0]$*, in the equilibrium, the incumbent and the entrant sets the same threshold* $s_I = s_E = 0$*; in each period, the authors with new papers always submit to either journal first with Probability* $1/2$*, and then to the other one after rejection.*

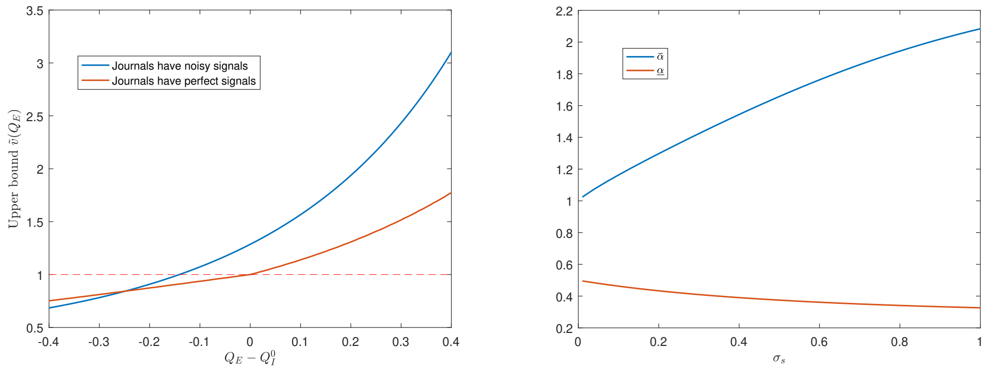



Figure 1.7: The quality follows a normal distribution: $f(q) = \phi(q, 0, \sigma_q)$. The parameters are: $\sigma_q = 1$.

### Impact of Author's Perception of Quality

In contrast to the previously discussed scenario of coarse authors, I now assume that the author knows their type, as defined in Section 1.2. To characterize the author's perception of quality, I assume that their type $\theta$ follows a normal distribution $\mathcal{N}(0, \sigma_\theta^2)$ and the quality conditional on the type $q|\theta$ follows $\mathcal{N}(\theta, \sigma_q^2)$. I set $\sigma_\theta^2 + \sigma_q^2 = 1$, so that when $\sigma_\theta$ approaches 1, authors know the quality perfectly, and when $\sigma_\theta$ approaches 0, we have the fully coarse case.

If the entrant sets a slightly higher threshold $s_E > s_I$ than the incumbent, the market splits, and two equilibria can exist with cutoffs $\bar{\theta}$ and $\underline{\theta}$, where $\bar{\theta} > \underline{\theta}$. In one equilibrium, authors with type $\theta > \bar{\theta}$ choose the entrant first, and the rest choose the incumbent. In the other equilibrium, authors with type $\theta > \underline{\theta}$ choose the entrant first. The former equilibrium favors the incumbent, while the latter favors the entrant. However, it requires authors with $\theta \in (\underline{\theta}, \bar{\theta})$ to change their strategy simultaneously to transition from one equilibrium to the other.

With partial information about quality, it is difficult to predict which equilibrium will be selected if multiple ones exist. For instance, if the entrant sets the same threshold as the incumbent, $s_E = s_I$, one equilibrium is that all authors choose the incumbent first regardless of their type, and the entrant receives only the rejected papers. In this situation, the incumbent publishes papers with higher average quality, and authors go to the incumbent first because the possibility of acceptance is the same. However, suppose that at some point, only the low-type authors choose the incumbent while the rest choose the entrant. In this case, the average quality of papers published in the entrant journal exceeds that of the incumbent journal. The equilibrium then transitions to the one favoring the entrant, and the next transition can occur at any time.

To investigate the stability of the equilibrium in which the incumbent holds the advantage, I introduce the concept of "inertia". I assume that, in each period, only a certain proportion of the authors can change their strategies, while the rest stick to their last choices. I then find the minimal level of "inertia" necessary for the incumbent to maintain its advantage. If this value is high, the entry barriers reduce for the entrant.

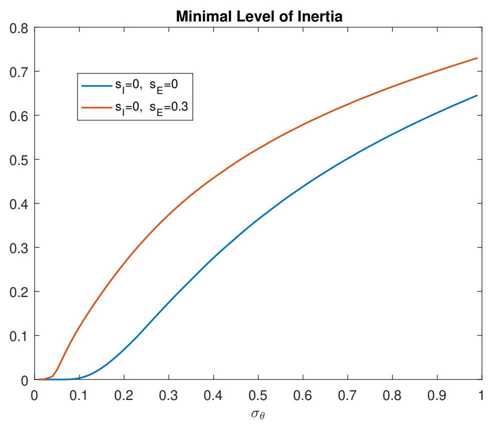



Figure 1.8: The minimal level of inertia under different perfectness of the authors' perception about the quality. The value function is linear, $v(Q) = \max\{0, Q\}/Q_I^0$.

Figure 1.8 displays the minimum level of inertia required to establish entry barriers for the entrant. As shown, when $\sigma_\theta$ approaches 0, the required inertia is negligible, which is consistent with previous findings. However, as the authors become more sophisticated about quality, the required inertia increases, making it more difficult for the incumbent to maintain their advantage.

The increase in required inertia can be attributed to authors possessing clearer private information about quality. When authors have a better understanding of their paper's quality, those who perceive their papers to be high-quality will be more likely to submit to the journal that provides higher value, whether it is the incumbent or the entrant. As more authors possess this information, a greater proportion of authors will be willing to switch journals. Consequently, a higher level of inertia is needed to prevent authors from doing so. Therefore, clearer private information raises the minimum level of inertia, which in turn reduces the entry barriers for the entrant.

## 1.6 Conclusion

The model in this study investigates a noisy market with information asymmetry and dynamic learning. I find that: i) without knowing the quality of "talent" perfectly, high-type seller try to sell more times compared to low-type ones. In contrast, knowing it perfectly, they will always try; and ii) there can be multiple equilibria but the one favoring the incumbent will be selected, which triggers the existence of the entry barrier. It shows that the entrant can not compete with the incumbent for both the market share and the quality of "talent". Moreover, the noisy perception of quality will make the entry barrier higher.

The existence of entry barriers found in this study has some inspiration in policymaking. The key factor to generate the barriers is the noisy perception. Therefore, one direction of reducing the barriers is to implement accurate perceiving technology or to request certification. A second direction is to diminish the information asymmetry in the market. For instance, the journals can require the authors to reveal their submission history. This has already appeared in the market where top journals have their affiliation journals. When the author resubmits her paper rejected by top journals to their affiliation, she should also hand in the previous referee's report. It mitigates the information asymmetry between top journals and their affiliations. However, it exacerbates the problem for other field journals because they will not know the authors' history or they have no commitment power to ask authors to reveal. A third direction of breaking the vicious cycle is to change the convention radically. One example is the draft lottery system in the North American sports league. It leads high-potential rookies to go to weak teams so that the oligopoly becomes harder to form.

# 1.7 Appendix

## 1.7.1 Author's Noisy Perception

If the author has perfect information about the quality of her paper before submission ($f(q|\theta)$ is a delta function), her problem simplifies to determining whether

$$\pi_A(q,h) = v[1 - \Phi(s_A, q, \sigma_s)] - c > 0$$

In this case, the author does not learn anything from rejection, and her payoff does not depend on her submission history. If the quality of her paper is higher than a certain threshold $q^*$, which satisfies $v[1-\Phi(s_A, q^*, \sigma_s)] = c$, then she will submit the paper until it is accepted or there is no chance left. This feature differs from the previous case, where the author has a rough perception of quality.

A preliminary result is that $q^*$ must be lower than $q_A$ in the equilibrium. If $q^*$ is higher than $q_A$, the editor would accept the paper regardless of the signal it receives, knowing that the paper's quality is higher than the threshold. In this case, the author would also submit the paper even if its quality is lower than $q^*$. Therefore, although the journals know the quality is at least $q^*$, it could be rejected before. In other words, selection bias still exists.

In contrast, without perfect perception, she learns after each submission, and the high-type author tries more times and stops after several submissions. This section analyzes how this ignorance affects the agents' behavior.

Consider a case where the author's type $\theta$ follows a normal distribution: $\mu(\theta) = \phi(\theta, 0, \sigma_\theta)$. The quality follows a normal distribution conditional on $\theta$: $f(q|\theta) = \phi(q, \theta, \sigma_q)$. $\sigma_q$ is a measure of the author's ignorance level. There are two homogeneous journals A in the field, which yields $v$ to the author for the publication. The submission cost is $c$. Their standard of quality $q_A$ is 0. The journal observes a noisy signal conditional on the quality $s = q + \epsilon$, $\epsilon \sim \mathcal{N}(0, \sigma_s)$.

When $\sigma_q$ is close to 0, the author has a precise perception of quality. The analysis at the beginning indicates that $\theta^*(\emptyset)$ are close to $\theta^*(A)$. As $\sigma_q$ increases, the author with a relatively lower type stops after getting a rejection. As shown in Figure 1.9, the difference between $\theta^*(A)$ and $\theta^*(\emptyset)$ becomes larger. Another finding is the trend of $\theta^*(\emptyset)$ depends on the benefit-cost ratio ($v/c$). When the author is extremely ignorant, her type implies little information. If $v/c$ is high, the ignorant author without getting rejected wants to try regardless of her type. Thus, $\theta^*(\emptyset)$ decreases as $\sigma_q$ increases. In contrast, if $v/c$ is low, the ignorant author has less

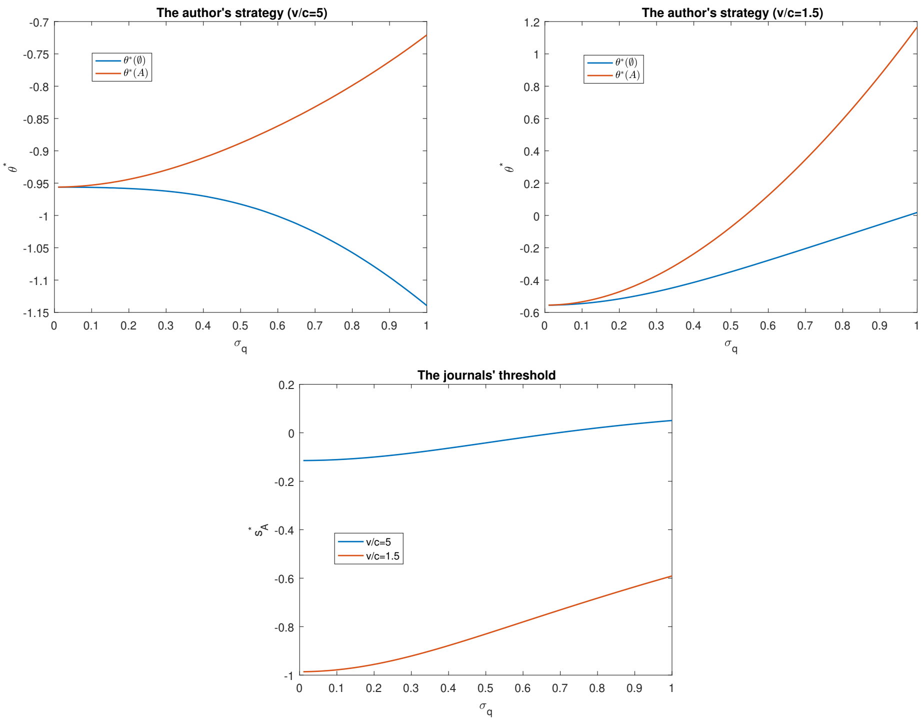



Figure 1.9: The left graph is the acceptance rate of a paper with quality $q$. The right graph is the author's payoff if the quality is $q$.

willing to have a try.

The journals are more likely to receive papers of low quality as the author becomes ignorant. Thus, they raise their thresholds.

### 1.7.2 Market Differentiation

In Section 1.4, I present the idea that introducing an ordinary class of journals splits the market of authors, makes the top-class journals easy to select those papers with high quality, and thus is beneficial to efficiency. This section further discusses it by analyzing the situation of two journals with three cases: 1. both set a low standard of quality; 2. both set a high standard of quality; 3. one set a high standard, and the other set a low one.

More specifically, consider a case where the author's type $\theta$ follows a normal distribution: $\mu(\theta) = \phi(\theta, 0, 1)$. The quality follows a normal distribution conditional on $\theta$: $f(q|\theta) = \phi(q, \theta, 1)$. Assume that the paper with a quality $q > 0$ is valuable and should be published. A paper with higher quality is more valuable. There are two journals A and B in the field. The submission cost is $c = 0.1$. The journal observes a noisy signal conditional on the quality $s = q + \epsilon$, $\epsilon \sim \mathcal{N}(0, 1)$. In case 1, both set a baseline standard of quality $q_A = q_B = 0$. A publication in either of them yields 1 to the author. In case 2, both set an aggressive standard $q_A = q_B = 1$. A publication on either of them yields 2. In case 3, journal A sets $q_A = 1$ while journal B sets $q_A = 0$. A publication on journal A yields 2, and 1 on journal B.

I define efficiency in two aspects: i) whether the paper published is worthwhile. I use the following $W_1$ to measure the global quality of the papers in the market.

$$W_1 = \mathbb{E}[q|Accepted] \cdot Pr[Accepted] = \int_{-\infty}^{+\infty} q\phi(q, 0, 2)Pr[Accepted|q]dq$$

A higher $W_1$ means that the market generates more valuable knowledge, and ii) whether a high-quality paper is easier to be published in a journal bringing higher payoff, and a relatively low-quality paper is easier to be published in a journal bringing lower payoff.

First, I solve the equilibrium in these 3 cases.

*Case 1*

The author with a type $\theta > -1.65$ submits her paper to either journal first. If she gets rejected, she submits to the other journal if her type $\theta > -1.39$. Both journals set the threshold $s_A = s_B = 0.16$.

*Case 2*

The author with a type $\theta > -0.75$ submits her paper to either journal first. If she gets rejected, she submits to the other journal if her type $\theta > -0.58$. Both journals set the threshold $s_A = s_B = 1.58$.

*Case 3*

The author with a type $\theta > 0.71$ submits her paper to journal A first. If she gets rejected, she submits to journal B. The author with a type $\theta \in (-1.73, 0.71)$ submits her paper to journal B first. If she gets rejected, she submits to journal A if her type $\theta > -0.42$. Journal A sets the threshold $s_A = 1.31$ while journal B sets $s_B = 0.08$.

Secondly, I analyze market efficiency in two aspects mentioned. From the first aspect, I compute the value of $W_1$ under three cases: 0.4788, 0.3948, and 0.4786. The first and third cases generate a similar amount of knowledge. In the first case, more papers are published and some of them are of low quality, compared to the third case. This is because journal A has a higher standard of quality in case 3. The left graph in Figure 1.10 also shows that the yellow curve is slightly below the blue one because journal A accepts fewer papers but with higher quality. In case 2, $W_1$ is lower because of the fact that far fewer papers are published. This is not only

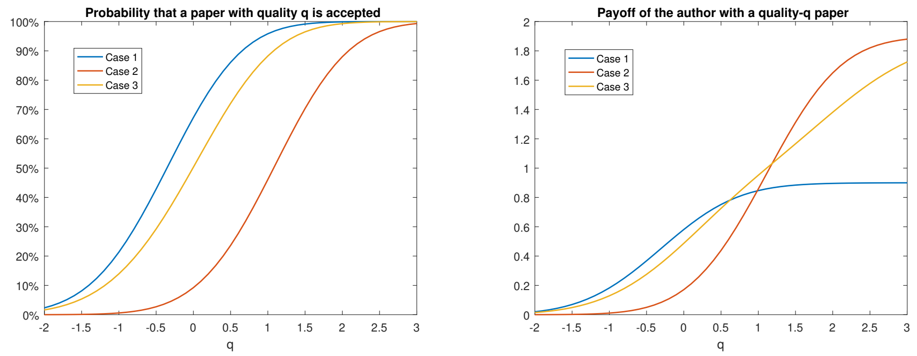



Figure 1.10: The left graph is the acceptance rate of a paper with quality $q$. The right graph is the author's payoff if the quality is $q$.

because journal B has a high standard but also because journals should set higher thresholds to correct selection bias. The right graph shows the author's payoff if the quality is $q$. In case 1, the author with a high-quality paper does not get rewarded while in case 2, only the substantial-high-quality paper brings a reward. In case 3, the extreme situation is improved by splitting the authors.

From the second aspect, in case 3, journal A's threshold $s_A$ is lower compared to case 2, and journal B's threshold $s_B$ is lower compared to case 1. The authors find it easier to publish either a high-quality or a relatively low-quality paper in the corresponding journal. Still, it is because of the splitting which weakens selection bias.

### 1.7.3 Entry Barrier in the Long Run

In Section 1.5, I find the existence of entry barriers. The next question is "Is it robust?" In other words, if we tremble authors' behavior, will the barriers still exist? This section introduces "mutation". It is assumed that there is some probability that the old authors pass away. Not being familiar with the payoffs from submitting, the new coming authors just randomly choose one journal. The answer to the question is the barriers reduce compared to the case without mutation, but when the number of authors is large, it takes an extremely long time for the entrant to transcend it.

First, instead of using the absolute value function $v(Q)$, I define the continuous function $R(Q_I^t, Q_E^t)$ to be the authors' relative payoff of a publication in the entrant's journal compared to the incumbent's given the average quality of their papers published respectively. More precisely, given $Q_I^t$ and $Q_E^t$, if the authors' payoff of a publication in the incumbent's journal is 1, the payoff from the entrant is $R(Q_I^t, Q_E^t)$. $R(Q_I^t, Q_E^t)$ is assumed to be increasing in $Q_E^t$, but decreasing or $Q_I^t$.

Note that the absolute value function is just a special case of the generalized relative value function.

Next, I define the deterministic dynamic[5] for the authors. Specifically, they compute the expected payoffs from submitting to the incumbent or the entrant based on the previous period. In each period $t$, they submit to the entrant if doing so yields a higher expected payoff than submitting to the entrant,

$$z_t = b(z_{t-1}) = \begin{cases} N & if\ \pi_I(t) \geq \pi_E(t), \\ 0 & otherwise \end{cases}$$

where $z_t$ is the number of authors who choose to submit to the incumbent first in period $t$, and $\pi_I(t)$ ($\pi_E(t)$) is the expected payoff from submitting to the incumbent (the entrant),

$$\pi_I(t) = \int f(q)\big[1 - \Phi\big(s_I, q, \sigma_s\big)\big]dq$$

$$\pi_E(t) = R(Q_I^{t-1}, Q_E^{t-1}) \int f(q)\big[1 - \Phi\big(s_E, q, \sigma_s\big)\big]dq$$

Finally, assume that in each period, with Probability $\epsilon$, each author changes her submission order, which is the mutation. Then, I define the long run equilibrium according to definitions 1 and 2 in Kandori et al. [1993]. First, one has a stochastic process of $z_t$,

$$z_t = b(z_{t-1}) + x_t - y_t$$

where $x_t$ and $y_t$ are binomial distributions,

$$x_t \sim Bin(N - b(z_{t-1}), \epsilon),\ \ y_t \sim Bin(b(z_{t-1}), \epsilon)$$

Then, one gets a Markov chain of $z_t$. Let $P$ be the Markov matrix, in which the element

$$p_{ij} = Pr[z_{t+1} = j | z_t = i]$$

Let $\mu_\epsilon = \big(\mu_\epsilon(1), \mu_\epsilon(2), ..., \mu_\epsilon(N)\big)$ be the stationary distribution of $z_t$, which is $\mu_\epsilon P = \mu_\epsilon$.

**Definition 1.** *Denote the limit distribution*

$$\mu^* = \lim_{\epsilon \to 0} \mu_\epsilon.$$

[5]This deterministic dynamic can be generalized to any one satisfying: $sign\{b(z_{t-1}) - z_{t-1}\} = sign\{\pi_I(t) - \pi_E(t)\}$

*Always submitting to the incumbent (entrant) first is the long run equilibrium if* $\mu^*(N) = 1$ *(*$\mu^*(0) = 1$*).*

I look for the knife-edge situation in which if multiple pure-strategy equilibria exist (authors always submitting to the incumbent or to the entrant), both can be stable against mutations in the long run. To induce the corresponding relative value function $\hat{R}$, we consider the mixed-strategy equilibrium: for any thresholds $s_I$ and $s_E$ set by the journals, half of the authors with new papers choosing the incumbent first and the other half choosing the entrant first. Then, their average quality of the papers published are

$$Q_I = \frac{\int qf(q)[1/2 + \Phi(s_E, q, \sigma_s)/2][1 - \Phi(s_I, q, \sigma_s)]dq}{\int f(q)[1/2 + \Phi(s_E, q, \sigma_s)/2][1 - \Phi(s_I, q, \sigma_s)]dq}$$

$$Q_E = \frac{\int qf(q)[1/2 + \Phi(s_I, q, \sigma_s)/2][1 - \Phi(s_E, q, \sigma_s)]dq}{\int f(q)[1/2 + \Phi(s_I, q, \sigma_s)/2][1 - \Phi(s_E, q, \sigma_s)]dq}$$

The authors' indifference condition indicates that the expected payoffs of submitting to either journal are the same. Therefore, $\hat{R}$ is defined as

$$\hat{R}(Q_I, Q_E) := \frac{\int f(q)[1 - \Phi(s_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s_E, q, \sigma_s)]dq}$$

**By Kandori et al. [1993] Theorem 3,** *if* $R < \hat{R}$*, always submitting to the incumbent first is the long run equilibrium regardless of the thresholds the journals set. Otherwise, always submitting to the entrant is.*

Similarly to $\tilde{v}$, $\tilde{R}(Q_I, Q_E)$ can be defined as the upper bound in the relative value form:

$$\tilde{R}(Q_I, Q_E) := \frac{\int f(q)[1 - \Phi(s_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s_E, q, \sigma_s)]dq}$$

where

$$Q_I = \frac{\int qf(q)[1 - \Phi(s_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s_I, q, \sigma_s)]dq}$$

$$Q_E = \frac{\int qf(q)[1 + \Phi(s_I, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)]dq}{\int f(q)[1 + \Phi(s_I, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)]dq}$$

Under the relative value $\hat{R}$, regardless of the thresholds the journals set, neither journal will definitely be the disadvantaged side by receiving only the rejected papers in the long run. However, the following proposition shows that without mutation, the latecomer is always the disadvantage side in a market potentially allowing two journals to compete fairly.

**Proposition 8.** *$\hat{R}(Q_I, Q_E) < \tilde{R}(Q_I, Q_E)$ for any $Q_I$ and $Q_E$.*

*How "long" does it need to transit to the long run equilibrium?*

Even if the relative value is above $\hat{R}$, it takes time to have enough proportion of authors to mutate and to transit from the status quo to another equilibrium. First, a higher relative value requires fewer mutations. Thus, the time needed to change the status quo is shorter. Another factor is the number of authors. With more authors, it asks for more mutations to reach the turning proportion. Therefore, it becomes longer to transit from the status quo.

Consider an example with $s_I = s_E = 0$ and $\epsilon = 0.1$. Table 1.1 shows the expected periods for transition to the equilibrium favoring the entrant under some bundles of parameters. It shows that when the number of authors is large, the entrant needs to wait billions of periods to become their first option.

| $N$ | $R/\hat{R}$ | $T$ | | $N$ | $R/\hat{R}$ | $T$ |
|---|---|---|---|---|---|---|
| 20 | 1.1 | $3.7 \times 10^8$ | | 2 | 1.5 | 2 |
| 20 | 1.2 | $1.8 \times 10^7$ | | 4 | 1.5 | 6 |
| 20 | 1.3 | $1.2 \times 10^6$ | | 6 | 1.5 | 15 |
| 20 | 1.4 | $8.9 \times 10^4$ | | 10 | 1.5 | 89 |
| 20 | 1.5 | $7.8 \times 10^3$ | | 20 | 1.4 | $7.8 \times 10^3$ |
| 20 | 1.6 | 770 | | 30 | 1.5 | $6.9 \times 10^5$ |
| 20 | 1.7 | 80 | | 50 | 1.5 | $5.4 \times 10^9$ |
| 20 | 1.8 | 8 | | 100 | 1.5 | $3.0 \times 10^{19}$ |

Table 1.1: $N$: number of authors, $R/\hat{R}$: the ratio of the relative value compare to the fair one, $T$: expected periods for transition to the equilibrium favoring the entrant.

### 1.7.4 Proofs

**Proof of lemma 1:**

The left hand side of (1.1) is increasing and continuous in $s$. Moreover,

$$\lim_{s \to +\infty} \mathbb{E}_{\beta_A}[q|s] = +\infty, \quad \lim_{s \to -\infty} \mathbb{E}_{\beta_A}[q|s] = -\infty$$

Therefore, there exists a $s_A$ such that

$$\mathbb{E}_{\beta_A}[q|s_A] = q_A. \quad \blacksquare$$

**Proof of lemma 2:**

Because $\gamma(q|\theta,h)$ satisfies the MLRP, $\pi_A(\theta,h)$ is monotonically increasing. Along with

$$\lim_{\theta\to+\infty}\pi_A(\theta,h)=v-c>0,\ \lim_{\theta\to-\infty}\pi_A(\theta,h)=-c<0,\forall h\in H$$

there exists a unique $\theta^*_A(h)$ such that $\pi_A(\theta^*_A(h),h)=0$. Additionally, $\pi_A(\theta^*_A(h),h)\geq 0$ if $\theta\geq\theta^*_A(h)$.

$\frac{\gamma(q|\theta,A^{i+1})}{\gamma(q|\theta,A^i)}\propto\Phi(s_A,q,\sigma_s)$ is decreasing in $q$. As a result, either $\gamma(q|\theta,A^{i+1})$ is always lower than $\gamma(q|\theta,A^i)$, or they are single crossing. Under both cases, $\pi_A(\theta,A^{i+1})\geq\pi_A(\theta,A^i)$. Therefore, $\theta^*_A(A^{m-1})>...>\theta^*_A(A)>\theta^*_A(\emptyset)$. ■

**Proof of proposition 1:**

According to lemma 2, given $s_A$, $\theta^*_A(h)$ are well-defined and continuous in $s_A$ because $\pi_A$ is continuous in $s_A$. Then, the journal receiving a paper forms a belief $\beta_A$ based on (1.3) and (1.4). According to lemma 1, the journal could find the optimal threshold noted as $\omega(s_A)$. The left hand side of (1.1) and $\beta_A$ are continuous in $\theta^*_A(h)$. Therefore, $\omega(s_A)$ is continuous in $s_A$. Obviously, $\omega(s_A)$ is bounded. As a result, there exists a fixed point $\omega(s^*_A)=s^*_A$ according to Brouwer fixed-point theorem.

Secondly,

$$\pi_A(\theta^*(h),h=A^i)=v\frac{\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)[1-\Phi(s_A,q,\sigma_s)]dq}{\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)dq}-c=0$$

Define

$$\begin{aligned}G(s_A):=&\,v\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)[1-\Phi(s_A,q,\sigma_s)]dq-c\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)dq\\ =&\,(v-c)\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)dq-v\int f(q|\theta^*(h))\Phi^{i+1}(s_A,q,\sigma_s)dq\\ =&\,i(v-c)\int F(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)\phi(s_A,q,\sigma_s)dq\\ &-(i+1)v\int F(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)\phi(s_A,q,\sigma_s)dq\end{aligned}$$

Then,

$$\begin{aligned}G'(s_A)=&\,i(v-c)\int f(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)\phi(s_A,q,\sigma_s)dq\\ &-(i+1)v\int f(q|\theta^*(h))\Phi^i(s_A,q,\sigma_s)\phi(s_A,q,\sigma_s)dq\end{aligned}$$

Since $F(q|\theta^*(h))$ is log-concave, $F/f$ is increasing in $q$. Then,

$$\frac{F(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)}{\int F(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)dq} \quad FOSDs \quad \frac{f(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)}{\int f(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)dq}$$
$$\Rightarrow \frac{\int f(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)\Phi(s_A,q,\sigma_s)dq}{\int f(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)dq} > \frac{\int F(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)\Phi(s_A,q,\sigma_s)dq}{\int F(q|\theta^*(h))\Phi^{i-1}(s_A,q,\sigma_s)dq}$$
$$\Rightarrow G'(s_A) < 0$$

Therefore, as $s_A$ increases, $\pi_A(\theta,h)$ decreases. Then. $\theta^*_A(h)$ increases, meaning that only high-type authors find it optimal to submit. Therefore, $\omega(s_A)$ decreases because the journal is more likely to receive a high-quality paper from high-type author. Thus, the equilibrium is unique because $\omega(s_A)$ is a decreasing function. ■

**Proof of proposition 2:**

If $m=1$, one can find $s^*_A$ and $\theta^*_A(\emptyset)$. Then, if $m=2$, one consider the expected quality if journals keep the standard at $s^*_A$.

$$\frac{\int_{-\infty}^{+\infty} q \int_{\theta^*_A(A)}^{+\infty} \gamma(q|\theta,A)d\theta dq}{\int_{-\infty}^{+\infty}\int_{\theta^*_A(A)}^{+\infty} \gamma(q|\theta,A)d\theta dq}, \ where\ \gamma(q|\theta,A) \propto f(q|\theta)\Phi(s^*_A,q,\sigma_s)$$

If $c \to 0$, $\theta^*_A(A) \to -\infty$. Along with $\gamma(q|\theta,\emptyset)$ first-order stochastic dominating $\gamma(q|\theta,A)$, the expected quality must be lower than $q_A$ if $c \to 0$. Thus, one can find $\underline{c}_1 \in (0,v]$ such that the expected quality is higher than $q_A$.

Then, for $m>2$, one repeats above process to find $\underline{c}_{m-1}$. $\underline{c} = \inf_{1\le i\le m-1} \underline{c}_i$. ■

**Proof of proposition 3:**

$\int_{-\infty}^{+\infty} f(q|\theta^*_A(h),h)[1-\Phi(s_A,q,\sigma_s)]dq = c/v$ and the left hand side is increasing in $\theta^*_A(h)$. Therefore, as $v/c$ increases, $\theta^*_A(h)$ decreases. It lowers the paper's expected quality. Thus, to make it equal to $q_A$, journals raise the threshold. ■

**Proof of lemma 3:**

The left hand side of (1.5) is increasing and continuous in $s$. Moreover,

$$\lim_{s\to+\infty} \mathbb{E}_{\tilde{\beta}_A}[q|s] = +\infty,\ \lim_{s\to-\infty} \mathbb{E}_{\tilde{\beta}_A}[q|s] = -\infty,\ \forall f \in \mathcal{F}$$

Therefore, there exists a $s_A$ $(s_B)$ such that

$$\mathbb{E}_{\tilde{\beta}_A}[q|s_A] = q_A,\ \mathbb{E}_{\tilde{\beta}_B}[q|s_B] = q_B.\ ■$$

**Proof of lemma 4:**

Because $\gamma(q|\theta,\tilde{h})$ satisfies the MLRP, $\pi_A(\theta,\tilde{h})$ is monotonically increasing. Along with

$$\lim_{\theta\to+\infty}\pi_A(\theta,\tilde{h}) = v-c>0,\quad \lim_{\theta\to-\infty}\pi_A(\theta,\tilde{h}) = -c<0,$$

there exists a unique $\theta^*_A(\tilde{h})$ such that $\pi_A(\theta^*_A(\tilde{h}),\tilde{h})=0$. Similarly, a unique $\theta^*_B(\tilde{h})$ exists such that $\pi_B(\theta^*_B(\tilde{h}),\tilde{h})=0$.

$\frac{v\phi(s_A,q,\sigma_s)}{\phi(s_B,q,\sigma_s)}$ is monotone in $q$. Then, the functions $v[1-\Phi(s_A,q,\sigma_s)]$ and $[1-\Phi(s_B,q,\sigma_s)]$ are single crossing. As a result, either $\pi_A(\theta,\tilde{h})$ is always higher than $\pi_B(\theta,\tilde{h})$, which corresponds to $\theta^*(\tilde{h})=-\infty$, or $\pi_A(\theta,\tilde{h})$ and $\pi_B(\theta,\tilde{h})$ crosses at a $\theta^*(\tilde{h})\in(-\infty,+\infty)$.

Finally, based on the definition of $\theta^*(\tilde{h})$, $\theta^*_A(\tilde{h})$ and $\theta^*_B(\tilde{h})$, there could be only two cases: 1. $\theta^*(\tilde{h})>\theta^*_A(\tilde{h})>\theta^*_B(\tilde{h})$; 2. $\theta^*(\tilde{h})\le\theta^*_A(\tilde{h})\le\theta^*_B(\tilde{h})$. In the first case, when $\theta\ge\theta^*(\tilde{h})$, $\pi_A(\theta,\tilde{h})>0$ and $\pi_A(\theta,\tilde{h})\ge\pi_B(\theta,\tilde{h})$. When $\theta\in[\theta^*_B(\tilde{h}),\theta^*(\tilde{h}))$, $\pi_B(\theta,\tilde{h})\ge0$ and $\pi_B(\theta,\tilde{h})>\pi_A(\theta,\tilde{h})$. When $\theta<\theta^*_B(\tilde{h})$, $\pi_A(\theta,\tilde{h})<0$ and $\pi_B(\theta,\tilde{h})<0$. In the second case, when $\theta\ge\theta^*_A(\tilde{h})$, $\pi_A(\theta,\tilde{h})\ge0$ and $\pi_A(\theta,\tilde{h})>\pi_B(\theta,\tilde{h})$. When $\theta<\theta^*_A(\tilde{h})$, $\pi_A(\theta,\tilde{h})<0$ and $\pi_B(\theta,\tilde{h})<0$. ■

**Proof of proposition 4:**

Proof: According to lemma 2, given $s_A$ and $s_B$, $\theta^*_A(\tilde{h})$, $\theta^*_B(\tilde{h})$ and $\theta^*(\tilde{h})$ are well-defined and continuous in $s_A$ and $s_B$ because $\pi_A$ and $\pi_B$ are continuous in $s_A$ and $s_B$ respectively. Then, the journal receiving a paper forms the believes $\tilde{\beta}_A$ and $\tilde{\beta}_B$ based on Bayes' rule. According to lemma 3, journals could find the optimal threshold noted as $\omega(s_A,s_B)=(s'_A,s'_B)$. The left hand side of (1.5) and $\tilde{\beta}_A$ are continuous in $\theta^*_A(\tilde{h})$. Therefore, $\omega(s_A,s_B)$ is continuous in $s_A$. Similarly, $\omega(s_A,s_B)$ is continuous in $s_B$. Obviously, $\omega(s_A,s_B)$ is bounded. As a result, there exists a fixed point $\omega(s^*_A,s^*_B)=(s^*_A,s^*_B)$ according to Brouwer fixed-point theorem. ■

**Proof of proposition 5:**

Proof: The first case is obvious where

$$s^*_B=\arg_{s_B}\left\{\mathbb{E}_{\beta_B}[q|s_B]=\frac{\int q\beta_B(q)\phi(s_B,q,\sigma_s)dq}{\int\beta_B(q)\phi(s_B,q,\sigma_s)dq}=q_B\right\},$$

$$\beta_B(q)=\int\mu(\theta)f(q|\theta)d\theta$$

$$s^*_A=\arg_{s_A}\left\{\mathbb{E}_{\beta_A}[q|s_A]=\frac{\int q\beta_A(q)\phi(s_A,q,\sigma_s)dq}{\int\beta_A(q)\phi(s_A,q,\sigma_s)dq}=q_A\right\}>s^*_B,$$

$$\beta_A(q) = \frac{\Phi(s_B^*, q, \sigma_s) \int \mu(\theta) f(q|\theta) d\theta}{\int \Phi(s_B^*, q, \sigma_s) \int \mu(\theta) f(q|\theta) d\theta dq}$$

Then, $\pi_B(\theta, \emptyset) > \pi_A(\theta, \emptyset)$ which means that the author will submit her paper to B first.

For the second case,

$$s_A^* = \arg_{s_A} \left\{ \mathbb{E}_{\beta_A}[q|s_A] = \frac{\int q \beta_A(q) \phi(s_A, q, \sigma_s) dq}{\int \beta_A(q) \phi(s_A, q, \sigma_s) dq} = q_A \right\},$$

$$\beta_A(q) = \int \mu(\theta) f(q|\theta) d\theta$$

$$s_B^* = \arg_{s_B} \left\{ \mathbb{E}_{\beta_B}[q|s_B] = \frac{\int q \beta_B(q) \phi(s_B, q, \sigma_s) dq}{\int \beta_B(q) \phi(s_B, q, \sigma_s) dq} = q_B \right\},$$

$$\beta_B(q) = \frac{\Phi(s_A^*, q, \sigma_s) \int \mu(\theta) f(q|\theta) d\theta}{\int \Phi(s_A^*, q, \sigma_s) \int \mu(\theta) f(q|\theta) d\theta dq}$$

To ensure that $s_A^* < s_B^*$ $(\pi_B(\theta, \emptyset) < \pi_A(\theta, \emptyset))$, $q_B$ should not be too low. $s_B^*$ is increasing in $q_B$. Therefore, there is a $\Delta$ such that $s_A^* = s_B^*$ when $q_B = q_A - \Delta$. ■

**Proof of lemma 5:**

$$Q_I^0 = \frac{\int q f(q)[1 - \Phi(s_I^0, q, \sigma_s)] dq}{\int f(q)[1 - \Phi(s_I^0, q, \sigma_s)] dq}$$

To have

$$Q_I^0 = Q_E = \frac{\int q f(q)[1 + \Phi(s_I^0, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)] dq}{\int f(q)[1 + \Phi(s_I^0, q, \sigma_s)][1 - \Phi(s_E, q, \sigma_s)] dq},$$

$s_I^0 < s_E$ because $f(q)$ first-order stochastic dominates $\frac{f(q)[1+\Phi(s_I^0, q, \sigma_s)]}{\int f(q)[1+\Phi(s_I^0, q, \sigma_s)] dq}$. Then,

$$\tilde{v}(Q_I^0) = \frac{\int f(q)[1 - \Phi(s_I^0, q, \sigma_s)] dq}{\int f(q)[1 - \Phi(s_E, q, \sigma_s)] dq} > 1 \blacksquare$$

**Proof of lemma 6:**

$$\frac{\tilde{v}(Q_E)}{\bar{v}(Q_E)} = \frac{\frac{\int f(q)[1-\Phi(s_I^0,q,\sigma_s)]dq}{\int f(q)[1-\Phi(s_E,q,\sigma_s)]dq}}{\frac{\int qf(q)[1+\Phi(s_I^0,q,\sigma_s)][1-\Phi(s_E,q,\sigma_s)]dq}{\int f(q)[1+\Phi(s_I^0,q,\sigma_s)][1-\Phi(s_E,q,\sigma_s)]dq} \Big/ \frac{\int qf(q)[1-\Phi(s_I^0,q,\sigma_s)]dq}{\int f(q)[1-\Phi(s_I^0,q,\sigma_s)]dq}}$$

$$= \frac{\int qf(q)[1-\Phi(s_I^0,q,\sigma_s)]dq}{\int qf(q)[1-\Phi(s_E,q,\sigma_s)]dq} \cdot \frac{\frac{\int qf(q)[1-\Phi(s_E,q,\sigma_s)]dq}{\int f(q)[1-\Phi(s_E,q,\sigma_s)]dq}}{\frac{\int qf(q)[1+\Phi(s_I^0,q,\sigma_s)][1-\Phi(s_E,q,\sigma_s)]dq}{\int f(q)[1+\Phi(s_I^0,q,\sigma_s)][1-\Phi(s_E,q,\sigma_s)]dq}}$$

$$> 1$$

The first item in the right hand side is higher than 1 because $s = s_I^0$ maximizes $\int qf(q)[1-\Phi(s,q,\sigma_s)]dq$ according to the definition of $s_I^0$. The second item is strictly higher than 1 because $f(q)$ first-order stochastic dominates $\frac{f(q)[1+\Phi(s_I^0,q,\sigma_s)]}{\int f(q)[1+\Phi(s_I^0,q,\sigma_s)]dq}$. ■

**Proof of proposition 6:**

If the incumbent sets the threshold $s_I^0$, the entrant sets $s_E^1$ and all the authors submit to the former first, the latter's utility is

$$u_E^0 = \int qf(q)\Phi(s_I^0,q,\sigma_s)[1-\Phi(s_E^1,q,\sigma_s)]dq$$

If the authors submit to the entrant first, its utility under different threshold $s_E$ is

$$u_E^1(s_E) = \int qf(q)[1-\Phi(s_E,q,\sigma_s)]dq$$

Define

$$s_E^l := \inf\{s_E | u_E^1(s_E) > u_E^0\}$$

and

$$s_E^h := \sup\{s_E | u_E^1(s_E) > u_E^0\}$$

We need to find the value of $\alpha$ such that when $s_E \in [s_E^l, s_E^h]$, $v(Q_E) < \tilde{v}(Q_E)$.

By lemma 6, $v(Q_E) < \tilde{v}(Q_E)$ for any $Q_E$ if $\alpha = 1$. $v(Q_E)$ is increasing in $\alpha$ if $Q_E > Q_I^0$ and decreasing if $Q_E < Q_I^0$. Then $\bar{\alpha}$ and $\underline{\alpha}$ are defined as

$$\bar{\alpha} = \inf\left\{\alpha > 1 \middle| v(Q_E) > \tilde{v}(Q_E), \exists s_E \in [s_E^l, s_E^h]\right\}$$

$$\underline{\alpha} = \sup\left\{\alpha \in [0,1) \middle| v(Q_E) > \tilde{v}(Q_E), \exists s_E \in [s_E^l, s_E^h]\right\}$$

Finally, the authors with new papers submit to the incumbent first because

$v(Q_E) < \tilde{v}(Q_E)$ under thresholds $s_I^0$ and $s_E^1$. If $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, the entrant finds it optimal to set threshold $s_E^1$ and to be the second option of the authors because even if it can be the first option by setting a threshold outside $[s_E^l, s_E^h]$, the payoff will be lower. The incumbent finds it optimal to set threshold $s_I^0$ because it maximizes its payoff if it is always the authors' first option. ■

**Proof of lemma 7:** For any threshold $s_I$ set by the incumbent

$$Q_I = \frac{\int qf(q)\mathbb{1}\{q \geq s_I\}dq}{\int f(q)\mathbb{1}\{q \geq s_I\}dq}$$

If the entrant sets the same threshold $s_E = s_I$,

$$Q_E = \frac{\int qf(q)[1 + \mathbb{1}\{q < s_I\}]\mathbb{1}\{q \geq s_E\}dq}{\int f(q)[1 + \mathbb{1}\{q < s_I\}]\mathbb{1}\{q \geq s_E\}dq} = \frac{\int qf(q)\mathbb{1}\{q \geq s_I\}dq}{\int f(q)\mathbb{1}\{q \geq s_I\}dq} = Q_I \quad ■$$

**Proof of proposition 7:**

First, if both journals set the same threshold, the authors with new papers are indifferent between them. Therefore, they randomly decide the submission order.

Then, if the incumbent sets $s_I = 0$ and $\alpha \in [\underline{\alpha}^0, \bar{\alpha}^0]$, the entrant finds it optimal to set the same threshold $s_E = 0$ because even if it can be the authors' first option by setting a threshold outside $[s_E^l, s_E^h]$, the payoff will be lower.

Finally, the total utility of these two journals gets maximum when the incumbent set threshold $s_I = 0$. According to lemma 7, the entrant can always guarantee the situation in which it sets the same threshold as the incumbent, and vice versa. Thus, in the equilibrium, both journals should have the same payoff, because otherwise the lower-utility journal can always deviate by choosing the same threshold as the other. Then, in the next period, both journals go back to situation maximizing their utility where $s_I = s_E = 0$. ■

**Proof of proposition 8:**

Given any $Q_I$ and $Q_E$,

$$\hat{R}(Q_I, Q_E) := \frac{\int f(q)[1 - \Phi(\hat{s}_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(\hat{s}_E, q, \sigma_s)]dq}$$

where $\hat{s}_I$ and $\hat{s}_E$ are determined according to

$$Q_I = \frac{\int qf(q)[1/2 + \Phi(\hat{s}_E, q, \sigma_s)/2][1 - \Phi(\hat{s}_I, q, \sigma_s)]dq}{\int f(q)[1/2 + \Phi(\hat{s}_E, q, \sigma_s)/2][1 - \Phi(\hat{s}_I, q, \sigma_s)]dq}$$

$$Q_E = \frac{\int qf(q)[1/2 + \Phi(\hat{s}_I, q, \sigma_s)/2][1 - \Phi(\hat{s}_E, q, \sigma_s)]dq}{\int f(q)[1/2 + \Phi(\hat{s}_I, q, \sigma_s)/2][1 - \Phi(\hat{s}_E, q, \sigma_s)]dq}$$

Then, consider

$$\tilde{R}(Q_I, Q_E) = \frac{\int f(q)[1 - \Phi(s'_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s'_E, q, \sigma_s)]dq}$$

where $s'_I$ and $s'_E$ are determined according to

$$Q_I = \frac{\int qf(q)[1 - \Phi(s'_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s'_I, q, \sigma_s)]dq}$$

$$Q_E = \frac{\int qf(q)[1 + \Phi(s'_I, q, \sigma_s)][1 - \Phi(s'_E, q, \sigma_s)]dq}{\int f(q)[1 + \Phi(s'_I, q, \sigma_s)][1 - \Phi(s'_E, q, \sigma_s)]dq}$$

Because $f(q)$ first-order stochastic dominates $\frac{f(q)[1+\Phi(\hat{s}_E,q,\sigma_s)]}{\int f(q)[1+\Phi(\hat{s}_E,q,\sigma_s)]dq}$, $s'_I < \hat{s}_I$. Then, $\frac{f(q)[1+\Phi(\hat{s}_I,q,\sigma_s)]}{\int f(q)[1+\Phi(\hat{s}_I,q,\sigma_s)]dq}$ first-order stochastic dominates $\frac{f(q)[1+\Phi(s'_I,q,\sigma_s)]}{\int f(q)[1+\Phi(s'_I,q,\sigma_s)]dq}$, $s'_E > \hat{s}_E$. Therefore,

$$\int f(q)[1 - \Phi(s'_I, q, \sigma_s)]dq > \int f(q)[1 - \Phi(\hat{s}_I, q, \sigma_s)]dq$$

and

$$\int f(q)[1 - \Phi(s'_E, q, \sigma_s)]dq < \int f(q)[1 - \Phi(\hat{s}_E, q, \sigma_s)]dq$$

Then,

$$\tilde{R}(Q_I, Q_E) = \frac{\int f(q)[1 - \Phi(s'_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(s'_E, q, \sigma_s)]dq} > \frac{\int f(q)[1 - \Phi(\hat{s}_I, q, \sigma_s)]dq}{\int f(q)[1 - \Phi(\hat{s}_E, q, \sigma_s)]dq} = \hat{R}(Q_I, Q_E) \blacksquare$$

# Chapter 2

# Adverse Selection with Dynamic Learning

## 2.1 Introduction

Many models of adverse selection in economics assume that one side possesses all the information while the other side learns about the state from history or external signals.[1] However, in many real-world scenarios, neither party initially holds an information advantage, but information asymmetry deepens over time. For example, in the labor market, a graduate and potential employer are unaware of the graduate's value at the outset, but the graduate can learn about her value through feedback from job interviews. This study explores situations where information asymmetry does not exist at the outset and whether it has any qualitative impact on the outcome. Other real-life examples where this may occur include high-tech firm acquisitions, startups seeking seed funds, and securities trading in the secondary market.

In this study, a search model is used to analyze a game in which Graduate sequentially searches among a group of Employers. Graduate's ability can be high or low, and both parties share the same prior belief of Graduate's ability before the search begins. Each interview provides both Graduate and Employer with a noisy signal of either good or bad. Graduate updates her belief using Bayes' rule based on her record of past signals, while each Employer only knows the signal he receives and is unaware of Graduate's past signals or how many Employers she has sampled.

[1] One seller bargains with one buyer (Deneckere and Liang [2006], Sobel and Takahashi [1983], Fuchs and Skrzypacz [2013]); one buyer searches among multiple sellers (Lauermann and Wolinsky [2016], Zhu [2012]); one seller waits for buyers (Kaya and Kim [2018], Daley and Green [2012]).

Sampled Employer then makes an offer, and Graduate has three options: accept the offer and end the search, reject the offer and continue sampling, or leave the market and receive an outside option.

Adverse selection is a critical factor that impacts market efficiency. Graduate, especially after receiving a bad signal, faces a trade-off between stopping immediately (by either accepting the offer or choosing the outside option) or searching for another Employer, possibly followed by a good signal and a better offer. Once she believes she could gain from continuation, she chooses the latter. Then, sampled Employer conjectures that Graduate might have received bad signals before. In other words, being sampled is not good news for Employers. Consequently, even if he receives a good signal, it may not be wise to offer a high price, which leads to the adverse selection where the high-type Graduate who is more likely to generate good signals does not receive an offer high enough, and thus may not enter the market at the beginning.

If Graduate knows her type ex-ante, information asymmetry can cause the market to collapse. This is because the high-type Graduate knows Employers will not offer a price higher than her outside option. Thus, she does not enter the market at the beginning. Then, the market is left with the low-type Graduate who takes a chance on the occurrence of good signals. Being aware of this, Employers offer her the outside option, which makes the low-type Graduate also leave the market. However, in the absence of initial information asymmetry, the market can avoid collapse and allow for efficient trades, as long as search costs are not extremely high. Unlike in cases where high-type Graduate avoids entering the market and low-type Graduate receives offers based on her poor signals, both types of Graduate have an incentive to search and learn about their true type when they share the same prior belief with Employers. The level of information asymmetry is not significant enough to create adverse selection. Thus, a surplus can be shared between Employers and Graduate, leading to mutually beneficial trades.

As search costs decrease to some level, Graduate has the option to reject low offers after receiving a bad signal and continue searching for a higher offer. This leads sampled Employers to consider whether Graduate has received bad signals and rejected previous low offers, reducing their expected value from the trade and resulting in a lower offer. This is known as the adverse selection effect, which lowers the average deal price and prolongs the search time for a trade. The effect is particularly significant in small markets with a low number of Employers and negligible search costs, leading to market inefficiency.

Nevertheless, in a large market with negligible search costs, efficiency recovers. As Graduate searches many times and receives bad signals, she is almost certain that she is of low type. However, she still believes that there is an extremely small probability that she is of high type, and sampled Employers are aware of the market being filled with this almost low-type Graduate. Therefore, giving the lowest possible offer, the outside option, is not an equilibrium, because sampled Employers can deviate by slightly sharing some surplus to largely increase the possibility of acceptance. This triggers competition among Employers, and eventually, the surplus of a low-type Graduate is totally shared, leading to increased market efficiency.

The result obtained in this study is qualitatively distinct from the scenario where Graduate has complete information about her type prior to searching. This disparity arises due to the presence of noise in the market. As a consequence of this uncertainty, after bad signals, high-type Graduate is inclined to continue searching under the belief that better opportunities may arise. In other words, the adverse selection effect never reaches the level in the complete information case. This, in turn, incentivizes Employers to compete with each other to offer more attractive deals.

The rest of the article is organized as follows: Section 2.2 presents the search model. Section 2.3 and 2.4 solve the equilibrium and discuss market efficiency. Section 2.5 concludes the study and provides scope for future research. All proofs are in appendix 2.6.5.

### 2.1.1 Related Literature

This study contributes to the literature on dynamic information asymmetry in the analysis of evolving information asymmetry.

Several existing studies of dynamic adverse selection have a similar model structure, Jovanovic [1979], Stern [1990], Kaya and Kim [2018], Moreno and Wooders [2016], Zhu [2012], Lauermann and Wolinsky [2016]. Among them, Jovanovic [1979] and Stern [1990] assumes that workers' type is complete information but the idiosyncratic matching is unknown. Kaya and Kim [2018] assumes that Employers arrive following a Poisson distribution and receive a noisy signal, while Graduate knows the state ex ante. If the prior probability of Graduate being of the high type is high enough, all Employers offer a high offer $c_H$ which is the cost of Graduate with high type regardless of the signals they receive. If it is low enough, all Employers offer the continuation payoff of Graduate of the low type regardless of the signals they receive. Neither strategy is optimal when Graduate does not know the state ex ante. Zhu [2012], Moreno and Wooders [2016] and Lauermann and Wolinsky [2016] present the search model where the side with information disadvantage is sampled and receives noisy signals about the state. The difference with the model in this study is information asymmetry evolves as time goes on. Its insignificance in the early periods plays a major role.

Hwang [2018] studies a case where Graduate and Employers have the same prior information in the beginning. At some time, Graduate is informed of the state. Martel et al. [2018] considers that Graduate of the good type receives a good signal with some probability as time goes on and Graduate of the bad type never receives a good signal. In both models, Employers receive no information about the state. Their offers form a "U-shape": at the early stages, Employers offer an average price; in the middle, the price decreases due to the adverse selection effect; and finally, the price increases because Graduate who is left is more likely to be of the good type. This study assumes that both Employers and Graduate observe the signal, and Graduate keeps a record. Employer offers a price based on the signal he receives. Sampled Employer realizes that Graduate is less likely to be high type because otherwise, she would have received good signals and made a trade.

Wolinsky [1986] and Anderson and Renault [1999] study the hold-up problem by introducing heterogeneity or differentiation across products. They find that consumers have incentives to search not only because they are looking for a better price but also a product they like. In this study, Graduate's value is identical across all Employers although she does not know it perfectly. Moreover, the noise in the information system plays a key role in deterring the hold-up problem.

## 2.2 The Model

In this model, Graduate (referred to as "she") sequentially searches among $N$ Employers (referred to as "he"), with Graduate's type $\theta$ (state) being either high ($H$) or low ($L$). High-type Graduate brings Employers a higher value $v_H$ than a low-type graduate brings ($v_L$).

At the beginning of the search process, Graduate and all Employers have the same prior belief that Graduate is of type $H$ with Probability $q_0 \in (0, 1)$. In each round of the search, Graduate meets one randomly selected Employer and both observe a signal $m$ that can take two values, $g$ or $b$. We use $\mu_\theta^m$ to denote the probability that the signal $m$ occurs when Graduate's type is $\theta$. We assume that

$\mu_H^g > \mu_L^g$, meaning that the high type is more likely to generate the high signal.

After observing the signal, Employer proposes a wage offer $p$, and Graduate has three choices: i) accept the offer, and the search process ends; ii) reject the offer and continue to search for another Employer; or iii) reject the offer and exit the labor market. During the search process, Graduate updates her belief about her type based on past signals using Bayes' rule. Each Employer only observes the signal from the current round and knows neither how many Employers Graduate has already sampled nor the past signals.

If Graduate accepts an offer from an Employer, Employer's utility is $v_H - p$ if Graduate's type is $H$, and $v_L - p$ otherwise. If Graduate rejects all offers and exits the labor market or reaches the end of the search process, Employers' utilities are 0. Graduate receives her outside option (by starting her own business), $c_H$ if her type is $H$, and $c_L$ otherwise. The search process is costly for Graduate, with a cost of $s > 0$ for each Employer she samples. Therefore, Graduate's terminal payoff is $p - ns$ if she samples $n$ Employers and accepts the last offer, and $c_\theta - ns$ if she samples $n$ Employers and exits the labor market.

It is assumed that $c_L < v_L < c_H < v_H$, which means that high-type Graduate brings high value to Employer, the trade is always beneficial to social welfare (working in a firm is less risky than starting one's own business), and there is no price to ensure the trade (otherwise, the result is trivial).

*Strategy*

Each Employer's strategy is a probability distribution function $\sigma^E : \{g, b\} \times \mathbb{R}_+ \to [0, 1]$, where $\sigma^E(m, p)$ means that the probability that Employer receiving the signal $m$ offers a price $p$.[2] $M = (m_1, m_2, ...)$ denotes Graduate's record of signals. Let $\mathcal{M}$ be the set of all possible histories. The strategy of Graduate is a probability distribution function $\sigma^G : \mathcal{M} \times \mathbb{R}_+ \to \Delta^2$, where $\Delta^2$ is a unit 2-simplex and $\sigma^G(M, p) = (\sigma_1^G, \sigma_2^G, \sigma_3^G)$ means that the probability that Graduate accepts the offer $p$ after receiving a series of signals $M$ is $\sigma_1^G(M, p)$, stops searching $\sigma_2^G(M, p)$, and continues searching $\sigma_3^G(M, p)$.

*Belief*

When an Employer is sampled, he develops a belief about Graduate's history $\phi : \mathcal{M} \to [0, 1]$, where $\phi(M)$ represents the probability that Graduate has a record of signals $M$. Denote $q(M)$ the posterior probability that the state is $H$ conditional on the history $M$ ($q(\emptyset) = q_0$).

[2]The focus is on the symmetric case where all Employers play the same strategy.

*Other Notations*

Let $\mathcal{M}_g$ be the set of histories where the last signal is $g$ and $\mathcal{M}_b$ be the set of histories where the last signal is $b$. A history $M = b^m g^n$ means that Graduate has received $m$ bad signals and $n$ good signals. $M = \emptyset$ means that Graduate has not started searching yet.

Denote $v(M)$ as the expected value conditional on the history $M$, that is, $v(M) = v_H \cdot q(M) + v_L \cdot (1 - q(M))$. Denote $c(M) = c_H \cdot q(M) + c_L \cdot (1 - q(M))$ the expected payoff from stopping searching conditional on the history $M$. Let $x(M)$ be the probability of a good signal appearing in the next sampling, that is, $x(M) = \mu_H^g \cdot q(M) + \mu_L^g \cdot (1 - q(M))$.

Let $U(M)$ be Graduate's continuation payoff given the history $M$. It should be the largest one among the offer, the outside option, and the continuation payoff in the next period minus search costs.

$$\begin{aligned} U(M) = {} & x(M)\mathbb{E}_{\sigma^E(g,p)}\Big[\max\{p, c(M,g), U(M,g)\}\Big] \\ & + (1 - x(M))\mathbb{E}_{\sigma^E(b,p)}\Big[\max\{p, c(M,b), U(M,b)\}\Big] - s \end{aligned}$$

If $|M| = N$, which corresponds to the case of the last sampling, $U(M) = 0$.

Denote the threshold $R(M) := \max\{c(M), U(M)\}$ as the higher value between the outside option and the continuation payoff.

*Equilibrium*

The perfect Bayesian equilibrium concept is used. A tuple $(\sigma^E, \sigma^G, \phi)$ is a perfect Bayesian equilibrium if

1. Given $\sigma^G$ and $\phi$, $\sigma^E(m,p) > 0$ only if $p$ maximizes Employer's (VNM-)expected utility upon receiving the signal $m$.

$$p \in \arg\max_{p'} \mathbb{E}_{\phi(M_m)}[(v(M_m) - p')\sigma_1^G(M_m, p')]$$

2. Given $\sigma^E$, $\sigma_1^G(M,p) > 0$ only if the offer is weakly higher than the larger of the outside option and the continuation payoff. That is $p \geq R(M)$. $\sigma_1^G(M,p) = 1$ if the inequality is strict. Similarly, $\sigma_2^G(M,p) > 0$ only if $R(M) = c(M) \geq p$. $\sigma_3^G(M,p) > 0$ only if $R(M) = U(M) \geq p$.

3. Given $\sigma^G$ and $\sigma^E$, $\phi$ is derived through Bayes' rule.

### 2.2.1 Preliminary Observation

Given $\sigma^G$ and $\phi$, it is not wise to offer a price except from $R(\mathcal{M}) := \{R(M) : M \in \mathcal{M}\}$. If that is not the case, Employer chooses a price $p \notin R(\mathcal{M})$. He could choose $R(M') = \max\{R(M) : R(M) < p, M \in \mathcal{M}\}$ instead, such that the probability that Graduate accepts the offer is not changed while the payoff increases.

However, the threshold $R(\mathcal{M})$ is determined by the price which causes a loop. The following lemma shows that, given Employers' strategy, the thresholds $R(\mathcal{M})$ are well-defined.

**Lemma 8.** *Given $\sigma^E$, there is a unique value for each $R(M)$. Moreover, $\sigma^E(g,p) > 0$ only if $p \in R(\mathcal{M}_g)$ and $\sigma^E(b,p) > 0$ only if $p \in R(\mathcal{M}_b)$.*

The above analysis restricts Employers' strategy space to $R(\mathcal{M})$. Moreover, if Employer's surplus is positive by offering $p$, $p = R(M) < \mathbb{E}_{\phi(M')}[v(M')]$, Graduate with history $M$ should accept it with Probability 1, $\sigma_1^G(M,p) = 1$ because otherwise Employer could slightly increase the offer. Then, given $\sigma^E$ and $\sigma^G$, the explicit expression of $\phi$ is obtained through Bayes' rule. That is, after receiving a signal $m$, the probability that Employer faces Graduate with history $M$ is $\phi(M)$. First, denote $L(M)$ as the likelihood that $M$ is reached. $L(\emptyset) = 1$. Then,

$$L(M,g) = L(M)x(M) \sum_{M' \in \mathcal{M}} \sigma^E(m, R(M'))\sigma_3^G(M, R(M'))$$

$$L(M,b) = L(M)(1 - x(M)) \sum_{M' \in \mathcal{M}} \sigma^E(m, R(M'))\sigma_3^G(M, R(M'))$$

where $m$ is the last signal in $M$. The likelihood that Graduate reaches the node $(M,g)$ ($(M,b)$) equals the likelihood that history $M$ is reached times the probability a good (bad) signal occurs times the probability that Graduate continues searching.

Since Employers are sampled randomly by Bayes' rule, if $m = g$,

$$\phi(M_g) = \frac{L(M_g) \cdot \frac{1}{N}}{\sum_{M'_g \in \mathcal{M}_g} L(M'_g) \cdot \frac{1}{N}} = \frac{L(M_g)}{\sum_{M'_g \in \mathcal{M}_g} L(M'_g)}$$

If $m = b$,

$$\phi(M_b) = \frac{L(M_b)}{\sum_{M'_b \in \mathcal{M}_b} L(M'_b)}$$

## 2.3 Two-Employer Equilibrium

This section analyzes the equilibrium of the search model proposed in this study.

In the conventional case where Graduate knows her type ex ante, the market collapses due to the combination of Akerlof's lemons (Akerlof [1970]) and the Diamond paradox (Diamond [1971]). High-type Graduate anticipates that Employers will not offer a price higher than her outside option $c_H$, so she opts out of the market. Consequently, only low-type Graduate remains, who takes a chance on the occurrence of good signals. However, Employers are aware of this and offer her the outside option $c_L$, leading to low-type Graduate also leaving the market.

In contrast, without perfect knowledge of the state initially, when search costs are not extremely high and the value generated by the market is sufficiently high, there could be trades. The adverse selection effect diminishes, and Graduate has an incentive to learn her value by searching in the market because she does not know it ex ante. In turn, Employers are willing to share some surplus because the information asymmetry is not that significant when the trade is made. Specifically, they can make the trade once Employer offers a price equal to Graduate's continuation payoff, provided it is lower than Employer's expected value from the trade, and Graduate's gain from the trade can compensate for search costs.

In this market, sampled Employer believes that Graduate has not sampled many Employers before him because the trade would have already been made. This suggests that the adverse selection effect is not significant, and Employer's expected value from the trade is high. Therefore, Employer has an incentive to offer a high enough price to induce Graduate to accept the offer and make the trade.

To provide a clearer illustration of this property, let us consider the case of two Employers where a good signal only occurs when Graduate is of high type, that is, $\mu_H^g > \mu_L^g = 0$.[3] An example is Graduate obtaining a good assessment if he passes all the tests set by Employer, which only Graduate of high quality can achieve. However, he could make mistakes and fail some tests, and obtain a bad assessment. Under this case, the focus is on the strategy of Employer receiving a bad signal because if Employer receives a good signal, he knows that Graduate is of high type, and offers $c_H$.

To find the equilibrium, I first study Employers' behavior by varying the search costs from high to low. Then, given Employers' strategy, Graduate searches the market if and only if the continuation payoff is higher than the outside option. The

[3] I put the complete analysis of the general case $\mu_H^g > \mu_L^g > 0$ in Appendix 2.6.1.

following lemmas show that high search costs mitigates the adverse selection effect, and Employers are willing to give a high offer. However, this effect is exacerbated when search costs are low, and Employers' offer decreases.

**Lemma 9.** *(Employers' behavior) Given* $N = 2$, $q_0$, $\mu_H^g > \mu_L^g = 0$, $c_H$ *and* $c_L$, *Employer offers a price* $p_g = c_H$ *when he receives a good signal and*

1. *If* $s \geq \bar{s}_1 := x(b)(c_H - v(b))$, *Employer offers a price* $p_b = R(b) = \max\{c(b), c_H - \frac{s}{x(b)}\}$ *when he receives a bad signal. He believes Graduate history is* $(b)$ *with Probability 1.*

2. *If* $\bar{s}_1 > s \geq \bar{s}_2 := x(b)c_H + (1 - x(b))c(b) - v(b)$, *Employer mixes between* $R(b) = \frac{v(b)+s-x(b)c_H}{1-x(b)}$ *and* $R(b^2) = c(b^2)$ *when he receives a bad signal. He offers* $R(b)$ *with Probability* $y$,

$$y = \frac{R(b) - c(b) + s}{(1 - x(b))(R(b) - c(b^2))}$$

*The likelihood ratio of his belief* $\phi(b^2)/\phi(b) = (1 - x(b))(1 - y)$ *and* $\phi(gb) = 0$.

3. *If* $s < \bar{s}_2$, *Employer mixes between* $R(b) = c(b)$ *and* $R(b^2) = c(b^2)$ *when he receives a bad signal. He offers* $R(b)$ *with Probability* $y$,

$$y = \frac{s}{(1 - x(b))(R(b) - c(b^2))}$$

*He believes that Graduate continues searching with Probability* $z$ *after first sampling with an offer* $R(b^2)$ *and stops with Probability* $1 - z$. *The likelihood ratio of his belief* $\phi(b^2)/\phi(b) = (1 - x(b))z(1 - y)$ *and* $\phi(gb) = 0$.

$$z = \frac{v(b) - c(b)}{[c(b) - c(b^2)](1 - x(b))(1 - y)}$$

When search costs are high, Employers are willing to share some of their surplus by offering $R(b)$ as long as the expected value from the trade is high enough ($v(b) > R(b)$). In the first round of sampling, there is no information asymmetry. Although sampled Employer does not know ex ante Graduate's order of sampling, he knows he ranks in the first position after being selected if he believes others offer the same prices.

However, if search costs decrease to some value $\bar{s}_1$, Graduate's continuation payoff increases to a level that $U(b) = R(b) = v(b)$, which means that Employer receiving the bad signal gets a negative payoff if he offers $R(b)$. Therefore, the equilibrium falls into the second case where Employers mix between a high offer $R(b)$ and a low offer $R(b^2)$. Graduate with the history $(b)$ accepts only the high offer $R(b)$, and rejects the low one $R(b^2)$ because she believes that she could receive a better offer by sampling another Employer. This exacerbates the adverse selection effect. Sampled Employer receiving a bad signal wonders whether the history is $(b)$ or $(b^2)$, which further reduces the probability of offering the high price ($y$ decreases as $s$ decreases).

If search costs decrease even further to some value $\bar{s}_2$, Graduate's continuation payoff decreases to her outside option, $U(b) = R(b) = c(b)$. In this case, she becomes indifferent between continuing or stopping after receiving an offer $R(b^2)$.

**Lemma 10.** *(Graduate's behavior) Given $N = 2$, $q_0$, $\mu_H^g > \mu_L^g = 0$, $c_H$ and $c_L$,*

1. *If $\bar{s}_1 \leq s < \underline{s}_1 := \frac{(c_H - c(\emptyset))x(b)}{1 - x(\emptyset) + x(b)}$, Graduate searches once. She accepts Employer's offers $R(b)$ after receiving a bad signal and offer $c_H$ after a good signal.*

2. *If $\min\{\bar{s}_1, \underline{s}_1, \underline{s}_2\} > s \geq \bar{s}_2$,*

$$\underline{s}_2 := \frac{1 - x(\emptyset)}{x(\emptyset) - x(b)}[v(b) - (1 - x(b))c(b) - x(b)c_H]$$

*Graduate searches at least once. She accepts the offers $c_H$ after a good signal and $R(b)$ after a bad signal only in her first sampling, but accepts the offers $c_H$, $R(b)$ and $R(b^2)$ in her second sampling.*

3. *For other cases, Graduate does not search.*

For Graduate, if the continuation payoff at the beginning $U(\emptyset)$ is higher than her outside option $c(\emptyset)$, which is true if the search cost is not too high ($s < \underline{s}_1$), then she has an incentive to search. If search costs are too large, she will not search.

As search costs decrease and Employers start to mix ($s < \bar{s}_1$), the continuation payoff $U(\emptyset)$ becomes lower than $c(\emptyset)$ again. She does not search. However, as search costs become negligible, the continuation payoff increases. She searches as long as Employers offer something higher than her outside option.

By taking into account both the behavior of Employers and Graduate, we can determine the equilibrium in the relationship between market value and search costs. There exist two critical thresholds for the market value. If the market value exceeds the first threshold, adverse selection is not a concern, and all trade occurs during the initial search. If the market value falls between the two thresholds, adverse selection may still be present, but trade is still possible. However, if the market value is below the second threshold, then no trade occurs.

**Proposition 9.** *Given $N = 2$, $q_0$, $\mu^g_H > \mu^g_L = 0$, $c_H$, $c_L$, and $s < \underline{s}_1 = \frac{[c_H - c(\emptyset)]x(b)}{1-x(\emptyset)+x(b)}$, let*

$$f_1(s) := c_H - \frac{s}{x(b)}$$

*and*

$$f_2(s) := \frac{x(\emptyset) - x(b)}{1 - x(\emptyset)} s + (1 - x(b))c(b) + x(b)c_H$$

1. *If $v(b) \geq f_1(s)$, Employer offers a price $c_H$ when he receives a good signal, and $R(b)$ when he receives a bad signal. Graduate accepts the offers under both circumstances and all trades are made in the first sampling.*

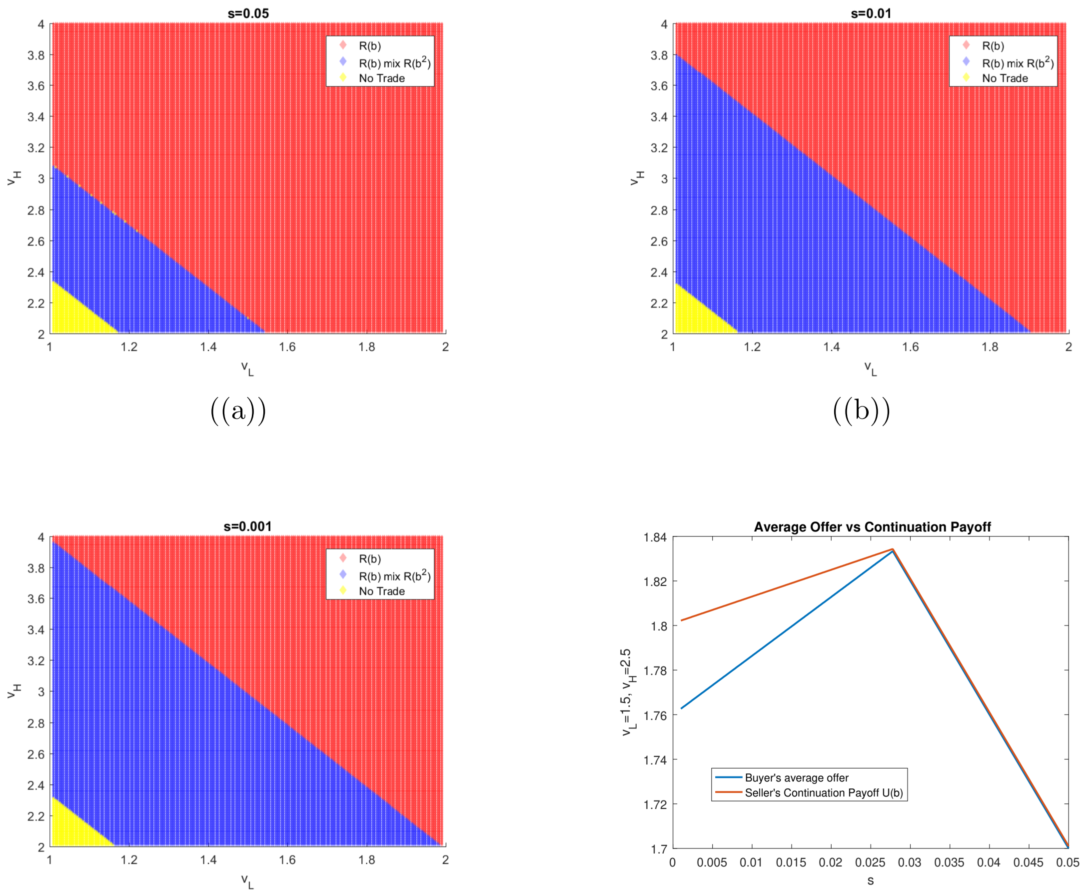
2. *If $f_2(s) < v(b) < f_1(s)$, Employer offers a price $c_H$ when he receives a good signal, and mixes between $R(b)$ and $R(b^2) = c(b^2)$ when he receives a bad signal. Graduate only accepts the offers $c_H$ and $R(b)$ in her first sampling, but accepts the offers $c_H$, $R(b)$ and $R(b^2)$ in her second sampling.*

3. *If $v(b) \leq f_2(s)$, there is no trade.*

**Example 4.** *Consider a two-Employer case ($N = 2$). The probability that a good signal occurs under the state $H$ is $\mu^g_H = 0.5$. $\mu^g_L = 0$. The outside option is as follows: $c_H = 2$ and $c_L = 1$. The prior $q_0 = 0.5$.*

*Figure 2.1 depicts the impact of search costs on equilibria. The red area in the figure corresponds to the equilibrium where Employers offer $R(b)$ after receiving the bad signal, while the blue area represents the mixed-strategy equilibrium. The yellow area indicates that Graduate opts not to search for a job.*

*As search costs decrease, the blue area expands, indicating that Employers start to reduce their offers due to adverse selection effects. Panel (d) shows how Employers' average bidding price changes in response to changes in search costs. When search costs exceed 0.028, Employers offer $R(b)$, which is equal to Graduate's continuation payoff $U(b)$. If the cost decreases below 0.028, Employers mix between $R(b)$ and $R(b^2)$, causing the average bidding price to decrease. Additionally, the adverse selection effect increases the likelihood of Employers offering a lower price, which widens the gap between the average bidding price and Graduate's continuation payoff.* □




((a)) ((b)) ((c)) ((d))

Figure 2.1: A two-period search process with different search costs.

### Negligible search costs

In the limit where search costs approach zero, the offers made under both good and bad signals should converge, as Graduate would reject any lower offer until a good signal emerges. The following lemma is a good tool, which shows that it is indeed the case when the offer under the bad signal is not too low.

**Lemma 11.** *Let $p_g$ ($p_b$) be Employers' optimal price under a good (bad) signal.*

1. *If the equilibrium is of pure-strategy, when $p_b \geq R(b^{N-1})$, $p_g - p_b \to 0$ as $s \to 0$.*

2. *If the equilibrium is of mixed-strategy, when all prices $p_b$ supported are higher than $R(b^{N-1})$, $\mathbb{E}[p_g] - \mathbb{E}[p_b] \to 0$ as $s \to 0$.*

## 2.4 Discussion

This section analyzes how market efficiency is affected by the number of Employers and the informativeness of signals.

### 2.4.1 Number of Employers

When there are more Employers in the market, their presence has a twofold effect on market efficiency. First, Graduate's bargaining power increases, prompting her to sample more Employers and wait for a good signal. This, in turn, increases her continuation payoff and threshold, which requires Employers to offer higher prices. However, because Graduate receiving bad signals prefers to sample more Employers, the adverse selection effect is exacerbated, leading to a decrease in the average offer made by Employers as they correct the selection bias.

To verify these points, we revisit the simplified case $\mu_H^g > \mu_L^g = 0$. We use an algorithm that is a generalization of Lemma 9 and Lemma 10 to find the equilibrium. In general, we use the thresholds from high to low ranking to find the equilibrium. If Employer find that their utility is negative by offering the high-ranking thresholds, they mix them with the low-ranking thresholds. After obtaining a candidate, Graduate's continuation payoff is checked to determine if it is higher than the outside option given any history.

**Algorithm 1.** *We start from Employers' side with a decreasing trend of search costs.*

*1. Add $R(b)$ in Employers' strategy support.*

*Graduate's threshold $R(b)$ is*

$$R(b) = \max\{c(b), U(b)\} = \max\{c(b), x(b)c_H + (1 - x(b)) \max\{R(b), c(b^2), U(b^2)\} - s\}$$

*It is easy to verify that $R(b)$ is decreasing with $s$. If $s$ is higher than $\bar{s}_1$ defined previously, $v(b) > R(b)$. Then, it is optimal for Employers to offer $R(b)$. Otherwise, Employer gets a negative payoff by offering $R(b)$. He adds a lower price.*

*Suppose that Employers' strategy support is $\{R(b^{n_1}), R(b^{n_2}), ..., R(b^{n_k})\}$.*

*2. Add $R(b^{n_{k+1}})$ in Employers' strategy support if any $R(b^{n_i}) = v(b^{n_i}),\ i \in \{n_1, ..., n_k\}$.*

$$n_{k+1} = \min\{n :\ R(b^n) < v(b^n),\ n_k < n \leq N\}$$

*Employer mixes between these prices and the probability allocated to each of them is determined by the indifference condition.*

*3. Graduate searches as long as continuation payoff is higher than outside option.*

*Given Employers' strategy and search costs, Graduate's continuation payoff of each history $U(b^i)$ ($i < n_k$) is computed. It should be guaranteed that $U(b^i) > c(b^i)$. If $U(b^i) = c(b^i)$, Graduate mixes between continuing and stopping if the offer is lower than $U(b^i)$. Graduate searches at the beginning as long as $U(\emptyset) \geq c(\emptyset)$.* ■

**Example 5.** *Consider a searching process where the good signal occurs only under the state $H$ with Probability $\mu_H^g = 0.5$. The outside option is as follows: $c_H = 2$ and $c_L = 1$. The prior $q_0 = 0.5$. Search costs $s$ approach 0.*

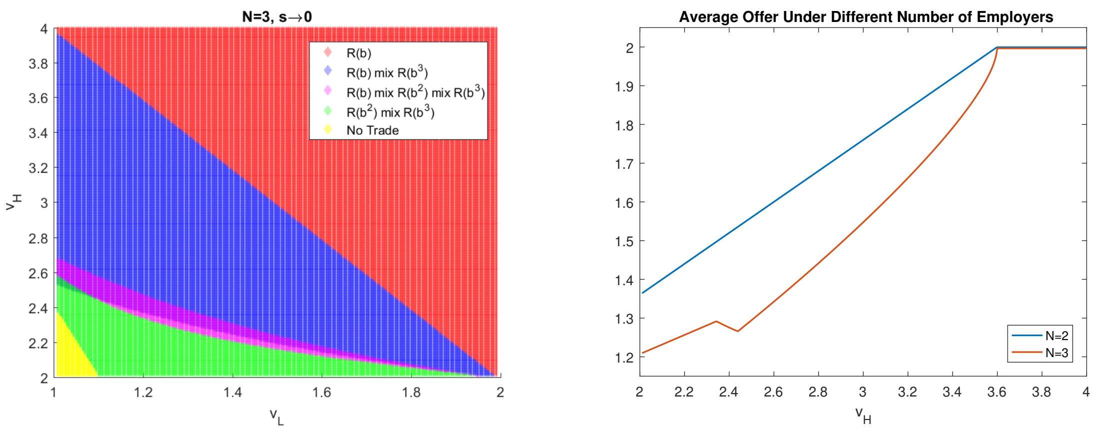



Figure 2.2: The left graph shows the equilibria with three Employers. The right is the average offer under two or three Employers given $v_L = 1.2$.

*Employers tend to decrease their offers due to the adverse selection effect. Comparing the left graph of Figure 2.2 and panel (c) of Figure 2.1, when there are three Employers, the offer $R(b^3)$ is included in their strategy. Then, Graduate could sample all three Employers where the searching period for trade is longer. The right graph shows that, if the value from the market is high enough, Employers offer a high price $R(b)$. Then, according to lemma 11, the deal price is just $p_g = c_H$ regardless of the number of Employers. If the value is not that high, the average offer is lower when there are more Employers.* □

### 2.4.2 Large Market with Negligible Friction

This section considers the asymptotic case where the number of employers $N$ approaches infinity and search costs $s$ approach zero. According to lemma 11, if $R(b^N)$

is not in Employers' strategy set, $p_b$ converges to $p_g = c_H$. They find it optimal to offer $c_H$ even after receiving a bad signal if the value from trade is sufficiently high ($v(b) > c_H$). Otherwise, Employer should offer $R(b^N)$ (which converges to $c_L$ as $N \to +\infty$) with some probability, but not with Probability 1[4]. As Employers mix between $R(b^{N-1})$ and $R(b^N)$, this increment raises Graduate's continuation payoff and it triggers the competition among Employers by sharing more surplus, which further raises Graduate's continuation payoff. Finally, this competition ends at a point when they give out all the surplus.

**Proposition 10.** *Given $q_0$, $\mu_H^g > \mu_L^g = 0$ and $s \sim o(1/NlogN)$, even if $v_H - c_H \to 0$, as $N \to +\infty$, Graduate of low type secures a payoff $v_L$ almost surely, and Graduate of high type secures a payoff $c_H$ almost surely.*

Remark:

This finding is qualitatively different from the observation of the hold-up situation. The key reason is the noise in the market. For Graduate, although no good signal shows up, she still believes that she is of high type with extremely small probability. It creates the small increment between $R(b^{N-1})$ and $R(b^N)$, which eventually accumulates to be the whole surplus. However, if the market is left with low-type Graduate knowing her type perfectly, $R(b^{N-1})$ and $R(b^N)$ are both her outside option $c_L$. Thus, no surplus will be shared. The following study shows that without any noise in the information system, the market collapses again.

### 2.4.3 Informativeness of Signals

The informativeness of signals is a crucial factor that affects market efficiency. Let $\mu := \mu_H^g - \mu_L^g$, where $\mu$ represents the degree of informativeness, where $\mu = 1$ means that the signals are completely informative, and $\mu = 0$ means that they are completely uninformative.

In the case of completely informative signals, there is no information asymmetry between Employers and Graduate. Sampled Employer sets prices $p_g = R(g) = c_H$ when receiving a good signal and $p_b = R(b) = c_L$ when receiving a bad signal, which is Graduate's outside option. Graduate would prefer to choose the outside option at the beginning rather than enter the market. Similarly, for completely uninformative

[4] If this is not the case, Graduate will search until the last Employer if the good signal never occurs. Then, sample Employer can deviate from offering $R(b^N)$ to $R(b^{N-1})$ (the increment is positive but small as $N \to +\infty$), to capture Graduate in her second to last search period.

signals, sampled Employer sets prices $p_g = p_b = q_0 c_H + (1 - q_0)c_L$, which is the expected outside option ex ante. Similarly, Graduate would not enter the market.

As the signals become more informative—in other words—$\mu$ increases from 0 to 1, there are two effects. First, Graduate wants to take advantage of the information asymmetry by prolonging the search process. If Employers are willing to share some surplus by offering a price higher than Graduate's outside option, they can make a mutually beneficial trade that improves market efficiency. However, the second effect is that information asymmetry diminishes as the signals become more informative. When $\mu$ is close to 0, the first effect dominates, and when $\mu$ is close to 1, the second effect dominates.

**Example 6.** *In the case of a two-Employer case ($N = 2$) with $\mu_L^g = 0$, the outside options are $c_H = 2$ and $c_L = 1$.*

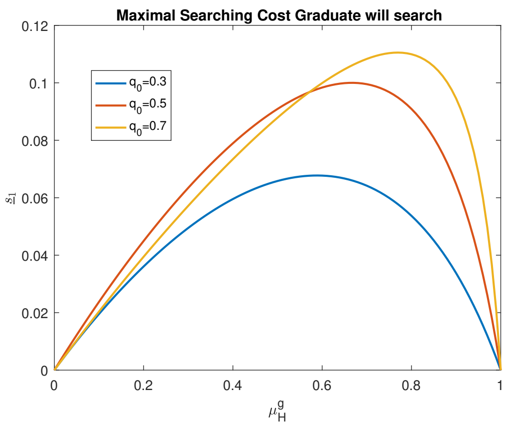



Figure 2.3: The relation between $\underline{s}_1$ and $\mu_H^g$.

*From lemma 10, we know that Graduate does search only if search costs $s < \underline{s}_1$. In Figure 2.3, $\underline{s}_1$ increases as $\mu_H^g$ increases from 0, which means that Graduate gains from information asymmetry to compensate for her search costs. Thus, she is more willing to enter the market. However, as $\mu_H^g$ is close to 1, Graduate can not profit by information asymmetry any more. She has less willingness to search.*

## 2.5 Conclusion and Further Discussion

This study contributes to the literature of dynamic information asymmetry in the aspect of the evolving information asymmetry. The key assumption different from current literature is that Graduate does not know the state ex ante, and the major finding is that, when search costs are sufficiently low, the market can be efficient. These findings are robust if the offering protocol is changed (appendix 2.6.3).

### 2.5.1 Further Discussion

In addition, this study also considers the case where Employer comes to Graduate, instead of Graduate sampling Employers, in appendix 2.6.4. When information asymmetry is not significant, arriving Employer believes that they are the first Employer Graduate has ever met, and so they are willing to offer a high price (Graduate's continuation payoff) as long as it is lower than their reservation value. However, as the number of Employers increases or Employers arrive more frequently, Graduate's continuation payoff will exceed Employer's reservation value, causing Graduate to reject Employer's offer and bringing the adverse selection problem to Employers arriving later. As a result, Employer who arrives later decreases his offer, which in turn decreases Graduate's payoff from continuation. This pull-push effect becomes stable until a new equilibrium is reached. For Employers who arrive early, they offer their reservation value and Graduate accepts with some probability. For those who arrive later, they either mix between a high and a low offer when the adverse selection effect is not so significant, or they only offer a low offer.

If Employers arrive with Probability 1 in each period, the only difference from the searching model is that Employers know when they arrive. This additional information weakens Graduate's bargaining power because Employer knows how much time is left to Graduate, enabling their offer to be accepted with a higher probability and benefiting market efficiency.

Finally, this study assumes that Graduate only receives information from the search process. However, in reality, Graduate may receive additional information about the state before the search process. For instance, Graduate received a private signal $m_S \in \{G, B\}$ before the search process. Let $\mu_\theta^{m_S}$ denote the probability that the signal $m_S$ shows up when the state is $\theta$. Then, when $\mu_H^G = \mu_L^G$, it is just the case discussed in this study. When $\mu_H^G = 1$ and $\mu_L^G = 0$, it corresponds to the case that Graduate knows the state ex ante. The general cases are just lying between the two extreme ones. Most findings in the current model should hold (appendix 2.6.2).

## 2.6 Appendix

### 2.6.1 General Case: $N = 2$

Section 2.3 focuses on the case $\mu_H^g > \mu_L^g = 0$, and solves the equilibria of the general case. More precisely, the conditions under which each possible equilibrium could exist are provided.

*1. $p_g = R(g)$, $p_b = R(b)$*

In this equilibrium, there should be three conditions. First, $p_b = R(b)$ should be lower than $v(b)$ to let Employer's payoff be positive.

$$v(b) \geq R(b) \geq c(b) \tag{1.1}$$

Secondly, $p_g = R(g)$ should be lower than $v(g)$.

$$v(g) \geq R(g) \geq c(g) \tag{1.2}$$

Thirdly, Graduate should have incentives to search at the beginning.

$$c(\emptyset) \leq U(\emptyset) = x(\emptyset)R(g) + (1 - x(\emptyset))R(b) - s \tag{1.3}$$

To compute the value of $R(g)$ and $R(b)$, the following equations are used:

$$\begin{aligned} R(g) &= \max\{c(g), U(g)\} \\ &= \max\{c(g), x(g)\max\{c(g^2), R(g)\} + (1 - x(g))\max\{c(gb), R(b)\} - s\} \end{aligned}$$

$$\begin{aligned} R(b) &= \max\{c(b), U(b)\} \\ &= \max\{c(b), x(b)\max\{c(bg), R(g)\} + (1 - x(b))\max\{c(b^2), R(b)\} - s\} \end{aligned}$$

*2. $p_g = R(g)$, $p_b : R(b)$ mix $R(b^2)$*

Suppose that Employer receiving the bad signal offers $R(b)$ with Probability $y \in (0, 1)$ and $R(b^2) = c(b^2)$ with Probability $1 - y$. There should be four conditions. First, he finds it indifferent between offering $R(b)$ and $R(b^2)$.

$$[v(b) - R(b)]\phi(b) + [v(b^2) - R(b)]\phi(b^2) = [v(b^2) - c(b^2)]\phi(b^2) \tag{2.1}$$

where $\phi(b) : \phi(b^2) = 1 : (1 - x(b))(1 - y)$.

Secondly, $U(b)$ should be higher than $c(b)$ to maintain Graduate's incentive to search.

$$c(b) \leq U(b) = x(b)R(g) + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s \tag{2.2}$$

Thirdly, Employer receiving the good signal should have higher payoff by offering $R(g)$ rather than $R(bg) = c(bg)$.

$$[v(g) - R(g)]\phi(g) + [v(bg) - R(g)]\phi(bg) \geq [v(bg) - c(bg)]\phi(bg) \tag{2.3}$$

where $\phi(g) : \phi(bg) = x(\emptyset) : (1 - x(\emptyset))x(b)(1 - y)$.

Fourthly, Graduate should have incentives to search at the beginning.

$$c(\emptyset) \leq U(\emptyset) = x(\emptyset)R(g) + (1 - x(\emptyset))R(b) - s \tag{2.4}$$

To compute the value of $y$, $R(g)$ and $R(b)$, the following equations along with (2.1) are used.

$$R(g) = \max\{c(g), x(g)\max\{c(g^2), R(g)\} + (1 - x(g))[y\max\{c(gb), R(b)\} + (1 - y)c(gb)] - s\}$$

$$R(b) = \max\{c(b), x(b)R(g) + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s\}$$

*3. $p_g = R(g) = c(g)$, $p_b = R(b^2) = c(b^2)$*

There should be four conditions. First, Employer receiving the good signal should have higher payoff by offering $R(g)$ rather than $R(bg) = c(bg)$.

$$[v(g) - R(g)]\phi(g) + [v(bg) - R(g)]\phi(bg) \geq [v(bg) - c(bg)]\phi(bg) \tag{3.1}$$

where $\phi(g) : \phi(bg) = x(\emptyset) : (1 - x(\emptyset))x(b)$.

Secondly, Employer receiving the bad signal should have higher payoff by offering $R(b^2)$ rather than $R(b)$.

$$[v(b) - R(b)]\phi(b) + [v(b^2) - R(b)]\phi(b^2) \leq [v(b^2) - c(b^2)]\phi(b^2) \tag{3.2}$$

where $\phi(b) : \phi(b^2) = 1 : (1 - x(b))$.

Thirdly, $U(b)$ should be higher than $c(b)$ to maintain Graduate's incentive to search.

$$c(b) \leq U(b) = x(b)R(g) + (1 - x(b))c(b^2) - s \tag{3.3}$$

Fourthly, Graduate should have an incentive to search at the beginning.

$$c(\emptyset) \leq U(\emptyset) = x(\emptyset)R(g) + (1 - x(\emptyset))R(b) - s \tag{3.4}$$

To compute the value of $R(b)$, one just uses the following equation.

$$R(b) = \max\{c(b), x(b)R(g) + (1 - x(b))c(b^2) - s\}$$

*4. $p_g : R(g)$ mix $R(bg)$, $p_b : R(b)$ mix $R(b^2)$*

Suppose that Employer receiving the bad signal offers $R(b)$ with Probability $y \in (0, 1)$ and $R(b^2) = c(b^2)$ with Probability $1-y$, and Employer receiving the good signal offers $R(g)$ with Probability $z \in (0, 1)$ and $R(bg) = c(bg)$ with Probability $1-z$. There should be four conditions. First, he finds it indifferent between offering $R(b)$ and $R(b^2)$.

$$[v(b) - R(b)]\phi(b) + [v(b^2) - R(b)]\phi(b^2) = [v(b^2) - c(b^2)]\phi(b^2) \tag{4.1}$$

where $\phi(b) : \phi(b^2) = 1 : (1 - x(b))(1 - y)$.

Secondly, he finds it indifferent between offering $R(g)$ and $R(bg)$.

$$[v(g) - R(g)]\phi(g) + [v(bg) - R(g)]\phi(bg) = [v(bg) - c(bg)]\phi(bg) \tag{4.2}$$

where $\phi(g) : \phi(bg) = x(\emptyset) : (1 - x(\emptyset))x(b)(1 - y)$.

Thirdly, $U(b)$ should be higher than $c(b)$ and $U(g)$ should be higher than $c(g)$ to maintain Graduate's incentive to search.

$$\begin{aligned} c(b) \leq U(b) = {} & x(b)[zR(g) + (1 - z)c(bg)] \\ & + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s \end{aligned} \tag{4.3}$$

$$\begin{aligned} c(g) \leq U(g) = {} & x(g)[z \max\{c(g^2), R(g)\} + (1 - z)c(g^2)] \\ & + (1 - x(g))[y \max\{c(gb), R(b)\} + (1 - y)c(gb)] - s \end{aligned} \tag{4.4}$$

Fourthly, Graduate should have incentives to search at the beginning.

$$c(\emptyset) \leq U(\emptyset) = x(\emptyset)R(g) + (1 - x(\emptyset))R(b) - s \tag{4.5}$$

To compute the value of $y$, $z$, $R(g)$ and $R(b)$, the following equations along with (4.1) and (4.2) are used.

$$\begin{aligned} R(g) = \max\{c(g),\, x(g)[z \max\{c(g^2), R(g)\} + (1 - z)c(g^2)] \\ + (1 - x(g))[y \max\{c(gb), R(b)\} + (1 - y)c(gb)] - s\} \end{aligned}$$

$$R(b) = \max\{c(b), x(b)[zR(g) + (1 - z)c(bg)] + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s\}$$

*5. $p_g = R(bg)$, $p_b : R(b)$ mix $R(b^2)$*

Suppose that Employer receiving the bad signal offers $R(b)$ with Probability $y \in (0, 1)$ and $R(b^2) = c(b^2)$ with Probability $1-y$. There should be four conditions. First, he finds it indifferent between offering $R(b)$ and $R(b^2)$.

$$[v(b) - R(b)]\phi(b) + [v(b^2) - R(b)]\phi(b^2) = [v(b^2) - c(b^2)]\phi(b^2) \tag{5.1}$$

where $\phi(b) : \phi(b^2) = 1 : (1 - x(b))(1 - y)$.

Secondly, $U(b)$ should be higher than $c(b)$ to maintain Graduate's incentive to search.

$$c(b) \leq U(b) = x(b)c(bg) + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s \tag{5.2}$$

Thirdly, Employer receiving the good signal should have higher payoff by offering $R(bg) = c(bg)$ rather than $R(g)$.

$$[v(g) - R(g)]\phi(g) + [v(bg) - R(g)]\phi(bg) \leq [v(bg) - c(bg)]\phi(bg) \tag{5.3}$$

where $\phi(g) : \phi(bg) = x(\emptyset) : (1 - x(\emptyset))x(b)(1 - y)$.

Fourthly, Graduate should have incentives to search at the beginning.

$$c(\emptyset) \leq U(\emptyset) = x(\emptyset)R(g) + (1 - x(\emptyset))R(b) - s \tag{5.4}$$

To compute the value of $y$, $R(g)$ and $R(b)$, the following equations along with

(5.1) are used.

$$R(g) = \max\{c(g), x(g)c(g^2) + (1 - x(g))[y \max\{c(gb), R(b)\} + (1 - y)c(gb)] - s\}$$

$$R(b) = \max\{c(b), x(b)c(bg) + (1 - x(b))[yR(b) + (1 - y)c(b^2)] - s\}$$

**Example 7.** *Consider a two-Employer case ($N = 2$). The probability that a good signal occurs under the state $H$ is $\mu_H^g = 0.8$. The probability that a good signal occurs under the state $L$ is $\mu_L^g = 0.2$. The outside option is as follows: $c_H = 3$ and $c_L = 1$. The prior $q_0 = 0.5$. Figure 2.4 shows the equilibrium under different combinations of $v_L$ and $v_H$.*

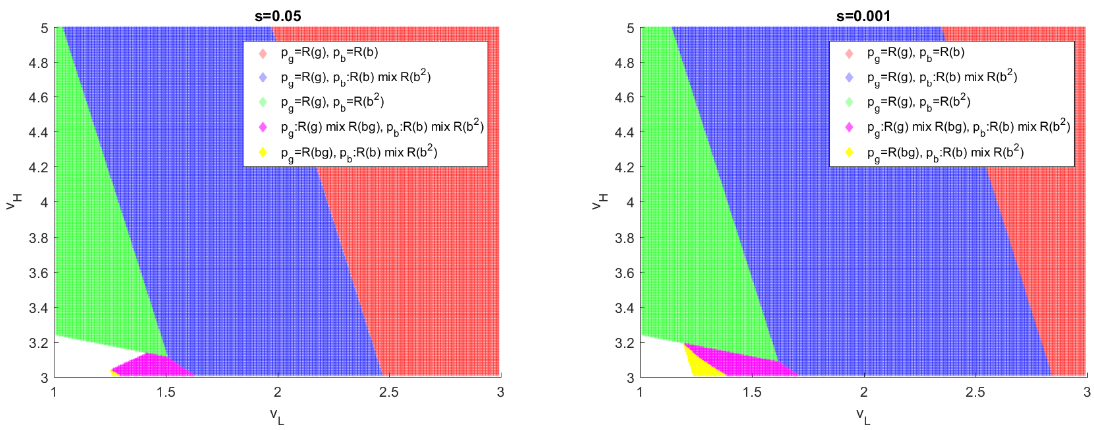



Figure 2.4: A two-period searching process with different search costs.

### 2.6.2 Continuous Signal Structure and Graduate's Heterogeneous Prior Belief

The search model is extended in two directions: i) Graduate has some private information $q$ about her type $\theta$. $q \in [0, 1]$ is the probability that she believes her type is $H$; all Employers just know the prior distribution of $q$, $\gamma(q)$ before the search starts, and ii) Employer observes one signal $s \in [0, 1]$ conditional on Graduate's type $\theta$ with continuous probability $f(s|\theta)$. $F(s|H)$ first-order stochastic dominates $F(s|L)$. He chooses any offer between $c_L$ and $v_H$ based on the signal they receive. Thus, his strategy is a mapping from the signal to an offer, $p(s)$.

**Example 8.** *In a two-Employer case ($N = 2$), the outside options are: $c_H = 1$ and $c_L = 0$; the values are: $v_H = 1.5$ and $v_L = 0.5$; the signal structure: $f(s|H) = 2s$,*

*$f(s|L) = 2(1-s)$; the prior distribution $\gamma(q)$ is a truncated normal distribution with mean 0.8 and standard deviation 0.1.*

*Figure 2.5 illustrates Employer's strategy in the equilibrium. There is a jump in $p(s)$. When $s$ is low, selected Employer believes he meets Graduate having been given a lower offer in her last sampling with high probability. Thus, he gives a relatively low offer targeting this kind of Graduate. When $s$ is high, it is less likely that this signal is from Graduate on her second search. Thus, Employer can target Graduate who searches for the first time with a high offer.*

*When search costs are large, Graduate quits more often after receiving a bad signal. Employers are willing to give a higher offer. So from the graph, larger search costs make the jump come earlier.*

*Besides the jump, the red curve is overall above the blue one. This is because lower search costs increase Graduate's continuation payoff. Thus, targeting Graduate with the same history, Employer should raise his offer.*

*The jump come earlier compared with the four-Employer case. The reason is that Employer is more likely to meet Graduate having sampled more times when there are more Employers in the market. When $s$ is low where Employers target Graduate in her last sampling, her expected outside option is lower when the market is large. Therefore, the blue curve is above the red one. Given a high $s$ where Employers target Graduate in her first sampling, more Employers in the market make Graduate's continuation payoff high. Employer should raise his offer.* □

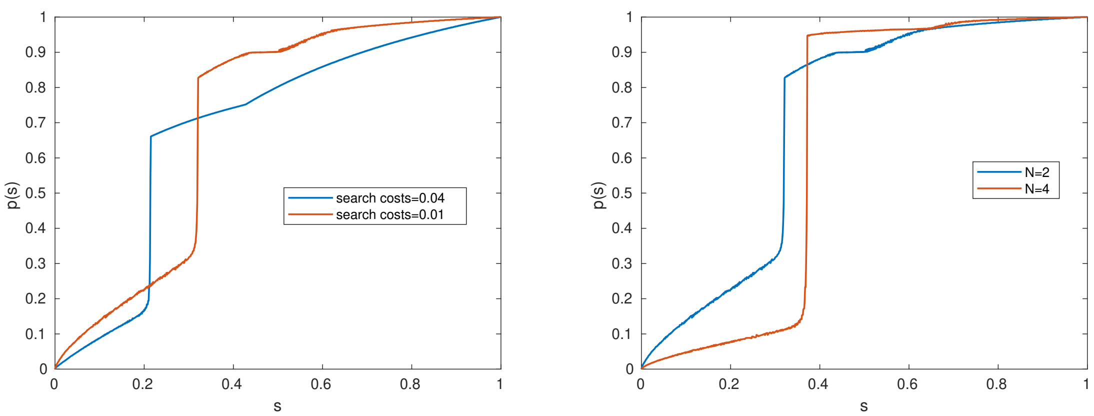



Figure 2.5: Employers' strategy.

### 2.6.3 Random Proposals

This study assumes that Employers propose the offer. All the bargaining power is with Employers. This section introduces the random proposal protocol to generate the offers (Wilson [2001]), and Graduate and Employer make the trade if they both accept the offer.

More specifically, the settings of the model are unchanged except that Employer offers a price after observing the signal. Instead, nature draws a price $p \in [c_L, v_H]$ according to some continuous density function $f$ strictly positive on $[c_L, v_H]$. Graduate and Employer simultaneously decide whether to accept $p$. If both of them accept $p$, they make the trade. Otherwise, Graduate either continues searching or leaves the market.

Then, the strategy of Employer is a probability distribution function $\tilde{\sigma}^E : \{g, b\} \times \mathbb{R}_+ \to [0, 1]$, where $\tilde{\sigma}^E(m, p)$ means that the probability that Employer receiving the signal $m$ accepts the offer $p$. Let $\tilde{U}(M)$ be Graduate's continuation payoff given the history $M$. It should be the larger one between the offer, the outside option, and the continuation payoff in the next round minus search costs. Denote the threshold $\tilde{R}(M) := \max\{c(M), \tilde{U}(M)\}$ as the larger one between the outside option and the continuation payoff. Then,

$$\begin{aligned}\tilde{U}(M) =& x(M) \int_{c_L}^{v_H} [\max\{p, \tilde{R}(M, g)\}\tilde{\sigma}^E(g, p) + \tilde{R}(M, g)(1 - \tilde{\sigma}^E(g, p))]f(p)dp \\ &+ (1 - x(M)) \int_{c_L}^{v_H} [\max\{p, \tilde{R}(M, b)\}\tilde{\sigma}^E(b, p) + \tilde{R}(M, b)(1 - \tilde{\sigma}^E(b, p))]f(p)dp - s\end{aligned}$$

If $|M| = N$, which corresponds to the case of the last sampling, $\tilde{U}(M) = 0$.

The strategy of Graduate is a probability distribution function $\tilde{\sigma}^G : \mathcal{M} \times \mathbb{R}_+ \to [0, 1] \times [0, 1]$. $\tilde{\sigma}^G(M, p) = (\tilde{\sigma}^G_1, \tilde{\sigma}^G_2)$ means that the probability that Graduate accepts the offer $p$ after receiving a series of signals $M$ is $\tilde{\sigma}^G_1(M, p)$. If the trade is not made (either Graduate or Employer rejects the offer), Graduate stops searching with Probability $\tilde{\sigma}^G_2(M, p)$, or continues with Probability $1 - \tilde{\sigma}^G_2(M, p)$.

*Equilibrium*

The perfect Bayesian equilibrium concept is used. A tuple $(\tilde{\sigma}^E, \tilde{\sigma}^G, \tilde{\phi})$ is a perfect Bayesian equilibrium if

1. Given $\tilde{\sigma}^G$ and $\tilde{\phi}$, $\tilde{\sigma}^E(m, p) > 0$ only if Employer's (VNM-)expected utility

upon receiving the signal $m$ and accepting the offer $p$ is positive.

$$\mathbb{E}_{\tilde{\phi}(M_m)}[(v(M_m) - p)\tilde{\sigma}_1^G(M_m, p)] \geq 0$$

$\tilde{\sigma}^E(m, p) = 1$ if it is strictly positive.

2. Given $\tilde{\sigma}^E$, $\tilde{\sigma}_1^G(M, p) > 0$ only if the offer is weakly higher than the threshold $\tilde{R}(M)$. That is $p \geq \tilde{R}(M)$. $\tilde{\sigma}_1^G(M, p) = 1$ if the inequality is strict. $\tilde{\sigma}_2^G(M, p) > 0$ only if $\tilde{U}(M) \leq c(M)$. $\tilde{\sigma}_2^G(M, p) = 1$ if the inequality is strict.

3. Given $\tilde{\sigma}^E$ and $\tilde{\sigma}^G$, $\tilde{\phi}$ is derived through Bayes' rule.

Given $\tilde{\sigma}^E$ and $\tilde{\sigma}^G$, the explicit expression of $\tilde{\phi}$ is obtained through Bayes' rule. That is, after receiving a signal $m$, the probability that Employer faces Graduate with history $M$ is $\tilde{\phi}(M)$. First, denote $\tilde{L}(M)$ as the likelihood that $M$ is reached. $\tilde{L}(\emptyset) = 1$. Then,

$$\tilde{L}(M, g) = \tilde{L}(M)x(M) \int_{c_L}^{v_H} [1 - \tilde{\sigma}^E(m, p)\tilde{\sigma}_1^G(M, p)][1 - \tilde{\sigma}_2^G(M, p)]f(p)dp$$

$$\tilde{L}(M, b) = \tilde{L}(M)(1 - x(M)) \int_{c_L}^{v_H} [1 - \tilde{\sigma}^E(m, p)\tilde{\sigma}_1^G(M, p)][1 - \tilde{\sigma}_2^G(M, p)]f(p)dp$$

where $m$ is the last signal in $M$. The likelihood that Graduate reaches the node $(M, m)$ equals the likelihood that history $M$ is reached times the probability a signal $m$ occurs times the probability that Graduate continues searching.

Since Employers are sampled randomly, by Bayes' rule, if $m = g$,

$$\tilde{\phi}(M_g) = \frac{\tilde{L}(M_g)}{\sum_{M'_g \in \mathcal{M}_g} \tilde{L}(M'_g)}$$

If $m = b$,

$$\tilde{\phi}(M_b) = \frac{\tilde{L}(M_b)}{\sum_{M'_b \in \mathcal{M}_b} \tilde{L}(M'_b)}$$

Under the random proposal protocol, Employer will not capture all the surplus. The minimal value he could receive from the market is $v_L$. Therefore, he accepts any price lower than $v_L$ because he receives a non-negative payoff by doing that. This is true even when Graduate knows the state ex ante. There is no hold-up problem under this protocol. The focus is on how the adverse selection affects the agents' behavior.

Lower search costs or a larger number of Employers exacerbates the adverse selection effect. As search costs decrease or the number of Employers increases, Graduate's continuation payoff increases and she prefers to continue searching rather than accept the offer, especially after receiving a bad signal. Then, sampled Employer rejects the high offer to avoid trading with Graduate who has a bad history.

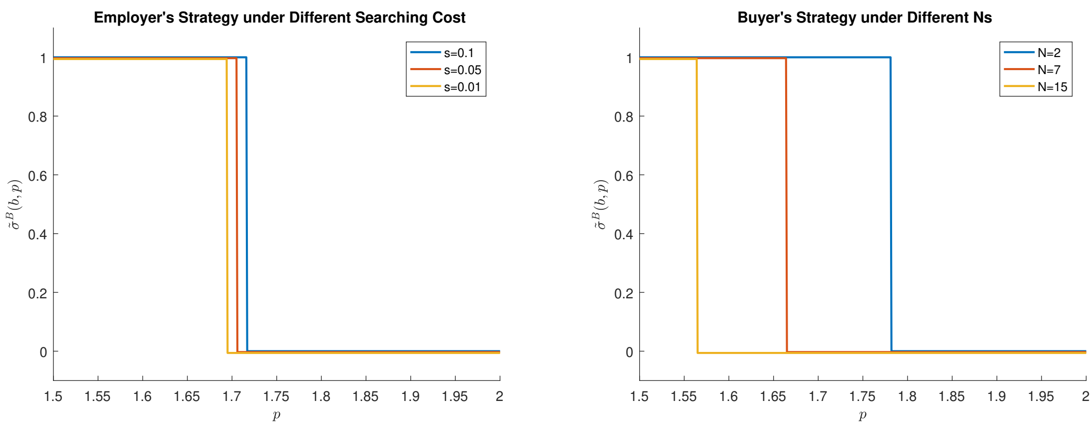



Figure 2.6: The parameter is set as: $v_H = 2.5$, $v_L = 1.5$, $c_H = 2$, $c_L = 1$, $\mu_H^g = 0.5$, $\mu_L^g = 0$, $q_0 = 0.5$.

The simplified case is considered: $\mu_H^g > \mu_L^g = 0$ such that Employer receiving the good signal will accept any offer because he knows Graduate is of high type and any price lower than $v_H$ brings him a positive payoff. Then, the focus is on the strategy of Employer receiving the bad signal. Figure 2.6 shows that, as search costs decrease or the number of Employers increases, Employer tends to reject the high offers.

As shown in Section 2.3, Graduate is willing to enter the market when search costs are low enough, $s < \underline{s}_1$. Graduate's gain from the market decreases as Employer rejects the high offer. Combining the analysis above, Graduate is less willing to enter the market when the number of Employers is large, or in other words, the adverse selection effect is significant. In Figure 2.7, the threshold $\underline{s}_1$ first increases and then decreases. There are two effects: i) Graduate is more likely to make the trade as there are more Employers; thus, her gain from the market increases which gives her incentives to enter, and ii) the adverse selection effect, which lets Employer reject the low offer and decreases Graduate's gain from the market.

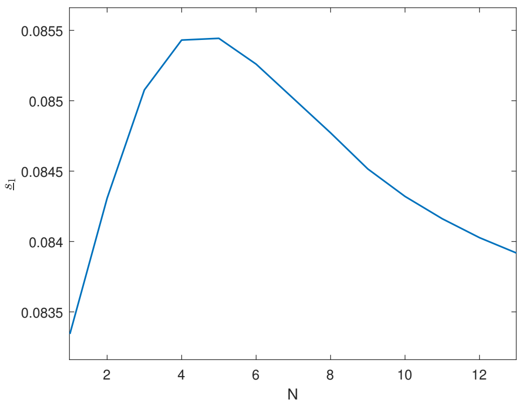



Figure 2.7: The parameter is set as: $v_H = 2.5$, $v_L = 1.5$, $c_H = 2$, $c_L = 1$, $\mu_H^g = 0.5$, $\mu_L^g = 0$, $q_0 = 0.5$.

### 2.6.4 Employers Arrive Randomly

This study assumes that Graduate can always sample another Employer after she rejects an offer whilst in real cases, she needs to wait for some time until another Employer comes along. This section considers a finite discrete-time situation $t = 1, 2, ...N$ where Employers arrive with some probability.

The basic settings are the same as those in Section 2.2, except that now one Employer comes to Graduate with Probability $\lambda$ in each time interval $t$. Once they meet, they observe a signal. Employer offers a price, and Graduate has three choices: i) Graduate accepts the offer and the process is over, ii) she rejects the offer and waits for another Employer, and iii) she rejects the offer and stops waiting. It is assumed that Employer knows the Time $t$ when he arrives but he knows neither how many Employers Graduate has met nor the past signals she has observed. Once the trade is made in Time $t$, Employer's utility is $v_H - p$ if the state is $H$, and $v_L - p$ otherwise. The utility of Graduate is $p$. If Graduate chooses to stop searching or no trade is made after Graduate samples all Employers, Employers' utilities are 0, and Graduate's utility is $c_H$ if she is of type $H$, and $c_L$ otherwise, which is her outside option.

*Strategy*

The strategy of Employers is a probability distribution function $\sigma^E : \{g, b\} \times \mathbb{N}^+ \times \mathbb{R}_+ \rightarrow [0, 1]$, where $\sigma^E(m, t, p)$ means that the probability that Employer receiving the signal $m$ offers a price $p$ in Time $t$. Denote $M^t = (m^1, m^2, ..., m^t)$ Graduate's record of signals in Time $t$. $m^i \in \{g, b, o\}$, $o$ means that no Employer comes. Let $\mathcal{M}^t$ be the set of all possible histories in Time $t$. The strategy of

Graduate is a probability distribution function $\sigma^G : \mathcal{M}^t \times \mathbb{R}_+ \to \Delta^2$, where $\Delta^2$ is a unit 2-simplex and $\sigma^G(M^t, p) = (\sigma_1^G, \sigma_2^G, \sigma_3^G)$ means that the probability that Graduate accepts the offer $p$ (if no Employer comes, $p = 0$) after receiving a series of signals $M^t$ in Time $t$ is $\sigma_1^G(M^t, p)$, stops waiting $\sigma_2^G(M^t, p)$, and continues waiting $\sigma_3^G(M^t, p)$.

*Belief*

When Employer comes in Time $t$, he forms a belief about Graduate's history $\phi : \mathcal{M}^t \to [0, 1]$, where $\phi(M^t)$ is the probability that Graduate has a history $M^t$ in Time $t$.

*Graduate's continuation payoff*

Let $U(M^t)$ be Graduate's continuation payoff given the history $M^t$. It should be the largest one among the offer, the outside option, and the continuation payoff in the next round.

$$\begin{aligned} U(M^t) =& \lambda x(M^t)\mathbb{E}_{\sigma^E(g,t+1,p)}[\max\{p, c(M^t, g), U(M^t, g)\}] \\ &+ \lambda(1 - x(M^t))\mathbb{E}_{\sigma^E(b,t+1,p)}[\max\{p, c(M^t, b), U(M^t, b)\}] \\ &+ (1 - \lambda) \max\{c(M^t, o), U(M^t, o)\} \end{aligned}$$

$U(M^N) = 0$ for any history $M^N$. Denote $R(M^t) := \max\{c(M^t), U(M^t)\}$.

*Equilibrium*

A tuple $(\sigma^E, \sigma^G, \phi)$ is a perfect Bayesian equilibrium if

1. Given $\sigma^G$ and $\phi$, $\sigma^E(m, t, p) > 0$ only if $p$ maximizes Employer's (VNM-)expected utility upon receiving the signal $m$ in Time $t$.

$$p \in \arg\max_{p'} \mathbb{E}_{\phi(M_m^t)}[(v(M_m^t) - p')\sigma_1^G(M_m^t, t, p')]$$

2. Given $\sigma^E$, $\sigma_1^G(M^t, p) > 0$ only if the offer is weakly higher than the threshold $R$. That is $p \geq R(M^t)$. $\sigma_1^G(M^t, p) = 1$ if the inequality is strict. Similarly, $\sigma_2^G(M^t, p) > 0$ only if $R(M^t) = c(M^t) \geq p$. $\sigma_3^G(M^t, p) > 0$ only if $R(M^t) = U(M^t) \geq p$.

3. Given $\sigma^G$ and $\sigma^E$, $\phi$ is derived through Bayes' rule.

Given $\sigma^G$ and $\phi$, it is weakly dominated to offer a price except from $\{0, R(\mathcal{M}^t)\}$ in Time $t$. The threshold $R(\mathcal{M}^t)$ can be well defined. Then, given $\sigma^E$ and $\sigma^G$, the explicit expression of $\phi$ is obtained through Bayes' rule. Define $L(M^t)$ to be the likelihood that $M^t$ is reached in Time $t$. $L(\emptyset) = 1$. Then[5],

$$L(M^t, g) = \lambda L(M^t) x(M^t) \sum_{\tilde{M}^t \in \mathcal{M}^t} \sigma^E(m^t, t, R(\tilde{M}^t)) \sigma_3^G(M^t, R(\tilde{M}^t))$$

$$L(M^t, b) = \lambda L(M^t)(1 - x(M^t)) \sum_{\tilde{M}^t \in \mathcal{M}^t} \sigma^E(m^t, t, R(\tilde{M}^t)) \sigma_3^G(M^t, R(\tilde{M}^t))$$

$$L(M^t, o) = (1 - \lambda) L(M^t)$$

By Bayes' rule, if $m = g$,

$$\phi(M_g^t) = \frac{L(M_g^t)}{\sum_{\tilde{M}_g^t \in \mathcal{M}_g^t} L(\tilde{M}_g^t)}$$

If $m = b$,

$$\phi(M_b^t) = \frac{L(M_b^t)}{\sum_{\tilde{M}_b^t \in \mathcal{M}_b^t} L(\tilde{M}_b^t)}$$

The continuation payoff $U(M^t)$ is always higher than the expected outside option $c(M^t)$ if $t < N$. Therefore, Graduate will wait until the end.

### Equilibrium Deduced from Evolving $\lambda$

An algorithm can be followed to find an equilibrium. Two-period case is used to illustrate the intuition. When $\lambda \to 0$, Graduate seldom meets Employer. Therefore, Employer can set prices $R(g) = R(o, g) = c(g)$ and $R(b) = R(o, b) = c(b)$ after receiving $g$ and $b$ respectively. As $\lambda$ increases, Graduate has incentives to reject the offer under the bad signal price in Time 1 because he can wait for a good signal in Time 2 and receive a higher offer. This effect increases her continuation payoff. Therefore, $R(b)$ increases.

Until $\lambda$ reaches some point $\lambda_1$, $R(b)$ equals $v(b)$, in which case Employer who arrives in Time 1 (B[1]) and receives $b$ gets 0 if he offers $R(b)$. As $\lambda$ continues to increase, if Employer who arrives in Time 2 (B[2]) still offers $R(o, m) = c(m)$, he knows that besides Graduate who has not met a Employer in Time 1, he may also face Graduate with a bad signal from Time 1 because B[1] receiving $b$ does not offer any price higher than $v(b)$ in which case Graduate will reject him. Then, offering

[5] Just for notation, $\sigma^E(\emptyset, t, p) = 0$.

$R(o, m)$ is still optimal if $\forall m \in \{g, b\}$,

$$\begin{aligned}&\phi(o,m)(v(m)-c(m))+\phi(b,m)(v(b,m)-c(m))\geq\phi(b,m)(v(b,m)-c(b,m))\\ \Leftrightarrow\ &\phi(o,m)(v(m)-c(m))\geq\phi(b,m)(c(m)-c(b,m))\end{aligned} \tag{2.1}$$

The left hand side is the payoff from offering $R(o, m)$. The right hand side is the payoff from offering $R(b, m)$. The inequality means that he has no incentive to deviate. If that is not the case, either B[2] receiving $g$ or $b$ will mix between $R(o, m)$ and $R(b, m)$.

**Lemma 12.** *If $\lambda > \lambda_1$ and (2.1) is not satisfied, there exists a threshold of prior $\bar{q}$ such that when $q_0 < \bar{q}$, B[2] receiving $g$ mixes between $R(o, g)$ and $R(b, g)$ while B[2] receiving $b$ offers $R(o, b)$. When $q_0 \geq \bar{q}$, B[2] receiving $b$ mixes between $R(o, b)$ and $R(b^2)$ while B[2] receiving $g$ offers $R(o, g)$.*

The reason that the initial prior affects who mixes is because if B[2] thinks the state is less likely to be $H$ ex ante, then he will be reluctant to offer a high price $R(o, g)$ even if he receives a good signal because he may face Graduate with a bad signal. If $q_0$ is high, then B[2] with a bad signal believes that he is in a worse condition. Graduate would trade in Time 1 if the state was $H$ where a good signal was more likely to occur. If she does not, he may face Graduate with a bad signal.

Focusing on the second case where B[2] with a bad signal mixes according to $z_1 R(o, b) \oplus (1 - z_1) R(b^2)$, the mixing will lower Graduate's continuation payoff in Time 1 and keeps her threshold equal to $v(b)$. It can be written as

$$R(b) = U(b) = \lambda\{x(b)c(g) + (1 - x(b))[z_1 c(b) + (1 - z_1)c(b^2)]\} + (1 - \lambda)c(b) = v(b)$$

Then, B[1] receiving $b$ offers $R(b) = v(b)$ and his utility is 0 if Graduate accepts. To keep B[2] with $b$ indifferent between $R(o, b)$ and $R(b^2)$, B[1] with $b$ offers $R(b)$ with Probability $y_1$ and 0 with Probability $1 - y_1$. Then, the indifference condition for B[2] with $b$ is

$$\phi(o, b)[v(b) - c(b)] + \phi(b, b)[v(b^2) - c(b)] = \phi(b, b)[v(b^2) - c(b^2)]$$

where $\phi(o, b) : \phi(b, b) = (1 - \lambda)\lambda(1 - x(\emptyset)) : \lambda(1 - x(\emptyset))(1 - y_1)\lambda(1 - x(b))$. The indifference condition is equivalent to

$$(1-\lambda)[v(b)-c(b)] = \lambda(1-y_1)(1-x(b))[c(b)-c(b^2)]$$

The equilibrium holds until $\lambda$ reaches some point $\lambda_2$ where $z_1$ becomes 0. As $\lambda$ continues to increase, B[2] receiving $b$ offers $R(b^2)$ optimally. If (2.1) is not satisfied for B[2] receiving $g$, he has incentives to mix.

**Lemma 13.** *If $\lambda > \lambda_2$, $q_0 < \bar{q}$ and (2.1) is not satisfied for $m = b$, B[2] receiving $b$ mixes between $R(o,b)$ and $R(b^2)$ while B[2] receiving $g$ offers $R(b,g)$. If $\lambda > \lambda_2$, $q_0 > \bar{q}$ and (2.1) is not satisfied for $m = g$, B[2] receiving $g$ mixes between $R(o,g)$ and $R(b,g)$ while B[2] receiving $b$ offers $R(b^2)$.*

To obtain the algorithm for the general case $N \geq 2$, Individual Rationality constraint and Incentive Compatibility constraint are defined.

*Individual Rationality constraint (IR)*

Given the equilibrium strategy $\sigma^{G*}$ and $\phi^*$,

$$\max_{p\in\{R(M^t)\}} \mathbb{E}_{\phi^*(M^t)}[(v(M^t)-p)\sigma_1^{G*}(M^t,p)] \geq 0$$

*Incentive Compatibility constraint (IC)*

Given the equilibrium strategy $\sigma^{G*}$ and $\phi^*$, $\forall p' \in \{R(M^t)\}$,

$$\max_{p\in\{R(M^t)\}} \mathbb{E}_{\phi^*(M^t)}[(v(M^t)-p)\sigma_1^{G*}(M^t,p)] \geq \mathbb{E}_{\phi^*(M^t)}[(v(M^t)-p')\sigma_1^{G*}(M^t,p')]$$

The constraint is binding if it is a equality.

**Algorithm 2.** *An equilibrium can be deduced from increasing $\lambda$ by following such a process.*

1. *After setting $\lambda = 0$, where Employer who arrives in Time $t$ and receives a signal $m$ offers $R(o,...,o,m)$, Graduate accepts the offer with Probability 1. Then, $\lambda$ increases continuously.*

2. *$\lambda$ stops increasing until the IR constraint of Employer who arrives in Time 1 (B[1]) and receives a signal $b$ is binding. He decreases the probability of offering $R(b)$ and mixes it with offering 0 until either of the two cases happen:*

*Case 1: A sequential Employer's IC constraint is binding. Then, $\lambda$ continues to increase and Employer whose IC constraint is binding mixes the corresponding strategies to keep the B[1] with b's IR constraint binding. $\lambda$ continues to increase.*

*Case 2: A sequential Employer's IR constraint is binding. He decreases the probability of offering $R(b)$ and mixes it with offering 0 to keep the B[1] with b's IR constraint binding. $\lambda$ continues to increase.*

3. *As $\lambda$ increases, four cases could happen.*

   *Case 1: The binding IR constraint is not binding. The corresponding Employer who mixes turns to a pure optimal strategy. $\lambda$ continues to increase.*

   *Case 2: Another IR constraint is binding. Then, go to Step 2.*

   *Case 3: The binding IC constraint is not binding. The corresponding Employer who mixes turns to a pure optimal strategy.*

   *Case 4: Another IC constraint is binding. The corresponding Employer mixes such that the current binding constraints keep binding. $\lambda$ continues to increase until either of the four cases happen.*

IR constraint of Employer who arrives in Time 1 and receives a signal $b$ is always first binding. It is because the threshold $R(M_b^t)$ is decreasing in $t$ where $M_b^t = (o, ..., o, b)$. $R(M_b^N) = c(b)$. $R(M_g^t) = c(g)$.

$$\begin{aligned} R(M_b^{N-1}) =& \lambda[x(b)c(g) + (1 - x(b))R(M_b^N)] + (1 - \lambda)c(M_b^{N-1}) \\ >& c(b) = R(M_b^N) \end{aligned}$$

$$\begin{aligned} R(M_b^{N-2}) =& \lambda[x(b)c(g) + (1 - x(b))R(M_b^{N-1})] \\ &+ (1 - \lambda)\lambda[x(b)c(g) + (1 - x(b))R(M_b^N)] + (1 - \lambda)^2 c(M_b^{N-2}) \\ >& \lambda[x(b)c(g) + (1 - x(b))R(M_b^N)] + (1 - \lambda)c(M_b^{N-1}) \\ =& R(M_b^{N-1}) \end{aligned}$$

Then, by applying the mathematical induction, we have $R(M_b^t) > R(M_b^{t+1})$.

**Example 9.** *Consider a three-period searching process ($N = 3$). The probability that a good signal occurs under the state $H$ is $\mu_H^g = 0.5$. The probability that a good signal occurs under the state $L$ is $\mu_L^g = 0$. The value and cost from trading* *is as follows:* $v_H = 3.2$*,* $v_L = 1.1$*,* $c_H = 3$ *and* $c_L = 1$*. The prior* $q_0 = 0.5$*. In this example, once Employer receives the good signal, he is sure that the state is* $H$*. Thus, he offers* $c_H$*.*

*Phase 1: when* $\lambda$ *is low,* $\lambda \in (0, 0.31)$*, Employer in Time 1 (B[1]) receiving the bad signal offers* $R(b)$*. B[2] offers* $R(o, b)$ *and B[3] offers* $R(o, o, b)$*.*

*Phase 2: as* $\lambda$ *increases,* $\lambda \in [0.31, 0.38)$*, B[1]'s IR condition is binding. Then, he mixes between offering* $R(b)$ *and 0. B[2]'s IR condition is binding and he mixes between offering* $R(o, b)$ *and 0.*

*Phase 3: as* $\lambda$ *continues increasing,* $\lambda \in [0.38, 0.44)$*, B[3]'s IC condition is binding. Then, he mixes between offering* $R(o, o, b)$ *and* $R(o, b, b)$*.*

*Phase 4: when* $\lambda \in [0.44, 0.6)$*, B[2]'s IR condition is not binding. Then, he plays the pure strategy by offering* $R(o, b)$*.*

*Phase 5: when* $\lambda \in [0.6, 0.75)$*, B[3]'s IC condition is not binding. Then, he plays the pure strategy by offering* $R(b, o, b)$*. B[1] lowers the probability on* $R(b)$*. Then, B[2]'s IC condition is binding. Then, he mixes between offering* $R(o, b)$ *and* $R(b^2)$*.*

*Phase 6: when* $\lambda \in [0.75, 0.82)$*, B[2]'s IR condition is binding. Then, he mixes between offering* $R(o, b)$ *and 0.*

*Phase 7: when* $\lambda \in [0.82, 1]$*, B[3]'s IC condition is binding. Then, he mixes between offering* $R(o, b, b)$ *and* $R(b, b, b)$*.* $\square$

### Comparison With Graduate Knows the State ex ante

If Graduate knows the state ex ante, an equilibrium similar to Kaya and Kim [2018] is obtained. If the prior $q_0$ is high enough, all Employers offer $c_H$ regardless of the signals they receive because Graduate with type $H$ only accepts offers higher than $c_H$, and if Employer offers the threshold of Graduate of the low type, he can trade only with the low type. When the threshold is higher than $v_L$, it is dominant to offer $c_H$. If the prior $q_0$ is low enough, all Employers offer the threshold of Graduate of the low type regardless of the signals they receive.

Neither strategy is optimal when Graduate does not know the state ex ante. One reason is the adverse selection effect. It lets Employer arriving in each period to shade his offer. This effect is accumulated backwards. Thus, in the early stages, Graduate's threshold is far lower than Employer's reservation value. Therefore, even if Graduate's type is of high probability to be $H$, it is optimal for Employer receiving a bad signal to offer a price equal to Graduate's threshold. The second reason is information asymmetry becomes significant as time goes. In the early stages, when information asymmetry is not significant, Employer who arrives and receives a good signal can trade with an offer far lower than $c_H$. That is why even when the prior is low, the trade can be done once a good signal occurs.

### 2.6.5 Proofs

**Proof of lemma 8:**

Since any price $p \notin R(\mathcal{M})$ is dominated, $\sigma^E(m,p) > 0$ only if $p \in R(\mathcal{M}_m)$. Then, for any $\sigma^E$, we can define a mapping $T$ on the value of the threshold $R(\mathcal{M})$. For any $R(\mathcal{M}) \in [c_L, c_H]^{|\mathcal{M}|}$, $T[R(\mathcal{M})]$ is defined by the larger one between Graduate's expected outside option and the continuation payoff.

$$T[R](M) = \max\{c(M), U(M)\}$$

where $U(M)$ is determined backwards

$$\begin{aligned} U(M) = {} & x(M)\mathbb{E}_{\sigma^E(g,p)}[\max\{p, c(M,g), U(M,g)\}] \\ & + (1 - x(M))\mathbb{E}_{\sigma^E(b,p)}[\max\{p, c(M,b), U(M,b)\}] - s \end{aligned}$$

and if $|M| = N$, $U(M) = 0$.

We define a distance $d$ on the space $\{R(\mathcal{M})\}$ as the Euclidean distance. Then, $(\{R(\mathcal{M})\}, d)$ is a non-empty complete metric space. To prove the uniqueness of the threshold, we shall apply the Banach fixed-point theorem, which requires the mapping $T$ to be a contraction mapping. We have

$$\begin{aligned} U(M) \leq \frac{c_H - s}{c_H}\Big( & x(M)\mathbb{E}_{\sigma^E(g,p)}[\max\{p, c(M,g), U(M,g)\}] \\ & + (1 - x(M))\mathbb{E}_{\sigma^E(b,p)}[\max\{p, c(M,b), U(M,b)\}]\Big) \end{aligned}$$

Let $\delta = \frac{c_H - s}{c_H} < 1$. Then, for any two thresholds $R_1$ and $R_2$, $d\Big(T[R_1], T[R_2]\Big) \leq \delta d\big(R_1, R_2\big)$, which means that the mapping $T$ is a contraction mapping. ■

**Proof of lemma 9:**

If Employer offers $p_g = c_H$ and $p_b = R(b)$,

$$R(b) = \max\{c(b), U(b)\} = \max\{c(b), x(b)c_H + (1 - x(b))R(b) - s\} \Rightarrow R(b) = c_H - \frac{s}{x(b)}$$

Employer finds it optimal to offer $p_b = R(b)$ only if $v(b) \geq R(b)$. That is

$$v(b) \geq c_H - \frac{s}{x(b)} \Leftrightarrow s \geq x(b)(c_H - v(b)) =: \bar{s}_1$$

Otherwise, he mixes between $R(b)$ and $R(b^2) = c(b^2)$. He offers $R(b)$ with Probability $y$, and $R(b^2)$ with Probability $1 - y$. Graduate with history $(b)$ only accepts the offers $R(b)$, but accepts both with history $(b, b)$. Then, the value $U(b)$ is

$$U(b) = x(b)c_H + (1 - x(b))[yR(b) + (1 - y)R(b^2)] - s$$

If $U(b) > c(b)$, then $R(b) = U(b)$. And Employer is indifferent between offering $R(b)$ and $R(b^2)$. That is

$$\begin{aligned}&(v(b) - R(b))(1 - x(\emptyset)) + (v(b^2) - R(b))(1 - x(\emptyset))(1 - x(b))(1 - y)\\&= (v(b^2) - R(b^2))(1 - x(\emptyset))(1 - x(b))(1 - y)\end{aligned}$$

With the above two equations,

$$R(b) = \frac{v(b) + s - x(b)c_H}{1 - x(b)}$$

and

$$y = \frac{R(b) - c(b) + s}{(1 - x(b))(R(b) - c(b^2))}$$

If $U(b)$ reaches $c(b)$, then $R(b) = U(b) = c(b)$. One gets

$$y = \frac{s}{(1 - x(b))(R(b) - c(b^2))}$$

And Graduate with history $(b)$ receiving offer $R(b^2)$ finds it indifferent between continuing or stopping. She continues with Probability $z$. Then, Employer is indifferent between offering $R(b)$ and $R(b^2)$. That is

$$\begin{aligned}&(v(b) - c(b))(1 - x(\emptyset)) + (v(b^2) - c(b))(1 - x(\emptyset))(1 - x(b))(1 - y)\\&= (v(b^2) - c(b^2))(1 - x(\emptyset))(1 - x(b))(1 - y)z\end{aligned}$$

So,

$$z = \frac{v(b) - c(b)}{[c(b) - c(b^2)](1 - x(b))(1 - y)}$$

■

**Proof of lemma 10:**

If Employer gives the highest offer $p_g = c_H$ and $p_b = R(b)$, Graduate will search only if

$$U(\emptyset) = x(\emptyset)c_H + (1 - x(\emptyset))R(b) - s > c(\emptyset)$$

That is

$$s < \frac{(c_H - c(\emptyset))x(b)}{1 - x(\emptyset) + x(b)} =: \underline{s}_1$$

If $s < \underline{s}_1$ and Employer mixes between $R(b)$ and $R(b^2)$, Graduate will search at the beginning only if

$$U(\emptyset) = x(\emptyset)c_H + (1 - x(\emptyset))[yR(b) + (1 - y)U(b)] - s > c(\emptyset)$$

Since $R(b) = U(b)$ in this case, the above inequality is equivalent to

$$v(b) > \frac{x(\emptyset) - x(b)}{1 - x(\emptyset)}s + (1 - x(b))c(b) + x(b)c_H$$

where one can get $\underline{s}_2$.

We still need to prove $U(b) > c(b)$, which is

$$R(b) > c(b) \Leftrightarrow v(b) > (1 - x(b))c(b) + x(b)c_H - s$$

It is obvious that this inequality holds if the previous one holds.

Finally, if Employer offers only $R(b) = c(b)$ and $R(b^2) = c(b^2)$, Graduate gets nothing but the expected outside option. Then, she does not search. ■

**Proof of proposition 9:**

According to lemma 10, Graduate does not searches if $s > \underline{s}_1$. According to lemma 9, Employers offer $R(b)$ if $s \geq \bar{s}_1$. This gives the first boundary $f_1(s)$. Then, $v(b) > f_1(s)$ is corresponding to the first case of lemma 9 and lemma 10.

The definition of $\underline{s}_2$ forms the second boundary $f_2(s)$. Note that if $\underline{s}_2 > 0$, $\bar{s}_2 < 0$. If $s < \underline{s}_1$, $f_1(s) > f_2(s)$. Then, $v(b) \in (f_2(s), f_1(s))$ is corresponding to the second case of lemma 9 and lemma 10.

If $v(b) < f_2(s)$, $\underline{s}_2 < 0$, $\bar{s}_2 > 0$. According to lemma 10, Graduate does not search and there is no trade. ■

**Proof of lemma 11:**

If the equilibrium is of pure-strategy and $p_b \geq R(b^{N-1})$,

$$p_b \geq R(b^{N-1}) = \max\{c(b^{N-1}), U(b^{N-1})\} \geq x(b^{N-1})p_g + (1 - x(b^{N-1}))p_b - s$$
$$\Leftrightarrow p_g - p_b \leq \frac{s}{x(b^{N-1})}$$

As $s \sim o(1/NlogN)$, the right hand side converges to 0.

If all prices $p_b$ supported are higher than $R(b^{N-1})$,

$$\mathbb{E}[p_b] \geq R(b^{N-1}) = x(b^{N-1})\mathbb{E}[p_g] + (1 - x(b^{N-1}))\mathbb{E}[p_b] - s$$
$$\Leftrightarrow \mathbb{E}[p_g] - \mathbb{E}[p_b] < \frac{s}{x(b^{N-1})}$$

As $s \sim o(1/NlogN)$, the right hand side converges to 0. ■

**Proof of proposition 10:**

We first prove that there is no pure strategy equilibrium where Employer offers $R(b^N)$ after receiving a bad signal. If it is not the case, we prove that offering $R(b^{N-1})$ is better than $R(b^N)$. We compare the expected payoff of offering $R(b^{N-1})$

$$\phi(b^{N-1})[v(b^{N-1}) - R(b^{N-1})] + \phi(b^N)[v(b^N) - R(b^{N-1})]$$

and offering $R(b^N)$

$$\phi(b^N)[v(b^N) - R(b^N)]$$

where $\phi(b^N)/\phi(b^{N-1}) = 1 - x(b^{N-1})$.

As $N \to +\infty$ and $s \sim o(1/NlogN)$, $R(b^{N-1}) \to c_L$, $R(b^N) \to c_L$, $x(b^{N-1}) \to 0$, $v(b^{N-1}) \to v_L$ and $v(b^N) \to v_L$. It is easy to verify that the expected payoff of offering $R(b^{N-1})$ is higher than offering $R(b^N)$.

Then, we prove that the mixed-strategy equilibrium where Employer mixing between $R(b^{N-1})$ and $R(b^N)$ exists as $v_H - c_H \to 0$. Assume that he offers $R(b^{N-1})$ with Probability $y$. The value of $R(b^{N-1})$ is

$$R(b^{N-1}) = x(b^{N-1})c_H + (1 - x(b^{N-1}))[yR(b^{N-1}) + (1 - y)R(b^N)] - s$$

The indifference condition claims that the expected payoff of offering $R(b^{N-1})$ and offering $R(b^N)$ should be the same.

$$[v(b^{N-1}) - R(b^{N-1})]\phi(b^{N-1}) + [v(b^N) - R(b^{N-1})]\phi(b^N) = [v(b^N) - R(b^N)]\phi(b^N)$$

where $\phi(b^N)/\phi(b^{N-1}) = [1 - x(b^{N-1})](1 - y)$.

With these two equations, we get

$$R(b^{N-1}) = \frac{v(b^{N-1}) - x(b^{N-1})c_H}{1 - x(b^{N-1})} + s$$

and

$$y = 1 - \frac{x(b^{N-1})}{1 - x(b^{N-1})} \frac{c_H - v(b^{N-1})}{v(b^{N-1}) - c(b^{N-1})}$$

As $N \to +\infty$, $R(b^{N-1}) \to v_L$ and $y \to 1$.

Next, we need to prove Employer has no incentive to offer $R(b^i)$, $i = 1, 2, ..., N-2$. We prove that it is not optimal to offer $R(b^{N-2})$ and one can use the same procedure to prove the rest.

First, the value of $R(b^{N-2})$ is

$$R(b^{N-2}) = x(b^{N-2})c_H + [1 - x(b^{N-2})]R(b^{N-1}) - s$$

Then, the expected payoff of offering $R(b^{N-2})$ is

$$[v(b^{N-2}) - R(b^{N-2})]\phi(b^{N-2}) + [v(b^{N-1}) - R(b^{N-2})]\phi(b^{N-1}) + [v(b^N) - R(b^{N-2})]\phi(b^N)$$

Compare it with the expected payoff of offering $R(b^{N-1})$. The difference is

$$[v(b^{N-2}) - R(b^{N-2})]\phi(b^{N-2}) - [R(b^{N-2}) - R(b^{N-1})][\phi(b^{N-1}) + \phi(b^N)]$$

where $\phi(b^{N-2}) : \phi(b^{N-1}) : \phi(b^N) = 1 : [1-x(b^{N-2})] : [1-x(b^{N-2})][1-x(b^{N-1})](1-y)$.

As $N \to +\infty$, $s \sim o(1/NlogN)$ and $y \to 1$. So, $\phi(b^N)$ is negligible compared to $\phi(b^{N-1})$ and $\phi(b^{N-2})$. Along with $v(b^{N-2}) = x(b^{N-2})v_H + [1 - x(b^{N-2})]v(b^{N-1})$, the difference becomes

$$x(b^{N-2})(v_H - c_H) + (c_H - v_L)[x(b^{N-1}) - x(b^{N-2})]$$

We have $x(b^i) = \mu_H^g q(b^i)$ and

$$q(b^{i+1}) = \frac{(1 - \mu_H^g)q(b^i)}{(1 - \mu_H^g)q(b^i) + (1 - q(b^i))}$$

The difference being lower than 0 is equivalent to $v_H - c_H < \mu_H^g(c_H - v_L)$, which is satisfied when $v_H - c_H \to 0$.

Since $s \sim o(1/NlogN)$, without an offer $c_H$, Graduate searches to the last two

Employers whatever her type. For a high-type Graduate, with Probability 1, she receives a good signal as $N \to +\infty$. For low-type Graduate, she accepts the offer $v_L$ in the last two rounds of searching. Since $y \to 1$, $v_L$ is given with Probability 1 after a bad signal.

Finally, we need to verify that Graduate searches until the final Employer. It is obvious that $R(b^i) = U(b^i) > c(b^i), ß = 1, .., N-1$, and

$$U(\emptyset) > q_0 \cdot c_H + (1-q_0) \cdot v_L - N \cdot s > c(\emptyset) \blacksquare$$

**Proof of lemma 12:**

If (2.1) is not satisfied, then two cases could occur.

Case 1: there exists a $y > 0$ such that

$$\phi(o,b)(v(b) - c(b)) = \phi(b,b)(1-y)(c(b) - c(b^2))$$

and

$$\phi(o,g)(v(g) - c(g)) > \phi(b,g)(1-y)(c(g) - c(b,g))$$

where the B[2] receiving $b$ mixes.

Case 2: there exists a $y > 0$ such that

$$\phi(o,g)(v(g) - c(g)) = \phi(b,g)(1-y)(c(g) - c(b,g))$$

and

$$\phi(o,b)(v(b) - c(b)) > \phi(b,b)(1-y)(c(b) - c(b^2))$$

where the B[2] receiving $g$ mixes.

Which case happens depends on:

$$d_1 - d_2 = \frac{\phi(o,b)(v(b) - c(b))}{\phi(b,b)(c(b) - c(b^2))} - \frac{\phi(o,g)(v(g) - c(g))}{\phi(b,g)(c(g) - c(b,g))}$$

If it is negative, it leads to case 1. Otherwise, it leads to case 2.

$$\begin{aligned}
d_1 - d_2 &= \frac{\phi(o,b)(v(b)-c(b))}{\phi(b,b)(c(b)-c(b^2))} - \frac{\phi(o,g)(v(g)-c(g))}{\phi(b,g)(c(g)-c(b,g))} \\
&= \frac{(1-\lambda)\lambda(1-x(\emptyset))(v(b)-c(b))}{\lambda^2(1-x(\emptyset))(1-x(b))(c(b)-c(b^2))} - \frac{(1-\lambda)\lambda x(\emptyset)(v(g)-c(g))}{\lambda^2(1-x(\emptyset))x(b)(c(g)-c(b,g))} \\
&= \frac{(1-\lambda)[(v_H-c_H)q(b)+(v_L-c_L)(1-q(b))]}{\lambda(1-x(b))(c_H-c_L)(q(b)-q(b,b))} \\
&\quad - \frac{(1-\lambda)x(\emptyset)[(v_H-c_H)q(g)+(v_L-c_L)(1-q(g))]}{\lambda(1-x(\emptyset))x(b)(c_H-c_L)(q(g)-q(b,g))}
\end{aligned}$$

With

$$q(b)-q(b,b) = q(b) - \frac{q(b)\mu_H^b}{q(b)\mu_H^b+(1-q(b))\mu_L^b} = \frac{q(b)(1-q(b))(\mu_L^b-\mu_H^b)}{1-x(b)}$$

$$q(g)-q(b,g) = q(g) - \frac{q(g)\mu_H^b}{q(g)\mu_H^b+(1-q(g))\mu_L^b} = \frac{q(g)(1-q(g))(\mu_L^b-\mu_H^b)}{1-x(g)}$$

$$x(b) = q(b)\mu_H^g + (1-q(b))\mu_L^g$$

$$x(g) = q(g)\mu_H^g + (1-q(g))\mu_L^g$$

$$q(b) = \frac{q_0\mu_H^b}{q_0\mu_H^b+(1-q_0)\mu_L^b}$$

$$q(g) = \frac{q_0\mu_H^g}{q_0\mu_H^g+(1-q_0)\mu_L^g}$$

Then

$$\begin{aligned}
&d_1 - d_2 < 0 \\
\Leftrightarrow\ & \frac{(v_H-c_H)q(b)+(v_L-c_L)(1-q(b))}{(1-x(b))(q(b)-q(b,b))} - \frac{x(\emptyset)[(v_H-c_H)q(g)+(v_L-c_L)(1-q(g))]}{(1-x(\emptyset))x(b)(q(g)-q(b,g))} < 0 \\
\Leftrightarrow\ & q_0 > \bar{q} = \frac{(v_L-c_L)\mu_L^g\mu_L^b(\mu_H^g-\mu_H^b)}{(v_L-c_L)(\mu_H^g\mu_L^g-\mu_H^b\mu_L^b)(\mu_H^g-\mu_L^b)+k\mu_H^b\mu_H^g(\mu_L^b-\mu_L^g)} \quad \blacksquare
\end{aligned}$$

**Proof of lemma 13:**

As $\delta > \delta_2$, case 1 in the proof of lemma 12 becomes

$$\phi(o,b)(v(b)-c(b)) < \phi(b,b)(1-y)(c(b)-c(b^2))$$

and

$$\phi(o,g)(v(g)-c(g)) = \phi(b,g)(1-y)(c(g)-c(b,g))$$

while case 2 becomes

$$\phi(o,g)(v(g)-c(g)) < \phi(b,g)(1-y)(c(g)-c(b,g))$$

and

$$\phi(o,b)(v(b)-c(b)) = \phi(b,b)(1-y)(c(b)-c(b^2)) \blacksquare$$

# Chapter 3

# Markets with Behavioral Agents

## 3.1 Introduction

Markets are not perfect, and one of the reasons for this is the agents with bounded rationality. Bounded rationality is the idea that people's decision-making abilities are limited by their cognitive abilities and the information available to them, which can lead to suboptimal decisions. Cognitive limitations refer to the fact that people's cognitive abilities are finite, which can result in biases and errors in decision-making. The complexity of the information environment can also make it difficult for individuals to process all of the available information and make optimal decisions. This chapter revisits the models discussed in the previous chapters in these two ways.

Section 3.2 revisits the market for talent, the "author-paper-journal" market. As the information disadvantage side, publishing qualified papers requires editors to avoid cognitive errors, which is different because the only information they can rely on is the signal they observe. First, editors should recognize that receiving a paper is an additional signal: the paper might have been rejected. He should correct this selection bias effect by only accepting papers with better signals. Otherwise, he overestimates the quality of the paper and sets a lower threshold of the signal than a rational editor. Secondly, editors should be aware that the author's decision to (re)submit or quit is based on her type, not randomizing. If the editor is unaware of the fact that the paper comes from a high-type author probably, he tends to set a higher threshold.

Moreover, if the incumbents deviate from setting optimal thresholds due to some cognitive errors, it reduces the level of selection bias effect in the market, as papers rejected by the incumbents are either of too low quality to be published elsewhere, or of sufficient quality to be published. This offers entrants the chance to challenge the incumbents' status.

Section 3.3 revisits the Graduate-Employer market. Compared to Graduate, selected Employers need to make a much more complicated decision, let alone the only information they have the feedback from interview. Thus, Employers sometimes rely on previous experiences to determine optimal strategies. However, misusing information could lead to irrational action. The objective is to find which pieces of information are essential for the agents to make an optimal decision and how it affects agents' behavior if they misuse them.

Before entering the market, Employer learns how to bid from the historical document which records previous Employers' offers, corresponding signals, corresponding Graduates' types, and acceptance outcomes. These are things Employer should know to make an optimal decision, from which he determines: i) the probability of Graduate's type given a particular signal; ii) the probability of his offer being accepted conditional on the signal and Graduate's type.

Then, I introduce a type of Employer who lacks sophistication and does not consider Graduate's type when determining the second probability. This unsophisticated Employer fails to recognize that low-type Graduates are more likely to accept his offer than high-type Graduates. Consequently, he tends to overbid due to his overexpectation of the value from trading. Furthermore, the adverse selection effect exacerbates this overbidding tendency. This finding can be applied to explain overbidding in corporate acquisitions, where the value of the target is difficult to observe and bidding firms may be influenced by hubris (Coff [2002], Hayward and Hambrick [1997], Roll [1986]).

Secondly, I consider another type of unsophisticated Employer who does not take the signal into account when determining the second probability. This Employer assumes that Graduates behave in a stationary manner conditional on her type, and expects that high-type or low-type Graduate plays a mixed-strategy where she accepts the offer with some probability. He may underbid compared to rational Employers because he can not differentiate between higher prices offered to Employers who received good signals and lower prices offered to those who received bad signals since he groups the data together. If the latter were accepted with high probability, unsophisticated Employer observes from the data that he can still make the trade by offering a lower price. Thus, he mistakenly believes that he can increase his payoff by underbidding. It decreases the deal prices in the market, which further weakens the willingness of Employer to offer a high price because of the adverse selection effect.

### 3.1.1 Related Literature

This study uses the analogy-class approach (Jehiel [2005]) to characterize bounded rational Employers, where they bundle the nodes into the analogy-classes. A real situation is presented to show how Employers form different kinds of coarse analogy-classes. In one case, Employer forms an expectation of the value from the trade, which is unconditional on whether Graduate will accept their offer. Eyster and Rabin [2005] applies this approach in the static adverse selection model, and Esponda [2008] modifies this approach by allowing Employers to adjust their behavior to correct the bias between the experience and the belief. In the other case, Employer has access only to how the behavior of Graduate depends on their type which corresponds to the payoff-relevant analogy partition (Jehiel and Koessler [2008]). They reason that the behavior of Graduate is stationary conditional on their type.

## 3.2 Bounded Rational Editor

This section discusses possible mistakes that an editor could make due to their bounded rationality. The first type of naive editor does not realize that "being sampled" contains some information, while the second type of bounded rational editor does not know that the author has private information (i.e., their type) and takes action based on probabilities given each history.

#### Editor with "Sampling Curse"

To illustrate the result, the following simplified case is illustrated: the type of the author $\theta$ is either high $(H)$ or low $(L)$. There are two journals. Table 3.1 lists the likelihood $l(\theta, h)$ of the combination of the author's type $\theta$ and his history $h$, where $\mu_\theta$ is the prior distribution of type and $P^R_\theta$ is probability that the paper of the type-$\theta$ author is reject. $P^R_H < P^R_L$, which means that the high-type author is less likely to get rejection. It is assumed that the submission costs are negligible so that the author will try again regardless of his type.

| $\theta \backslash h$ | $\emptyset$ | (A) |
|---|---|---|
| $H$ | $\mu_H$ | $\mu_H \cdot P^R_H$ |
| $L$ | $\mu_L$ | $\mu_L \cdot P^R_L$ |

Table 3.1: Likelihood of the combination of the author's type $\theta$ and his history $h$.

The naive editor knows the prior distribution of the authors' type and the conditional probability of the author's history given her type,[1] but does not realize that "receiving a paper" itself is an extra signal. He computes the probability,

$$Pr\left[\theta = H, h = \emptyset\right] = Pr\left[\theta = H\right] Pr\left[h = \emptyset \middle| \theta = H\right] = \frac{\mu_H}{1 + P_H^R}$$

and

$$Pr\left[\theta = L, h = \emptyset\right] = Pr\left[\theta = L\right] Pr\left[h = \emptyset \middle| \theta = L\right] = \frac{\mu_L}{1 + P_L^R}$$

However, for the sophisticated editor being aware of "sampling curse", he computes the conditional probability

$$\begin{aligned} Pr\left[\theta = H, h = \emptyset \middle| sampled\right] &= \frac{\frac{1}{2}l(H,\emptyset)}{\frac{1}{2}l(H,\emptyset) + \frac{1}{2}l(L,\emptyset) + \frac{1}{2}l(H,(A)) + \frac{1}{2}l(L,(A))} \\ &= \frac{\mu_H}{1 + \mu_H P_H^R + \mu_L P_L^R} \end{aligned}$$

and

$$\begin{aligned} Pr\left[\theta = L, h = \emptyset \middle| sampled\right] &= \frac{\frac{1}{2}l(L,\emptyset)}{\frac{1}{2}l(H,\emptyset) + \frac{1}{2}l(L,\emptyset) + \frac{1}{2}l(H,(A)) + \frac{1}{2}l(L,(A))} \\ &= \frac{\mu_L}{1 + \mu_H P_H^R + \mu_L P_L^R} \end{aligned}$$

It is easy to verify that $Pr\left[\theta = H, h = \emptyset\right] > Pr\left[\theta = H, h = \emptyset | sampled\right]$ and $Pr\left[\theta = L, h = \emptyset\right] < Pr\left[\theta = L, h = \emptyset | sampled\right]$ and it is also true for history $h = (A)$. The naive editor overestimates the author's type, and furthermore, the paper's quality. Thus, compared to the sophisticated editor, he tends to set a lower threshold of the signal.

**Proposition 11.** *When search costs are negligible $c \to 0$, the naive editor sets a threshold $\tilde{s}_A < s_A^*$.*

### Editor with Analogy-based Expectation

Another kind of bounded rational editor who does not know that the author has private information (her type) is considered. He only observes that a proportion of the authors quit after being rejected, without realizing that the author takes action according to her type (the high-type author resubmits but the low-type quits). Then, he believes that the author rejected mixes between resubmitting and stopping. The

[1] In a real situation, the naive editor perceives that from asking his colleagues how many times their papers have been rejected and whether they will submit again.

analogy-based approach (Jehiel [2005]) is used to characterize the solution concept. More specifically, the naive editor bundles authors with the same history (whatever her type) into analogy classes, and he only tries to learn their average behavior in each analogy class.

*Analogy-based Expectation Equilibrium (ABEE)*

A tuple $(\gamma, \hat{\tau}, \hat{\eta}_A, \hat{\beta}_A, \xi, \alpha)$ is an ABEE if

1. Bounded rational editors put the authors with the same history $h$ but different types into the same analogy class $\alpha(h)$, and form an analogy-based expectation that the author (re)submits with Probability $\xi(h)$, and stops with Probability $1-\xi(h)$.

$$\xi(h) = \int_{\hat{\tau}(\theta,h)=A} \mu(\theta|h)d\theta$$

$\mu(\theta|h)$ is the distribution of authors' type conditional on their history $h$.

2. Given signal $s$ and belief $\hat{\beta}_A$, class-A journals accept a paper ($\hat{\eta}_A(s) = Ac$) if and only if the expected quality is higher than $q_A$,

$$\mathbb{E}_{\hat{\beta}_A}[q|s] \geq q_A$$

3. Given her history $h$, the author calculates the expected payoff of submitting her paper to a class-A journal. That is,

$$\hat{\pi}_A(\theta, h) = v \int \gamma(q|\theta, h) \int_{\hat{\eta}_A(s)=Ac} \phi(s, q, \sigma_s)dsdq - c$$

If $\hat{\pi}_A(\theta, h) \geq 0$ and the author has not tried all journals, she submits her paper to a journal of class-A she has not tried before ($\hat{\tau}(\theta, h) = A$). Otherwise, she stops ($\hat{\tau}(\theta, h) = stop$).

4. Given $\xi$ and $\hat{\eta}_A$, $\gamma(q|\theta, h)$ and $\hat{\beta}_A(q)$ are derived by Bayes' rule.

$$\hat{\beta}_A(q) = \sum_{i=0}^{m-1} \hat{L}(q, h = A^i) \Bigg/ \sum_{i=0}^{m-1} \int \hat{L}(q, h = A^i)dq$$

$$\hat{L}(q, h = A^i) = \frac{1}{m} \cdot \int \mu(\theta) f(q|\theta)d\theta \cdot \left(\prod_{j=0}^{i} \xi(A^j)\right) \cdot \left(\int_{\hat{\eta}_A(s)=Rj} \phi(s, q, \sigma_s)ds\right)^i$$

In this case, the bounded rational editor is unaware of the fact that the paper in hand probably comes from a high-type author. Thus, compared to the sophisticated editor, he tends to set a higher threshold of the signal.

**Proposition 12.** *The bounded rational editor with analogy-based expectation sets a threshold* $\hat{s}_A > s_A^*$.

### 3.2.1 Bounded Rational Incumbents

The previous sections show that the editor's decision can be biased by different mistakes he makes. In this section, I study the impact of such biases on entry barriers when the incumbents are not fully rational. I find that entry barriers are reduced regardless of the direction of the bias. This is because, under both types of biases, the selection bias effect is less significant, making receiving rejected papers less disadvantageous.

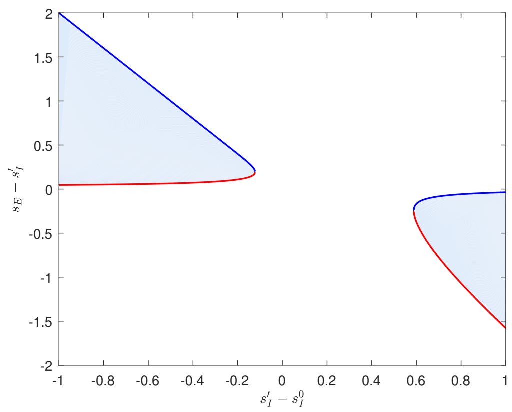



Figure 3.1: The quality follows a normal distribution: $f(q) = \phi(q, 0, \sigma_q)$. The signal conditional on quality follows a normal distribution $\phi(s, q, \sigma_s)$. The value function is linear, $v(Q) = \max\{0, Q\}/Q_I^0$. The parameters are: $\sigma_q = 1$, $\sigma_s = 0.1$.

If the incumbents are of the first kind of naivety (neglecting the selection bias effect) and set lower thresholds, the rejected papers are of poor quality and are harder to be republished by the entrant. Figure 3.1 shows the shaded area where the entrant's threshold lies, which makes it the first option for authors. If the incumbents set a threshold $s_I'$ lower than the optimal threshold $s_I^0$, the entrant can challenge them by setting a slightly higher threshold, ensuring that most papers published have not been rejected previously. This results in both the average quality and acceptance rate being competitive with the incumbents.

On the other hand, if the incumbents are of the second kind of naivety (ignorance about the authors' private information) and set higher thresholds, rejected papers are not necessarily of low quality, and receiving them is also not unacceptable. In this case, the entrant can challenge the incumbents by setting a slightly lower threshold, keeping the average quality similar to the incumbents but bringing a higher acceptance rate.

## 3.3 Bounded Rational Employers

Determining the probability of the history $\phi(M)$ and calculating the Graduate's threshold $R(M)$ for each history $M$ is an arduous task for Employers, especially when they only receive a single signal $m$. Furthermore, comprehending the relationship between their offer, the probability of the trade, and the conditional expectation of the trade's value adds an additional layer of complexity. To assist agents in decision-making, providing access to historical data allows them to learn from the data. This section presents a scenario in which agents can make optimal decisions given sufficiently complete historical data. Additionally, it analyzes how agents behave when they misuse the data.

Before entering the market, Employers review historical data comprising former Employers' offers, the signal they received, the corresponding state, and whether the offer was accepted. They do not know which Graduate met which Employer or in which round Employer was sampled. Here is an example.

| Offer Price | ... | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 3 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Signal | ... | b | g | g | b | g | b | g | b | g | b | ... |
| State | ... | H | H | H | L | H | H | L | L | L | H | ... |
| Accept or Not | ... | NA | NA | A | A | NA | A | NA | A | A | A | ... |

After collecting the information, Employer wants to find the answers of two questions: i) what is the probability that Graduate is of high type; ii) what is the probability she will accept the offer $p$. Therefore, first, he calculates the probability of state $H$ if signal $g$ or $b$ occurs by utilizing data. For instance, suppose that there were $m_g$ good signals among all records, and $n_g$ of them were linked to state $H$. In that case, the posterior probability that the state is $H$ given a good signal $Pr[H|g]$ is $n_g/m_g$. The probability $Pr[H|b]$ is defined similarly. For example, suppose there are eleven records as follows, with the upper line indicating the signals and the lower line indicating the corresponding types:

| Signal | g | g | b | b | g | b | g | g | b | b | g |
|---|---|---|---|---|---|---|---|---|---|---|---|
| State | H | H | L | L | H | H | H | L | L | L | L |

Employer observes that once a good signal occurs, there is 4/6 probability that the state is $H$, and once a bad signal occurs, the probability is 1/5.

Secondly, he calculates the probability that an offer $p$ was accepted given the signal $m$ and the state $\theta$, denoted as $\beta(p|\theta, m)$. There exists some perturbation in Employers' offer. The perturbation is sufficiently small such that it does not affect Graduate's behavior but it gives Employer $\beta(p|\theta, m)$ on each price $p$.

| Offer Price | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Signal | b | g | g | b | g | b | g | b | g | b |
| State | H | H | H | L | H | H | L | L | L | H |
| Accept or Not | NA | NA | A | A | NA | A | NA | A | A | A |

In the above case, Employer observes that, once a good signal occurs and the state is $H$, Graduate accepts Offer 2 with Probability 1/3. Therefore, $\beta(2|H, g) = 1/3$. Additionally, $\beta(2|L, g) = 1/2$, $\beta(2|H, b) = 2/3$, $\beta(2|L, b) = 1$.

Given these two probabilities, Employer chooses a price $p$ such that

$$\pi_R(p|m) = (v_H - p)Pr[H|m]\beta(p|H, m) + (v_L - p)Pr[L|m]\beta(p|L, m)$$

is maximized after receiving a signal $m$.

This information structure is sufficient for Employers to make an optimal decision. Knowing the function $\beta(p|\theta, m)$ enables Employers to find an optimal offer along with the $Pr[\theta|m]$ because now the function $\beta(p|\theta, m)$ can select Graduate with different histories under both states $\theta = H$ and $\theta = L$. Thus, as the document includes sufficiently big data, Employer behaves in the same way as the one in the rational model.

**Proposition 13.** $\pi_R(p|m)$ *converges in probability to rational Employer's payoff* $\mathbb{E}_{\phi(M_m)}[(v(M_m) - p)\sigma_1^G(M_m, p)]$.

The idea of the proof is demonstrated by rewriting $\mathbb{E}_{\phi(M_m)}[(v(M_m)-p)\sigma_1^G(M_m, p)]$

as follows:

$$\mathbb{E}_{\phi(M_m)}[(v(M_m)-p)\sigma_1^G(M_m,p)] = \sum_{M_m\in\mathcal{M}_m}(v(M_m)-p)\sigma_1^G(M_m,p)\phi(M_m)$$
$$= (v_H - p)\left(\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)\right)\frac{\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)\sigma_1^G(M_m,p)}{\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)}$$
$$+ (v_L - p)\left(\sum_{M_m\in\mathcal{M}_m}(1-q(M_m))\phi(M_m)\right)\frac{\sum_{M_m\in\mathcal{M}_m}(1-q(M_m))\phi(M_m)\sigma_1^G(M_m,p)}{\sum_{M_m\in\mathcal{M}_m}(1-q(M_m))\phi(M_m)}$$

Then, the empirical perception $Pr[H|m]$ converges to $\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)$, and $\beta(p|H,m)$ to $\frac{\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)\sigma_1^G(M_m,p)}{\sum_{M_m\in\mathcal{M}_m} q(M_m)\phi(M_m)}$.

The next two subsections discuss how Employers' behavior changes when Employers misuse the data.

### 3.3.1 Employers with Private-Information Analogy Classes

This section assumes that when calculating the probability of an offer $p$ being accepted by Graduate, Employer only refers to the signal $m$, denoted as $\beta^I(p|m)$.

| Offer Price | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| Signal | b | g | g | b | g | b | g | b | g | b |
| Accept or Not | NA | NA | A | A | NA | A | NA | A | A | A |

Employer observes that once a good signal occurs, Graduate accepts Offer 2 with Probability 2/5. Once a bad signal occurs, Graduate accepts Offer 2 with Probability 4/5.

This coarse Employer chooses a price $p$ such that

$$\pi_I(p|m) = (v_H - p)Pr[H|m]\beta^I(p|m) + (v_L - p)Pr[L|m]\beta^I(p|m)$$

is maximized after receiving a signal $m$. To observe how this differs from the rational case, one can notice the equivalency that irrational Employer forms an expectation of the value from trading,

$$\mathbb{E}^I[v|m] = v_H \cdot Pr[H|m] + v_L \cdot Pr[L|m],$$

which is **unconditional on whether his offer will be accepted by Graduate**. He faces a supply function $\beta^I(p|m)$, and maximizes $\pi_I(p|m) = (\mathbb{E}^I[v|m]-p)\beta^I(p|m)$.

In contrast, for a rational Employer, if he offers $p$, the conditional expected value of the trade is

$$\mathbb{E}^R[v|m] = \frac{v_H \cdot Pr[H|m]\beta(p|m,H) + v_L \cdot Pr[L|m]\beta(p|m,L)}{Pr[H|m]\beta(p|m,H) + Pr[L|m]\beta(p|m,L)}$$

He maximizes the payoff $\pi_R(p|m) = (\mathbb{E}^R[v|m] - p)\beta^I(p|m)$.

In general, $\beta(p|m,L) \neq \beta(p|m,H)$ because low-type Graduate is more likely to generate bad signals and underrates her expectation of offers from the market. Consequently, she accepts offers with higher probability, which makes the difference between the expected value of the trade for rational and coarse Employers. Rational Employers realize this fact and update their belief in their expectation when making offers. They know that, in some sense, their offers screen Graduate's history. In contrast, coarse Employers do not, leading them to overvalue the gain from the trade.

To formalize the analysis, the analogy-based approach is used to formally model this kind of defection of the agents' ability and this study terms them Employers with private-information analogy classes.[2] Bounded rational Employers, who bundle all the nodes (histories) $M_g$ into an analogy-class $\alpha_g$ and all the nodes $M_b$ into the other analogy-class $\alpha_b$, are considered. This grouping allows Employers to use the information contained in the signal $m$ to form expectations regarding Graduate's acceptance probabilities, which are now represented as mixed strategies. Specifically, after receiving a signal $m$, Employer expects that Graduate will accept an offer $p$ with Probability $\beta(p|m) = \sum_{M_m \in \alpha_m} \phi(M_m)\sigma_1^G(M_m, p)$. Employer then chooses a price $p$ that maximizes the expected profit:

$$\mathbb{E}_{\phi(M_m)}[(v(M_m) - p)\beta^I(p|m)] = (\mathbb{E}_{\phi(M_m)}[v(M_m)] - p)\beta^I(p|m)$$

As the data set grows larger, it can be shown that the expression $(\mathbb{E}^I[v|m] - p)\beta^I(p|m)$ converges to the above expression.

*Analogy-based Expectation Equilibrium (ABEE)*

A tuple $(\sigma^E, \sigma^G, \phi, \{\alpha_g, \alpha_b\}, \{\beta^I(\cdot|g), \beta^I(\cdot|b)\})$ is an ABEE of Employer with private-information analogy classes if

1. Employers form an analogy-based expectation that Graduate accepts the offer

[2] This part is technical and skipping it does not affect the understanding of the result.

$p$ upon receiving the signal $m$. That is[3]

$$\beta^I(p|m) = \sum_{M_m \in \alpha_m} \phi(M_m)\sigma_1^G(M_m, p)$$

2. Given $\beta$ and $\phi$, $\sigma^E(m,p) > 0$ only if $p$ maximizes Employer's expected utility upon receiving the signal $m$.

$$p \in \arg\max_{p'} (\mathbb{E}_{\phi(M_m)}[v(M_m)] - p')\beta^I(p'|m)$$

3. Given $\sigma^E$, $\sigma_1^G(M,p) > 0$ only if the offer is weakly higher than the larger one between the outside option and the continuation payoff, that is, the threshold $R$. That is $p \geq R(M)$. $\sigma_1^G(M,p) = 1$ if the inequality is strict. Similarly, $\sigma_2^G(M,p) > 0$ only if $R(M) = c(M) \geq p$. $\sigma_3^G(M,p) > 0$ only if $R(M) = U(M) \geq p$.

4. Given $\sigma^G$ and $\sigma^E$, $\phi$ is derived through Bayes' rule.

Due to bounded rationality, coarse Employer may not realize that a higher offer can attract Graduates with both good and bad histories. As a result, when the average value $\mathbb{E}_{\phi(M_m)}[v(M_m)]$ or $\mathbb{E}^I[v|m]$ is sufficiently high, coarse Employers tend to overbid, particularly when adverse selection effects are significant, such as when search costs are low. Rational Employers shade their offers to avoid trading with Graduate who receives bad signals and waits for a high offer, which makes Graduate of the high type who often receives good signals unable to accept. In contrast, coarse Employer perceives an increase in the average value of trades as such type of Graduate is more adverse selected.

To investigate the tendency of coarse Employer to overbid when faced with significant adverse selection effects, we revisit the scenario where a good signal is received only by high-type Graduates, that is, $\mu_H^g > \mu_L^g = 0$. If $p_b^* = R(b)$ is a rational equilibrium, the sampled Employer knows he is the first one Graduate meets and $\mathbb{E}[v|b] = v(b)$. Graduate's threshold $R(b)$ is the same as those under the rational case. In other words, there is no adverse selection, and coarse Employers

[3] There exists some perturbation in Employers' strategy. $\sigma_\epsilon^E(m,p)$ is a perturbation of $\sigma^E(m,p)$. That is, $\forall p$, $\sigma_\epsilon^E(m,p) > 0$ and $\sigma_\epsilon^E(m,p) \to \sigma^E(m,p)$ as $\epsilon \to 0$. Then,

$$\beta^I(p|m) = \frac{\sum_{M_m \in \alpha_m} \phi(M_m)\sigma_\epsilon^E(m,p)\sigma_1^G(M_m,p)}{\sum_{M_m \in \alpha_m} \phi(M_m)\sigma_\epsilon^E(m,p)} = \sum_{M_m \in \alpha_m} \phi(M_m)\sigma_1^G(M_m,p)$$

behave in the same way as rational Employers. In case this assumption does not hold, Algorithm 1 can be utilized to obtain the ABEE.

It is worth noting that coarse Employers may deem it optimal to offer high-ranking thresholds, even if rational Employers do not. Specifically, rational Employers consider it individually rational to offer $R(b^n)$ when $v(b^n) - R(b^n) \geq 0$. For coarse Employers, however, the condition becomes $\mathbb{E}^I[v|b] - R(b^n) \geq 0$. In the event that offering $R(b^n)$ is a pure strategy equilibrium for rational Employers, coarse Employers also find it profitable to offer $R(b^n)$ as $\mathbb{E}^I[v|b] \geq v(b^n)$.

Moreover, as the adverse selection effect becomes more significant and $\mathbb{E}^I[v|b]$ increases sufficiently, coarse Employers tend to overbid. This phenomenon is evident when the following condition holds:

$$(\mathbb{E}^I[v|b] - R(b^{n-1}))\phi(b^{n-1}) + (\mathbb{E}^I[v|b] - R(b^{n-1}))\phi(b^n) > (\mathbb{E}^I[v|b] - R(b^n))\phi(b^n)$$
$$\Leftrightarrow (\mathbb{E}^I[v|b] - R(b^{n-1}))\phi(b^{n-1}) > (R(b^{n-1}) - R(b^n))\phi(b^n)$$

Coarse Employers find it optimal to increase their offers, which indicates that they tend to overbid.

**Proposition 14.** *Suppose $\mu_L^g = 0$ and $p_b$ is the minimal offer from rational Employer receiving the bad signal in the equilibrium deduced by algorithm 1, then there exists an ABEE for coarse Employer with private-information analogy classes where his minimal offer after receiving the bad signal is weakly higher than $p_b$.*

**Example 10.** *(A case where the adverse selection effect is not significant) Consider a rational equilibrium where $p_g = R(g)$ and $p_b = R(b^2)$.*

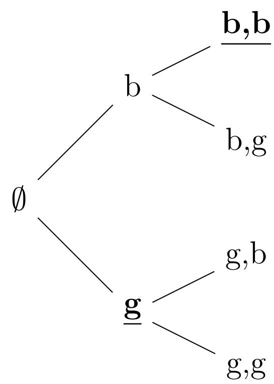



Figure 3.2: The equilibrium where rational Employers' strategy is $p_g = R(g)$ and $p_b = R(b^2)$.

*When receiving a bad signal, coarse Employers find it profitable to offer $R(b)$ when*

$$[\mathbb{E}^I[v|b] - R(b)]\phi^*(b) + [\mathbb{E}^I[v|b] - R(b)]\phi^*(b^2) > [\mathbb{E}^I[v|b] - R(b^2)]\phi^*(b^2)$$
$$\Leftrightarrow [\mathbb{E}^I[v|b] - R(b)]\phi^*(b) > [R(b) - R(b^2)]\phi^*(b^2)$$

*However, since $v(b) > \mathbb{E}^I[v|b]$ and rational Employer does not deviate by choosing $R(b)$,*

$$[v(b) - R(b)]\phi^*(b) + [v(b) - R(b)]\phi^*(b^2) < [v(b) - R(b^2)]\phi^*(b^2)$$
$$\Leftrightarrow [v(b) - R(b)]\phi^*(b) < [R(b) - R(b^2)]\phi^*(b^2)$$

*Offering $R(b)$ yields less payoff than $R(b^2)$ for coarse Employer also. So, coarse Employer receiving a bad signal does not increase his offer because the average value is not high enough.* □

### 3.3.2 Employers with Payoff-Relevant Analogy Classes

This section assumes that when calculating the probability of an offer $p$ being accepted by Graduate, Employer only refer to the state $\theta$, denoted as $\beta^P(p|\theta)$.

| Offer Price | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|
| State | H | H | H | L | H | H | L | L | L | H |
| Accept or Not | NA | NA | A | A | NA | A | NA | A | A | A |

In the above example, Employer observes that if the state is $H$, Graduate accepts Offer 2 with Probability 1/2. If the state is $L$, Graduate accepts Offer 2 with Probability 3/4.

This coarse Employer chooses a price $p$ such that

$$\pi_P(p|m) = (v_H - p)Pr[H|m]\beta^P(p|H) + (v_L - p)Pr[L|m]\beta^P(p|L)$$

is maximized after receiving a signal $m$. Specifically[4], he bundles all the nodes (histories) $(H, M)$ into an analogy-class $\alpha_H$ and all the nodes $(L, M)$ into the other analogy-class $\alpha_L$. He expects that the Graduate plays a mixed-strategy where they accept an offer $p$ with Probability $\beta^P(p|\theta)$. He reasons that Graduate's behavior is stationary conditional on the state. In this model, the state is related to the

[4] This part is technical and skipping it does not affect the understanding of the result.

ex post payoff from the trade. Therefore, this study terms them Employers with payoff-relevant analogy classes.

*Analogy-based Expectation Equilibrium (ABEE)*

A tuple $(\sigma^E, \sigma^G, \phi, \{\alpha_H, \alpha_L\}, \{\beta^P(\cdot|H), \beta^P(\cdot|L)\})$ is an ABEE of Employers with payoff-relevant analogy classes if

1. Employers form an analogy-based expectation that Graduate accepts the offer $p$ conditional on the state

$$\beta^P(p|\theta) = \frac{\sum_{m\in\{b,g\}} \sum_{(\theta,M_m)\in\alpha_\theta} \phi(\theta, M_m)\sigma^E_\epsilon(m,p)\sigma^G_1(M_m,p)}{\sum_{m\in\{b,g\}} \sum_{(\theta,M_m)\in\alpha_\theta} \phi(\theta, M_m)\sigma^E_\epsilon(m,p)}$$

   where $\phi(\theta, M_m)$ is the probability that the node $(\theta, M_m)$ is reached.[5] $\sigma^E_\epsilon(m,p)$ is a perturbation of $\sigma^E(m,p)$. That is, $\forall p$, $\sigma^E_\epsilon(m,p) > 0$ and $\sigma^E_\epsilon(m,p) \to \sigma^E(m,p)$ as $\epsilon \to 0$.

2. Given $\beta$ and $\phi$, $\sigma^E(m,p) > 0$ only if $p$ maximizes Employer's expected utility upon receiving the signal $m$.

$$p \in \arg\max_{p'} (v_H - p')Pr[H|m]\beta^P(p'|H) + (v_L - p')Pr[L|m]\beta^P(p'|L)$$

   where $Pr[\theta|m]$ is the conditional probability of the state upon receiving the signal $m$. That is

$$Pr[\theta|m] = \frac{\sum_{M_m\in\mathcal{M}_m} \phi(\theta, M_m)}{\sum_{M_m\in\mathcal{M}_m} [\phi(H, M_m) + \phi(L, M_m)]}$$

3. Given $\sigma^E$, $\sigma^G_1(M,p) > 0$ only if the offer is weakly higher than the larger one between the outside option and the continuation payoff, that is, the threshold $R$. That is $p \geq R(M)$. $\sigma^G_1(M,p) = 1$ if the inequality is strict. Similarly, $\sigma^G_2(M,p) > 0$ only if $R(M) = c(M) \geq p$. $\sigma^G_3(M,p) > 0$ only if $R(M) = U(M) \geq p$.

4. Given $\sigma^G$ and $\sigma^E$, $\phi$ is derived through Bayes' rule.

In the simplified case where $\mu^g_H > \mu^g_L = 0$, coarse Employer tends to offer a lower price after receiving a bad signal compared to rational Employer. A two-period searching case is used to illustrate intuition. Suppose, in the pure strategy

[5] $\phi(M_m) = \phi(H, M_m) + \phi(L, M_m)$.

equilibrium, rational Employer offers $p_b = R(b)$ after receiving the bad signal. He offers $p_g = c_H$ after receiving the good signal, as shown in Figure 3.3.

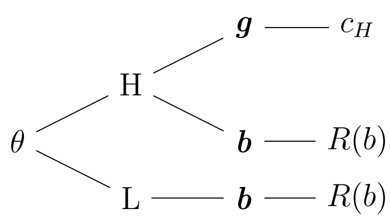



Figure 3.3: The signal structure of the two-Employer case.

We can focus on the state H and obtain the supply functions. Graduate with history $(b)$ is offered $R(b)$ and accepts it with Probability 1, while Graduate with history $(g)$ is offered $c_H$ and also accepts it with Probability 1.

$$\beta(R(b)|H,b) = 1,\ \beta(c_H|H,g) = 1$$

However, coarse Employer can not know that $R(b)$ and $c_H$ are offered by Employers with different signals since the data is bundled in the same category. Therefore, he learns that high-type Graduate not only accepts $c_H$ with Probability 1 but also accepts $R(b)$ with Probability 1[6],

$$\beta^P(R(b)|H) = 1,\ \beta^P(c_H|H) = 1$$

As a result, he mistakenly believes he could still trade with Graduate even if he offers a low price.

**Proposition 15.** *Suppose $\mu_L^g = 0$ and $p_b$ is the maximal offer from rational Employer receiving the bad signal in the equilibrium deduced by algorithm 1, then there exists an ABEE for coarse Employer with payoff-relevant analogy classes where his maximal offer after receiving the bad signal is weakly lower than $p_b$, and he could offer a price other than $c_H$ after receiving the good signal.*

[6]More specifically, there is perturbation in Employer's strategy, so we have the supply function $\beta$ on each price. A negligible proportion of Graduate with history $(g)$ was offered $R(b)$ and rejected it. A negligible proportion of Graduate with history $(b)$ was offered $c_H$ and accepted it.

$$\beta(R(b)|H,g) = 0,\ \beta(c_H|H,b) = 1$$

Then,

$$\beta^P(R(b)|H) \to 1,\ \beta^P(c_H|H) = 1$$

This proposition demonstrates that coarse Employer lowers his offer compared to the rational one. This finding holds true even in the general case. For instance, as coarse Employer receiving the good signal starts to mix between $p_g = R(g)$ and $p_b = R(b)$, $\beta^P(p_b|H)$ and $\beta^P(p_b|L)$ decrease because Graduate receiving a good signal rejects the offer $p_b$ regardless of the state. Moreover, the good signal is more likely to occur when the state is $H$. Therefore, $\beta^P(p_b|H)$ decreases in a greater extent than $\beta^P(p_b|L)$ does. This change further decreases the offer from coarse Employer receiving a bad signal. Lower deal prices exacerbate the adverse selection effect, which further makes Employer lower their offer.

An example to show the difference between the behavior of rational and coarse Employer is used. In comparison to rational Employers, coarse Employers tend to be too pessimistic because they underestimate the probability of trading with high-type Graduates.

**Example 11.** *In a two-period searching process ($N = 2$), the probability that a good signal occurs under the state $H$ is $\mu_H^g = 0.6$. The probability that a good signal occurs under the state $L$ is $\mu_L^g = 0.2$. The value and cost from trading are as follows: $v_H = 2.4$, $v_L = 1.2$, $c_H = 2$ and $c_L = 1$. The prior of the state being $H$ is $q_0 = 0.5$, and search costs are $s = 0.01$.*

*The equilibrium with rational Employers is he offers $p_g^* = R(g) = 1.75$ after receiving the good signal, and mixes between $p_{b1}^* = R(b) = 1.54$ and $p_{b2}^* = R(b^2) = c(b^2) = 1.2$ after receiving the bad signal, where the probability $y$ allocated on $p_{b1}^*$ is* 0.74.

*With* $\beta(p|\theta, m) = \frac{\sum_{R(M_m)\leq p} \phi(\theta, M_m)}{\sum_{M_m} \phi(\theta, M_m)}$,[7]

$$\beta(p_{b2}^*|H, b) = \frac{\phi(H, b^2)}{\phi(H, b) + \phi(H, b^2)} = 0.096$$

$$\beta(p_{b2}^*|L, b) = \frac{\phi(L, b^2)}{\phi(L, b) + \phi(L, b^2)} = 0.175$$

*If Employer wants to increase the possibility for trade, he offers a higher price $p_{b1}^*$ after receiving a bad signal. Then,*

$$\beta(p_{b1}^*|H, b) = \frac{\phi(H, b) + \phi(H, b^2)}{\phi(H, b) + \phi(H, b^2)} = 1$$

[7]Refer to the proof of proposition 13.

$$\beta(p_{b1}^*|L,b) = \frac{\phi(L,b)+\phi(L,b^2)}{\phi(L,b)+\phi(L,b^2)} = 1$$

*Employers with payoff-relevant analogy classes perceive the probability that the offer was accepted.*

$$\beta^P(p_{b1}^*|L) = \beta^P(p_{b1}^*|H) = 1$$

*They have no incentive to offer the higher price $p_g^*$ because they believe offering $p_{b1}^*$ gives the same probability of trading. However, Graduate receiving the good signal rejects the offer $p_{b1}^*$. $\beta^P(p_{b1}^*|L)$ and $\beta^P(p_{b1}^*|H)$ decrease. Then, after receiving the bad signal, coarse Employer lowers his offer by allocating more probability to $p_{b2}^*$, which further leads $\beta^P(p_{b1}^*|L)$ and $\beta^P(p_{b1}^*|H)$ to decrease. Coarse Employer receiving the good signal finds it not optimal to offer $p_{b1}^*$.*

*Finally, the ABEE for them is $p_g^{**} = R(g) = 1.75$ and $p_b^{**} = R(b^2) = c(b^2) = 1.2$. If Employer wants to raise his offer to $p_b' = R(b) = 1.54$ after receiving a bad signal, he calculates*

$$\beta^P(p_b^{**}|H) = 0.29,\ \ \beta^P(p_b^{**}|L) = 0.44$$

$$\beta^P(p_b'|H) = 0.4,\ \ \beta^P(p_b'|L) = 0.8$$

*$\beta^P(p_b'|L) - \beta^P(p_b^{**}|L)$ is larger than $\beta^P(p_b'|H) - \beta^P(p_b^{**}|H)$. It can be compared with the difference between $\beta(p_{b1}^*|L,b) - \beta(p_{b2}^*|L,b)$ and $\beta(p_{b1}^*|H,b) - \beta(p_{b2}^*|H,b)$. It means that Employer with payoff-relevant analogy classes believes a higher offer $p_b'$ is more likely to attract those Graduates of low type, which leads to $\pi_P(p_b'|b) = -0.1 < \pi_P(p_b^{**}|b) = 0.096$.* □

### 3.3.3 Efficiency comparison

This section compares the market efficiency of three types of employers - Employers with private-information analogy classes, rational Employers, and Employers with payoff-relevant analogy classes - based on the ranking of offers in the market from high to low. Generally, higher offers increase trade and improve market efficiency. I use a simplified search model where the good signal only occurs when a graduate is of high type ($\mu_H^g > \mu_L^g = 0$). The model assumes negligible search costs so that Graduate would prefer to search rather than choose the outside option. The study also considers the possibility of an inefficient market where Employer only offers the lowest possible price $p_b = R(b^N) = c(b^N)$.

In Figure 3.4, the red area corresponds to the case in which the market is inefficient with rational Employer. The blue area corresponds to Employer with

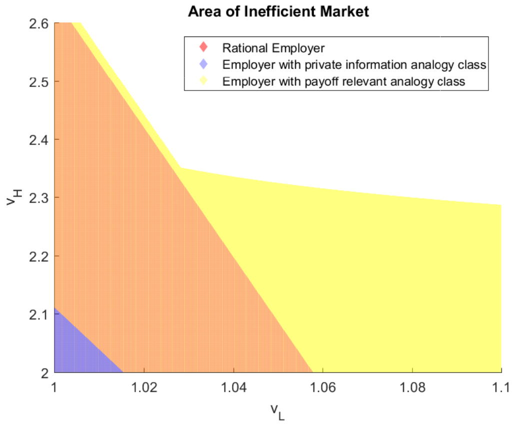



Figure 3.4: Market efficiency comparison among three kinds of Employers. The parameter is set as: $q_0 = 0.5$, $\mu_H^g = 0.7$, $c_L = 1$ and $c_H = 2$. The red, yellow and blue areas are overlapping.

private-information analogy classes. The yellow area corresponds to Employers with payoff-relevant analogy classes. Comparing the three areas, the blue one is the smallest, which means that the market with Employer with private-information analogy classes is the most efficient because he tends to overbid. In contrast, the yellow area is the largest, which coincides with the finding that the offer from Employers with payoff-relevant analogy classes is the lowest. Thus, the market with this kind of coarse Employer is the most inefficient.

## 3.4 Conclusion

This study contributes to the literature of markets with noise and information asymmetry by the introduction of bounded rational agents. I find that by making different mistakes, unsophisticated agents' behavior can deviate in different directions. These deviations can reduce the entry barrier of the market and can affect the market efficiency.

## 3.5 Appendix

### 3.5.1 Proofs

**Proof of proposition 11:**

Given $s_A$, the naive editor computes the probability,

$$\begin{aligned} Pr\left[\theta, h = A^i\right] &= Pr\left[\theta\right] Pr\left[h = A^i \middle| \theta\right] \\ &= \mu(\theta) \cdot \int f(q|\theta)\Phi^i(s_A, q, \sigma_s)dq \Bigg/ \sum_{j=0}^{m-1} \int f(q|\theta)\Phi^j(s_A, q, \sigma_s)dq \end{aligned}$$

The sophisticated editor computes the probability,

$$Pr\left[\theta, h = A^i \middle| sampled\right] = \frac{\int \mu(\theta) f(q|\theta)\Phi^i(s_A, q, \sigma_s)dq}{\sum\limits_{j=0}^{m-1} \iint \mu(\theta') f(q|\theta')\Phi^j(s_A, q, \sigma_s)dqd\theta'}$$

We have

$$\frac{Pr\left[\theta, h = A^i \middle| sampled\right]}{Pr\left[\theta, h = A^i\right]} = Y \sum_{j=0}^{m-1} \int f(q|\theta)\Phi^j(s_A, q, \sigma_s)dq, \ \forall i$$

$Y$ is a constant. Since $f(q|\theta)$ satisfies MLRP and $\Phi^j(s_A, q, \sigma_s)$ is decreasing in $q$, the right hand side is decreasing in $\theta$. Since

$$1 = \sum_{j=0}^{m-1} \int Pr\left[\theta, h = A^i \middle| sampled\right] d\theta = \sum_{j=0}^{m-1} \int Pr\left[\theta, h = A^i\right] d\theta,$$

$Pr\left[\theta, h = A^i \middle| sampled\right]$ and $Pr\left[\theta, h = A^i\right]$ are single-crossing in $\theta$.

The naive editor forms a belief on the quality $\tilde{\beta}_A(q)$

$$\tilde{\beta}_A(q) = \frac{\sum\limits_{i=0}^{m-1} \int \gamma(q|\theta, h = A^i) Pr\left[\theta, h = A^i\right] d\theta}{\sum\limits_{i=0}^{m-1} \iint \gamma(q|\theta, h = A^i) Pr\left[\theta, h = A^i\right] d\theta dq},$$

while the sophisticated editor forms the unbiased belief $\beta_A(q)$,

$$\beta_A(q) = \frac{\sum\limits_{i=0}^{m-1} \int \gamma(q|\theta, h = A^i) Pr\left[\theta, h = A^i | sampled\right] d\theta}{\sum\limits_{i=0}^{m-1} \iint \gamma(q|\theta, h = A^i) Pr\left[\theta, h = A^i | sampled\right] d\theta dq},$$

which is equivalent to (1.4). Since $Pr\left[\theta, h = A^i \middle| sampled\right]$ and $Pr\left[\theta, h = A^i\right]$ are single-crossing in $\theta$, $\tilde{\beta}_A(q)$ FOSDs $\beta_A(q)$. The naive editor's optimal threshold is lower than the sophisticated's, $\tilde{\omega}(s_A) < \omega(s_A)$ ($\tilde{\omega}$ and $\omega$ are defined in the same way as in the proof of proposition 1). Thus, in equilibrium, the naive editor sets a lower threshold $\tilde{s}_A < s^*_A$. ■

**Proof of proposition 12:**

$$\hat{\beta}_A(q|h = A^i) = f(q|h = A^i) \propto \int_{-\infty}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)$$

while

$$\beta_A(q|h = A^i) = f(q|h = A^i, \theta > \theta^*_A(h)) \propto \int_{\theta^*_A(A^i)}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)$$

Thus, $\beta_A(q|h = A^i)$ FOSDs $\hat{\beta}_A(q|h = A^i)$ for any $i$.

Secondly,

$$Pr[h = A^i] \propto \iint_{\theta^*_A(A^i)}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)dq$$

$$\hat{Pr}[h = A^i] \propto \left(\prod_{j=0}^{i} \xi(A^j)\right) \iint_{-\infty}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)dq$$

$$\xi(A^j) = \frac{\iint_{\theta^*_A(A^j)}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^j(s_A, q, \sigma_s)dq}{\iint_{\theta^*_A(A^{j-1})}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^{j-1}(s_A, q, \sigma_s)dq}$$

Then,

$$\begin{aligned} & \frac{\hat{Pr}[h = A^{i+1}]}{Pr[h = A^{i+1}]} \Bigg/ \frac{\hat{Pr}[h = A^i]}{Pr[h = A^i]} \\ = \quad & \frac{\iint_{\theta^*_A(A^i)}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^{i-1}(s_A, q, \sigma_s)dq}{\iint_{-\infty}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^{i-1}(s_A, q, \sigma_s)dq} \frac{\iint_{-\infty}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)dq}{\iint_{\theta^*_A(A^i)}^{+\infty} \mu(\theta)f(q|\theta)d\theta \cdot \Phi^i(s_A, q, \sigma_s)dq} \\ = \quad & \frac{Pr[\theta > \theta^*_A(A^i)|h = A^{i-1}]}{Pr[\theta > \theta^*_A(A^i)|h = A^i]} > 1 \end{aligned}$$

Therefore, $\frac{\hat{Pr}[h=A^i]}{Pr[h=A^i]}$ is increasing in $i$. In other words, $\hat{Pr}[h = A^i]$ and $Pr[h = A^i]$ are single-crossing.

Since

$$\hat{\beta}_A(q) = \sum_{i=0}^{m-1} \hat{\beta}_A(q|h = A^i)\hat{Pr}[h = A^i], \;\; \beta_A(q) = \sum_{i=0}^{m-1} \beta_A(q|h = A^i)Pr[h = A^i],$$

$\beta_A(q)$ FOSDs $\hat{\beta}_A(q)$. The naive editor's optimal threshold is lower than the sophisticated's, $\hat{\omega}(s_A) > \omega(s_A)$ ($\hat{\omega}$ and $\omega$ are defined in the same way as in the proof of proposition 1). Thus, in equilibrium, the naive editor sets a higher threshold $\hat{s}_A > s_A^*$. ■

**Proof of proposition 13:**

First, let $s \in S$ be the possible record of signals Graduate could have. Let $l^H(s)$ ($l^L(s)$) be the number of historical samples where the record of signals is $s$ and the state is $H$ ($L$). Without loss of generality, suppose we are calculating $Pr[H|b]$.

$$Pr[H|b] = \frac{\sum_{s \in S} n_b(s) l^H(s)}{\sum_{s \in S} n_b(s) l^H(s) + \sum_{s \in S} n_b(s) l^L(s)}$$

where $n_b(s)$ is the number of bad signals in the record $s$.

Then, for those records with the most number of signals, they must show up in pairs, which means that if $(M, b) \in S$, then $(M, g) \in S$ and vice versa. That is because once Graduate rejects when the history $M$, both $(M, b)$ and $(M, g)$ could happen. Moreover, we have $l^H(M, g) = l^H(M)\mu_H^g$ and $l^H(M, b) = l^H(M)\mu_H^b$. So, $l^H(M, g) + l^H(M, b) = l^H(M)$. Similarly, $l^L(M, g) + l^L(M, b) = l^L(M)$. Then, we have

$$\begin{aligned} n_b(M, b) l^H(M, b) + n_b(M, g) l^H(M, g) &= (n_b(M) + 1) l^H(M, b) + n_b(M) l^H(M, g) \\ &= n_b(M) l^H(M) + l^H(M, b) \end{aligned}$$

and

$$n_b(M, b) l^L(M, b) + n_b(M, g) l^L(M, g) = n_b(M) l^L(M) + l^L(M, b)$$

Repeat the above process to those records with the second most signals. Repeat until all records with a good signal at the end are eliminated. After that, we obtain

$$Pr[H|b] = \frac{\sum_{M_b \in \mathcal{M}_b} l^H(M_b)}{\sum_{M_b \in \mathcal{M}_b} l^H(M_b) + \sum_{M_b \in \mathcal{M}_b} l^L(M_b)}$$

When the population of the sampling is infinity,

$$Pr[H|b] \to \frac{\sum_{M_b \in \mathcal{M}_b} \phi(\theta, M_b)}{\sum_{M_b \in \mathcal{M}_b} \phi(H, M_b) + \sum_{M_b \in \mathcal{M}_b} \phi(L, M_b)}$$

where $\phi(H, M_m)$ is probability that the node $(H, M_m)$ is reached. In general, we can write

$$Pr[\theta|m] \to \frac{\sum_{M_m} \phi(\theta, M_m)}{\sum_{M_m} \phi(H, M_m) + \sum_{M_m} \phi(L, M_m)}$$

Secondly, When the data set gets big, the empirical acceptance rate approaches to:

$$\beta(p|\theta, m) \to \frac{\sum_{R(M_m)\leq p} \phi(\theta, M_m)}{\sum_{M_m} \phi(\theta, M_m)}$$

There, the optimization problem for rational Employer can be written as follows:

$$\begin{aligned}
&\mathbb{E}_{\phi(M_m)}[(v(M_m) - p)\sigma_1^G(M_m, p)] \\
&= (v_H - p)\frac{\sum_{R(M_m)\leq p} \phi(H, M_m)}{\sum_{M_m} \phi(H, M_m) + \sum_{M_m} \phi(L, M_m)} \\
&\quad + (v_L - p)\frac{\sum_{R(M_m)\leq p} \phi(L, M_m)}{\sum_{M_m} \phi(H, M_m) + \sum_{M_m} \phi(L, M_m)} \\
&= (v_H - p)\frac{\sum_{M_m} \phi(H, M_m)}{\sum_{M_m} \phi(H, M_m) + \sum_{M_m} \phi(L, M_m)} \frac{\sum_{R(M_m)\leq p} \phi(H, M_m)}{\sum_{M_m} \phi(H, M_m)} \\
&\quad + (v_L - p)\frac{\sum_{M_m} \phi(L, M_m)}{\sum_{M_m} \phi(H, M_m) + \sum_{M_m} \phi(L, M_m)} \frac{\sum_{R(M_m)\leq p} \phi(L, M_m)}{\sum_{M_m} \phi(L, M_m)} \\
&= (v_H - p)Pr[H|m]\beta(p|H, m) + (v_L - p)Pr[L|m]\beta(p|L, m) = \pi_R(p|m)
\end{aligned}$$

■

**Proof of proposition 14:**

If $p_b = R(b)$, then the equilibrium of the rational agents is also the coarse agents'.

If $p_b = R(b^n)$ where $n > 1$, then the IR constraint for the rational agents holds, which is

$$v(b^n) - R(b^n) \geq 0$$

Additionally, if $p$ is the pure strategy or the minimum price in the mixed-strategy, we have

$$\mathbb{E}_{\phi(M_b)}[v(M_b)] > v(b^n)$$

which means that the IR constraint for the coarse agents holds. We could enumerate $R(b^i)$ for $i < n$ to check if there exists any ABEE with a higher offer. ■

**Proof of proposition 15:**

Suppose the maximal price offered by rational Employer receiving the bad signal is $\bar{p}_b = R(b^n)$. If coarse Employer receiving the good signal offers $c_H$, then coarse Employer's strategy is the same as that of rational Employer because in this case,

$$\beta(\bar{p}_b|H,b) = \beta^P(\bar{p}_b|H) \textit{ and } \beta(\bar{p}_b|L,b) = \beta^P(\bar{p}_b|L)$$

Otherwise, coarse Employer receiving the good signal allocates some probability to $R(b^m)$. If $m < n$, offering $R(b^m)$ is not optimal for coarse Employer receiving the bad signal because $R(b^m)$ would be rejected by Graduate receiving the good signal and

$$\beta(R(b^m)|H,b) > \beta^P(R(b^m)|H)$$

Since offering $R(b^m)$ is not optimal for rational Employer (following the algorithm 1), it is also not optimal for coarse Employer. Then, coarse Employer receiving the good signal would not offer $R(b^m)$ because $\beta^P(R(b^m)|H) = 0$, which means that it could not be an ABEE.

Then, let $\sigma_\epsilon^E(g,p)$ be the perturbation of Employer's strategy after receiving the good signal satisfying

$$\sigma_\epsilon^E(g, R(b^k)) = \epsilon, \ \forall k < n$$

And let $\sigma_\epsilon^E(b,p)$ be the perturbation of Employer's strategy after receiving the bad signal satisfying

$$\sigma_\epsilon^E(b, R(b^k)) = \epsilon^2, \ \forall k < n$$

Then, we have

$$\beta^P(R(b^k)|H) = 0, \ \forall k < n$$

which makes Employer receiving the bad signal have no incentive to offer a price higher than $\bar{p}_b$. ■

# Bibliography

Philippe Aghion and Patrick Bolton. Contracts as a barrier to entry. *The American economic review*, pages 388–401, 1987.

George A Akerlof. The market for âlemonsâ: Quality uncertainty and the market mechanism. *The quarterly journal of economics*, 84(3):488–500, 1970.

Peter J Alexander. Entry barriers, release behavior, and multi-product firms in the music recording industry. *Review of Industrial Organization*, 9:85–98, 1994.

Simon P Anderson and Regis Renault. Pricing, product diversity, and search costs: A bertrand-chamberlin-diamond model. *The RAND Journal of Economics*, pages 719–735, 1999.

Ofer H Azar. A model of the academic review process with informed authors. *The BE Journal of Economic Analysis & Policy*, 15(2):865–889, 2015.

Marcello Bofondi and Giorgio Gobbi. Informational barriers to entry into credit markets. *Review of Finance*, 10(1):39–67, 2006.

William A Brock and Steven N Durlauf. Discrete choice with social interactions. *The Review of Economic Studies*, 68(2):235–260, 2001.

Russell W Coff. Human capital, shared expertise, and the likelihood of impasse in corporate acquisitions. *Journal of Management*, 28(1):107–128, 2002.

Russell Cooper and Andrew John. Coordinating coordination failures in keynesian models. *The Quarterly Journal of Economics*, 103(3):441–463, 1988.

Christopher Cotton. Submission fees and response times in academic publishing. *American Economic Review*, 103(1):501–09, 2013.

Brendan Daley and Brett Green. Waiting for news in the market for lemons. *Econometrica*, 80(4):1433–1504, 2012.

Giovanni Dell'Ariccia. Asymmetric information and the structure of the banking industry. *European Economic Review*, 45(10):1957–1980, 2001.

Giovanni Dell'Ariccia, Ezra Friedman, and Robert Marquez. Adverse selection as a barrier to entry in the banking industry. *The RAND Journal of Economics*, pages 515–534, 1999.

Raymond Deneckere and Meng-Yu Liang. Bargaining with interdependent values. *Econometrica*, 74(5):1309–1364, 2006.

Peter A Diamond. A model of price adjustment. *Journal of economic theory*, 3(2): 156–168, 1971.

Glenn Ellison. Evolving standards for academic publishing: A q-r theory. *Journal of political Economy*, 110(5):994–1034, 2002.

Ignacio Esponda. Behavioral equilibrium in economies with adverse selection. *American Economic Review*, 98(4):1269–91, 2008.

Erik Eyster and Matthew Rabin. Cursed equilibrium. *Econometrica*, 73(5):1623–1672, 2005.

William Fuchs and Andrzej Skrzypacz. Bargaining with deadlines and private information. *American Economic Journal: Microeconomics*, 5(4):219–43, 2013.

Mathew LA Hayward and Donald C Hambrick. Explaining the premiums paid for large acquisitions: Evidence of ceo hubris. *Administrative science quarterly*, pages 103–127, 1997.

Yael V Hochberg, Alexander Ljungqvist, and Yang Lu. Networking as a barrier to entry and the competitive supply of venture capital. *The Journal of Finance*, 65 (3):829–859, 2010.

Ilwoo Hwang. Dynamic trading with developing adverse selection. *Journal of Economic Theory*, 176:761–802, 2018.

Philippe Jehiel. Analogy-based expectation equilibrium. *Journal of Economic theory*, 123(2):81–104, 2005.

Philippe Jehiel and Frédéric Koessler. Revisiting games of incomplete information with analogy-based expectations. *Games and Economic Behavior*, 62(2):533–557, 2008.

Boyan Jovanovic. Job matching and the theory of turnover. *Journal of political economy*, 87(5, Part 1):972–990, 1979.

Michihiro Kandori, George J Mailath, and Rafael Rob. Learning, mutation, and long run equilibria in games. *Econometrica*, 61(1):29–56, 1993.

Ayça Kaya and Kyungmin Kim. Trading dynamics with private buyer signals in the market for lemons. *The Review of Economic Studies*, 85(4):2318–2352, 2018.

Corinne Langinier. Are patents strategic barriers to entry? *Journal of Economics and Business*, 56(5):349–361, 2004.

Stephan Lauermann and Asher Wolinsky. Search with adverse selection. *Econometrica*, 84(1):243–315, 2016.

Derek Leslie. Are delays in academic publishing necessary? *American Economic Review*, 95(1):407–413, 2005.

Robert Marquez. Competition, adverse selection, and information dispersion in the banking industry. *The Review of Financial Studies*, 15(3):901–926, 2002.

Jordan Martel, Kenneth S Mirkin, and Brian Waters. Learning by owning in a lemons market. *Available at SSRN 2798088*, 2018.

David Martimort, Jérôme Pouyet, and Thomas Trégouët. Contracts as a barrier to entry: Impact of buyerâs asymmetric information and bargaining power. *International Journal of Industrial Organization*, 79:102791, 2021.

Paul Milgrom and John Roberts. Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica: Journal of the Econometric Society*, pages 1255–1277, 1990.

Diego Moreno and John Wooders. Dynamic markets for lemons: Performance, liquidity, and policy intervention. *Theoretical Economics*, 11(2):601–639, 2016.

Michele Muller-Itten. Gatekeeping under asymmetric information. *Available at SSRN 3186803*, 2017.

Sharon Oster. The optimal order for submitting manuscripts. *American Economic Review*, 70(3):444–448, 1980.

Richard Roll. The hubris hypothesis of corporate takeovers. *Journal of business*, pages 197–216, 1986.

Robert C Seamans. Threat of entry, asymmetric information, and pricing. *Strategic Management Journal*, 34(4):426–444, 2013.

Joel Sobel and Ichiro Takahashi. A multistage model of bargaining. *The Review of Economic Studies*, 50(3):411–426, 1983.

Steven Stern. The effects of firm optimizing behaviour in matching models. *The Review of Economic Studies*, 57(4):647–660, 1990.

Hal R Varian. A model of sales. *The American economic review*, 70(4):651–659, 1980.

Charles A Wilson. Mediation and the nash bargaining solution. *Review of Economic Design*, 6(3):353–370, 2001.

Asher Wolinsky. True monopolistic competition as a result of imperfect information. *The Quarterly Journal of Economics*, 101(3):493–511, 1986.

H Peyton Young. The evolution of conventions. *Econometrica: Journal of the Econometric Society*, pages 57–84, 1993.

Haoxiang Zhu. Finding a good price in opaque over-the-counter markets. *The Review of Financial Studies*, 25(4):1255–1285, 2012.