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Cooperative game theory models for allocation problems applied to the distribution of CO_2 emission permits

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Contents

1 State of the Art **37**

1.1 Introduction 37

1.2 Climate change abatement negotiations 38

1.3 Game Theory in climate negotiations 48

1.4 Bankruptcy situations 50

1.5 Conclusion 53

2 Preliminary and first application **55**

2.1 Introduction 55

2.2 Bankruptcy rules 56

2.3 Bankruptcy approaches 59

2.3.1 Direct approach 59

2.3.2 Axiomatic approach 60

2.3.3 Game Theoretic approach 64

2.4 Application on CO_2 emission 68

2.4.1 Bankruptcy rules applied to emission allocations 68

2.4.2 CEA allocations 76

2.5 Conclusion 80

3 Weighted bankruptcy method for GHG emissions permit **83**

3.1 Introduction 83

3.2 Weighted rules for bankruptcy situations 84

3.3 Weighted Constrained Equal Awards (WCEA) applied to emissions reduction problem 86

3.4 Surplus and technology transfer 95
3.5 WCEA Algorithmic approach 99
3.6 Conclusion 107

4 Double weighted bankruptcy problems without priority 109
4.1 Introduction 109
4.2 A Double Weighted Constrained Equal Awards rule (DWCEA) 111
4.3 A DWCEA solution algorithm with no priority over criteria 112
4.4 DWCEA applied to CO_2 emissions permits 117
4.5 Allocations results 126
4.6 Detailed algorithms 130
4.7 Conclusion 132

5 Double weighted bankruptcy problems with priority 135
5.1 Introduction 135
5.2 Double weighted bankruptcy method with priority 137
5.3 A DWCEA solution algorithm with priority over criteria 138
5.4 DWCEA^P applied to CO_2 emissions permits 141
5.5 Fairness 148
5.5.1 Fairness index 148
5.5.2 Fairness index applied to CO_2 allocations 150
5.6 Detailed Algorithms 153
5.7 Conclusion 154

Conclusion & Perspectives 160

Bibliography 169

List of Figures

1.1 Emissions allocation methods [93] 42

2.1 Luxembourg allocations 74

2.2 Czech Republic allocations 75

2.3 Germany allocations 75

2.4 UK Allocations CO_2 emissions permits 79

2.5 Spain Allocations CO_2 emissions permits 80

2.6 Allocations for high emitters countries 80

3.1 EU countries emissions 88

3.2 EU countries GDP 88

3.3 UK allocations CO_2 emissions permits 91

3.4 Poland allocations CO_2 emissions permits 93

3.5 Estonia allocations CO_2 emissions permits 94

3.6 Sweden allocations CO_2 emissions permits 94

4.1 Percentage distribution of total EU CO_2 emissions, GPD and Renewable energy production of each EU country over the interval time of five years from 2010 to 2014 (all data were collected from the World Bank Open Data project in 2020, see https://data.worldbank.org/indicator/). 118

4.2 Percentages of total CO_2 emission permits for the eight most powerful countries (Sh-Sh power index $\geq 3\%$) for the four allocation rules considered in this paper. Countries are ordered according to the Sh-Sh power index (solid line) 125

4.3 Percentages of total CO_2 emission permits for EU countries with Sh-Sh power index strictly smaller than 3% for the four allocation rules considered in this paper. Countries are ordered according to the Sh-Sh power index (solid line) 126

5.1 Player Satisfaction rate average of 2010-2014 for CEA and weighted allocation methods. 151

5.2 Gini index average of 2010-2014 for CEA and weighted allocations methods. 153

List of Tables

1 Acronyms used throughout this manuscript 35

1.1 Some fairness principles to allocate emissions rights 46

1.2 Criteria's list respecting fairness and efficiency principles 47

2.1 The correspondence among rules and axioms 64

2.2 Allocations CO_2 emissions in 2010 versus bankruptcy allocations 69

2.3 Allocations CO_2 emissions in 2011 versus bankruptcy allocations 70

2.4 Allocations CO_2 emissions in 2012 versus bankruptcy allocations 71

2.5 Allocations CO_2 emissions in 2013 versus bankruptcy allocations 72

2.6 Allocations CO_2 emissions in 2014 versus bankruptcy allocations 73

2.7 Luxembourg emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014 73

2.8 Czech Republic emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014 74

2.9 Germany emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014 75

2.10 The CEA rule for 28-EU 78

3.1 The CO_2 emissions allocations generated by WCEA rule for the 28-EU countries 90

3.2 A comparative table among WCEA allocations and CEA allocations 92

3.3 A weighted bankruptcy situation and the corresponding WCEA allocations 97

3.4 Computation of utility for every $S \subseteq N$ 97

3.5 Computation of the TT-game for every $S \subseteq N$ 99

3.6 A double weighted bankruptcy situation (E,c,w) with $N = \{1,2,3\}$ as set of agents and estate $E = 20$. 104

3.7 Parameters computed at each the iteration It of Algorithm 1 on the weighted bankruptcy situation of Example 5, with $\lambda^S \leftarrow \{2, \frac{5}{2}, 5\}$. 106

4.1 A double weighted bankruptcy situation (E,c,w^1,w^2) with $N = \{1,2,3\}$ as set of agents and estate $E = 20$. 115

4.2 Parameters computed at each the iteration It of Algorithm 6 on the double weighted bankruptcy situation of Example 6, with $\lambda_1^S \leftarrow \{2, \frac{5}{2}, 5\}$ and $\lambda_2^S \leftarrow \{\frac{1}{2}, 4, 5\}$. 116

4.3 Parameters computed at each the iteration It of Algorithm 6 on the double weighted bankruptcy situation of Example 6, with $\lambda_1^S \leftarrow \{2, \frac{5}{2}, 5\}$ and $\lambda_2^S \leftarrow \{\frac{1}{2}, 4, 5\}$. 117

4.4 Outcomes of the K-means clustering application over allocation records provided by alternative allocation methods. Names in bold indicate countries that are totally satisfied at least for 3 years. 121

4.5 Clusters combinations that are able to approve a decision according to the EU council voting systems. 123

4.6 Pearson correlation coefficients for CO_2 allocation vectors provided by the four allocation rules considered in this paper over the interval 2010-2014 and for two groups of countries (groups are based on the Sh-Sh power index, cut-off 3%). 124

4.7 Allocation vectors provided by CEA solution for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2), ... 127

4.8 Allocation vectors provided by the WCEA solution with GDP as unique weight for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2). 128

4.9 Allocation vectors provided by the WCEA solution with Renewable energy production as unique weight for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2). 129

4.10 Allocation vectors provided by the DWCEA solution for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2). 130

5.1 A double weighted bankruptcy situation (E,c,w^1,w^2) with $N=\{1,2,3,4,5\}$ as set of agents and estate $E=35$. 140

5.2 Exection steps of DWCEA^P solution giving priority to w^1 for Example 8 . . 140

5.3 DWCEA^P allocations with priority on GDP from 2010 to 2014. Bold text represents countries that have received all their claims. 142

5.4 DWCEA^P allocations with priority on renewable energy from 2010 to 2014. Bold text represents countries that have received all their claims. 144

5.5 Outcomes of the K-means clustering application over DWCEA^P method. Names in red color indicate countries that are totally satisfied at least for 2 years. 147

5.6 DWCEA^P Clusters are able to approve a decision according to the EU council voting systems. 147

5.7 Player Satisfaction rate for weighted allocations method and CEA rule applied on CO_2 emission permits. 152

5.8 Gini index for weighted allocations methods and CEA rule applied on CO_2 emission permits. Bold text represents the smallest Gini index 152

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Resume

Au cours des dernières décennies, le changement climatique et le réchauffement de la planète ont été les principaux problèmes environnementaux auxquels la communauté internationale est confrontée. Ils sont définis comme des problèmes environnementaux mondiaux en raison de leur dimension planétaire qui a un impact sur tous les pays du monde. En effet, les gazes, notamment le dioxyde de carbone, accumulés dans l'atmosphère et distribués de manière indivisible, provoquent des dommages irréversibles. Le temps nécessaire à la réparation de ces dommages pourrait durer des centaines d'années. La littérature sur la coopération internationale en matière de changement climatique s'est organisée autour de différentes formes de coopération internationale [94].

Les négociations [90] visant à réduire les émissions de gaz à effet de serre sont cruciales, compte tenu de la menace que représentent ces émissions pour la planète et l'humanité. Les négociations visent à trouver un accord entre les pays pour réduire leur taux d'émission de carbone. Cependant, aucun pays ne peut influencer le système de changement climatique indépendamment des autres. Les programmes de recherche ont commencé à s'intéresser au changement climatique au début des années 1990, en essayant de trouver des politiques d'émissions pour limiter les émissions de gaz à effet de serre.

La première conférence internationale sur le réchauffement de la planète a eu lieu à Rio de Janeiro en 1992. La Convention-cadre des Nations unies sur les changements climatiques (CCNUCC), une organisation intergouvernementale chargée de faire face aux problèmes de changement climatique, organise chaque année une conférence à laquelle participent 197 pays. Cette conférence est connue sous le nom de Conférence des Parties (COP). Elle a pour objectif de limiter à moins de 2°C, avant 2050, l'augmentation

de la température moyenne mondiale par une réduction de 50 % des émissions de gaz à effet de serre. Bien que plusieurs accords et négociations aient été enregistrés lors des conférences sur le climat, aucun accord à signer n'a pu être atteint. Approuvée par 200 pays en septembre 2013, la première partie du 5e rapport d'évaluation (AR5) du Groupe d'experts intergouvernemental sur l'évolution du climat (GIEC) affirme que l'influence humaine a été la principale cause du réchauffement climatique au XXe siècle, avec un taux de certitude de 95 %. La communauté internationale s'est engagée à maintenir le réchauffement climatique en dessous de 2°C, considérant que la survie et le développement des petits pays insulaires en développement ainsi que des pays les moins avancés seraient menacés au-delà de 1,5°C. Cependant, compte tenu des engagements actuels, on estime que les projections d'émissions nous conduiront à une augmentation de 2,7°C de la température mondiale au cours de ce siècle. Divers indicateurs thermiques et de température montrent une augmentation au fil du temps. Le CO_2 et plusieurs autres gaz à effet de serre persistent dans l'atmosphère pendant une très longue période. Nous appauvrissons la capacité de l'atmosphère à absorber ces gaz. Pour évaluer le réchauffement, les scientifiques utilisent la méthode des "scénarios d'émission", qui crée une projection cohérente de ce à quoi l'avenir pourrait ressembler. Ces scénarios s'appuient sur des hypothèses qui reposent principalement sur la croissance démographique, les choix énergétiques tels que le charbon, le solaire ou les énergies renouvelables, la croissance économique et l'innovation technologique.

L'un des principaux défis auxquels les participants sont confrontés est les questions politiques, telles que les négociations mondiales sur le changement climatique, et la détermination d'un système qui soit à la fois [8,43,93] "équitable" et "efficace" afin d'être validé par tous les gouvernements ou du moins la plupart d'entre eux.

La CCNUCC a fourni des indications très approximatives sur la manière dont la charge de la stabilisation des émissions de GES devait être répartie et a donné la priorité aux considérations d'équité et d'efficacité (article 3.1 de la convention sur le climat). Cependant, l'équité et l'efficacité ont été définies de manière générale comme impliquant cinq considérations (article 4a de la convention sur le climat):

- Différences dans les points de départ et les approches des parties, leurs structures économiques et leurs ressources.

- La nécessité de maintenir une croissance économique forte et durable.
- Les technologies disponibles.
- Le besoin de contributions équitables.
- Autres circonstances individuelles.

D'autre part, le principe d'efficacité est principalement lié à la politique économique de réduction des émissions, comme la minimisation du coût total de la réduction ou l'optimisation du partage des charges. Cependant, certains travaux ont souligné que l'efficacité peut également être traitée comme un type d'équité. L'un des critères suivant le principe d'efficacité est le critère métrique, selon lequel les permis d'émission sont divisés proportionnellement à l'inverse de l'intensité des émissions.

Les émissions doivent être réduites afin de trouver un équilibre avec la capacité naturelle de notre planète à les absorber. Nous devons passer à des ressources énergétiques propres et renouvelables. Cela implique une gestion durable des forêts et des autres ressources qui absorbent le carbone. Nous devons donc agir rapidement, tous ensemble et de différentes manières.

En ce sens, l'étude d'une politique de partage responsable des émissions de gaz à effet de serre, en particulier des émissions de CO_2, devient l'objectif principal des recherches sur le changement climatique. Dans les négociations climatiques, la théorie des jeux est une méthode intéressante utilisée pour établir un accord entre les pays. Les jeux coopératifs et les jeux non coopératifs, deux branches de la théorie des jeux, sont étudiés pour trouver les stratégies à adopter afin de réduire les émissions de gaz à effet de serre.

D'une part, la théorie des jeux non coopératifs permet de comprendre l'interaction des comportements stratégiques. Cette approche est cohérente pour étudier la coopération en matière de changement climatique. En effet, en l'absence d'une entité internationale permettant de surveiller ou de réguler le comportement des États, la conclusion d'un accord ne peut être que le résultat d'un choix volontaire des États. C'est ce que la théorie des jeux non coopératifs permet d'analyser. D'un autre côté, s'intéresser à la coopération internationale en matière de changement climatique revient à déterminer la taille de l'accord et à

réunir un grand nombre de pays qui peuvent le signer et en respecter les termes. Ainsi, les jeux coopératifs permettent la formation de coalitions, petites coalitions contenant un ou plusieurs joueurs ou grande coalition incluant tous les joueurs, afin de maximiser le gain de chaque joueur. Étant donné que tous les pays souffrent des conséquences des changements climatiques, il est impératif de trouver un accord qui soit ratifié par tous les pays participant aux négociations.

Compte tenu de la forte demande d'émissions de gaz à effet de serre, du taux d'absorption limité de la planète et de la consommation par les pays en 8 mois de toutes les ressources que les écosystèmes peuvent produire en une seule année, il semble évident que nous sommes confrontés à un problème de revendication. Les problèmes de revendication ou problèmes de faillite sont des jeux coopératifs et visent à allouer une quantité donnée d'un bien divisible entre un groupe d'agents. Bien que chaque agent ait sa propre demande, le bien divisible ne peut pas satisfaire toutes les demandes des agents. Dans notre contexte, le bien à diviser est la quantité autorisée d'émissions de CO_2, et les agents sont les pays de l'Union européenne, où, chacun de ces pays a sa propre quantité de CO_2 réclamée.

Une situation de faillite peut se produire lorsqu'un bien à répartir entre plusieurs agents ne peut pas satisfaire toutes leurs revendications, besoins ou demandes. Dans cette situation, chaque agent a une demande non négative (besoin ou demande), et la somme de toutes les demandes est supérieure au montant disponible.

Le problème de la faillite ne suppose que des biens divisibles et de nombreux demandeurs. Il peut être considéré comme une approche utile car il permet de modéliser différentes situations réelles et peut être appliqué à de nombreux problèmes connexes. L'exemple le plus pertinent est un exemple financier, où une entreprise, en faillite, doit être liquidée et ses créanciers doivent être remboursés, mais la valeur de l'entreprise ne peut pas couvrir toutes les demandes des créanciers. Un exemple social concerne le partage d'un bien entre plusieurs héritiers lorsque ce bien ne peut honorer les engagements du défunt. Dans le secteur économique, on peut définir une situation de faillite par le montant des taxes ou encore lorsque la demande d'un produit donné dépasse l'offre disponible.

Pour le secteur écologique, lorsqu'il y a une ressource rare à partager entre les pays comme la nourriture, les fournisseurs médicaux, le budget carbone [35] ou les conflits de

ressources en eau [51] en fonction d'une demande croissante en eau. En situation de faillite, l'objectif principal est de trouver une bonne répartition des ressources entre les agents. L'article fondateur [62] définit des règles d'allocation équitable pour associer à chaque problème de réclamation une division entre les réclamants et la ressource disponible et considère les méthodes d'attribution proportionnelle et d'attribution égale sous contrainte comme les deux concepts les plus pertinents utilisés dans le problème réel. Plusieurs solutions peuvent être impliquées dans ce type de situations générées par différentes méthodes. Une étude des règles, axiomes et méthodes de base [81, 82] caractérise les problèmes de réclamation et présente les propriétés à respecter. La première est la faisabilité, la somme allouée ne doit pas dépasser la ressource disponible, à qui elle doit être totalement allouée, efficacement. La non-négativité et la limitation des réclamations prévoient qu'aucune allocation ne doit être négative et qu'aucune ne doit dépasser la réclamation initiale demandée.

Un résumé [83] modélise les problèmes de réclamations comme des jeux coopératifs et se concentre sur l'inventaire des règles de division et le modèle de population variable.

Une analyse axiomatique [42] pour les méthodes de faillite les plus utilisées ; la règle proportionnelle, la règle d'attribution égale contrainte, la règle de pertes égales contraintes et la règle du Talmud suppose que la règle de résolution dépend de propriétés que cette règle doit satisfaire. En ce sens, chaque règle peut être définie par des ensembles d'axiomes indépendants et pour chaque type de problème, il existe une caractérisation de règle satisfaisante.

La recherche économique identifie ce type de problèmes selon deux approches fondamentales.

- L'approche de la théorie des jeux, dans ce cas, le problème de la faillite est évalué comme un jeu TU ou comme un problème de négociation [2].
- La deuxième approche est l'approche axiomatique, où les solutions sont établies par les propriétés de la règle [82].

Une nouvelle caractérisation des règles de faillite a été faite par Dagan [21]. Dans ce dernier, deux règles de faillite, *le principe d'attribution égale contrainte et le principe*

du vêtement contesté, sont étudiées par une approche axiomatique et suppose que ces deux règles considèrent non pertinente toute réclamation au-dessus du total à distribuer. Un autre nouvel aspect de la faillite [46] définit pour un groupe d'agents ayant non seulement des créances mais aussi leur propre patrimoine. En conséquence, chaque agent est considéré comme un demandeur et un emprunteur simultanément.

Considérant la question du changement climatique comme une situation de faillite, différents chercheurs ont étudié la taxation des émissions, le budget des émissions, le droit des émissions avec différentes méthodes de faillite et affirment que l'approche des réclamations est une alternative raisonnable et faisable pour allouer les émissions autorisées de CO_2 par les pays ou les groupes de pays.

L'approche de [35] se concentre sur la limitation du CO_2 global cumulé sur la période 2000 et 2050, avec la probabilité de dépasser le seuil de 2°C au-dessus des niveaux préindustriels [55], en considérant le budget carbone global comme un patrimoine à diviser entre les parties qui demandent les droits d'émission maximum. Ensuite, un modèle d'allocation efficace est proposé en tenant compte de différents scénarios de risque.

La méthodologie proposée analyse l'application d'un large groupe de solutions théoriques tout en respectant les critères d'équité et de stabilité. Dans ce contexte, l'analyse empirique [55] inclut diverses allocations globales de CO_2 provenant des prévisions de revendications associées aux scénarios RCP (Representative Concentration Pathways). Cette étude montre que les solutions d'allocation égale et de contrainte a-minimum répondent au critère d'équité.

L'étude [44] des protocoles internationaux équitables pour la réduction des émissions de GES formule des axiomes pour les règles d'allocation basés sur des principes normatifs, comme la population du pays, les émissions historiques et les émissions habituelles. Les combinaisons de ces axiomes caractérisent les règles d'allocation d'égalité par habitant avec et sans responsabilité historique. Les allocations prévues par ces règles contrastent avec l'allocation suggérée par le protocole de Kyoto, où les allocations égales par habitant autorisent davantage d'émissions dans les pays en développement à forte population. Les pays développés dont les émissions historiques sont élevées sont clairement désavantagés.

Compte tenu de la complexité des situations et de l'hétérogénéité de l'agent, les méthodes de faillite ne peuvent ni englober tous les aspects du problème ni proposer une allocation équitable et efficace acceptée par tous. En ce sens, les problèmes de faillite pondérés sont des problèmes de faillite où chaque agent a non seulement des créances, mais aussi un poids. Le premier document de travail dans ce domaine de recherche a été réalisé par une étude axiomatique des problèmes de faillite sans respecter l'égalité de traitement des égaux. L'allocation de chaque agent ne dépend pas seulement de sa demande mais aussi d'une priorité ou d'un poids.

Quatre méthodes de faillite pondérées [13] ont été étudiées pour caractériser le problème des laissez-passer de musée et définir une distribution des revenus issus de la vente des laissez-passer. La première partie de ce travail est une étude axiomatique des quatre nouvelles règles pondérées : 1) la règle proportionnelle ajustée contrainte pondérée, 2) la règle des prix égaux contrainte pondérée, 3) la règle des pertes égales contrainte pondérée et 4) la règle proportionnelle contrainte pondérée. Ensuite, une implémentation de ces méthodes pondérées aux problèmes de passe-musée, dans la municipalité de Gênes, est faite.

Un autre problème résolu par les méthodes de faillite pondérée est celui des bassins fluviaux transfrontaliers [56], en raison de l'augmentation de la demande en eau. L'objectif de ce travail est de partager les bassins fluviaux disponibles entre les pays riverains de manière équitable et raisonnable. Cette dernière étude introduit une nouvelle méthode de faillite pondérée appliquée à un cas réel, un fleuve partagé entre la Turquie, la Syrie et l'Irak, le fleuve Tigre sous deux situations différentes : groupe homogène et groupe hétérogène. L'idée de cette méthode est que les allocations sont distribuées en fonction de la contribution des agents et de leur poids.

Formellement, on caractérise une situation de faillite par un ensemble de $N = \{1, 2, .., n\}$ agents ayant un vecteur $c = (c_i, \ldots, c_n)$ de créances, donc chaque agent $i \in N$ a sa propre créance $c_i \geq 0$. Nous désignons par B^N la classe de toutes les situations de faillite avec N comme ensemble d'agents. Un *problème de faillite* [62, 83] ou un *problème de réclamations* standard est défini par un tuple (N, E, c) (ou, simplement, (E, c) si l'ensemble N est déjà clairement identifié), où $N = \{1, 2, ..., n\}$ est un ensemble d'agents, et la succession

$E \in \mathbb{R}_{>0}$ est telle que $0 < E < \sum_{i \in N} c_i$, avec $c \in \mathbb{R}^N_{\geq 0}$ comme vecteur de revendications. Ainsi, le problème est de diviser E entre les agents de N, chaque agent $i \in N$ ayant sa propre réclamation $c_i \geq 0$. Ce type de problèmes doit satisfaire deux propriétés triviales mais nécessaires :

- $E > 0$: Le domaine doit être positif et non nul.
- $E < \sum_{i=1}^{n} c_i$: C'est la condition de base d'une situation de faillite et cela signifie que toutes les réclamations des agents ne seront pas satisfaites.

Un **vecteur d'allocations**, également appelé *vecteur d'allocations*, est un vecteur de solution $x = (x_i, \ldots, x_n) \in \mathbb{R}^N$, donnant à chaque agent $i \in N$ un montant x_i respectant :

- Rationalité : $x_i \geq 0$;
- Limitation de la demande : $x_i \leq c_i$;
- Efficacité : $\sum_{i=1}^{n} x_i = E$.

En ce sens, chaque agent reçoit une quantité d'allocation non négative inférieure ou égale à sa propre demande. Et le patrimoine doit être totalement divisé entre les agents.

Une **règle d'allocation**, pour les situations de faillite est une carte $\phi : B^N \to \mathbb{R}^N_+$ qui induit pour chaque situation de faillite dans B^N un vecteur d'attribution $x \in \mathbb{R}^N_+$tel que $\sum_{i=1}^{n} x_i = E$.

Dans les situations de faillite, plusieurs règles existent pour diviser la masse selon des critères spécifiques et en fonction du contexte de la demande. La méthode la plus courante, la **Règle proportionnelle, *(P)*** [80] prévoit des indemnités proportionnelles aux réclamations. Chaque agent dispose d'une part de la succession proportionnelle à sa réclamation. Formellement, chaque agent i reçoit une allocation λc_i.

$$P_i(E, c) = \lambda \times c_i \tag{1}$$

où

$$\lambda \in \mathbb{R}_+$$

et

$$\sum_{i=1}^{n} \lambda c_i = E \tag{2}$$

La règle des prix égaux sous contrainte, ***(CEA)*** [21, 52] offre des parts égales de la succession à tous les agents, à condition que la somme des allocations ne dépasse pas le montant total de la succession et que chaque agent i ne puisse avoir plus que sa réclamation.

$$CEA_i(E,c) = min\{\lambda, c_i\} \tag{3}$$

où

$$\lambda \ in \mathbb{R}_+$$

et

$$\sum_{i=1}^{n} min\{\lambda, c_i\} = E \tag{4}$$

La règle d'égalité des pertes sous contrainte, ***(CEL)*** est un algorithme symétrique à la règle précédente. La méthode CEL [52] choisit le vecteur de récompenses pour lequel tous les agents subissent des pertes égales, à condition que personne ne reçoive un montant négatif.

$$CEL_i(E,c) = max\{c_i - \lambda, 0\} \tag{5}$$

où

$$\lambda \ in \mathbb{R}_+$$

et

$$\sum_{i=1}^{n} max\{c_i - \lambda, 0\} = E \tag{6}$$

La règle du Talmud, ***(T)*** [2] combine les deux dernières règles, la règle CEA et la règle CEL. Elle utilise ces deux règles et a pour point de bascule la moitié de la somme des créances. En d'autres termes, si la succession est inférieure à la moitié de la somme des réclamations, elle agit comme la règle CEA avec $c = \frac{c}{2}$. Sinon, chaque agent i reçoit la moitié de sa réclamation et la règle CEL est appliquée avec le reste de la succession. Formellement : $\quad If \quad E \leq \frac{\sum_{i=1}^{n} c_i}{2}$

$$T_i(E,c) = CEA_i(E, \frac{c}{2}) \tag{7}$$

Otherwise $\quad If \quad E \geq \frac{\sum_{i=1}^{n} c_i}{2}$

$$T_i(E,c) = \frac{c_i}{2} + CEL_i(E - \sum_{i=1}^{n} \frac{c_i}{2}, \frac{c}{2}) \tag{8}$$

Puisque tous les pays émettent CO_2, il est évident de superviser leurs allocations et de définir une politique commune de permis d'émission. Dans cette thèse, nous étudions le problème d'allocation des émissions de CO_2 comme un problème de revendication. Le domaine est défini comme la quantité d'émissions de CO_2 autorisée par le protocole de Kyoto, et l'ensemble des agents $N = \{1,\ldots,28\}$ sont des pays de l'Union européenne (UE) et chaque pays a sa propre espérance d'émission de CO_2. Cependant, le protocole de Kyoto a limité les émissions excessives en réduisant la quantité de CO_2 autorisée de 22 % par rapport aux émissions totales de 1990. En ce sens, chaque pays doit planifier une politique d'émission respectant le seuil imposé de permission de CO_2. Nous étudions les permis d'émission CO_2 des pays de l'UE sur 5 ans, de 2010 à 2014, en appliquant les règles Proportionnelle, CEA, CEL, et Talmud. Chaque année, nous considérons l'émission réelle du pays comme une demande et nous supposons que chaque pays s'attend à obtenir sa demande (émission) avec l'une des méthodes d'allocation tant que le seuil défini n'est pas dépassé. Nous pouvons qualifier un pays de satisfait si l'allocation générée par l'une des méthodes utilisées est égale à sa demande.

Selon les demandes et les allocations des pays générées par les règles de faillite, afin de faciliter l'analyse, nous procédons à la division des pays en trois groupes. La répartition entre les trois groupes est déterminée à partir de la colonne CEL et de la colonne CEA des tableaux d'allocation (Tableau 2.2, Tableau 2.3, Tableau 2.4, Tableau 2.5 et Tableau 2.6). Les pays ne recevant pas le droit d'émettre selon la règle CEL sont classés dans le premier groupe. Nous pouvons remarquer que ce groupe est composé de pays ayant de faibles revendications comme Malte, la Lettonie, Chypre, le Luxembourg, la Lituanie, la Slovénie, l'Estonie, la Croatie, la Slovaquie, l'Irlande, le Danemark, le Portugal, la Hongrie, la Bulgarie et la Suède. Les pays restants sont répartis dans le groupe 2 ou le

groupe 3 en fonction de la colonne CEA. Si un pays reçoit la totalité de sa créance, il est considéré comme un membre du deuxième groupe. Ce groupe est composé de la Finlande, de l'Autriche, de la Grèce, de la Roumanie, de la Belgique, de la République tchèque et des Pays-Bas, ces pays sont caractérisés par des demandes moyennes. Les pays ayant des créances importantes comme l'Espagne, la Pologne, la France, l'Italie, le Royaume-Uni et l'Allemagne font partie du troisième groupe. Tous les membres du troisième groupe reçoivent le même montant.

Notez qu'au fil des années, les pays peuvent changer de groupe, par exemple en 2010 et 2011, la Finlande appartient au premier groupe, mais en 2012, 2013 et 2014, elle est considérée comme un pays du deuxième groupe. Avant 2013, la Slovaquie, l'Irlande, le Danemark, le Portugal, la Hongrie, la Bulgarie et la Suède font partie du premier groupe avec de faibles demandes. Depuis 2013, ces pays font partie du deuxième groupe.

En analysant les résultats des allocations , on peut affirmer que :

- La *règle CEA* satisfait totalement les pays du premier et du second groupe et propose les mêmes allocations aux pays à forte demande.
- Sous la *règle CEL*, les pays à faible demande ne sont pas autorisés à émettre tandis que les pays à demande moyenne obtiennent moins de la moitié de leurs demandes. Cependant, les pays du troisième groupe obtiennent la quasi-totalité de leurs demandes.
- La *règle proportionnelle* propose à tous les pays une allocation supérieure à la moitié de leurs demandes.
- En considérant la règle *Talmud*, le premier et le deuxième groupe recevront la moitié de leurs demandes. Les membres du troisième groupe reçoivent des allocations supérieures à la moitié de leurs demandes et ces allocations sont meilleures que celles générées par la règle *Proportionnelle*.

L'étude des émissions de CO_2 sur 5 ans permet de comprendre les stratégies d'émission des pays et de générer des allocations basées sur des méthodes de faillite. Tant que les

demandes varient d'une année à l'autre, les allocations évoluent en conséquence. En étudiant ces différentes règles d'allocation, nous pouvons conclure que les pays du premier groupe sont autorisés à produire la moitié de leurs demandes selon la règle *Talmud*, tandis qu'ils peuvent produire plus de la moitié de leurs demandes selon la règle *proportionnelle*, mais sont discriminés par la règle *CEL*. Seule la règle *CEA* permet de satisfaire pleinement ces petites demandes. Les pays du deuxième groupe sont également favorisés par la règle *CEA*, qui leur offre la quasi-totalité de leurs demandes, mais la règle *proportionnelle* leur attribue plus de la moitié de leurs demandes, et la règle *Talmud* leur donne le droit d'émettre la moitié de leurs demandes. Alors que le *CEL* leur alloue moins de la moitié de leurs demandes. Le troisième groupe est favorisé par le *CEL*, qui propose des allocations proches de leurs demandes. Les règles *proportionnelle* et *Talmud* proposent des allocations qui dépassent la moitié des demandes, tandis que la *CEA* propose la même allocation quelle que soit la demande.

Sur la base de ces résultats, nous poursuivons notre étude en nous focalisant sur la méthode CEA d'allocation des permis d'émission CO_2 entre les pays.

Dans le troisième chapitre, nous nous intéressons à l'allocation des permis d'émission CO_2 via une nouvelle famille de règles de faillite, la règle de faillite pondérée, et en particulier la méthode des prix égaux sous contrainte pondérée. Dans le chapitre précédent, nous utilisons des méthodes classiques et considérons les pays de l'UE-28 comme des joueurs et la quantité de CO_2 émise de 2010 à 2014 comme des réclamations. Cependant, dans la méthode pondérée, nous ajouterons un nouveau paramètre, le PIB, et distribuerons les permis d'émission de CO_2 en fonction des demandes et des poids.

Dans une situation de faillite, le processus d'allocation ne prend en compte que les créances des agents, cependant, les agents peuvent avoir des droits, des besoins ou des obligations et peuvent être caractérisés par différentes caractéristiques qui doivent être prises en compte. Ces paramètres sont définis dans notre travail comme des poids.

Soit une *situation de faillite pondérée* comme le triplet (E,c,w) où $w \in \mathbb{R}^N_+$ un vecteur de poids. Nous désignons par WB^N la classe de toutes les situations de faillite pondérée (E,c,w) avec $N = \{1,\ldots,n\}$ comme un ensemble fini d'agents. Une solution pour les situations de faillite pondérée est alors une carte $\psi : WB^N \to \mathbb{R}^N_+$. Les émissions de CO_2

en particulier et les problèmes liés au changement climatique ont été abordés par le CEA car il génère, d'une part, des allocations stables [41], d'autre part, un équilibre entre l'équité et la proportionnalité en donnant une distribution qui tient compte des demandes de chaque région. En outre, la règle du CEA satisfait un ensemble de principes, tels que l'égalité de traitement entre égaux, l'anonymat, la monotonie des ressources, la composition et la super-modularité (une plus grande demande subit une plus grande réduction) que les pays souhaiteraient mettre en œuvre dans l'allocation du domaine. Non seulement ces principes s'inscrivent naturellement dans le contexte des permis d'émission de CO_2, mais ils sont également considérés comme des exigences minimales en matière d'équité.

Pour toutes ces raisons, nous concentrons notre travail sur la version pondérée de la règle du CEA [59] pour allouer les permis d'émission de CO_2 entre les pays de l'UE. Nous considérons les émissions historiques de CO_2 comme des créances et le PIB de chaque pays comme des pondérations. Un pays ne peut obtenir ni plus que ses demandes CO_2 ni plus que $\lambda \times GDP$, où λ est un coefficient défini pour permettre l'équilibre budgétaire des allocations. Ce dernier doit respecter la relation entre les demandes et le PIB de tous les pays. Contrairement à la règle CEA, qui favorise les pays à faible demande et satisfait pleinement leurs demandes, tandis que les pays à forte demande recevront moins que leurs demandes, nous pouvons remarquer que la règle WCEA est bénéfique pour les pays ayant des technologies de production efficaces. Les pays ayant une faible intensité d'émission, $\lambda_i = \frac{c_i}{w_i}$, bénéficient d'une meilleure allocation que la règle CEA. Les pays qui revendiquent des quantités élevées (plus de 200.000Mt d'émissions de CO_2) recevront moins que leur demande selon la règle du CEA, cependant la règle du WCEA propose de meilleures allocations. Le Royaume-Uni, par exemple, reçoit toujours moins que ce qu'il demande selon la règle du CEA. En 2012, ce pays a demandé à émettre 468,573Mt de CO_2, et a obtenu le droit d'émettre 236,057Mt de CO_2 selon la règle du CEA, c'est-à-dire presque la moitié de la demande. Au lieu de cela, le CEA pondéré lui a attribué 391,0327Mt de permis d'émission de CO_2, soit plus des deux tiers de la demande. De plus, en 2014, elle aurait pu être totalement satisfaite par la règle pondérée, comme le montre la figure 3.3.

D'autre part, les pays ayant à la fois une forte demande et une forte intensité d'émission λ, ne trouveront pas d'incitation à adopter la méthode pondérée. Si l'on considère la règle

du CEA, la Pologne reçoit moins que sa demande, mais cette allocation est supérieure à la quantité allouée par la règle du WCEA, voir Fig. 3.4.

En effet, les pays dont la demande est faible ne représentent pas une charge dans le processus d'allocation et sont pleinement satisfaits de la règle du CEA. En revanche, s'ils n'adoptent pas une politique de gestion de l'intensité des émissions, ils sont pénalisés par la règle du CEA. C'est le cas de l'Estonie, dont la demande est faible, qui est totalement avantagée par la règle CEA, alors que la règle WCEA génère une allocation inférieure à la moitié de la demande (Figure 3.5).

D'autres pays, comme la Suède, sont capables de créer un équilibre entre leurs demandes et leurs intensités énergétiques. Cet équilibre génère une allocation complète selon les deux règles, le CEA et le WCEA. La figure (3.6) montre les demandes de la Suède, les allocations CEA et les allocations WCEA pour les quatre années étudiées. Elle met en évidence deux aspects de la politique utilisée, tout d'abord, on peut noter une stratégie de réduction des émissions au fil des années. Ensuite, pour chaque année, la demande est totalement satisfaite par les deux méthodes.

Grâce à cette étude comparative, nous pouvons observer que les pays qui revendiquent une faible allocation de permis d'émission CO_2 doivent l'utiliser pour produire plus de richesse, afin d'obtenir plus de droits d'émission. D'autre part, les pays les plus riches, avec un PIB élevé, sont considérés comme des pays ayant une bonne utilisation de CO_2.

Les allocations de la WCEA reflètent le fait que les pays ayant une grande efficacité des ressources et des niveaux élevés de technologie reçoivent leurs demandes. Cela résulte d'une bonne capacité à émettre de faibles quantités de CO_2 par unité de PIB. Cependant, certains pays à faible demande ne sont pas satisfaits en raison du manque d'efficacité dans l'utilisation de CO_2 dans la création de richesse et l'implication dans les nouvelles technologies.

Dans les questions mondiales en général et les négociations sur le changement climatique en particulier, les pays cherchent à coopérer et à se mettre d'accord sur une stratégie opérationnelle commune. Une négociation réussie entraîne des conséquences positives pour tous les participants, tandis qu'un échec reflète un manque de coopération et génère

de graves conséquences.

Dans ce travail, nous introduisons un nouveau TU-game dans le but d'étudier le problème de la mise en œuvre d'incitations économiques pour transférer des technologies des pays les plus efficaces, avec un faible $\frac{c_i}{w_i}$, vers les pays les moins efficaces, avec un ratio $\frac{c_i}{w_i}$ élevé. Ce jeu peut être décrit comme un jeu de transfert de technologie (TT-) où le profit de chaque coalition est donné par le profit total de la coalition qui est obtenu à partir du plus haut niveau de technologie disponible pour les membres de la coalition. Dans le quatrième chapitre, nous nous concentrons sur l'allocation de permis d'émission CO_2 qui doivent être partagés entre les pays de l'UE, et où chaque pays a sa propre réclamation de permis d'émission CO_2 pour prendre en considération deux paramètres exogènes visant à assurer un équilibre entre les activités économiques et la production d'énergie renouvelable. A notre connaissance, il s'agit de la première étude où le problème d'allocation de CO_2 a été étudié comme un problème de réclamation d'un point de vue multicritère. Plus précisément, dans ce travail, nous introduisons une application préliminaire d'une nouvelle méthode d'allocation pour les situations de faillite à double pondération basée sur la combinaison de deux critères distincts : la capacité des pays à utiliser efficacement leurs permis d'émission de CO_2, et la capacité des pays à produire de l'énergie de manière durable via des sources renouvelables. Plus précisément, en utilisant une version étendue de la solution CEA pour les situations de faillite, nous tirons parti de la demande de permis d'émission CO_2 des pays de l'UE en tenant compte à la fois de la croissance économique des pays de l'UE (en termes de PIB) et de leurs politiques durables, mesurées par la quantité d'énergie renouvelable produite par chaque pays. Les situations de faillite pondérées [13] tiennent compte d'un paramètre supplémentaire pour déterminer les vecteurs d'allocation. Dans cette section, nous introduisons un cadre plus riche pour les situations de faillite, en considérant deux vecteurs de poids à la fois, en plus du vecteur des créances.

Nous désignons par $(E, c = (c_i)_{i\in N}, w^1 = (w_i^1)_{i\in N}, w^2 = (w_i^2)_{i\in N})$ un *problème de faillite à double pondération* où $E \in \mathbb{R}_{\geq 0}$ est un domaine non négatif, N est un ensemble fini de réclamants, $c = (c_i)_{i\in N}$ est le vecteur des créances non négatives tel que $\sum_{i\in N} c_i \geq E$, et $w^1 = (w_i^1)_{i\in N}$ et $w^2 = (w_i^2)_{i\in N}$ sont deux vecteurs de poids positifs entiers.

Nous désignons par $\mathscr{DW}$ la famille de tous les problèmes de faillite à double pondéra-

tion. Une règle d'allocation $\phi : \mathscr{DW} \to \mathbb{R}^N_{\geq 0}$ est une carte qui associe à tout problème de faillite doublement pondéré $(E, c, w^1, w^2)\mathscr{DW}$ un vecteur d'allocation dans $\mathbb{R}^N$ tel que $\sum_{i \in N} \phi_i(E, c, w^1, w^2) = E$. Dans les problèmes de faillite pondérée, chaque agent est caractérisé par une réclamation et des poids. Ces poids ont des impacts différents selon le paramètre choisi et son taux de conformité. Par conséquent, ils peuvent avoir un impact sur les résultats de l'allocation.

Dans les problèmes de décision multicritères, nous avons fréquemment un large choix de vecteurs d'allocation qui satisfont les contraintes spécifiées par les relations (4.1) et (4.2) pour certains λ_1^* et λ_2^*. Par conséquent, afin de sélectionner un vecteur d'allocation spécifique, nous introduisons une approche computationnelle pour trouver des paramètres pour λ_1^* et λ_2^* qui ne nécessite pas de supposer une priorité spécifique sur les critères représentés par les deux vecteurs de poids. En considérant à la fois le PIB et l'énergie renouvelable comme poids dans l'algorithme 6 pour fournir une méthode DWCEA, les pays respectant un seuil spécifique des deux poids reçoivent leurs demandes totales de permis d'émission, comme c'est le cas pour le Danemark, la Suède, l'Autriche et l'Espagne en 2010, et pour la France et le Royaume-Uni en 2014. Par rapport à l'allocation générée par la solution WCEA basée uniquement sur le PIB comme poids, nous pouvons remarquer que l'allocation DWCEA augmente la quantité de permis d'émission pour 13 pays en 2010, 16 pays en 2011 et 2012 et 19 pays en 2013 et 2014. Au contraire, comparé aux résultats obtenus par la WCEA basée sur l'énergie renouvelable comme poids unique, nous pouvons observer une amélioration des allocations pour 17 pays pour 2010 et 2011, et au moins 14 pays de 2012 à 2014. En outre, en 2010, le Danemark et la Suède sont entièrement satisfaits quelle que soit la méthode d'allocation utilisée, Malte et l'Autriche, en 2010, obtiennent une quantité égale par la DWCEA et la WCEA basée sur l'énergie renouvelable uniquement, et l'Irlande, la Finlande et la France, en 2014, reçoivent la même quantité de permis d'émission selon la DWCEA et la WCEA basée sur le PIB uniquement.

Nous constatons également des similitudes entre les allocations de certains pays au cours des cinq années étudiées. Pour être plus précis, nous avons étudié ces similitudes à l'aide d'une technique de clustering non supervisé, à savoir la méthode des K-means, basée sur la notion de 1-distance pour mesurer la similitude de la distribution des allocations de

chaque pays au fil des années à partir du centre de référence de chaque cluster : [77]. Cette technique de clustering non supervisée vise à regrouper les enregistrements (pays) de l'ensemble de données en K clusters distincts en fonction de leurs similarités, et chaque enregistrement ne peut se trouver que dans un seul cluster à la fois. Les groupes en fonction de la répartition des allocations sur les cinq années données par la solution du CEA : on remarque que les pays des groupes G1 et G2 ont des demandes faibles et moyennes et sont totalement satisfaits par rapport à leurs demandes de 2010 à 2014. En revanche, dans le groupe 3, seuls les Pays-Bas, dont la répartition des émissions est moyenne, sont totalement satisfaits sur 5 ans. Les autres membres du groupe G3 reçoivent le même montant malgré leurs différentes demandes (sauf l'Espagne, qui est totalement satisfaite depuis 2013).

Les groupes formés selon la distribution de l'allocation sur les cinq ans par la solution WCEA basée sur le PIB uniquement, sont présentés dans le tableau 4.4(3). On observe que le groupe G1 est défini par les pays à faible PIB, le groupe G2 par les pays à PIB intermédiaire, et le dernier groupe, G3, est formé par les pays à PIB élevé. Deux pays du groupe G2(le Danemark et la Suède) et un seul pays du groupe G3(la France) sont totalement satisfaits tout au long des cinq années. Ces groupes montrent donc une forte association entre les demandes de permis d'émission et le PIB, dont l'effet joue en faveur des pays dont la production économique interne est importante.

Dans le tableau 4.4(4), concernant les groupes sur les distributions d'allocation sur les cinq années fournies par la solution WCEA basée sur l'énergie renouvelable uniquement sur les cinq années, le groupe G3 contient des pays à fortes émissions, qui sont également caractérisés par des niveaux élevés de production d'énergie renouvelable. Dans ce groupe, l'Espagne et l'Allemagne obtiennent leurs revendications pour toute la période étudiée. Même si l'effet des activités productrices de CO_2 est toujours prédominant, ici, les pays avec une forte production d'énergie renouvelable montrent une distribution d'allocation similaire dans le temps, et ceux qui sont entièrement satisfaits pendant au moins trois ans sont dans les groupes avec des demandes intermédiaires et élevées et aussi une forte production d'énergie renouvelable en même temps. Enfin, le tableau 4.4(5) présente les clusters générés sur la distribution des allocations au fil des années par la méthode d'allocation DWCEA. Dans ce cas, les clusters sont plus homogènes car ils sont générés par un équili-

bre entre leurs revendications, leurs niveaux économiques et leurs attitudes à l'égard de l'utilisation de l'énergie verte. Selon la distribution de l'allocation DWCEA dans le temps, seuls l'Irlande, le Danemark, la Suède et l'Autriche reçoivent la totalité de leurs demandes pendant au moins trois ans en raison de leur poids dans la production d'énergie renouvelable. Ainsi, le compromis entre le PIB et la production d'énergie renouvelable dans la méthode d'allocation DWCEA semble jouer en faveur des pays caractérisés par des niveaux intermédiaires de demandes CO_2 et de PIB et, en même temps, de bons niveaux de production d'énergie renouvelable. Dans le cinquième chapitre, nous traitons le problème des émissions de CO_2 comme une situation de faillite bi-critère, en donnant une priorité différente à chacun des poids. Nous avons sélectionné deux critères importants pour les pays, le premier représente un paramètre économique et décrit la richesse d'un pays (le PIB) et le second est une question environnementale et écologique à travers la production d'énergies renouvelables. Ces deux paramètres sont des points stratégiques lors des négociations, et chaque pays peut argumenter en faveur de l'un d'entre eux en fonction de sa politique et de ses stratégies environnementales. Dans notre implémentation des allocations de permis d'émission de CO_2, nous avons commencé notre expérience en priorisant le premier paramètre (PIB) par rapport à la production d'énergie renouvelable, puis nous avons expérimenté notre méthode avec la priorité du second paramètre (énergie renouvelable) par rapport au premier. La méthode de faillite à double pondération avec priorité sur les critères, DWCEA^P, suppose que chaque poids (critère) a une priorité différente. Contrairement à la méthode présentée dans le chapitre précédent, avec cette méthode, un des poids a plus d'influence et a donc plus d'impact sur les allocations finales. Plus ce critère est respecté, plus l'agent en bénéficiera.

La présentation formelle de cette méthode ne change pas de celle de la méthode DWCEA sans priorité sur les poids, sauf qu'ici, on ne parle pas de w^1 et w^2 mais plutôt de poids prioritaire w^P et de poids non prioritaire w^{NP}. En considérant un problème de faillite doublement pondérée (E,c,w^P,w^{NP}), un *Double Weighted Constrained Equal Awards (DWCEA)*, associe à toute situation de faillite doublement pondérée un vecteur d'allocation particulier $x \in \mathbb{R}^N_{\geq 0}$ satisfaisant les conditions suivantes:

$$x_i(E,c,W) = min(c_i, \lambda^{P^*} w^P, \lambda^{NP^*} w^{NP}) \tag{9}$$

where $\lambda^{P^*}, \lambda^{NP^*} \in R^N$ such as

$$\sum_{i \in N} min(c_i, \lambda^{P^*} w^P, \lambda^{NP^*} w^{NP}) = E \tag{10}$$

En analysant les allocations générées par le DWCEA^P avec le PIB comme priorité, comme le montre le tableau 5.3, le Danemark est totalement satisfait tout au long de la période étudiée. La Suède et l'Autriche reçoivent leurs demandes tout au long des années étudiées, sauf en 2011.

En premier lieu, nous comparons nos résultats avec les allocations générées par la WCEA sur la base du PIB comme poids unique, présentées dans le tableau 4.8. Nous pouvons observer qu'au fil des ans, le nombre de pays qui préfèrent opter pour une méthode à double pondération avec une priorité fixée sur le PIB augmente, ils étaient 16 en 2010, 17 en 2011, et plus de 20 en 2012-2014.

Par rapport aux allocations DWCEA sans priorité, dans le tableau 4.10, nous pouvons remarquer qu'en 2010, 15 pays obtiennent de meilleures allocations avec la méthode DWCEA^P avec le PIB comme critère prioritaire, en 2011, 21 pays peuvent améliorer leurs allocations en utilisant la méthode DWCEA^P avec priorité au PIB. Nous pouvons également noter que la plupart des pays à faible et moyenne demande ont une amélioration des émissions en optant pour une priorisation du PIB.

Nous pouvons souligner que plus de $\frac{4}{5}$ du domaine est détenu par les 7 pays qui ont une plus grande demande d'émissions mais ce montant est encore inférieur à ce que ces pays peuvent obtenir en utilisant le DWCEA sans priorité et le WCEA basé sur le PIB. Selon le tableau 5.4 où la priorité est donnée aux énergies renouvelables, les allocations du Danemark couvrent entièrement ses demandes de 2010 à 2014. La Suède a pu être satisfaite pendant 4 ans (2010, 2012, 2013 et 2014). D'autres pays comme l'Irlande, la Finlande, l'Espagne et l'Italie obtiennent leurs demandes pour au moins 2 ans.

Nous commençons une étude comparative avec l'allocation générée par une méthode à double pondération considérant la production d'énergie renouvelable comme un paramètre unique, Tableau 4.9 et nous pouvons remarquer que pour toutes les années étudiées, la

majorité des pays, plus de 18 pays en 2010-2012, préfèrent utiliser une méthode à double pondération considérant la production d'énergie renouvelable comme une priorité. En 2013, 15 pays ont obtenu la même allocation avec les deux méthodes. En regardant le tableau 4.10, nous pouvons remarquer que 14 pays en 2010, 9 pays en 2011, et 12 pays en 2012 à 2014 reçoivent de meilleures allocations par la méthode de double pondération avec priorité aux énergies renouvelables que la méthode de double pondération sans priorité. Nous comparons les allocations générées par DWCEA^P avec le PIB comme priorité aux allocations basées sur la production d'énergie renouvelable comme critère de priorité et nous remarquons qu'en 2010, 10 pays préfèrent donner la priorité à la production d'énergie renouvelable, 11 pays en 2011, et 12 pays en 2013 et 2014. À l'issue de chaque méthode d'allocation étudiée, nous avons utilisé la méthode k-means pour définir des différents clusters et ensuite nous avons évaluer le poids de ces groupes dans les négociations du conseil de l'union européen. Pour chaque méthode, nous avons défini les coalitions des groupes qui peuvent passer un consensus ou une loi. Nous avons aussi étudié à la fin du dernier chapitre les indices de satisfactions des pays en utilisant les méthodes de l'index de Gini et la méthode du taux de satisfaction des joueurs.

Introduction

In recent decades, climate change and global warming have been the major environmental issues facing the international community. They are called global environmental problems because of their planetary dimension impacting all countries worldwide. In fact, gases, in particular carbon dioxide, accumulated in the atmosphere and distributed indivisibly, cause irreversible damages. The time required for repairing these damages could last hundreds of years. The literature on international cooperation on climate change has been organized around various international cooperation forms [94].

Negotiations [90] to reduce greenhouse emissions are crucial, considering the threat of these emissions to the planet and humanity. Negotiations aim to find an agreement between countries to reduce their carbon rate emission. However, no country can influence on the climate change system independently from the others. Research programs started to be interested by climate change in the early 1990s, trying to find emissions policies to limit greenhouse emissions.

The first international conference on global warming occurred in Rio de Janeiro in 1992. The United Nations Framework Convention on Climate Change (UNFCCC), an intergovernmental organization to face climate change problems, annually organizes a conference where 197 countries are included. This conference is known as the Conference of Parties (COP). It targets to limit under 2°C, before 2050, the increase of global average temperature through a reduction of the greenhouse gases emissions by 50% [65]. Although, several agreements and negotiations have been recorded during the climate conferences no agreements to sign could be reached.

Approved by 200 countries in September 2013, the first part of the 5th Assessment Report (AR5) of the Intergovernmental Panel on Climate Change (IPCC) states that human influence was the main cause of global warming in the 20th century, with a 95% certainty. The international community has made a commitment to keep global warming below 2°C, considering the fact that survival and development of developing small island as well as least developed countries, would be threatened beyond 1.5°C. However, in view of current commitments, it is estimated that emission projections will lead us to a 2.7°C rise in global temperature during this century. A variety of thermal and temperature indicators show an increase over time. CO_2 and several other greenhouse gases persist in the atmosphere for a very long time. We are depleting the capacity of the atmosphere to absorb these gases. To assess warming, scientists use the ”emission scenario” method, which creates a coherent projection of what the future may resemble. These scenarios rely on assumptions that are based on primarily population growth, energy choices such as coal, solar or renewable energy, economic growth and technological innovation.

Emissions must be reduced in order to find a balance with the natural capacity of our planet to absorb them. We must move to clean and renewable energy resources. This involves sustainable management of forests and other carbon-absorbing resources. We must therefore act quickly, all together and with different manners.

In this sense, studying a responsible share policy to the greenhouse gases emissions, in particular CO_2 emissions, becomes the main goal of climate change researches. In climate negotiations, game theory is an interesting method used for establishing an agreement among countries. Cooperative games and non-cooperative games, two branches of game theory, are studied to find strategies to adopt in order to reduce greenhouse gases emissions.

On one hand, non-cooperative game theory allows to understand the strategic behaviors interaction. This approach is consistent to study cooperation for climate change. In fact, in absence of an international entity for monitoring or regulating States behavior, the conclusion of an agreement can only be the result of a state voluntary choice. This is what non-cooperative game theory allows to analyze.

On another hand, taking an interest in international cooperation on climate change comes down to determine the size of the agreement and bring together a large number

of countries that can sign and abide by its terms. Hence, cooperative games allow the formation of coalitions, small coalitions containing one or more players or grand coalition including all players, in order to maximize each player's gain. Since all countries suffer from consequences of climate changes, it is imperative to find an agreement which ratifies all the countries participating in the negotiations.

Taking into account the strong demand for greenhouse gas emissions, the limited absorption rate of the planet and the countries 8-months-consumption of all the ressources that ecosystems can produce in a signle year, it seems obvious that we are facing a claim problem. Claim problems or bankruptcy problems are cooperative games and aim to allocate a given quantity of a divisible good among a group of agents. Although each agent has his own demand, however the divisible good can not satisfy all agents demands. In our context, the good to be divided is the permitted CO_2 emissions quantity, and the agents are European Union countries, where, each one of these countries has his own CO_2 quantity claimed.

This report is organized in 5 chapters:

- Chapter 1 : State of the art, it presents bankruptcy problems and some applications domains. Followed by a small survey about climate change negotiations with the different approaches used to limit the CO_2 emissions.
- Chapter 2 : Preliminary and first application, it links our research context on the allocation of emissions and bankruptcy situations. In this chapter, we apply different bankruptcy method to distribute CO_2 emissions permits.
- Chapter 3 : Weighted bankruptcy method for CO_2 emissions permit, it presents the weighted bankruptcy problems and propose an allocation analyzing not only claims but also players weight. We will also present a transfer technology game among players.
- Chapter 4 : Double weighted bankruptcy method, it is a new version of the weighted bankruptcy method considering two weights with the same priority. We adopt this method to CO_2 emission permits problem and define groups through k-means clustering method to study their vote impact in EU council.

- Chapter 5 : Double weighted bankruptcy method with priority, it presents a new allocation method considering two different weights with different priority. Based on algorithmic approach, we implement the CO_2 emission permits problem twice by giving, in first time, priority to GDP, then to Renewable energy production. And as a final step, We introduce some fairness index and we make a comparative study among the different methods presented throughout this manuscript.

Abbreviations	Complete definition
CBDR	Common But Differentiated Responsibilities
CEA	Constrained Equal Awards
CEL	Constrained Equal Losses
COP	Conference of the parties
CO_2	Carbon dioxide
DEA	Data Envelopment Analysis
DFS	Demand Fraction Satisfaction rate
DWCEA	Double Weighted Constrained Equal Awards
EU	European Union
GDP	Gross Domestic Product
GHG	Greenhouse gas
GT	Game theory
IEA	International environmental agreement
KP	Kyoto Protocol
P	Proportional
PS	Player Satisfaction rate
PWCEA	Priority-augmented Weighted Constrained Equal Awards
T	Talmud
Transfer Utility game	TU-game
Technology-Transfer game	TT-game
UNFCCC	The United Nations Framework Convention on Climate Change
WCEA	Weighted Constrained Equal Awards
ZSG	Zero Sum Game

Table 1 – Acronyms used throughout this manuscript

Chapter 1

State of the Art

Contents

1.1 Introduction 37

1.2 Climate change abatement negotiations 38

1.3 Game Theory in climate negotiations 48

1.4 Bankruptcy situations 50

1.5 Conclusion 53

1.1 Introduction

Because of global warming, population equality will not be reached. Some will be more affected than others. Note that those who have emitted the most greenhouse gases are not necessarily the ones who are exposed to the climate change effects.

Research has revealed that 90% of emissions are generated on 8% of the planet surface, and that more than half of the land will experience extreme warming by the end of the century. On the other hand, the industrialized countries as Western Europe and northeastern North America, for example - that emit the most greenhouse gases will experience relatively less climate impact. Others who emit less CO_2, such as Africa and Central Asia, will experience greater impacts in terms of warming.

Since everyone profits from cutting global warming and everyone has an advantage in putting the burden of reducing emissions on someone else, climate change is considered a global public good. Different research paths were explored in order to obtain a common agreement for all countries.

This chapter is dedicated to present the literature review associated to the main theoretical developments on this issue. It is organized as follows:

- Climate change abatement negotiations section introduces climate change negotiations literature, including several allocation approaches and relevant parameters.
- Game theory in climate negotiations section is a survey of cooperative games in climate change negotiation survey and basics of this branch of game.
- Bankruptcy situations section presents bankruptcy situations associated to CO_2 emissions permits, presenting some climate studies as claims problem.

1.2 Climate change abatement negotiations

Global warming is an increasing threat to the planet environment, led by greenhouse gas emissions. Climate researchers [38] are working on a tipping point to limit warming to 2°C above the pre-industrial level by 2100. Otherwise global warming will lead to a definite increase in environmental catastrophes [87]. The stakes are high and further actions are urgently needed to avoid an imminent disaster. In order to prevent dangerous long-term effects of climate change, in 1992, the United Nations Framework Convention on Climate Change (UNFCCC) was held in Rio de Janeiro, where participating countries committed to taking measures to avoid climate change with the main target of *"...stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system [...] within a time-frame sufficient to allow ecosystems to adapt naturally to climate change* (UNFCCC, 1992). This international environmental pledge has the goal to achieve a stabilization of GHG concentrations in the atmosphere based on a fair way of the distribution right of GHG emission.

The United Nations Framework Convention on Climate Change (UNFCCC 1992) recognised the importance of an equitable solution through Article 3.1: *The Parties should protect the climate system for the benefit of present and future generations of humankind, on the basis of equity and in accordance with their common but differentiated responsibilities and respective capabilities. Accordingly, the developed country Parties should take the lead in combating climate change and the adverse effects thereof. (UNFCCC, Article 3.1; emphasis added)*

More than 178 countries, almost all the world countries, showed their intention to stabilize their GHG emissions and have joined this framework. The signatories countries are qualified as "Parties". Since 1995, once a year, their representatives meet during the COP. During these COPs, countries may ratify agreement or present their strategies on GHG emissions reductions. They also evaluate the evolution of their commitments and the implementation of the framework convention.

On the occasion of the 3rd COP in 1997, the most known accord, Kyoto Protocol, was signed. It provides 5% emissions reduction for 37 developed countries over the period 2008-2012 compared to 1990 levels. Later at COP15, the Kyoto Protocol was extended till 2020 and specified an average reduction of 22% of committed countries emissions compared to 1990 levels. However, this agreement neither fixed a procedure nor defined specific measures that each signatory country must follow to reach its emission target.

In 2015, the first universal climate agreement was signed by 195 countries, including the biggest polluters: China and the United States. That agreement is known as the Paris Agreement. It stipulates:

- First to limit the rise in temperatures to less than 2°C, or even 1.5°C by reducing greenhouse gas emissions. Each country has made voluntary and non-binding pledges. These reduction rates are determined at the national level and are set according to the constraints of each country.

- Second, a carbon neutral world by the end of the century. In other words, the amount of CO_2 emitted must be equal to the one naturally absorbed by trees, soils and oceans.

- Third, it offers a financial aid to the poorest countries: 100 billion dollars per year between 2015 and 2020 to help the poorest countries.
- Finally, the signatory countries must meet every 5 years to take stock of the situation.

Since then, CO_2 emissions have increased, the U.S. has left the agreement, and the COVID pandemic has been a distraction.

In this section, we introduce the main approaches used in climate change negotiation, some alternatives of emissions reductions and different allocations methods used and indicators studied in the literature.

Some studies [9, 64] work on identifying agents that should be considered the financial burden holders for new GHG emission allowances and entities that should hold the authorization to emit GHG. They distinguish two different types of questions: i) a philosophical question studying which type of entity has the moral right to emit GHG, an individual entity or a state, and, ii) a practical question focusing on which type of entity should be allowed the legal right to emit GHG when designing systems to abate climate change.

To define a fair way to distribute GHG emission rights, [9] criticizes three criteria ;

- The first and most applied one in practice is grandfathering, where the quota of an actor (state, firm, individual or group) depends on its current share of emissions and these rights must be distributed for free.
- Secondly, the equality criterion based on the egalitarian approach, where all actors are entitled to an equal amount of GHG emissions to emit and equal right to be protected from pollution.
- The last criterion studied is the historical responsibility, which stipulates that the GHG emission right should reflect the historical fact of GHG emissions since the industrial revolution and emission reductions are allocated according to actor's responsibility on temperature increase. This criterion presents the principle of *'common but differentiated responsibilities' (CBDR)* and defined by the United Nations Framework Convention on Climate Change as *'the developed country Parties should*

take the lead in combating climate change and the adverse effects thereof' (United Nations 1995, p. 5)

Paper [86] assumes that power and stream are the main contributors of CO_2 emissions and presents five allocation possibilities that dominate the negotiation: grandfathering, egalitarianism, historical responsibility, equal treatment right for new entrants and Economic activity where less developed countries claim the right to emit more than their past emissions. He then applies three allocation methods: uniform reduction, burden sharing and per capita allocation, for CO_2 emissions permits among European Union. The uniform reduction consists of a 22% reduction in CO_2 emissions for each state. The burden-sharing approach gives each member state the right to reduce GHG emissions by its respective percentage relative to 1990. Finally, the total 1990 EU GHG emissions reduced by 22% will be allocated to each Member State according to their population (per capita).

During the Kyoto Protocol negotiations, countries proposed several types of burden-sharing agreements [68]. Some members chose a general or symmetrical approach and opted for a single percentage of emission reductions. Others have proposed a differentiated approach to country obligations.

France, Switzerland and EU propose [68] a *Convergence* approach which consists to converge per capita GHG emissions in all countries to stabilize atmospheric GHG concentrations at an acceptable level. Therefore, developed countries must significantly reduce their per capita emissions, while developing countries can benefit from an increase until they reach a certain level.

Brazil recommends a historical responsibility approach which reveals differences in the GHG contributions of different countries cumulated over time. It is generally reflected by the quantities of GHGs emitted by developed countries. However, we cannot consider fair and equitable to use the quantities emitted as the main criterion for determining emission permits.

Poland, Estonia, Russia and South Korea focus on GDP per capita criterion to divide commitments among countries. This indicator can be described as a capacity-to-pay or capacity indicator in these proposals.

New Zealand invites to cost-effectiveness approach in burden-sharing agreement. It stipulates an equal and optimal distribution of marginal abatement costs among countries.

Scholars [35] use an emission scenario method to create a coherent story of what the future might look like. To assess warming, scenarios rely on assumptions, primarily based on:

- Population growth: the more people there are, the more energy is used
- Energy choices: coal, solar, wind, renewable energy
- Economic growth
- Technological innovation

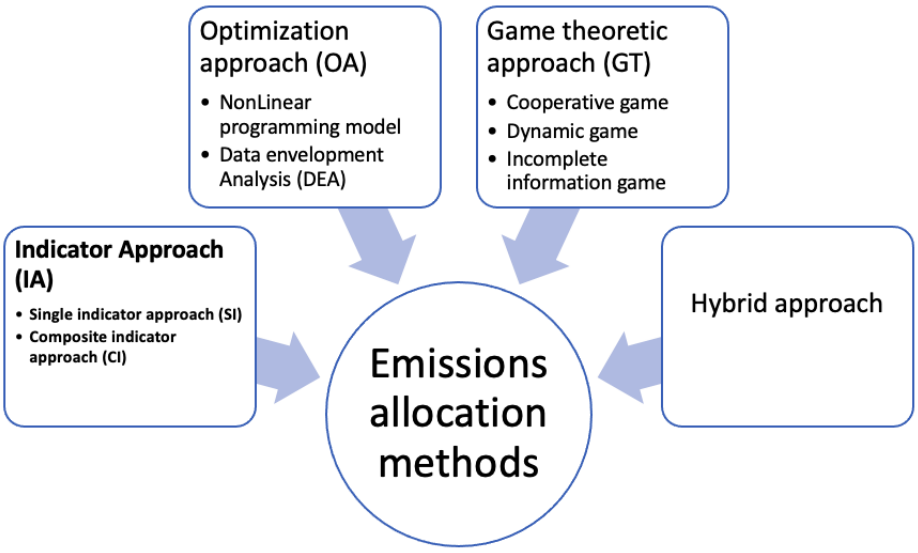

Figure 1.1 – Emissions allocation methods [93]

Studying allocations methods for CO_2 emissions, a new classification into four groups [93]; indicator approach, optimization approach, game theoretic approach and hybrid approaches, has been highlighted, as depicted in Figure 1.1. The most commonly used approach to allocate emissions is the indicator approach. The latter allocates emission permits according to a single [71] or composite indicators [68]. An example of a composite indicator we can focus on is the Triptych approach [66] a sector approach that takes into account

technological criteria at the industrial sector level as well as a range of other structural differences, including population size and growth, standard of living, economic structure and fuel mix in electricity generation. In 1997, this approach was discussed in the negotiations on differentiated commitments by the Member States of the European Union. Taking into account per capita indicator, [67] proposes sharing principles model of the available carbon budget based on grandfathering distribution. By studying the population indicator [38], he noticed that southern countries with a sufficient emission threshold are favored. Instead, the GDP and emission criteria favor the northern countries.

According to the indicator chosen, certain criteria are fulfilled. For example, the population indicator has been strongly promoted in the distribution of CO_2 emissions at the national level. It meets the egalitarian criterion by ensuring an equal distribution per capita [40].

The emissions indicator on allocation systems guarantees grandfathering and is the most popular at the corporate level due to its low data requirement, greater acceptability, and carbon reduction potential [74]. It states that emissions rates are allocated in proportion to the entity's historical emissions. In addition to grandfathering, various allocation criteria that rely the emissions indicator have also been suggested, such as sovereignty, historical responsibility, and polluter pays.

One of the most widely used indicators is GDP, which can satisfy three criteria horizontal equity, ability to pay and economic activity [71, 93] . The horizontal equity criterion strives to allocate CO_2 emission permits in a way that equalizes the net welfare change, based on an estimate of the cost functions of carbon reduction. The ability-to-pay criterion suggests that countries with high GDP should bear a greater responsibility for CO_2 emissions abatement than poorer countries. The economic activity criterion specifies that CO_2 emission permits are allocated in proportion to GDP [89] and so all countries have equal rights to maintain their standard of living. Another interesting parameter is emission intensity [72] which represents the CO_2 emissions per unit of GDP.

Earlier studies have explored an optimization approach, either using linear programming models or non-linear models, in CO_2 emissions allocation research. Data Envelopment Analysis (DEA), is a linear model used to allocate CO_2 emissions at different levels.

At the national level, [37] proposed a zero-sum gain DEA (ZSG- DEA) model to redistribute CO_2 emission permits between Annex I and non-Annex I countries[1]. At the regional level, DEA models investigate the optimal allocation of CO_2 emissions among different provinces in China [92]. Non-linear programming models [29, 61] have also been studied for the allocation of GHG emission reductions around the world, such as the Regional Integrated Climate and Economic Model (RICE), whose advantage is that it integrates climate and economic models.

The game theoretic approach has been naturally used in the negotiation process to reach a fair allocation of permits. Often, negotiations between participants seem like a game in which each player seeks to obtain more permits. The allocation results can be viewed as an equilibrium solution of the game. GT can model negotiations among countries as a cooperative game, dynamic game or incomplete information game, having emissions permits allocations as equilibrium. The next section is about cooperative games.

Finally, the hybrid approach can be presented as a mixture of various methods that cannot be associated with the three groups presented above (indicator approach, game theory approach and optimal approach). The main idea of this approach is to use several indicators without constructing a composite indicator, e.g. the multistage scheme proposed by [6] which allows for a gradual increase in the number of countries covered and in the level of reductions depending on their participation and allocation rules. Another example [76] of a hybrid approach is the multi-sectoral convergence based on the Triptych method, the per capita convergence scheme and the multi-stage scheme.

One of the main challenges faced by participants in political issues, such as the global climate change negotiations, is determining a system that is both [8, 43, 93] "fair" and "effective" in order to be validated by all or at least most governments.

The UNFCCC provided very rough guidance on how the burden of stabilizing GHG emissions was to be distributed and gave priority to considerations of equity and efficiency (Article 3.1 of the climate convention). However, equity and efficiency were broadly defined as involving five considerations (Article 4a of the climate convention):

[1] https://unfccc.int/process/parties-non-party-stakeholders/parties-convention-and-observer-states

- Differences in the starting points and approaches of the parties, their economic structures and resources.
- The need to maintain strong and sustainable economic growth.
- Available technologies.
- The need for equitable contributions.
- Other individual circumstances.

Fairness principal, shown in Table 1.1, is often associated to the general concept of distributive justice and can be defined based on three different but complementary notions [69]:

- *Equality*: Equality can be supported by respecting sovereignty criterion which, in the context of international negotiations, requires that all states are equal and should have the same rights and duties.
- *Equity* : The concept of equity is relative [91]; an allocation is said to be fair to a group of individuals if it is relative to the rights and needs of each subgroup. It is articulated on equal per capita emissions, historical responsibility, ability to pay and preserving future development opportunities [54].
- *Exemption (for parties who do not have the capacity to contribute)*. In a global context, the range of variation among "responsibility" and "capacity" is so wide that it results in unfair burdens on the poorest. At some threshold, the principle of equity becomes the principle of "exemption"; i.e the exemption of any uncompensated obligation. In global climate change negotiations, the poorest countries are temporarily exempted and committed to contribute, at least in the future.

On the other hand, the principle of efficiency is primarily related to the economic policy of reducing emissions, such as minimizing the total cost of reduction or optimizing burden sharing. However, some works [92] have pointed out that efficiency can also be treated as a type of equity. One of the criteria following the efficiency principle is the metric

Criterion	Interpretations
Egalitarian/ Grandfathering	Each one (people/ firm) has equal right to pollute and to be protected from pollution
Sovereignty	All nations have the right to maintain their level of emission w.r.t population size
Basic needs	Countries have a minimum threshold of emissions to fulfill the basic needs of the population
Horizontal	Equal right for nations with the same economic circumstances
Vertical	Ability to pay depends on national economic wellbeing
Polluter pays	More abatement to historical emissions nations (1990)
Historical responsibility	Proportional to accumulated past emissions and contribution to the damages (1850-1995)
Ability to pay	Countries with highest GDP should pay for the abatement

Table 1.1 – Some fairness principles to allocate emissions rights

criterion, according to which emission permits are divided proportionally to the inverse of the emission intensity.

In general, single-criteria approaches to allocating emissions allowances are of limited interest since they are not accepted by all countries [85]. Yet individual countries often propose them. This is the case for many developing countries, which suggest the population-based rules, while developed countries prefer the grandfathering rule. Nevertheless, few authors [69] have developed multi-criteria approaches to allocate commitments fairly and effectively across countries. A combination of approaches is preferable, as fairness and efficiency have different levels and consequences for countries. It is important to emphasize that the choice of the used method implies that certain criteria must be respected.

To illustrate the real potential for application, criteria's choices are made as realistically as possible, but they must be the result of a negotiation process between the decision-makers of the different countries. In addition, the choice of criteria used to establish fairness and efficiency distribution patterns is a complex and delicate step. The process of

developing the criteria involves, on the one hand, a good understanding and ownership of the problem by the parties, and on the other hand, a significant degree of formalism in the mathematical design of the problem, as shown in Table 1.2.

Principles	Criteria
Egalitarianism	Population
Basic needs	GDP GDP per capita
Polluter pays	Emissions per capita
Historical responsibility	Cumulated emissions
Sovereignty	Current emissions
Metric	Emissions/GDP emissions/Energy consumption

Table 1.2 – Criteria's list respecting fairness and efficiency principles

Many concepts have been employed among which we can cite the work of Llavador [50] who proposes an inter-generational North-South world and the study's [39] which presents a climate architecture to manage carbon's budget policy for 13 major economies and to fix emissions consumption. There is also some scholars about coalition formation for cooperative game [24], multidimensional systems based on mutli-criteria methodology and mathematical programming, decision tools studying policy exercise and benefit-cost analysis [14].

Game theory models have been extensively used in the literature in an attempt to give a formal definition of the possible relations and decisions of the involved countries. Researchers explored both non-cooperative and cooperative game theory approaches in order to provide valuable suggestions for policy negotiation process. These models investigate the incentives of different nations to commit themselves to a climate agreement and examine the extent to which climate agreements are stabilized.

In the next section, we introduce some game theoretic works applied to climate change negotiations and we present several notions on game theory used in this thesis.

1.3 Game Theory in climate negotiations

Throughout the recent years, global issues such as financial markets, trade, biodiversity, international relations and negotiations on climate change have been tackled by game theory models.

Game theory is a set of mathematical models used to study situations of conflict and cooperation. Therefore, applied to climate negotiations, several of its models provide a formalization of strategic interactions to assist the decisions of parties in certain circumstances. In this context, different assumptions are made, such as the rationality of countries and the uniqueness of the objective for all, namely climate protection.

The literature on international environmental agreements has approached the problem from the perspective of non-cooperative game theory. A CO_2 emission permits allocation is seen as an outcome that is obtained at the equilibrium of a strategic game [11]. The two fundamental papers [3, 12] equate international cooperation with the formation of an environmental cartel and study, in a framework of simultaneous and sequential decisions, the choice of a set of countries to participate in this cartel. The countries are considered symmetric, the global public good is additive and involves benefits perceived by all and costs assumed individually. On the basis of these two contributions, a large literature has developed, seeking in particular to introduce heterogeneity into the game [88], uncertainty and/or risk aversion [7], and incentive mechanisms such as transfer [33] or ratification mechanisms [17]. This literature, however, does not focus on the actual negotiation process. Countries are supposed to choose whether or not to participate in an international agreement and they do not need to negotiate or interact to make their choice. Some contributions [10, 19] overcome this problem and analyze the negotiation process in the form of strategic approaches. However, these approaches do not tackle the paradox of cooperation and possible free-rider behavior. They assume that stable and profitable agreements exist and study the outcomes of strategic bargaining.

On another hand, for decision-making and strategic choices, negotiations are based on theoretical game models. Dependency between countries means that if one country shows its interest to cooperate, it promotes opportunities for further cooperation and influences the

future actions of other countries. These models [47] recognize that individual decisions are affected by the expected actions of others, leading to more effective and efficient incentive-based policy mechanisms for policymakers.

In this context, Forgo [31] stated the best field of application for various game theory models is the climate change negotiations, where the conflicting aspects of the issue are evident not only to scholars but also to affected individuals. Negotiations goal is to find a ground for cooperation and understanding and to stop the pursuit of self-interest and to focus on the available options and the possible strategies.

Several assumptions [16] can be made about cooperation and participation in collective action. For example, negotiations could reveal players' utilities to compensate for the lack of information on response functions. Or, unequally informed countries would have an incentive to act collectively by revealing information about their costs and benefits to the players. Negotiations could change the perception of payoffs and provide incentives for cooperation. The second hypothesis likens the negotiations to a communication's pre-game. Countries would signal during the negotiations what they are going to play. Then, negotiations would be the site of interaction of the parties involved and would be successful to the extent that the communication pre-game is self-signaling and self-committing. A third hypothesis, based on the success of some international environmental agreements, such as the 1982 Convention on the Law of the Sea, the Montreal Protocol on the Protection of the Ozone Layer, and the Kyoto Protocol, examines the influence of executive presidents in the negotiations. This later would be a mediator who knows the utility functions of each participant and suggests a draft agreement acceptable to all. Negotiations would then be successful if there is a suggested agreement from which the players have no interest in deviating. The failure of the negotiations would be interpreted by the non-existence of this draft, or by the inability of the chairman to produce it. This hypothesis evokes the concept of correlated equilibrium defined by Aumann [1]. This equilibrium takes into account the essential dimension of the coordination of players' behaviors. The strategic correlation mechanism would be exercised in the form of a draft agreement describing correlated strategies from which the coalitions would have no interest in deviating. Once implemented, this agreement would allow the achievement of a Pareto-improving situation.

There is a large literature devoted to game theoretic models applied to climate change negotiations. On one hand, there is literature based on static or repeated games [3, 30]. On the other hand, there is work modeling climate change as a dynamic game, where the GHG stock can vary over time [27, 53]. Others [60] considers simultaneous move on non cooperative. Similarly, others [73] have provided a very broad view of the economics and methodology of climate change investigation and game theory. Multiple modeling and survey works have been devoted to negotiations i.e., [15, 18, 31].

1.4 Bankruptcy situations

A bankruptcy situation can occur when a good to divide among several agents can not satisfy all their claims, needs, or demands. In this situation, each agent has a non-negative claim(need/ demand), and the sum of all claims is over the available amount.

Bankruptcy problem assumes only divisible good and many claimants, it can be considered as a useful approach since it can model different real situations and can be applied to many related problems. The most pertinent example is a financial example, where a firm, going bankrupt, must be liquidated and its creditors must be refunded, but the worth of the firm can not cover all the creditors' claims. A social example concerns the division of a heirloom among several heirs when this good can not honor the deceased's commitments. In economic sector, we can define a bankruptcy situation by taxes amounts or also when demands for a given product exceeds the available supply.

For ecological sector, when there is a scarce resource to be shared among countries like food, medical suppliers, carbon budget [35] or water resource conflicts [51] in accordance with a growing demand for water.

In bankruptcy situation, the main goal is to find a good resource distribution among agents. The seminal article [62] defines fair allocation rules to associate for each claim problem a division among claimants and the available resource and considers proportional and constrained equal awards methods as the two most relevant concepts used in real issue. Several solutions can be involved in this type of situations generated by different methods.

A survey of the basic rules, axioms and methods [81, 82] characterizes claims problems and presents proprieties to be honored. The first one is feasibility, allocations sum should not exceed the available resource to whom it should be totally allocated, efficiently. Non negativity and claims boundedness provide that no allocation should be negative and none exceed the requested initial claim as well.

A summary [83] models claims problems as cooperative games and focuses on inventory of division rules and variable population model.

An axiomatic analysis [42] for the most used bankruptcy methods; proportional rule, constrained equal awards rule, constrained equal losses rule and the Talmud rule assumes that the resolution rule depends on properties that this rule has to satisfy. In this sense, each rule can be defined by sets of independent axioms and for every problems type, there is a satisfying rule characterization.

Economic research identifies this type of problems in two fundamental approaches.

- The game theoretic approach, in this case, bankruptcy problem is assessed as a TU game or as a bargaining problem [2].
- The second approach is the axiomatic one, where solutions are established by the rule's properties [82].

A new characterization of bankruptcy rules was made by Dagan [21]. In the latter, two bankruptcy rules, *the Constrained Equal Awards and the Contested Garment principle*, are studied by an axiomatic approach and assumes that these two rules consider irrelevant any claim above the total to be distributed. Another new aspect of bankruptcy [46] define for a group of agents having not only claims but also having their owns estate. In consequence, every agent is considered as a claimant and as a borrower simultaneously.

Since climate change issue is considered as bankruptcy situation, different scholars studied emissions taxation, emissions budget, emissions right with different bankruptcy methods and states that claims approach is a reasonable and feasible alternative to allocate allowable CO_2 emissions by countries or groups of countries.

The approach in [35] focuses on limiting the global cumulative CO_2 over the period 2000 and 2050, with the probability of exceeding the threshold of 2°C above pre-industrial levels [55], considering the global carbon budget as Estate to be divided among parties who ask for the maximum emission rights. Then, an efficient allocation model is proposed taking into account different risk scenarios.

A proposed methodology [25] analyzes the application of a large group of theoretical solutions while respecting the criteria of equity and stability. In this context, the empirical analysis includes various global CO_2 allocations from [55] and claims forecasts associated with the RCP (Representative Concentration Pathways) scenarios.This study shows that both the equal allocation and a-minimum constrained solutions meet the equity criterion.

The study [44] of equitable international protocols for GHG emissions reduction formulates axioms for allocation rules based on normative principles, as country population, historical emissions, and habitual emissions. Combinations of these axioms characterize the per capita equality allocation rules with and without historical responsibility. The allocations under these rules contrast with the allocation suggested by the Kyoto Protocol, where equal per capita allocations allow more emissions in developing countries with large populations. Developed countries with high historical emissions are clearly at a loss.

Given the complexity of the situations and the heterogeneity of the agent, bankruptcy methods can neither encompass all aspects of the problem nor offer a fair and an efficient allocation accepted by all. In this sense, Weighted bankruptcy problems are bankruptcy problems where every agent has not only claims, but also a weight. The first working paper [59] in this research area made by an axiomatic study of bankruptcy problem without respecting Equal Treatment of Equals. Every agent's allocation does not depend only from his demand but also from a priority or a weight.

Four weighted bankruptcy methods [13] were investigated to characterize the museum pass problem and define an income distribution from the passes sale. The first part of this work is an axiomatic study of the four new weighted rules; 1) the weighted constrained adjusted proportional rule, 2) the weighted constrained equal awards rule, 3) the weighted constrained equal losses rule and 4) the weighted constrained proportional rule. Then, an implementations of these weighted methods to museum pass problems, in the Municipality

of Genova, is done.

Another problem resolved by weighted bankruptcy methods is transboundary river basins problems [56], due to the increasing of water demands. The goal of this work is to share the available river basins among the riparian countries equitably and reasonably. The latter study introduces a new weighted bankruptcy method applied to a real case, a shared river among Turkey, Syria and Iraq, the Tigris River under two different situations: homogeneous group and heterogeneous group. The idea of this method is that the allocations are distributed according to agents' contribution and their weights.

1.5 Conclusion

Climate change has been a central issue in negotiations among countries for a long time. In spite of the different studies conducted and the various aspects discussed, countries have not yet been able to agree on a common protocol that needs to be respected. In this thesis, we treat this topic from a new perspective. We consider the emission right as a bankruptcy situation where the quantity of emissions is insufficient to meet the continuous demands for emissions. In the next chapter, we focus on bankruptcy situations by presenting the different rules and approaches. Then, we establish a comparative study of CO_2 emissions permits according to four bankruptcy methods in the European Union countries.

Chapter 2

Preliminary and first application

Contents

2.1 Introduction 55

2.2 Bankruptcy rules 56

2.3 Bankruptcy approaches 59

2.3.1 Direct approach 59

2.3.2 Axiomatic approach 60

2.3.3 Game Theoretic approach 64

2.4 Application on CO_2 emission 68

2.4.1 Bankruptcy rules applied to emission allocations 68

2.4.2 CEA allocations 76

2.5 Conclusion 80

2.1 Introduction

Greenhouse gas emissions are increasing and becoming a real problem for society. In view of the rising amount of gas emission and the limited absorption rate of the planet on the one hand[1], and the fact that countries in 2020 have already consumed in 8 months[2] all the

[1] https://www.unep.org/interactive/emissions-gap-report/2020/

[2] https://www.nytimes.com/2020/08/19/climate/earth-overshoot-day.html

resources that ecosystems can produce in a single year, it becomes obvious that we are facing a *claims problem* [25, 35].

In our study, we evaluate CO_2 emissions permits as a bankruptcy problem and we apply some of its methods to resolve this situation. This chapter is organized as follow:

- In the first section, we introduce bankruptcy rules that we use in our empirical study.
- Then, we define different bankruptcy approaches to tackle a problem.
- In the last part of this chapter, we apply the presented rules on CO_2 emission permits problem to define a good allocation among 28-EU countries.

2.2 Bankruptcy rules

Bankruptcy rules aim to divide a good or an estate among a set of agents when the estate amount can not fully satisfy all agent's claims.

Formally, an estate $E \in \mathbb{R}_+$ has to be divided among a set of $N = \{1,2,..,n\}$ agents having a vector $c = (c_i, \ldots, c_n)$ of claims, hence every agent $i \in N$ has his own claim $c_i \geq 0$. We denote as B^N the class of all bankruptcy situations with N as the set of agents. A standard *bankruptcy problem* [62, 83] or a *claims problem* is defined by a tuple (N,E,c) (or, simply, (E,c) if the set N is already clearly identified), where $N = \{1,2,..,n\}$ is a set of agents, and the estate $E \in \mathbb{R}_{>0}$ is such that $0 < E < \sum_{i \in N} c_i$, with $c \in \mathbb{R}^N_{\geq 0}$ as vector of claims. So, the problem is to divide E among the agents of N, each agent $i \in N$ having her/his own claim $c_i \geq 0$. This type of problems has to satisfy two trivial but required properties:

- $E > 0$: The estate must be positive and not null.
- $E < \sum_{i=1}^{n} c_i$: This is the basic condition of bankruptcy situation and means that all agents claims will not be satisfied.

An **awards vector**, also called an *allocations vector*, is a solution vector $x = (x_i, \ldots, x_n) \in \mathbb{R}^N$, given to each agent $i \in N$ an amount x_i respecting:

- Rationality: $x_i \geq 0$;
- Claim boundedness: $x_i \leq c_i$;
- Efficiency: $\sum_{i=1}^{n} x_i = E$.

In this sense, each agent receives a non-negative allocation's quantity smaller or equal to his own claim. And the estate must totally be divided among agents.

An **allocation rule**, or a *rule*, for bankruptcy situations is a map $\phi : B^N \to \mathbb{R}_+^N$ that induces for every bankruptcy situation in B^N an awards vector $x \in \mathbb{R}_+^N$ such that $\sum_{i=1}^{n} x_i = E$.

In bankruptcy situations, several rules exist to divide the estate according to specific criteria and depending on the application's context. In this manuscript, we present only four rules [35]; the proportional, the constrained equal awards, the constrained equal losses, and the Talmud rules. Other rules, like Concede-and-Divide, Random Arrival Rule, Constrained Egalitarian Rule, are discussed in [83].

The most common method, the **Proportional rule,** ***(P)*** [80] provides awards that are proportional to claims. Each agent has a ratio of the estate proportional to his claim. Formally, each agent i receives an allocation λc_i.

$$P_i(E, c) = \lambda \times c_i \tag{2.1}$$

where

$$\lambda \in \mathbb{R}_+$$

and

$$\sum_{i=1}^{n} \lambda c_i = E \tag{2.2}$$

The Constrained Equal Awards rule, ***(CEA)*** [21, 52] offers equal shares of the estate to all agents, provided that allocations sum does not exceed the total amount of the estate and that each agent i may not have more than his claim.

$$CEA_i(E,c) = min\{\lambda, c_i\} \tag{2.3}$$

where

$$\lambda \in \mathbb{R}_+$$

and

$$\sum_{i=1}^{n} min\{\lambda, c_i\} = E \tag{2.4}$$

This method can be defined as an iterative procedure. In the first iteration, all agents receive the same amount of estate, which must be smaller or equal than the smallest claim among the agents. Then, the agents with the smallest claim leave the game. A new iteration starts by equally allocating the remaining part of the estate, if any, among the remaining agents and paying attention to allocate to each agent no more than the second smallest claim. Then, the agents claiming the second smallest claim leave the game, and the procedure is repeated among the remaining agents until there is no more estate to be shared.

The Constrained Equal Losses rule, ***(CEL)*** is a symmetric algorithm to previous rule. The CEL [52] chooses the awards vector at which all agents incur equal losses subject to no one receiving a negative amount.

$$CEL_i(E,c) = max\{c_i - \lambda, 0\} \tag{2.5}$$

where

$$\lambda \in \mathbb{R}_+$$

and

$$\sum_{i=1}^{n} max\{c_i - \lambda, 0\} = E \tag{2.6}$$

Here, the estate and the loss is allocated equally to all agents as long as their collective loss is not equal to the smallest claim. Then, the agent who has the smallest claim receives 0 and leaves the game. As the estate is not finished and the decrease proceeds, losses sustain for the remaining players until their collective loss is equal to the second smallest claim. The second smallest claim's agent withdraws and so on.

The Talmud rule, *(T)* [2] combines the two last rules, the CEA rule and the CEL rule. It uses these two rules and has as switchpoint the half of claims sum. In other words, if the estate is less than the half of claims sum, it acts as CEA rule with $c = \frac{c}{2}$. Otherwise, every agent i receives his claim's half and the CEL rule is applied with the estate remain. Formally: *If* $E \leq \frac{\sum_{i=1}^{n} c_i}{2}$

$$T_i(E,c) = CEA_i(E, \frac{c}{2}) \tag{2.7}$$

Otherwise *If* $E \geq \frac{\sum_{i=1}^{n} c_i}{2}$

$$T_i(E,c) = \frac{c_i}{2} + CEL_i(E - \sum_{i=1}^{n} \frac{c_i}{2}, \frac{c}{2}) \tag{2.8}$$

2.3 Bankruptcy approaches

In addition to bankruptcy rules, we can distinguish three main approaches that can be adopted when examining any problem area.

2.3.1 Direct approach

The intuitive approach would be to use allocation rules. A rule may be used directly, or it may be a formal representation of ideas about how to deal with the situation being studied. It can be expressed as a formula, a geometric construction, a system of equations, or an algorithm. In order to evaluate the relevance of a rule, it is often tested with examples. It is usually suggested to resource allocation situations that require particular resolutions, and we examine whether the recommendations made by the rule correspond to these expectations.

A crucial challenge in implementing this approach is to specify a set of examples that are representative of the range of situations that may be encountered. It is often critical to identify a set of examples that satisfy a single desirable recommendation for each agent,

and where these set of examples constitute a powerful test set to evaluate the rules in their entirety and to differentiate between them.

2.3.2 Axiomatic approach

This approach is based on the properties of rules, formulated as axioms. An axiom is a mathematical formula that respects the way a rule should process a certain class of situations.

An axiomatic study examines properties, logical relationships, and implications in various combinations. The results may imply either incompatible properties and thus an impossibility theorem or a family of compatible properties that can describe a family of rules and thus an axiomatic characterization of the rule.

On the other hand, a characterization theorem can determine that a particular family of rules satisfies a given list of properties. A central point in the axiomatic approach is to determine the border separating those that are compatible and those that are not. If they are compatible, it is necessary to define a complete description of the rules, or families of rules, that satisfy them.

A useful characterization generates axioms that express conceptually distinct ideas, in order to emphasize the idea of the characteristic of rules. And no axiom should be entailed by the conjunction of the others, the characteristic axioms should be logically independent.

We can organize axioms on categories [83], for example, we can define **universal** axioms such as "equal treatment" (if two agents have the same claim, they must receive the same allocation) and "efficiency" (all the good has to be allocated). However, other axioms are inspired by the particular characterization of the analyzed situation, these axioms are qualified as **model-specific** axioms.

We can also distinguish between **punctual** and **relational** axioms. The first category applies to each problem in the domain of a rule separately, "point by point", such as balance and equal treatment. The second category applies to pairs, triples... of problems that are

connected in some specific way i.e. the increase of the good to share does not imply claims decrease.

Axioms from the literature

In the literature on the axiomatic approach to bankruptcy [25], axioms are in general divided into two specific categories: **minimum requirements** and **additional principles**.

The minimum requirements are shared by several rules and guarantee no discrimination between agents. They are considered natural and basic fairness requirements. It concerns **anonymity**, **equal treatment**, **order preservation** and **resource monotonicity**.

Concerning additional principles, some axioms are specific to a particular rules like **super-modularity**, **composition down** and **composition up**.

In this thesis, we present a selection of minimum requirements and additional principles. Then we characterize each one of the presented rules with the corresponding axioms in Table 2.1. More definitions of bankruptcy axioms are explained in the survey [82].

Anonymity [35]

A rule ψ satisfies anonymity if only claims matter and not identities.

Formally, let Π^N be the class of bijections from N into itself : for each (E,c), each $\pi \in \Pi^N$ and each $i \in N$, $\psi_{\pi(i)}(E,c') = \psi_i(E,c)$, where $c' \equiv (c_{\pi(j)})_{j\in N}$

Equal treatment [35]

A rule ψ satisfies equal treatment, if two agents or more having the same claims, have to receive always the same allocation.

Formally, $\forall i, j \in N$such that $c_i = c_j$ it implies $\psi_i(E,c) = \psi_j(E,c)$.

Order preservation **[2,83]**

This axiom assumes that if agent j has a larger claim than agent i:

- agent i should be allocated at most as much as agent j : **order preservation in awards** [83].

  Formally, for each (E, c) and each pair$(i, j) \subseteq N$, if $c_i \leq c_j$, then $\psi_i(E,c) \leq \psi_j(E,c)$. And, $c_i - \psi_i(E,c) \leq c_j - \psi_j(E,c)$

- And the loss of agent i should be at most as large as the loss of agent j: **order preservation in losses** [2].

Formally, for each (E, c) and each $i, j \in N$, if $c_i \geq c_j$, then $\psi_i(E,c) \geq \psi_j(E,c)$. And, $c_i - \psi_i(E,c) \geq c_j - \psi_j(E,c)$

Resource monotonicity **[83]**

A rule ψ satisfies resource monotonicity, if the estate increases from E to E', each agent should receive at least as much as he received with the previous situation.

Formally, let $E' > E > 0$ and if $\sum_{i \in N} c_i \geq E'$ then $\forall i \in N$, $\psi_i(E', c) \geq \psi_i(E, c)$.

Consistency **[2,91]**

A rule ψ satisfies consistency if after solving a problem(E,c,N), a group of a subset $S \subset N$ reassess the allocation of what they received, they obtain the same initial allocation.

Formally, $\forall S \subset N$ and $\forall i \in S$, it implies $\psi_i(E,c,N) = \psi_i(\sum_{i \in S} \psi_i(E,c,N), c_S, S)$.

Independence of claim truncation **[21]**

A rule ψ satisfies independence of claim truncation if the allocation vector is not affected by truncating the claim. If agents claim an amount over of available resources, then their claim has to be condensed.

Formally: Let $C_i^T = min\{E, c_i\}$ $\forall_i \in N, \psi_i(E,c) = \psi_i(E, c_i^T)$.

Super-modularity **[22]**

Super-modularity requires that agents with larger claims face a larger reduction in allocation.

Formally, for each (E,c), all $E' \in \mathbb{R}_+$ and each $i,j \in N$ such that $c > E' > E$ and $c_i \geq c_j$, then $\psi_i(E',c) - \psi_i(E,c) \geq \psi_j(E',c) - \psi_j(E,c)$.

Composition down **[58,91]**

Composition down stipulates that if the estate value decreases, agents allocations have to be reevaluated. There are two options giving the same allocations: the first solution is to cancel the initial allocation and to use the rule to obtain new allocations according to the new value of the estate. The second is to use the initially calculated allocations as claims with the newly revised value of the estate and apply the allocation rule.

Formally, for each (E,c), and each $E' < E$, we have $\psi(E',c) = \psi(E', \psi(E,c))$

Composition up **[58,91]**

Composition up provides that the estate increase, agents allocations have to be reevaluated. This axiom specifies two methods that generate the same allocations. The first method is to cancel the previous allocations and to divide the new estate among agents. The second option is to offer agents their initial allocations, then adjust their claims based on what they received and reapply the allocation rule with the new value of the estate.

Formally, for each (E,c) and each $E' > E$ such that $\sum_{i \in N} c_i \geq E'$>E, $\psi(E',c) = \psi(E,c) + \psi(E' - E, c - (\psi(E,c))$

Self-duality **[2]** Claim problem can be seen from two perspectives: dividing what is available or dividing what will be missed, this axiom ensures that both approaches generate the same allocations.

Formally, for each (E,c), and each $i \in N$, $\psi_i(E,c) = c_i - \psi(\sum_{i=1}^{n} c_i - E, c)$

Minimal rights first **[20]** This axiom secures a minimum allocation right for each agent.

Formally, for each (E,c), the vector of minimal rights is $m(E,c) = (m_i(E,c))_{i \in N}$ where $m_i(E,c) = max\{0, E - \sum_{N/i} c_j\}$. Then, $\psi(E,c) = m(E,c) + \psi(E - \sum_{i \in N} m_i(E,c), c - m(E,c))$

According to different studies [35, 42, 81], some rules share the same axioms while others are characterized by specific axioms, as shown in Table 2.1. All the rules respect the

Principles	P	CEA	CEL	T
Anonymity	Yes	Yes	Yes	Yes
Consistency	Yes	Yes	Yes	Yes
Composition down	Yes	Yes	Yes	No
Composition up	Yes	Yes	Yes	No
Equal treatment	Yes	Yes	Yes	Yes
Minimal right first	No	No	Yes	Yes
Order preservation	Yes	Yes	Yes	Yes
Resource monotonicity	Yes	Yes	Yes	Yes
Super-modularity	Yes	Yes	Yes	Yes
Self-duality	Yes	No	No	Yes

Table 2.1 – The correspondence among rules and axioms

minimum requirement axioms such as anonymity, equal treatment, consistency. However, the Talmud rule does not respect the composition axioms, the Proportional rule does not satisfy independence of claim truncation, and minimal right first axioms, CEA and CEL do not respect self-duality axiom.

2.3.3 Game Theoretic approach

This approach models the bankruptcy situation as a game, i.e. a situation with agents having conflicting interests, and then applies a solution concept from game theory. Various game theory concepts and techniques have been proposed to model bankruptcy problems, and the literature examines both cooperative and non-cooperative games for solving claims problems.

Considering cooperative game, we can distinguish two important classes associated to claims problems: *bargaining games* [23] and *coalitional game* [2, 62].

If the allocation generated by a given division rule in the claim problems matches with an allocation given by the bargaining solution, we assume that the rule matches the solution. By studying bargaining games [23,81,83], there are some correspondences with allocations rules:

- The constrained equal awards rule and the Nash bargaining solution [23];
- The constrained equal awards rule and the lexicographic egalitarian solution [81];

- The proportional rule and the weighted Nash solution with weights equal to claims [23];
- The proportional rule operated from truncated claims and Kalai-Smorodinsky solution [23];
- The constrained equal losses rule operated from truncated claims and the extended equal losses bargaining solution [81];
- For $| N |= 2$. Concede-and-divide and the Perles-Maschler solution [83].

Several works in the literature propose different conflicts model as games such as cooperative game [2, 23, 62] and strategic game [34, 62].

If the allocation generated by a given division rule in the claim problems matches with an output given by the coalitional game, we assume that the rule matches the solution. By studying coalitional games [2, 20, 62, 83], there are some correspondences with allocations rules:

- the Talmud rule corresponds to the Kernel and the prenucleolus [2];
- the random arrival corresponds to Shapley value [62] ;
- the adjusted proportional rule corresponds to the τ value [20];
- The constrained equal awards corresponds to the Dutta-Rey solution [26];

On the other hand, several strategic games, Game of stakes, Game of Rules, and Sequential Game of Offers are considered as the claims problem and the rules generate several allocation vectors that can be associated with non-cooperative equilibrium outcomes of the games.

Cooperative approach: formal description

In this section, we present the basic notions of game theory and cooperative games in particular. These notions allowed us to better develop the solutions proposed in this thesis.

In cooperative game theory, players can commit to behaving in an optimal manner. In this approach, it is assumed that players can reach binding agreements. The different groups of players are called **coalitions**.

A coalition is a not empty subset of players $S \subseteq N, S \neq (\emptyset)$. A coalition S can be a singleton if it includes only one player $i \in N$, $S = \{i\}$ and the grand coalition includes all players ($S = N$). More general, a coalition can be composed by subsets of N.

Let be v : $2^N \rightarrow \mathbb{R}$ a characteristic function which associates with each nonempty coalition $S \subseteq N$ a value $v(S) \in \mathbb{R}$, and such that $v(\emptyset) = 0$.

A game (N, v) is *monotonic* if $\forall S, T \subseteq N$ and $S \subseteq T$

$$v(S) \leq v(T) \tag{2.9}$$

A game (N, v) is *superadditive* if $\forall S, T \subseteq N$ and $S \cap T = \emptyset$

$$v(S \cup T) \geq v(S) + v(T) \tag{2.10}$$

A game (N, v) is *essential* if each player obtains better utility function on joining the grand coalition than playing alone:

$$v(N) \geq \sum_{i \in N} v(\{i\}) \tag{2.11}$$

A game (N, v) is *Symmetric* if the utility function of any coalition is dependent on its size. Considering that $card(S) = s$, then there is a function f such that :

$$\forall S \subset N, v(S) = f(s) \tag{2.12}$$

A game (N, v) is *positive* if the utility function of any coalition is positive or zero.

$$\forall S \subset N; v(S) \geq 0 \tag{2.13}$$

A game (N, v) is *convex* if $\forall S, T \subseteq N$

$$v(S \cup T) + v(S \cap T) \geq v(S) + v(T) \tag{2.14}$$

An *imputation* for a game v is a vector $x \in \mathbb{R}^N$ that satisfies, for all $i \in N$:

$$\sum_{i \in N} x_i = v(N) \tag{2.15}$$

and

$$x_i \geq v(\{i\}) \tag{2.16}$$

A **cooperative game with transferable utility** (a TU game) [4] is defined as a finite set of players N and a function $v : 2^N \leftarrow \mathbb{R}, v(\emptyset) = 0$, and defines for each coalition of players $S \in 2^N$ the maximum transferable utility wealth $v(S)$ that they could share among themselves if they cooperate together.

A **simple game** is a pair (N, v) where N is a set of agents, or *voters*, and v is the *characteristic function* of the game that associates to each *coalition* $S \subseteq N$ a value $v(S) \in \{0, 1\}$ such that $v(N) = 1$ and $v(\emptyset) = 0$. The standard interpretation of a simple game is that $v(S) = 1$ means that coalition S is winning, according to some voting rule, while $v(S) = 0$ means that S is losing, according to the same voting rule

The **Shapley and Shubik power index** [75] is defined as the map ϕ associating to any simple game (N, v) a vector of numbers representing the P-power of voters [28], that is interpreted as the voters expected shares of a fixed prize for winning the elections, and is computed as the expected marginal contribution of each voter $i \in N$ over all possible permutation of voters, i.e. $\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!}(v(S \cup \{i\}) - v(S))$, where s is the cardinality of coalition S.

The Core's notion warns against coalition deviations. Even if the distribution is pareto optimal, nothing prevents a players subgroup to deviate from the grand coalition in order to have more profit. The *core* of a cooperative TU game (N, v) is a set of allocations which are efficient and satisfy group rationality and individual rationality. In cooperative game,

the core of v is defined $\forall S \subset N$

$$C(v) = \{x \in \mathbb{R}^N : \sum_{i \in N} x_i = v(N), \sum_{i \in S} x_i \geq v(S)\}. \tag{2.17}$$

2.4 Application on CO_2 emission

Since all countries emit CO_2, it is obvious to supervise their allocations and to define a common emissions permits policy. In this section, we study CO_2 emissions allocation problem as a claim problem. The estate is defined as the amount of CO_2 emissions permitted according to Kyoto Protocol, and the set of agents $N = \{1, \ldots, 28\}$ are European Union (EU) countries and each country has its own CO_2 emission expectation. Though, Kyoto Protocol limited excessive emissions by reducing the amount of CO_2 permitted by 22% the total emissions of 1990. In this sense, each country has to plan an emission policy respecting the imposed threshold CO_2 permission. In the following, we apply the different bankruptcy rules presented above to define the best method that is appropriate for the CO_2 emission permit problem.

2.4.1 Bankruptcy rules applied to emission allocations

We study the EU countries CO_2 emission permits over 5 years, from 2010 to 2014, by applying the Proportional, the CEA, the CEL, and the Talmud rules. Each year, we consider the country real emission as a claim and we suppose that each country expects to obtain its claim (emission) with one of the allocation methods as long as the defined threshold is not exceeded. We summarize the allocations generated by these methods in Table 2.2 for 2010, Table 2.3 for 2011, Table 2.4 for 2012, Table 2.5 for 2013 and Table 2.6 for 2014.

We may describe a country as satisfied if the allocation generated by one of the used methods is equal to its demand.

According to countries claims and allocations generated by bankruptcy rules, in order to facilitate the analysis, we proceed in splitting countries into three groups. The partition

Country	Claims_{2010}	Proportion	CEA	CEL	Talmud
Malta	2,560	1,680	2,560	0,000	1,280
Cyprus	7,708	5,059	7,708	0,000	3,854
Latvia	8,075	5,299	8,075	0,000	4,038
Luxembourg	10,968	7,198	10,968	0,000	5,484
Lithuania	13,469	8,839	13,469	0,000	6,735
Slovenia	15,335	10,064	15,335	0,000	7,668
Estonia	18,108	11,884	18,108	0,000	9,054
Croatia	20,172	13,239	20,172	0,000	10,086
Slovakia	36,241	23,784	36,241	0,000	18,121
Ireland	40,055	26,287	40,055	0,000	20,028
Bulgaria	44,114	28,951	44,114	0,000	22,057
Denmark	46,641	30,610	46,641	0,000	23,321
Portugal	48,137	31,591	48,137	0,000	24,069
Hungary	50,223	32,960	50,223	0,000	25,112
Sweden	52,024	34,142	52,024	0,000	26,012
Finland	62,082	40,743	62,082	0,000	31,041
Austria	67,502	44,300	67,502	0,901	33,751
Romania	79,413	52,117	79,413	12,812	39,707
Greece	83,857	55,034	83,857	17,256	41,929
Belgium	110,824	72,732	110,824	44,223	55,412
Czech Republic	111,579	73,227	111,579	44,978	55,790
Netherlands	183,053	120,134	183,053	116,452	91,527
Spain	270,911	177,794	220,420	204,310	151,069
Poland	316,264	207,558	220,420	249,663	196,422
France	353,033	231,689	220,420	286,432	233,191
Italy	405,361	266,031	220,420	338,760	285,519
UK.	493,208	323,683	220,420	426,607	373,366
Germany	758,860	498,026	220,420	692,259	639,018

Table 2.2 – Allocations CO_2 emissions in 2010 versus bankruptcy allocations

between the three groups is determined from both CEL column and CEA column of allocation tables (Table 2.2, Table 2.3, Table 2.4, Table 2.5 and Table 2.6). Countries not receiving the right to emit according to CEL rule are classified in the first group. We could notice that this group is composed by countries with low claims like Malta, Latvia, Cyprus, Luxembourg, Lithuania, Slovenia, Estonia, Croatia, Slovakia, Ireland, Denmark, Portugal, Hungary, Bulgaria, and Sweden. Remaining countries are splited into group 2 or group 3 according to the CEA column. If a country receives its full claim, it is considered as a member of the second group. The second group is composed by Finland, Austria, Greece, Romania, Belgium, Cezch Republic, and Netherlands, these countries are characterized by medium claims. Countries with large claims like Spain, Poland, France, Italy, UK, and

Country	Claims_{2011}	Proportion	CEA	CEL	Talmud
Malta	2,54	1,73	2,54	0,00	1,21
Latvia	7,29	4,98	7,29	0,00	3,65
Cyprus	7,43	5,07	7,43	0,00	3,71
Luxembourg	10,94	7,46	10,94	0,00	5,47
Lithuania	13,79	9,41	13,79	0,00	6,89
Slovenia	15,09	10,30	15,09	0,00	7,55
Estonia	18,61	12,70	18,61	0,00	9,30
Croatia	19,81	13,52	19,81	0,00	9,90
Slovakia	34,52	23,56	34,53	0,00	17,26
Ireland	35,63	24,31	35,63	0,00	17,82
Denmark	40,65	27,73	40,65	0,00	20,32
Portugal	47,62	32,50	47,62	0,00	23,81
Hungary	47,84	32,65	47,84	0,00	23,91
Bulgaria	49,35	33,67	49,35	0,00	24,67
Sweden	51,73	35,30	51,73	0,00	25,87
Finland	56,82	38,77	56,82	0,62	28,41
Austria	65,02	44,37	65,02	8,82	32,51
Greece	79,84	54,48	79,84	23,64	39,92
Romania	84,88	57,92	84,88	28,68	42,44
Belgium	99,94	68,20	99,94	43,74	49,97
Czech Republic	106,91	72,95	106,91	50,71	53,45
Netherlands	174,17	118,84	174,17	117,97	87,08
Spain	270,55	184,61	227,37	214,35	170,84
Poland	317,00	216,30	227,37	260,80	217,29
France	331,80	226,40	227,37	275,61	232,10
Italy	397,99	271,57	227,37	341,79	298,29
UK	447,83	305,57	227,37	391,63	348,13
Germany	732,50	499,81	227,37	676,30	632,79

Table 2.3 – Allocations CO_2 emissions in 2011 versus bankruptcy allocations

Germany are in the third group. All the third group members receive the same amount.

Notice that over years, countries may switch groups, for example in 2010 and 2011, Finland belongs to the first group, however, in 2012, 2013 and 2014, it is considered as a country in the second group. Before 2013, Slovakia, Ireland, Denmark, Portugal, Hungary, Bulgaria, Sweden are part of the first group with low claims. Since 2013, these countries are in the second group.

By analyzing the allocations tables (Table 2.2, Table 2.3, Table 2.4, Table 2.5, Table 2.6), we can state that :

Country	$Claims_{2012}$	Proportion	CEA	CEL	Talmud
Malta	2,681	1,868	2,681	0,000	1,341
Cyprus	6,920	4,921	6,920	0,000	3,532
Latvia	7,063	4,822	7,063	0,000	3,460
Luxembourg	10,664	7,430	10,664	0,000	5,332
Lithuania	13,832	9,638	13,832	0,000	6,916
Slovenia	14,782	10,300	14,782	0,000	7,391
Estonia	17,624	12,280	17,624	0,000	8,812
Croatia	17,994	12,538	17,994	0,000	8,997
Slovakia	32,765	22,830	32,765	0,000	16,383
Ireland	35,592	24,800	35,592	0,000	17,796
Denmark	36,428	25,382	36,428	0,000	18,214
Hungary	44,583	32,062	44,583	0,000	23,007
Bulgaria	44,708	31,065	44,708	0,000	22,292
Portugal	46,014	31,152	46,014	0,000	22,354
Sweden	47,048	32,782	47,048	0,000	23,524
Finland	49,134	34,236	49,134	0,000	24,567
Austria	62,273	43,391	62,273	9,634	31,137
Greece	80,043	55,772	80,043	27,404	40,022
Romania	81,723	56,943	81,723	29,084	40,862
Belgium	95,107	66,269	95,107	42,468	47,554
Czech Republic	101,030	70,396	101,030	48,391	50,515
Netherlands	170,310	118,668	170,310	117,671	85,155
Spain	264,779	184,493	236,057	212,140	173,055
Poland	299,931	208,986	236,057	247,292	208,207
France	333,228	232,186	236,057	280,589	241,504
Italy	369,469	257,438	236,057	316,830	277,745
UK	468,573	326,492	236,057	415,934	376,849
Germany	739,861	515,520	236,057	687,222	648,137

Table 2.4 – Allocations CO_2 emissions in 2012 versus bankruptcy allocations

- The *CEA rule* satisfies totally countries in the first and the second groups and propose the same allocations to countries with high demands.

- Under the *CEL rule*, low demand countries are not allowed to emit while medium demand countries get less than half of their demands. However, third group countries obtain almost of their claims.

- The *proportional rule* proposes to all countries an allocation more than half of their claims.

- Considering the *Talmud* rule, the first and the second groups will receive half of their demands. The members of the third group receive allocations, greater than half of

Country	Claims_{2013}	Proportion	CEA	CEL	Talmud
Malta	2,340	1,800	2,340	0,000	1,170
Cyprus	5,948	4,576	5,948	0,000	2,974
Latvia	7,081	5,448	7,081	0,000	3,541
Luxembourg	10,051	7,733	10,051	0,000	5,026
Lithuania	12,640	9,724	12,640	0,000	6,320
Slovenia	14,151	10,887	14,151	0,000	7,076
Croatia	17,550	13,502	17,550	0,000	8,775
Estonia	19,893	15,304	19,893	0,000	9,947
Slovakia	33,091	25,458	33,091	1,076	16,546
Ireland	34,855	26,815	34,855	2,840	17,428
Denmark	38,533	29,645	38,533	6,518	19,267
Bulgaria	39,600	30,466	39,600	7,585	19,800
Hungary	42,141	32,421	42,141	10,126	21,071
Sweden	44,847	34,502	44,847	12,832	22,424
Portugal	45,427	34,949	45,427	13,412	22,714
Poland	302,278	34,949	242,088	13,412	22,714
Finland	47,220	36,328	47,220	15,205	23,610
Austria	62,486	48,073	62,486	30,471	31,243
Greece	69,482	53,455	69,482	37,467	34,741
Romania	70,945	54,581	70,945	38,930	35,473
Belgium	96,970	74,603	96,970	64,955	48,485
Czech Republic	98,675	75,914	98,675	66,660	49,338
Netherlands	173,255	133,291	173,255	141,240	123,207
Spain	237,035	182,360	237,035	205,020	186,987
France	334,097	257,033	242,088	302,082	284,049
Italy	345,318	265,666	242,088	313,303	295,270
UK	458,250	352,548	242,088	426,235	408,202
Germany	757,313	582,628	242,088	725,298	707,265

Table 2.5 – Allocations CO_2 emissions in 2013 versus bankruptcy allocations

their demands and these allocations are better than the allocations generated by the *Proportional rule*.

The study of CO_2 emissions over 5 years makes possible to understand the emission strategies of countries and to generate allocations based on bankruptcy methods. As long as the demands vary from year to year, the allocations change accordingly. In the following, we study the evolution of allocations over time for three countries belonging to each of the three groups defined above: Luxembourg, the Czech Republic and Germany.

First, we analyze a country with low emissions, Luxembourg, and we focus on its emissions from 2010 to 2014. Table 2.7 groups Luxembourg's claims and the allocations

Country	Claims_{2014}	Proportion	CEA	CEL	Talmud
Malta	2,347	1,800	2,347	0,000	1,170
Cyprus	6,062	4,576	6,062	0,000	2,974
Latvia	6,975	5,448	6,975	0,000	3,541
Luxembourg	9,659	7,733	9,659	0,000	5,026
Lithuania	12,838	9,724	12,838	0,000	6,320
Slovenia	12,812	10,887	12,812	0,000	7,076
Estonia	19,519	13,502	19,519	0,000	8,775
Croatia	16,843	15,304	16,843	0,000	9,947
Slovakia	30,678	25,458	30,678	1,076	16,546
Ireland	34,066	26,815	34,066	2,840	17,428
Bulgaria	42,416	29,645	42,416	6,518	19,267
Denmark	33,498	30,466	33,498	7,585	19,800
Portugal	45,053	32,421	45,053	10,126	21,071
Hungary	42,086	34,502	42,086	12,832	22,424
Sweden	43,421	34,949	43,421	13,412	22,714
Finland	47,301	34,949	47,301	13,412	22,714
Austria	58,712	36,328	58,712	15,205	23,610
Romania	70,003	48,073	70,003	30,471	31,243
Greece	67,319	53,455	67,319	37,467	34,741
Belgium	93,351	54,581	93,351	38,930	35,473
Czech Republic	96,475	74,603	96,475	64,955	48,485
Netherlands	167,303	75,914	167,303	66,660	49,338
Spain	233,977	133,291	233,977	141,240	123,207
Poland	285,740	182,360	248,389	205,020	186,987
France	303,276	257,033	248,389	302,082	284,049
Italy	320,411	265,666	248,389	313,303	295,270
UK	419,820	352,548	248,389	426,235	408,202
Germany	719,883	582,628	248,389	725,298	707,265

Table 2.6 – Allocations CO_2 emissions in 2014 versus bankruptcy allocations

Years	Claims	Proportional	CEA	CEL	Talmud
2010	10,968	7,198	10,968	0,000	5,484
2011	10,939	7,464	10,939	0,000	5,472
2012	10,664	7,430	10,664	0,000	5,332
2013	10,051	7,733	10,051	0,000	5,026
2014	9,659	7,733	9,659	0,000	5,026

Table 2.7 – Luxembourg emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014

generated by the several bankruptcy methods as shown in Figure 2.1. We remark that:

- *The CEA rule* gives preferential treatment to this country by giving it all its demand.

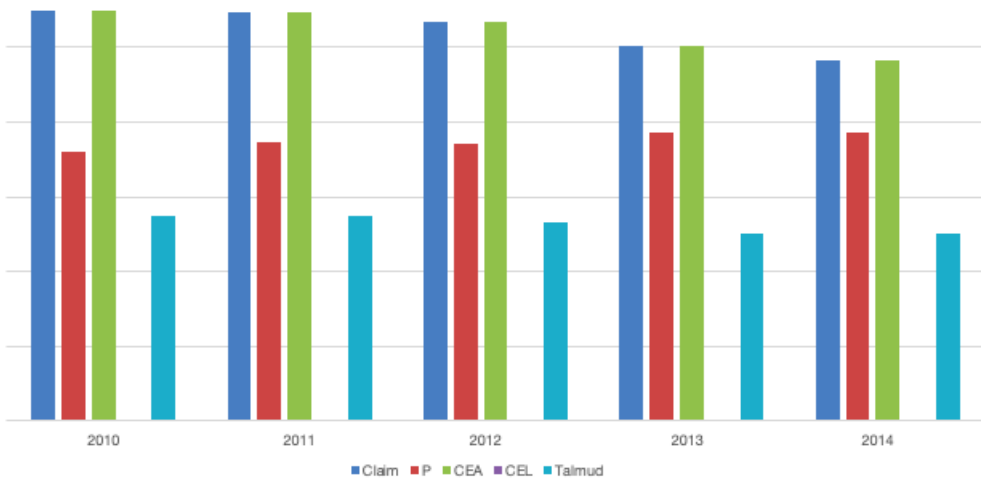

Figure 2.1 – Luxembourg allocations

- Even if a country with low demand adopts a strategy to reduce its emissions, *the CEL rule* always discriminates this country by not providing it the right to emit.

- When Luxembourg decreases its claim, *the Proportional rule* increases its allocation.

Year	Claim	Proportional	CEA	CEL	Talmud
2010	111,579	73,227	111,579	44,978	55,790
2011	106,908	72,948	106,908	50,709	53,454
2012	101,030	70,396	101,030	48,391	50,515
2013	98,675	75,914	98,675	66,660	49,338
2014	96,475	74,603	96,475	64,955	48,485

Table 2.8 – Czech Republic emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014

Table 2.8 collects Czech Republic's emissions from 2010 to 2014, and potential allocations generated by bankruptcy rules, Figure 2.2. This country with medium emission has opted for an emission reduction strategy, and we can notice that:

- Czech Republic can have all of its demand by choosing *the CEA rule*.

- As long as claims decrease,*CEL* allocations increase.

- At the same time, *Talmud* still allocate the claim's half.

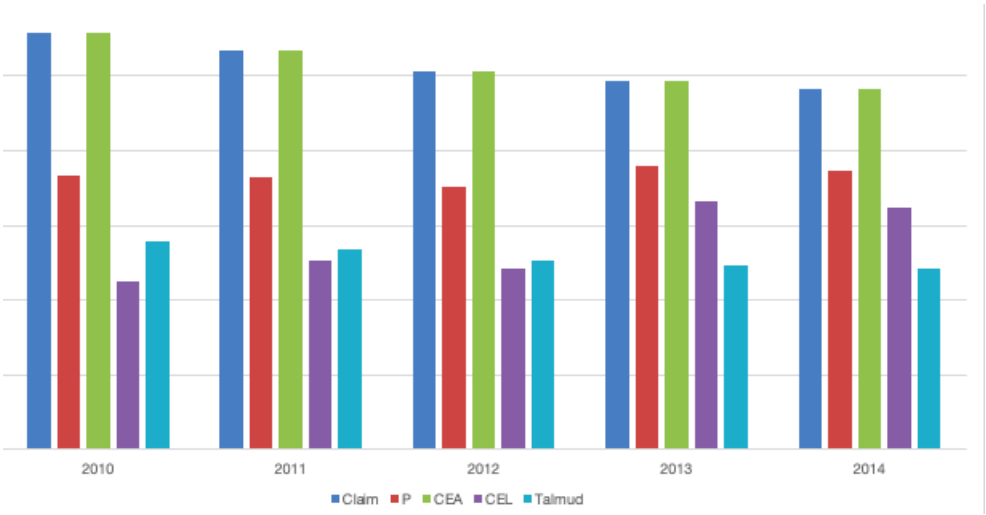

Figure 2.2 – Czech Republic allocations

Year	Claim	Proportional	CEA	CEL	Talmud
2010	758,860	498,026	220,420	692,259	639,018
2011	732,498	499,814	227,373	676,299	632,7942
2012	739,861	515,520	236,057	687,222	648,137
2013	757,313	582,628	242,088	725,298	707,265
2014	719,883	582,628	248,389	715,289	707,265

Table 2.9 – Germany emissions and allocations using the four bankruptcy rules (Proportional, CEA, CEL , Talmud) from 2010 to 2014

In the third group characterized by high emissions countries, we studied Germany's emissions(Table 2.9).

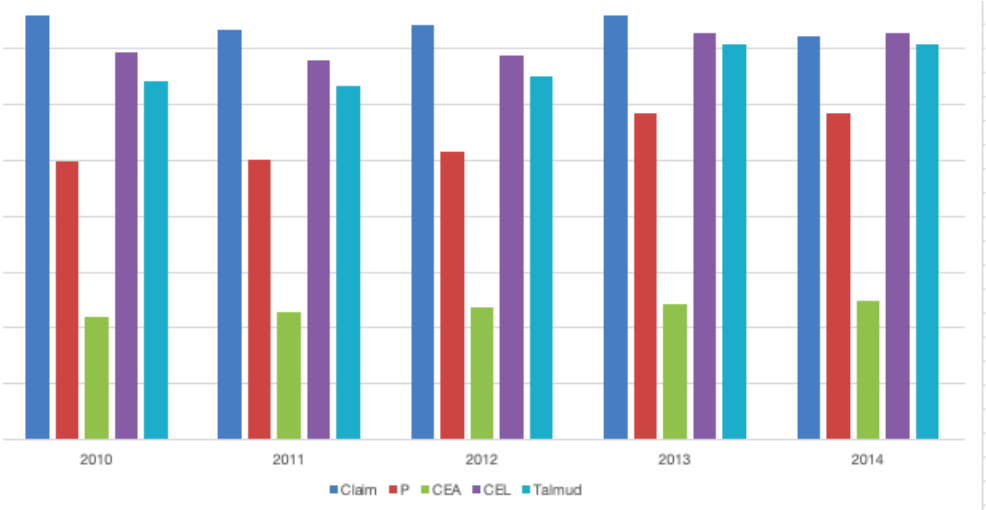

Figure 2.3 – Germany allocations

From a global point of view as seen in Figure 2.3, Germany reduced its carbon emissions over the studied period, as consequence:

- All allocations have been increased.
- The CEL method provides the best allocations vector to Germany.
- CEA rule allocates less than half of Germany's claim.

By studying these different allocation rules, we can conclude that the countries of the first group are allowed to produce half of their demands according to the *Talmud*, while they can produce more than half of their demands according to the *proportional* rule, but are discriminated by the *CEL* rule. Only the *CEA* rule fully satisfies these small demands. The countries in the second group are also favored by the *CEA* rule, which offers them almost all of their requests, however, the *proportional* rule allocates them more than half of their requests, and the *Talmud* gives them the right to issue half of their requests. Whereas the *CEL* allocates them less than half of their demands. The third group is favored by the *CEL*, which proposes allocations close to their demands. The *proportional* and the *Talmud* rules propose allocations that exceed half of the requests, while the *CEA* proposes the same allocation whatever the claim.

Based on these results, we continue our study by focusing on the CEA method for allocating CO_2 emission permits between countries

2.4.2 CEA allocations

In previous CO_2 emissions allocations [41] study, CEA rule has been considered as a stable allocation method for resource conflict problems. To formalize carbon budget [8], CEA is proposed as a fairness solution, in the sens it generates an equal distribution of the cumulative CO_2 emission.

On the other hand, in a study of water resources allocation problem [57], CEA rule has been adopted as *an equitable and reasonable* allocation method.

More recently, [25] argues that CEA has been the most equitable alternative, since it **Lorenz dominates** all other methods. On the other hand, CEL favors large claimants and is therefore judged as the least equitable alternative. From a practical point of view, CEL is difficult to implement because it may allocate zero emission rights to some countries, which is not very plausible.

Therefore, we adopt the CEA method to study of CO_2 emission permits because it satisfies the relevant principles in the theoretical analysis of claims problems [83] such as the principles of resource monotonicity, super-modularity, composition-up, and composition-down. This rule is fair and satisfies the stability principle. Moreover, it can be chosen in a voting process negotiations with a larger number of groups.

Country	$Claim_{2010}$	CEA_{2010}	$Claim_{2011}$	CEA_{2011}	$Claim_{2012}$	CEA_{2012}	$Claim_{2013}$	CEA_{2013}	$Claim_{2014}$	CEA_{2014}
Malta	2,560	2,560	2,541	2,541	2,681	2,681	2,340	2,340	2,347	2,347
Cyprus	7,708	7,708	7,426	7,426	6,920	6,920	5,948	5,948	6,062	6,062
Latvia	8,075	8,075	7,294	7,294	7,063	7,063	7,081	7,081	6,975	6,975
Luxembourg	10,968	10,968	10,939	10,939	10,664	10,664	10,051	10,051	9,659	9,659
Lithuania	13,469	13,469	13,788	13,788	13,832	13,832	12,640	12,640	12,838	12,838
Slovenia	15,335	15,335	15,090	15,090	14,782	14,782	14,151	14,151	12,812	12,812
Estonia	18,108	18,108	18,606	18,606	17,624	17,624	19,893	19,893	19,519	19,519
Croatia	20,172	20,172	19,809	19,809	17,994	17,994	17,550	17,550	16,843	16,843
Slovakia	36,241	36,241	34,525	34,525	32,765	32,765	33,091	33,091	30,678	30,678
Ireland	40,055	40,055	35,632	35,632	35,592	35,592	34,855	34,855	34,066	34,066
Bulgaria	44,114	44,114	49,347	49,347	44,708	44,708	39,600	39,600	42,416	42,416
Denmark	46,641	46,641	40,645	40,645	36,428	36,428	38,533	38,533	33,498	33,498
Portugal	48,137	48,137	47,623	47,623	46,014	46,014	45,427	45,427	45,053	45,053
Hungary	50,223	50,223	47,843	47,843	44,583	44,583	42,141	42,141	42,086	42,086
Sweden	52,024	52,024	51,734	51,734	47,048	47,048	44,847	44,847	43,421	43,421
Finland	62,082	62,082	56,816	56,816	49,134	49,134	47,220	47,220	47,301	47,301
Austria	67,502	67,502	65,020	65,020	62,273	62,273	62,486	62,486	58,712	58,712
Romania	79,413	79,413	84,880	84,880	81,723	81,723	70,945	70,945	70,003	70,003
Greece	83,857	83,857	79,842	79,842	80,043	80,043	69,482	69,482	67,319	67,319
Belgium	110,824	110,824	99,944	99,944	95,107	95,107	96,970	96,970	93,351	93,351
Czech Republic	111,579	111,579	106,908	106,908	101,030	101,030	98,675	98,675	96,475	96,475
Netherlands	183,053	183,053	174,168	174,168	170,310	170,310	173,255	173,255	167,303	167,303
Spain	270,911	220,420	270,548	227,373	264,779	236,057	237,035	237,035	233,977	233,977
Poland	316,264	220,420	316,997	227,373	299,931	236,057	302,278	242,088	285,740	248,389
France	353,033	220,420	331,805	227,373	333,228	236,057	334,097	242,088	303,276	248,389
Italy	405,361	220,420	397,994	227,373	369,469	236,057	345,318	242,088	320,411	248,389
UK	493,208	220,420	447,829	227,373	468,573	236,057	458,250	242,088	419,820	248,389
Germany	758,860	220,420	732,498	227,373	739,861	236,057	757,313	242,088	719,883	248,389

Table 2.10 – The CEA rule for 28-EU

In Table 2.10, we compare the EU claims and allocations generated by *CEA rule* over 5 years (2010-2014).

We notice that countries with small and medium emissions receive all their claims. If a country follows an emission reduction strategy, it receives its claim or obtain a better allocation. Countries with large claims are not totally satisfied and receive the same allocation, despite they have different claims.

According to the CEA rule, emission permit allocations depends on claims variations. Thus, countries following a strategy to reduce emissions, over years, should receive more adequate emission permits with their demands. In other words, a decrease of the emission in year y can generate an allocation increase in year $y+1$.

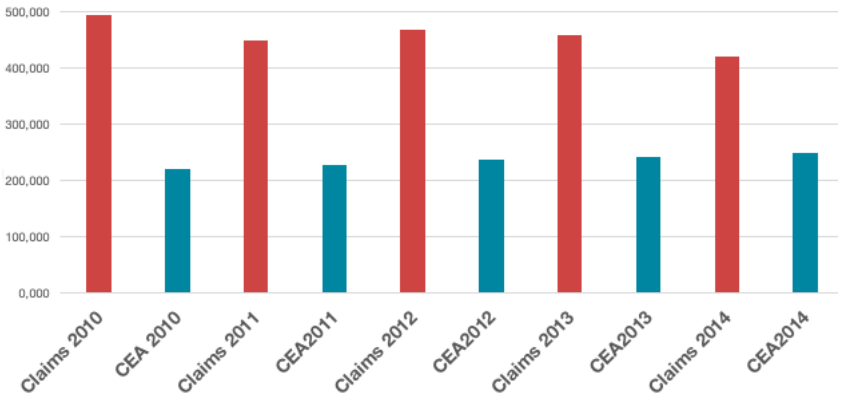

Figure 2.4 – UK Allocations CO_2 emissions permits

This is the case of UK, whose in 2010 claims 493.208Kt of CO_2 and, according to the CEA rule, receives 220.420Kt as emissions permits. However, a few years later (see Fig 2.4), when UK decrease his claims and asks for 419.820Kt, i.e. a lower amount than in 2010, he receives 248.338Kt of CO_2 permits, i.e. greater than in 2010.

Till 2012, Spain demands are higher than 250.000Kt, so it did not receive its full demand. However, when this country follows a decreasing strategy and its demand becomes less than 250.000Kt, it is fully satisfied.

It is worthy to notice that countries with high demands (see Figure 2.6: Spain, Poland, France, Italy UK and Germany, obtain the same quantity permits despite they have different demands. For example, Germany emissions is two times higher than France demands and

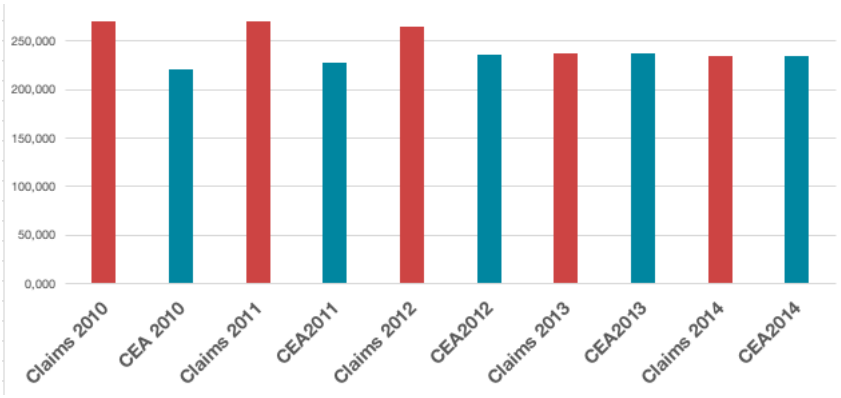

Figure 2.5 – Spain Allocations CO_2 emissions permits

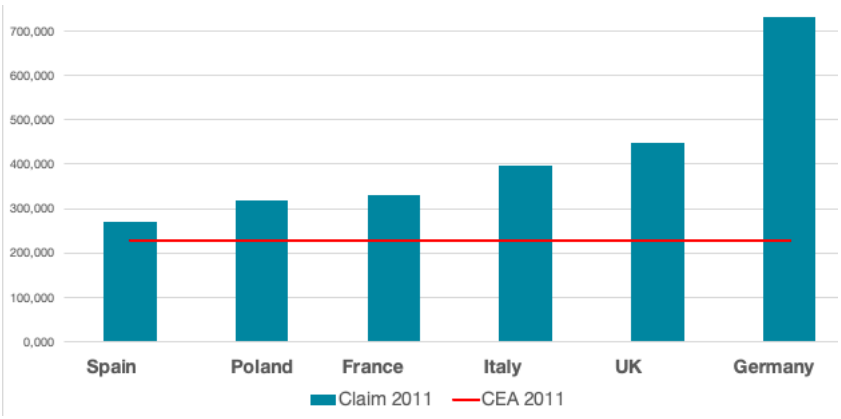

Figure 2.6 – Allocations for high emitters countries

three times higher than Spain.

2.5 Conclusion

In this chapter, we introduce important notions about bankruptcy situation by presenting the fundamental basics and some rules. We define the three approaches of bankruptcy problems as direct approach, axiomatic approach and game theoretic approach. Then, we apply four bankruptcy rules on the CO_2 emission permits problem. We notice that the CEA rule promotes low emission countries by satisfying their demands, and high emission countries share allocations on equal parts. However, the CEL supports these latter by allocating them a large part of the estate. Based on these results and the literature on studies of resource allocation problems, we continued our analysis using the CEA rule.

This emission sharing problem considers only CO_2 emissions claims to share emissions rights, but countries have other important features that should be considered. In the next chapter, we will introduce a new parameter in the bankruptcy situation and study a new situation to allocate CO_2 emissions permits.

Chapter 3

Weighted bankruptcy method for GHG emissions permit

Contents

3.1 Introduction 83

3.2 Weighted rules for bankruptcy situations 84

3.3 Weighted Constrained Equal Awards (WCEA) applied to emissions reduction problem 86

3.4 Surplus and technology transfer 95

3.5 WCEA Algorithmic approach 99

3.6 Conclusion 107

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3.1 Introduction

The establishment of international agreements aimed to reduce greenhouse gas emissions is a current problem [94], due to the fact that greenhouse gases present a big threat to the planet. Negotiation models [90], in particular, have been widely studied with the goal to

find an agreement among all countries for reducing their carbon-emission rate. However, no country can influence the climate change system independently of the others. Each one of these countries would benefit from the reduction of global warming, but no one wants to bear the cost of emission reductions. One of the main issues in this research stream is the problem of finding fair and efficient protocols to allocate the permits for CO_2 emissions.

In this chapter, we are interested in allocating CO_2 emission permits via a new family of bankruptcy rules, the weighted bankruptcy rule, and in particular the weighted constrained equal awards method. In the previous chapter, we use classical methods and consider the EU-28 countries as players and the quantity of CO_2 emitted from 2010 to 2014 as claims. However, in the weighted method, we will add a new parameter, the GDP, and distribute the CO_2 emission permits depending on claims and weights.

3.2 Weighted rules for bankruptcy situations

Given a bankruptcy situation (E,c) where $c \in \mathbb{R}^N_+$ is a vector representing agent's claims and such that $c_i \geq 0$ for all $i \in N$, and $E \in \mathbb{R}$ is the estate to be divided among the agents and such that $0 < E < \sum_{i=1}^{N} c_i$.

In a bankruptcy situation, the allocation process takes into account only the agents claims, however, agents can have rights, needs, or obligations and may be characterized by different features that should be considered. These parameters are defined in our work as weights.

Let a *weighted bankruptcy situation* as the triple (E,c,w) where $w \in \mathbb{R}^N_+$ a weight vector. We denote by WB^N the class of all weighted bankruptcy situations (E,c,w) with $N = \{1,\ldots,n\}$ as a finite set of agents. A solution for weighted bankruptcy situations is then a map $\psi : WB^N \to \mathbb{R}^N_+$.

In this manuscript, we present only four weighted bankruptcy methods and for our application, we will only focus on one method, the Weighted Constrained Equal Awards rule.

The solution called *Weighted* Proportional rule is a weighted version of the Proportional rule. However, due to claim *boundedness* and the estate's size, it is not possible to distribute allocations proportionally to claims multiplied by weights w. Thus, adjustments are required and *WP* rule is defined as follows:

$$WP_i(E,c,w) = min\{\lambda w_i c_i, c_i\} \tag{3.1}$$

where the parameter $\lambda \in \mathbb{R}_+$ such that

$$\sum_{i\in N} min\{\lambda w_i c_i, c_i\} = E \tag{3.2}$$

The *weighted* Constrained Equal Awards rule [13, 59] is a weighted definition of CEA rule. The estate is distributed in proportion to a fixed vector of positive weights, provided that no agent receives more than its claim. If the estate is large enough for some number $k \in \{1..n\}$, the agents with the k smallest $\frac{c}{w}$ ratio are fully satisfied and the others allocations are fixed according to their weights [45, 83]. The formal definition is as follows:

$$WCEA_i(E,c,w) = \min\{c_i, \hat{\lambda} w_i\} \tag{3.3}$$

where the parameter $\hat{\lambda} \in \mathbb{R}$ is such that

$$\sum_{i\in N} \min\{c_i, \hat{\lambda} w_i\} = E \tag{3.4}$$

The weighted version of CEL is dual to *weighted* Constrained Equal Awards rule and is defined as:

$$WCEL_i(E,c,w) = max\{c_i - \frac{\lambda}{w_i}, 0\} \tag{3.5}$$

where $\lambda \in \mathbb{R}_+$ such that achieve a efficiency.

The weighted version of Talmud is a hybrid of *weighted* CEA and *weighted* CEL rules. It recommends for each $(E,c,w) \in \mathbb{R}^N$and each $i \in N$

If

$$\sum_{i=1}^{n} \frac{c_i}{2} \geq E$$

$$WT_i(E,c,w) = min\{\frac{c_i}{2}, w_i\lambda\} \tag{3.6}$$

Otherwise If

$$\sum_{i=1}^{n} \frac{c_i}{2} \leq E$$

$$WT_i(E,c,w) = max\{\frac{c_i}{2}, c_i - \frac{\lambda}{w_i}\} \tag{3.7}$$

and in each case, λ achieve a balance.

Weighted rules have long been studied in different forms: Moulin [59] provides an axiomatic characterization of the weighted version proportional rule, the CEA rule, and the CEL rule. Hokari [81] gives a characterization of the weighted version of the Talmud rule. New axiomatic studies [13] are investigated for four rules: the weighted constrained adjusted proportional rule, the WCEA rule, the WCEL rules, and the weighted constrained proportional rule, and then applied to the museum passage problem. Others [79] define a new version of the WCEA rules with augmented priority and propose a generalized version of the bankruptcy problem with multiple divisible and indivisible resources.

3.3 Weighted Constrained Equal Awards (WCEA) applied to emissions reduction problem

Depending on the field of research, it is necessary to match the adequate method to the studied problem. The P rule is the most commonly used in allocation problems. It divides the carbon budget proportionally to the demand of each country. However, given the criteria of historical responsibility and polluter pays (explained in Table 1.1), some countries (such as the United States) refuse this allocation method and are not ready to engage in discussions with countries with a historical significant emissions [25].

The CEL rule recommends an equal distribution of losses (L = C - E), and since no one can emit a negative amount which implies not allocating emission rights to the

poorest countries, this rule is ethically and economically not applicable to CO_2 allocation problems [35].

The CO_2 emissions in particular and the problems related to climate change have been addressed by the CEA since it generates, on one hand, stable allocations [41] , on other hand, a balance between equity and proportionality [25] by giving a distribution taking into account the demands of each region. In addition, the CEA rule satisfies a set of principles, such as equal treatment of equals, anonymity, monotonicity of resources, composition, and super-modularity (a larger demand suffers a larger reduction) that countries would like to implement in the allocation of the estate. Not only these principles fit naturally in the context of CO_2 emissions permits, but they are also considered as minimum requirements for equity.

For all these reasons, we focus our work on the weighted version of the CEA rule [59] to allocate CO_2 emissions permits among EU countries. We consider historical emissions of CO_2 as claims and the GDP of each country as weights.

In the CO_2 emissions literature [38,70,71,93], GDP has been widely studied as parameter to allocate CO_2 among countries. It may reflect the economic growth of countries and is the most common measure of "wealth production", i.e. it can indicate both the ability to pay and the benefits derived from carbon-based development. The economy of countries is based on energy consumption since the transformation of resources into products and services requires the use of energy [36]. Therefore, it reflects the global economic activity of a country. As a consequence, countries with a low $\frac{CO_2}{GDP}$ ratio are assumed to be more efficient due to lower CO_2 emissions per unit of GDP produced.

In this study, we consider that every country having a quantity of CO_2 produces a defined amount of GDP according to this emission.

A country may obtain neither more than its CO_2 demands nor more than $\lambda \times GDP$, where λ is a coefficient defined to allow the budgetary balance of allocations. This latter should respect the relation among claims and GDP of all countries.

In the improvement of our CO_2 emissions allocations application based on the Kyoto

Protocol requirement, we add the GDP [1] criterion as a new parameter, (in Figure 3.2), and we analyze the way in which the CO_2 emission permits, (see Figure 3.1), would be allocated among the EU-28 countries using the WCEA rule.

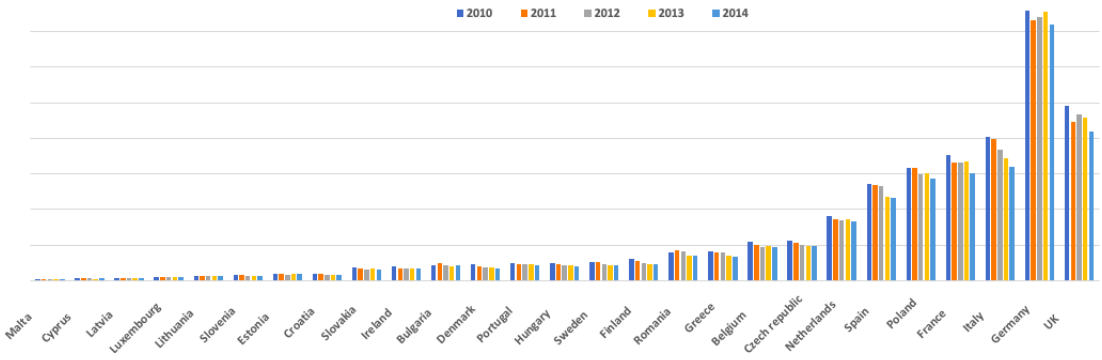

Figure 3.1 – EU countries emissions

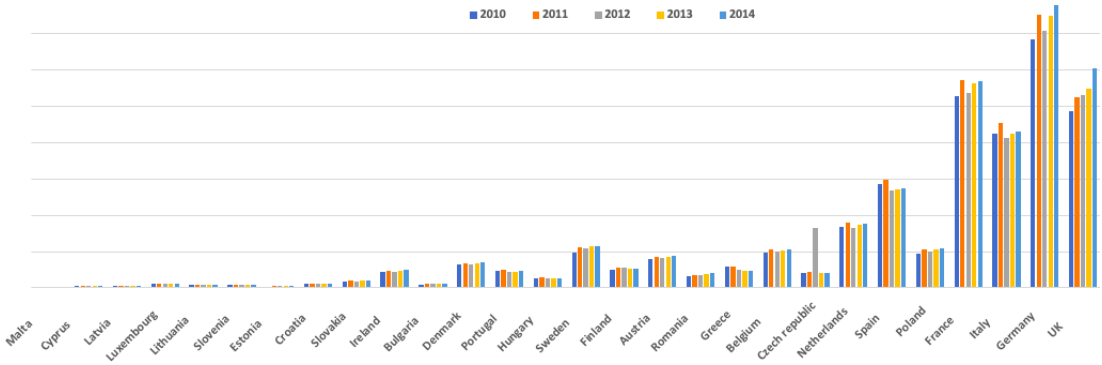

Figure 3.2 – EU countries GDP

To this end, we consider the problem of CO_2 emissions permits for a year y, $y = 2010, \ldots, 2014$, as a weighted bankruptcy situation (E, c^y, w^y), where the EU-28 countries are considered as agents, and 78% of the cumulative amount of CO_2 produced in EU in 1990 (equal [41] to 2434.658 Mt) represents the estate. Each country i has a claim c_i^y which describes the real amount of CO_2 emitted during year y, and a weight w_i^y which describes the GDP quantity of this country in year y, as stated in World Bank Open Data[2].

Note that in a weighted bankruptcy situation, a country i asks for the quantity c_i of CO_2 emission permits in order to achieve a w_i of GDP . Nevertheless, country i may not obtain

[1] https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?end=2014&locations=EU&start=2010
[2] https://data.worldbank.org/indicator/

a proportion of CO_2 emission permits larger than $\hat{\lambda} w_i$, where $\hat{\lambda}$ is the maximum emission intensity per unit of GDP calculated according to the relation (3.3).

Country	$Claim_{2010}$	$WCEA_{2010}$	$Claim_{2011}$	$WCEA_{2011}$	$Claim_{2012}$	$WCEA_{2012}$	$Claim_{2013}$	$WCEA_{2013}$	$Claim_{2014}$	$WCEA_{2014}$
Malta	2,560	1,281	2,54	1,311	2,681	1,353	2,340	2,048	2,347	1,563
Cyprus	7,708	3,745	7,43	3,781	6,920	4,131	5,948	5,948	6,062	4,359
Latvia	8,075	3,481	7,29	3,891	7,063	3,678	7,081	4,863	6,975	3,648
Luxembourg	10,968	7,796	10,94	8,273	10,664	8,309	10,051	10,510	9,659	9,228
Lithuania	13,469	5,439	13,79	5,994	13,832	6,294	12,640	9,372	12,838	6,750
Slovenia	15,335	7,035	15,09	7,072	14,782	6,809	14,151	9,715	12,812	6,943
Estonia	18,108	2,856	18,61	3,195	17,624	3,385	19,893	5,075	19,519	3,250
Croatia	20,172	8,766	19,81	8,600	17,994	8,325	17,550	11,728	16,843	8,018
Slovakia	36,241	13,113	34,52	13,537	32,765	13,722	33,091	19,883	30,678	6,943
Ireland	40,055	32,519	35,63	32,954	35,592	33,134	34,855	34,855	34,066	34,066
Bulgaria	44,114	7,415	49,35	7,916	44,708	7,918	39,600	11,258	42,416	7,893
Denmark	46,641	46,641	40,65	40,645	36,428	36,428	38,533	38,533	33,498	33,498
Portugal	48,137	34,914	47,62	33,764	46,014	31,782	45,427	45,427	45,053	31,947
Hungary	50,223	19,182	47,84	19,410	44,583	18,781	42,141	27,302	42,086	19,494
Sweden	52,024	52,024	51,73	51,734	47,048	47,048	44,847	44,847	43,421	43,421
Finland	62,082	36,306	56,82	37,732	49,134	37,707	47,220	47,220	47,301	37,926
Austria	67,502	57,417	65,02	59,440	62,273	60,140	62,486	62,486	58,712	58,712
Romania	79,413	24,418	84,88	25,419	81,723	25,216	70,945	38,675	70,003	27,754
Greece	83,857	43,860	79,84	39,680	80,043	36,086	69,482	48,430	67,319	32,976
Belgium	110,824	70,846	99,94	72,660	95,107	73,134	96,970	69,482	93,351	73,843
Czech republic	111,579	30,398	106,91	31,428	101,030	30,461	98,675	42,280	96,475	28,912
Netherlands	183,053	122,542	174,17	123,225	170,310	121,763	173,255	98,675	167,303	122,378
Spain	270,911	209,750	270,55	205,165	264,779	196,247	237,035	173,255	233,977	191,561
Poland	316,264	70,227	317,00	72,912	299,931	73,498	302,278	105,847	285,740	75,847
France	353,033	353,033	331,80	331,805	333,228	333,228	334,097	302,278	303,276	303,276
Italy	405,361	311,348	397,99	313,840	369,469	304,476	345,318	334,097	320,411	299,357
UK	493,208	357,662	447,83	361,187	468,573	391,032	458,250	345,318	419,820	419,820
Germany	758,860	500,647	732,50	518,086	739,861	520,573	757,313	485,250	719,883	541,275

Table 3.1 – The CO_2 emissions allocations generated by WCEA rule for the 28-EU countries

Now that we have allocated CO_2 permits with the WCEA, we establish a comparative study (Table 3.2) among the new allocations generated by WCEA rule and those generated by the CEA rule2.10 and presented in the previous chapter.

Unlike the CEA rule, which favors low-demand countries and fully satisfies their demands, while high-demand countries will receive less than their demands, we can notice that the WCEA rule is beneficial for countries with efficient production technologies. Countries having low emission intensity, $\lambda_i = \frac{c_i}{w_i}$, benefits by a better allocation than CEA rule.

Countries claiming high quantities (more than 200.000Mt of CO_2 emissions) will receive less than their claim according to the CEA rule, however the WCEA rule proposes better allocations. The UK, as an example, receives always less than its claims according to CEA rule. In 2012, this country asked to emit 468,573Mt of CO_2, and obtained the right to emit 236,057Mt of CO_2 based on CEA rule, i.e. almost half of the claim. Instead, the weighted CEA allocated it 391,0327Mt of CO_2 emission permits, i.e. more than two-thirds of the claim. Moreover, in 2014, it could have been totally satisfied by the weighted rule, as shown in Figure 3.3.

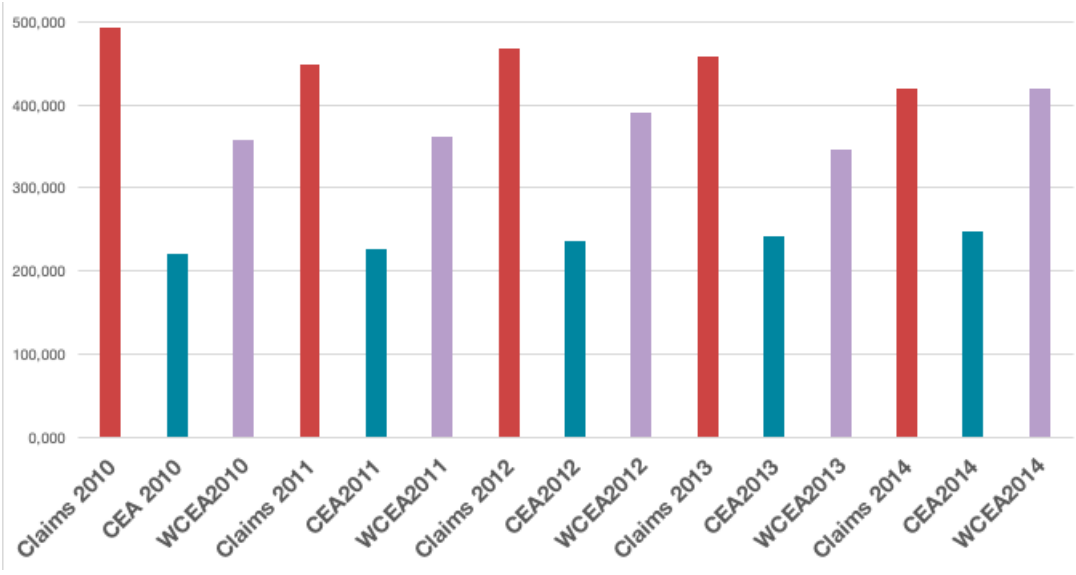

Figure 3.3 – UK allocations CO_2 emissions permits

On the other hand, countries having both high demand and high emission intensity λ, will not find an incentive to adopt the weighted method. Considering the CEA rule, Poland

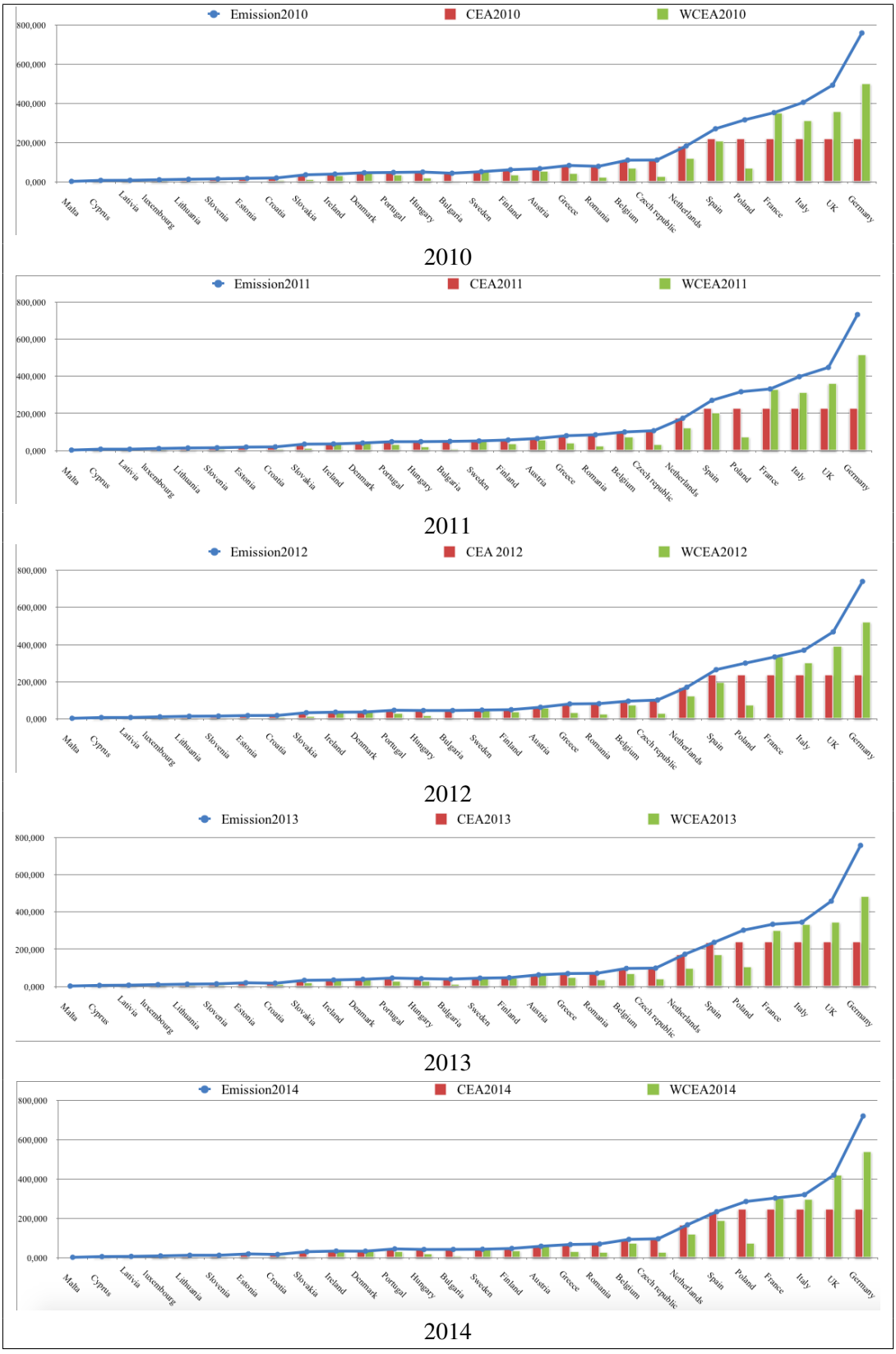

Table 3.2 – A comparative table among WCEA allocations and CEA allocations

receives less than its demand, but this allocation is larger than the allocated quantity by the WCEA rule (see Fig. 3.4).

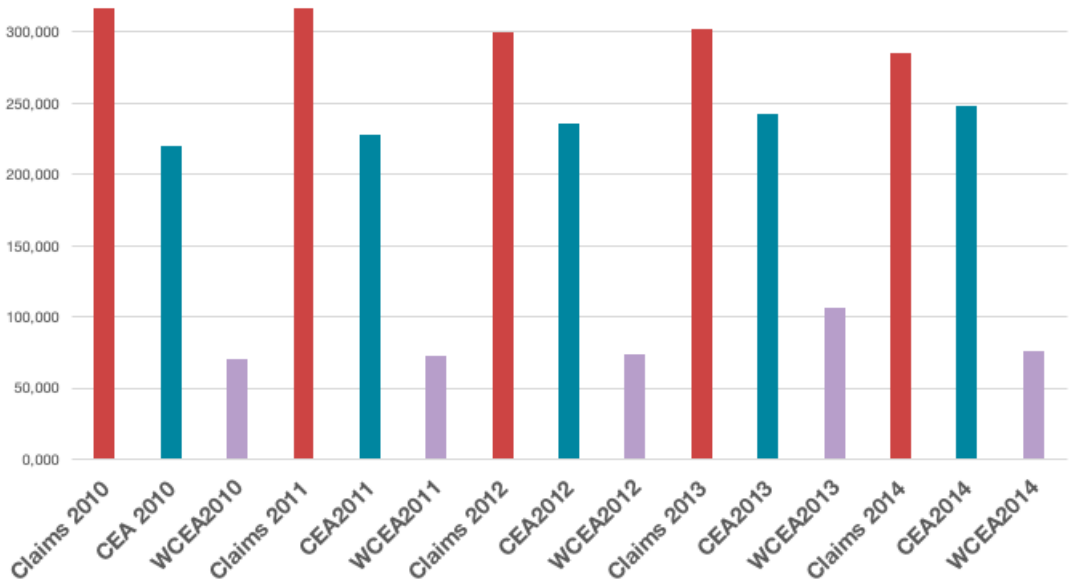

Figure 3.4 – Poland allocations CO_2 emissions permits

Indeed, countries with small demands do not represent a burden in the allocation process and are fully satisfied with the CEA rule. Instead, if they do not adopt a policy to manage emission intensity, they will be penalized with the WCEA rule. This is the case of Estonia, having a small demand, is totally advantaged with the CEA, while the WCEA rule generates an allocation less than the demand's half (Figure 3.5).

Other countries, like Sweden, are able to create a balance among their demands and their energy intensities. This balance generates full allocation according to both rules, the CEA and the WCEA. Figure 3.6) shows Sweden's claims, CEA allocations and WCEA allocations for the four studied years. It highlights two aspects of the used policy, first, we may notice a reduction strategy of emission over the years. Then, for each year the demand is totally satisfied by both methods.

Through this comparative study, we can observe that countries, claiming a low CO_2 allocations permits, have to use it to produce more wealth, in order to obtain more emission rights. On the other hand, the richest countries, with a high GDP, are considered as countries with a good use of CO_2.

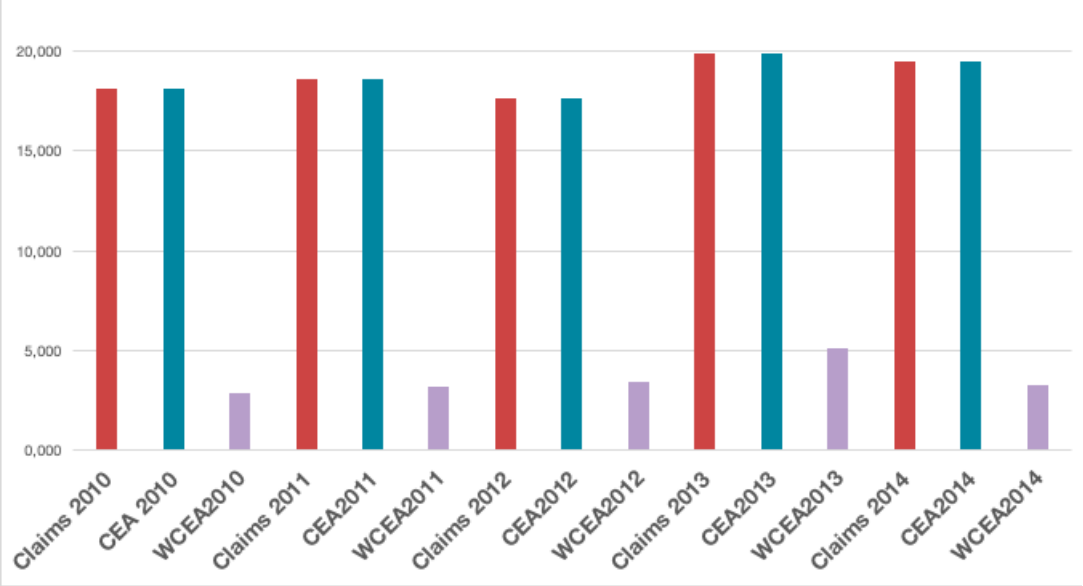

Figure 3.5 – Estonia allocations CO_2 emissions permits

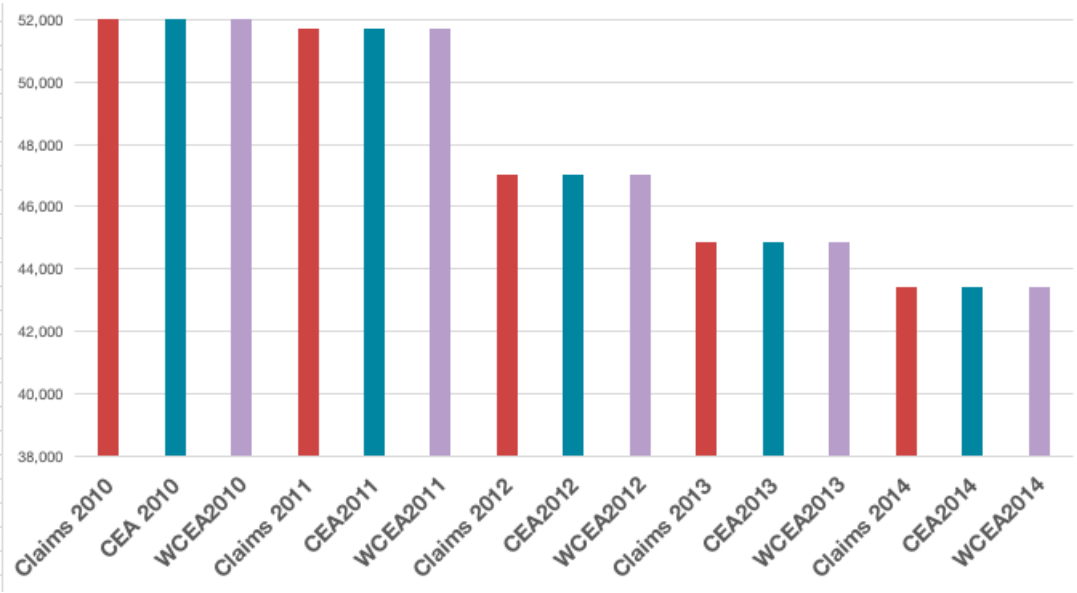

Figure 3.6 – Sweden allocations CO_2 emissions permits

In climate change negotiations, we seek to create a cooperative strategy among countries and encourage them to ratify a common policy. We propose, in the next section, a technology transfer policy from efficient (low-emission) countries to the most polluting (high-emission) countries, which can be balanced by benefit transfers among cooperating countries.

3.4 Surplus and technology transfer

The WCEA allocations reflect the fact that countries with high resources efficiency and high levels of green technology receive their claims. It results from a good ability to emit low amounts of CO_2 per unit of GDP. However, some countries with low demand are not satisfied due to the lack of efficiency in the use of CO_2 in wealth creation and involvement in new technologies.

In global issues in general and climate change negotiations in particular, countries seek to cooperate and agree on a common operational strategy. A successful negotiation results in positive consequences for all participants, while a failure reflects a lack of cooperation and generates serious consequences.

In this section, we introduce a new TU-game with the aim of studying the problem of implementing economic incentives to transfer technology from the most efficient countries, with a low $\frac{c_i}{w_i}$, to the least efficient countries, with a high $\frac{c_i}{w_i}$ ratio. This game can be described as a technology transfer (TT-) game [84] where the profit of each coalition is given by the total profit of the coalition that is obtained from the highest level of technology available to the coalition's members.

Definition 1 *Considering a weighted bankruptcy situation (E,c,w) and a vector of allocations $WCEA(E,c,w) = (x_1,x_2,\ldots,x_n)$.*

For each player $i \in N$, let $\lambda_i = \frac{c_i}{w_i}$ and let for all coalition $S \subseteq N$, $\lambda(S) = \min_{i\in S: x_i - c_i = 0} \lambda_i$ (provided that $\lambda(S) = 0$ if $x_i - c_i \neq 0$ for all $i \in S$).

The corresponding Technology-Transfer (TT-)game is defined as the TU-game $(N,\tilde{v})$ such that for all $S \subseteq N$

$$\tilde{v}(S) = \begin{cases} \sum_{i\in S: c_i - x_i > 0} \left(\frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i} \right) & \text{if } \lambda(S) \neq 0, \\ 0 & \text{otherwise.} \end{cases} \tag{3.8}$$

The profit $\tilde{v}(S)$ of a coalition $S \subset N$ corresponds to the best technology among players in

S. The best emission rate $\lambda(S)$ is defined as the minimum λ_i among players $i \in S$ receiving all their claimed quantities; $\lambda(S) = min_{i \in S: x_i = c_i} \lambda_i$.

Specifically, in coalition S, each player i whose did not receive all his claim $x_i < c_i$ where x_i is the WCEA allocation of CO_2 emission permits, generates a profit rate $\frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i}$ and the sum of these profits corresponds to the worth (profit) of the coalition $\tilde{v}(S)$.

A transfer technological game is considered from countries $i \in N$ having a $\lambda_i \leq \hat{\lambda}$ to countries $j \in N$ having $\lambda_j > \hat{\lambda}$ where $\hat{\lambda}$ is the limit of emission intensity, for all coalition of the game $S \subset C$, each player in S will receive a positive profit rate $\frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i} \geq 0$ if and only if $\lambda(S) \neq 0$.

The *transfer* of technology within a coalition is permitted on only one sens from countries $i \in N$ with $\lambda_i \leq \hat{\lambda}$ to countries $j \in N$ with $\lambda_j > \hat{\lambda}$, where $\hat{\lambda}$ is the boundary of emission intensity introduced in relation (3.3).

This TT-games was introduced in the paper [84] where we state that these type pf games are monotonic and superadditive, however they are not convex in general.

Proposition 1 *TT-games are monotonic and superadditive.*

Proof We omit the straightforward proof that TT-games are monotonic. Consider a TT-game $(N, \tilde{v})$ corresponding to the allocation generated by the weighted CEA rule $WCEA(E, c, a) = (x_1, x_2, \ldots, x_n)$. Consider any coalitions $S, T \subseteq N$ with $S \cap T = \emptyset$. If $\tilde{v}(S) = 0$ or $\tilde{v}(T) = 0$ (or both), by monotonicity of $\tilde{v}$, it directly follows that $\tilde{v}(S) + \tilde{v}(T) \leq \tilde{v}(S \cup T)$. Otherwise, if $\tilde{v}(S) > 0$ and $\tilde{v}(T) > 0$, we have that

$$\tilde{v}(S) + \tilde{v}(T) \leq \sum_{i \in S \cup T: c_i - x_i > 0} \left(\frac{x_i}{\min\{\lambda(S), \lambda(T)\}} - \frac{x_i}{\lambda_i} \right) = \tilde{v}(S \cup T),$$

where the equality follows from the fact that $\min\{\lambda(S), \lambda(T)\} = \lambda(S \cup T)$. So, we have proved that $\tilde{v}(S)$ is superadditive.

The following example show that TT-games are not convex in general.

Example 2 *Let $(N,\tilde{v})$ be the TT-game corresponding to the weighted bankruptcy situation (E,c,w) with $E = 20$ and the other parameters as shown in Table 3.3. It is easy to check by relations (3.3) and (3.4) that $\hat{\lambda} = \frac{1}{2}$, yielding the weighted CEA allocation shown in the last column of Table 3.3.*

$i \in N$	$c_i(CO_2)$	w_i (GDP)	λ_i	x_i (WCEA)
1	4	2	2	1
2	21	7	3	$\frac{7}{2}$
3	1	1	1	$\frac{1}{2}$
4	10	25	$\frac{2}{5}$	10
5	5	15	$\frac{1}{3}$	5

Table 3.3 – A weighted bankruptcy situation and the corresponding WCEA allocations

$\tilde{v}(1) = 0$	$\tilde{v}(1,2) = 0$	$\tilde{v}(1,3) = 0$	$\tilde{v}(1,4) = 2$	$\tilde{v}(1,5) = \frac{5}{2}$
$\tilde{v}(2) = 0$	$\tilde{v}(2,3) = 0$	$\tilde{v}(2,4) = \frac{91}{12}$	$\tilde{v}(2,5) = \frac{28}{3}$	
$\tilde{v}(3) = 0$	$\tilde{v}(3,4) = \frac{3}{4}$	$\tilde{v}(3,5) = 1$		
$\tilde{v}(4) = 0$	$\tilde{v}(4,5) = 0$			
$\tilde{v}(5) = 0$				
$\tilde{v}(1,2,3) = 0$	$\tilde{v}(2,3,5) = \frac{31}{3}$	$\tilde{v}(1,2,4) = \frac{115}{12}$	$\tilde{v}(2,3,4) = \frac{25}{3}$	$\tilde{v}(1,4,5) = \frac{5}{2}$
$\tilde{v}(1,4,3) = \frac{11}{4}$	$\tilde{v}(1,5,3) = \frac{7}{2}$	$\tilde{v}(1,2,5) = \frac{71}{6}$		
$\tilde{v}(1,2,3,4) = \frac{31}{3}$	$\tilde{v}(1,2,3,5) = \frac{77}{6}$			
$\tilde{v}(1,2,3,4,5) = \frac{77}{6}$				

Table 3.4 – Computation of utility for every $S \subseteq N$

Agent 4 and agent 5 are totally satisfied agents where $c_j = x_i$ for $j \in \{4,5\}$. However agent 5 has a better technology in a coalition than agent 4 as seen in Table 3.4. To study convexity (2.14), let $S = \{2,3,5\}$ and $T = \{3,4\}$. Then:

$\tilde{v}(S) = \left(\frac{\frac{7}{2}}{\frac{1}{3}} - \frac{\frac{7}{2}}{3}\right) + \left(\frac{\frac{1}{2}}{\frac{1}{3}} - \frac{\frac{1}{2}}{1}\right) = \frac{31}{3}$,

$\tilde{v}(T) = \frac{\frac{1}{2}}{\frac{2}{5}} - \frac{\frac{1}{2}}{1} = \frac{3}{4}$,

$\tilde{v}(S \cap T) = \tilde{v}(\{3\}) = 0$,

$\tilde{v}(S \cup T) = \tilde{v}(\{2,3,4,5\}) = \tilde{v}(\{2,3,5\}) = \frac{31}{3}$

Then, $\tilde{v}(S) + \tilde{v}(T) > \tilde{v}(S \cup T) + \tilde{v}(S \cap T)$.

So, $\tilde{v}$ is not convex.

Due to the non-convexity of the TT game (as shown in the previous example), we can

neither guarantee balancedness nor guarantee that the Shapley value is in the core of TT games.

However, based on the previous definition of TT-game, we can implement a rule that provides an allocation always in the core of a TT-game. Considering a weighted bankruptcy situation (E,c,w) and a corresponding TT-game $(N,\tilde{v})$, the following definition guarantees balancedness [84]:

Let $i^* \in \arg\min_{i\in N: x_i - c_i = 0} \lambda_i$. Define the allocation $z \in \mathbb{R}^N$ such that for each $i \in N$

$$z_i = \begin{cases} \frac{x_i}{\lambda(N\setminus\{i^*\})} - \frac{x_i}{\lambda_i} & \text{if } c_i - x_i > 0, \\ 0 & \text{if } c_i - x_i = 0 \text{ and } i \neq i^*, \\ \tilde{v}(N) - \sum_{i\in N\setminus\{i^*\}} z_i & \text{if } i \neq i^*. \end{cases} \tag{3.9}$$

Proposition 3 *TT-games are balanced.*

Proof Consider a TT-game $(N,\tilde{v})$ corresponding to the allocation generated by the weighted CEA rule $WCEA(E,c,w) = (x_1,x_2,\ldots,x_n)$. Allocation z is clearly efficient, as $\sum_{i\in N} z_i = \tilde{v}(N)$. Let $P = \{i \in N : c_i - x_i > 0\}$, and let $I = N \setminus P$. If $P \neq \emptyset$, from the fact that $\lambda(N\setminus\{i^*\}) \leq \lambda_i$, we have that $z_i \geq 0$ for each $i \in P$. Moreover, by the fact $\lambda(N\setminus\{i^*\}) \geq \lambda(N) = \lambda_{i^*}$, we also have that

$$\tilde{v}(N) = \sum_{i\in P} \frac{x_i}{\lambda_{i^*}} - \frac{x_i}{\lambda_i} \geq \sum_{i\in P} \Big(\frac{x_i}{\lambda(N\setminus\{i^*\})} - \frac{x_i}{\lambda_i} \Big),$$

and then it immediately follows that $z_{i^*} = \tilde{v}(N) - \left(\frac{x_i}{\lambda(N\setminus\{i^*\})} - \frac{x_i}{\lambda_i} \right) \geq 0$. Then, for each $S \subseteq N$ such that $S \cap P = \emptyset$ or $S \cap I = \emptyset$, we have that $\sum_{i\in S} z_i \geq 0 = \tilde{v}(S)$.

Now, let $S \subseteq N$ be such that $S \cap P \neq \emptyset$ and $S \cap I \neq \emptyset$. If $i^* \notin S$, we have that

$$\tilde{v}(S) = \sum_{i\in S\cap P} \frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i} \leq \sum_{i\in S\cap P} \frac{x_i}{\lambda(N\setminus\{i^*\})} - \frac{x_i}{\lambda_i} = \sum_{i\in S} z_i, \tag{3.10}$$

where the first equality follows directly from relation (3.8), and the inequality follows from

$z(1) = 2$	$z(1,2) = \frac{115}{12}$	$z(1,3) = \frac{11}{4}$	$z(1,4) = 2$	$z(1,5) = \frac{9}{2}$
$z(2) = \frac{91}{12}$	$z(2,3) = \frac{25}{3}$	$z(2,4) = \frac{91}{12}$	$z(2,5) = \frac{121}{12}$	
$z(3) = \frac{3}{4}$	$z(3,4) = \frac{3}{4}$	$z(3,5) = \frac{13}{4}$		
$z(4) = 0$	$z(4,5) = \frac{5}{2}$			
$z(5) = \frac{5}{2}$				
$z(1,2,3) = \frac{31}{3}$	$z(2,3,5) = \frac{65}{6}$	$z(1,2,4) = \frac{115}{12}$	$z(2,3,4) = \frac{25}{3}$	$z(1,4,5) = \frac{9}{2}$
$z(1,4,3) = \frac{11}{4}$	$z(1,5,3) = \frac{21}{4}$	$z(1,2,5) = \frac{145}{12}$		
$z(1,2,3,4) = \frac{31}{3}$	$z(1,2,3,5) = \frac{77}{6}$			
$z(1,2,3,4,5) = \frac{77}{6}$				

Table 3.5 – Computation of the TT-game for every $S \subseteq N$

the fact that $\lambda(S) \geq \lambda(N \setminus \{i^*\})$. Otherwise, if $i^* \in S$, we have that

$$\tilde{v}(S) = \sum_{i \in S \cap P} \frac{x_i}{\lambda(S)} - \frac{x_i}{\lambda_i} \leq \sum_{i \in S \cap P} \frac{x_i}{\lambda(N \setminus \{i^*\})} - \frac{x_i}{\lambda_i} + z_{i^*} = \sum_{i \in S} z_i, \tag{3.11}$$

where the inequality follows from the previous arguments and the fact that we have shown earlier that $z_{i^*} \geq 0$. So, we have proved that $z \in C(\tilde{v})$.

Example 4 *Considering the same bankruptcy game as in Example 3.3. The Table 3.5 groups all possible coalitions among $N = 5$ agents and presents the TT-game for each coalition $S \subseteq N$.*

Table 3.5 shows that the game presented in the Example 2 has a non empty Core and for each coalition $S \in N, z(S) \geq \tilde{v}(S)$.

3.5 WCEA Algorithmic approach

The main idea of the WCEA rule is, on one hand, to find an allocation that does not exceed either agents demand nor a predefined weight threshold. On other hand, these allocations must respect the balance constraint.

In this section, we present an algorithmic approach (Algorithm 1) for the computation of WCEA in a weighted bankruptcy situation. Each player receives a $\lambda = \frac{c_i}{w_i}$ generated by his own claim and his weight, then this vector of λ is sorted in an incremental way. With

the objectif to find a $\hat{\lambda}$ that respects rationality, claim boundedness and efficiency, we start our search by the median of λ^S vector affected to temporary parameters λ^{Prev}. This starting value, reduces by half the number of iterations since to reach the (upper or lower) bound of the list of λ^S, we will go through $\frac{N+1}{2}$ players. At each iteration, we compute the sum attributed allocations, $S = \sum_{i \in N} min(c_i, w_i \lambda^{Prev})$, which can take three possible values:

Algorithm 1 Weighted Constrained Equal Awards rule

Input: Estate E, Set of player N, Claims vector c, weight vectors w
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

```
ForEach {i ∈ N} :{λ_i = c_i / w_i}
λ^S ← SortIncrλ
moy = mediane{λ^S}
λ^Prev ← moy
ForEach {i ∈ n } : {x_i = min(c_i, w_i λ^Prev)}
S = Σ_{i=1}^N x_i
if S=E then
   λ̂ = λ^Prev
   ForEach {i ∈ N } : {x_i = min(c_i, w_i λ̂)}
end if
It ← 0
while It ≤ (n+1)/2 AND S ≠ E do
   if S > E then
      Algorithm 2
   else
      Algorithm 3
   end if
end while
return x
```

1) $S = E$, the efficiency constraint is respected and the allocations sum S is equal to the estate E. The $\hat{\lambda} = \lambda^{Prev}$ and we obtain the final allocations.

2) $S > E$, in case of the allocations sum S is higher than the estate E, λ must be decreased (Algorithm 2). The λ^{Prev}, used to compute the allocations, will obtain the value of the first λ^S lower than the last computed λ^{Prev}. Then, we recompute S and we re-iterate the procedure until we obtain $S = E$ or we arrive to lower or upper bound of the vector of λ^S. In every iteration we can face three situations:

   – If the allocations' sum is equal to estate, $S = E$, $\hat{\lambda}$ is equal to λ^{Prev}.

- If the allocations' sum is still higher to estate, we need to decrements the last used λ^{Prev}
- Or the allocations sum toggles to a lower sum than the estate, so we pass to a new procedure the LastComputing Algorithm 4.

3) $S < E$, a situation where the allocations sum is lower than estate, we have to increment λ used in the allocation process (Algorithm 3). The λ^{Prev} value will change to the first higher λ^S than the previous computed λ^{Prev}. Then, the allocations sum has to be recomputed with the new value of λ^{Prev} until we satisfy the efficiency constraint or we arrive to upper or lower bound of λ^S vector. We can face three different results :

- Allocations satisfy the balance condition, i.e. $\sum_{i=1}^{N} x_i = E$, the $\hat{\lambda}$ is equal to the last computed λ^{Prev}.
- allocations' sum is lower than the estate, we continue to increment the λ^{Prev}
- If the allocations sum switch from lower than the estate to higher than the estate, we have to trigger the LastComputing Algorithm 4.

Algorithm 2 Allocations' sum is higher than Estate

Input: Estate E, Set of player N, Claims vector c, weight vectors w
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

```
while S > E AND It < (N+1)/2 do
    It ++;
    λ^Prev ← First(λ^S < λ^Prev)
    x_i = min(c_i, w_i λ^Prev)
    S = Σ_{i=1}^{N} x_i
    if S = E then
        λ̂ = λ^Prev
        ForEach {i ∈ N } : {x_i = min(c_i, w_i λ̂)}
    end if
    if S < E then
        Algo : LastComputing
    end if
end while
return x
```

When all λ^S are tested and the allocations sum does not satisfy the efficiency constraint where $\sum_{i \in N} x_i = E$ or between two iterations, allocations sum pass from lower to

Algorithm 3 Estate is higher than allocations' sum

Input: Estate E, Set of player N, Claims vector c, weight vectors w
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

```
while S < E AND It < (N+1)/2 do
    It ++;
    λ^Prev ← First(λ^S > λ^Prev)
    x_i = min(c_i, w_i λ^Prev)
    S = Σ_{i=1}^N x_i
    if S = E then
        λ̂ = λ^Prev
        ForEach {i ∈ N } : {x_i = min(c_i, w_i λ̂)}
    end if
    if S > E then
        Algo : LastComputing
    end if
end while
return x
```

higher than estate and inversely, we pass to the LastComputing procedure4 . In the first step, we need to know from which algorithm the call of this procedure has been triggered. Each algorithm, Algorithm 2 and Algorithm 3 has its own instructions but both are based on the same idea.

If the allocations sum is higher than estate, we have to fix a new λ^{inf}, this parameter will be defined as the first sorted λ^S lower than λ^{Prev}.

Remember that λ^{Prev} is the last λ with which we have computed the allocations and calculated the sum S. We consider $\hat{\lambda} \in [\lambda^{inf}, \lambda^{Prev}]$, and we assign to a λ^{test}, new temporary parameter, a value in this range. As the execution progresses, the searching boundary is reduced by changing one of the bounds each time, until a balance situation is reached. In a situation where the estate is higher than allocations sum, we have to introduce a new parameter λ^{sup} which is the first λ^S higher than λ^{Prev}.

Similarly to the previous situation, we can define the range of $\hat{\lambda}$ as $[\lambda^{Prev}, \lambda^{sup}]$ and we start the variation of λ^{test} to compute allocations. When allocations sum is equal to the estate, we can consider the last λ^{test} used as $\hat{\lambda}$ and each agent will obtain his allocation.

Algorithm 4 LastComputing Algorithm

Input: Estate E, Set of player N, claims vector c, weights vector w
Output: Allocation vector x

if $S > E$ **then**
  $\lambda^{inf} \leftarrow max\{0, First(\lambda^S < \lambda^{Prev})\}$
  $\lambda^{test} \leftarrow \frac{\lambda^{inf}+\lambda^{Prev}}{2}$
  $S = \sum_{i=1}^{N} \min(c_i, w_i\lambda^{test})$
  while $S \neq E$ **do**
    if $S > E$ **then**
      $\lambda^{Prev} \leftarrow \lambda^{test}$
    else
      $\lambda^{Inf} \leftarrow \lambda^{test}$
    end if
    $\lambda^{test} \leftarrow \frac{\lambda^{inf}+\lambda^{Prev}}{2}$
    $S = \sum_{i=1}^{N} \min(c_i, w_i\lambda^{test})$
  end while
else if $S < E$ **then**
  $\lambda^{sup} \leftarrow min\{\lambda^{Prev}, First(\lambda^S > \lambda^{Prev}\}$
  $\lambda^{test} \leftarrow \frac{\lambda^{sup}+\lambda^{Prev}}{2}$
  $S = \sum_{i=1}^{N} min(c_i, w_i\lambda^{test})$
  while $S \neq E$ **do**
    if $S < E$ **then**
      $\lambda^{Prev} \leftarrow \lambda^{test}$
    else
      $\lambda^{Sup} \leftarrow \lambda^{test}$
    end if
    $\lambda^{test} \leftarrow \frac{\lambda^{sup}+\lambda^{Prev}}{2}$
    $S = \sum_{i=1}^{N} \min(c_i, w_i\lambda^{test})$
  end while
else
  $\hat{\lambda} \leftarrow \lambda^{test}$
  for $i \in N$ **do**
    $x_i \leftarrow min(c_i, w_i\hat{\lambda})$
  end for
end if
return x_i

Example 5 *Consider the weighted bankruptcy situations* (E,c,w) *with* $N = \{1,2,3\}$ *as the set of agents,* $E = 20$ *and the other parameters as shown in Table 3.6.*

Algorithm 5 Function Firstλ^S

Input: λ^{Prev}, λ^S : a vector of sorted λ
Output: X

```
if S > E then
  %We are looking for the first λ < λ^Prev
  i ← 0
  while λ^S[i] < λ^Prev do
    i ← ++
  end while
  if i=0 then
    X = 0
  else
    X ← λ^S[i−1]
  end if
else
  %We are looking for the first λ > λ^Prev
  i ← N
  while λ^S[i] > λ^Prev do
    i ← −−
  end while
  if i=N then
    X = λ^Prev + 1
  else
    X ← λ^S[i+1]
  end if
end if
return X
```

Agent i	c_i	w_i	$\lambda_i = \frac{c_i}{w_i}$
1	6	3	2
2	10	2	5
3	20	8	$\frac{5}{2}$

Table 3.6 – A double weighted bankruptcy situation (E,c,w) with $N = \{1,2,3\}$ as set of agents and estate $E = 20$.

At the first iteration of Algorithm 1 the vectors $\lambda^S = (2, \frac{5}{2}, 5)$ *is defined to sort the ratio* $(\frac{c_i}{w_i})_{i\in N}$*, and the value moy* $=$ *mediane*$\{2, \frac{5}{2}, 5\} = \frac{5}{2}$ *is the median of* λ^S*. This value moy is assigned to the variable* λ^{Prev}*. The provisional allocation for the three agents is computed as* $x_1 = \min\{6, 3 \times \frac{5}{2} = 6$ *for agent* 1*,* $x_2 = \min\{10, 2 \times \frac{5}{2} = 5$ *for agent* 2 *and* $x_3 = \min\{20, 8 \times \frac{5}{2} = 20$ *for agent* 3*. For the sum of allocations* $S = 31$ *is larger than the estate* $E = 20$*, the procedure calls Algorithm 2. Now, at the new iteration (It* $= 1$ *in*

Table 3.7), the procedure selects the largest value of λ^S *vector which is also smaller than* $\lambda^{Prev} = \frac{5}{2}$*, and such a value is used to set the new* λ^{Prev}*. So, at* $It = 1$*,* $\lambda^{Prev} = 2$*. A new allocation for* λ^{Prev} *is computed, and the procedure continues as detailed in in Table 3.7, till the iteration number* $It = 2$ *exceeds the fixed limit of the vector* $It \geq \frac{N+1}{2} = 2$*. At this point, the LastComputing process is called to calculate the final allocation. New temporary parameters are introduced* λ^{inf} *and* λ^{test}*. The first one selects the maximum among* 0 *and First* $(\lambda^S < \lambda^{Prev})$*, generated by Algorithm 7 . The* λ^{test} *is calculated as the average of the* λ^{inf} *and* λ^{Prev}*. This procedure is repeated until the allocations sum is equal to the estate, i.e.* $S = E$*. The relevant parameters calculated at each iteration of Algorithm 1 are shown in Table 3.7.*

$It = 0$: $\lambda^{Prev} = mediane\{2, \frac{5}{2}, 5\} = \frac{5}{2}$

Agent	c_i	$w^i \times \lambda^{Prev}$	x_i
1	6	$3 \times \frac{5}{2} = 7{,}5$	6
2	10	$2 \times \frac{5}{2} = 5$	5
3	20	$8 \times \frac{5}{2} = 20$	20
			$S = 31 > E$

$It = 1$: $\lambda^{Prev} = 2$

Agent	c_i	$w_i \times \lambda^{Prev}$	x_i
1	6	$3 \times 2 = 6$	6
2	10	$2 \times 2 = 4$	4
3	20	$8 \times 2 = 16$	16
			$S = 26 > E$

$It = 2 \geq \frac{N+1}{2}$: LastComputing process, $\lambda^{inf} = 0$, $\lambda^{test} = \frac{0+2}{2} = 1$

Agent	c_i	$w_i^1 \times \lambda^{test}$	x_i
1	6	$3 \times 1 = 3$	3
2	10	$2 \times 1 = 2$	2
3	20	$8 \times 1 = 8$	8
			$S = 13 < E$

LastComputing process: $\lambda^{inf} \leftarrow \lambda^{test} = 1$, $\lambda^{test} = \frac{1+2}{2} = \frac{3}{2}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{3}{2} = \frac{9}{2}$	6
2	10	$2 \times \frac{3}{2} = 3$	1
3	20	$8 \times \frac{3}{2} = 12$	2,5
			$S = 19{,}5 < E$

LastComputing Process: $\lambda^{inf} \leftarrow \lambda^{test} = \frac{3}{2}$, $\lambda^{test} = \frac{\frac{3}{2}+2}{2} = \frac{7}{4}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{7}{4} = 5{,}25$	5,25
2	10	$2 \times \frac{7}{4} = 3{,}5$	3,5
3	20	$8 \times \frac{7}{4} = 14$	14
			$S = 21{,}75 > E$

LastComputing Process: $\lambda^{inf} \leftarrow \lambda^{test} = \frac{3}{2}$, $\lambda^{test} = \frac{\frac{3}{2}+2}{2} = \frac{7}{4}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{7}{4} = 5,25$	5,25
2	10	$2 \times \frac{7}{4} = 3,5$	3,5
3	20	$8 \times \frac{7}{4} = 14$	14
			$S = 21,75 > E$

LastComputing Process: $\lambda^{Prev} \leftarrow \lambda^{test} = \frac{7}{4}$, $\lambda^{test} = \frac{\frac{3}{2}+\frac{7}{4}}{2} = \frac{13}{8}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{13}{8} = 4,87$	4,87
2	10	$2 \times \frac{13}{8} = 3,25$	3,25
3	20	$8 \times \frac{13}{8} = 13$	13
			$S = 21,12 > E$

LastComputing Process: $\lambda^{Prev} \leftarrow \lambda^{test} = \frac{13}{8}$, $\lambda^{test} = \frac{\frac{3}{2}+\frac{13}{8}}{2} = \frac{25}{16}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{25}{16} = 4,68$	4,68
2	10	$2 \times \frac{25}{16} = 3,12$	3,12
3	20	$8 \times \frac{25}{16} = 12,5$	12,5
			$S = 20,3 > E$

LastComputing Process: $\lambda^{Prev} \leftarrow \lambda^{test} = \frac{25}{16}$, $\lambda^{test} = \frac{\frac{3}{2}+\frac{25}{16}}{2} = \frac{49}{32}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{49}{32} = 4,59$	4,59
2	10	$2 \times \frac{49}{32} = 3,06$	3,06
3	20	$8 \times \frac{49}{32} = 12,25$	12,25
			$S = 19,9 < E$

LastComputing Process: $\lambda^{inf} \leftarrow \lambda^{test} = \frac{49}{32}$, $\lambda^{test} = \frac{\frac{49}{32}+\frac{25}{16}}{2} = \frac{99}{64}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{99}{64} = 4,64$	4,64
2	10	$2 \times \frac{99}{64} = 3,09$	3,09
3	20	$8 \times \frac{99}{64} = 12,37$	12,37
			$S = 20,1 > E$

LastComputing Process: $\lambda^{Prev} \leftarrow \lambda^{test} = \frac{99}{64}$, $\lambda^{test} = \frac{\frac{49}{32}+\frac{99}{64}}{2} = \frac{197}{128}$

Agent	c_i	$w_i \times \lambda^{test}$	x_i
1	6	$3 \times \frac{197}{128} = 4,61$	4,61
2	10	$2 \times \frac{197}{128} = 3,09$	3,09
3	20	$8 \times \frac{197}{128} = 12,37$	12,37
			$S = 20 = E$

Table 3.7 – Parameters computed at each the iteration *It* of Algorithm 1 on the weighted bankruptcy situation of Example 5, with $\lambda^S \leftarrow \{2, \frac{5}{2}, 5\}$.

3.6 Conclusion

In this chapter, we introduce a new bankruptcy method to allocate CO_2 emissions permits among the EU-28 which considers not only countries' claims but also countries' GDP productions. We propose a cooperative approach to solicit countries with low emission intensity to transfer efficient techniques to countries with high emission intensity, through the allocation characteristics provided by the WCEA rule, the core of these games is non-empty. On the other hand, in the study of country characteristics and weights, we observed that the GDP index is often considered a good measure for comparing the environmental performance of countries with similar social and economic conditions. However, in order to take into account country heterogeneity, a more comprehensive study of the environmental performance of country policies requires the study of more parameters. In the next chapter, we present a more generalized method to allocate CO_2 emission permits, considering more weights.

Chapter 4

Double weighted bankruptcy problems without priority

Contents

4.1 Introduction 109

4.2 A Double Weighted Constrained Equal Awards rule (DWCEA) . . 111

4.3 A DWCEA solution algorithm with no priority over criteria 112

4.4 DWCEA applied to CO_2 emissions permits 117

4.5 Allocations results 126

4.6 Detailed algorithms 130

4.7 Conclusion 132

This chapter has been accepted for publication in the journal Games [78].

4.1 Introduction

Faced with the urgency of finding solutions for climate change, studies have covered several areas of research. The critical point is still to reach an agreement that all the countries are willing to sign.

In practice, alternative methods have been proposed to allocate CO_2 emissions among countries and taking into account criteria [93] like energy, the Gross Domestic Product (GDP), population, energy production, etc.

In this chapter, we focus on the allocation of CO_2 emissions permits that must be shared among the EU countries, and where each country has its own CO_2 emissions permits claim to take into consideration two exogenous parameters aimed at providing a balance between economic activities and production of Renewable energy. As far as we know, this is the first study where the CO_2 allocation problem has been investigated as a claims problem from a multi-criteria point of view. Specifically, in this work, we introduce a preliminary application of a new allocation method for double weighted bankruptcy situations based on the combination of two distinct criteria: the ability of countries to efficiently use their CO_2 emission permits, and the capacity of countries to produce energy in a sustainable manner via Renewable sources. More precisely, using an extended version of the CEA solution for bankruptcy situations, we leverage the request of CO_2 emission permits of EU countries taking into account both the economic growth of the EU countries (in terms of their GDP) and their sustainable policies, measured as the amount of Renewable energy produced by each country.

The remaining of the chapter is organized as follows.

- The Section 2 introduces a new family of bi-criteria allocation methods for double weighted bankruptcy situations, namely, the Double Weighted Constrained Equal Awards (DWCEA) methods and presents an algorithm to compute a specific allocation protocol.
- Then, Section 3 presents an algorithm approach to characterize the Double Weighted Constrained Equal Awards method and two examples describing the several steps of this algorithm.
- The Section 4 is devoted to the application of the proposed protocol introduced in Section 3 to allocate CO_2 emissions permits among EU countries over the years 2010-2014, and to the comparison of the results provided by the alternative solutions considered in this study.

4.2 A Double Weighted Constrained Equal Awards rule (DWCEA)

Previous studies on bankruptcy situations and their solutions can be seen as an allocation approach where only one criterion, agent's claim, is considered. Weighted bankruptcy situations [13] keep into account one more parameter to determine allocation vectors. In this section, we introduce a richer framework for bankruptcy situations, considering two vectors of weights at once, in addition to the vector of claims.

We denote by $(E, c = (c_i)_{i\in N}, w^1 = (w_i^1)_{i\in N}, w^2 = (w_i^2)_{i\in N})$ a *double weighted bankruptcy problem* where $E \in \mathbb{R}_{\geq 0}$ is a non-negative estate, N is a finite set of claimants, $c = (c_i)_{i\in N}$ is the vector of non-negative claims such that $\sum_{i\in N} c_i \geq E$, and $w^1 = (w_i^1)_{i\in N}$ and $w^2 = (w_i^2)_{i\in N}$ are two vector of positive weights.

We denote by $\mathscr{DW}$ the family of all double weighted bankruptcy problem. An allocation rule $\phi : \mathscr{DW} \to \mathbb{R}^N_{\geq 0}$ is a map that associate to any double weighted bankruptcy problem $(E, c, w^1, w^2) \in \mathscr{DW}$ an allocation vector in $\mathbb{R}^N$ such that $\sum_{i\in N} \phi_i(E, c, w^1, w^2) = E$.

Some interesting properties for an allocation rule for double weighted bankruptcy situations are the following. *Equal treatment*: an allocation rule ϕ satisfies equal treatment if for all $(E, c, w^1, w^2) \in \mathscr{DW}$, if $i, j \in N$ are such that $c_i = c_j$, $w_i^1 = w_j^1$ and $w_i^2 = w_j^2$, then $\phi_i(E, c, w^1, w^2) = \phi_j(E, c, w^1, w^2)$. *Composition*: An allocation rule ϕ satisfies composition if for all $(E, c, w^1, w^2) \in \mathscr{DW}$ such that $w_i^1 = w_i^2 = 1$ for all $i \in N$ and for all $0 \leq E' \leq E$ we have $\phi(E, c, w^1, w^2) = \phi(E', c, w^1, w^2) + \phi(E - E', c - \phi(E', c, w^1, w^2), w^1, w^2)$. *Invariance under claims truncation*: An allocation rule ϕ satisfies invariance under claims truncation if for all $(E, c, w^1, w^2) \in \mathscr{DW}$ we have $\phi(E, c, w^1, w^2) = \phi(E, c^E, w^1, w^2)$, where $c^E \in \mathbb{R}^N$ is such that $c_i^E = \min\{c_i, E\}$ for all $i \in N$.

In an extension of the WCEA rule to situations with two weight vectors, we introduce in this chapter a solution *double Weighted Constrained Equal Awards (DWCEA)*, which associates to any doubly weighted bankruptcy situation a particular allocation vector $x \in \mathbb{R}^N_{\geq 0}$ satisfying the following conditions:

$$x_i(E, c, w^1, w^2) = min(c_i, \lambda_1^* w^1, \lambda_2^* w^2) \tag{4.1}$$

where $\lambda_1^*, \lambda_2^* \in R^N$ such as

$$\sum_{i \in N} min(c_i, \lambda_1^* w^1, \lambda_2^* w^2) = E \tag{4.2}$$

In weighted bankruptcy problems, each agent is characterized by a claim and weights. These weights have different impacts depending on the chosen parameter and its compliance rate. Therefore, they can impact allocation results.

In multi-criteria decision problems, we frequently have a wide choice of allocation vectors that satisfy the constraints specified by the relations (4.1) and (4.2) for some λ_1^* and λ_2^*. Hence, in order to select a specific allocation vector, we introduce a computational approach to find parameters for λ_1^* and λ_2^* that does not require assuming a specific priority on the criteria represented by the two weights vectors.

In the end of this chapter, we also make use of some concepts from cooperative game theory. Specifically, we compare allocation vectors for bankruptcy situations with the vector provided by the *Shapley and Shubik (Sh-Sh) power index* [75] for a simple game representing the voting rule at the EU Council [48].

4.3 A DWCEA solution algorithm with no priority over criteria

The algorithm studied in this section is based on the general principle that a DWCEA solution should grant the same importance to the different criteria represented by the two weight vectors. Based on this assumption, we introduce Algorithm 6 providing a solution for problem (4.1) under the constraint (4.2) with the objective to find feasible λ_1^* and λ_2^* without arbitrarily promoting the use of one of the two criteria to drive the choice of these parameters.

Algorithm 6 Double Weighted method's algorithm

Input: Estate E, Set of player N, Claims vector c, weight vectors w_1, w_2
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

ForEach $i \in N$: $\lambda_i^1 = \frac{c_i}{w_i^1}$, $\lambda_i^2 = \frac{c_i}{w_i^2}$
$\lambda_1^S \leftarrow SortIncr\lambda^1$, $\lambda_2^S \leftarrow SortIncr\lambda^2$
$moy = \frac{median\lambda_1^S + median\lambda_2^S}{2}$
$\lambda_1^{Prev} \leftarrow moy$, $\lambda_2^{Prev} \leftarrow moy$
ForEach $i \in N$: $x_i = \min(c_i, w_i^1\lambda_1^{Prev}, w_i^2\lambda_2^{Prev})$
$S = \sum_{i \in N} x_i$
if $S = E$ **then**
  $\lambda_1^* = \lambda_1^{Prev}$
  $\lambda_2^* = \lambda_2^{Prev}$
  ForEach $i \in N$:
  $x_i = min(c_i, w_i^1\lambda_1^*, w_i^2\lambda_2^*)$
end if
$It \leftarrow 0$
while $It \leq \frac{N}{2} + 1$ **AND** $S \neq E$ **do**
  if $S > E$ **then**
    Algorithm 8
  end if
  if $S < E$ **then**
    Algorithm 9
  end if
end while
return x

Algorithm 6 starts by calculating, for every player $i \in N$, a value $\lambda_{1,i} = \frac{c_i}{w_i^1}$ and a value $\lambda_{2,i} = \frac{c_i}{w_i^2}$, then it sorts these values in incremental order, yielding vectors λ_1^S and λ_2^S. In the initial iteration, the same value $moy = \frac{median\lambda_1^S + median\lambda_2^S}{2}$ is affected to parameters λ_1^{Prev} and λ_2^{Prev}, which are the temporary parameters used to compute the allocation. Starting the search for parameters λ_1^* and λ_2^* from the median *moy* ensures a kind of neutrality for the importance of both criteria in the allocation computation, and it guarantees a minimal worst-case number of iterations before reaching the final λ_1^* and λ_2^*, since this procedure will be iterated at most $\frac{N}{2} + 1$ times. At each iteration of the algorithm, the sum of the allocations, $S = \sum_{i \in N} \min(c_i, w_i^1\lambda_1^{Prev}, w_i^2\lambda_2^{Prev})$, can take three possible values:

1) $S = E$, i.e., the sum of allocations is equal to the estate. In this case, we immediately have that $\lambda_1^* = \lambda_1^{Prev}$ and $\lambda_2^* = \lambda_2^{Prev}$.

2) $S > E$, i.e., the sum of allocations is strictly larger than the estate. So, λ_1 and/or λ_2

must be decreased. In this situation, Algorithm 6 selects the first sorted $\lambda_1^S < \lambda_1^{Prev}$ as X and the first sorted $\lambda_2^S < \lambda_2^{Prev}$ as Y. Then, we choose the biggest value among X and Y to change the corresponding λ_j^{Prev} value, $j \in \{1,2\}$, and we keep the same value for the other $\lambda_{k\neq j}^{Prev}$; then, we recompute S and we re-iterate the procedure until we obtain $S = E$, as shown in Algorithm 8.

3) $S < E$, i.e., the sum of allocations is strictly smaller than the estate, we have to increase λ_1 and/ or λ_2. So, λ_j^{Prev}, $j \in \{1,2\}$, will be updated to the smallest value among X, the first $\lambda_1^S > \lambda_1^{Prev}$, and Y, the first $\lambda_2^S > \lambda_2^{Prev}$, and we keep the same value for the other $\lambda_{k\neq j}^{Prev}$; then, we recompute S and we re-iterate the procedure until we obtain $S = E$, as shown in Algorithm 9.

After a number of iterations $It > \frac{N}{2} + 1$, half of the agents' demands are considered, and if the sum of allocations does not satisfy the efficiency constraint, $E = \sum_{i\in N} \min(c_i, w_i^1 \lambda_1^{Prev}, w_i^2 \lambda_2^{Prev})$, the algorithm starts the *LastComputing* procedure. The same *LastComputing* procedure is also called at an iteration It such that $S > E$ at iteration $It - 1$, but $S < E$ at iteration It (or, inversely, $S < E$ at iteration $It - 1$ and $S > E$ at iteration It). Basically, as shown in the corresponding pseudo−code, the Algorithm10 transforms the double weighted bankruptcy situation with two weight vectors in a single weighted bankruptcy situation, keeping the parameter λ_j^{Prev}, $j \in \{1,2\}$, that did not change from the previous iteration.

It is obvious that the solution provided by Algorithm 6 satisfies the equal treatment property. It also satisfies the composition property, as any solution satisfying conditions (4.1) and (4.2) when applied to a double weighted bankruptcy problem (E, c, w^1, w^2) with $w^1 = w^2$ yields the allocation provided by the WCEA solution applied to the weighted bankruptcy problem (E, c, w^1), and the WCEA solution satisfies the composition property on the class of weighted bankruptcy problems (see [13, 59]). Instead, we cannot guarantee that the solution provided by Algorithm 6 satisfies the property of invariance under claims truncation, as the use of vector c^E instead of c may affect the procedure to compute λ_1^{Prev} and λ_2^{Prev} along the iterations of Algorithm 6.

We now introduce an example of calculations provided by Algorithm 6.

Example 6 *Consider the double weighted bankruptcy situations* (E,c,w^1,w^2) *with* $N=\{1,2,3\}$ *as the set of agents,* $E=20$ *and the other parameters as shown in Table 4.1.*

Agent i	c_i	w_i^1	$\lambda_i^1=\frac{c_i}{w_i^1}$	w_i^2	$\lambda_i^2=\frac{c_i}{w_i^2}$
1	6	3	2	12	$\frac{1}{2}$
2	10	2	5	2	5
3	20	8	$\frac{5}{2}$	5	4

Table 4.1 – A double weighted bankruptcy situation (E,c,w^1,w^2) with $N=\{1,2,3\}$ as set of agents and estate $E=20$.

At the first iteration of Algorithm 6 the vectors $\lambda_1^S=(2,\frac{5}{2},5)$ *and* $\lambda_2^S=(\frac{1}{2},4,5)$ *are defined to sort the ratios* $(\frac{c_i}{w_i^1})_{i\in N}$ *and* $(\frac{c_i}{w_i^2})_{i\in N}$*, respectively, and the value* $moy=\frac{\frac{5}{2}+4}{2}=\frac{13}{4}$ *is calculated as the average of the two medians of* λ_1^S *and* λ_2^S*. This value moy is assigned to the variables* λ_1^{Prev} *and* λ_2^{Prev}*. The provisional allocation for the three agents is computed as* $x_1=\min\{6,3\times\frac{13}{4},12\times\frac{13}{4}\}=6$ *for agent* 1*,* $x_2=\min\{10,2\times\frac{13}{4},2\times\frac{13}{4}\}=6,5$ *for agent* 2 *and* $x_3=\min\{20,8\times\frac{13}{4},5\times\frac{13}{4}\}=16,25$ *for agent* 3*. For the sum of allocations* $S=28,75$ *is larger than the estate* $E=20$*, the procedure calls Algorithm 8 (see the last section). Now, at the new iteration (*$It=1$ *in Table 4.2), the procedure selects the largest value between* λ_1^S *and* λ_2^S *which is also smaller than* $moy=\frac{13}{4}$*, and such a value is used to set the new* λ_j^{Prev} *for the corresponding weight* $j\in\{1,2\}$*, while the value* $\lambda_{k\neq j}^{Prev}$ *is maintained equal to its value at the previous iteration. So, at* $It=1$*,* $\lambda_1^{Prev}=\frac{5}{2}$ *and* $\lambda_2^{Prev}=moy=\frac{13}{4}$*. A new allocation for* λ_1^{Prev} *and* λ_2^{Prev} *is computed, and the procedure continues as detailed in in Table 4.2, till the sum of allocations becomes strictly smaller than the estate* $E=20$ *(this happens at the iteration* $It=3$ *of Table 4.2. At this point, the largest (and closest to moy) value between the* λ_1^{Prev} *and* λ_2^{Prev} *is selected (so,* $\lambda_1^{Prev}=2$ *in the specific case), and the LastComputing process is called to calculate the final allocation corresponding to the WCEA allocation defined by relations (3.3) with claims vector* $(\min\{c_i,w_i^1\lambda_1^{Prev}\})_{i\in N}$*. The relevant parameters calculated at each iteration of Algorithm 6 are shown in Table 4.2.*

$It = 0$: $\lambda_1^{Prev} = \lambda_2^{Prev} = moy = \frac{\frac{5}{2}+4}{2} = \frac{13}{4}$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times \frac{13}{4} = 9,75$	$12 \times \frac{13}{4} = 39$	6
2	10	$2 \times \frac{13}{4} = 6,5$	$2 \times \frac{13}{4} = 6,5$	6,5
3	20	$8 \times \frac{13}{4} = 26$	$5 \times \frac{13}{4} = 16,25$	16,25
				$S = 28,75 > E$

$It = 1$: $X \leftarrow \frac{5}{2}$, $Y \leftarrow \frac{1}{2}$, $\lambda_1^{Prev} = \frac{5}{2}$, $\lambda_2^{Prev} = moy$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times \frac{5}{2} = 7,5$	$12 \times \frac{13}{4} = 39$	6
2	10	$2 \times \frac{5}{2} = 5$	$2 \times \frac{13}{4} = 6,5$	5
3	20	$8 \times \frac{5}{2} = 20$	$5 \times \frac{13}{4} = 16,25$	16,25
				$S = 27,25 > E$

$It = 2$: $X \leftarrow 2$, $Y \leftarrow \frac{1}{2}$, $\lambda_1^{Prev} = 2$, $\lambda_2^{Prev} = moy$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times 2 = 6$	$12 \times \frac{13}{4} = 39$	6
2	10	$2 \times 2 = 4$	$2 \times \frac{13}{4} = 6,5$	4
3	20	$8 \times 2 = 16$	$5 \times \frac{13}{4} = 16,25$	16
				$S = 26 > E$

$It = 3$: $X \leftarrow 2$, $Y \leftarrow \frac{1}{2}$, $\lambda_1^{Prev} = 2$, $\lambda_2^{Prev} = \frac{1}{2}$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times 2 = 6$	$12 \times \frac{1}{2} = 6$	6
2	10	$2 \times 2 = 4$	$2 \times \frac{1}{2} = 1$	1
3	20	$8 \times 2 = 16$	$5 \times \frac{1}{2} = 2,5$	2,5
				$S = 9,5 < E$

LastComputing Process: $\lambda_1^* \leftarrow \lambda_1^{Prev}$, $\lambda_2^* \leftarrow 2$

Agent	c_i	$w_i^1 \times \lambda_1^*$	$w_i^2 \times \lambda_2^*$	x_i
1	6	$3 \times 2 = 6$	$12 \times 2 = 24$	6
2	10	$2 \times 2 = 4$	$2 \times 2 = 4$	4
3	20	$8 \times 2 = 16$	$5 \times 2 = 10$	10
				$S = 20 = E$

Table 4.2 – Parameters computed at each the iteration *It* of Algorithm 6 on the double weighted bankruptcy situation of Example 6, with $\lambda_1^S \leftarrow \{2, \frac{5}{2}, 5\}$ and $\lambda_2^S \leftarrow \{\frac{1}{2}, 4, 5\}$.

Example 7 *Consider the double weighted bankruptcy situations* (E, c, w^1, w^2) *with* $N = \{1,2,3\}$ *as the set of agents,* $E = 30$ *and the other parameters as shown in Table 4.1. Similar to Example 7, we show all the iterations of Algorithm 6 in Table 4.3. Notice that at iterations* $It = 0$ *and* $It = 1$ *it is called Algorithm 9, instead of Algorithm* 8 *as in Example 6, for* $S < E$ *at* $It = 0$ *and* $S > E$ *at* $It = 1$.

$It = 0$: $\lambda_1^{Prev} = \lambda_2^{Prev} = moy = \frac{\frac{5}{2}+4}{2} = \frac{13}{4}$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times \frac{13}{4} = 9,75$	$12 \times \frac{13}{4} = 39$	6
2	10	$2 \times \frac{13}{4} = 6,5$	$2 \times \frac{13}{4} = 6,5$	6,5
3	20	$8 \times \frac{13}{4} = 26$	$5 \times \frac{13}{4} = 16,25$	16,25
				$S = 28,75 < E$

$It = 1$: $X \leftarrow 5, Y \leftarrow 4, \lambda_1^{Prev} = moy, \lambda_2^{Prev} = 4$

Agent	c_i	$w_i^1 \times \lambda_1^{Prev}$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times \frac{13}{4} = 9,75$	$12 \times 4 = 48$	6
2	10	$2 \times \frac{13}{4} = 6,5$	$2 \times 4 = 8$	6,5
3	20	$8 \times \frac{13}{4} = 26$	$5 \times 4 = 20$	20
				$S = 32,5 > E$

LastComputing Process: $\lambda_1^* \leftarrow \lambda_1^{Prev} = \frac{13}{4}, \lambda_2^* \leftarrow \frac{7}{2}$

Agent	c_i	$w_i^1 \times \lambda_1^*$	$w_i^2 \times \lambda_2^{Prev}$	x_i
1	6	$3 \times \frac{13}{4} = 9,75$	$12 \times \frac{7}{2} = 42$	6
2	10	$2 \times \frac{13}{4} = 6,5$	$2 \times \frac{7}{2} = 7$	6,5
3	20	$8 \times \frac{13}{4} = 26$	$5 \times \frac{7}{2} = 17,5$	17,5
				$S = 20 = E$

Table 4.3 – Parameters computed at each the iteration *It* of Algorithm 6 on the double weighted bankruptcy situation of Example 6, with $\lambda_1^S \leftarrow \{2, \frac{5}{2}, 5\}$ and $\lambda_2^S \leftarrow \{\frac{1}{2}, 4, 5\}$.

4.4 DWCEA applied to CO_2 emissions permits

In the context of global climate negotiations, an imperative step is to find an agreement or a strategy to be adopted for the reduction of CO_2 emissions. However, CO_2 quantities permits are limited and countries have to find a common consensus over methods to allocate emission permits. In this section, we analyse EU countries claims on CO_2 emission permits taking into account the limits in emission of CO_2 recommended by the Kyoto protocol [86].

In order to retrospectively determine an allocation method which can be considered both efficient and equitable by EU countries, we focus on data provided by the World Bank Open Data project[1]. Precisely, for each of the five years from 2010 to 2014, we consider a double weighted bankruptcy situation $(N, E^y, c^y, w^{y,1}, w^{y,2})$, $y = 2010, \ldots, 2014$, with the 27 EU countries as the set N of agents, and the following features as vectors of claims and weights: the actual CO_2 emission data from 2010 to 2014, as the vector of claims c^y; the quantity of GDP over the same time interval, as the first weight vector $w^{y,1}$ reflecting the economic growth rate of a country; finally, the production of Renewable energy, as

[1] https://data.worldbank.org/indicator/

the second weight vector $w^{y,2}$ quantifying the sustainability of policies adopted by each country. Figure 4.1 summarizes the total fraction of these three parameters for the 27 EU countries during the period $2010-2014$. According to the Kyoto protocol [86], that imposed by 2010 a reduction of 22% of the CO_2 total emissions in 1990, we set the estate E of each of the five bankruptcy problems considered over the years from 2010 to 2014, equal to the 78% of the total amount of CO_2 emitted by all EU countries in 1990.

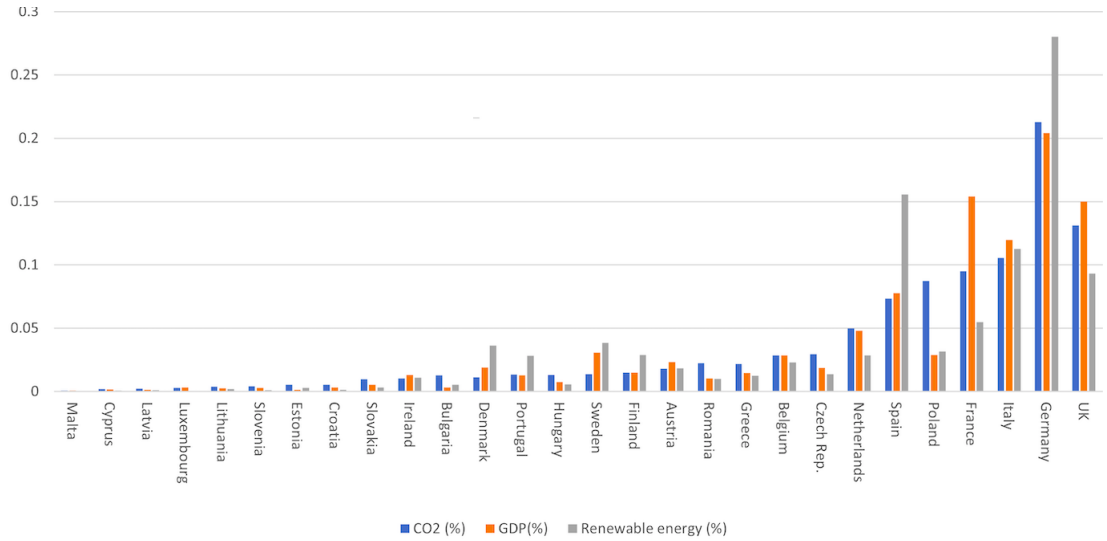

Figure 4.1 – Percentage distribution of total EU CO_2 emissions, GPD and Renewable energy production of each EU country over the interval time of five years from 2010 to 2014 (all data were collected from the World Bank Open Data project in 2020, see https://data.worldbank.org/indicator/).

We applied Algorithm 6 to each bankruptcy situation $(N, E^y, c^y, w^{y,1}, w^{y,2})$ to obtain the DWCEA allocations for years $y = 2010, \ldots, 2014$, as well as the CEA solution over the bankruptcy situations (E^y, c^y), the WCEA solution over the weighted bankruptcy problems $(E^y, c^y, w^{y,1})$ and $(E, c, w^{y,2})$, respectively, for each year from 2010 to 2014. The allocation of CO_2 emission permits generated by those four solutions are reported in Tables 4.7-4.10.

As expected, the allocation yielded by the CEA solution completely satisfies small claims of emission permits. Instead, countries with high demands (i.e, Germany, UK, Italy and France), are drastically limited in their claims and receive the same amount of emission permits. The allocation provided by the WCEA solution based on GDP as the unique weight, favours countries with high GDP by giving more than the half of the estate to the

four countries with the highest GDP (i.e., Germany, UK, France and Italy), while using the Renewable energy as the unique weight, it completely satisfies some countries with intermediate emissions claims, (e.g., Denmark, Sweden, Finland and Portugal), as well as countries having high claims and a good Renewable energy production (e.g., Germany and Spain).

Considering both the GDP and Renewable energy as weights in Algorithm 6 to provide a DWCEA method, the countries respecting a specific threshold of both weights receive their total claims of emission permits, as it happens for Denmark, Sweden, Austria and Spain in 2010, and for France and UK in 2014. Compared to the allocation generated by the WCEA solution based on GDP as the only weight, we can notice that the DWCEA allocation increases the amount of emission permits for 13 countries in 2010, 16 countries in 2011 and 2012 and 19 countries in 2013 and 2014. Instead, compared to the results obtained by the WCEA based on Renewable energy as unique weight, we can observe an improvement in allocations for 17 countries for 2010 and 2011, and at least 14 countries from 2012 to 2014. Furthermore, in 2010, Denmark and Sweeden are completely satisfied whatever allocation method is used, Malta and Austria, in 2010, obtain an equal amount by DWCEA and WCEA based on Renewable energy only, and Ireland, Finland and France, in 2014, receive the same amount of emissions permits according to DWCEA and WCEA based on GDP.

We also notice similarities among allocations of some countries over the five studied years. To be more specific, we investigated those similarities by means of an unsupervised clustering technique, namely, the K-means method, based on the 1-distance notion to measure the similarity of each country's allocation distribution over the years from each cluster's reference centre [77]. This unsupervised clustering technique aims at grouping records (countries) of the data set into K distinct clusters according to their similarities, and each record can only be found in one cluster at a time. Note that countries' emissions are naturally divided in small, medium ,and high emission levels. For this reason, we applied the K-means method with $K = 3$, to define three distinct groups reflecting the impact on CO_2 emissions of different EU countries. The clustering on CO_2 emission records over the five years is reported in Table 4.4(1) and shows a group G1 of countries with low emis-

sions, a group G2 formed by countries with medium emissions and, finally, a group G3 containing high emissions countries.

The results of the clustering procedure using the CO_2 emissions and the allocation distributions over the five years provided by the considered solutions CEA, WCEA with GDP as weight, and WCEA with renewable energy as weight, and DWCEA are shown in Table 4.4(2), 4.4(3), 4.4(4), and 4.4(5), respectively. Table 4.4(2) shows the clusters in accordance with the allocation distribution over the five years yielded by the CEA solution: notice that countries in groups G1 and G2 have low and medium demands and are totally satisfied with respect to their claims from 2010 to 2014. However, in Group 3, only the Netherlands, which has a medium emission distribution, is totally satisfied over 5 years. Other members of cluster G3 receive the same amount despite their different claims (except Spain, who has been totally satisfied since 2013).

Groups formed according to the allocation distribution over the five years by the WCEA solution based only on GDP, are presented in Table 4.4(3). We observe that group G1 is defined by countries with a low GDP, group G2 by countries with intermediate GDP, and the last group, G3, is formed by countries with high GDP. Two countries from group G2 (i.e., Denmark and Sweden), and only one country from group G3 (i.e., France), are totally satisfied throughout the five years. So, these groups show a high association between emissions permits claims and GDP, whose effect plays in favour of countries with a large internal economic production.

In Table 4.4(4), concerning the clusters on allocation distributions over the five years provided by the WCEA solution based on the renewable energy only over the five years, group G3 contains countries with high emissions, which are also characterized by high levels of renewable energy production. In this group, Spain and Germany obtain their claims for all studied period. Even if the effect of the activities producing CO_2 is still predominant, here, countries with a high production of renewable energy show a similar allocation distribution over time, and those that are fully satisfied for at least three years are in groups with intermediate and high claims and also high renewable energy production at the same time.

Table 4.4(1): Clustering EU country emissions

Group 1	Group 2	Group 3
Malta	Romania	Spain
Cyprus	Greece	Poland
Latvia	Belgium	France
Luxembourg	Czech republic	Italy
Lithuania	Netherlands	Germany
Slovenia		
Estonia		
Croatia		
Slovakia		
Ireland		
Bulgaria		
Denmark		
Portugal		
Hungary		
Sweden		
Finland		
Austria		

Maps: EU country emissions

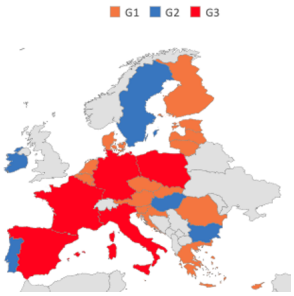

Table 4.4(2): Clustering CEA allocations

Group 1	Group 2	Group 3
Malta	**Romania**	**Netherlands**
Cyprus	**Greece**	Poland
Latvia	**Belgium**	France
Luxembourg	**Czech republic**	Italy
Lithuania	**Denmark**	Germany
Slovenia	**Bulgaria**	Spain
Estonia	**Portugal**	
Croatia	**Hungary**	
Slovakia	**Sweden**	
Ireland	**Finland**	
	Austria	

Maps: CEA allocations

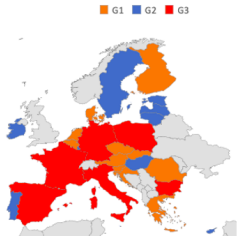

Table 4.4(3): Clustering WCEA GDP allocations

Group 1	Group 2	Group 3
Malta	Austria	Spain
Cyprus	Greece	**France**
Latvia	Belgium	Italy
Luxembourg	Czech republic	Germany
Lithuania	Netherlands	
Slovenia	Poland	
Estonia	Ireland	
Croatia	**Denmark**	
Slovakia	Portugal	
Bulgaria	**Sweden**	
Romania	Finland	
Hungary		

Maps: WCEA GDP allocations

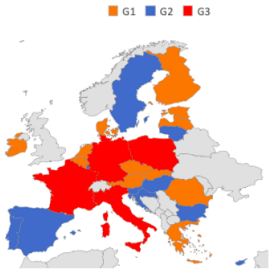

Table 4.4(4): Clustering WCEA renewable energy

Group 1	Group 2	Group 3
Malta	Austria	Italy
Cyprus	Greece	Spain
Latvia	Belgium	Germany
Luxembourg	Czech republic	
Lithuania	Netherlands	
Slovenia	Poland	
Estonia	Finland	
Croatia	**Denmark**	
Slovakia	**Portugal**	
Bulgaria	**Sweden**	
Hungary	France	
Ireland	Romania	

Maps: WCEA renewable energy

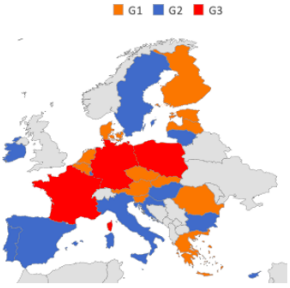

Table 4.4(5): Clustering DWCEA allocations

Group 1	Group 2	Group 3
Malta	**Austria**	Italy
Cyprus	Greece	Spain
Latvia	Belgium	Germany
Luxembourg	Czech republic	France
Lithuania	Netherlands	
Slovenia	Poland	
Estonia	Finland	
Croatia	**Denmark**	
Slovakia	Portugal	
Bulgaria	**Sweden**	
	Hungary	
	Ireland	
	Romania	

Maps: DWCEA allocations

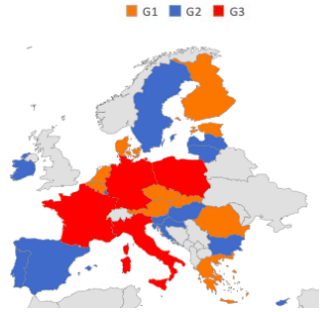

Table 4.4 – Outcomes of the K-means clustering application over allocation records provided by alternative allocation methods. Names in bold indicate countries that are totally satisfied at least for 3 years.

Finally, Table 4.4(5) presents the clusters generated by the allocation distribution over years by the DWCEA allocation method. In this case, clusters are more homogeneous because they are generated by a balance among their claims, their economic levels, and their attitudes to employ green energy. According to the DWCEA allocation distribution over time, only Ireland, Denmark, Sweden, and Austria receive their full claims for at least three years due to their weight on renewable energy production. So, the trade-off between GDP and renewable energy production in the DWCEA allocation method seems to play in favour of countries characterized by intermediate levels of CO_2 claims and GDP and, at the same time, good levels of renewable energy production.

The clusters provided by the K-means analysis can be interpreted as groups of countries equally affected by each proposed allocation method over years. So, it seems reasonable to assume that countries in a cluster are guided by common interests and similar goals when faced with the possibility to accept or not a proposed allocation method. Therefore, we analysed the ability of these groups to influence the decision making process in the EU, and in particular on the voting process in the EU Council, which is the main collegiate body defining the overall political priorities of the EU, over the five year interval 2010–2014. For this reason, we focused on the ability of the different clusters of countries to impose a decision according to the voting system adopted during the same period by the EU Council. According to the EU rules (following the Treaty of Lisbon, effective since 2009, and operative for the voting rule of the EU Council since 2014), a decision is approved by the EU Council if it is supported by a coalition of at least 55% of countries representing at least 65% of the EU population (and keeping into account that a coalition may block a decision if it contains at least four countries globally representing at least 35% of the EU population). Using the calculator provided by the EU Council[2], we found the alternative combinations of clusters that may lead to the approval of a decision according to the voting system of the EU Council, which are summarized by Table 4.5. Considering the clusters generated by K-means just on the emissions distributions over the five years, and assuming that all countries within a cluster may only cooperate within the same cluster, or together with all the countries in another cluster, a decision can be approved when all countries in cluster G1 and G3 cooperate, or when all countries cooperate together (the

[2]https://www.consilium.europa.eu/fr/council-eu/voting-system/voting-calculator

Emissions	CEA	$WCEA_{GDP}$	$WCEA_{Renewable}$	DWCEA
G1+G3	G1+G3	G1+G3	G1+G2	G1+G2
G1+G2+G3	G2+G3	G2+G3	G1+G2+G3	G1+G2+G3
	G1+G2+G3	G1+G2+G3		

Table 4.5 – Clusters combinations that are able to approve a decision according to the EU council voting systems.

grand coalition). Concerning the clusters obtained on the CEA allocation distributions over years, the approval of a new agreement is reached by the collaboration of group G3 with groups G2 or G1, respectively, or when all countries of the three groups cooperate to form a grand coalition. Looking at clusters formed with the allocation distributions yielded by the WCEA solution based only on GDP, we observe the same winning coalitions as for the CEA solution, whereas for clusters obtained over the WCEA solution based on renewable energy, an agreement could be reached either by the coalition of countries in clusters G1 and G2, or by all countries together. A similar configuration of winning coalitions of clusters is obtained with the allocation distribution provided by the DWCEA solution, where again the cluster G3 plays a less relevant role to form winning coalitions in the EU Council. So, even if countries in cluster G3 are never fully satisfied for at least three years according to the DWCEA solution, their possible common interest to increase their own allocation would be prevented by the voting mechanism of the EU Council, which makes cluster G3 non-decisive in forming a winning coalition. A similar argument holds for countries in cluster G1 that are never totally satisfied, as well as for the group formed by clusters G1 and G3 together. On the contrary, the group formed by G1 and G2 together could form a blocking coalition against the use of the DWCEA solution, but the fact that group G2 contains many countries that are totally satisfied using the DWCEA method, makes the formation of such a coalition less likely.

As we already observed, the CEA rule fully satisfies countries with a low claim of CO_2 emissions permits, but it might encounter an objection to its application from countries with larger claims, who see their demands of CO_2 emissions permits strongly reduced. In order to mitigate this effect, and improve the acceptability of an allocation by countries with larger claims, the DWCEA solution can be seen by EU governments and populations as a fair compromise, keeping into account both the efficiency in production of countries, as

	CEA	WCEA_{GDP}	WCEA_{Ren}	DWCEA
8 EU countries (Sh-Sh $\geq 3\%$)	0.69	0.95	0.82	0.90
17 EU countries (Sh-Sh $< 3\%$)	0.89	0.77	0.79	0.78

Table 4.6 – Pearson correlation coefficients for CO_2 allocation vectors provided by the four allocation rules considered in this paper over the interval 2010-2014 and for two groups of countries (groups are based on the Sh-Sh power index, cut-off 3%).

represented by the GDP, and their environmental impact, measured by the rate of renewable energy production. To measure the level of acceptability of the different allocations of CO_2 emission permits, we compared the allocation vectors over the years 2010–2014 with the Sh–Sh power index [75] computed on the simple game representing the EU Council voting rule. This power index is used as a benchmark to represent the actual shares of power of the EU countries according to the voting rule established by the Lisbona treatment (the data of the Sh–Sh power index used in this paper are from the paper [48], Table A1 on page 7, and refer to the population data of 2015). It is in fact well established that the Sh–Sh power index in a simple game is an appropriate measure of the P-power of voters (see, for instance, [28, 48]), which is a measure for evaluating the outcome that each voter in a simple game can expect before playing the game.

Precisely, to assess the association between the Sh–Sh power index distribution and the CO_2 emission permits allocation vectors, we computed the Pearson correlation between the vector yielded by the Sh–Sh power index of the EU Council and the sum of CO_2 allocations over the interval 2010–2014 for the most powerful EU countries having Sh–Sh power index larger or equal to 3% of the total power, as well as for the remaining countries (see Table 4.6; the choice of the cut-off follows from the fact that countries with a Sh–Sh index larger or equal than 3% form the smallest set of countries having two-thirds of the total power).

Figures 4.2 and 4.3 show the distribution (in percentage) of total CO_2 allocation vectors over the five years for the most powerful EU countries and the least powerful ones, respectively. In Figure 4.2, we observe the systematic cut operated by the CEA rule (dotted line) on the demand of CO_2 permits for the six most powerful countries (precisely, for Poland, Spain, Italy, UK, France and Germany), while the the adoption of the DWCEA rule shows an increase of the association between the allocation of CO_2 and the Sh-Sh power index

of more than 20% in terms of the Pearson correlation (see Table 4.6 for the correlation coefficients of the different CO_2 allocation vectors versus the Sh-Sh index vector). On the other hand, for the group of countries with Sh-Sh index smaller than 3%, the loss of correlation with the Sh-Sh power index faced by the DWCEA allocation with respect to the CEA allocation is much less important, and is in the order of 10% (see Table 4.6). So, we argue that the loss of acceptability of the DWCEA versus the CEA for the groups of less powerful countries, is largely compensated by the double gain of acceptability for the most powerful ones, and further justified by the use of criteria based on production efficiency and environmental preservation, that may further encourage small countries to adopt better technologies to improve their energetic policies.

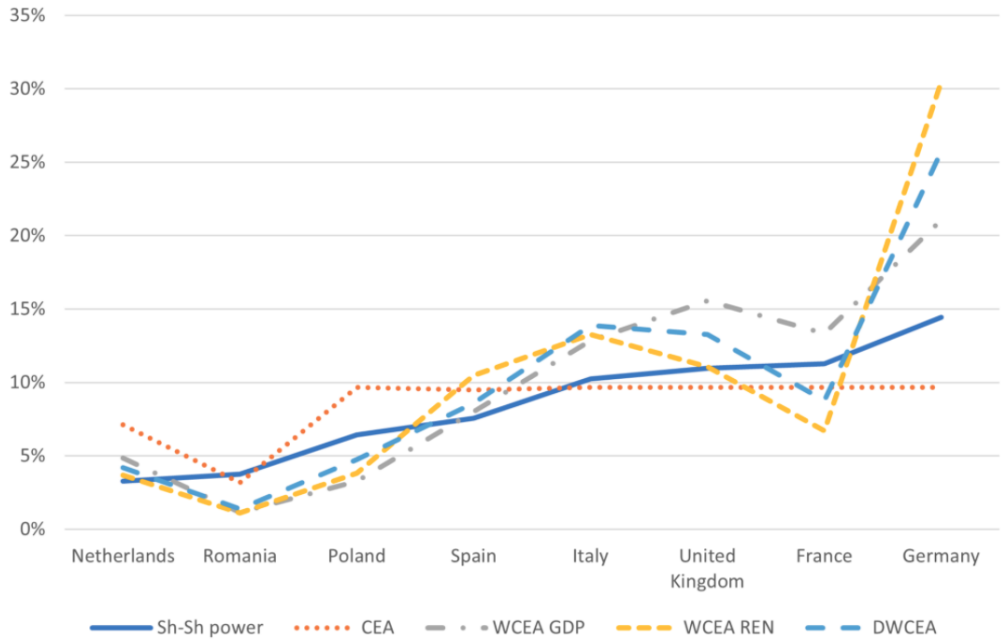

Figure 4.2 – Percentages of total CO_2 emission permits for the eight most powerful countries (Sh-Sh power index $\geq 3\%$) for the four allocation rules considered in this paper. Countries are ordered according to the Sh-Sh power index (solid line)

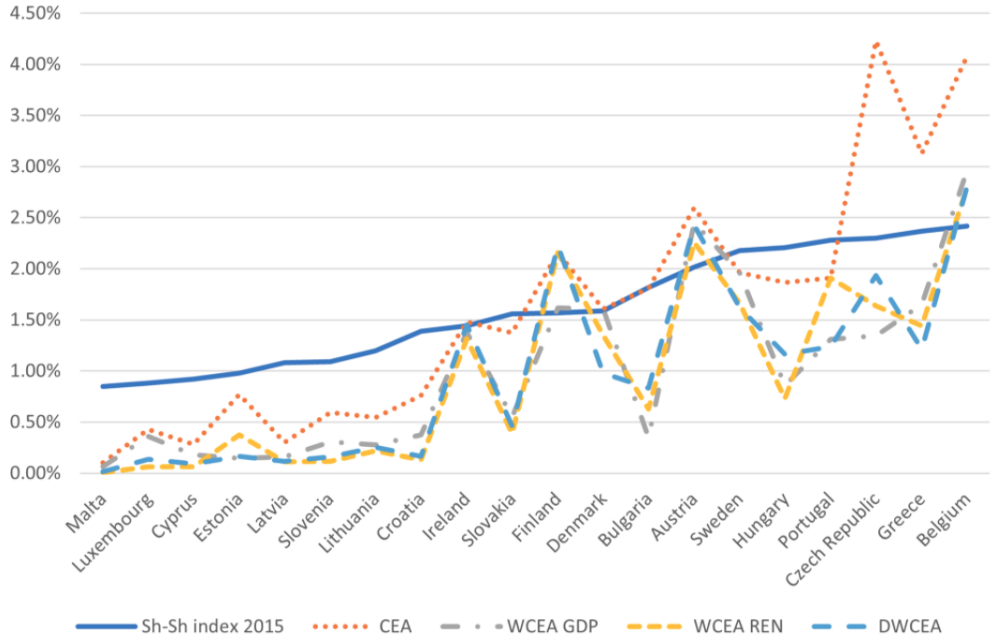

Figure 4.3 – Percentages of total CO_2 emission permits for EU countries with Sh-Sh power index strictly smaller than 3% for the four allocation rules considered in this paper. Countries are ordered according to the Sh-Sh power index (solid line)

4.5 Allocations results

In this section, we provide all the allocations results generated by the CEA, WCEA with GDP as the single weight, WCEA with renewable energy as the single weight, and DWCEA from the year 2010 to 2014.

Country	CEA2010	CEA2011	CEA2012	CEA2013	CEA2014
Austria	67,502	65,02	62,273	62,486	58,712
Belgium	110,824	99,944	95,107	96,97	93,351
Bulgaria	44,114	49,347	44,708	39,6	42,416
Croatia	20,172	19,809	17,994	17,55	16,843
Cyprus	7,708	7,426	6,92	5,948	6,062
Czech republic	111,579	106,908	101,03	98,675	96,475
Denmark	46,641	40,645	36,428	38,533	33,498
Estonia	18,108	18,606	17,624	19,893	19,519
Finland	62,082	56,816	49,134	47,22	47,301
France	220,42	227,373	236,057	242,088	248,389
Germany	220,42	227,373	236,057	242,088	248,389
Greece	83,857	79,842	80,043	69,482	67,319
Hungary	50,233	47,843	44,583	42,141	42,086
Ireland	40,055	35,632	35,592	34,855	34,066
Italy	220,42	227,373	236,057	242,088	248,389
Latvia	8,075	7,294	7,063	7,081	6,975
Lithuania	13,469	13,788	13,832	12,64	12,838
Luxembourg	10,968	10,939	10,664	10,051	9,659
Malta	2,56	2,541	2,681	2,34	2,347
Netherlands	183,053	174,168	170,31	173,255	167,303
Poland	220,42	227,373	236,057	242,088	248,389
Portugal	48,137	47,623	46,014	45,427	45,053
Romania	79,413	84,88	81,723	70,945	70,003
Slovakia	36,241	34,525	32,765	33,091	30,678
Slovenia	15,335	15,09	14,782	14,151	12,812
Spain	220,42	227,373	236,057	237,035	233,977
Sweden	52,024	51,734	47,048	44,847	43,421
UK	220,42	227,373	236,057	242,088	248,389

Table 4.7 – Allocation vectors provided by CEA solution for each year in the interval 2010 – 2014 (values are in Millions of tons of CO_2),

Country	WCEA[1]2010	WCEA[1]2011	WCEA[1]2012	WCEA[1]2013	WCEA[1]2014
Austria	57,417	59,44	60,14	62,486	58,712
Belgium	70,846	72,66	73,134	69,482	73,843
Bulgaria	7,415	7,916	7,918	11,258	7,893
Croatia	8,766	8,6	8,325	11,728	8,018
Cyprus	3,745	3,781	4,131	5,948	4,359
Czech republic	30,398	31,428	30,461	42,28	28,912
Denmark	46,641	40,645	36,428	38,533	33,498
Estonia	2,856	3,195	3,385	5,075	3,25
Finland	36,306	37,732	37,707	47,22	37,926
France	353,033	331,805	333,228	302,278	303,276
Germany	500,647	518,086	520,573	485,25	541,275
Greece	43,86	39,68	36,086	48,43	32,976
Hungary	19,182	19,41	18,781	27,301	19,494
Ireland	32,519	32,954	33,134	34,855	34,066
Italy	311,348	313,84	304,476	334,097	299,357
Latvia	3,481	3,891	3,678	4,863	3,648
Lithuania	5,439	5,994	6,294	9,372	6,75
Luxembourg	7,796	8,273	8,309	10,51	9,228
Malta	1,281	1,311	1,353	2,048	1,563
Netherlands	122,542	123,225	121,763	98,675	122,378
Poland	70,227	72,912	73,498	105,847	75,847
Portugal	34,914	33,764	31,782	27,302	31,947
Romania	24,418	25,419	25,216	38,675	27,754
Slovakia	13,113	13,537	13,722	19,883	6,943
Slovenia	7,035	7,072	6,809	9,715	6,943
Spain	209,75	205,165	196,247	173,255	191,561
Sweden	52,024	51,734	47,048	44,847	43,421
UK	357,657	361,187	391,03	363,443	419,818

Table 4.8 – Allocation vectors provided by the WCEA solution with GDP as unique weight for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2).

Country Name	WCEA^{2}2010	WCEA^{2}2011	WCEA^{2}2012	WCEA^{2}2013	WCEA^{2}2014
Austria	67,502	58,969	50,324	48,690	49,245
Belgium	66,504	72,797	68,655	66,084	65,710
Bulgaria	7,864	22,121	14,280	16,645	15,340
Croatia	1,850	2,278	2,889	3,821	5,126
Cyprus	0,785	1,584	1,747	1,913	1,7473
Czech republic	33,488	47,027	40,346	38,469	40,034
Denmark	46,641	9,058	36,428	38,533	33,498
Estonia	10,940	10,224	9,754	6,986	7,507
Finland	62,082	56,816	49,134	47,22	47,301
France	166,551	174,478	165,301	151,862	159,724
Germany	758,86	732,498	739,861	756,900	719,883
Greece	32,940	14,122	39,069	46,824	42,449
Hungary	30,466	11,190	16,537	15,066	15,649
Ireland	33,661	35,632	30,273	29,423	31,314
Italy	278,077	330,482	342,210	345,318	320,411
Latvia	1,237	1,690	2,732	3,6391	4,465
Lithuania	3,991	5,624	5,166	5,880	6,129
Luxembourg	1,710	1,637	1,421	1,469	1,604
Malta	0,011	0,089	0,177	0,205	0,413
Netherlands	119,325	109,138	84,350	70,613	63,918
Poland	85,718	96,155	100,880	85,585	97,343
Portugal	48,137	47,623	46,014	45,427	45,053
Romania	4,486	36,768	19,439	30,377	45,872
Slovakia	7,369	10,865	9,319	8,805	11,134
Slovenia	2,485	2,830	2,923	2,814	2,861
Spain	270,911	270,548	264,779	237,035	233,977
Sweden	52,024	15,683	47,048	44,847	43,421
UK	239,037	256,729	243,602	284,206	323,526

Table 4.9 – Allocation vectors provided by the WCEA solution with Renewable energy production as unique weight for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2).

Country	DWCEA2010	DWCEA2011	DWCEA2012	DWCEA2013	DWCEA2014
Austria	67,5	65,02	59,287	44,810	58,712
Belgium	72,61	82,104	80,883	31,792	73,549
Bulgaria	8,59	10,012	36,428	38,533	7,861
Croatia	2,02	2,569	3,403	4,172	7,986
Cyprus	0,86	1,786	2,058	2,398	4,342
Czech republic	36,56	39,747	47,532	82,840	28,798
Denmark	46,64	10,217	19,482	9,2545	33,498
Estonia	3,9	4,040	4,107	4,789	3,237
Finland	49,56	47,721	48,781	87,009	37,775
France	181,84	196,786	194,740	190,366	303,276
Germany	683,42	655,230	631,695	622,818	539,121
Greece	35,96	15,927	22,901	39,811	32,845
Hungary	26,18	12,620	38,566	44,847	19,416
Ireland	36,75	35,632	35,592	34,855	34,066
Italy	303,6	372,737	369,469	345,318	298,166
Latvia	1,35	1,907	3,219	3,997	3,634
Lithuania	4,36	6,343	6,086	7,371	6,722
Luxembourg	1,87	1,847	1,674	1,841	9,191
Malta	0,012	0,100	0,208	0,379	1,557
Netherlands	130,28	123,091	99,371	34,755	121,891
Poland	93,58	92,213	89,186	226,032	75,545
Portugal	47,66	42,702	9,607	18,886	31,820
Romania	4,9	32,148	43,789	61,035	27,644
Slovakia	8,05	12,254	10,978	11,038	13,988
Slovenia	2,71	3,191	3,443	3,528	6,915
Spain	270,91	259,474	238,137	88,516	190,798
Sweden	52,02	17,688	47,048	37,522	43,421
UK	260,966	289,548	286,982	356,265	418,879

Table 4.10 – Allocation vectors provided by the DWCEA solution for each year in the interval $2010-2014$ (values are in Millions of tons of CO_2).

4.6 Detailed algorithms

In this section, we provide the pseudo-code of all relevant routines and functions that are used by Algorithm 6 to compute an allocation of the DWCEA solution as explained in Section 3.

Algorithm 7 Function Firstλ_j^S for $j = 1$ or $j = 2$

Input: λ^{Prev}, λ_j^S : a vector of sorted λ_j for $j = 1$ or $j = 2$
Output: X

if $S > E$ **then**
  %We are looking for the first $\lambda < \lambda^{Prev}$
  $i \leftarrow 0$
  while $\lambda_j^S[i] < \lambda_j^{Prev}$ **do**
    $i \leftarrow ++$
  end while
  $X \leftarrow \lambda_j^S[i-1]$
else
  %We are looking for the first $\lambda > \lambda^{Prev}$
  $i \leftarrow N$
  while $\lambda_j^S[i] > \lambda_j^{Prev}$ **do**
    $i \leftarrow --$
  end while
  $X \leftarrow \lambda_j^S[i+1]$
end if
return X

Algorithm 8 Allocations' sum is higher than Estate

Input: Estate E, Set of players N, Claims vector c, weight vectors w_1, w_2
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

while $S > E$ and $It \leq \frac{N}{2} + 1$ **do**
  It ++;
  $X \leftarrow First\lambda_1^S$
  $Y \leftarrow First\lambda_2^S$
  if $X > Y$ **then**
    $\lambda_1^{Prev} \leftarrow X$
  else
    $\lambda_2^{Prev} \leftarrow Y$
  end if
  ForEach $i \in N$: $x_i = \min(c_i, w_i^1\lambda_1^{Prev}, w_i^2\lambda_2^{Prev})$
  $S = \sum_{i \in N} x_i$
  if $S = E$ **then**
    $\lambda_1^* = \lambda_1^{Prev}$
    $\lambda_2^* = \lambda_2^{Prev}$
    ForEach $i \in N$:
    $x_i = min(c_i, w_i^1\lambda_1^*, w_i^2\lambda_2^*)$
  else
    LastComputing
  end if
end while
return x

Algorithm 9 Estate is higher than allocations' sum

Input: Estate E, Set of players N, Claims vector c, weight vectors w_1, w_2
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

while $S < E$ and $It \leq \frac{N}{2} + 1$ **do**
  It ++;
  $X \leftarrow First\lambda_1^S$
  $Y \leftarrow First\lambda_2^S$
  if $X < Y$ **then**
    $\lambda_1^{Prev} \leftarrow X$
  else
    $\lambda_2^{Prev} \leftarrow Y$
  end if
  ForEach $i \in N$: $x_i = \min(c_i, w_i^1\lambda_1^{Prev}, w_i^2\lambda_2^{Prev})$
  $S = \sum_{i \in N} x_i$
  if $S = E$ **then**
    $\lambda_1^* = \lambda_1^{Prev}$
    $\lambda_2^* = \lambda_2^{Prev}$
    ForEach $i \in N$:
    $x_i = min(c_i, w_i^1\lambda_1^*, w_i^2\lambda_2^*)$
  else
    LastComputing
  end if
end while
return x

Algorithm 10 LastComputing Algorithm DWCEA

Input: Estate E, Set of players N, Claims vector c, weight vectors w_1, w_2
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

if $(\lambda_1^{Prev} = LowerBound\lambda_1^S)$ **OR** $(\lambda_1^{Prev} = UpperBound\lambda_1^S)$ **then**
  $M \leftarrow min(c, \lambda_1^{prev} w^1)$
  $x \leftarrow WCEA(E, M, w^2)$
else
  $M \leftarrow min(c, \lambda_2^{prev} w^2)$
  $x \leftarrow WCEA(E, M, w^1)$
end if
return x

4.7 Conclusion

The most critical point, in climate change study, is to promote all countries to limit their emissions by finding a common strategy. In this chapter, we propose a new bi-criteria

method in order to allocate CO_2 emissions permits among EU countries. By giving the same importance to two parameters(GDP and production of Renewable energy), the DWCEA method ensures a balance between efficiency and green policies. Using basic clustering techniques on allocation records over the years 2010-2014, we showed that the DWCEA allocation method generates allocation distributions over time having similar behaviours within cluster of countries characterized by specific levels of decision influence within the EU Council voting system, and prevent the cluster of larger countries to claim for extra shares of emissions permits.

We also showed that the allocation suggested by the DWCEA solution could encounter a larger level of acceptability, with respect to the other solutions, by most influential or powerful EU countries (according to the well established Shapley and Shubik measure of power [75]), and without drastically affecting the acceptability by less influential countries. The justification of the trade-off in favour of larger and most powerful countries is based on the use of criteria aimed at taking into account both efficiency in production and environmental preservation.

Chapter 5

Double weighted bankruptcy problems with priority

Contents

5.1 Introduction 135

5.2 Double weighted bankruptcy method with priority 137

5.3 A DWCEA solution algorithm with priority over criteria 138

5.4 DWCEA^P applied to CO_2 emissions permits 141

5.5 Fairness 148

5.5.1 Fairness index 148

5.5.2 Fairness index applied to CO_2 allocations 150

5.6 Detailed Algorithms 153

5.7 Conclusion 154

5.1 Introduction

Representing the CO_2 emissions negotiations as a bankruptcy situation requires taking into account, not only the demands of the participating countries but also some other criteria.

The presented DWCEA method, in the previous chapter, transforms the situation into a bi-criteria problem, while providing the same importance to both selected parameters. However, in multi-criteria situations, we can face the criteria priority problem. Hence, the order of criteria may influence the output of the study. In the same way, the double weighted bankruptcy situation generates several solutions according to criteria priority.

In this chapter, we deal with the CO_2 emission problem as a bi-criteria bankruptcy situation, giving a different priority to each of the weights. We have selected two important criteria for countries, the first one represents an economic parameter and describes the wealth of a country (the GDP) and the second one is an environmental and ecological issue through the production of renewable energies. These two parameters are strategic points during negotiations, and each country can argue in favor of one of them according to its environmental policy and strategies. In our implementation of CO_2 emission permit allocations, we started our experiment by prioritizing the first parameter (GDP) over the renewable energy production and then experimented our method with the priority of the second parameter (Renewable Energy) with respect to the first one.

This chapter is organized as follows:

- First, we introduce a formal description of a new version of a prioritized double weighted bankruptcy method. This one gives priority to one parameter over another.

- In Section 3, we make an algorithmic approach to define the prioritized double weighted method. This algorithm proposes according the priority weight, several steps to find an allocations vector satisfying efficiency constraint.

- Then, we adopt the new method to CO_2 emissions permits problem. We consider two situations where a priority is given to the first weight and in a second time, a priority is given to the other weight. We define several countries groups using a clustering method to highlights the impact of prioritized parameters on the voting process.

- In the last part, we introduce three fairness indicators to study the rate of fairness in the proposed bankruptcy methods.

5.2 Double weighted bankruptcy method with priority

The double weighted bankruptcy method with priority over criteria, DWCEA^P, assumes that each weight (criteria) has a different priority. Unlike the method presented in the previous chapter, with this method one of the weights has more influence and therefore has more impact on the final allocations. The more this criteria is respected the more the agent will benefit.

The formal presentation of this method does not change from the formal presentation of the DWCEA method without priority over weights, except that here, we don't discuss w^1 and w^2 but rather priority weight w^P and non-priority weight w^{NP}. Considering a double weighted bankruptcy problem (E, c, w^P, w^{NP}), a *Double Weighted Constrained Equal Awards (DWCEA)*, associates to any doubly weighted bankruptcy situation a particular allocation vector $x \in \mathbb{R}^N_{\geq 0}$ satisfying the following conditions:

$$x_i(E, c, W) = min(c_i, \lambda^{P^*} w^P, \lambda^{NP^*} w^{NP}) \tag{5.1}$$

where $\lambda^{P^*}, \lambda^{NP^*} \in R^N$ such as

$$\sum_{i \in N} min(c_i, \lambda^{P^*} w^P, \lambda^{NP^*} w^{NP}) = E \tag{5.2}$$

The proposed method differs from the priority-augmented weighted constrained equal awards (PWCEA) method [79] in which agents are divided into priority classes and the estate is first shared among the applicants in the first priority class using the WCEA method, and if there is any part of the estate left after allocating the first priority class, it is shared among the applicants in the second priority class. And the distribution continues until the estate is depleted.

The main difference between DWCEA^P and PWCEA is that in the DWCEA^P method with priority, priority is assigned to weights. These latter represent the agents characteristics that need to be taken into account during allocations. Whereas the weights in the PWCEA method represent the priority classes of agents.

The idea of DWCEA^P with priority method is to define the weight that will be promote and try to find an allocation vector respecting the most the ratio $\frac{c}{w^P}$. While studying our new method, we observed that it respects some basic axioms of bankruptcy problems . First, Equal Treatment (ET), if two agents $i, j \in N$ have the same demand $c_i = c_j$, the same weights $w_i^P = w_j^P$ and $w_i^{NP} = w_j^{NP}$ then they should receive the same allocation $x_i = x_j$. Then, Invariance Under Claim (IUC), for all claims exceeds the estate, cutting off each claim from the estate does not influence the allocation vector.

5.3 A DWCEA solution algorithm with priority over criteria

In this section, we present an algorithmic approach of DWCEA^P. The goal of Algorithm 11 is to find two feasible λ^{P^*} and λ^{NP^*} where λ^P is a ratio of $\frac{c}{w^P}$ and λ^{NP} is obtained from $\frac{c}{w^{NP}}$ such that w^P is the priority weight and w^{NP} is the non priority weight.

Algorithm 11 Double Weighted method Priority

Input: Estate E, Set of player N, Claims vector c, weight vectors w_1, w_2
Output: an allocation vector $x \in \mathbb{R}^N_{\geq 0}$

PriorityChoiceProcedure
ForEach $(i \in N)$: $\lambda_i^P = \frac{c_i}{w_i^P}$, $\lambda_i^{NP} = \frac{c_i}{w_i^{NP}}$
$\lambda^{P,S} \leftarrow SortIncr\lambda^P$, $\lambda^{P,S} \leftarrow SortIncr\lambda^{NP}$
$It \leftarrow 0$
while $It \leq N$**AND** $S < E$ **do**
  It++;
  $j \leftarrow 0$
  while $j \leq N$**AND** $S < E$ **do**
    j++;
    ForEach $i \in N$: $x_i = \min(c_i, w_i^P \lambda_{It}^P, w_i^{NP} \lambda_j^{NP})$
    $S = \sum x_i$
  end while
end while
if S==E **then**
  $\lambda^{P^*} = \lambda_{It}^P$
  $\lambda^{NP^*} = \lambda_j^{NP}$
  ForEach$(i \in N) : x_i = min(c_i, w_i^P \lambda^{P^*}, w_i^{NP} \lambda^{NP^*})$
else
  $FinalProcedure$
end if
return X

At the beginning of Algorithm 11, a procedure to define which weight is promoted is launched. If we promote the first weight, all the value of w^1 are inherited by w^P and w^{NP} receives all w^2 values. If the choice of priority is for the second weight, then the procedure is reversed, $w^P = w^2$ and $w^{NP} = w^1$, as shown in Algorithm 12. For each player $i \in N$, $\lambda_i^P = \frac{c_i}{w_i^P}$ and $\lambda_i^{NP} = \frac{c_i}{w_i^{NP}}$ are defined and sorted in incremental order providing $\lambda^{P,S}$ and $\lambda^{NP,S}$. The *It* counter is the priority weight counter and it starts with the lowest $\lambda^{P,S}$. The counter j is the counter of the non-priority weight, and starts with the smallest $\lambda^{NP,S}$ until it reaches the largest one. Before increasing the counter of the non-priority weight, the allocation vector is calculated and the sum $S = \sum_{i \in N} \min(c_i, w_i^P \lambda_{It}^P, w_i^{NP} \lambda_j^{NP})$ is checked:

- If $S = E$, all the estate is allocated and we have an allocation vector respecting the efficiency constraint. In this situation, $\lambda^{P^*} = \lambda_{It}^P$ and $\lambda^{NP^*} = \lambda_j^{NP}$.

- If $S < E$, the counter of the priority weight *It* must increase and the counter j is reset to the smallest $\lambda^{NP,S}$ in order to check all possibilities until obtain an all the estate is allocated $S = E$, the sum of allocations exceeds the estate $S > E$ or we test all λ_{It}^P and λ_j^{NP} values ($j = N$ and $IT = N$) and we do not reach an efficient allocation solution.

- If $S > E$, a new procedure *FinalProcedure* is triggered. It is developed in Algorithm 13. It starts by checking if all the values of all λ_{It}^P and λ_j^{NP} have been tested, if this is the case, it determines a value m_i for each player $i \in N$ the minimal value between its request c_i and its priority weight w_i^P according to the last λ_{It}^P and thus transforms the situation of two weights to a situation of bankruptcy with only one weight $WCEA(E, m, w^{NP})$ with the vector $m = (m_1, m_2 \ldots m_n)$. If this is not the case, it looks for an intermediate λ^{test} value of the last two λ_j^{NP} and λ_{j-1}^{NP} tested until finding an allocation vector respecting the efficiency constraint.

Example 8 *Consider the double weighted bankruptcy situation* (E, c, w^1, w^2) *with* $N = \{1,2,3,4,5,6\}$ *as the set of agents, the estate* $E = 35$ *and the other parameters as shown in Table 5.1. In this example, we highlight the different steps of DWCEA*^{P} *method while* w^1 *is the priority criteria.*

Agent i	c_i	w_i^1	$\lambda_i^1 = \frac{c_i}{w_i^1}$	w_i^2	$\lambda_i^2 = \frac{c_i}{w_i^2}$
1	12	3	4	4	3
2	14	7	2	2	7
3	3	1	3	4	0.75
4	20	20	1	40	0.5
5	2	4	0.5	8	0.25
6	12	3	4	6	2

Table 5.1 – A double weighted bankruptcy situation (E,c,w^1,w^2) with $N = \{1,2,3,4,5\}$ as set of agents and estate $E = 35$.

Agent_i	c_i	$w_i^P \times \lambda^P$
1	12	$3 \times 0.5 = 1.5$
2	14	$7 \times 0.5 = 3.5$
3	3	$1 \times 0.5 = 0.5$
4	20	$20 \times 0.5 = 10$
5	2	$4 \times 0.5 = 2$
6	12	$3 \times 0.5 = 1.5$

Table A: Computation of the priority weight $w^P = w^1$ according to $\lambda^P_{=} 0.5$

$w_i^{NP} \times 0.25$	$w_i^{NP} \times 0.5$	$w_i^{NP} \times 0.75$	$w_i^{NP} \times 2$	$w_i^{NP} \times 3$	$w_i^{NP} \times 7$
$4 \times 0.25 = 1$	$4 \times 0.5 = 2$	$4 \times 0.75 = 3$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 7 = 28$
$2 \times 0.25 = 0.5$	$2 \times 0.5 = 1$	$2 \times 0.75 = 1.5$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 7 = 14$
$4 \times 0.25 = 1$	$4 \times 0.5 = 2$	$4 \times 0.75 = 3$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 7 = 28$
$40 \times 0.25 = 10$	$40 \times 0.5 = 20$	$40 \times 0.75 = 30$	$40 \times 2 = 80$	$40 \times 3 = 120$	$40 \times 7 = 280$
$8 \times 0.25 = 2$	$8 \times 0.5 = 4$	$8 \times 0.75 = 6$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 7 = 56$
$6 \times 0.25 = 1.5$	$6 \times 0.5 = 3$	$6 \times 0.75 = 4{,}5$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 7 = 42$

Table B: Computation of the non priority weight $w^{NP} = w^2$ according to all the sorted λ^2

Agent_i	$X^{\lambda^{NP}=0.25}$	$X^{\lambda^{NP}=0.5}$	$X^{\lambda^{NP}=0.75}$	$X^{\lambda^{NP}=2}$	$X^{\lambda^{NP}=3}$	$X^{\lambda^{NP}=7}$
1	1	1.5	1.5	1.5	1.5	1.5
2	0.5	1	1.5	3.5	3.5	3.5
3	0.5	0.5	0.5	0.5	0.5	0.5
4	10	10	10	10	10	10
5	2	2	2	2	2	2
6	1.5	1.5	1.5	1.5	1.5	1.5
$\sum_{i \in n} x_i$	15,5	16,5	17	19	19	19

Table C: Computation of allocations vector and efficiency constraint check for $\lambda^P_{=} 0.5$

Agent_i	c_i	$w_i^P \times \lambda^P$
1	12	$3 \times 1 = 3$
2	14	$7 \times 1 = 7$
3	3	$1 \times 1 = 1$
4	20	$20 \times 1 = 20$
5	2	$4 \times 1 = 4$
6	12	$3 \times 1 = 3$

Table D: Computation of the priority weight $w^P = w^1$ according to $\lambda^P_{=} 1$

Agent_i	$X^{\lambda^{NP}=0.25}$	$X^{\lambda^{NP}=0.5}$	$X^{\lambda^{NP}=0.75}$	$X^{\lambda^{NP}=2}$	$X^{\lambda^{NP}=3}$
1	1	2	3	3	3
2	0.5	1	1.5	4	6
3	1	1	1	1	1
4	10	20	20	20	20
5	2	2	2	2	2
6	1.5	3	3	3	3
$\sum_{i \in n} x_i$	16	29	30.5	33	**35**

Table E: Computation of allocations vector and efficiency constraint check for $\lambda^P = 1$

Table 5.2 – Exection steps of DWCEA^P solution giving priority to w^1 for Example 8

Since we define w^1 as the priority weight, let's sort $\lambda_i^P = \frac{c_i}{w_i^1}$ and $\lambda_i^{NP} = \frac{c_i}{w_i^2}$ into incremental order such as $\lambda^P = \{0.5,1,2,3,4,4\}$ and $\lambda^{NP} = \{0.25,0.5,0.75,2,3,7\}$. In the Table 5.2, we illustrate all steps of the proposed method considering the priority to the first weight w^1. In Table A, we compute for every agent his priority criteria w^P and the first classified $\lambda^P = 0,5$. In Table B, we compute for every agent his non priority criteria w^{NP} and all the λ^{NP}in incremental order. Then, we define allocations in each distribution generated by Table A and Table B, while the allocations sum does not reach the estate, as shown in Table C. Since we tested all the value of λ^{NP} in Table C and the estate is not totally allocated, we increase the value of λ^P to the second lowest value i.e $\lambda^P = 1$, as shown in Table D and we have to compute the allocations with this new value, Table E.

5.4 DWCEA^P applied to CO_2 emissions permits

In order to reach a consensus among countries, the study of CO_2 emission permits should not be limited to a single parameter nor should recognize the importance of certain parameters in establishing countries' strategies. Throughout this manuscript, we have treated the problem of emissions by different bankruptcy methods, considering the estate as an emission quantity defined by the Kyoto protocol [86], the agents as the European countries, and the claims are associated with the emitted quantity of CO_2 for the year 2010 to the year 2014. The different developed and explained methods dealt with several aspects of the situation. However, one issue has yet been unaddressed, which remains the priority of the criteria, defined as weights in our study. We based our work on the data of the World Data Bank[1], and we define double weighted bankruptcy situation $(N, E^y, c^y, w^{P,y}, w^{NP,y})$ where the 27-EU countries are the set N of agents, the CO_2 emission data as the vector of claims c^y, the quantity of GDP as the first weight vector and the production of renewable energy, as the second weight vector for each year $y = 2010,\ldots,2014$. Then, we focus on the two possible orders of criteria priority. First, we assume that an economic parameter such as GDP is a priority criterion, w^P, and summarize its allocations in the Table 5.3. And then, we generate a new distribution of CO_2 permits by giving priority to an environmental criterion, w^{NP}, which is the production of renewable energies, as shown in the Table 5.4.

[1] https://data.worldbank.org/indicator/

Country	claims$_{2010}$	DWCEA$^{P=GDP}_{2010}$	claims$_{2011}$	DWCEA$^{P=GDP}_{2011}$	claims$_{2012}$	DWCEA$^{P=GDP}_{2012}$	claims$_{2013}$	DWCEA$^{P=GDP}_{2013}$	claims$_{2014}$	DWCEA$^{P=GDP}_{2014}$
Malta	2,56	0,014	2,54	0,13	2,68	0,28	2,34	0,38	2,35	1,30
Cyprus	7,708	0,987	7,43	2,40	6,92	2,76	5,95	3,59	6,06	4,35
Latvia	8,075	1,555	7,29	2,56	7,06	3,81	7,08	3,50	6,98	3,64
Luxembourg	10,968	2,151	10,94	2,48	10,66	2,25	10,05	2,76	9,66	5,05
Lithuania	13,469	5,018	13,79	6,48	13,83	6,52	12,64	6,74	12,84	6,74
Slovenia	15,335	3,124	15,09	4,28	14,78	4,63	14,15	5,29	12,81	6,93
Estonia	18,108	3,357	18,61	3,45	17,62	3,50	19,89	3,65	19,52	3,24
Croatia	20,172	2,326	19,81	3,45	17,99	4,57	17,55	7,18	16,84	8,00
Slovakia	36,241	9,265	34,53	14,64	32,77	14,21	33,09	14,31	30,68	14,02
Ireland	40,055	38,230	35,63	**35,63**	35,59	34,31	34,86	34,78	34,07	**34,07**
Bulgaria	44,114	8,717	49,35	8,56	44,71	8,20	39,60	8,10	42,42	7,88
Denmark	46,641	**46,641**	40,65	**40,65**	36,43	**36,43**	38,53	**38,53**	33,50	**33,50**
Portugal	48,137	41,047	47,62	16,93	46,01	32,91	45,43	32,85	45,05	31,89
Hungary	50,223	22,551	47,84	16,93	44,58	19,45	42,14	19,65	42,09	19,46
Sweden	52,024	**52,024**	51,73	23,73	47,05	**47,05**	44,85	**44,85**	43,42	**43,42**
Finland	62,082	42,683	56,82	40,80	49,13	41,63	47,22	39,23	47,30	37,86
Austria	67,502	**67,502**	65,02	64,27	62,27	**62,27**	62,49	**62,49**	58,71	**58,71**
Romania	79,413	5,640	84,88	27,48	81,72	26,11	70,95	27,83	70,00	27,71
Greece	83,857	41,416	79,84	21,37	80,04	37,37	69,48	34,85	67,32	32,92
Belgium	110,824	83,289	99,94	78,56	95,11	75,73	96,97	75,69	93,35	73,72
Czech republic	111,579	35,737	106,91	33,98	101,03	63,85	98,68	30,42	96,48	28,86
Netherlands	183,053	144,065	174,17	133,24	170,31	126,08	173,26	125,92	167,30	122,17
Spain	270,911	246,590	270,55	221,84	264,78	203,21	237,04	197,87	233,98	191,23
Poland	316,264	82,561	317,00	78,84	299,93	76,10	302,28	76,17	285,74	75,72
France	353,033	209,409	331,81	264,01	333,23	261,61	334,10	285,21	303,28	**303,28**
Italy	405,361	349,632	397,99	339,34	369,47	315,27	345,32	309,55	320,41	298,84
Germany	758,86	588,581	732,50	560,18	739,86	539,04	757,31	545,21	719,88	540,34
UK	493,208	300,547	447,83	388,46	468,57	385,53	458,25	398,08	419,82	**419,82**

Table 5.3 – DWCEA^P allocations with priority on GDP from 2010 to 2014. Bold text represents countries that have received all their claims.

Analyzing allocations generated by the DWCEA^P with GDP as a priority, shown in Table 5.3, Denmark is totally satisfied all the studied period. Sweden and Austria receive their claims throughout the studied years except in 2011.

In the first place, we compare our results with allocations generated by WCEA based on GDP as unique weight, shown in Table 4.8. We can observe that over the years, the number of countries that prefer to opt to a double weighted method with a priority set on the GDP is increasing, they were 16 in 2010, 17 in 2011, and more than 20 in 2012-2014.

Compared to DWCEA allocations without priority, in Table 4.10, we can notice that in 2010, 15 countries obtain better allocations with DWCEA^P with GDP as priority criteria, in 2011, 21 countries can ameliorate their allocations by using DWCEA^P priority to GDP. We can also note that most of the countries with low and medium demands have an improvement of the emissions by opting for a prioritization of the GDP.

We can point out that more than $\frac{4}{5}$ of the estate is held by the 7 countries that have more emissions demand but this amount is still lower than what these countries can obtain using the DWCEA without priority and WCEA based on GDP.

Country	$claim_{2010}$	$DWCEA_{2010}^{P=REN}$	$claim_{2011}$	$DWCEA_{2011}^{P=REN}$	$claim_{2012}$	$DWCEA_{2012}^{P=REN}$	$claim_{2013}$	$DWCEA_{2013}^{P=REN}$	$claim_{2014}$	$DWCEA_{2014}^{P=REN}$
Malta	2,56	0,013	2,54	0,10	2,68	0,19	2,34	0,20	2,35	0,45
Cyprus	7,708	0,934	7,43	1,75	6,92	1,89	5,95	1,91	6,06	1,90
Latvia	8,075	1,472	7,29	1,86	7,06	2,95	7,08	3,64	6,98	4,61
Luxembourg	10,968	2,035	10,94	1,81	10,66	1,53	10,05	1,47	9,66	1,74
Lithuania	13,469	4,749	13,79	6,20	13,83	5,58	12,64	5,88	12,84	6,67
Slovenia	15,335	2,957	15,09	3,12	14,78	3,16	14,15	2,82	12,81	3,11
Estonia	18,108	3,527	18,61	4,13	17,62	4,41	19,89	6,99	19,52	4,11
Croatia	20,172	2,202	19,81	2,51	17,99	3,12	17,55	3,82	16,84	5,58
Slovakia	36,241	8,769	34,53	11,98	32,77	10,06	33,09	8,81	30,68	12,11
Ireland	40,055	**40,055**	35,63	**35,63**	35,59	32,68	34,86	29,44	**34,07**	34,07
Bulgaria	44,114	9,159	49,35	10,24	44,71	10,32	39,60	15,74	42,42	9,98
Denmark	46,641	**46,641**	40,65	**40,65**	36,43	**36,43**	38,53	**38,53**	33,50	**33,50**
Portugal	48,137	43,126	47,62	12,34	46,01	41,41	45,43	45,43	45,05	40,41
Hungary	50,223	23,693	47,84	12,34	44,58	17,85	42,14	15,07	42,09	17,02
Sweden	52,024	**52,024**	51,73	17,29	47,05	**47,05**	44,85	**44,85**	43,42	**43,42**
Finland	62,082	44,845	56,82	48,79	49,13	**49,13**	**47,22**	47,22	47,30	**47,30**
Austria	67,502	**67,502**	65,02	**65,02**	62,27	54,33	62,49	48,72	58,71	53,57
Romania	79,413	5,338	84,88	32,87	81,72	20,99	70,95	30,39	70,00	35,11
Greece	83,857	39,197	79,84	15,57	80,04	42,18	69,48	46,85	67,32	41,71
Belgium	110,824	79,137	99,94	80,27	95,11	74,12	96,97	66,12	93,35	71,48
Czech republic	111,579	37,547	106,91	40,63	101,03	43,56	98,68	38,49	96,48	36,57
Netherlands	183,053	141,991	174,17	120,34	170,31	91,07	173,26	70,65	167,30	69,54
Spain	270,911	259,081	270,55	265,27	264,78	255,69	237,04	237,04	233,98	233,98
Poland	316,264	86,743	317,00	94,27	299,93	95,76	302,28	85,63	285,74	95,94
France	353,033	198,188	331,81	192,38	333,23	178,47	334,10	151,94	303,28	173,76
Italy	405,361	330,899	397,99	364,39	369,47	**369,47**	345,32	**345,32**	320,41	**320,41**
Germany	758,86	618,394	732,50	669,86	739,86	678,26	757,31	**757,31**	719,88	684,65
UK	493,208	284,443	447,83	283,07	468,57	263,00	458,25	284,36	419,82	351,96

Table 5.4 – $DWCEA^P$ allocations with priority on renewable energy from 2010 to 2014. Bold text represents countries that have received all their claims.

According to Table 5.4 where the priority is given to renewable energy, Denmark allocations fully cover its demands from 2010 to 2014. Sweden could be satisfied for 4 years (2010, 2012, 2013, and 2014). Other countries like Ireland, Finland, Spain, and Italy obtain their claims for at least 2 years.

We start a comparative study with allocation generated by a double weighted method considering Renewable energy production as a unique parameter, Table 4.9 and we can notice that for all studied years, the majority of countries, more than 18 countries in 2010-2012, prefer to use a double weighted method considering the production of Renewable energy as a priority. In 2013, 15 countries obtained the same allocation with both methods. By looking at the Table 4.10, we can remark that 14 countries in 2010, 9 countries in 2011, and 12 countries in 2012 to 2014 receive better allocations by double weighted method with priority to Renewable energy than the double weighted method without priority. We compare the allocations generated by DWCEA^P with GDP as a priority to allocations based on Renewable energy production as priority criteria and we notice that in 2010, 10 countries prefer giving priority to Renewable energy production, 11 countries in 2011, and 12 countries in 2013 and 2014.

Looking at the allocations, we can see similarities between countries over the 5 years studied. To this end, we employed the same unsupervised clustering technique used in the previous chapter the K-means method.We therefore group the countries into $K = 3$-distinct clusters based on their similarities.

This clustering process was applied to the DWCEA^P methods, considering first the GDP as priority criteria and then the renewable energy production as priority weight. The results are stored in the Table 5.5 where we find in the first line the clusters generated by the method favoring the GDP (table 5.5.1) and in the second line of the table the clusters resulting from the method which offers the priority to the renewable energies (Table 5.5.2).

According to the allocation distribution over the five years yielded by the DWCEA^P solution based on GDP as a priority, we notice that all countries with high GDP are classified in group G3, and countries having a low and medium GDP are in groups G1 and G2. Only some countries in G2, i.e. Denmark, Sweden, and Austria, are fully satisfied for at least 2 years. At the same time, countries in G3 dominate most of the estate.

In Table 5.5.2, concerning the clusters on allocation distributions over the five years provided by the DWCEA^P solution based on the Renewable Energy as a priority criterion over the five years, the distribution of the groups is the most heterogeneous among the other classifications. The majority of countries are grouped in the first cluster G1. This group contains countries with different emission levels: low (Malta, Cyprus), medium (Bulgaria, Hungary) and high (Czech republic), countries with different GDP rates and involvement in renewable energy production as well as fully satisfied countries like Denmark, Sweden, Finland, and Austria. Group G3 includes only two countries: Italy and Germany. These two countries are characterized by a high demand for emissions but also an important GDP rate and are among the most involved countries in the production of renewable energies.

Once we have defined the different groups of the weighted method with priorities, we can study the impact of the votes of these groups in the EU Council. Through the calculator available from the Council of the EU [2], we have determined the alternative combinations of groups that can reach an agreement or a decision. These results are summarized in the Table 5.6. Countries clusters giving the priority to GDP can achieve a decision if G1 and G3 cooperate and work together to find a consensus. In this situation, G2 could foster the negotiation, since if the other two groups reach an agreement, the G2 votes do not change the adoption of the voted strategy.

In the case where the groups are formed under the priority of renewable energy production, G1 and G2 are the decision making groups. If they can come up with a common strategy, they can vote on it and approve it without the agreement of G3. For this reason, the G3 group has an interest in participating in the negotiations.

[2] https://www.consilium.europa.eu/fr/council-eu/voting-system/voting-calculator/

Clustering DWCEA^P allocations

Table 5.5.1: Allocations based on GDP priority

Group1	Group2	Group3
Malta	Denmark	Spain
Cyprus	Sweden	France
Latvia	Finland	Italy
Luxembourg	Austria	Germany
Lithuania	Belgium	
Slovenia	Czech republic	
Estonia	Netherlands	
Croatia	Poland	
Slovakia		
Ireland		
Bulgaria		
Portugal		
Hungary		
Romania		
Greece		

Maps : DWCEA^P

Maps based on GDP priority

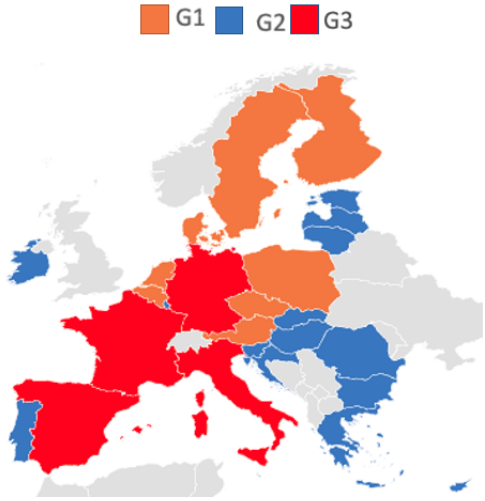

Table 5.5.2: Allocations based on Renewable energy production priority

Group1	Group2	Group3
Malta	Netherlands	Italy
Cyprus	Spain	Germany
Latvia	Poland	
Luxembourg	France	
Lithuania		
Slovenia		
Estonia		
Croatia		
Slovakia		
Ireland		
Bulgaria		
Denmark		
Portugal		
Hungary		
Sweden		
Finland		
Austria		
Romania		
Greece		
Belgium		
Czech republic		

Maps based on Renewable energy production priority

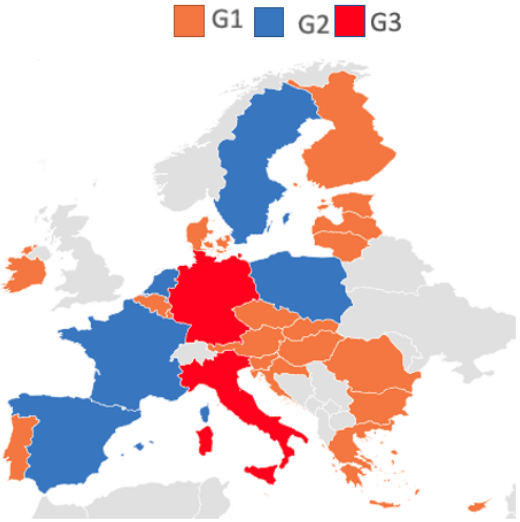

Table 5.5 – Outcomes of the K-means clustering application over DWCEA^P method. Names in red color indicate countries that are totally satisfied at least for 2 years.

GDP as priority criteria	Renewable energy as priority criteria
G1+G3	G1+G2
G1+G2+G3	G1+G2+G3

Table 5.6 – DWCEA^P Clusters are able to approve a decision according to the EU council voting systems.

5.5 Fairness

The allocation rules respect certain fairness criteria [32]. In the case of two agents requesting the same amount of resources, it is logical to give each of them the same portion of the resource. In other cases, a minimum quantity of the resource must be guaranteed to ensure the maximum number of allocated claimant. Studying fairness involves understanding whether one allocation method is more equitable than another. However, fairness does not necessarily imply an equal distribution of resources. In some situations, the allocation of more resources to some agents may be appropriate.

In this section, we present some classical methods to measure the satisfaction of an agent with an allocation. Then, we apply these methods to the allocations generated by our methods. In the problem of resource allocation, it is important on the one hand to find the best solution respecting certain constraints and on the other hand to evaluate its fairness through a fairness index. To this end, based on the work of Jain [43], we estimate the individual satisfaction rate as the proportion of demand that is satisfied by an allocation.

5.5.1 Fairness index

Considering the agent i with demand c_i and an allocation x_i, the **Demand Fraction Satisfaction (DFS) Rate** [32] of i is:

$$DFS_i = \frac{x_i}{c_i} \tag{5.3}$$

This rate represents the percentage of demand that is satisfied and varies between 0 and 1, where rate = 0 is a totally unsatisfied agent while rate = 1 is a totally satisfied agent.

In order to define a new fairness index, the **Player Satisfaction Rate**, first we should present the TU-game of bankruptcy problems. The game is a pair (N,v) where $N = \{1,...,n\}$ denotes the set of players and $v : 2^N \to \mathbb{R}$ is the characteristic function, (by convention, $v(\emptyset) = 0$). The value of each coalition S of players is given by:

$$v(S) = max\{E - \sum_{i \in N \setminus S} c_i, 0\} \tag{5.4}$$

Where E is the estate to be divided satisfying the condition $E \geq 0$ is and $c \in \mathbb{R}_+^N$ is a vector of claims such that $\sum_{i \in N} c_i > E$.

The **Player Satisfaction Rate** [32] allows defining an alternative satisfaction rate, taking into account the demands of other players. This method should satisfy:

- On the one hand the relativity of the demand: a user is fully satisfied when he receives his maximal right. The maximal right for player i is

$$max_i = v(N) - v(N \setminus \{i\}) \tag{5.5}$$

such that $If \quad c_i > E$

$$v(N) - v(N \setminus \{i\}) = E \tag{5.6}$$

Otherwise $If \quad c_i \leq E$

$$v(N) - v(N \setminus \{i\}) = c_i \tag{5.7}$$

- And on the other hand the relative null satisfaction: a user has a null satisfaction when he receives exactly his minimal right. The minimal right for player i is

$$min_i = v(i) \tag{5.8}$$

such that

$$v(i) = max\{E - \sum_{i \in N \setminus \{i\}} c_i, 0\} \tag{5.9}$$

Formally, the **Player Satisfaction Rate** is defined as :

$$PS_i = \frac{x_i - min_i}{max_i - min_i} \tag{5.10}$$

The Player Satisfaction rate $i \in [0,1]$ such that $PS_i = 0$ presents the dissatisfaction and is equal to 1 when $\sum_{i \in N} c_i = E, \forall i \in N$.

The **Gini index** [49], or **Gini coefficient**, is an indicator of economic inequality in a population, measuring the distribution of income or wealth. The coefficient ranges from

perfect equality G=0 to perfect inequality G=1.

$$G = \frac{R}{2M} \tag{5.11}$$

Where $M = \frac{1}{n}\sum_{i\in N} x_i$ and $R = \frac{1}{n^2}\sum_{i\in N}\sum_{j\in N}|x_i - x_j|$

5.5.2 Fairness index applied to CO_2 allocations

In this section, we study the several fairness index on the presented methods throughout this thesis.

In our CO_2 emissions permits problem, we face a situation where $\forall i \in N, \sum_{j\in N\setminus\{i\}} c_j > E$, such that N is the set of EU countries, c is the vector of CO_2 demands and E is the quantity to be shared defined by the Kyoto Protocol, and according to Equation 5.9, the minimal right of a country i $min_i = 0$, and the maximal right $max_i = c_i$. As a result, Equation 5.10 becomes:

$$PS_i = \frac{x_i - 0}{c_i - 0} \tag{5.12}$$

Therefore, according to Equation 5.12 and Equation 5.3, the Player Satisfaction rate is equivalent to the Demand Fraction Satisfaction rate.

In Table 5.7 and in Table 5.8, we have defined the average of the PS rate and the average of the Gini index, respectively, over the 5 studied years for the weighted methods introduced in this manuscript such as WCEA with GDP as weight, WCEA with renewable energies as weight, DWCEA and $DWCEA^P$ for GDP and for renewable energies, as well as the classical bankruptcy method, CEA.

As expected, we can state that countries with a small demand have a $PS = 1$ and therefore their allocations are fully satisfied under the CEA method. If we focus only on the weighted methods, we can note that there are 3 countries fully satisfied with the DWCEA based on the GDP method and 4 countries with the DWCEA based on Renewable energies. We can observe that the PS rate of Denmark is always equal to 1 whatever the allocation method used.

Each weighted allocation method favors some countries over others. For this reason, we focused on the average of each method to rank the satisfaction rate of countries. As we can see in Figure 5.1, the CEA is the most egalitarian method than weighted CEA. The WCEA based on GDP is the most fairness weighted method, then DWCEA and DWCEA^{P}_{GDP}. According to PS rate and DFS rate, the WCEA based on Renewable energy and the DWCEA^{P}_{Ren} are the most unsatisfying methods.

The second fairness index studied is the Gini index. We determine this index for allocations generated by each weighted method and the CEA method from 2010 to 2014, in Table 5.8. The Gini index for CEA is the lowest index for 4 years, and it implies that the relative differences are on average small compared to the average of the demands and the inequalities between countries are low. However, the closer Gini index to 1, the higher inequality is. This is the case for allocations generated by the $WCEA_{Ren}$ method in 2010, 2011, the $DWCA^{P}_{Ren}$ in 2012, the $WCEA_{GDP}$ in 2013 and the DWCEA in 2014 .

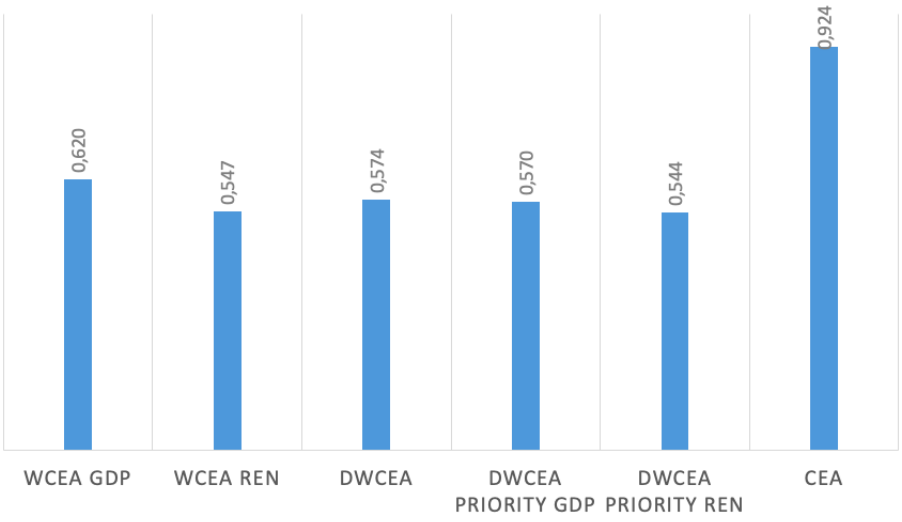

Figure 5.1 – Player Satisfaction rate average of 2010-2014 for CEA and weighted allocation methods.

Year	$WCEA_{GDP}$	$WCEA_{Ren}$	DWCEA	$DWCEA^{P}_{GDP}$	$DWCE^{P}_{Ren}$	CEA
Malta	0,555	0,074	0,240	0,176	0,079	1
Cyprus	0,598	0,236	0,491	0,435	0,253	1
Latvia	0,493	0,385	0,437	0,420	0,406	1
Luxembourg	0,807	0,150	0,371	0,286	0,164	1
Lithuania	0,464	0,404	0,487	0,474	0,438	1
Slovenia	0,478	0,194	0,387	0,343	0,211	1
Estonia	0,172	0,489	0,181	0,184	0,246	1
Croatia	0,451	0,178	0,334	0,286	0,192	1
Slovakia	0,407	0,287	0,417	0,401	0,311	1
Ireland	0,921	0,891	0,950	0,983	0,953	1
Bulgaria	0,177	0,346	0,185	0,189	0,256	1
Denmark	1	1	1	1	1	1
Portugal	0,702	0,847	0,581	0,671	0,790	1
Hungary	0,422	0,388	0,409	0,434	0,378	1
Sweden	1	0,861	0,798	0,892	0,867	1
Finland	0,728	1	0,765	0,777	0,916	1
Austria	0,933	0,867	0,965	0,998	0,913	1
Romania	0,336	0,363	0,352	0,300	0,328	1
Greece	0,486	0,473	0,413	0,444	0,497	1
Belgium	0,731	0,688	0,768	0,781	0,749	1
Czech republic	0,433	0,389	0,413	0,376	0,383	1
Netherlands	0,700	0,513	0,729	0,750	0,565	1
Spain	0,775	1	0,813	0,830	0,980	0,909
Poland	0,239	0,307	0,251	0,256	0,302	0,774
France	1	0,495	0,884	0,806	0,541	0,712
Italy	0,830	0,889	0,871	0,880	0,946	0,646
Germany	0,699	1	0,734	0,748	0,919	0,317
UK	0,835	0,594	0,837	0,834	0,646	0,516

Table 5.7 – Player Satisfaction rate for weighted allocations method and CEA rule applied on CO_2 emission permits.

Year	$WCEA_{GDP}$	$WCEA_{Ren}$	DWCEA	$DWCEA^{P}_{GDP}$	$DWCE^{P}_{Ren}$	CEA
2010	0,670	0,694	0,670	0,683	0,683	**0,492**
2011	0,559	0,727	0,651	0,714	0,621	**0,440**
2012	0,613	0,604	0,590	0,600	0,622	**0,484**
2013	0,574	0,569	0,572	0,570	0,568	**0,481**
2014	**0,553**	0,581	0,592	0,586	0,579	0,568

Table 5.8 – Gini index for weighted allocations methods and CEA rule applied on CO_2 emission permits. Bold text represents the smallest Gini index

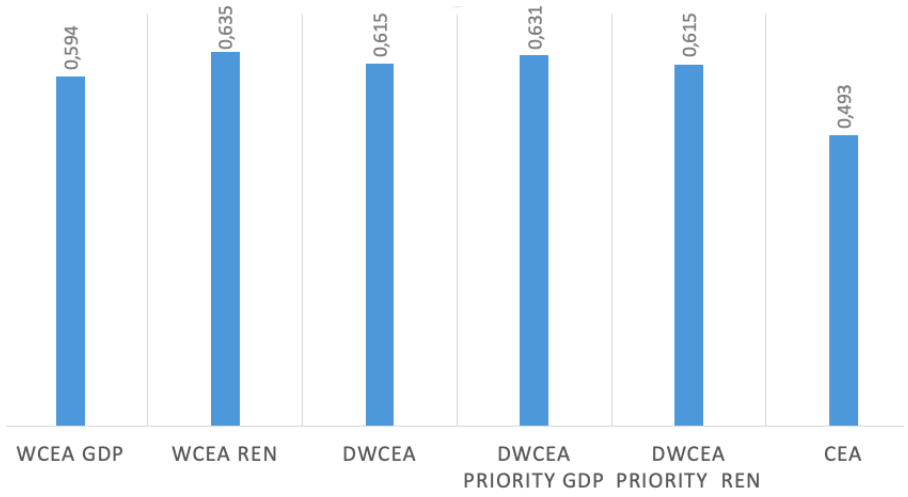

Figure 5.2 – Gini index average of 2010-2014 for CEA and weighted allocations methods.

To conclude this section, in the problem we present throughout this thesis, we can observe that according to the PS ratio and the Gini index, the allocation method without weight is the most egalitarian one and if we include a weight, it is preferable that it would be the GDP. When considering the double weights methods, the PS ratio prompts a method without priority while the Gini index incentives a method that focuses on renewable energy production.

5.6 Detailed Algorithms

In this section, we present the detailed algorithms related to *Priority Choice Procedure* Algorithm 12 and *Final Procedure* Algorithm 13. These later are important procedures for the correct execution of the DWCEA^P method presented in Section 2 and defined by the Algorithm 11.

Algorithm 12 Priority Choice Procedure

Input: weight vectors w^1, w^2, choice $\in 1,2$
Output: Priority weight w^P, Non Priority weight w^{NP}

if $choice = 1$ **then**
  $w^P \leftarrow w^1$
  $w^{NP} \leftarrow w^2$
else
  $w^P \leftarrow w^2$
  $w^{NP} \leftarrow w^1$
end if

Algorithm 13 Final Procedure

Input: Estate E, Priority weight w^P, priority λ^P, Non Priority weight w^{NP}, Non Priority λ^{NP} ,
Output: An allocation vector $x \in \mathbb{R}^N_{\geq 0}$

if $It > N$ **And** $j > N$ **then**
  $m_i \leftarrow min(c_i, \lambda^P_{It} w^P_i)$
  $WCEA(E, m, w^{NP})$
else
  $\lambda^{test} \leftarrow \frac{\lambda^{NP}_{j-1} + \lambda^{NP}_j}{2}$
  ForEach $(i \in N) : x_i = min(c_i, w^P_i \lambda^P_{It}, w^{NP}_i \lambda^{test})$
  $S = \sum x_i$
  while $S \neq E$ **do**
    if S>E **then**
      $\lambda^{test} \leftarrow \frac{\lambda^{test} + \lambda^{NP}_{j_1}}{2}$
    else
      $\lambda^{test} \leftarrow \frac{\lambda^{test} + \lambda^{NP}_j}{2}$
    end if
    ForEach $(i \in N) : x_i = min(c_i, w^P_i \lambda^P_{It}, w^{NP}_i \lambda^{test})$
    $S = \sum x_i$
  end while$\lambda^{NP^*} \leftarrow \lambda^{test}$ $\lambda^{P^*} \leftarrow \lambda^P_{It}$
end if
ForEach$(i \in N) : x_i = min(c_i, w^P_i \lambda^P *, w^{NP}_i \lambda^{NP^*})$
return X

5.7 Conclusion

Sharing a good among several agents is a hard process. There are many sharing methods based on different principles, that make the choice of which method to use not evident. In this chapter, we have introduced a new bankruptcy method that allows the allocation

of an estate by taking into consideration the demands and two different criteria, each one of them has a different priority. We present an algorithmic approach of $DWCEA^P$ and apply it to CO_2 emission permits problem. We consider the GDP and the production of Renewable energy as the most relevant criteria in this allocation problem and we generate two solutions by giving priority to each of these two criteria. A comparative study is made with these results and we studied the k-means clustering method on these allocations vectors. Countries with a high implication on Renewable energy obtain better allocations when we promote this criterion and countries without Renewable energy production prefer to promote the GDP. Regardless of the method studied, countries should work together to build a common reduction strategy. In the final section of this chapter, we introduce some fairness indicators. A fairness analysis of the different methods studied allows pointing a research direction that can lead to an agreement between the countries.

Conclusion & Perspectives

This dissertation tackles a current problem that affects all humanity. Climate change, in general, and CO_2 emissions, in particular, is a topic that we have been dealing with for several years. Negotiations between countries have not stopped trying to find solutions but unfortunately in vain. Each country claims the right to emit but the emission strategies differ from one country to another. Some would like to keep their emission rate with the pretext of the right to live, and others would like to improve their living standards and blame the former polluters for the current situation. The work conducted in this thesis addresses the issue of emissions rights among European countries, defining the problem of allocation permits as a bankruptcy problem. The data used in this manuscript are real country data extract from the World Data Bank.

First, we have defined the different parameters of corresponding bankruptcy situations, such as the estate which is based on the quantity of CO_2 emissions permits specified in Kyoto Protocol, agents are the EU countries and claims are the CO_2 emissions emitted for each year from 2010 to 2014. We, then, applied the four most well-used bankruptcy methods to our problem, to finally identify the most reliable and fair method. The Constrained Equal Awards is an allocation rule that divides the estate between agents, starting by incrementally allocating the demands. As a result, countries with small demands are fully satisfied and all countries with large demands will receive the same allocations. Nevertheless, we can not allocate emission permits only according to countries demands, and it is important to take into account more information that characterizes the studied countries.

In this context, the rest of our work is based on the addition of parameters (weights) and the definition of methods in order to find a solution that can be the target of a common

consensus among a large number of countries.

In chapter 3, we have redefined the emissions problem by including a new parameter representing an economic indicator of the countries, the GDP. By implementing a weighted method of the CEA, we noticed that the allocations of CO_2 emission permits do not have the same distribution aspect anymore. Countries with low emission rates are no longer totally favored but rather countries with a good ability to emit low amounts of CO_2 per unit of GDP receive their demands. Then, as we consider that a successful negotiation generates positive consequences for all participants, we proposed a technology transfer game to transfer technology from more efficient countries to less efficient ones. The profit generated by each coalition is given from the highest level of technology available to the coalition members.

The last two chapters of the manuscript develop further methods for bankruptcy situations, where we have introduced a second parameter and thus study a bankruptcy situation with one claim and two weights for each agent.

Chapter 4 presents an algorithmic approach to the bankruptcy problem with two weights without giving priority to weights. Allocations generated by DWCEA method no longer depend on demands but represent a fair balance respecting the efficiency of the countries production, the GDP, and their environmental impact, measured by the Renewable energy production rate. Using the k-means classification method, we form groups of similar countries and study the weight of their votes in the European Council. Finally, we compare the allocation vectors over the years studied with the power index Sh-Sh computed on the simple set representing the EU Council voting rule, in order to determine the level of acceptability of different CO_2 emission permit allocations.

The last chapter deals with bankruptcy problems with two weights and provides a method to assign priority to one of the two weights. An algorithmic approach has been proposed to characterize the double weighted allocation method with priority. Then, we apply this method to the problem of allocation CO_2 emission permits, first by favoring the GDP and then by emphasizing the production of Renewable energies. Finally, we present some fairness index and we make a comparative study of fairness of all methods presented in this thesis.

Believing that an agreement can be signed only if it combines the right decision criteria, we think that weighted methods are the best methods that can describe the situation. The more countries' parameters are considered, the more allocations can be fair. Certainly, the classical bankruptcy method is more egalitarian and offers more opportunity to small demands, but it does not take into account the production or the rate of involvement in renewable energies or the population. Stopping on a single parameter does not reflect the real situation and does not allow to reach an agreement. The more informations are integrated into the study, the more a consensus, that has a high probability of being implemented, can be reached. For these reasons, we think that the double weight methods are the most favorable method to this problem and especially the method without priority, DWCEA. This method offers equal importance to all parameters and therefore it does not create a discrimination among countries with a high GDP production or those that are oriented on the production of renewable energies. This method proposes allocations according to a balance of all parameters. In addition, according to the two fairness indices studied, the DWCEA is consistently ranked among the most egalitarian and fair methods.

Although this thesis presents new methods for dealing with bankruptcy situations, there are a number of interesting directions for future research.

- First, it is obvious to generalize double-weighted methods to m-weighted methods with $m > 2$. A more general method can take into account a large number of player characteristics and thus propose allocations that that would be more representative of the agents situation. In a bankruptcy problem, in order to obtain a fair share, it would be worth considering all aspects of the situation, namely the relevant characteristics of the agents. In this thesis, we notice that each time we add a new weight, the vector of allocations is transformed by creating a balance between the different studied parameters.

- In bankruptcy problem, each rule can be considered as adapted to a particular situation. For example, to compensate the seller in an auction where several buyers cancel their offers simultaneously. The loss is divided among the buyers according to the CEL rule. This rule is also known as poll tax, in the context of taxation. The proportional method was adopted [5] in problems with constraints and claims (PCC) that

contain bankruptcy and PERT problems as subsets. The Talmud division rule has been discussed in the problem of collusion in an oligopoly [63], in various concepts of game theory [2] and in the problem of arbitration of rights [62]. It is interesting to develop new two-weights methods based on other bankruptcy methods such as proportional, CEL and Talmud rules.

- The new methods presented in this thesis are based on an algorithmic approach. Nevertheless, bankruptcy methods are generally characterized by axioms [81–83]. For future work, it might be worthy to study the double weighted methods with an axiomatic approach based on the previous work.

- In CO_2 emission permits problems, we consider the GDP and Renewable energy production as weights, however, countries can be characterized by several other relevant criteria [93] such as population, demography, historical responsibility or emission intensity. Depending on the parameters chosen, allocations attributed to countries are not the same, which makes the selection of parameters very important in a situation where the agreement of all participants is sought. Therefore, it seems interesting to review our study with several other parameters and to examine the impact of each of them in the proposed allocations and in the voting mechanism within the EU Council.

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RÉSUMÉ

Dans cette thèse, nous abordons le problème des émissions de CO_2 comme un problème de faillite. Dans notre étude, nous considérons que les pays de l'union européen sont des agents qui réclament le droit d'émettre une quantité supérieure à celle décidée par le protocole de KYOTO. Cette étude concerne les émissions des pays pendant 5 années, de 2010 au 2014. Dans un premier temps, nous avons utilisé les méthodes classiques de faillites afin de déterminer les allocations de chaque pays. En regardant séparément chacune des méthodes et son évolution dans le temps, nous avons constaté que la méthode des prix égaux sous contrainte est la méthode la plus adapté à notre problématique. Ensuite, nous avons introduit comme poids un premier paramètre représentant le PIB et nous avons remarqué l'influence d'ajout de paramètre dans la distribution des allocations. Dans un second temps, nous avons étudié le problème de faillite avec deux poids (le PIB et la production des énergies renouvelables) d'une approche algorithmique, d'une part pour la méthode ne favorisant aucun critère. Cette méthode offre les meilleures allocations aux pays qui respectent un seuil d'équilibre entre les émissions de CO_2, la production de PIB et la production des énergies renouvelables. D'autre part, nous avons implémenter une méthode donnant une priorité à l'un des poids et avons testé les deux situations où le PIB est favorable et ensuite une situation où la production des énergies renouvelable est propriétaire. Tout au long de notre étude, nous avons comparé les résultats des allocations générées par les différentes méthodes employées et avons classifié les pays en groupes similaires grâce à la méthode de classification k-means, dans le but de mettre en avant l'impact des votes des groupes lors des négociations et des prises des décisions au conseil européen.

MOTS CLÉS

Les droits des émissions de CO_2, Les allocations, Problème de faillite, méthode pondérée, étude bi-critère, PIB, énergies renouvelables, conseil européen

ABSTRACT

In this thesis, we approach the problem of CO2 emissions as a bankruptcy problem. In our study, we consider that the countries of the European Union (EU) are agents that claim the right to emit. However, the sum of all claims is more than the amount of CO_2 emissions decided by the KYOTO protocol. This study concerns the emissions of the countries for 5 years, from 2010 to 2014. First, we used the classical bankruptcy methods to determine the allocations of each country. By looking separately at each method and its evolution over time, we found that the constrained equal awards (CEA) method is the most appropriate method for our problem. Then, we introduced as a weight a first parameter representing the GDP and we noticed the influence of adding parameters in the distribution of allocations. In a second step, we studied the bankruptcy problem with two weights (GDP and renewable energy production) from an algorithmic approach, on the one hand, for the method that does not favor any criteria. This method offers the best allocations to countries that respect a balance threshold between CO_2 emissions, GDP production, and renewable energy production. On the other hand, we have implemented a method giving priority to one of the weights and have tested the two situations where GDP is favorable and then a situation where renewable energy production is prioritized. Throughout our study, we compared the results of the allocations generated by the different methods used and classified the countries into similar groups using the k-means classification method, with the aim of highlighting the impact of the groups' votes during negotiations and decisions making at the European Council.

KEYWORDS

CO_2 emission rights, allocations, bankruptcy problem, weighted method, bi-criteria study, GDP, renewable energies, European Council