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Theoretical and Phenomenological Implications of Discreteness

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Theoretical and Phenomenological Implications of Discreteness

Lautaro Amadei

Abstract

Several approaches to quantum gravity entertain the idea that smooth spacetime is an emergent approximation of a fundamental, discrete Planckian structure. Which are the theoretical and observational consequences of this hypothesis?

Currently, we cannot test the geometry of the Universe down to the Planck scale. Still, we can look for signs of discreteness and its consequences in our prevailing theoretical and observational models.

In the first part of this thesis, we propose that a discrete microstructure at the Planck scale provides a natural solution to the black hole information loss problem if taken at face value. The fundamental discrete degrees of freedom provide a large reservoir for information to be encoded at the end of black hole evaporation. We put forward a conservative and natural perspective of the black hole evaporation puzzle where information is not lost or destroyed but simply degraded into correlations (inaccessible to low-energy observers) with the microscopic structure of the geometry at the Planck scale.

In the second part, we propose a model of inflation driven by a relaxation of an initially Planckian cosmological constant via diffusion due to friction with a fundamentally discrete Planckian structure. We show that this model can generate a nearly scale-invariant spectrum of primordial adiabatic perturbations and its tilt that matches observations. Moreover, this is done without the introduction of an inflaton field with an arbitrary potential. Furthermore, this process admits a well-defined semi-classical interpretation and avoids the trans-Planckian problem often found in the standard treatment of structure formation.

Résumé

La relativité générale et la théorie quantique sont souvent citées comme étant les deux piliers de la physique théorique moderne. À grande échelle, les phénomènes sont dominés par l'interaction gravitationnelle, où les observations sont décrites avec précision par la relativité générale jusqu'à des échelles de l'ordre du millimètre[1], tandis que les échelles submillimétriques jusqu'à $10^{-19}\,m$ sont bien modélisées par la théorie quantique.

La leçon centrale de la relativité générale (RG) est que la gravité est géométrie et , par conséquent, dans une théorie fondamentale, aucune *background structure* ne devrait être privilégiée. Cela entre en conflit avec les besoins de la théorie quantique qui, dans ses formulations standard, nécessite un *background* fixe et une séparation préférentielle entre le temps et l'espace.

La théorie quantique fournit un cadre général pour toutes les théories décrivant les interactions fondamentales. Elle a passé de nombreux tests expérimentaux avec une précision remarquable (e.g. [2, 3]), et est considérée comme une théorie bien établie, à l'exception des discussions en cours sur ses interprétations possibles. Jusqu'à présent, la seule interaction qui n'a pas été entièrement intégrée dans ce cadre est l'interaction gravitationnelle.

En outre, une théorie purement classique de la gravitation peut expliquer toutes les données d'observation dont nous disposons actuellement. Un exemple remarquable en est l'observation récente des ondes gravitationnelles[4, 5, 6, 7, 8], une conséquence longtemps prédite de la relativité générale d'Einstein.

La quête d'une théorie quantique de la gravité n'est pas, pour l'instant, motivée par des résultats expérimentaux. Il n'y a actuellement aucune observation qui indique sans ambiguïté des phénomènes que nos théories actuelles ne peuvent expliquer. Ainsi, nous ne disposons d'aucune expérience pour nous aider sur la voie d'une théorie quantique de la gravité.

Où peut-on s'attendre à ce que les effets de la gravitation quantique deviennent inévitables ? Quelles sont les échelles auxquelles nous devrions nous attendre à ce que ces effets soient pertinents ? Dans une théorie de la gravité quantique universellement valide, de véritables effets gravitationnels quantiques peuvent se produire à n'importe quelle échelle, le comportement classique apparaissant à une limite appropriée. Ce-

pendant, il devrait exister des échelles où les effets de la gravité quantique deviennent non-négligeables.
Ceci est intimement lié à de vieilles questions de physique : existe-t-il une limite fondamentale à la résolution des structures au-delà de laquelle nous ne pouvons pas accéder?
L'espace-temps est-il continu? Ou cette image n'est qu'une bonne *low-energy approximation* qui émerge d'une structure discrète plus fondamentale?
L'analyse dimensionnelle fournit un indice de la réponse à ces questions. Trois constantes sont censées jouer un rôle dans la gravité quantique : la constante de Planck $\hbar$, la constante gravitationnelle G, et la vitesse de la lumière c. En 1899, Planck s'est rendu compte que ces trois constantes pouvaient être combinées de manière unique pour produire des quantités avec des unités de masse, de longueur et de temps. Ce sont la masse de Planck, M_{Pl}, la longueur de Planck l_P et le temps de Planck t_P :

$$
\begin{aligned}
M_{\text{Pl}} &= \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ g \approx 1.22 \times 10^{19}\ \text{GeV} \\
t_P &= \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ s \\
l_P &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ cm
\end{aligned} \tag{1}
$$

Notez que la masse de Planck est plutôt grande par rapport aux échelles microscopiques. Mais cela ne signifie rien en soi, cette masse doit être contenue dans une région de dimension linéaire l_P pour que les effets de la gravité quantique soient non négligeables.
Plus précisément, les échelles de Planck sont atteintes lorsque la longueur d'onde Compton d'une particule élémentaire est de l'ordre de son rayon de Schwarzschild :

$$
\frac{\hbar}{M_{\text{Pl}} c} \approx \frac{2 M_{\text{Pl}} G}{c^2}, \tag{2}
$$

ce qui signifie que la courbure d'une telle particule élémentaire n'est pas négligeable et conduit à la création d'un trou noir microscopique.
Cet argument, dû à Matvei Bronstein[9], qui reconnaissait déjà en 1936 le formidable défi que représentait la quantification de la gravité, dans sa version moderne va dans le sens suivant[10, 11] :
Supposons que l'on veuille déterminer la position x d'une particule avec une précision L. Alors, en raison du principe d'incertitude de Heisenberg, $\Delta x > \frac{\hbar}{\Delta p}$ et il s'ensuit donc que $\Delta p > \frac{\hbar}{L}$.
Puisque la valeur moyenne p^2 est plus grande que la dispersion $(\Delta p)^2$, nous avons

que $p^2 > \left(\frac{\hbar}{L}\right)^2$.
À son tour, un grand moment implique une grande énergie : dans la limite relativiste (par exemple, une particule accélérée au CERN), nous avons que $E \sim cp$, où c est la vitesse de la lumière, ce qui explique pourquoi nous devons construire des accélérateurs de plus en plus grands pour prouver des distances plus en plus petites. Nous voyons donc que, pour localiser la particule avec précision, nous avons besoin de grandes énergies.
Considérons maintenant la relativité générale. Une énergie E gravite avec une masse $M \sim \frac{E}{c^2}$. Lorsque l'énergie est suffisamment grande, et que la masse est concentrée à l'intérieur d'une sphère de rayon $R_S = \frac{GM}{c^2}$, un trou noir se forme. Ainsi, la région initiale L que nous voulions localiser sera cachée derrière l'horizon du trou noir. Nous concluons alors que L ne peut être diminué que jusqu'à une valeur minimale, qui est atteinte lorsque le rayon de l'horizon est égal à L, c'est-à-dire $R_S = L$.
En d'autres termes, la longueur minimale dans laquelle nous pouvons localiser une particule sans qu'elle soit cachée derrière son horizon est donnée par

$$L = R_S = \frac{GM}{c^2} = \frac{GE}{c^4} = \frac{Gp}{c^3} = \frac{G\hbar}{Lc^3} \implies L = \ell_{\mathrm{Pl}} \tag{3}$$

Cet Gedankenexperiment nous dit donc qu'il n'est pas possible de localiser quoi que ce soit avec une précision meilleure que la longueur de Planck.
L'échelle de Planck est remarquablement éloignée des échelles auxquelles nous sommes habitués : le LHC sonde des régimes de 10^{-14} fois la masse de Planck, la longueur de Planck est de 10^{-20} fois le diamètre d'un proton. Il est donc clair que l'exploration directe de l'échelle de Planck est impossible avec nos expériences actuelles. Néanmoins, la présence d'une longueur minimale et/ou d'une microstructure discrète à l'échelle de Planck est pertinente en physique des trous noirs et en physique de l'Univers primitif. Que se passe-t-il lorsque la courbure de l'espace-temps est planckienne ? En d'autres termes, que se passe-t-il à proximité de la singularité à l'intérieur des trous noirs et de la singularité du Big Bang ? Si les attentes d'une longueur minimale sont alors satisfaites, la courbure de l'espace-temps ne peut être supérieure à $\mathbf{R} \sim \ell_{\mathrm{Pl}}^{-2}$, les théoriciens s'attendent donc à ce que les singularités soient guéries dans une théorie entièrement quantique-gravitationnelle. Ces idées font des trous noirs et de la cosmologie des bancs d'essai parfaits pour les conséquences de la discrétisation à l'échelle de Planck.
L'existence d'une structure discrète de *background* est un trait partagé par de nombreuses incarnations de la gravité quantique : *Loop Quantum Gravity, String Theory, Causal Set, Dynamical Triangulations* entre autres. Dans toutes ces formulations, l'espace-temps *smooth* résulte d'un *coarse graining* où les détails des relations entre les constituants discrets sont perdus. Il est donc raisonnable de s'attendre à ce qu'une

géométrie *smooth* donnée corresponde à de nombreux micro-états fondamentaux différents.
Je passerai brièvement en revue le cas de la LQG, qui est non seulement la formulation de la gravité quantique que je connais le mieux, mais aussi celle qui a motivé notre travail.
Loop Quantum Gravity (LQG) est une approche *background independent* d'une théorie quantique de la gravité, basée sur la quantification canonique non-perturbative de la relativité générale. Dans ce cadre, il n'y a pas de *background structure* préférée et l'espace-temps lui-même est une entité dynamique qui doit être quantifiée.
L'un des résultats clés de la LQG est que les opérateurs de l'espace-temps, tels que l'aire et le volume, acquièrent des spectres discrets : les états des degrés de liberté gravitationnels peuvent être envisagés en termes d'états de réseaux de spins, dont chacun admet l'interprétation d'un état propre de la géométrie qui est discret au niveau fondamental.
De manière plus détaillée, on veut travailler dans le cadre hamiltonien, nous considérons donc la décomposition $3+1$ de l'espace-temps. Les détails de la décomposition $3+1$ et de la formulation Hamiltonienne peuvent être trouvés dans [12].
On introduit une foliation de l'espace-temps en termes d'hypersurfaces tridimensionnelles Σ. Dans cette formulation, les dix composantes de la métrique sont remplacées par les six composantes de la métrique induite sur les hypersurfaces q_{ab}, le *shift vector* N^a et la fonction de lapse N. Afin de préparer la quantification, on doit considérer la courbure extrinsèque K_{ab} de l'hypersurface Σ, qui joue le rôle de momenta canoniquement conjugué à la 3-métrique q_{ab}[1].
Nous introduisons ensuite la triade E_i^a, qui est un ensemble de trois champs vectoriels définis par la relation :

$$q^{ab} = E_i^a E_j^b \delta_{ij}. \tag{4}$$

Pour quantifier le champ gravitationnel, au lieu d'utiliser la triade, nous utilisons comme variable la triade *densitisé* définie par

$$\tilde{E}_i^a = \sqrt{\det(q)} E_i^a. \tag{5}$$

L'autre ensemble de variables est une connexion $SU(2)$ A_a^i qui est liée à la connexion de spin w_a^i et à la courbure extrinsèque de Σ par

$$A_a^i = w_a^i + \gamma K_a^i, \tag{6}$$

où γ est un paramètre sans unites appelé "paramètre de Barbero-Immirzi".

[1] En fait, le momenta π_{ab} canoniquement conjugué à la métrique q_{ab} est donné par : $\pi^{ab} = q^{-1/2}\left(K^{ab} - Kq^{ab}\right)$, où $K = K^{ab} q_{ab}$.

Étant donné une 2-surface S qui est paramétrée par x^1, x^2 avec $x^3 = 0$, l'aire de la surface en termes de triade est donnée par :

$$A_S = \int_S dx^1 dx^2 \sqrt{\tilde{E}_i^3 \tilde{E}^{i3}} \tag{7}$$

L'aire peut être promue en un opérateur qui, agissant sur les états du réseau de spin, donne les valeurs propres :

$$\hat{A}_S \psi_s = 8\pi l_P^2 \beta \sum_\alpha \sqrt{j_\alpha (j_\alpha + 1)} \psi_s, \tag{8}$$

où la somme est prise sur tous les bords du réseau qui passent par S et j_α étiquette la représentation $SU(2)$ portée par les bords.
Ainsi, nous voyons que LQG prédit l'existence d'une aire minimale donnée par :

$$A_{min} = 4\pi\sqrt{3}\beta l_P^2 \tag{9}$$

Un argument similaire peut être avancé pour l'opérateur de volume[13, 14], montrant qu'il possède du spectre discret.
Des efforts considérables ont été consacrés à l'étude de la limite de basse énergie de la théorie. Alors que les difficultés dues à la *background independence* de la théorie empêchent toujours une caractérisation précise de la limite de basse énergie et qu'il n'est donc pas possible actuellement de décrire en détail la nature de la structure pré-géométrique qui peut survivre dans la limite semi-classique, on a appris qu'une géométrie lisse devrait émerger des états prégéométriques planckiens via des observateurs *coarse-grained* insensibles aux détails UV.
Il existe des exemples d'états dans la théorie qui semblent ne jouer aucun rôle important dans la limite du continuum, mais qui devraient être produits par des processus dynamiques. Par exemple, les boucles fermées et les nœuds trivalents des réseaux de spins dégénèrent en états avec des quanta de volume évanouissants qui, en principe, peuvent survivre dans la limite du continuum[15].
Le calcul de l'entropie du trou noir dans la LQG est un autre exemple de cette caractéristique : l'entropie peut être calculée en comptant les réseaux de spin (*microstates*) compatibles avec une géométrie lisse (*macrostates*) décrivant un trou noir de masse M.
Cette caractéristique de la LQG (et en général des théories de gravité quantique où l'espace-temps est remplacé par une notion discrète) sera centrale dans notre proposition de solution au problème de l'information des trous noirs : le nombre infini de micro-états planckiens dégénérés compatibles avec un espace-temps plat et lisse constituera le réservoir parfait pour le stockage de l'information.
Les approches modernes de la gravité quantique ont été développées avec l'espoir

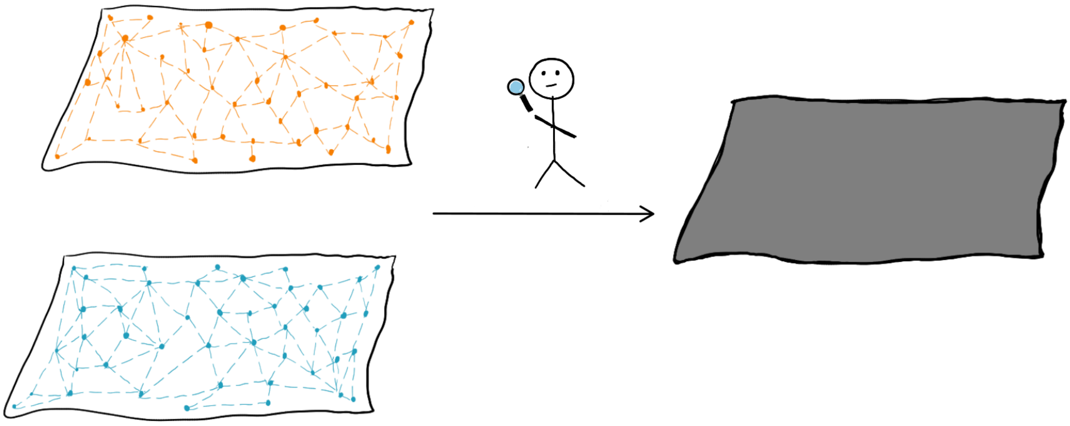

FIGURE 1 – Un observateur aux capacités limitées (*coarse-grained observer*) est incapable de distinguer deux micro-états dans la théorie fondamentale. Chaque géométrie lisse dans la description à basse énergie correspond à une myriade d'états dans la théorie fondamentale.

que la cohérence avec la relativité générale et la théorie quantique suffirait à trouver une théorie quantique cohérente de la gravité, repoussant ainsi la confrontation avec l'observation à un stade ultérieur.
Cet espoir est encore loin de se réaliser, et le nombre sans cesse croissant d'approches différentes de la gravité quantique nous rappelle l'importance capitale de tester les idées par rapport aux observations.
L'un des principaux problèmes rencontrés lors de la construction d'une théorie quantique complète de la gravité est le manque de données d'observation disponibles pour guider les efforts théoriques.
Les approches de la formulation de la théorie que nous avons décrites jusqu'à présent adoptent une démarche "de bas en haut" : elles commencent par des hypothèses sur la nature de l'espace-temps à une échelle éloignée de ~ 15 ordres de grandeur des échelles qui sont actuellement accessibles expérimentalement, et à partir de là, la théorie devrait remonter jusqu'à la limite dans laquelle elle devrait retrouver la phénoménologie connue.
Dans d'autres domaines où les observations sont abondantes, on peut proposer des modèles phénoménologiques qui expliquent les données. Bien que ces modèles soient peut-être incohérents sur le plan conceptuel, ils peuvent servir de guide pour l'identification de la théorie correcte et conceptuellement satisfaisante.
Un exemple emblématique est la théorie de l'interaction de Fermi. En 1934, Fermi a proposé un modèle phénoménologique pour expliquer la désintégration β[16, 17]. À l'époque, Fermi ne disposait pas d'une description ultraviolette complète des in-

teractions faibles, il a donc modifié la QED pour tenir compte de la désintégration des neutrons. La description de Fermi s'est avérée très précise pour décrire les phénomènes observés. Si l'extrapolation de la théorie de Fermi pour expliquer d'autres interactions s'est avérée infructueuse, son échec même a ouvert la voie à une compréhension plus fondamentale des interactions faibles.
L'explication de Fermi sur la désintégration β peut être considérée en termes modernes comme (peut-être le premier) un exemple d'une EFT (*Effective Field Theory*) : elle est extrêmement utile pour décrire les processus à des énergies bien inférieures à la masse du boson W.
Cette approche "de haut en bas" est celle qui nous intéresse. En particulier, nous voulons étudier la phénoménologie possible associée à la présence d'un substrat discret sous-jacent.
Nous pensons qu'une voie prometteuse pour réaliser cette idée est l'étude des implications observationnelles (et théoriques) possibles d'un espace-temps fondamentalement discret. À quoi peut ressembler cette évidence ? Quel peut être le témoin de la discrétisation ? Quel est le mouvement brownien de notre temps ? Dans la deuxième partie de cette thèse (voir la section 6), nous proposons un modèle de formation de structure[18] qui est la conséquence directe d'une diffusion de type brownien planckien dans l'Univers primitif.
Permettez-moi enfin de mentionner que, comme nous l'avons dit plus haut, nous n'avons pas observé de phénomènes quantiques-gravitationnels. Mais à mesure que les expériences deviennent plus précises et capables de tester de nouveaux aspects de la réalité physique, des tensions apparaissent avec les théories physiques établies. Ces divergences apparentes n'ont, bien sûr, rien de nouveau : elles se produisent depuis que les humains tentent de comprendre l'univers.
Néanmoins, si nous avons de la chance, ces divergences ne sont pas dues à une expérience défectueuse ou à une mauvaise compréhension de la théorie, mais elles sont la conséquence de nouveaux phénomènes physiques que les théories disponibles ne peuvent expliquer.
La valeur non nulle de la constante cosmologique est généralement citée comme une conséquence des effets gravitationnels quantiques. En outre, il existe actuellement une tension entre l'observation de la constante de Hubble H_0 en temps cosmologique tardif et précoce. La valeur mesurée de H_0 dans l'univers tardif est en désaccord avec les valeurs de l'univers précoce de 5σ[19, 20, 21]. De manière intéressante, il a été avancé que ces phénomènes peuvent être la conséquence de la microstructure discrète de l'espace-temps à l'échelle de Planck[22, 23, 24, 25].
Cette thèse contient deux projets principaux réalisés au cours des trois dernières années, nés de ces idées. La première partie propose une solution nouvelle pour le paradoxe de l'information dans les trous noirs, et la seconde partie une vue sur la

formation des structures en cosmologie à partir de la gravité quantique.

Partie I : Paradoxe de l'information des trous noirs

Les premiers indices du comportement thermodynamique des trous noirs sont apparus au début des années 70 avec Bekenstein, Bardeen, Wheeler et Hawking, entre autres. Il a été démontré que les solutions des trous noirs à l'équilibre de l'équation d'Einstein satisfont des lois analogues à celles de la thermodynamique.

Mais ce n'est qu'avec les travaux fondamentaux de Hawking qu'un paradoxe apparent est apparu. L'article révolutionnaire de Hawking a montré, à l'aide de considérations semi-classiques, que les trous noirs perdent leur masse par rayonnement thermique. De plus, le taux d'émission dépend inversement de la masse du trou noir, ce qui suggère que les trous noirs s'évaporent en un temps fini. Si le trou noir disparaît complètement, toute l'information autrefois contenue dans le trou noir disparaît avec lui, ne laissant dans son sillage qu'un bain thermique de particules de Hawking. Ainsi, un état initial pur s'est transformé en un état final mixte. En contradiction flagrante avec la mécanique quantique, l'évolution entre l'état initial et l'état final n'est pas unitaire et l'information est perdue : l'information codée dans l'état final est insuffisante pour retrouver l'état initial. Ce phénomène est connu sous le nom de paradoxe de l'information du trou noir. Dans la section 3.2, nous donnons une discussion plus détaillée et approfondie du paradoxe.

Plusieurs propositions visant à résoudre ce paradoxe apparent sont apparues dans la littérature au cours des 50 dernières années. Certaines de ces approches, cherchant à restaurer l'unitarité dans la dynamique, proposent des écarts dramatiques de la physique connue dans des régimes où les modèles semi-classiques sont considérés comme une bonne approximation.

D'autres propositions, plus conservatrices, envisagent l'idée que la solution provienne d'une nouvelle physique apparaissant à l'échelle de Planck, où la description semi-classique devrait s'effondrer. Nous passons en revue les mérites et les inconvénients de chaque proposition dans la section 3.3.

Dans la partie II, nous proposons une solution conservatrice et nouvelle au problème. Dans les approches de la gravité quantique où la description de l'Univers comme un espace-temps lisse est considérée comme une approximation d'une structure discrète plus fondamentale à l'échelle de Planck, toute description en termes de champs lisses est vouée à manquer une partie de ces degrés de liberté discrets et donc à rompre l'unitarité. De ce point de vue, si ces degrés de liberté fondamentaux sont pris en compte, le processus de formation d'un trou noir et son évaporation ultérieure peuvent être décrits par une évolution unitaire. Cette idée, proposée pour la première fois [15], est démontrée dans un contexte de gravitation quantique explicite dans la section 6.78, suivant [26, 27].

L'argument essentiel de la proposition consiste à noter que la formation et l'évaporation éventuelle d'un trou noir peuvent être formulées dans les mêmes termes que le processus de combustion d'un papier.
Considérons l'impression, la lecture et la combustion de cette page. L'état initial du système est très particulier de notre point de vue : en tant qu'observateurs macroscopiques, nous pouvons distinguer les particules d'encre qui forment les lettres *CMJ* de celles des lettres *LBC*.
S'il était placé dans un boîtier fermé, le texte imprimé sur cette page persisterait pendant une période assez longue et les générations futures pourraient juger du contenu de ces mots. L'espace de phase disponible pour les particules qui forment le papier et les mots qui y sont écrits restent plus ou moins constants dans le temps.
Si, toutefois, nous décidions de mettre le feu au papier, les informations qui y sont inscrites seraient-elles perdues à jamais ? Pouvons-nous récupérer le texte écrit en regardant ses cendres ?
Jusqu'à présent, les théories physiques décrivant notre monde sont toutes réversibles dans le sens où nous pouvons prédire l'état futur d'un ensemble de variables à partir de leur état initial. Inversement, nous pouvons retrouver le passé à partir des valeurs des variables dans le futur. Naïvement, cette caractéristique se heurte à l'intuition dans le cas du papier qui brûle : les informations autrefois contenues dans le papier semblent perdues à jamais. Cependant, il existe une explication physique du phénomène qui préserve la réversibilité. Une fois le papier enflammé, l'information se transforme en corrélations entre les molécules qui sont maintenant libres de se diffuser dans l'atmosphère. L'information n'est alors pas perdue mais dégradée en corrélations inaccessibles pour nous, *coarse-grained* observateurs.
Dans les premières sections de la partie II, nous soutenons que la formation et l'évaporation des trous noirs peuvent être encadrées de la même manière : une description unitaire et réversible du processus est disponible lorsque les degrés de liberté planckiens sont considérés. Les corrélations établies avec la microstructure planckienne sont manquées lorsque l'on considère des champs lisses qui ne tiennent pas compte de ces degrés de liberté discrets et que l'on récupère la description non unitaire habituelle de la QFT à basse énergie. Ces corrélations sont alimentées par la présence d'une région de haute courbure inévitable. Cette région de courbure planckienne autour de l'endroit où il y a classiquement une singularité agit comme le feu dans l'exemple du papier brûlé : elle ouvre une énorme, nouvelle et inexplorée région de l'espace des phases en mettant le système en contact avec l'échelle de Planck.
Dans les sections 4.2-4.10 nous introduisons un modèle quantique-gravitationnel dans lequel ces idées sont réalisées. Le modèle est basé sur les techniques de la cosmologie quantique à boucles, qui est présentée dans les sections 4.2 ainsi que dans l'annexe B.

Nous soutenons que ce modèle cosmologique quantique contient les ingrédients essentiels qui fournissent la solution dans le cas d'un trou noir : la présence de degrés de liberté UV fournis par les $\epsilon - sectors$. Les résultats peuvent alors être immédiatement extrapolés au cas du trou noir.
L'étude de cette question dans les modèles polymères cosmologiques, outre le fait qu'elle présente moins de difficultés techniques que son homologue du trou noir, fournit une interprétation conceptuelle claire de la physique en jeu. Les résultats pour le cas plus compliqué techniquement (mais analogue) du trou noir seront présentés ailleurs[28].
Nous montrons que pour des observateurs *coarse-grained* insensibles aux degrés de liberté planckiens, un état initialement pur développe des corrélations entre les degrés de liberté à basse énergie et les degrés de liberté planckiens lorsqu'il évolue à travers une région à forte courbure.
Ces corrélations sont inaccessibles aux observateurs de basse énergie, pour lesquels la dynamique est non unitaire, même si la dynamique de la théorie sous-jacente est unitaire. Ces observateurs *coarse-grained* voient l'entropie croître entre l'état initial et l'état final, et cette entropie n'est qu'une mesure de l'ignorance de ces observateurs.

Partie II : observables du CMB à partir de la granularité planckienne.
Dans la partie III, nous étudions une autre conséquence possible d'un espace-temps discret. Nous avons proposé un modèle d'inflation piloté par une constante cosmologique décroissante due à la friction avec une structure de *background* discret.
Dans ce modèle, les inhomogénéités observées dans le CMB proviennent de celles associées à un espace-temps discret. Nous obtenons le spectre observé approximativement invariant d'échelle des perturbations scalaires avec le *blue-tilt* correspondante.
Le modèle prédit également un rapport tenseur/scalaire extrêmement faible. Ces prédictions phénoménologiques ne dépendent que des paramètres connus de la physique du modèle standard.
Dans la section 6.1, nous présentons la gravité unimodulaire comme la limite à basse énergie d'une théorie dans laquelle l'espace-temps est fondamentalement discret et construit à partir de blocs de construction discrets à 4 dimensions : les quanta de 4 volumes.
Bien que le modèle phénoménologique que nous proposons puisse également être réalisé dans le cadre de la relativité générale, la gravité unimodulaire offre un point de vue conceptuel unique : elle décrit la limite de basse énergie d'une théorie fondamentalement discrète. En tant que telle, elle incorpore la friction comme une violation de la conservation du tenseur énergie-momentum, c'est-à-dire $\nabla^a \mathbf{T}_{ab} \neq 0$.
Il s'agit d'un ingrédient crucial : le signe habituel d'une structure de *background* est la friction, la possibilité de perdre de l'énergie dans le chaos des micro-états non

capturés par la théorie effective.
Dans la section 6.2, nous étudions la dynamique d'un espace-temps cosmologique avec un terme cosmologique décroissant de façon exponentielle (par analogie avec les systèmes dissipatifs standard).

$$\Lambda(t) = \Lambda_0 \exp\left(-\beta M_{\rm Pl} t\right) \tag{10}$$

où $M_{\rm Pl}$ est la masse de Planck, β un paramètre sans dimension, $\Lambda_0 \sim M_{\rm Pl}^2$ et t le temps cosmologique (unimodulaire) lié au temps *comoving* τ par

$$dt = a^3 d\tau \tag{11}$$

Nous montrons que, tant que $\beta M_{\rm Pl} < 1$, l'Univers subit une phase d'inflation alimentée par la désintégration de Λ. De plus, pour β suffisamment petit, la phase d'inflation dure suffisamment longtemps pour résoudre les problèmes d'horizon et de planéité indépendamment des conditions initiales.
Dans la section C, nous montrons que les conséquences observationnelles du modèle ne dépendent pas fortement de la forme fonctionnelle exacte de Λ. En particulier, tant que Λ est presque constant pour un nombre suffisamment long de *e-folds* de telle sorte que les échelles observées aujourd'hui dans le CMB ont été créées à l'échelle de Planck pendant l'inflation, puis que Λ décroît rapidement, les prédictions observationnelles du modèle restent du même ordre de grandeur.

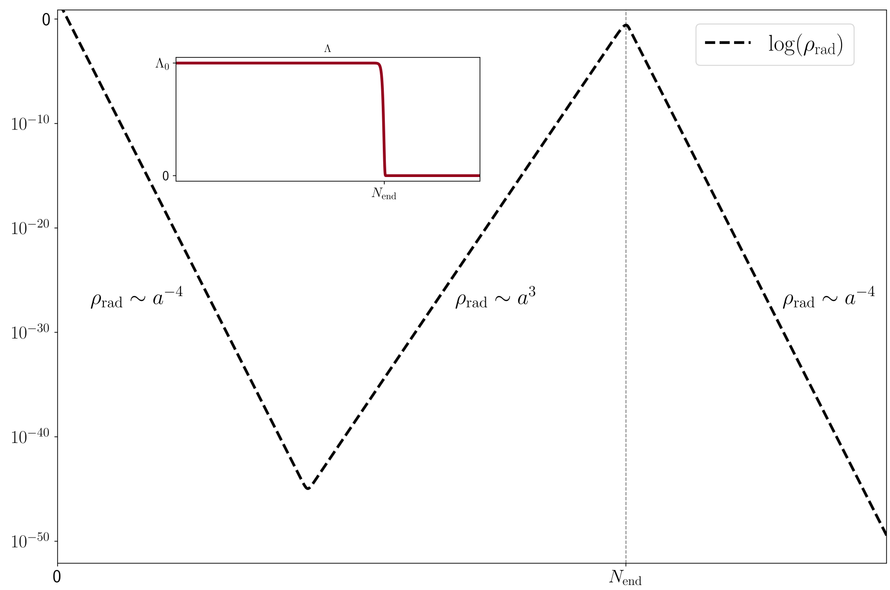

FIGURE 2 – Dynamique de *background* avec constante cosmologique décroissant de façon exponentielle

Dans la section 6.2.2, nous étudions la dynamique du champ de Higgs couplé à la

gravité. Nous montrerons que la discrétisation fondamentale s'imprime sur le degré de liberté scalaire naturel du modèle standard, le boson de Higgs, qui portera ces inhomogénéités jusqu'au CMB.
Dans la section 6.3, nous décrivons en détail le mécanisme de formation de structure proposé dans [18]. Nous commençons par étudier les équations dynamiques des fluctuations du champ scalaire (6.48).
Nous proposons que ces fluctuations proviennent d'un processus stochastique source par la discrétisation associée aux effets de gravité quantique à l'échelle de Planck : nous supposons que pendant son roulement vers le bas du potentiel, les modes zéro du Higgs interagissent avec la granularité à l'échelle de Planck et diffusent de l'énergie dans les modes avec le nombre d'onde physique k/a. Plus précisément, nous ajoutons un terme de friction à-la Langevin à l'équation du mode scalaire homogène pour tenir compte de cette interaction(6.71). Nous obtenons alors une équation d'équilibre reliant l'énergie perdue par le Higgs qui diminue son potentiel au travail nécessaire pour produire les fluctuations (6.73).
De plus, dans la section 6.4, nous montrons également que si nous supposons qu'une partie de ces fluctuations se condensent pour former des trous noirs primordiaux, ces trous noirs ne sont pas dilués par la phase d'inflation, et le modèle produit la quantité de matière noire froide requise par les observations (6.95).
Dans la section 6.5, nous discutons des différences entre le modèle de formation de structure que nous proposons et le compte standard basé sur les fluctuations du vide. Une différence essentielle entre le mécanisme de formation de structure que nous proposons et le compte standard est que notre modèle admet une interprétation semiclassique correspondant à la version linéarisée des équations de champ d'Einstein semiclassiques.

$$\mathbf{G}_{ab} = 8\pi G \langle \Psi | \mathbf{T}_{ab} | \Psi \rangle \tag{12}$$

Dans notre modèle, les fluctuations naissent au moment du franchissement de l'horizon et diffèrent donc du compte standard où les conditions initiales sont données par le vide de Bunch-Davies qui est défini asymptotiquement dans le passé, ce qui donne lieu au problème *trans-Planckian*.

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A mi familia, en Mendoza y Córdoba, sin la que nada de esto hubiera sido posible.

Амине, за все.

Marseille, December 2021 L. A.

Contents

Introduction Part I

1 Discreteness in Quantum Gravity

General Relativity and Quantum Theory are often quoted to be the two pillars of modern theoretical physics. At large scales, phenomena is dominated by the gravitational interaction, where observations are accurately described by General Relativity down to milimeter-scales [1], while sub-millimiter scales down to $10^{-19}\,m$ are well modelled by Quantum Theory.

The central lesson in General Relativity (GR) is that gravity is geometry, whence, in a fundamental theory, no background structure [1] should be preferred. This clashes with the needs of Quantum Theory, which in its standard formulations need a fixed background and a preferential splitting between time and space.

Quantum Theory provides a general framework for all theories describing particular interactions. It has passed plenty of experimental tests with remarkable precision (e.g. [2, 3]), and is considered a well-established theory, except for the ongoing discussion about its possible interpretations. So far the only interaction which has not been fully accommodated in this framework is the gravitational interaction.

Moreover, a purely classical theory of gravitation can explain every piece of observational data we currently have. One outstanding example of this is the recent observation of gravitational waves [4, 5, 6, 7, 8], a long-predicted consequence of Einstein's General Relativity.

The quest to find a quantum theory of gravity is not, as yet, motivated by experimental results. There are currently no observations that unambiguously point to phenomena that our current theories cannot explain. Thus, we have no experiment to help us in the path towards a quantum theory of gravity.

[1] It is a widely shared opinion that the most outstanding feature of General Relativity is its manifestly *background independence*. Usually confused with *general covariance*, these concepts are related but certainly non-equivalent. See Giulini in [29] for an insightful discussion.

Where do we expect quantum gravitational effects to become unavoidable? Which are the scales we should expect these effects to be relevant? In a universally valid theory of Quantum Gravity, genuine gravitational quantum effects can occur on any scale, with classical behavior arising at a suitable limit. However, there should exist scales where quantum gravity effects become non-negligible.

This is intimately related to old questions in physics: there exist a fundamental limit to the resolution of structures beyond which we cannot access? Is spacetime continuous? Or this picture is just a good low energy approximation that emerges from a more fundamental, discrete structure?[10, 30]

One hint towards the answer to these questions comes from dimensional analysis. There are three constants expected to play a role in Quantum Gravity: Planck's constant $\hbar$, gravitational constant G, and the speed of light c. In 1899 Planck realized that these 3 constants could be combined uniquely to yield quantities with mass, length, and time units. They are the Planck mass, M_{Pl}, Planck length l_P and Planck time t_P.

$$\begin{aligned} M_{\text{Pl}} &= \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\ g \approx 1.22 \times 10^{19}\ \text{GeV} \\ t_{\text{Pl}} &= \sqrt{\frac{\hbar G}{c^5}} \approx 5.40 \times 10^{-44}\ s \\ \ell_{\text{Pl}} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-33}\ cm \end{aligned} \tag{1.1}$$

Note that the Planck mass is rather large by microscopic standards. But this by itself does not mean anything, this mass must be contained in a region of linear dimension l_P for the quantum gravity effects to be non-negligible.

In more detail, Planck scales are attained when the Compton wavelength of an elementary particle is of the order of its Schwarzschild radius:

$$\frac{\hbar}{M_{\text{Pl}} c} \approx \frac{2 M_{\text{Pl}} G}{c^2}, \tag{1.2}$$

which means that the curvature of such elementary particle is not negligible and leads to creation of a microscopic black hole.

This argument, due to Matvei Bronstein[9], who already in 1936 recognized the formidable challenge of quantizing gravity, in its modern version goes along these lines[10, 11]:

Suppose we want to determine the position x of a particle with precision L. Then, due to Heisenberg's uncertainty principle $\Delta x > \frac{\hbar}{\Delta p}$ and thus it follows that $\Delta p > \frac{\hbar}{L}$.

Since the mean value p^2 is larger than the dispersion $(\Delta p)^2$ we have that $p^2 > \left(\frac{\hbar}{L}\right)^2$.

In turn, large momentum implies large energy: in the relativistic limit (for example, a particle being accelerated at CERN), we have that $E \sim cp$, where c is the speed of light, which is why we need to build larger and larger accelerators to prove smaller distances. We see then that, in order to precisely localize the particle we need large energies.

Let us now consider General Relativity. An energy E gravitates with a mass $M \sim \frac{E}{c^2}$. When the energy is large enough, and the mass is concentrated inside a sphere of radius $R_S = \frac{GM}{c^2}$, a black hole forms. Thus the initial region L we wanted to localize will be hidden behind the black hole horizon. We conclude then that L can be decreased only to a minimum value, which is reached when the horizon radius equals L, that is, $R_S = L$.

In other words, the minimal length within which we can localize a particle without it being hidden behind its horizon is given by

$$L = R_S = \frac{GM}{c^2} = \frac{GE}{c^4} = \frac{Gp}{c^3} = \frac{G\hbar}{Lc^3} \implies L = \ell_{\mathrm{Pl}} \tag{1.3}$$

This Gedankenexperiment tells us, then, that it is not possible to localize anything with a precision better than the Planck length.

The Planck scale is remarkably far from the scales we are used to: the LHC probes regimes 10^{-14} times the Planck mass, the Planck length is 10^{-20} times the diameter of a proton. Thus, it is clear that the direct exploration of the Planck scale is impossible with our current experiments.

Nevertheless, the presence of a minimal length and/or a discrete microstructure at the Planck scale is relevant in black hole physics and the physics of the early Universe. What happens when the curvature of the spacetime is Planckian? In other words, what happens close to the singularity inside black holes and the singularity at the Big Bang? If the expectations of a minimal length are then met, the curvature of the spacetime cannot be larger than $\mathbf{R} \sim \ell_{\mathrm{Pl}}^{-2}$, thus theoreticians expect that singularities to be cured in a fully quantum-gravitational theory. These ideas transform black holes and cosmology into the perfect testbeds for the consequences of discreteness at the Planck scale.

The existence of an underlying discrete structure is a trait shared by many incarnations of Quantum Gravity: Loop Quantum Gravity, String Theory, Causal Set, Dynamical Triangulations, among others. In all of these formulations, smooth spacetime arises from a coarse graining where the details of the relationships between the discrete constituents is lost. It is reasonable then to expect that a given smooth geometry corresponds to many different fundamental microstates.

I will briefly review here the case of LQG, which is not only the formulation of quantum gravity I'm more familiar with but also the one that motivated our work.

Loop Quantum Gravity (LQG) is an approach to a background independent Quantum Theory of Gravity based on the non-perturbative canonical quantization of General Relativity. In this framework, there is no preferred background structure and the spacetime itself is a dynamical entity that has to be quantized.

One of the keys results in LQG is that spacetime operators such as Area and Volume acquire discrete spectra[2]: states of the gravitational degrees of freedom can be spanned in terms of spin-network states, each of which admits the interpretation of an eigenstate of geometry which is discrete at fundamental level.

In more detail, one wants to work within the Hamiltonian framework thus we consider the familiar $3+1$ split of spacetime. The details of the $3+1$ decomposition and the Hamiltonian formulation can be found in [12]. One introduces a foliation of the spacetime in terms of 3-dimensional hypersurfaces Σ. In this formulation, the ten components of the metric are replaces by the six components of the induced metric on the hypersurfaces q_{ab}, the shift vector N^a and the lapse function N. In order to prepare to quantization, one must consider the extrinsic curvature K_{ab} of the hypersurface Σ, which plays the role of momenta canonically conjugated to the 3-metric q_{ab}[3].

Next we introduce the triad E^a_i, which is a set of three vector fields defined by the

[2]To be precise, Area and Volume operators are operators acting on the Hilbert space $\mathscr{L}^2(\mathscr{A}/\mathscr{G})$, where $\mathscr{A} \in SU(2)$-connections on Σ, a spatial hypersurface, and $\mathscr{G}$ the Gauss constraint. We will call this space the Kinematical Space. Thus, to get the physical Hilbert space, we need to factor out the Hamiltonian constrain. It is generally assumed that the discrete spectra of these geometrical operators translate from the Kinematical space to the physical space. However, Dittrich and Thiemann showed that discreteness at kinematical level not always translates to the physical level, although in a toy model with few degrees of freedom[31].

[3]In fact, the momenta π_{ab} canonically conjugated to the metric q_{ab} is given by: $\pi^{ab} = q^{-1/2}\left(K^{ab} - Kq^{ab}\right)$, where $K = K^{ab}q_{ab}$.

relation:

$$q^{ab} = E_i^a E_j^b \delta_{ij}. \tag{1.4}$$

For quantizing the gravitational field, instead of using the triad we use as variable the *densitized* triad defined by

$$\tilde{E}_i^a = \sqrt{\det(q)} E_i^a. \tag{1.5}$$

The other set of variables is a $SU(2)$-connection A_a^i which is related to the spin connection w_a^i and the extrinsic curvature of Σ by

$$A_a^i = w_a^i + \gamma K_a^i, \tag{1.6}$$

where γ is a dimensionless parameter called the "Barbero-Immirzi parameter".

Given a 2-surface S that is parametrized by x^1, x^2 with $x^3 = 0$, the area of the surface in terms of the triad is given by:

$$A_S = \int_S dx^1 dx^2 \sqrt{\tilde{E}_i^3 \tilde{E}^{i3}} \tag{1.7}$$

The Area can be promoted to an operator that acting on the spin-network states gives the eigenvalues:

$$\hat{A}_S \psi_s = 8\pi l_P^2 \beta \sum_\alpha \sqrt{j_\alpha (j_\alpha + 1)} \psi_s, \tag{1.8}$$

where the sum is taken over all edges of the network that go through S and j_α label the $SU(2)$ representation carried by the edges.

Thus, we see that LQG predicts the existence of a minimal area given by:

$$A_{\text{min}} = 4\pi\sqrt{3}\beta\ell_{\text{Pl}}^2 \tag{1.9}$$

A similar argument can be made for the volume operator[13, 14], showing that posses discrete spectra.

Considerable effort has been devoted to the study of the low energy limit of the theory. While the difficulties due to the fundamental background independence of the theory still prevent a precise characterization of the low energy limit and thus it is not currently possible to describe in detail the nature of the pre-geometric structure that may survive in the semiclassical limit, it has been learned that a smooth geometry should emerge from the Planckian pre-geometric states via coarse-grained observers insensitive to the UV details.

There are examples of states in the theory that appear to play no important role in the continuum limit but are expected to be produced through dynamical processes. For instance, closed loops and trivalent spin-network nodes degenerate states with vanishing volume quanta which in principle can survive in the continuum limit without spoiling it[15].

The black hole's entropy account in LQG is another example of this theory's feature[32, 33]: the entropy can be calculated by counting the spin-networks (*microstates*) compatible with a smooth (*macroscopic*) geometry describing a macroscopic black hole of mass M.

This feature of LQG (and in general of quantum gravity theories where spacetime is replaced by a discrete notion) will be central in our proposal for a solution to the black hole information problem: the infinite number of degenerated Planckian microstates compatible with flat smooth spacetime will provide the perfect reservoir for information to be stored.

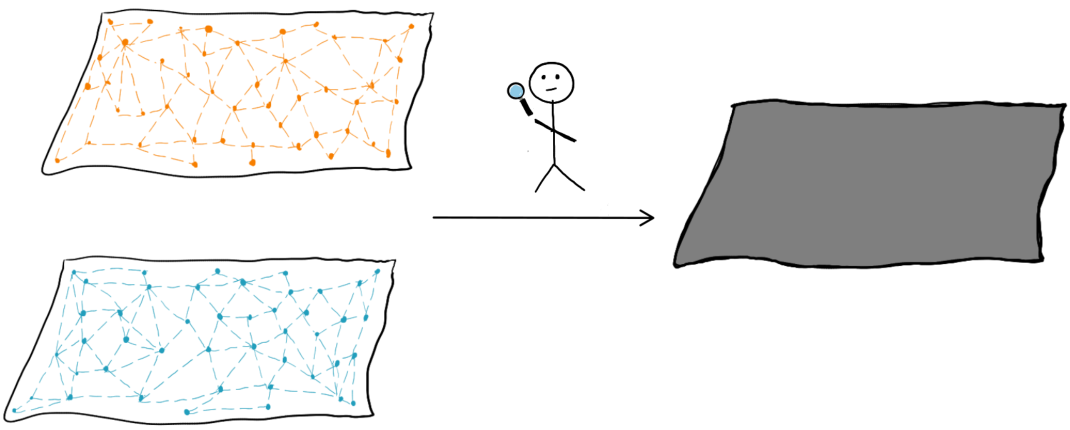

Figure 1.1 – An observer with limited capabilities (coarse-grained observer) is unable to distinguish between two microstates in the fundamental theory. Each smooth geometry in the low-energy description correspond to a myriad of states in the fundamental theory.

Modern approaches to quantum gravity have been developed with the hope that consistency with general relativity and quantum theory would be sufficient to find a coherent quantum theory of gravity, thus postponing the confrontation with observation to a later stage[4].

This hope is still far from becoming true, and the ever-growing number of different approaches to quantum gravity reminds us of the uttermost importance of testing ideas against observations.

One of the biggest problems when constructing a complete quantum theory of gravity is the lack of availability of observational data to help guide the theoretical efforts.

The approaches to the formulation of the theory we described so far take an "bottom to top" approach: they start with some assumptions about the nature of spacetime at a scale ~ 15 orders of magnitude away from the scales that are currently accesible experimentally, and from there the theory should make its way back to the limit in which it should recover the known phenomenology.

In other areas where observations are abundant one can propose phenomenological models that explain the data. Although perhaps these models are inconsistent conceptually, they may serve as guidance for the identification of the correct and conceptually satisfactory theory.

An emblematic example is the theory of Fermi's interaction. In 1934 Fermi proposed a phenomenological model to explain β-decay[16, 17]. At the time, Fermi didn't have a complete ultraviolet description of weak interactions, so he tweaked QED to account for neutron decay. Fermi's account proved to be very accurate in describing the observed phenomena. While the extrapolation of Fermi's theory to explain other interactions proved unsuccessful, its very failure paved the way for a more fundamental understanding of weak interactions.

Fermi's account of β-decay can be thought of in modern terms as (possibly the first) an example of an EFT: it is extremely useful to describe processes at energies well below the mass of the W boson.

This "top to bottom" approach is the one we are interested in. In particular, we want to investigate the possible phenomenology associated with the presence of an underlying discrete substratum.

[4] In this thesis I will not review the various attempts to quantize gravity (Or those, certainly fewer, but for that not less compelling attempts to *gravitize* quantum mechanics[34]) or the reasons why looking for such a theory seems reasonable. For that the reader is referred to the various textbooks and reviews available in the literature[35, 36, 37, 38, 39].

We believe that one promising avenue to realize this idea is studying the possible observational (and theoretical) implications of a fundamentally discrete spacetime. How may this evidence look? Which can be the tell-tale of discreteness? Which is the Brownian motion of our time? In the second part of this thesis (see Section 6), we propose a model of structure formation[18] which is the direct consequence of Planckian Brownian-like diffusion in the early Universe.

Let me finally mention that, as mentioned above, we have not observed quantum-gravitational phenomena,but as experiments become more precise and able to test new aspects of the physical reality, tension with the established physical theories arises. These apparent discrepancies are, of course, nothing new: they have been occurring since humans attempt to understand the universe.

Nevertheless, if we are lucky enough, these discrepancies are not due to a faulty experiment or a misunderstanding of the theory, but they are the consequence of new physical phenomena that the available theories cannot explain.

Several of these apparent discrepancies are present in our current understanding of cosmology. For example, observations of the rotation curves of galaxies [5] suggest that the visible matter of the Universe accounts only for 15% of the total matter content of the Universe while the remaining 85% corresponds to a yet unseen type of matter: dark matter.

The standard account of structure formation[43] in contemporary cosmology relies on the amplification of quantum fluctuations during an inflationary epoch. These quantum effects are assumed to break the background's homogeneity and source the inhomogeneities that, according to the standard account, will evolve to form all the stars, galaxies, clusters, and everything we observe in the Universe.

The standard treatment has several unsatisfactory features, among which is the so-called *quantum-to-classical transition*[44, 45, 46]: near the end of the inflationary epoch, the quantum nature of the system is disregarded, and quantum uncertainties are replaced (usually implicitly) by classical density fluctuations. This transition in the framework still constitutes an unexplained phenomenon[47, 48, 49].

In the majority of incarnations of the framework, the inflationary epoch is driven by a yet unseen (i.e., a scalar field which is not part of the standard model of particle physics[6]) scalar field evolving under an arbitrary potential. In each of these examples, the primordial fluctuations are created asymptotically far in the past at

[5] Observational evidence for dark matter can be found in gravitational lensing, in the CMB, in galaxies clusters, etc.[40, 41, 42]

[6] There are proposals of inflation fueled by the Higgs scalar[50]

trans-Planckian lengths, which is usually quoted as the *trans-Planckian* problem.

In the model we propose in Part III these problems are surmounted. The fluctuations, born at the Planck scale due to the intrinsic granularity of spacetime, can be treated in an explicit semi-classical setting[18].

The non-zero value of the cosmological constant is usually quoted as a consequence of quantum gravitational effects. Also, there is currently a tension between the late and early cosmological time observation of the Hubble constant H_0. The measured value of H_0 in the late universe disagrees with the early Universe values by 5σ[19, 20, 21]. Interestingly, it has been argued that these phenomena can be the consequence of the discrete spacetime microstructure at the Planck scale[22, 23, 24, 25].

This dissertation contains two main projects carried out during the last three years born from these ideas. The first part proposes a novel solution for the information paradox in black holes, and the second part a view on structure formation in cosmology from quantum gravity.

1.1 Part I: Black Hole Information Paradox

The first hints towards the thermodynamical behavior of black holes came in the early 70s with Bekenstein, Bardeen, Wheeler, and Hawking among others. It was shown that equilibrium black hole solutions of Einstein's equation satisfy laws analogous to those of thermodynamics.

But it wasn't until Hawking's seminal work that an apparent paradox arose. Hawking's ground-breaking paper showed, using semi-classical considerations, that black holes lose mass via thermal radiation. Moreover, the emission rate depends inversely on the black hole's mass, suggesting that black holes evaporate in finite time. If the black hole completely disappears, all the information once contained inside the black hole disappears with it, leaving only a thermal bath of Hawking's particles in its wake. Thus, an initial pure state has evolved into a mixed final state. In stark contradiction with quantum mechanics, the evolution between the initial and final state is not unitary and information is lost: the information encoded in the final state is insufficient to recover the initial state. This is known as the black hole information paradox. In Section 3.2 we give a more detailed and thorough discussion of the paradox.

Several proposals to solve this apparent paradox have appeared in the literature over the last 50 years. Some of these approaches, seeking to restore unitarity in the dynamics, propose dramatic departures of known physics in regimes where semi-classical models are thought to be a good approximation.

Other, more conservative proposals entertain the idea of the solution coming from new physics arising at the Planck scale, where the semi-classical description is expected to break down. We do a review of the merits and drawbacks of each proposal in Section 3.3.

In Part II we propose a conservative and novel solution to the problem. In approaches to quantum gravity where the description of the Universe as smooth spacetime is thought of emerging as an approximation of a more fundamental, discrete structure at the Planck scale, any description in terms of smooth fields is bound to miss part of these discrete degrees of freedom and thus break unitarity. Under this point of view, if these fundamental degrees of freedom are taken into account, the process of black hole formation and subsequent evaporation can be described by a unitary evolution. This idea, first proposed [15], is shown in an explicit quantum gravitational context in Section 6.78, following [26, 27].

The essential argument in the proposal is to note that the formation and eventual evaporation of a back hole can be framed in the same terms as the process of burning a paper.

Let's consider printing, reading, and burning this page. The system's initial state is highly special from our perspective: as macroscopic observers we can distinguish the ink particles that form the letters *CMJ* from the ones in the letters *LBC*.

If placed in an enclosed case, the printed text on this page would persist for a fairly long time and be left for future generations to judge the content of these words. The phase space available to the particles forming the paper and the words written on it remain more or less constant through time.

If, however, we decided to set the paper on fire, is the information written on it lost forever? Can we recover the written text by looking at its ashes?

So far, the physical theories describing our world are all reversible in the sense that we can predict the future state of a set of variables from their initial state. Conversely, we can recover the past from the values of the variables in the future. Naively, this feature clashes with the intuition in the case of the burning paper: the information once contained in the paper appears forever lost. However, there is a physical account of the phenomenon that preserves reversibility. Once the paper is set on fire, the information goes into correlations between the molecules that are now free to diffuse into the atmosphere. Information is then not lost but degraded into correlations that are inaccessible to us, coarse-grained observers.

In the first sections of Part II, we argue that the formation and evaporation of black holes can be framed along the same lines: a unitary, reversible account of the process is available when Planckian degrees of freedom are considered. Correlations established with the Planckian microstructure are missed when considering smooth fields that disregard these discrete degrees of freedom and the usual low-energy, QFT non-unitary description is recovered. These correlations are fueled by the presence of a unavoidable high curvature region (the *would-be singularity*). This Planckian curvature region around the place where classicaly there is a singularity acts as the fire in the burning paper example: it opens a huge, new and unexplored region of phase space by bringing the system in contact with the Planck scale.

In Sections 4.2-4.10 we introduce a quantum-gravitational model where these ideas are realized. The model is based on the techniques of Loop Quantum Cosmology, which is presented in 4.2 as well as in the Appendix B.

We argue that this quantum cosmological model contains the essential ingredients which provide the solution in the black hole case: the presence of UV degrees of freedom provided by the so-called ϵ-sectors. The results can then be immediately extrapolated to the black hole case.

The study of this issue in cosmological polymer models, aside from having fewer technical difficulties than its black hole counterpart, provides a clear conceptual interpretation of the physics in play. The results for the more technically complicated (but analogous) black hole case will be reported elsewhere[28].

We show that for coarse-grained observers insensitive to Planckian degrees of freedom, an initially pure state develops correlations between low-energy and Planckian degrees of freedom when evolved through a region of high curvature.

These correlations are inaccessible for low-energy observers, for which the dynamics are non-unitary even though the dynamics of the underlying theory are unitary. These coarse-grained observers see the entropy grow between the initial and final state, and this entropy is just a measure of the ignorance of such observers.

1.2 Part II: CMB observables from Planckian granularity

In Part III we study another possible consequence of a discrete spacetime. We proposed a model of inflation driven by a decaying cosmological constant due to friction with a discrete background.

In this model, the inhomogeneities observed in the CMB arise from those associated with a discrete spacetime. We obtain the observed approximately-scale invariant spectrum of scalar perturbations with the correspondent blue tilt. The model also predicts a vanishingly small tensor-to-scalar ratio. These phenomenological predictions depend only on known parameters of standard model physics.

In section 6.1 we introduce unimodular gravity as the low-energy limit of a theory in which spacetime is fundamentally discrete and built from 4-dimensional discrete building block: quanta of 4-volume.

While the phenomenological model we propose can also be realized within General Relativity, Unimodular Gravity provides a unique conceptual point of view: it describes the low-energy limit of a fundamentally discrete theory. As such, it incorporates friction as violations of conservation of the energy-momentum tensor, that is $\nabla^a \mathbf{T}_{ab} \neq 0$.

This is a crucial ingredient: the usual smoking gun of a discrete background structure is friction, the possibility of energy to be lost into the chaos of microstates not captured by the effective theory.

In Section 6.2 we study the dynamics of a cosmological spacetime with an exponentially (in analogy with standard dissipative systems) decreasing cosmological term

$$\Lambda(t) = \Lambda_0 \exp\left(-\beta M_{\rm Pl}\, t\right) \tag{1.10}$$

where $M_{\rm Pl}$ is the Planck mass, β a dimensionless parameter, $\Lambda_0 \sim M_{\rm Pl}^2$ and t the cosmological (unimodular) time related to the comoving time τ by

$$dt = a^3 d\tau \tag{1.11}$$

We show that, as long $\beta M_{\rm Pl} < 1$, the Universe undergoes an inflationary phase fueled by the decaying Λ. Moreover, for small enough β the inflationary phase lasts enough to resolve both the horizon and the flatness problems independently of the initial conditions[7]. See Figure 2. We see also that the decay once $\beta M_{\rm Pl} \sim 1$ is largely independent of the value of β.

In Section C we show that the observational consequences of the model are not strongly dependent on the exact functional form of Λ. In particular, as long as Λ is nearly constant for a long enough number of *e-folds* in such a way that the scales observed today in the CMB were created at the Planck scale during inflation, and Λthen rapidly decays the observational predictions of the model remain of the same order of magnitude.

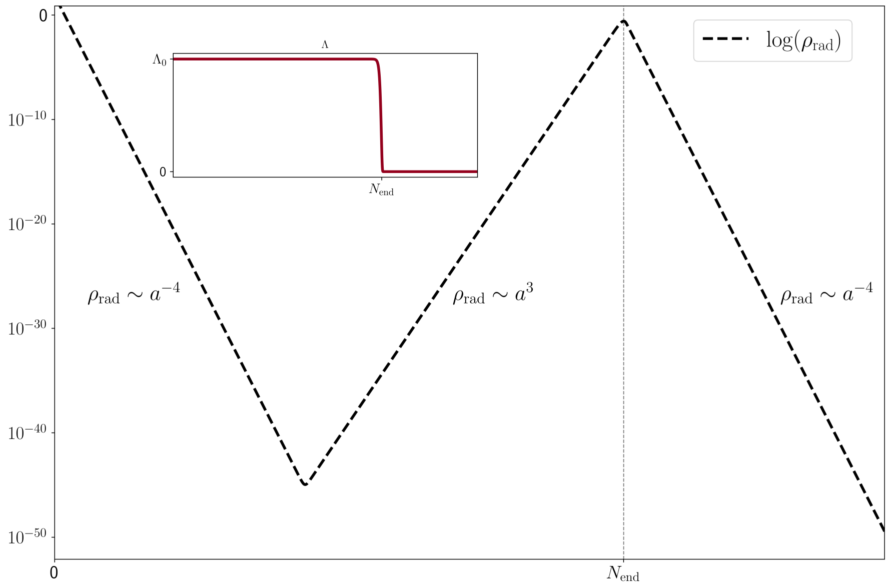

Figure 1.2 – Background dynamics with a decaying cosmological constant

In Section 6.2.2 we study the dynamics of the Higgs field coupled to gravity. We will show that the fundamental discreteness is imprinted on the natural scalar degree of freedom in the standard model, the Higgs boson, which will carry these inhomogeneities to the CMB.

In Section 6.3 we thoroughly describe the structure formation mechanism proposed in [18]. We start by studying the dynamical equations for the scalar field fluctuations (6.48).

These fluctuations are proposed to arise from a stochastic process source by the discreteness associated with quantum gravity effects at the Planck scale: we assume

[7] It is misleading to claim that inflation *solves* any of these problems. A tremendous reduction of the possible initial conditions is already in place when considering homogeneous spacetimes (and perturbations around them). This problem is beautifully stated and explained by Penrose in many of his books, see for example [51]

that during its rolling down the potential, the zero modes of the Higgs interacts with the granularity at the Planck scale and diffuses energy into the modes with physical wavenumber k/a. More precisely, we add a Langevin-type friction term to the equation for the homogeneous scalar mode to account for this interaction(6.71). We then obtain a balance equation linking the energy lost by the Higgs rolling down its potential to the work necessary to produce the fluctuations (6.73).

We can then carry on to compute the power spectrum of the scalar perturbations (6.89). Moreover we can compute the spectral index n_s (6.90). We see that the spectrum is compatible with observations for a friction coefficient $\gamma \sim 10^{-10}$.

Furthermore, in Section 6.4,we also show that if we assume that part of these fluctuations collapse to form primordial black holes, these black holes do not get diluted by the inflationary phase, and the model produces the amount of cold dark matter required by observations (6.95).

In Section 6.5 we discuss the differences between our propose model of structure formation and the standard account based on vacuum fluctuations.

A key difference between our proposed mechanism of structure formation and the standard account is that our model admits a semiclassical interpretation corresponding to the linearized version of the semiclassical Einstein field equations

$$\mathbf{G}_{ab} = 8\pi G \langle \Psi | \mathbf{T}_{ab} | \Psi \rangle \tag{1.12}$$

In our model fluctuations are born at horizon crossing and hence differs from the standard account where the initial conditions are given by the Bunch-Davies vacuum which is defined asymptotically in the past, giving rise to the *trans-Planckian* problem.

2 An Apology to Unimodular Gravity

Unimodular Gravity will be a reoccurring theme throughout this work. Here I will present a brief description of Unimodular Gravity and its unique conceptual value.

In a nutshell, Unimodular Gravity is a theory of gravity where the cosmological constant, rather as a parameter, appears as a Lagrange multiplier in the gravitational action.

This slight reformulation harbors a conceptually unique interpretation: the constraint imposed by the cosmological constant expresses the existence of fundamental elements of 4-volume[52, 18].

The molecular structure of matter has significant consequences in the macroscopic description: a giveaway of the presence of discrete constituents is the leak of energy from the macroscopic degrees of freedom to microscopic degrees of freedom not accounted for in the long-wavelength description.

For example, we describe macroscopic fluids using Navier-Stokes equations. This is just an effective description that ignores the molecular degrees of freedom. Nevertheless, their presence manifests in the viscosity and friction terms that lead to energy loss in the system. Energy from macroscopic degrees of freedom leaks into the microscopic molecular chaos. Analogously, if we think the continuous as a emergent feature in the low energy description, energy should be able to diffuse from the macroscopic degrees of freedom to the underlying granular structure.

How can this phenomenology be accommodated in gravity? GR's general covariance implies that the energy-momentum tensor is conserved. Here Unimodular Gravity enters the scene: the cosmological constant is a Lagrange multiplier which fixes the volume element to a constant value thus breaking down general diffeomorphisms down to volume preserving ones.

The action of Unimodular Gravity is given by

$$S = \int \left(\sqrt{g}\mathbf{R} + \lambda\left[\sqrt{g} - v^{(4)}\right]\right) dx^4 + S_m, \tag{2.1}$$

where S_m denotes the matter action, λ is a Lagrange multiplier and $v^{(4)}$ is a fixed background 4-volume form

$$\mathbf{v}^{(4)} \equiv v^{(4)} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{2.2}$$

The background volume element partially breaks diffeomorphisms down to volume-preserving ones. This allows for violations of energy-momentum conservation. More explicitly, varying the action (2.1) we obtain the unimodular equations of motion

$$\mathbf{R}_{ab} - \frac{1}{4}\mathbf{R}g_{ab} = 8\pi G\left(\mathbf{T}_{ab} - \frac{1}{4}\mathbf{T}g_{ab}\right) \tag{2.3}$$

which, using Bianchi identities can be rewritten as

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} + \underbrace{\left[\Lambda_0 + \int_\ell J\right]}_{\Lambda} g_{ab} = 8\pi G\mathbf{T}_{ab}, \tag{2.4}$$

where $\mathbf{J}_b \equiv (8\pi G)\nabla^a \mathbf{T}_{ab}$ is the energy-momentum violating current which stisfies $d\mathbf{J} = 0$.

Note that, if $\mathbf{J} = 0$ we recover *exactly* Einstein field equations[1].

What is the role of the background volume structure, and how interpret it? Pure gravity has no scale: from the gravity point of view, the hydrogen's Bohr radius is an oddity.

We can shed light on this question by assuming that, as the Navier-Stokes hydrodynamics description, general relativity is nothing but an effective macroscopic theory that emerges as the low-energy limit of a more fundamental, discrete description that comes to aid us.

These fundamental building blocks have to be carved out from four-volume elements

[1] Even when $\mathbf{J} \neq 0$, we can recast, at the level of the field equations, Unimodular Gravity as the Einstein field equations with a energy-momentum tensor formed by two interacting components. That is

$$\mathbf{G}_{ab} = 8\pi\tilde{\mathbf{T}}_{ab}, \tag{2.5}$$

where $\tilde{\mathbf{T}}_{ab} = \mathbf{T}_{ab} - \left[\Lambda_0 + \int_\ell J\right] g_{ab}$

to be compatible with Lorentz invariance. Moreover, these 4-dimensional building blocks will naturally produce a background fixed structure in the long-wavelength limit, thus recovering Unimodular gravity (See Part III and [53, 54] for a more detailed account).

In Part III, we study a possible mechanism where friction with this fundamental building block in the early universe produces a cosmological constant that relaxes to the observed values and leaves an observable imprint in the CMB.

Finally, it is important to remark that, while Unimodular Gravity plays a pivotal role in the conceptual and physical aspects of this thesis (and those of [26, 27, 18]), everything in this thesis can be reformulated in terms of general relativity (and polymerization of).

To be more precise, in Part II, unimodular gravity dramatically simplifies the calculations and the physical interpretation by providing a unique notion of time (the cosmological time). Nevertheless, the essential ingredients: the presence of different e-sectors, and the fact that wavefunctions supported on distinct lattices evolve differently, are also present in the more standard polymerizations (i.e., in the standard accounts of LQC). Moreover, as we argue in Section 4.3, the crucial feature in our proposal (ultraviolet degrees of freedom not accessible to coarse-grained observers) is present in every polymer model.

Concerning Part III, the situation is similar. While Unimodular Gravity laid the conceptual basis of our analysis, the model could have been realized in general relativity with interacting fluids as the source.

We think this is an exciting feature of the proposals described in this thesis: the conceptual ideas they are based on are independent of one's favorite theory of gravity and one's favorite quantum gravity approach. Indeed these scenarios could be realized in different approaches to quantum gravity.

Black Hole Information Paradox Part II

3 Black Hole Information Paradox

3.1 Entropy and the Second Law of Thermodynamics

The mathematical models that so far define our successful physical theories are all reversible in the sense that they can predict the future value of the variables they use from their initial values, while conversely the past can be uniquely reconstructed from the values of these variables in the future. The memory of the initial condition is not lost in the dynamics and their information content remains.

This is true for classical mechanics and field theory, and also true for quantum mechanics and quantum field theory as long as we do not invoke the postulate of the collapse of the wave function (i.e. as long as we do not intervene from the outside via a measurement [1]). In the quantum mechanical setting, this property boils down to the fact that evolution to the future is given by a unitary operator, which can always be undone via its adjoint transformation.

This property of our fundamental models has always troubled naive intuition when faced with situations that appear to be irreversible. For example: what would happen with these words if the computer collapses at this very moment. What if, after printed, this paper is burned. Common sense would answer that the information in these pages (if of any relevance) would be lost. However, the physicist, trained to firmly believe in the statement of the previous paragraph, would say that the information in these words is not lost but simply hidden (to the point of becoming unrecoverable) in the humongous number of microscopic variables that would describe the whole system.

[1] This is not the case in modifications of quantum mechanics where the collapse of the wave function happens spontaneously.
In such theories information is actually destroyed (for a discussion of black hole evaporation in such contexts see [55, 56, 57]).

In the case of burning the paper, these words remain 'written' (it would be claimed) in the multiple correlations between the degrees of freedom of the molecules in the gas of the combustion diffusing in the atmosphere while transferring the information to even larger and yet pristine portions of the very large phase space of an unbounded universe.

In the case where the computer collapses, a similar story can be told involving the dissipation of the bits into the environment. Of course the physicist cannot prove this; however, it is a consistent story in view of the strongly cherished principle of unitarity.

Such effective irreversibility is clearly captured in the second law of thermodynamics, stating that (for suitable situations involving a large number of degrees of freedom) entropy can only increase.

At the classical level this clashes, at first sight, with the Liouville theorem stating that the phase space volume of the support of a distribution in phase space is preserved by dynamical evolution.

However, nothing restricts the shape of this volume to evolve into highly intricate forms that a macroscopic observer might be unable to resolve. More precisely, suitable initial conditions that the observer agent regards as special (for instance, the macroscopic configurations of ink particles defining words in this paper before the fire reached them) come with an uncertainty according to the observers' limited measurement capabilities. This is idealized by a distribution in phase space occupying an initial phase space-volume of a regular shape (this ensemble of points represents the system in what follows).

Now as time goes, the apparent phase space volume (not the real volume which remains constant) would seem to grow to the agent just because of its intrinsic inability to separate the points in phase space that the systems occupies from the close neighbouring ones where the system is not. In this sense the arrow of time is only emergent macroscopically due to the special initial conditions, and the intrinsic coarse graining introduced by a macroscopic observer with its limitations.

We will argue that the general lines of this story remain the same when black hole evaporation is considered.

3.2 Information Loss

To be more precise, when considering evaporation spacetimes, there is an apparent breakdown of *retrodictability*: having the complete knowledge of the final state after the black hole evaporates is not sufficient to retrodict the full details of the initial state.

The initial state contains information (for example, the pages of a book thrown into the black hole before evaporation) that does not get encoded into the final state. At the quantum level, this is expressed as a non-unitary evolution between the early and late states. A pure initial state gets mapped into a mixed state after the black hole evaporates.

The paradox refers to an unmet expectation: this result is at odds with quantum mechanics' unitary (and deterministic) evolution.

There are, though, several caveats to the above conclusion. First, its validity is based on (sometimes subtle) assumptions that are not always explicit and (more often than not) depend on the background and prejudices of the physicist making them. A glance at the vast bibliography about the subject makes this evident.

For example, the string community mainly focuses on models guided by holographic inputs (EP=EPR [58], Replica wormholes [59, 60, 61], etc.), while physicists working on canonical approaches to gravity generally consider polymer models [62, 63, 64, 65].

Given this plethora of proposed solutions and bibliography about the subject, this section will introduce the black hole evaporation paradox, emphasizing those crucial features to our approach and trying to explicit the assumptions made along the way. This strategy will help us understand which assumptions are dropped in the different proposed solutions and the physical implication of those choices.

3.2.1 Classical Black Holes

An astronomical object of big enough mass will undergo an unstoppable gravitational collapse to form a black hole. The details of this collapse can be very complicated, depending on the details of the initial matter distribution.

Nevertheless, the singularity theorems, the black hole uniqueness theorems, and the cosmic censorship conjecture imply that the final configuration of (isolated) gravitational collapse is a stationary black hole characterized by a handful of macroscopic parameters: its mass$M$, charge Q, and angular momentum J.

In other words, every isolated (dynamical) black hole will eventually settle into a stationary black hole belonging to the Kerr-Newman family described by three parameters: M, Q, and J.

This feature of gravitational collapse is reminiscing of the mechanism in section 3.1:

an arbitrarily large set of initial conditions are dynamically driven into a final state described by a finite (and small) number of parameters. This behavior points towards a first clue unveiling the irreversible character of gravitational collapse. Information in the initial matter configuration appears to be lost in the final state.

We can draw an analogy with the classical thermodynamic example of expanding gas in a box. In this case, dynamics drive an infinite set of initial data into a final state (gas occupying the whole box volume) described by temperature T, pressure P, and volume V of the gas in an irreversible fashion. During the process, a coarse-grained observed saw entropy grow due to its limited capabilities.

We will argue in the following sections that the process of gravitational collapse, forming of a black hole, its eventual evaporation, and the resulting apparent paradox can be framed and understood under this paradigm.

In the early 70s, Bardeen and Hawking showed that stationary black holes obey laws analogous to those of thermodynamics. The area A of the black hole's horizon plays the role of the entropy S, and the surface gravity κ plays the role of the temperature T.

Identifying the black hole temperature with the surface gravity may seem spurious at this stage: a classical black hole has strictly absolute zero temperature. Indeed, classically a black hole does not emit any radiation. However, as we hinted before and will show in the next section, the situation changes dramatically when quantum effects are taken into account.

Before considering the effects of quantum fields evolving on black hole spacetimes, let us address the apparent loss of information we mentioned early this section.

Consider the two Cauchy surfaces in Figure 1: at early times, the Cauchy surface Σ_1 is entirely located outside the black hole, while at later times, the Cauchy surface Σ_2 has a part inside the horizon and a part outside. Thus, the evolution is not unitary for outside observers with no access to the information encoded in Σ_1^{in}.

In other words, pure states on Σ_1 evolve into pure states in Σ_2, but these become mixed when degrees of freedom in the black hole's interior are traced out.

This example in the classical case is evocative of what will happen in the semiclassical case: evolution between an initial Cauchy surface and a final non-Cauchy surface (like Σ_2^{out}) is non-unitary.

Wald [66] shows clearly and explicitly that even within the realms of a fully deterministic theory, a state in a non-Cauchy surface can be entirely determined from a state on

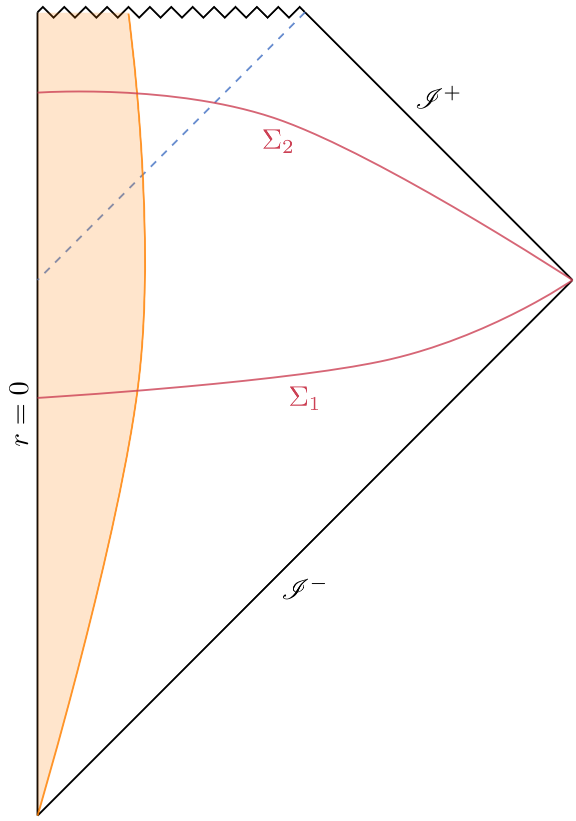

Figure 3.1 – Penrose diagram of a spherically symmetric collapsing star. Σ_1 is a early times Cauchy surface entirely located outside the horizon. Σ_2 is a late times Cauchy surfaces which has parts inside and outside the horizon (dashed blue line).

a Cauchy surface but not vice-versa. That is, the evolution is deterministic forward in time, but it is not retrodictable[67].

We only have the right to expect unitary (and deterministic) evolution from one state on a Cauchy surface to another.

Analogous to the outside observer case discussed before, the evolution of a state on a Cauchy surface to a state on a non-Cauchy surface carries an apparent loss of information.

3.2.2 QFT on Black Hole Spacetimes

As hinted in the introduction, Hawking showed that black holes radiate thermally and that the temperature of this radiation depends linearly on the surface gravity $T = \frac{\kappa}{2\pi}$.

In a regime where both are expected to be good approximations, general relativity combined with quantum field theory implies that large isolated black holes behave like thermodynamical systems in equilibrium. This is because they are objects close to equilibrium at the Hawking temperature that lose energy extremely slowly via Hawking radiation.

When perturbed, they come back to equilibrium to a new state, and the process satisfies the first law of thermodynamics with an entropy equal to 1/4 of the area A of the black hole horizon in Planck units. Under such perturbation (which in particular can also be associated to their slow evaporation), the total entropy of the universe can only increase, namely

$$\delta S = \delta S_{\text{matter}} + \frac{\delta A}{4} \geq 0, \tag{3.1}$$

where δS_{matter} represents the entropy of whatever is outside the black hole (including for instance the emitted radiation).

This result shed new light onto the analogy between the black hole laws and those of thermodynamics: now we can interpret $\frac{\kappa}{2\pi}$ as truly as the *physical* temperature of the black hole.

Hawking's calculation concerns test quantum fields on black hole spacetimes. In general, techniques of quantum fields on curved spacetimes cannot deal with backreaction, and currently, an appropriate framework is lacking. So a dynamical model of black hole evaporation, including the Hawking radiation's backreaction on the spacetime, is presently lacking.

Nevertheless, by considering energy conservation arguments, it can be deduced that the flux of positive energy towards future null infinity implies that the mass of the black hole must decrease.

The picture now changes radically: the black hole emits radiation at a temperature T proportional to the surface gravity, which in turn is inversely proportional to M, the black hole mass. As the mass of the black hole decreases, the temperature increases, leading to a more rapid emission rate leading to a runaway process that suggests that the black hole evaporates in a finite time, resulting in a spacetime as in Figure 3.2.

The point is that the irreversibility captured by (3.1) can once more be associated with the same ingredients present in the last section's discussion: the special nature of the initial conditions due to our biased criterion of macroscopic observers (low curvature and low densities in the past), high curvature and a huge new phase available in the Planckian regime near what would be the singularity in general relativity (the *would-be-singularity* from now on).

As emphasized by Penrose (see for instance [51]), among others, the arrow of time comes from the fact that we started with a spacetime that was well approximated by a low curvature one with some dilute matter distribution (gas and dust) that would first form large stars that one day can collapse to form black holes[2].

Before the formation of a black hole, the story of our system exploring larger and larger portions of the available phase space is the usual one involving molecules, atoms, and fundamental particles.

The perspective we want to stress here is that the story continues to be the same after the black hole forms, but now a new and huge new portion of phase space has is opened by the gravitational collapse: the internal *would-be-singularity* of the classical description beyond the event horizon.

Like the lighter setting the paper on fire and allowing for fast chemical reactions that degrade the ink in these words when burning the paper, the singularity brings the system in contact with the quantum gravity scale. The gravitational collapse ignites interactions with the Planckian regime inside the black hole horizon (see Footnote 1), and that must be (as in the burning paper) the key point for resolving the puzzle of information in black hole evaporation. This perspective was advocated in [15, 68].

As we pointed out before, the current theories cannot provide a complete, systematic treatment of the evaporation process. Here is, then, one of the places for assumptions to come in, leading us (or not, depending on which our assumptions are) to a paradox.

The calculation of Hawking radiation needs quantum field theory on curved spacetimes to be valid around the black hole horizon. Indeed, this poses no problem in the early evaporation phase. As we discussed in the introduction, the QFT on CS description is thought to break down at the Planck scale. [3].

[2]To these two specialty conditions one might also have to add one concerning the state of the hypothetical microscopic constituents at the Planck scale if the view we are advocating here and in [15, 68] is correct.

[3]There is a caveat here: when tracing backward from $\mathscr{I}^+$ a Hawking mode and its partner mode inside the horizon, we see that the entanglement comes from very short (transplanckian) distances across the horizon. So, as we discussed above: can QFT on CS be trusted in this regime? The issue of robustness of Hawking radiation against changes at transplanckian scales has been discussed lengthily[69, 70, 71, 72, 73, 74]. Although no conclusive answer is available, evidence suggests that the prediction of Hawking radiation is independent of physics at very short scales. Furthermore, an argument due to Fredenhagen and Haag, suggest that Hawking radiation arise as a consequence of the

What happens, then, close to the singularity and at very late stages of the evaporation, when the semicalssical description breaks down?[4].

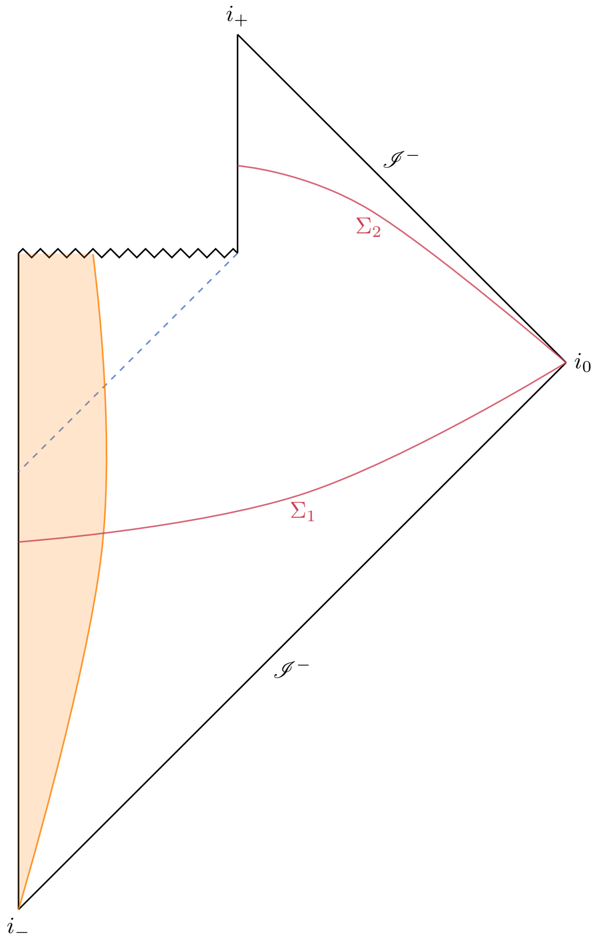

Figure 3.2 – Penrose diagram of a evaporation spacetime. Σ_2 is not a Cauchy surface.

3.3 The Paradox

Before answer the question posed above, let me introduce a set of assumptions that will lead to a first conflict to arise:

- **(0):** A black hole forms as a result of gravitational collapse
- **(1):** Quantum fied theory is valid near the black hole horizon

behavior of Hadamard state at (relatively) late times around the black hole horizon[75, 76].

[4]More precisely, when the curvature at the horizon becomes Planckian, $K = R_{abcd}R^{abcd} \sim \ell_{\rm Pl}^{-4}$

- **(2):** Quantum dynamics is always unitary

Of course, this list of assumptions can be made more explicit or concise depending on which characteristic one is looking to emphasize[5]. For example, we could replace assertion **(1)** by:

- **(1a):** Hawking's radiation results in the decrease of the black hole mass
  (1b): Due to Hawking's radiation the black hole completely evaporates
  (1c): If the black hole does not completely evaporate and leaves a Planckian remnant, then the number of its internal degrees of freedom is bounded by its mass[6]

Let me go back to the original set of proposed assumptions. When the first two conditions, **(0)** and **(1)**, are satisfied, we are led to the black hole formation and later evaporation depicted in Figure 3.2. First, due to the gravitational collapse of some matter distribution, **(0)** a black hole is formed. Then, since **(1)** quantum field theory on curved spacetimes is valid around the horizon, the black hole starts evaporating until it disappears in finite time . A pure quantum state on the Cauchy surface, Σ_1 evolves into a mixed state on the surface of Σ_3. Thus, in contradiction with assumption **(2)** the quantum evolution is not unitary.

Wald[80] (and many others[67, 57]) has vehemently argued that this inconsistency poses no problem to quantum mechanics. The key observation is that Σ_3 is not a Cauchy surface; thus, non-unitary evolution between Σ_1 and Σ_3 is to be expected. In Wald's own words

> *...within the semiclassical framework, the evolution of an initial pure state to a final mixed state in the process of black hole formation and evaporation*

[5]A minimal set of assertions is presented in [77]:

- **(GH)** All physical reasonable spacetimes are globally hyperbolic
- **(EvST)** Some evaporation spacetimes are physically reasonable

The paradox in these assumptions is put forward when invoking the Kodama-Wald Theorem[78, 79]: *No evaporation spacetime is globally hyperbolic.*

The majority of solutions we will discuss in this thesis involve rejecting the second assumption. In these cases, it's assumed that evaporation spacetimes are not physically reasonable by invoking the emergence of new physical processes at some scale (usually the Planck scale).

[6]This one can be replaced by:

- **(BHEn)** The statistical entropy of a black hole is the same as its Bekenstein-Hawking entropy

> *can be attributed to the fact that the final time slice fails to be a Cauchy surface for the spacetime. No violation of any of the local laws of quantum field theory occurs.[80]*

Maudlin also raised this issue in a recent provocative paper[67] in which he gives a conservative and straightforward solution to the so-called paradox: there is no paradox.

To restore unitary evolution between an initial and final slice in an evaporation spacetime, one must consider a final Cauchy surface after the evaporation event. This is possible by extending the definition of Cauchy surfaces to consider disconnected Cauchy surfaces (see Figure 3). Using this new disconnected family of Cauchy surfaces, we can foliate the entire evaporation spacetime with Cauchy surfaces: the spacetime is globally hyperbolic.

Yet, we mentioned earlier the Kodama-Wald theorem, which states that no evaporation spacetime is globally hyperbolic. What went wrong? The key is one assumption that goes into the Kodama-Wald theorem (as well as many standard theorems in General Relativity, in particular the Geroch splitting theorem [81]): the spacetime is a 4-dimensional smooth (Hausdorff, paracompact) Lorentzian manifold.

However, the spacetime depicted in Figure 3 fails to be a manifold at the Evaporation Event. This is what ultimately allows to foliate the evaporation spacetime into Cauchy surfaces.

As readily pointed out in [77, 57], the definition of disconnected Cauchy surfaces poses several technical problems. I will not discuss these issues here[7], but I will highlight a central idea in Maudlin's paper that resonates with our proposal: Maudlin's insight was that, in order to found an appropriate Cauchy foliation, the smooth manifold structure had to breakdown at the evaporation event.

While in this particular case this breakdown appears to be due to a pathological spacetime, it suggests that the description in terms of a smooth manifold is an emergent description only adequate at low energies which may not be sufficient to describe the whole process of evaporation. This is the point of view we adopt in this thesis and will be crucial in our proposal (see Section 4). A smooth (Lorentzian) manifold is just an emergent feature from a more fundamental, discrete structure.

In a concrete example, we have seen that one could restore the sought unitarity at the cost of losing the manifold structure at a point. However, Maudlin's scenario does not

[7] See [77, 57]

include is any expected effects from quantum gravity at Planckian (or any) scale.

Even when Maudlin's argument is compelling, it fails to address the crucial lesson of the problem: the black hole evaporation opens a path to the quantum gravitational regime. Unavoidably, the system will be in touch with the Planck scale before it completely evaporates.

Let us go back now to the discussion of **(2)**. As we mentioned before, unitarity is not to be expected when the final surface is non-Cauchy. Many attempts to restore unitarity in this scenario seem misguided at best: having a unitary evolution in this semiclassical approach is not compatible with quantum mechanics!

It has been argued that a non-unitary evolution will raise severe conflicts with energy conservation at ordinary (laboratory) scales. The most common paper cited to support this claim is by Banks, Peskin, and Susskind. Later, careful analysis showed that these claims were not laid on solid grounds and that many possibilities to avoid the conclusion exist[82].

More concretely, Unruh proposed a model where decoherence can take place without energy dissipation[83]. We realized this idea in a series of papers [26, 27], showing in a concrete quantum gravitational toy model that one can have an enormous increase in entanglement entropy at no energy cost (see Section 4).

Let us now turn the attention briefly to assumption **(0)**: Dropping this assumption amounts to say that a black hole never forms as a result of gravitational collapse and is replaced by another structure without an event horizon. Of course, if no black hole ever forms, there is no black hole information paradox.

I will no comment further about these proposals. Still, they generally imply that general relativity and/or quantum field theory should fail drastically in regimes expected to be a good approximation.

In what follows, we will always suppose that black holes can form due to gravitational collapse (i.e. that assumption **(0)** is fulfilled).

Another possibility is to consider stark departures from semiclassicality during evaporation (at early times). These proposals sought to restore unitarity in the final state by considering deviations from the semiclassical picture so that there is no entanglement between the degrees of freedom outside and inside the horizon.

As we mentioned before, a non-unitary evolution is a consequence of quantum field theory; attempting to restore unitarity in this way implies the breakdown of quantum field theory in low curvature regions (the curvature at the black hole's horizon is unbounded). In particular, when entanglement between Hawking pairs across the horizon is erased, the quantum field becomes singular at the horizon, it becomes a *firewall*[8]. This stark deviation from semiclassicality makes this type of proposal quite radical.

Another possibility presents when quantum gravitational effects enter the picture. As we argued in the introduction, assumption **(1)** cannot be trusted close to the singularity or at late stages of evaporation.

The *remnant* paradigm assumes that after the black hole evaporated down the Planck scale, quantum gravitational effects halt the black hole evaporation leaving behind a remnant where information can be stored. A spacetime with a stable remnant is depicted in Figure 3.3.

Following Chen et al. we consider the following broad definition for remnant:

> ***Definition:*** *A remnant is a localized late stage of a black hole under Hawking evaporation, which is either (i) absolutely stable, or (ii) long-lived.*[84]

Several avenues exist to realize concrete remnant scenarios: modified quantum mechanics (GUP-like scenarios), modified general relativity, quantum gravitational effects (usually coming from polymer models inspired in LQG), among others. It's not our aim to be exhaustive but to highlight the merits and limitations of the general remnant approach. For a comprehensive account, see [84].

Assuming the natural scale for quantum gravitational effects to become relevant to be the Planck scale, the remnant's mass can be estimated to be of the order of Planck's mass M_{Pl}.

From the perspective of an observer close to future null infinity, the Bondi mass of the black hole will decrease until it reaches the Planck scale.

Note that the initial mass of the black hole is, in principle, unbounded. In that case, we see a potential problem: the final remnant has to have an arbitrarily large number

[8]There is an extra assumption going into the firewall models: **(BHEn)** The statistical entropy of a black hole is the same as its Bekenstein-Hawking entropy.

In order words, the number of black hole states to which external degrees of freedom can be entangled is bounded by $N \sim e^{A/4}$. This assumption and the desire of restoring unitarity in a non-unitary process led to firewalls at the horizon. We will see that discrete approaches to quantum gravity suggest that **(BHEn)** is an unphysical assumption.

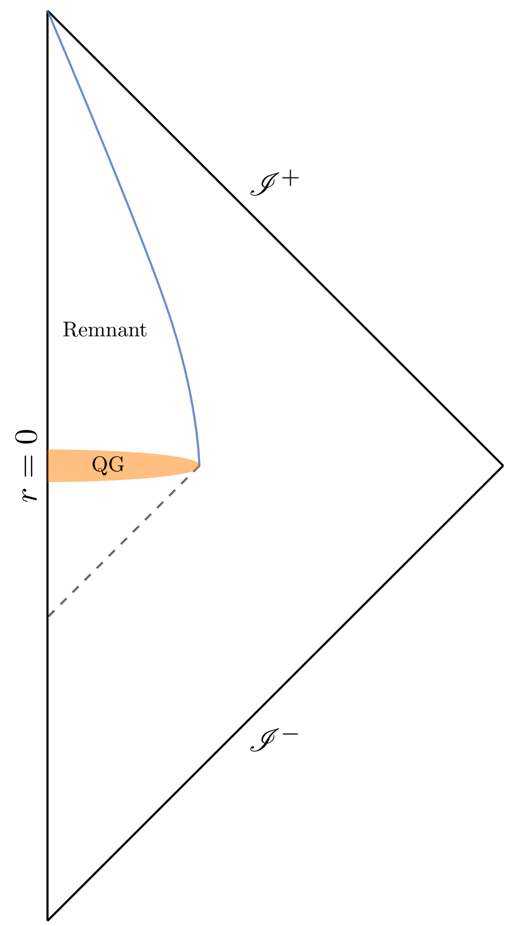

Figure 3.3 – Penrose diagram of a remnant scenario. After the black holes evaporates down to $\sim M_{\rm Pl}$ evaporation halts leaving behind a (stable) Planckian remnant. This is one of many possible realizations of the remnant scenario, for a lengthy discussion see [84].

of degrees of freedom to purify an arbitrarily large amount of radiation emitted during the evaporation phase.

An arbitrarily large number of degrees of freedom have to be accommodated in only a Planck mass of energy $E \sim M_{\rm Pl}$. In other words, these degrees of freedom need to carry arbitrarily small energy.

It has been argued then that, if remnants can be treated as point particles in an effective field theory with cutoff scale $\lambda_c >> \ell_{\rm Pl}$, the vacuum should be unstable and produce a virtually infinite number of Planckian black hole remnants. There are, however, several caveats in the conclusion that may suggest that this argument is not entirely correct[84].

In particular, it is not evident that remnants can be treated as point particles. Old black holes have a large interior volume[85], although they appear as point particles to outside observers.

A significant point we have not addressed yet is the behavior of fields close to the singularity inside the black hole. The presence of such singularity poses an obvious problem if we want to retain predictability: matter carrying information hits the singularity in a finite time and can no longer be accounted for. At the classical level, this behavior is usually thought to signal the theory's breakdown at these scales.

The consensus among theorists is that a quantum gravity theory will cure the singularity's pathological behavior, replacing it with an appropriate quantum gravitational notion[62], in which transitions across a Planckian curvature region is well defined. For example, see Figure 3.4.

As we will see in the following chapter, our view is that the singularity formation in General Relativity is a symptom of the breakdown of an effective low-energy description. This feature is not exclusive of General Relativity, the Navier-Stokes description being the classic example of this behavior. The hydrodynamic equations have solutions that develop shock waves, singularities in a variety of regimes. This signals that the theory is no longer a good description of the fluid's physics in those regimes. Nevertheless, the more fundamental kinetic theory is thought to still be valid in those situations.

Analogously, we argue that the breakdown of General Relativity at the singularity is just an omen of a more fundamental discrete theory, to which General Relativity is just a low-energy approximation.

Naively unitarity can be readily restored in a spacetime like 3.4. However, this approach suffers the same issue we discussed in the remnants framework. When quantum gravitational effects become relevant, an arbitrarily large amount of radiation has been radiated towards future null infinity. It is then not clear how an arbitrarily large amount of information can be encoded in extremely low energetic degrees of freedom.

From the perspective of non-perturbative quantum gravity, there is a natural resolution for this problem: a smooth spacetime is just a coarse-grained notion emerging from a multitude of microscopic degrees of freedom. In this viewpoint, flat-spacetime is a low-energy approximation that corresponds to myriads of pre-geometric Planckian states. While several quantum gravity approaches share this image, a concrete realization of this idea has been used to compute the entropy of a black hole. The

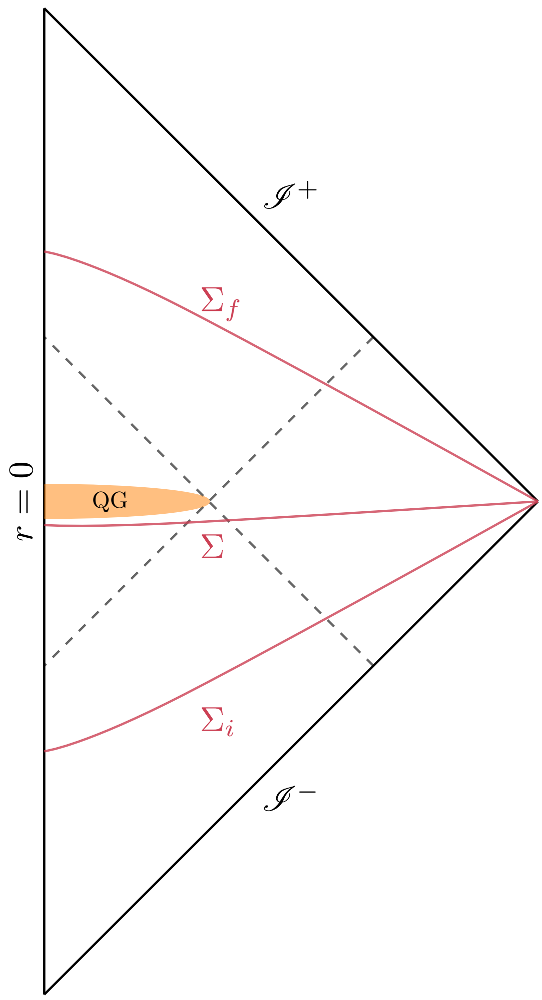

Figure 3.4 – Quantum gravity spacetime.

basic idea is to count the number of states in the fundamental theory (LQG) compatible with a given macroscopic black hole of mass M. This suggests that every smooth spacetime is highly degenerate in the fundamental theory.

This idea is crucial in a new paradigm to understand the black hole information paradox. Starting in [15], and later made concrete in [27, 26] it is proposed that correlations can be transferred to these Planckian degrees of freedom (which are degenerate and carry no energy) via interaction with matter degrees of freedom in high curvature regions. This is the heart of our proposal that we discuss in what follows.

4 Hawking's information puzzle: a solution from discreteness

This chapter overlaps with [26, 27].

Let us briefly describe our scenario introduced in[26, 27] with the help of Figure 4.1. The first assumption in the diagram is that there is evolution across the *would-be-singularity* (predicted by the classical dynamics) inside the black hole. This assumption is intrinsic in the representation in the figure; however, the scenario still makes sense if instead a baby universe is formed, i.e., if the *would-be-singularity* remains causally disconnected from the outside after evaporation. In that case the correlations established with the baby universe remain hidden forever to outside observers. The virtue of the present scenario in such case would be to give an identity to the degrees of freedom involved. The idea that the spacetime representation of the situation resembles the one in Figure 4.1 comes from the various results in symmetry reduced models for both cosmology [86, 87] as well as for black holes [88, 65, 89, 90, 91, 92, 63] and was first pictured in [62]. In such a context a 'scattering theory' representation (where an in-state evolves into an out-state) is possible even though the result (as we will argue) cannot be translated into the language of effective quantum field theory.

But what do we mean by a black hole in this evaporating context? In the asymptotically flat idealization, the black hole region is defined in classical general relativity as the portion of the spacetime $\mathscr{M}$ that is not part of the past of $\mathscr{I}^+$. Such definition needs to be modified in quantum gravity. In order to do that we introduce the notion of the semiclassical past $J^-_{\rm C}(\mathscr{I}^+)$ of $\mathscr{I}^+$ as the collection of events in the spacetime that can be connected to $\mathscr{I}^+$ by causal curves along which the Kretschmann scalar $K \equiv R_{abcd}R^{abcd} \le {\rm C}\ell_{\rm Pl}^{-4}$ for some constant C of order unity. The black hole region can be defined then as

$$B \equiv \mathscr{M} - J^-_{\rm C}(\mathscr{I}^+). \tag{4.1}$$

Its dependence on the constant C is not an important limitation in the discussion about information. Different C would lead to BH regions that coincide up to Planckian corrections.

The most clear physical picture emerges from the analysis of the Penrose diagram on the left panel of Figure 4.1 from the point of view of observers at future null infinity $\mathscr{I}^+$.

These observers are assumed to be at the center of mass Bondi frame of the BH formed via gravitational collapse. We also assume that the Bondi mass of the BH is initially $M \gg p$ at some retarded time u on $\mathscr{I}^+$ representing the time where the BH has achieved its quasi-equilibrium state and starts evaporating slowly via Hawking radiation, i.e., the BH is initially macroscopic.

Under such conditions the evaporation is very slow and we can trust the semiclassical description that tell us that the Bondi mass $M(u)$ will slowly decrease with u from this initial value M until times $u = u_0$ (see figure) with $M(u_0) \gtrapprox M_{\rm Pl}$ in a time of the order of M^3 in Planck units.

From this time on the details depend on a full quantum gravity calculation because the curvature around the BH horizon has become Planckian. Nevertheless, independently of such details we can safely say that the spacetime and the matter degrees of freedom encoded on $\mathscr{I}^+$ for $u > u_0$ must be in a superposition of states all of which are very close to flat space-time, as far as their geometry is concerned, with matter excitations very close to the vacuum because there is only at best an energy of the order of $E_{late} \approx M_{\rm Pl}$ to substantiate both. In addition these excitations must be correlated with the early Hawking radiation with energy $E_{early} \approx M - M_{\rm Pl}$ if unitarity is to hold.

The late degrees of freedom are often referred to as *purifying degrees of freedom*.

The possibility we put forward is that if smooth spacetime is an emergent notion from an underlying discrete physics, then the classical geometries of general relativity with quantum fields living on them would only restrict the fundamental Hilbert space to a subset containing very large (possibly infinite) number of states.

For instance the Minkowski vacuum unicity in standard quantum field theory would fail in the sense that the requirement that states look *flat* for (coarse grained) low energy observers—which are those for which an effective quantum field theory description in terms of smooth fields living on a smooth geometry is a suitable approximation—would still admit highly denegerate ensemble (all with total mass indistinguishable from zero by these observers).

Now, such low energy modes cannot be interpreted as the infrared excitations of fields mentioned in the previous paragraph (say low energy photons) because they are smooth low energy configurations.

These low energy degrees of freedom would correspond to defects in the Planckian fabric of quantum gravity which we simply are not sensitive to with our coarse low energy probes (like the molecular structure that escapes the smooth characterization of the Navier-Stokes effective theory of fluids).

Why should information be hidden in the UV and not be IR modes as in the remnant scenario mentioned last section? It is often believed that because the volume inside the black hole actually becomes very large (according to suitable definitions [85]) then modes that are correlated with the Hawking radiation are redshifted and become highly IR inside.

Although this is true for spherically symmetric Hawking quanta in the spherically symmetric Schwarzshild background—where such modes are indeed infinitely redshifted as detected by regular observers when they approach the singularity at $r = 0$—this conclusion fails when one considers no-spherical modes no matter how small the deviation from spherical symmetry is [1]. Therefore, generically all modes become UV close to the singularity.

We can draw a formal analogy with the Unruh effect as follows. The Unruh effect arises from the structure of the vacuum state $|0\rangle$ of a quantum field on Minkowski spacetime when written in terms of the modes corresponding to Rindler accelerated observers with their intrinsic positive frequency notion.

[1] In the Schwarzshild background, the frequency measured by a radially freely falling observer normal to the r =constant hypersurfaces goes like

$$\omega^2(r) = \frac{\ell^2}{r^2} + \frac{r}{2M}\mathscr{E}^2 + \mathscr{O}\left(\frac{r^2}{M^2}\right), \tag{4.2}$$

where $\mathscr{E} = -k\cdot\xi$ and $\ell = k\cdot\psi$ are the conserved quantities associated to the massless particle with wave vector k^a and ξ^a and ψ^a are the stationarity and rotation killing fields of the background. The qualitative behaviour approaching $r = 0$ would be the same for any other observer measuring ω (the divergence of ω is observer independent). Only exactly spherically symmetric modes with $\ell = 0$ would become IR at the singularity. However, this conclusion is no longer true if the BH rotates or if we consider that at the fundamental level states with exact spherical symmetry inside the BH are of measure zero. Notice that such UV divergence in the non-spherically symmetric Hawking partners implies large deviations from spherical symmetry near the singularity (if their back reaction would be taken into account). This should be kept in mind when modelling the situation with spherically symmetric mini-superspace quantum gravity models.

The vacuum takes the form

$$|0\rangle = \prod_k \left(\sum_n \exp\left(-n\frac{\pi\omega_k}{a}\right) |n,k\rangle_R \otimes |n,k\rangle_L \right), \tag{4.3}$$

where $|n,k\rangle_L$ and $|n,k\rangle_R$ define the particle modes—as viewed by an accelerated observer with uniform acceleration a—on the left and the right of the Rindler wedge [66]. Here we see from the form of the previous expansion that even when we are dealing with a pure state (if we define the density matrix $|0\rangle\langle 0|$), the reduced density obtained by tracing over one of the two wedges would produce a thermal state with $T = a/(2\pi)$. The statement in the perspective we propose on the purification of information in BH evaporation can we schematically represented (the following is certainly not a precise equation) by

$$\mathbf{U} \underbrace{|\text{flat}, 0\rangle}_{\text{quantum geometry}} \otimes \overbrace{|\phi\rangle}^{\text{matter fields}} = \prod_k \left(\sum_n \exp\left(-\frac{\beta}{2} n\omega_k\right) |\text{flat}, n\rangle \otimes |n,k\rangle \right), \tag{4.4}$$

where an initial state of flat quantum geometry $|\text{flat}, 0\rangle$ tensor a state representing initially diluted matter fields $|\phi\rangle$ evolves unitarily via $\mathbf{U}$ into the formation of a BH and the subsequent evaporation (Figure 4.1) which after complete evaporation is written as a superposition of flat quantum geometry states $|\text{flat}, n\rangle$—which are all indistinguishable from $|\text{flat}, 0\rangle$ to low energy agents and differ among them by quantum numbers n corresponding to quantities that are only measurable if one probes the state down to its Planckian structure—tensor product with normal n-particle excitations of matter fields representing Hawking radiation.

As mentioned above the previous equation is only schematic. Is main inappropriateness is the fact that the reduce density matrix obtained by tracing over the quantum geometry hidden degrees of freedom would give a thermal state at a fixed temperature T. This is at odds with the expectation that the Hawking radiation should contain a superposition of the thermal radiation emitted at different Hawking temperatures during the long history of the evaporation of the BH. But the point that this equation and the discussion of the previous paragraph should make clear is that the purification mechanism proposed here has nothing to do with the point like remnant scenario with all its problems associated to a long lasting particle-like remnant.

Here, to the future of the *would-be-singularity* in Figure 4.1, we simply have a quantum superposition of different quantum geometry states that all look flat to low energy observers. There are no localized remnant hiding the huge degeneracy inside; there is only a large superposition of states that are inequivalent in the fundamental quantum gravity Hilbert space but seem all the same for low energy agents.

Such degrees of freedom cannot be captured by any effective description in terms of smooth fields (EQFT) for the simple reason that they are discrete in their fundamental nature.

Notice that the degrees of freedom where information would be coded after BH evaporation do not satisfy the usual Einstein-Planck relationship $E = \hbar\omega$ or equivalently $p = h/\lambda$ (for some 'wavelength' λ or 'frequency' ω) and this might deceive intuition [2] . They are described as Planckian defects nevertheless they do not carry Plankian energy. The point is that such relationship only applies under suitable conditions which happen to be meet in many cases but need not be always valid.

One case is the one of degrees of freedom that can be thought of as waves moving on a preexistent spacetime. This is the case of particle excitations in the Fock space of quantum field theory or effective quantum field theories which are defined in terms of a preexistent spacetime geometry.

There is no clear meaning to the above intuitions in the full quantum gravity realm where the present discussion is framed. Even when such relations (linked to the usual uncertainty principle of quantum mechanics) should hold in a suitable sense—if the structure suggested by canonical quantization survives in quantum gravity as it should to a certain degree—they would apply to phase space variables encoding the true degrees of freedom of gravity that we expect (from the general covariance of general relativity) to be completely independent of a preexistent background geometry.

We will see that such degrees of freedom with such peculiar nature actually arise naturally in the toy model of quantum gravity that we analyze in this article.

[2]A nice counter example of this intuition is given by the case of a non relativistic charged particle in a two dimensional infinite perfect conductor in a uniform magnetic field normal to the conducting plane. The energy eigenvalues are given by the Landau levels $E_n = \hbar\omega_{\rm B}(n+1/2)$ where $\omega_{\rm B} = qB/(mc)$ is the Bohr magneton frequency, but they are infinitely degenerate. There are canonically conjugated variables (P, Q) associated to the particle that are cyclic, i.e., do not appear in the Hamiltonian. In this case one can produce wave packets that are as 'locallized' as wanted in the variable Q without changing the energy of the system.

Interestingly, this is a perfect example of system where one could have an apparent loss of information of the type we are proposing here (for a more realistic analog gravity model discussing the information paradox along the lines of the present scenario see [93]). If one scatters a second particle interacting softly with the charged particle on the plate so that the interaction does not produce a jump between different Landau levels, then correlations with the cyclic variables would be established without changing the energy of the system. This is the perfect model to illustrate the possibility of decoherence without dissipation.

It is presently hard to prove that such scenario is viable in a quantum theory of gravity simply because there is no such theoretical framework developed enough for tackling BH formation and evaporation in detail. However, the application of loop quantum gravity to quantum cosmology leads to a model with similar features, where evolution across the classical singularity is well defined [86].

The interior of a spherically symmetry black hole can be described by a homogeneous, non-isotropic cosmological model given by the Kantowski-Sachs geometry. It is foliated by spatially homogeneous hypersurfaces with topology $\mathbb{R} \times \mathbb{S}^2$ (For example, the part of the hypersurface Σ inside the horizon in 3.4).

The literature dealing with the loop quantization of such models is extensive[65, 63, 94, 92, 95]. The resulting quantum theory has the same structure as the homogeneous and isotropic cosmological models. This characteristic is crucial for us and justifies the use of homogeneous cosmological models: the Planckian degrees of freedom central in our argument are also present in the polymer models of black hole interior.

Incidentally, this highlights another feature of our proposal: one of the main assumptions - the presence of extra degrees of freedom at the Planck scale - is a natural assumption, ubiquitous in various approaches to quantum gravity. In particular, every polymer model inspired by canonical quantization techniques shares this feature.

The results in the rest of the chapter can be described briefly by making reference to the Figure 4.2 which should be compared with Figure 4.1. We will show that the evolution in loop quantum cosmology from a universe in an initially contracting state in the past of the *would-be-singularity* to an expanding universe in its future is perfectly unitary in the fundamental description.

Nevertheless, states in the Hilbert space of loop quantum cosmology contain quantum degrees of freedom which are hidden to low energy coarse grained observers. If these degrees of freedom are traced out of the initial density matrix then we will see that pure states (in the sense of the reduced density matrix) generically evolve into mixed states across the *would-be-singularity*. Information is lost into correlations with degrees of freedom that are Planckian and thus inaccessible to macroscopic observers.

These correlations are established in an inevitable way during the strong curvature phase of evolution across the big bang (just as expected in the BH scenario described above). As energy is conserved (energy is a delicate notion in cosmology but happens to be well defined in our model as we will see) the defects that purify the final state do not enter in the energy balance which realizes another crucial necessary ingredient of the general scenario (decoherence happens with negligible dissipation [83]).

Let us finish this introduction with a very brief description of the organization of the rest of the sections. The discussion is basically separated in two parts. In the first part we show that the scenario we have described is realized in unimodular quantum cosmology following the standard quantization prescription of loop quantum cosmology.

The model of this section corresponds exactly with the type of models in the standard literature [96]. In the second part of the paper we observe that there is natural extension of loop quantum cosmology based on the regularization ambiguity associated with the quantization of the Hamiltonian. This extension opens a new chanel for information to flow.

Although this second option is not necessary to illustrate our point (already realized in the standard theory in the first part) it gives a different identity to the defects which could lead to independent and thus useful insights.

4.1 Unimodular Cosmology

Unimodular Gravity is nearly as old as General Relativity itself, it was introduced by Einstein in 1919 [97] as an attempt to describe nuclear structure geometrically. In this work Einstein identifies also an appealing feature of the theory which is the fact that the cosmological constant arises as a dynamical constant of motion that needs to be added to the initial values of the theory. In unimodular gravity the cosmological constant is a constant of integration and not a universal or fundamental constant of nature. Interest in the theory was regained in the late 80's with the observation of Weinberg [98] that, for the above reason, semiclassical unimodular gravity provides a trivial resolution of the cosmological constant problem as vacuum energy simply does not gravitate. Unimodular gravity is the natural low energy description that emerges from the formal thermodynamical ideas of Jacobson [54] and represents the expected low energy regime of the causal set approach [53].

Another property of unimodular gravity (specially important for us here) is that it completely resolves the problem of time [99] in the cosmological FLRW context. More precisely, the theory comes with a preferred time evolution and a preferred Hamiltonian (the energy of the universe is well defined and directly linked with the value of the cosmological constant). The quantum theory is described by a Schroedinger like equation where states of the universe are evolved by a unitary evolution operator. Therefore, unlike the general situation in quantum gravity, the notion of unitarity is unambiguously defined in unimodular quantum cosmology. This is the main reason why unimodular gravity provides the perfect framework for the discussion of the central point of this work.

Here we specialize to homogeneous and isotropic cosmologies that are spatially flat (k=0), i.e., the spatial manifold Σ is topologically $\mathbb{R}^3$. What follows is the standard construction. For a detailed account of the Hamiltonian analysis in the cosmological framework see [100] . The FLRW metric is

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \underbrace{(dx^2 + dy^2 + dz^2)}_{\mathring{q}_{ab}}, \tag{4.5}$$

where $\mathring{q}_{ab}$ denotes the fiducial spacial metric. Since Σ is non-compact some integrals are infrared divergent and are regularized by restricting to a fixed fiducial cell $\mathcal{V}$ of fiducial volume V_0 with respect to the fiducial spacial metric

$$\mathring{q}_{ab} = \mathring{e}^i_a \mathring{e}^j_b\, \delta_{ij}, \tag{4.6}$$

where $\mathring{e}^i_a$ denotes a fiducial triad and the physical metric is given by $q_{ab} = a^2(t)\mathring{q}_{ab}$. The action of unimodular gravity in the FLRW minisuperspace setup is given by

$$S[a, \dot{a}, \lambda] \quad = \quad \frac{3}{8\pi G}\int_{\mathbb{R}} \left(\frac{V_0 a \dot{a}^2}{N} + \lambda V_0 (N a^3 - 1)\right) dt, \tag{4.7}$$

where λ is a Lagrange multiplier imposing the unimodular constraint $N = a^{-3}$ (i.e. $\sqrt{|g|} = 1$), and the first term is the Einstein-Hilbert action restricted to the FLRW geometries [3]. In order to use loop quantum cosmology techniques (for a discussion of the quantization in the full loop quantum gravity context see [101, 102]) one introduces the new canonical variables c and p via the basic Ashtekar-Barbero connection variables A^i_a and E^a_i, namely

$$E^a_i = p\left(\mathring{e}^a_i V_0^{-2/3}\right), \qquad A^i_a = c\left(\mathring{\omega}^i_a V_0^{-1/3}\right), \tag{4.8}$$

where $\mathring{\omega}^i_a$ is a fiducial reference connection. These variables are related to those in

[3]There is an overall minus sign the definition action with respect to standard treatments. This is done so that the the pure-geometry Hamiltonian is positive definite.

(4.7) via the equations

$$|p| = V_0^{2/3} a^2, \qquad c = V_0^{1/3} \frac{\gamma \dot{a}}{N}. \tag{4.9}$$

The action becomes

$$S[c,p,\lambda] = \frac{3}{8\pi G} \int_{\mathbb{R}} \gamma^{-1} c\dot{p} - N\gamma^{-2}\sqrt{|p|}c^2 + \lambda(N|p|^{\frac{3}{2}} - V_0)\,dt, \tag{4.10}$$

and c and p are canonically conjugated in the sense that

$$\{c,p\} = \frac{8\pi G\gamma}{3}. \tag{4.11}$$

The unimodular condition $N = a^{-3}$ fixes the lapse to $N = V_0/|p|^{3/2}$ and the unimodular Hamiltonian becomes

$$H = \frac{3V_0}{8\pi G} \frac{c^2}{\gamma^2 |p|}. \tag{4.12}$$

The proportionality of the Hamiltonian with V_0, and the fact that the four volume bounded by V_0 at two different times is given by $v^{(4)} = V_0 \Delta t$, implies that time evolution can be parametrized in terms the four volume elapsed from some reference initial slice. The associated Hamiltonian (conjugated to $v^{(4)}/(8\pi G)$) is

$$\Lambda = \frac{3c^2}{\gamma^2 |p|}, \tag{4.13}$$

and corresponds to the cosmological constant.

4.2 Quantization

The loop quantum cosmology quantization uses a non standard representation of the canonical variables where the variable c does not exist as a quantum operator, and the definition of the Hamiltonian requires a special regularization procedure known as the $\bar{\mu}$-scheme [96]. The quantization prescription is greatly simplified by the introduction of new canonically conjugated dynamical variables b and v defined as [103]

$$b \equiv \frac{c}{|p|^{\frac{1}{2}}} \qquad v \equiv \mathrm{sign}(p) \frac{|p|^{\frac{3}{2}}}{2\gamma\pi\ell_{\mathrm{Pl}}^2}, \tag{4.14}$$

with Poisson brackets [4]

$$\{b, v\} = 2\hbar^{-1}. \tag{4.15}$$

The variable v corresponds to the physical volume of the fiducial cell divided by $\ell_{\rm Pl}^2$; it has units of distance. The variable b is simply its conjugate momentum. In terms of these variables the gravitational (unimodular) Hamiltonian (4.12) integrated in a fiducial cell $\mathscr{V}$ becomes

$$H = \frac{3V_0}{8\pi G\gamma^2} b^2 \tag{4.16}$$

Note the extreme simplicity of the previous expression: the unimodular hamiltonian is just the analog of that of a free particle in one dimension with mass parameter $m = 4\pi\gamma^2/(3V_0)$ and momentum b. In the absence of matter, the Hamiltonian can be quantized in the Wheeler-DeWitt representation where the evolution in unimodular time is unitary and there is no singularity (the classical solutions correspond to De-Sitter geometries with arbitrary but positive cosmological constants). The singularity in the classical theory becomes real when matter is introduced.

In the loop quantum cosmology polymer representation, just as for c, there is no operator corresponding to b but only the operators corresponding to finite v translations [87]; from here on referred to as shift operators

$$\exp(i2kb) \triangleright \Psi(v) = \Psi(v - 4k). \tag{4.17}$$

For $k = q\sqrt{\Delta}\ell_{\rm Pl}$ and $q \in \mathbb{N}$, states that diagonalize the previous shift operators, denoted $|b_0;\Gamma^{\epsilon}_{\Delta}\rangle$, are labelled by a real value b_0 and by a graph $\Gamma^{\epsilon}_{\Delta}$. The graph is a 1d lattice of points in the real line of the form $v = 4n\sqrt{\Delta}\ell_{\rm Pl} + \epsilon$ with $\epsilon \in [0, 4\sqrt{\Delta}\ell_{\rm Pl})$ and $n \in \mathbb{N}$. The corresponding wave function is given by $\Psi_{b_0}(v) \equiv \langle v|b_0;\Gamma^{\epsilon}_{\Delta}\rangle = \exp(-i\frac{b_0 v}{2})\delta_{\Gamma^{\epsilon}_{\Delta}}$ where the symbol $\delta_{\Gamma^{\epsilon}_{\Delta}}$ means that the wavefunction vanishes when $v \notin \Gamma^{\epsilon}_{\Delta}$. It follows from (4.17) that

$$\exp(i2kb) \triangleright |b_0;\Gamma^{\epsilon}_{\Delta}\rangle = \exp(i2kb_0)\,|b_0;\Gamma^{\epsilon}_{\Delta}\rangle. \tag{4.18}$$

The states $|b;\Gamma^{\epsilon}_{\Delta}\rangle$ are eigenstates of the shift operators that preserve the lattice $\Gamma^{\epsilon}_{\Delta}$. Notice, that the eigenvalues are independent of the parameter ϵ. i.e. they are infinitely degenerate and span a non separable subspace of the quantum cosmology Hilbert space $\mathscr{H}_{lqg}$.

[4] The factor $\hbar^{-1}$ appears on the right hand side of the Poisson brackets due to the introduction of $\hbar$ (via $\ell_{\rm Pl}^2$) in the definition of the new variable v. This is done to match standard definitions [96].

A scale $\bar{\mu}$ is needed in order to define a regularization of (4.16) representing the Hamiltonian in $\mathscr{H}_{lqc}$. The reason is that there is no operators associated to b but only approximants constructed via the shift operators (4.17). The so-called $\bar{\mu}$-scheme [96] introduces a dynamical length scale $\bar{\mu}$ defined as

$$\overline{\mu}^2 = \frac{\ell_p^2 \Delta}{|p|}, \tag{4.19}$$

where Δ represents the so-called 'area-gap' which plays the role of a UV regulator. It is normally associated to the smallest non-vanishing area quantum in the full theory of loop quantum gravity. For the moment (as in the standard treatement) this is just a fixed parameter[5]. When translated into the variables (4.14) $\overline{\mu}$ corresponds to considering approximants to b constructed out of shift operators (4.17) with fixed $k \equiv \sqrt{\Delta}\ell_{\rm Pl}$. In terms of these one obtains the following regularization of the Hamiltonian (4.16) which is a well defined self-adjoint operator[6] acting on $\mathscr{H}_{lqg}$

$$H_\Delta \equiv \frac{3V_0}{8\pi G\gamma^2}\frac{1}{\Delta \ell_{\rm Pl}^2}\sin^2\left(\Delta^{\frac{1}{2}}\ell_{\rm Pl}\, b\right), \tag{4.21}$$

which coincides with (4.16) to leading (zero) order in $\ell_{\rm Pl}^2$. From (4.13) we obtain an operator associated to the (here dynamical) cosmological constant, namely

$$\Lambda_\Delta \equiv \frac{3}{\gamma^2}\frac{\sin^2\left(\Delta^{\frac{1}{2}}\ell_{\rm Pl}\, b\right)}{\Delta \ell_{\rm Pl}^2}. \tag{4.22}$$

In the pure gravity case, the cosmological constant is positive definite and bounded from above by the maximum value $\lambda_{\max} = 1/(\gamma^2 \ell_{\rm Pl}^2 \Delta)$. Negative cosmological constant solutions are possible when matter is added. The states (4.18) with $k = k_\Delta \equiv \sqrt{\Delta}\ell_{\rm Pl}$

[5] In Section 4.7, we will turn this quantity into a quantum operator acting on the microscopic sector of the Hilbert space that will be introduced.

[6] The Hamiltonian $\hat{H}_0$ (4.21) is symmetric, that is $\langle \Psi_1, \hat{H}_0 \Psi_2\rangle = \langle \hat{H}_0\Psi_1, \Psi_2\rangle$, with respect to the inner product $\langle \Psi_1, \Psi_2\rangle = \sum_v \overline{\Psi_1(v)}\Psi_2(v)$. The action of the Hamiltonian on $\Psi(v)$ is given by:

$$\hat{H}_0\Psi(v) = -3(2\gamma^2\Delta_s\ell_{\rm Pl}^2)^{-1}\left(\Psi(v+2\lambda) - 2\Psi(v) + \Psi(v-2\lambda)\right), \tag{4.20}$$

with $\lambda = 2\sqrt{\Delta_s}\ell_{\rm Pl}$. The key property is $\langle \Psi_1(v), \Psi_2(v+2\lambda)\rangle = \langle \Psi_1(v-2\lambda), \Psi_2(v)\rangle$ where v is in the support of both $\Psi_1(v)$ and $\Psi_2(v)$. This is the statement of the unitarity of the shift operators $\langle e^{-i2\lambda b}\Psi_1, \Psi_2\rangle = \langle \Psi_1, e^{i2\lambda b}\Psi_2\rangle$. The symmetric nature of the shift operators appearing in H_0 implies the result.

diagonalize the Hamiltonian, i.e.

$$H_\Delta \triangleright |b_0;\Gamma^\epsilon_\Delta\rangle = E_\Delta(b_0)\,|b_0;\Gamma^\epsilon_\Delta\rangle\,, \tag{4.23}$$

with energy eigenvalues

$$E_\Delta(b_0) = \frac{3V_0}{8\pi G\gamma^2}\frac{1}{\Delta\ell_{\rm Pl}^2}\sin^2\left(\Delta^{\frac{1}{2}}\ell_{\rm Pl}\,b_0\right). \tag{4.24}$$

States $|b_0;\Gamma^\epsilon_\Delta\rangle$ are also eigenstates of the cosmological constant with eigenvalue $\lambda_\Delta(b_0) = (8\pi G)E_\Delta(b_0)/V_0$. Notice that the energy eigenvalues do not depend on $\epsilon \in [0, 4\sqrt{\Delta}\ell_{\rm Pl})$. Thus, the energy levels are infinitely degenerate with energy eigenspaces that are non-separable. This is not something peculiar of our model but a general property of the non-standard representation of the canonical commutation relations used in loop quantum cosmology.

4.3 On the interpretation of the ϵ-sectors.

It is customary in the loop quantum cosmology literature to restrict to a fixed value of ϵ in concrete cosmological models, as the dynamical evolution does not mix different ϵ sectors. The terminology *'superselected sectors'* is used in a loose way in discussions. However, these sectors are not superselected in the strict sense of the term because they are not preserved by the action of all the possible observables in the model, i.e. there are non trivial Dirac observables mapping states from one sector to another. The explicit construction of such observables might be very involved in general (as it is the usual case with Dirac observables); nevertheless, it is possible to exhibit them directly at least in one simple situation: the pure gravity case. In that case the shift operator (4.17) with shift parameter δ commutes with the pure gravity Hamiltonian (the Hamiltonian constraint if we were in standard loop quantum cosmology) and maps the ϵ sector to the $\epsilon - 4\delta$ sector. The analogous Dirac observables in a generic matter model can be formally described with techniques of the type used for the definition of evolving constants of motion [104]. No matter how complicated this might be in practise, the point is well illustrated by our explicit example in the matter free case [7].

Thus, different ϵ sectors are not superselected and therefore the infinite degeneracy of the energy eigenvalues of the Hamiltonian (which again we exhibit explicitly in

[7]This point was independently communicated to us in the context of Dirac observables for isotropic LQC with a free matter scalar field [105].

the previous discussion only in the vacuum case) must be taken at face value. How can we understand this large degeneracy from the fact that there would be only a two fold one (associated with a contracting or expanding universe) if we had quantized the model using the standard Schroedinger representation or, in other words, the standard Wheeler-DeWitt quantization? The answer is to be found, we claim, in the notion of coarse graining: low energy observers only distinguish a two fold degeneracy for energy (or cosmological constant) eigenvalues: one the universe has a given cosmological constant, and two it is expanding or contracting. These are by the way the quantum numbers in the Wheeler-DeWitt quantization which plays the role in our context of the low energy effective quantum field theory formulation. Such coarse observers are declared to be insensitive to the huge additional degeneracy of energy eigenstates encoded in the quantum number ϵ. All these infinitely many states in the quantum cosmology representation must be considered as equivalent up to the two-fold degeneracy mentioned above.

In what follows, and for concreteness, we will consider combinations of states with two different values for ϵ only, i.e. on two different lattices. The idea of the previous paragraph will naturally produce a notion of coarse graining entropy associated to the intrinsic statistical uncertainty due to the inability for a low energy agent to distinguish these microscopically orthogonal states. Arbitrary superposition with N different ϵ-sectors would lead to similar results (the entropy capacity growing with the usual $\log(N)$). The $N=2$ case treated here makes some explicit calculations straightforward.

4.4 Matter couplings and a model capturing its essential features.

Here we discuss two simple matter models in order isolate the generic features of the influence of matter. At the end of the section we will define a simple and trivially solvable model capturing these features.

Perhaps the simplest matter model that would serve our purposes is the minimal and isotropic coupling to a Dirac Fermion defined in [106]. After symmetry reduction the action for matter is

$$S_{\mathrm{F}}(\eta,\bar{\eta}) = V_0 \int_{\mathbb{R}} d\tau \Big[\frac{i}{2} a(\tau)^3 \left(\bar{\eta}\gamma^0 \dot{\eta} - \dot{\bar{\eta}}\gamma^0 \eta \right) - mN(\tau)a(\tau)^3 \bar{\eta}\eta \Big], \tag{4.25}$$

from which we read the Fermionic contribution to the Hamiltonian

$$
\begin{aligned}
H_{\mathrm{F}} &= mN(\tau)\bar{\eta}\eta = -mN(\tau)p_\eta\gamma_0\eta \\
&= \frac{m}{a^3}p_\eta\gamma_0\eta,
\end{aligned}
\tag{4.26}
$$

where (p_η, η) are the Fermionic canonical variables $p_\eta \equiv (V_0 a)^{3/2}\psi^\dagger$ and $\eta \equiv (V_0 a)^{3/2}\psi$ [36], and in the second line we use the unimodular condition $N = a^{-3}$.

In the quantum theory the non trivial anticommutator is $\{\eta, p_\eta\} = \mathbf{1}$ with the rest equal to zero.

This is achieved by writing $\eta = \sum_s \left(a_s u^s e^{-imt} + b_s^\dagger v^s e^{imt}\right)$ with non trivial anti-commutation relations for the creation and annihilation opetrators $\{a_r, a_s^\dagger\} = \delta_{rs} = \{b_r, b_s^\dagger\}$, and $u^s e^{-imt}$ and $v_s e^{imt}$ a complete basis of solutions of the Dirac equation for positive and negative frequency respectively [107]. In our model we can have either the vacuum state, one or two Fermions which saturates the Pauli exclusion principle.

If we assume normal ordering the contribution to the unimodular energy is

$$
H_{\mathrm{F}} = \frac{mn}{a^3}. \tag{4.27}
$$

where $n = 0, 1, 2$ is the occupation number for the Fermion. If instead of the condition $N = a^{-3}$ we had used $N = 1$ (where time is comoving time) then the energy contribution would have been just m for which we have a clear physical intuition: a single fermion homogeneously distributed in the universe contributes to the Hamiltonian with its total mass. The factor $1/a^3$ in the previous expression comes from the unimodular condition.

In the case of Wheeler-de-Witt quantization the contribution of the fermion becomes singular at the big-bang $a = 0$. In loop quantum cosmology such a quantity remains bounded above due to loop quantum gravity discreteness. Indeed, using the inverse volume quantization given in reference [96] one has

$$
\hat{H}_{\mathrm{F}} \triangleright |\psi\rangle = -m\sum_\nu |\nu\rangle\, h_{\mathrm{F}}(\nu; \sqrt{\Delta}\ell_{\mathrm{Pl}})\Psi(\nu, \eta), \tag{4.28}
$$

where

$$
h_{\mathrm{F}}(\nu; \lambda) \equiv \frac{1}{4\lambda^2}\left(|\nu + 2\lambda|^{\frac{1}{2}} - |\nu - 2\lambda|^{\frac{1}{2}}\right)^2. \tag{4.29}
$$

We notice that $h_{\rm F}(\nu;\sqrt{\Delta}\ell_{\rm Pl}) < 1$ and decays like $1/\nu$ for $\nu \to \infty$[8]. One could in principle add this term to the free Hamiltonian and solve the unimodular time independent Schroedinger equation

$$(\hat{H}_0 + \hat{H}_{\rm F}) \triangleright |\psi\rangle = E|\psi\rangle\,. \tag{4.30}$$

Solutions can be interpreted in the sense of scattering theory starting with free wave packets for large ν picked around some value of the cosmological constant (4.22) or energy (4.24).

The case of a the coupling with a scalar field is formally very similar, specially in the simplified case where we assume it to be massless. Following [96] and using the unimodular condition $N = a^{-3}$ we get

$$H_\phi = -\frac{p_\phi^2}{8\pi^2\gamma^2\ell_{\rm Pl}^4\nu^2}. \tag{4.31}$$

This leads to

$$\hat{H}_\phi \triangleright |\psi\rangle = -m\sum_\nu |\nu\rangle\, h_\phi(\nu;\sqrt{\Delta}\ell_{\rm Pl})\Psi(\nu,\phi), \tag{4.32}$$

where

$$h_\phi(\nu;\lambda) \equiv \frac{p_\phi^2}{16\lambda^4}\left(|\nu+2\lambda|^{\frac{1}{2}} - |\nu-2\lambda|^{\frac{1}{2}}\right)^4. \tag{4.33}$$

The momentum p_ϕ commutes with the Hamiltonian and thus it is a constant of motion. If we consider an eigenstate of p_ϕ then the problem reduces again to a scattering problem with a potential decaying like $1/\nu^2$ when we consider solving the time independent Schroedinger equation

$$(\hat{H}_0 + \hat{H}_\phi) \triangleright |\psi\rangle = E|\psi\rangle\,. \tag{4.34}$$

Therefore, both the Fermion as well as the scalar field models (which are closer to a possibly realistic scenario) seem tractable with a slight generalization of the standard scattering theory to the discrete loop quantum cosmology setting. However, the main objective in this section is to illustrate an idea in terms of a concrete and simple toy model. With this idea in mind we will modify the structure suggested by the Fermion

[8] There is a great degree of ambiguity in writing the inverse volume operators. Perhaps the simplest is the one introduced in [108] that we will actually used in the concrete computations of the section 4.7. For more discussion on this see [109] and references therein.

coupling and the scalar field coupling and simply add an interaction term where the 'long distance interaction' term represented by the function $F(v;\lambda)$ is replaced by a short range analog $F(v;\lambda) \propto \delta_{v,0}$.

The qualitative properties of the scattering will be the same and the model becomes sufficiently trivial for straightforward analytic computations.

For that we consider an interaction that kicks in at $v = 0$:

$$\hat{H} = \hat{H}_0 + \mu \hat{H}_{\text{int}}, \tag{4.35}$$

where μ is a dimensionless coupling, $\hat{H}_0$ is the pure gravitational Hamiltonian, and $\hat{H}_{\text{int}}$ is

$$\hat{H}_{\text{int}} \triangleright |\psi\rangle \equiv \sum_v \left(\ell_p^{-4} \frac{V_0}{\sqrt{\Delta}}\right) |v\rangle \frac{\delta_{v,0}}{\sqrt{\Delta}} \Psi(0). \tag{4.36}$$

We have added by hand an interaction Hamiltonian that switches on only when the universe evolves through the *would-be-singularity* at the zero volume state. The key feature of the $\hat{H}_{\text{int}}$ is that—as its more realistic relatives matter Hamiltonians (4.28) and (4.32)—it breaks translational invariance and thus it leads to different dynamical evolution for different ϵ-sectors.

4.5 Solutions as a scattering problem

The scattering problem is very similar to the standard one in one-dimensional quantum mechanics; however, one needs to take into account the existence of the peculiar degeneracy of energy eigenvalues contained in the ϵ sectors; see Sections 4.2 and 4.3. We will consider, for simplicity, the superposition of only two states supported on two lattices respectively: the lattice $\Gamma^{\epsilon}_{\Delta}$ with $\epsilon = 0$ for the first one and the one with $\epsilon = 2\sqrt{\Delta}\ell_{\text{Pl}}$ for the second one. The degenerate eigenstates of the shift operators (4.18) with eigenvalues $\exp(i2kb)$ will be denoted

$$|b,1\rangle \equiv |b;\Gamma^0_\Delta\rangle, \quad \text{and} \quad |b,2\rangle \equiv |b;\Gamma^{2\sqrt{\Delta}\ell_{\text{Pl}}}_\Delta\rangle, \tag{4.37}$$

respectively, while we will denote by Γ^1 and Γ^2 the corresponding underlying lattices. The immediate observation is that states supported on Γ^2 (superpositions of $|b,2\rangle$) will propagate freely because they are supported on a lattice that does not contain the point $\nu = 0$ where the interaction is non trivial. On the other hand, states supported on Γ^1 (superpositions of $|b, 1\rangle$) will be affected by the interaction at the big bang. Before and after the big bang the universe's evolution of the second type of states is well described by the eigenstates of the *free* Hamiltonian described in Section 4.2. Such asymmetry of the interaction on different ϵ-sectors is not an artifact of the simplicity of the interaction Hamiltonian. This is just a consequence of the necessary breaking of the shift invariance for any realistic matter interaction as we argued in the previous section.

Therefore, the non trivial scattering problem concerns only states on the lattice $\Gamma^1 = \{\nu = 2n\sqrt{\Delta}\ell_{\mathrm{Pl}} \mid n \in \mathbb{Z}\}$ that is preserved by the Hamiltonian and contains the point $\nu = 0$. In order to solve the scattering problem we consider an in-state of the form

$$|\psi_k\rangle = |\nu\rangle \begin{cases} e^{-i\frac{k}{2}\nu} + A(k)\, e^{i\frac{k}{2}\nu} & (\nu \geq 0) \\ B(k)\, e^{-i\frac{k}{2}\nu} & (\nu \leq 0), \end{cases} . \tag{4.38}$$

where $\nu \in \Gamma^1$, and $A(k)$ and $B(k)$ are coefficients depending on k. For suitable coefficients, such states are eigenstates of the full Hamiltonian (4.35). Arbitrary solutions (wave packets) can then be constructed in terms of appropriate superpositions of these 'plane-wave' states.

We can compute the scattering coefficients $A(k)$ and $B(k)$ from the discrete (finite difference) time-independent Schrodinger equation

$$\left(\hat{H}_0 + \hat{H}_{\mathrm{int}}\right) \triangleright |\psi\rangle \quad = \quad E\,|\psi\rangle \tag{4.39}$$

which amounts to the following finite difference equation in the ν basis:

$$\sum_\nu \Big(-\frac{3V_0}{8\pi G\gamma^2}\frac{1}{2\Delta\ell_{\mathrm{Pl}}^2}\Big[\Psi(\nu - 4\sqrt{\Delta}\ell_{\mathrm{Pl}}) + \Psi(\nu + 4\sqrt{\Delta}\ell_{\mathrm{Pl}}) - 2\Psi(\nu)\Big] + \frac{V_0\mu}{\Delta\ell_{\mathrm{Pl}}^4}\delta_{\nu,0}\Psi(0) - E(k)\,\Psi(\nu)\Big)|\nu\rangle = 0. \tag{4.40}$$

The matching conditions on $v = 0$ are given by:

$$1 + A(k) = B(k)$$
$$-\frac{3}{16\pi G\gamma^2\Delta\ell_{\rm Pl}^2}\Big[\Psi(-4\sqrt{\Delta}\ell_{\rm Pl}) + \Psi(4\sqrt{\Delta}\ell_{\rm Pl}) - 2\Psi(0)\Big] + \frac{\mu}{\Delta\ell_{\rm Pl}^4}\Psi(0) = \frac{E(k)}{V_0}\Psi(0), \tag{4.41}$$

where the first equation comes from continuity at $v = 0$, the second equation from the time independent Shroedinger equation. The solution of the previous equations is

$$\begin{aligned} A(k) &= \frac{-i\Theta(k)}{1+i\Theta(k)} \\ B(k) &= \frac{1}{1+i\Theta(k)}. \end{aligned} \tag{4.42}$$

where

$$\Theta(k) \equiv \frac{16\pi\gamma^2}{3}\frac{\mu}{\sin(2k\sqrt{\Delta}\ell_{\rm Pl})}. \tag{4.43}$$

We consider an in-state of the form (valid for early times)

$$|\psi_{in}, t\rangle = \frac{\pi}{\sqrt{2\Delta}\ell_{\rm Pl}}\int db\big(\psi(b; b_0, v_0)\,|b,1\rangle + \psi(b; b_0, v_0)\,|b,2\rangle\big)\,e^{-iE_\Delta(b)t}, \tag{4.44}$$

where $\psi(b; b_0, v_0)$ is a wave function picked at some $b = b_0$ value and $v = v_0$. Notice that we are superimposing two wave packets supported on lattices Γ^1 and Γ^2 respectively.

We can now write the pure in-density matrix

$$\rho_{\rm in}(t) = \frac{\pi^2}{2\Delta\ell_p^2}\int db\,db'\,e^{i[E_\Delta(b)-E_\Delta(b')]t} \tag{4.45}$$
$$\times\Big[|b',1\rangle\,\psi(b'; b_0, v_0) + |b',2\rangle\,\psi(b'; b_0, v_0)\Big]\Big[\langle b,1|\,\overline{\psi}(b; b_0, v_0) + \langle b,2|\,\overline{\psi}(b; b_0, v_0)\Big].$$

which scatters into the out-density matrix

$$\rho_{\text{out}}(t) = \frac{\pi^2}{\Delta\ell_p^2}\int db\, db'\, e^{i[E_\Delta(b)-E_\Delta(b')]t} \tag{4.46}$$
$$\times\Big[\langle b,1|\overline{\psi}(-b;b_0,\nu_0)\overline{A}(-b) + \langle b,1|\overline{\psi}(b;b_0,\nu_0)\overline{B}(b) + \langle b,2|\overline{\psi}(b;b_0,\nu_0)\Big]$$
$$\times\Big[|b',1\rangle\,\psi(-b')e^{-ib'\overline{\nu}}A(-b') + |b',1\rangle\,\psi(b')e^{ib'\overline{\nu}}B(b') + |b',2\rangle\,\psi(b';b_0,\nu_0)\Big].$$

Let us assume that $\psi(b)$ is highly picked at some b_0 so that we can substitute the integration variables b and b' by b_0 and have a finite dimensional representation of the reduced density matrix after the scattering (this step is rather formal, it involves an approximation but it helps visualising the result). In the relevant 4×4 sector (with basis elements ordered as $\{|1,b_0\rangle, |1,-b_0\rangle, |2,b_0\rangle, |2,-b_0\rangle\}$) we get the matrix representation

$$\rho_{in} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad\to\quad \rho_{out} = \frac{1}{2}\begin{pmatrix} |B(b_0)|^2 & \overline{A}(-b_0)B(b_0) & B(b_0) & 0 \\ A(-b_0)\overline{B(b_0)} & |A(-b_0)|^2 & A(-b_0) & 0 \\ \overline{B(b_0)} & \overline{A(-b_0)} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.47}$$

4.6 Matter coupling produces a coarse-graining entropy jump at the big-bang

A reduced density matrix encoding the notion of coarse graining associated with the low energy equivalence of the ϵ-sectors is defined by tracing over the discrete degree of freedom labelling the component of the state in either the Γ_1 or the Γ_2 lattices. In other words, tracing over the two (macroscopically indistinguishable) ϵ-sectors, namely

$$\langle b|\,\rho^{\text{R}}\,|b'\rangle \equiv \sum_{i=1}^{2}\langle b,i|\,\rho\,|b',i\rangle\,. \tag{4.48}$$

In other words, the subspace of the Hilbert space we are working with is the one supported on two different epsilon sectors $\mathscr{H}(\Gamma^1)\oplus\mathscr{H}(\Gamma^2)\subset\mathscr{H}_{\text{lqc}}$ which, as the two terms are isomorphic $\mathscr{H}(\Gamma^1)\approx\mathscr{H}(\Gamma^2)\approx\mathscr{H}_0$, $\mathscr{H}(\Gamma^1)\oplus\mathscr{H}(\Gamma^2)\subset\mathscr{H}_{\text{lqc}}$ can be written as

$$\mathscr{H}_0\otimes\mathbb{C}^2\subset\mathscr{H}_{\text{lqc}}. \tag{4.49}$$

The coarse graining is defined by tracing over the $\mathbb{C}^2$ factor. This implies that from the previous 4×4 matrix we obtain the 2×2 reduced density matrices. The reduced density matrix ρ^{R}_{in} remains pure, explicitly

$$\rho^{\mathrm{R}}_{in} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{4.50}$$

Nevertheless, the reduced density matrix ρ^{R}_{out} is now mixed, namely

$$\rho^{\mathrm{R}}_{out} = \frac{1}{2}\begin{pmatrix} 1+|B(b_0)|^2 & \overline{A}(-b_0)B(b_0) \\ A(-b_0)\overline{B(b_0)} & |A(-b_0)|^2 \end{pmatrix}. \tag{4.51}$$

We can now compute the entanglement entropy. To first order in the cosmological constant the result is

$$\delta S = \log(2) - \frac{3\Delta\Lambda {\ell_{\mathrm{Pl}}}^2}{128\pi^2\gamma^2\mu^2} + \mathcal{O}(\Lambda^2\ell_{\mathrm{Pl}}^4) \tag{4.52}$$

The behaviour as a function of b is shown in Figure 4.4.

4.7 Quantum cosmology on a superposition of backgrounds.

In the first part of the paper we have seen how the fact that the Hilbert space of loop quantum cosmology is vastly larger than the standard Schroedinger representation implies (via coarse graining) that the coarse graining entropy would go up generically through the evolution across the big-bang *would-be-singularity*. In this section we explore another closely related feature that leads to an apparent non-unitary evolution when dynamics is probed by a low energy agent. This new key property of the fundamental quantum dynamics in loop quantum cosmology is tight to the fact that the Hamiltonian defining evolution can only be defined if an area-gap Δ is provided (see Section 4.2). The quantization of the Hamiltonian reviewed in 4.2 needs an input from a UV background structure. We will see here that the loop quantum cosmology model can be extended naturally to admit superpositions of such microscopic structures and that such extension generically leads to the dynamical development of correlations between the macroscopic and the microscopic degrees of freedom. If the microscopic degrees of freedom are assumed to remain hidden to low energy observers, then such correlations lead to an apparent violation of unitarity in the low energy description where pure states evolve into mixed states.

4.8 The UV input in quantum cosmology: revisiting the $\bar{\mu}$ scheme.

The $\bar{\mu}$ scheme was designed to avoid an inconsistency of an early model of loop quantum cosmology with the low energy limit (or large universe limit) of loop quantum cosmology [110]. The problem arises from the effective compatification of the connection variable c due to the polymer regularization of the Hamiltonian with a fixed fiducial scale μ which implies that c and $c+4\pi/\mu$ are dynamically identified. This leads to anomalous deviations from classical behaviour in situations where the variable c is classically expected to be unbounded for large universes. This can be seen clearly in the present situation where the unimodular Hamiltonian (4.16) is given, in (c, p) variables, by

$$H = \frac{3V_0}{8\pi G}\frac{c^2}{\gamma^2|p|}. \tag{4.53}$$

For non-vanishing energies (or equivalently non-vanishing cosmological constant) the conservation of the Hamiltonian implies that c grows as $|p| \propto a^2$, i.e., c grows without limits as the universe expands so that no matter how small μ is, anomalous effects due to the compactification of the c become relevant at macroscopic scales [111]. As no quantum gravity effects seem acceptable in the large universe regime for a model with finitely many degrees of freedom, this anomaly is seen as an inconsistency of the model.

The $\bar{\mu}$ scheme solves this inconsistency by 'renormalizing' the regulating scale μ as the universe grows (recall equation (4.19)). The interesting thing is that such renormalization is justified by quantum geometry arguments that link the mini superspace model of loop quantum cosmology to the geometry of a microscopic background state in the full theory. The argument uses explicitly the idea that the low energy degrees of freedom (dynamical variable of loop quantum cosmology) arise from the coarse graining of the fundamental ones in loop quantum gravity.

Here we review the construction of the $\bar{\mu}$ scheme as described in [96]. Consider a fundamental quantum geometry state $|s\rangle$ in the Hilbert space of loop quantum gravity, representing a microscopic state on top of which the quantum cosmological coarse grained dynamics will eventually be defined. Such underlying fundamental state will have to be approximately homogeneous and isotropic up to some scale $L > \ell_{\rm Pl}$ with respect to the preferred foliation defining the co-moving FLRW observers at low energies. If that is the case then such space slices can be divided into (approximately) cubic 3-cells of physical side length L which all have approximately equivalent quantum geometries. The area of a face of such cubic cells in Planck units will be denoted

Δ_s so that $L^2 = \ell_p^2 \Delta_s$. Note that Δ_s is a property of the underlying microstate: an area eigenstate if the microstate is an eigenstate, or an area expectation value if the state is sufficiently peaked on a quantum geometry and has small fluctuations around it. A simple realization is the one where Δ_s is an area eigenvalue, and the important assumption is that Δ_s is the same for all cells (this encodes the homogeneity of the microsocopic state). Consider the area of a large two dimensional surface (the face of a fiducial cell $\mathcal{V}$) whose area is measured by the low energy (coarse grained) quantity p used as configuration variable in loop quantum cosmology. We naturally would expect that $|p| \gg \ell_{\rm Pl}^2$ or alternatively that

$$N\ell_p^2 \Delta_s = |p|, \tag{4.54}$$

where N denotes the number of microscopic cells contained in the coarse grained surface (a face of $\mathcal{V}$), and $N \gg 1$. The fiducial cell has fiducial coordinate volume V_0 and hence fiducial side coordinate length $V_0^{1/3}$. Therefore, the fiducial coordinate length $\bar{\mu}$ of the microscopic homogeneity cells is given by the relation

$$N(\bar{\mu} V_0^{1/3})^2 = V_0^{2/3}. \tag{4.55}$$

Combining the previous two equations one recovers equation (4.19), namely

$$\bar{\mu}_s^2 \equiv \bar{\mu}^2 = \frac{\ell_p^2 \Delta_s}{|p|}, \tag{4.56}$$

i.e., the fiducial scale $\bar{\mu}$ is dynamical: as the universe grows (and $|p|$ becomes large), the underlying fiducial length scale decreases. The fiducial regularization scale (4.55) depends on the fundamental state $|s\rangle$ via the quantity Δ_s, hence we denote it $\bar{\mu}_s$. When such dynamical scale is used in the regularization of the quantum cosmology Hamiltonian the effective compactification scale for c grows like $|p|$ and the inconsistency previously discussed is avoided. This is transparent in terms of the new canonical pair (b, v). From equation (4.14) we have that $b = c\bar{\mu}_s/(\sqrt{\Delta_s}\ell_{\rm Pl})$, in contrast with c (see (4.53)), remains constant (see (4.16)) in the De Sitter universe. The quantization of the Hamiltonian presented in Section 4.2 introduces an effective compatification of the variable b whose dynamical effect is now only relevant when the cosmological constant approaches one in Planck units. This can be seen from (4.21). The cosmological constant is bounded from above by its natural value in Planck units due to the underlying quantum geometry structure while the anomalous IR behaviour is avoided (the problems exhibited in the model studied in [110] are also resolved).

The previous is the standard account of the motivation of the $\bar{\mu}$ scheme of [87] with the little twist (which is very important for us here) that Δ_s need not be the lowest area eigenvalue of loop quantum gravity. In the usual argument the microscopic state is thought to be built from a special homogeneous spin network (geometry eigenstate) with all spins equal to the fundamental representation. This implies that, in the above construction, $\Delta_s = \Delta_{1/2} \equiv 2\pi\gamma\sqrt{3}$. The observation here is that Δ_s can take different values according to the microscopic properties of the underlying quantum geometry state. One could take for instance all spins equal to the vector representation and then have $\Delta_s = \Delta_1 \equiv 4\pi\gamma\sqrt{2}$ instead, or take j arbitrary and use $\Delta_s = \Delta_j$. It is important to point out that such possibility can arise naturally in quantum cosmology models obtained in the group field theory framework [112, 113, 114].

As we have seen in Section 4.8, the field strength regularization, and hence the Hamiltonian, depend on the value Δ_s of the background (approximately homogeneous) spin network state $|s\rangle$ through the dynamical scale $\bar{\mu}_s$.

In this way, the dynamics of loop quantum cosmology establishes correlations with the a microscopic degree of freedom in the underlying loop quantum gravity fundamental state.

As such degree of freedom (the area eigenvalue Δ_s of the minimal homogeneity cells) is quantum, it is natural to model the system by a tensor product Hilbert space $\mathscr{H} \equiv \mathscr{H}_m \otimes \mathscr{H}_{\rm lqc}$ where $\mathscr{H}_m$ is the Hilbert space representing the microscopic degree of freedom encoded in the minimal homogeneous cell operator (whose eigenvalues we denote Δ_s), and $\mathscr{H}_{\rm lqc}$ the standard kinematical Hilbert space of loop quantum cosmology.

General states in $\mathscr{H}$ can be expressed as linear combinations of product states $|s\rangle \otimes \psi$ in the respective factor Hilbert spaces. The quantum Hamiltonian has a natural definition on such states and therefore on the whole of $\mathscr{H}$, namely

$$\hat{H} \rhd \left(|s\rangle \otimes \psi\right) = |s\rangle \otimes \ \hat{H}_{\Delta_s} \rhd \psi, \tag{4.57}$$

where $\hat{H}_{\Delta_s}$ is the usual loop quantum cosmology Hamiltonian in the $\bar{\mu}_s$ scheme, which in our particular case is defined in equation (4.21) with regulator $\Delta = \Delta_s$.

Notice that the previous extension of the standard loop quantum cosmology framework to the larger Hilbert space $\mathscr{H}$ is also natural from the perspective of the full theory. Indeed the generally accepted regularization procedure of the Hamiltonian constraint in loop quantum gravity (first introduced by Thiemann [115] and further developped in recent analysis—see Varadarajan and Ladda [116] and references therein) is state dependent in that the loops defining the regulated curvature of the connection are added on specific nodes of the state where the Hamiltonian is acting upon. This feature finds its analog in the action (4.57) where the regulating scale Δ_s depends on the state $|s\rangle \in \mathscr{H}_m$.

In order to simplify the following discussion we will restrict states in $\mathscr{H}_m$ even further and consider a subspace $\mathfrak{h} = \mathbb{C}^2 \subset \mathscr{H}_m$, i.e. we will model the situation where the underlying microscopic state is an arbitrary superposition of only two fixed microscopic homogeneous spin-network states. For example we take

$$\mathfrak{h} \equiv \mathrm{span}\Big[\,|+\rangle\,,|-\rangle\,\Big], \tag{4.58}$$

where $|\pm\rangle \in \mathscr{H}_m$ are two suitable orthogonal background states (these two states will be conveniently picked below). From the infinitely dimensional Hilbert space $\mathscr{H}_m$ we are now selecting a single *q-bit* subspace $\mathbb{C}^2$. The Hilbert space of our model is

$$\mathscr{H} = \mathfrak{h} \otimes \mathscr{H}_{\mathrm{lqc}}. \tag{4.59}$$

The factor $\mathfrak{h}$ represents additional microscopic (hidden to low energy observers) UV degrees of freedom, while $\mathscr{H}_{\mathrm{lqc}}$ encodes the data that under suitable circumstances (e.g. when the universe is large) represent the low energy cosmological degrees of freedom.

In this way we see that in addition to the intrinsic degeneracy of energy eigenvalues analyzed in the first part of this paper, there is another candidate for microscopic degree of freedom associated to the regularization of the Hamiltonian action via the $\bar{\mu}$-scheme. Both mechanisms are proper of the present loop quantum cosmology toy model but reflect generic properties of the full theory of loop quantum gravity. More generally, we expect similar features to be present in any quantum gravity approach where smooth geometry is only emergent from a discrete fundamental theory.

From now on we adopt the convenient notation $|s\rangle$ with $s = \pm$ for such prefered basis elements of $\mathfrak{h}$. With this notation, and using (4.21), the Hamiltonian (4.57) becomes

$$\begin{aligned}\hat{H}_0 \triangleright \big(|s\rangle \otimes |\psi\rangle\big) &= \frac{3V_0}{8\pi G\gamma^2}\frac{1}{\Delta_s \ell_{\mathrm{Pl}}^2}\Big(\sin(\sqrt{\Delta_s}\ell_{\mathrm{Pl}}\, b)\Big)^2 \triangleright |s\rangle \otimes |\psi\rangle \\ &= -\frac{3V_0}{8\pi G\gamma^2}\sum_v \frac{1}{2\Delta_s \ell_{\mathrm{Pl}}^2}\,|s\rangle \otimes |v\rangle \Big[\Psi(v - 4\sqrt{\Delta_s}\ell_{\mathrm{Pl}}) + \Psi(v + 4\sqrt{\Delta_s}\ell_{\mathrm{Pl}}) - 2\Psi(v)\Big],\end{aligned} \tag{4.60}$$

where $\Psi(v) \equiv \langle v|\psi\rangle$.

Now, the only special feature of the basis $|\pm\rangle$ is that it is preferred from the perspective of the regularization of the effective (unimodular) loop quantum cosmology Hamiltonian. Consequently, a natural question to the quantum theory is how the dynamics would look if the initial state is arbitrary in the factor $\mathfrak{h}$? More precisely, what if we consider the linear combination of two background spin networks $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \in \mathfrak{h}$ times some loop quantum cosmology wave function as depicted in Figure 4.5? To answer this question we consider a special initial state where correlations between the low energy and the UV degrees of freedom are not present. This will lead to a reduced density matrix—tracing out the microscopic space $\mathfrak{h}$ in (4.59)—that is *pure* initially, the form of such state is illustrated in Figure 4.5. At the same time we want to be able to diagonalize the Hamiltonian with such uncorrelated initial states; more precisely this boils down to diagonalizing both H_{Δ_+} and H_{Δ_-} in $\mathscr{H}_{\rm lqc}$. This implies that the factor $\psi(v) \in \mathscr{H}_{\rm lqc}$, in Figure 4.5, must be supported on a lattice $\Gamma^{\epsilon}_{\Delta}$ that is left invariant by the action of both H_{Δ_+} and H_{Δ_-} (left invariant in the sense that the shift operators in the definition of the Hamiltonian only relate points of $\Gamma^{\epsilon}_{\Delta}$ and never map points out). This can be achieved by assuming that $\sqrt{\Delta_+} = m\sqrt{\Delta_-}$ for some natural number m. For simplicity we will take $m = 2$ from now on [9]. The parameter ϵ will be taken so that the lattice $\Gamma^{\epsilon}_{\Delta}$ contains the point $v = 0$. This is a standard choice. With all this the invariant lattice, denoted Γ_{Δ_-}, is

$$\Gamma_{\Delta_-} \equiv \Gamma^{\epsilon=0}_{k=2\sqrt{\Delta_-}\ell_{\rm Pl}}. \tag{4.61}$$

Note that in the notation described below (4.37) we have that $\Gamma_{\Delta_-} = \Gamma_1 \cup \Gamma_2$.

The choices made above are not mandatory. One could have chosen a different initial state. The previous choice is particularly interesting here because it would lead to a reduced initial density matrix that is pure and hence and initially vanishing entanglement entropy. Other states would involve correlations and would therefore carry a non vanishing entropy load from the beginning. For the discussion that interests us here and for the analogy with black hole evaporation it is more transparent to set the entropy to zero initially.

An arbitrary (unimodular) loop quantum cosmology state associated to such choice

[9] One might be worried that is hard to achieve if one sticks to the form of the area spectrum of loop quantum gravity. This is however simply a model and the link with the full theory (remember) must be taken at the heuristic level. Nevertheless, solutions do exist for instance $m = 4$ for $j_+ = 3$ and $j_- = 1/2$.

of background state can be expressed as:

$$\begin{aligned}\Psi_{\rm in}(v,t) &= \langle v|\frac{1}{\sqrt{2}}\sum_s |s\rangle\otimes|\Psi_{\rm in}(t)\rangle \\ &= \frac{1}{\sqrt{2}}\sum_s |s\rangle\otimes\left[\delta_{\Gamma_{\Delta_-}}(v)\int_0^{\frac{\pi}{\sqrt{\Delta_-}\ell_{\rm Pl}}} dk\,\psi(k;b_0,v_0)\exp\Big(-iE_s(k)t\Big)\right] \end{aligned} \tag{4.62}$$

where $\psi(k;b_0,v_0)$ is a properly normalized function peaked at $k=b_0$ and $v=v_0$.

The initial state in the momentum representation is given by:

$$\begin{aligned}\Psi_{\rm in}(b,t) &= \sum_{v\in\Gamma^0_{\Delta_-}} \langle b,1\cup 2|v\rangle\,\langle v|\Psi_{\rm in}(t)\rangle \\ &= \frac{\pi}{\sqrt{\Delta_-}\ell_{\rm Pl}}\sum_s|s\rangle\otimes\psi(b;b_0,v_0)e^{-iE_s(b)t}\end{aligned} \tag{4.63}$$

where in the first line we used the natural extension of the notation introduced in (4.37) where $|b,1\cup 2\rangle$ means an eigenstate of the corresponding shift operators (4.18) supported on the lattice $\Gamma_{\Delta_-}=\Gamma_1\cup\Gamma_2$. Notice that we can also write

$$|b,1\cup 2\rangle = |b,1\rangle + |b,2\rangle\,, \tag{4.64}$$

keeping in mind that terms on the *r.h.s.* are individually eigenstates of the shift operators with twice the lattice spacing of Γ_{Δ_-}.

We also used

$$\sum_{v\in\Gamma^0_{\Delta_-}}\exp\left(i\frac{b-k}{2}v\right) = \frac{\pi}{\sqrt{\Delta_-}\ell_{\rm Pl}}\delta(b-k). \tag{4.65}$$

We can write then

$$\Psi_{\rm in}(t) = \sum_s\int {\rm D}b\,|s\rangle\otimes|b,1\cup 2\rangle\,\psi(b;b_0,v_0)e^{-iE_s(b)t}, \tag{4.66}$$

where

$$\mathrm{D}b \equiv \frac{\pi}{\sqrt{2\Delta_-}\ell_{\mathrm{Pl}}} db, \tag{4.67}$$

is the Haar measure on the circle of circumference $\pi/\sqrt{2\Delta_-}\ell_{\mathrm{Pl}}$. We notice from (4.66) that even when our initial state contains no correlations between the low energy degrees of freedom represented by b and the microscopic degrees of freedom encoded in $|s\rangle$ at $t=0$, quantum correlations between the two will develop with time due to the non trivial dependence of the energy spectrum with s.

Even when this is quite clear from (4.66) one can state this fact in an equivalent way by analysing the (pure) density matrix $\rho_{\mathrm{in}}(t) \equiv |\Psi_{\mathrm{in}}(t)\rangle \langle \Psi_{\mathrm{in}}(t)|$, whose matrix elements in the b basis are:

$$\begin{aligned}\rho_{\mathrm{in}}(t) \equiv \sum_{s,s'} \int \mathrm{D}b\, \mathrm{D}b' \left(\overline{\psi}(b;b_0,v_0)\psi(b';b_0,v_0)\; e^{i[E_s(b)-E_{s'}(b')]t}\right) \\ \times |b',1\cup 2\rangle\, |s'\rangle\, \langle b,1\cup 2|\, \langle s|\,. \end{aligned} \tag{4.68}$$

As coarse grained observers are assumed to be insensitive to the microscopic structure that is here encoded in the 'spin' quantum number s, low energy physical information is encoded in the reduced density matrix

$$\rho^R_{\mathrm{in}}(t) \equiv \sum_{s} \int \mathrm{D}b\, \mathrm{D}b' \left(\overline{\psi}(b;b_0,v_0)\psi(b';b_0,v_0)\; e^{i[E_s(b)-E_s(b')]t}\right) \tag{4.69}$$

$$\times |b',1\cup 2\rangle\, |s'\rangle\, \langle b,1\cup 2|\, \langle s|\,, \tag{4.70}$$

which can be simply be written as

$$\rho^R_{\mathrm{in}}(t) = \frac{1}{2}\sum_s |\Psi_s(t)\rangle\, \langle \Psi_s(t)|\,. \tag{4.71}$$

where

$$|\Psi_s(t)\rangle \equiv \int \mathrm{D}b\; \psi(b;b_0,v_0)\; e^{-iE_s(b)t}\; |b,1\cup 2\rangle \tag{4.72}$$

Notice that (4.71) is only pure at $t=0$ and becomes mixed due to the correlations evoked above as time passes. One can compute the entanglement entropy $S(t) \equiv -\mathrm{Tr}\left[\rho^R_{\mathrm{in}}(t)\log(\rho^R_{\mathrm{in}}(t))\right]$ which turns out to be given by the simple analytic expression

$$S(t) = -\log\left(1-\frac{\delta}{2}\right) - \frac{\delta}{2}\log\left(\frac{\delta}{1-\frac{\delta}{2}}\right), \tag{4.73}$$

where

$$\delta(t) \equiv 1 - \left|\int \mathrm{D}b\,\overline{\psi}(b;b_0,v_0)\psi(b;b_0,v_0)e^{i[E_+(b)-E_-(b)]t}\right|. \tag{4.74}$$

For generic wave packets $\psi_s(b)$ the entanglement entropy is a monotonic growing function of time which grows asymptotically to the maximally mixed situation $S_{\max} = \log(2)$ (see an example in Figure 4.6).

A more intuitive picture can be obtained from a suitable expansion of the energy eigenvalues (4.24) in powers of the label $b\ell_{\mathrm{Pl}}$

$$E_s(b) = \frac{3V_0}{8\pi G\gamma^2}\frac{1}{\Delta_s\ell_{\mathrm{Pl}}^2}\left(\sin(\sqrt{\Delta_s}\ell_{\mathrm{Pl}}b)\right)^2 = \frac{3V_0}{8\pi G\gamma^2}b^2 - \frac{V_0}{8\pi G\gamma^2}\Delta_s\ell_{\mathrm{Pl}}^2 b^4 + b^2\mathcal{O}(\ell_{\mathrm{Pl}}^4 b^4). \tag{4.75}$$

Such expansion makes sense in that it allows for the identification of the low energy effective Hamiltonian (the one that one would define in a purely Wheeler-DeWitt quantization) plus corrections that involve interactions with the underlying discrete structure of LQG here represented by the spin s degree of freedom.

Namely, we can read from the previous expansion

$$H_{\mathrm{eff}} \equiv H_{\mathrm{eff}}^0(b) + \Delta H(b,s), \tag{4.76}$$

where $\hat{H}_{\mathrm{eff}}^0(\hat{b}) \equiv \frac{6}{\gamma^2}\hat{b}^2$ is the Wheeler-DeWitt Hamiltonian and the additional term an interaction with the environment represented by the underlying discrete structure represented by the dependence on S_1 (a hidden degree of freedom from the low energy continuum perspective).

Of course the hats in the previous equation denote operators in a different representation (the continuum Schroedinger representation) that is not unitarily equivalent to the 'fundamental' polymer representation introduced in Section 4.2 and used in the LQC setup (recall for instance that the operator $\hat{b}$ does not even exist in the polymer representation).

The lack of purity for $t > 0$ of the reduced density matrix (4.71) is due to correlations that develop between the low energy degree of freedom b and the hidden microscopic degree of freedom s via this non trivial interaction Hamiltonian. This means that generically (i.e. for arbitrary initial states $\psi_s(b)$) the fundamental evolution would seem to violate unitarity, from the perspective of low energy observers, due to the decoherence with the microscopic quantum geometric structure. Notice however that for states $\psi_s(b)$ picked at sufficiently small $\bar{k}$, i.e., $\bar{k}\sqrt{\Delta_s}\ell_{\rm Pl} \ll 1$, we have from (4.22) that

$$\Lambda(b) \approx 3\gamma^{-2} b^2 \tag{4.77}$$

and the density matrix (4.71) is pure for all times.

More precisely, we can translate the criterion for the absence of decoherence with the underlying microscopic discrete structure in terms of the value of the cosmological constant of the given state. For an eigenstate of the Hamiltonian the relation is given by $\Lambda \equiv E_s(b)$. Therefore the criterion for the absence of decoherence in terms of the cosmological constant is

$$\Delta_s \ell_{\rm Pl}^2 \gamma^2 \Lambda \approx \ell_p^2 \Lambda \ll 1 \tag{4.78}$$

Interestingly for states with low values of the cosmological constant in natural units—equivalent semi-classically to the scalar curvature R in our matter free model—define a decoherence free subspace. When the cosmological constant does not satisfy the condition (4.78) decoherence with the microscopic structure is turned on and maximized for Λ of order one in Planck units: notice incidentally that due to the polymer quantization the cosmological constant is bounded by

$$\Lambda_{\rm max} = \frac{3}{\gamma^2 \Delta_{\frac{1}{2}} \ell_{\rm Pl}^2}. \tag{4.79}$$

For low values of Λ unitarity is recovered in the effective description that ignores the microscopic structure.

Decoherence takes place here due to an interaction between the low energy coarse degrees of freedom and the microscopic discreteness in the underlying quantum geometry background but in way (in our simple model) that the energy and (hence the cosmological constant) is conserved. However, the presence of decoherence suggest the possibility for a natural deviation of this idealized absence of dissipation: generically decoherence and dissipation often come together. Therefore, a surprising and unexpected consequence of our analysis is the suggestion of a natural channel for the relaxation of a large cosmological constant due to the possibility of dissipative effects associated to the decoherence pointed out here.

Incidentally, all this shows it is only in the limit of low values of E (small cosmological constant) that the coarse graining that leads from the full theory of loop quantum gravity to the minisuperspace description of loop quantum cosmology is well defined. This is not surprising and only confirms the usual intuition that drives the construction of models of loop quantum cosmology. However, it opens the door for a qualitative understanding of the necessity of decoherence effects in more general situations. For instance, the standard $\bar{\mu}_s$ construction suggests that coarse graining is weaker at the big bang where the Hamiltonian evolution (4.60) takes the universe through the $v = 0$ states. During this high (spacial) curvature phase it is natural to expect that the higher corrections in (4.75) (describing the interaction with the microscopic Planckian structure) can no longer be neglected.

Interestingly, there is another way to make decoherence go away. This is due to the asymptotic behaviour of the separation of area eigenvalues in loop quantum gravity which imply that for large Δ_s there are states such that $\Delta_s - \Delta_{s'} \approx \Delta_s \exp(-\pi\sqrt{2\Delta_s/3})$ [117]. Therefore, in the continuum limit $\Delta_s - \Delta_{s'} \ll 1$ the dynamical entanglement growth of our model can be made as small as wanted.

4.9 Matter coupling produces entanglement entropy jump at the big-bang

In the pure gravity case we can make decoherence be as small as wanted by choosing states with a cosmological constant that is sufficiently small. Here we show that this is no longer possible once matter is added and that there is a generic development of correlations with the UV degrees of freedom in the evolution across the *would-be-singularity*: an initially pure state (reduced low energy density matrix) evolves generically into a mixed state (reduced low energy density matrix) after the big-bang.

In order to see this in more detail we just need to write the matter Hamiltonians acting in the Hilbert space (4.59). One needs the natural generalization of the expressions written in Section 4.4 to the present context. For instance for the scalar field coupling equation (4.32) becomes

$$\hat{H}_\phi \triangleright \left(|s\rangle \otimes |\psi\rangle\right) = -m \sum_{v\in\Gamma} |s\rangle \otimes |v\rangle\, h_\phi(v; \sqrt{\Delta_s}\ell_{\rm Pl})\Psi(v,\phi), \tag{4.80}$$

where

$$h_\phi(\nu;\lambda) \equiv \frac{p_\phi^2}{16\lambda^4}\left(|\nu+2\lambda|^{\frac{1}{2}} - |\nu-2\lambda|^{\frac{1}{2}}\right)^4. \tag{4.81}$$

The momentum p_ϕ commutes with the Hamiltonian and thus is its a constant of motion. As before, if we consider an eigenstate of p_ϕ then the problem reduces again to a scattering problem with a potential decaying like $1/\nu^2$ when solving the time independent Schroedinger equation

$$\hat{H}_0 + \hat{H}_\phi \triangleright \left(|s\rangle \otimes |\psi\rangle\right) = E\left(|s\rangle \otimes |\psi\rangle\right). \tag{4.82}$$

From the discussion of 4.4 we can capture the basic qualitative effect of matter interaction by considering a simple solvable model where the matter contribution is concentrated at a single event a the big-bang. Non of the qualitative conclusions that follow depend on this simplification, and the more realistic free scalar field model can be dealt with (some results are shown in Section 4.7). With some extra effort one could actually analyze the a more realistic model (say the one defined by (4.80)) but the conclusion will remain the same. Therefore we consider

$$\hat{H} = \hat{H}_0 + \mu \hat{H}_{\text{int}}, \tag{4.83}$$

where μ is a dimensionless coupling, $\hat{H}_0$ is given in (4.60), and $\hat{H}_{\text{int}}$ is the generalization of (4.36)

$$\hat{H}_{\text{int}} \triangleright \left(|s\rangle \otimes |\psi\rangle\right) \equiv \sum_\nu \hat{O}|s\rangle \otimes |\nu\rangle \frac{\delta_{\nu,0}}{\sqrt{\Delta_s}} \Psi(0) \tag{4.84}$$

where $\hat{O}$ is a self adjoint operator in $\mathfrak{h} = \mathbb{C}^2$.

A natural and simple model for this operator is to choose

$$\hat{O} \equiv \ell_p^{-4} \frac{V_0}{\sqrt{\Delta_s}}. \tag{4.85}$$

This choice is formulated in the notation introduced below (4.60) and inspired by the analogy with a spin system. We have added by hand an interaction Hamiltonian that switches on only when the universe evolves through the *would-be-singularity* at the zero volume state. This encodes the idea of the intrinsic uncertainty of the peculiar construction of the mini-superspace model of loop quantum cosmology that we discussed in Section 4.8. The discrete local degrees of freedom must be important close to the big bang and symmetry reduction must fail in some way that can only be correctly described if a full quantum gravity theory is available. Here we model such unknown dynamics in the simplest fashion available to us here, which consists of including the possibility for the background state $|s\rangle$ (representing in spirit the underlying quantum geometry) to be modified by the dynamics via $\hat{H}_{\text{int}}$.

Here we proceed as in Section 4.5 while keeping in mind that, in the present case, there are two distinct cases at hand given by the two possible values $\Delta_{\pm}$. Let us consider an in-state of the form

$$|k,s\rangle = |s\rangle \otimes |v\rangle \begin{cases} e^{-i\frac{k}{2}v} + A_s(k)\, e^{i\frac{k}{2}v} & (v \geq 0) \\ B_s(k)\, e^{-i\frac{k}{2}v} & (v \leq 0), \end{cases}. \tag{4.86}$$

where $A_s(k)$ and $B_s(k)$ are coefficients depending on k and (in contrast with the case in Section 4.5) now also on $s = \pm 1$ (with $|\pm\rangle$ the eigenstates of $\hat{S}_z$). For suitable coefficients, such states are eigenstates of the Hamiltonian H_0 as well as the full Hamiltonian (4.35). Arbitrary solutions (wave packets) can then be constructed in terms of appropriate superpositions of these 'plane-wave' states.

$$\begin{aligned} A_s(k) &= \frac{-i\Theta_s(k)}{1+i\Theta_s(k)} \\ B_s(k) &= \frac{1}{1+i\Theta_s(k)}. \end{aligned} \tag{4.87}$$

where

$$\Theta_s(k) \equiv \frac{16\pi\gamma^2}{3} \frac{\mu}{\sin(2k\sqrt{\Delta_s}\ell_{\text{Pl}})}. \tag{4.88}$$

One can superimpose the previous eigenstates to produce wave packets (semiclassical states) for the wave function of the universe that are picked at some value v_0 of the rescaled volume (see footnote 4). Wave packets will evolve in time according to the Schroedinger equation which in our case is just a discrete analog of the one corresponding to a free particle in quantum mechanics with an interaction term at the 'origin' $v = 0$. If we start with a state that is sufficiently picked around v_0 for $v \gg \ell_{\text{Pl}}$ initially, then the state can be described in terms of the supperposition (4.66) where the explicit values of the coefficients $A_s(b)$ and $B_s(b)$ does not appear. Equation (4.44) is generalized to

$$\Psi_{\text{in}}(t \ll 0) = \int \mathrm{D}b \big(|b,1\rangle\, \psi(b;b_0,v_0) + |b,2\rangle\, \psi(b;b_0,v_0)\big)\, e^{-iE_-(b)t} \\ + \int \mathrm{D}b \big(|b,1\rangle\, \psi(b;b_0,v_0) + |b,2\rangle\, \psi(b;b_0,v_0)\big)\, e^{-iE_+(b)t}. \tag{4.89}$$

The coefficients (4.87) enter the expression of the scattered wave packet at late times which becomes

$$\Psi_{\text{out}}(t \gg 0) = \\ \int \mathrm{D}b\ |-\rangle \otimes |b,1\cup 2\rangle \Big[\psi(-b;b_0,v_0)A_-(-b) + \psi(b;b_0,v_0)B_-(b)\Big]\, e^{-iE_-(b)t} + \\ \int \mathrm{D}b\ |+\rangle \otimes \Big[\, |b,1\rangle \big(\psi(-b;b_0,v_0)A_+(-b) + \psi(b;b_0,v_0)B_+(b)\big) + |b,2\rangle\, \psi(b;b_0,v_0)\Big]\, e^{-iE_+(b)t}. \tag{4.90}$$

Note that the solution of the scattering problem for the $E_+(b)$ eigenvalues is asymmetric with respect to the components of the in state supported on Γ_1 and Γ_2. Indeed the states $|b,2\rangle$ are eigenstates of the Hamiltonian directly because they are not supported on $v = 0$ and hence they do not 'see' the interaction: this is capture by trivial scattering coefficients for this component.

4.10 Entropy associated with the entanglement with the UV degrees of freedom

From the previous initial state we can calculate (by tracing over the factor $\mathfrak{h}$, see (4.59)) the initial reduced density matrix

$$\rho^{\text{R}}_{\text{in}}(t) = \int \mathrm{D}b\, \mathrm{D}b'\, e^{i[E_+(b)-E_+(b')]t} \\ \times \Big[\, |b',1\rangle\, \psi(b';b_0,v_0) + |b',2\rangle\, \psi(b';b_0,v_0)\Big]\Big[\, \langle b,1|\overline{\psi}(b;b_0,v_0) + \langle b,2|\overline{\psi}(b;b_0,v_0)\Big] \\ + \int \mathrm{D}b\, \mathrm{D}b'\, e^{i[E_-(b)-E_-(b')]t} \\ \times \Big[\, |b',1\rangle\, \psi(b';b_0,v_0) + |b',2\rangle\, \psi(b';b_0,v_0)\Big]\Big[\, \langle b,1|\overline{\psi}(b;b_0,v_0) + \langle b,2|\overline{\psi}(b;b_0,v_0)\Big]. \tag{4.91}$$

The reduced density matrix after the big bang is

$$\rho^{\mathrm{R}}_{\mathrm{out}}(t) = \int \mathrm{D}b\, \mathrm{D}b'\, e^{i[E_+(b)-E_+(b')]t} \tag{4.92}$$
$$\Big[\, \langle b,1| \Big(\overline{\psi}(-b;b_0,v_0)\overline{A}_+(-b) + \overline{\psi}(b;b_0,v_0)\overline{B}_+(b)\Big) + \langle b,2|\,\overline{\psi}(b;b_0,v_0)\Big]$$
$$\Big[\, |b,1\rangle \left(\psi(-b';b_0,v_0)A_+(-b') + \psi(b';b_0,v_0)B_+(b')\right) + |b,2\rangle\, \psi(b';b_0,v_0)\Big] +$$
$$e^{i[E_-(b)-E_-(b')]t}\Big[\, |b',1\cup 2\rangle\, \psi(-b';b_0,v_0)A_-(-b') + |b',1\cup 2\rangle\, \psi(b';b_0,v_0)B_-(b')\Big]$$
$$\times \Big[\, \langle b,1\cup 2|\,\overline{\psi}(-b;b_0,v_0)\overline{A}_-(-b) + \langle b,1\cup 2|\,\overline{\psi}(b;b_0,v_0)\overline{B}_-(b)\Big],$$

where $\alpha_+ = 1/4$ and $\alpha_- = 1$ and δ_{s+} is unity when $s = +$ and vanishes when $s = -$.

Then the non vanishing entries of the reduced density matrix are

$$\begin{aligned}
\rho^{R\,11}_{out}(b_0,b_0) &= \frac{1}{4}\left(|B_+(b_0)|^2 + |B_-(b_0)|^2\right)\\
\rho^{22}_{out}(b_0,b_0) &= \frac{1}{4}\left(1 + |B_-(b_0)|^2\right)\\
\rho^{R\,12}_{out}(b_0,b_0) &= \frac{1}{4}\left(B_+(b_0) + |B_-(b_0)|^2\right) = \overline{\rho^{R\,21}_{out}(b_0,b_0)}\\
\rho^{R\,11}_{out}(-b_0,-b_0) &= \frac{1}{4}\left(|A_+(-b_0)|^2 + |A_-(-b_0)|^2\right)\\
\rho^{R\,22}_{out}(-b_0,-b_0) &= \frac{1}{4}\left(|A_-(-b_0)|^2\right)\\
\rho^{R\,12}_{out}(-11b_0,-b_0) &= \frac{1}{4}\left(|A_-(-b_0)|^2\right) = \overline{\rho^{R\,21}_{out}(-b_0,-b_0)}\\
\rho^{R\,11}_{out}(b_0,-b_0) &= \frac{1}{4}\Big(\overline{A}_+(-b_0)B_+(b_0) + \overline{A}_-(-b_0)B_-(b_0)\Big) = \overline{\rho^{R\,11}_{out}(-b_0,b_0)}\\
\rho^{R\,22}_{out}(b_0,-b_0) &= \frac{1}{4}\Big(\overline{A}_-(-b_0)B_-(b_0)\Big) = \overline{\rho^{R\,22}_{out}(-b_0,b_0)}\\
\rho^{R\,21}_{out}(b_0,-b_0) &= \frac{1}{4}\Big(\overline{A}_+(-b_0) + \overline{A}_-(-b_0)B_-(b_0)\Big) = \overline{\rho^{R\,12}_{out}(-b_0,b_0)}\\
\rho^{R\,12}_{out}(b_0,-b_0) &= \frac{1}{4}\Big(\overline{A}_-(-b_0)B_-(b_0)\Big) = \overline{\rho^{R\,21}_{out}(-b_0,b_0)}.
\end{aligned} \tag{4.93}$$

The matrix ρ^R_{out} is positive definite, $\mathrm{Tr}[\rho^R_{out}] = 1$ and $\rho^R_{out} = \rho^{R\dagger}_{out}$. In the case $b_0\ell_{\mathrm{Pl}} \ll 1$ we have

$$\Theta_s(b_0) \approx \frac{8\pi\gamma^2\mu}{3}\frac{1}{b_0\sqrt{\Delta_s}\ell_{\mathrm{Pl}}}. \tag{4.94}$$

We can now compute the entanglement entropy jump δS to first leading order in $b_0\ell_{\mathrm{Pl}}/\mu$. The result (expressed in terms of the cosmological constant in this regime, namely (4.77)) is

$$\delta S = \delta_0 S - \frac{3\Delta_- \ell_{\rm Pl}^2 \log(3)}{128\pi^2\gamma^2\mu^2}\,\Lambda + \mathcal{O}(\Lambda^2 \ell_{\rm Pl}^4), \tag{4.95}$$

where $\delta_0 S = 2\log(2) - \frac{3}{4}\log(3)$.

The previous equation shows that the entropy jump is non trivial at crossing the big-bang *would-be-singularity*, even in the low cosmological (low energy limit) where (according to the analysis of the previous section) decoherence with the microscopic Planckian structure can be neglected during the time the universe is large. Information is unavoidably degraded (it seems lost for low energy observers) during the singularity crossing.

The general entropy jump for arbitrary (not necessarily small Λ) can be computed explicitly. Its value is bounded by log(2) in our model. Finally, the energy is conserved through the big bang and during all the dynamical evolution for arbitrary values of b_0. The decoherence and entanglement which can be interpreted as an information loss happens without energy spending as required by the scheme put forward in [15].

4.11 Discussion

We have seen that one can precisely realize the scenario put forward in [15] for the resolution of Hawking's information loss paradox in quantum gravity in the context of loop quantum comology. The key feature making this possible is the existence of additional degrees of freedom with no macroscopic interpretation which unavoidably entangle with the macroscopic degrees of freedom during the dynamical evolution and lead to a reduce density matrix whose entropy grows. The fundamental description is unitary but the effective description—that does not take the microscopic degrees of freedom into account and hence is analogous to the QFT description of BH evaporation—evolves pure states into mixed states. The microscopic degrees of freedom in the toy model are not introduced by hand, their existence is intimately related to the peculiar choice of representation of the fundamental phase space variables that leads to singularity resolution [86]. Moreover, such representation mimics the one used in the full theory of loop quantum gravity [118] where also one expects such extra residual and microscopic degrees of freedom to exist and remain hidden to low energy coarse grained observers describing physics in terms of an effective QFT.

From a more general perspective we expect this scenario to transcend the framework of loop quantum gravity: in any approach to quantum gravity, where spacetime geometry is emergent [10] from more fundamental discrete degrees of freedom, the effect (precisely illustrated here by our toy model) would generically occur.

These results extrapolated to the context of black hole formation and evaporation suggest a simple resolution of the information paradox that avoids the pathological features of other proposals. For instance the possible development of firewalls [121, 122] or the risks of information cloning that holographic type of scenarios must deal with [123] are completely absent here. As decoherence in our model takes place without diffusion [83], the usual difficulties [124] with energy conservation in the purification process are avoided along the lines of [82, 83], yet in a concrete quantum gravity framework (hence without the problems faced by the QFT approach [125, 126]).

We notice that the possibility of decoherence illustrated in the present model also suggest the possibility of diffusion into the underlying Planckian structure, such diffusion might have, in suitable situations, important consequences at large scales as argued in a series of recent papers [25, 127, 128]. The present model is very simplistic and realizes an example where such diffusion is not possible due to (unimodular) energy conservation and the fact that the microscopic degrees of freedom do not contribute independently to the Hamiltonian. Nevertheless, one could generalize these models easily in order to include diffusion. This possibility is under current investigation and we plan to report the results elsewhere.

[10] For instance in the causal sets approach [53], or in the context of Jacobson's ideas about emergence [54] (where, incidentally, in both cases unimodular gravity is the natural emergent structure), in causal dynamical triangulations [119], group field theory [120], etc.

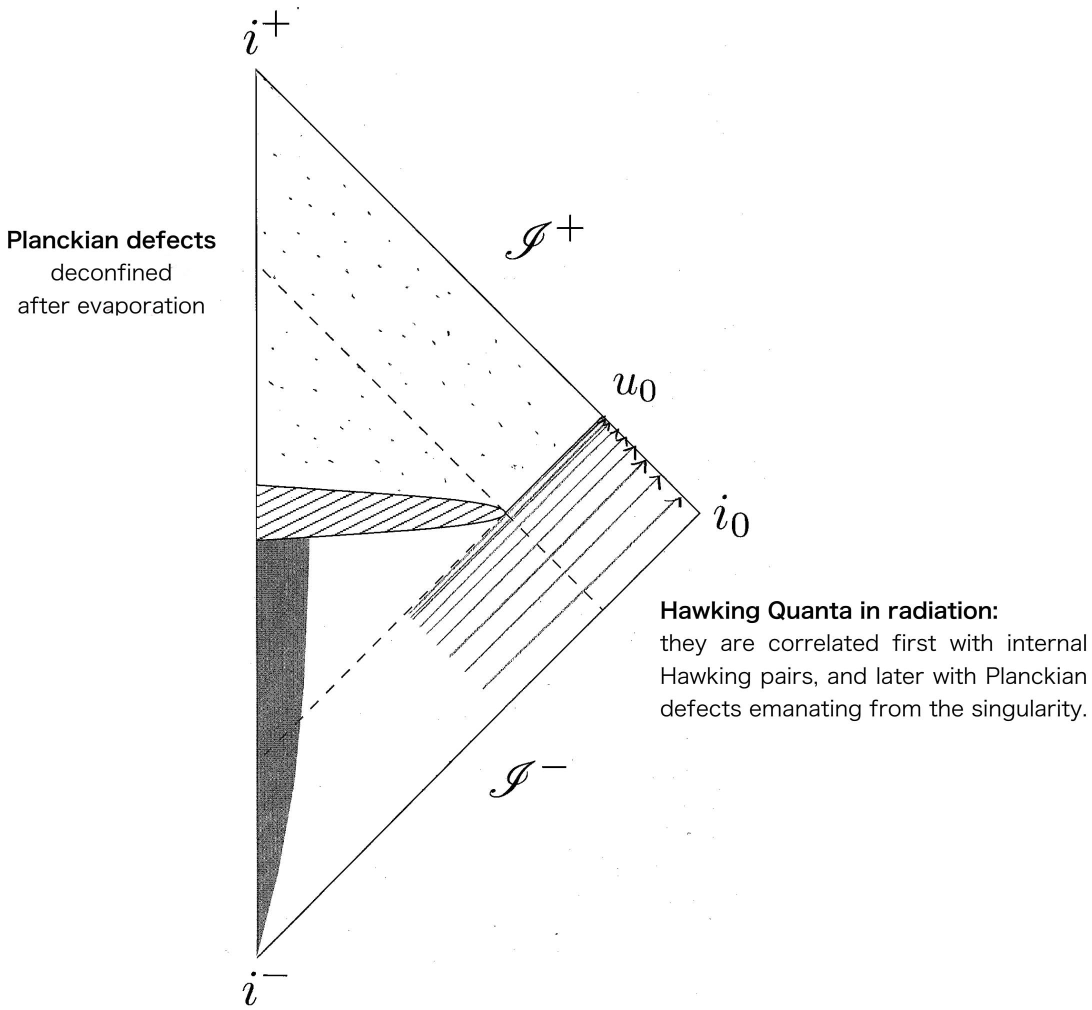

Figure 4.1 – Penrose diagram illustrating (effectively) the natural scenario, suggested by the fundamental features of LQG, for the resolution of the information puzzle in black hole evaporation [15]. The shaded region represents the *would-be-singularity* where high fluctuations in geometry and fields are present and where the low energy degrees of freedom of the Hawking pairs are forced to interact with the fundamental Planckian degrees of freedom.

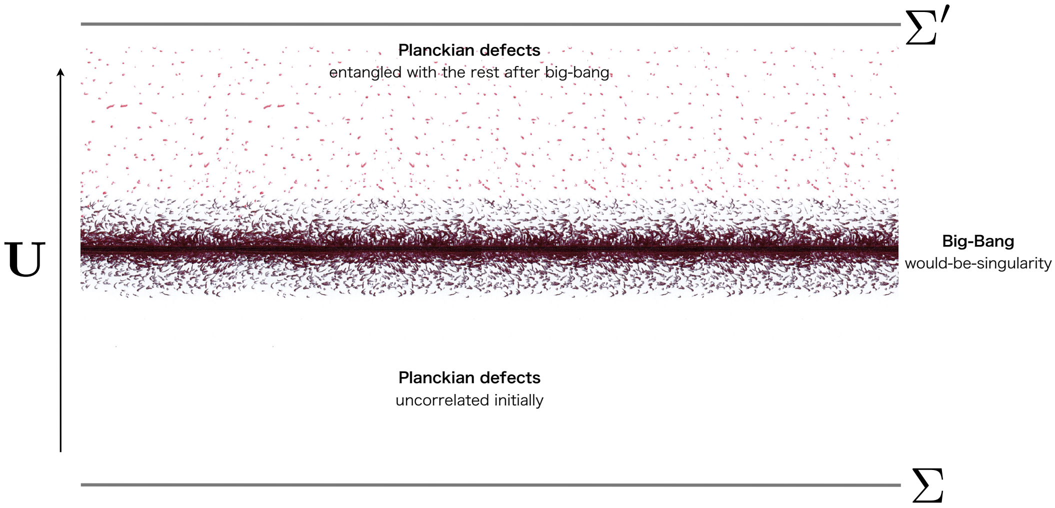

Figure 4.2 – Diagram illustrating (effectively) the natural scenario, suggested by the fundamental features of LQG, for the resolution of the information puzzle in black hole evaporation [15]. As in Figure 4.1, one should keep in mind the limitations of such spacetime representation of a process that is fundamentally quantum and hence only understandable in terms of superpositions of different spacetime geometries.

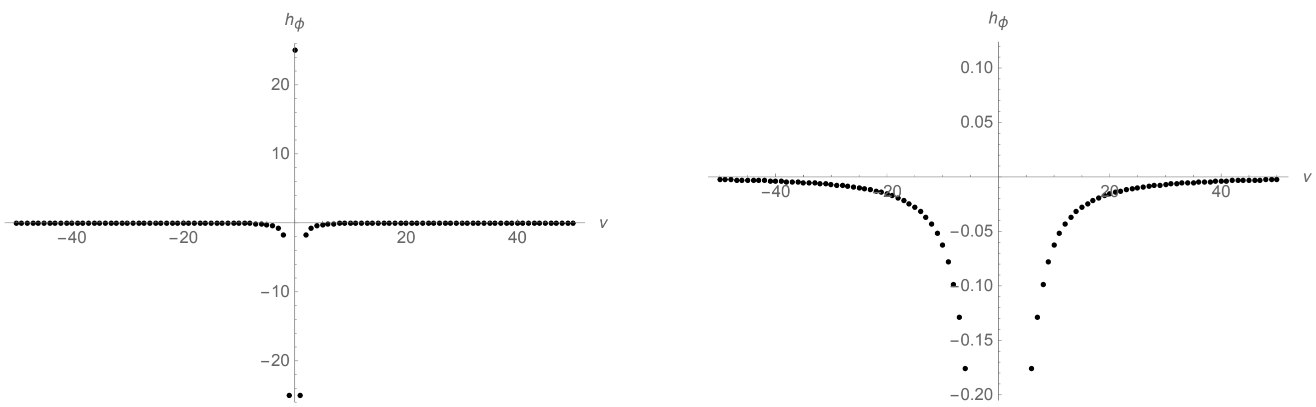

Figure 4.3 – The function $h_\phi(v;\lambda)$ evaluated on an epsilon sector containing $v=0$ for $p_\phi = 10$ in natural units and $\lambda = 1/2$ is plotted using two different ranges. On the left we see that the function is finite near $v=0$. On the right we can see that it behaves like $-v^{-2}$ for large values of v. This fucntion can be seen as the effective potential where an asymptotically free state of the universe (pure gravity with cosmological constant state or asymptotically DeSitter state) scatters. If the cosmological constant is negative there are bound states whose superposition can be used to define semiclassical universes oscillating in an endless series of big-bangs and big-crunches.

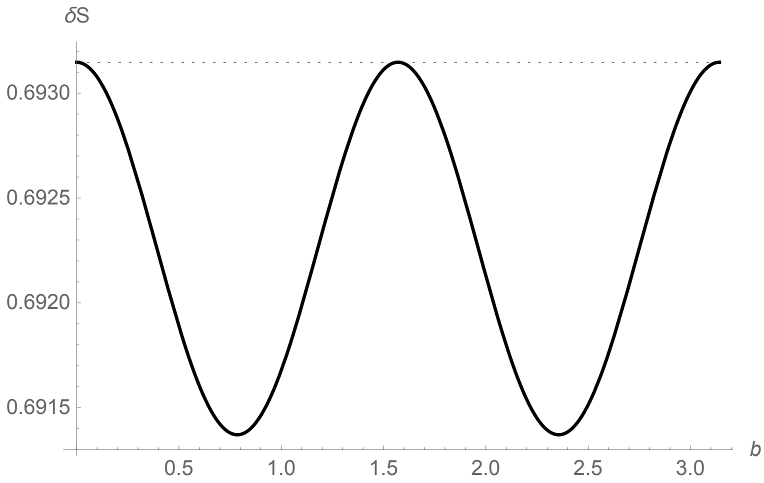

Figure 4.4 – The curve represented by a thin line is the entropy jump δS as a function of b in Planck units for $\gamma = \mu = \Delta = 1$. The small $b\ell_{\mathrm{Pl}}$ behaviour in (4.95) is apparent. The entropy is periodic for $b\ell_{\mathrm{Pl}} \in [0, \pi]$ as expected from (4.88). The dotted line represents the maximum possible entropy which is log[2] in our model.

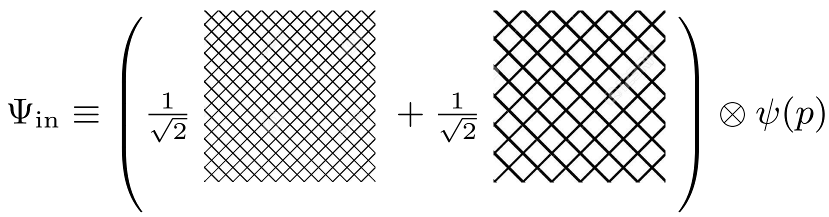

Figure 4.5 – Schematic representation of the state of interest. There are two different UV structures with dynamical implications via the $\bar{\mu}$-scheme. The state represented here has trivial correlations with the microscopic structure and would lead to a zero initial entanglement entropy state as defined by the reduced density matrix where the background state is traced out.

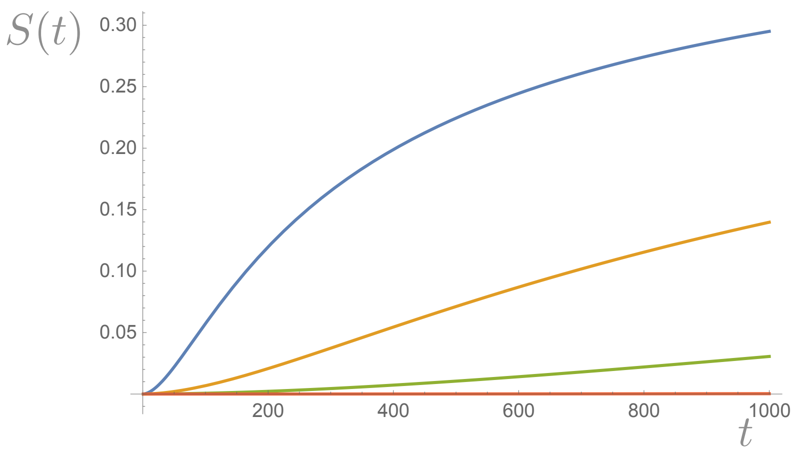

Figure 4.6 – Here we plot $S(t)$ as a function of time for a gaussian wave packet centred at $b = 2.5\ 10^{-2}$, $b = 5\ 10^{-2}$, $b = 7\ 10^{-2}$, and $b = 10^{-1}$ with width $\sigma = b$ respectively. Numerical integration plus the approximation (4.75) was used with the assumption $2(\Delta_+ - \Delta_-)/\gamma^2 = 1$, all in Planck units. As b grows the scalar curvature (the cosmological constant) grows and the rate at which entropy increases grows as well. For $b \ll 1$ an effective unitary evolution is recovered.

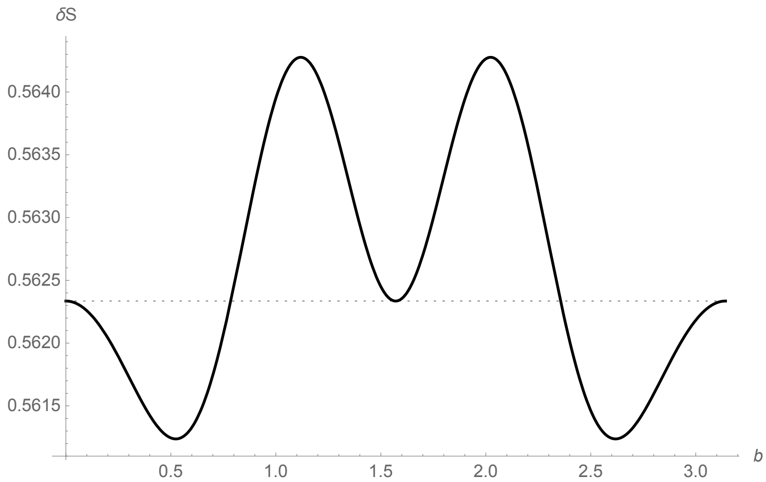

Figure 4.7 – The entropy jump δS as a function of b in Planck units for $\gamma = \mu = \Delta_- = 1$. The small $b\ell_{\mathrm{Pl}}$ behaviour in (4.95) is apparent. The entropy is periodic for $b\ell_{\mathrm{Pl}} \in [0, \pi]$ as expected from (4.88).

Structure Formation Part III

5 Inflation and Structure Formation

The standard Big Bang picture of cosmology (the ΛCDM model. See, for example [129, 130]) explain all the pieces of observational data to this date. Given the right set of parameters and initial data, all the observations at the cosmological scale are explained by the stardard model of cosmology. Despite its great success, the standard Big Bang picture has unsettling issues regarding initial conditions. The observed large-scale homogeneity and flatness seem to require very special, fine-tuned initial conditions at the very early Universe. These problems are usually captured[1] in the so-called **flatness problem** and the **horizon problem**, or its modern version which questions the origin of **superhorizon correlations**[2].

The **flatness problem** problem can be stated, in a rough way, as the question why the spatial curvature of the Universe is so small. In the standard hot big bang picture the curvature parameter Ω_k grows towards the present and, if extrapolating to the past, the initial value of the curvature parameter had to be even smaller. Inflation thus provide a dynamical origin for the vaningshily small initial value of the curvature parameter. To be more precise, in terms of the critical density $\rho_{\text{crit}} = 3M_{\text{Pl}}H^2$ the curvature parameter is given by

$$\Omega_k = \frac{\rho_{\text{crit}} - \rho}{\rho_{\text{crit}}} = \frac{(a_0 H_0)^2}{(aH)^2}\Omega_k^0 \tag{5.1}$$

CMB observations give an upper bound for the curvature parameter today $|\Omega_k^0| < 0.005$[134]. Ignoring the late dar matter domination, we can compute the Hubble

[1]Early works in the inflationary paradigm by Guth [131] were also motivated by the **monopole problem**: certain grand unified theories predict, if the temperature is high enough, the production of heavy, stable magnetic monopoles that have not been observed in nature. If produced during an inflationary phase, the monopole density would dilute by many orders of magnitude thus dissolving the tension with observations.

[2]For an introduction to the paradigm of comological inflation, see [132, 133]

radius during the standard cosmology model (radiation+matter)

$$\Omega_k = \frac{\Omega_k^0}{\Omega_m^0} \frac{a^2}{a + a_{\text{eq}}} \tag{5.2}$$

Using that at matter-radiation equality $a_{\text{eq}} = (1 + z_{\text{eq}})^{-1} = \frac{1}{3400}$ we have that, at *matter-radiation domination*

$$\left|\Omega_k \left(t_{\text{eq}}\right)\right| = \frac{\left|\Omega_k^0\right|}{\Omega_m^0} \frac{a_{\text{eq}}}{2} \lessapprox 10^{-6} \tag{5.3}$$

At early times, the dynamics of the Universe was dominated by radiation and doing a similar calculation we can see that

$$|\Omega_k \left(t_{\text{BBN}}\right)| < 10^{-16} \tag{5.4}$$

at *big bang nucleosynthesis* and

$$|\Omega_k \left(t_{\text{EW}}\right)| < 10^{-30} \tag{5.5}$$

at the *electroweak transition* .

We see that if extrapolated at even earlier times, the curvature parameter is even smaller.

During an inflationary epoch, the Huble radius shrinks, and thus any initial curvature will decrease. Thus, for a sufficiently long inflationary period the flatness problem is solved. In particular, during a pure de Sitter phase the Hubble radius shrinks exponentially in comoving time and thus we have that $\Omega_k \sim e^{-2Ht}$, where H is the (constant) Hubble rate during the de Sitter phase.

The **horizon problem** and the **problem of superhorizon perturbations** can be stated, in a nutshell, as the fact that in the standard hot big bang picture, most of the universe appears to *not* have been in casual contact and thus there is no dynamical mechanism possible that can explain the observed homogeneity and the presence of correlations between patches of the universe that were never, a priori, in causal contact. To put some numbers on this, let us do a quick back-of-the-enveloppe estimate. Let us consider the *comoving particle horizon* at time t[3]:

$$d_h(\eta) = \int_{t_{\rm bb}}^{t} \frac{\mathrm{d}t}{a(t)} = \int_{a_i}^{a} \frac{\mathrm{d}a}{a\dot{a}} = \int_{\ln a_i}^{\ln a} (aH)^{-1}\, \mathrm{d}\ln a \tag{5.6}$$

where $a_i = a(t_{\rm bb}) = 0$ corresponds to the Big Bang singularity. Notice that in the last term of (5.6) appears, via a simple change of variables, the *comoving Hubble radius*, $R_h = (aH)^{-1}$, which we already saw it plays an important role in the flatness problem. Then, we see that the dynamics of the Hubble radius determine the causal structure of the spacetime via (5.6).

In the standard hot big bang picture (and in general for ordinary matter sources) the comoving Hubble radius is a monotonically incresing function of comoving time t— e.g. for a dust-dominated universe $(aH)^{-1} \sim t^{1/3}$— and thus the integral is dominated by the late-time contributions. This already is suggesting the horizon problem, the amount of conformal time between the initial singularity and the CMB release (around $z = 1000$) was much smaller than the present conformal age of the universe. Then the problem becomes evident: how two distant patches of the universe have the same temperature if they didn't have time to enter into causal contact before the expansion of the universe separated them forever? In fact, in the standard ΛCDM model the CMB consist of ~ 40000 causally-disconnected patches. We can make this argument more precise by a quick back-of-the-enveloppe calculation[132, 133]. The value of the comoving horizon at CMB release can be shown to be $d_h\left(\eta_{\rm rec}\right) \approx 265$Mpc (where rec stands for recombination). Comparing this with the comoving distance to the last-scattering surface $d_A\left(\eta_{\rm rec}\right) = 15.1$Gpc we see that

$$\theta = \frac{2 d_h\left(\eta_{\rm rec}\right)}{d_A\left(\eta_{\rm rec}\right)} = 0.036\text{rad} \implies \theta \sim 2.0^\circ \tag{5.7}$$

or in other words, regions separated more than 2.0° in the sky whad to causal contact at the time the CMB was released.

A more nuanced version of the horizon problem is the fact that, even if we accept perfect homogeneity and flatness as a plausible initial state of the universe, the night sky is full of fluctuations that exhibit correlations over, a priori, acausal distances. Figure 5.1 illustrates this problem. Since in the standard cosmological models the Hubble radius $(aH)^{-1}$ is always incresing, any fluctuation of wavelenght λ that we see now inside the Hubble radius was, at early times, outside the Hubble radius. For ordinary matter, the Hubble radius is approximately equal to the particle horizon[136].

[3]For a detailed account of the different notions of horizon appearing in cosmology see [135]

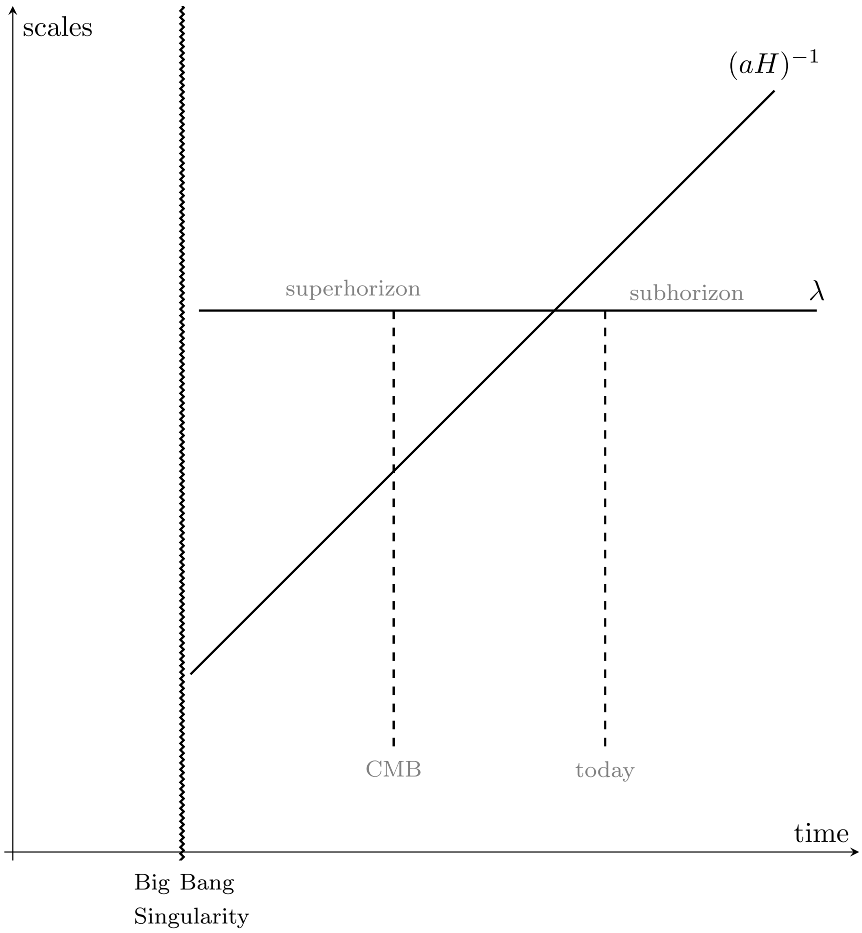

Figure 5.1 – A fluctuation of fixed (comoving) wavelenght λ that is today inside the horizon, at sufficiently early times it can be found outside the horizon in a universe described by the standard model of cosmology.

We saw that the particle horizon at CMB release was ~ 265Mpc, and thus scales larger than this would have been *outside* the horizon before recombination. But nevertheless we observe correlation all over the sky. This adds an extra layer to the horizon problem: not only the CMB is homogeneous at acausal scales, but it exhibits subtle correlations between fluctuations on these scales.

In this case the shrinking Hubble radius also alleviates the superhorizon correlation problem: it provides a mechanism which which puts into causal contact regions of the universe that were, in the standard hot big bang model, apparently separated by superhorizon scales.

In the Figure 5.2 we see how the picture is now modified: if the inflationary phase lasted long enough, the scales that we now observe inside th horizon, at sufficiently early times during the inflationary epoch they were inside the horizon and thus causal processes were able to establish nontrivial correlations.

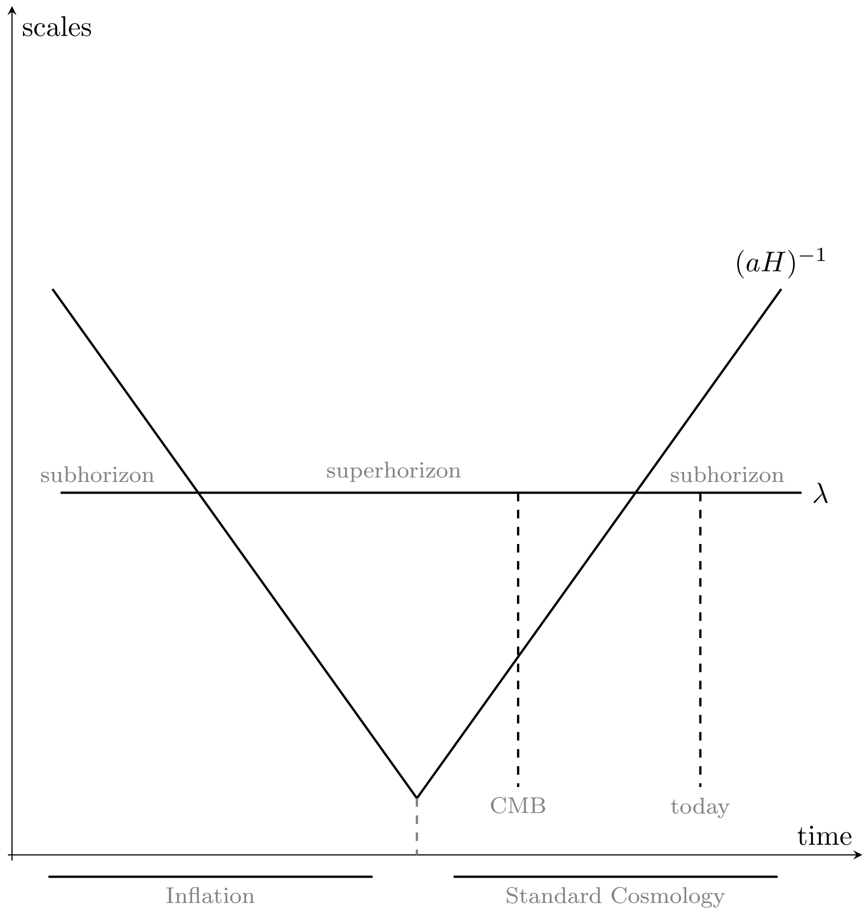

Figure 5.2 – A fluctuation of fixed (comoving) wavelenght λ in a universe that undergoes an inflationary phase at early times. Fluctuations at a scale we observe today inside the Hubble radius were, at sufficiently early times, inside the Hubble radius and exited the horizon during the inflationary epoch.

Is important to note that the apparent problems of the hot big bang model assume that we can extrapolate general relativity into the trans-Planckian regime. For instance, when computing (5.6) or when we compute the conformal time elapsed between the initial singularity and later times we included in the integral early times arbitrarily close to the initial singularity

$$\Delta\eta = \int_0^{\varepsilon} \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})} + \int_{\varepsilon}^{t} \frac{\mathrm{d}\tilde{t}}{a(\tilde{t})}. \tag{5.8}$$

When we derived that the conformal time between the initial singularity and the release of the CMB was finite and small we assumed that the first integral on the LHS of (5.8), which involves regimes in which where general relativity cannot be trusted, does not lead to large contribution to the total conformal time. Notice that this early-times integral involve regimes where even a classical notion of spacetime may be ill-defined. An easy way out of these so-called problems is to postulate that they will be solved in a complete theory of quantum gravity and thus we should not worry about them. Whereas this is a reasonable (and possible correct) approach, an early inflationary phase provides a simple apparent solution to these problems and, moreover, it is interesting to investigate if such an inflationary phase can arise as an effective description of quantum-gravitational phenomena at early times when the classical description breaks down. In fact, such an approach is taken in the following sections, where we show how an inflationary phase fueled by decay of the cosmological constant (thought of as an effective description of a quantum gravity phenomenon) provides not only the solution to the problems we mentioned in this section but also the seeds of structure formation.

Before finishing the section, let me disgress about the exceptional features of the early universe. Although the inflationary paradigm provides a solution to the apparent problems with the standard model of cosmology it *does not* address one of the most tantalizing and conceptually deep issues in physics: how our universe happened to start in an extraordinarily special initial state (*the Big Bang*), which had an extraordinarily low entropy with respect to its gravitational degrees of freedom but maximum entropy in every other respect[51, 137, 138][4]. The CMB give us a surprising feature of the early universe: the radiation coming from the CMB not only is extremely homogeneous but it is the best-measured black-body in nature [139, 140]. From the side of the matter degrees of freedom, this is telling us that the CMB is in *thermal equilibrium*. In other words, the matter degrees of freedom are in state of *maximum entropy*[5]. In the gravitational degrees of freedom the situation is the inverse. The gravitational influence at this time was extremely low[6] due to the uniformity in the matter distribution. This provided an enormous potential for entropy growth when the gravitational influence starts playing a role.

The model we present in the next section, while does not address these issues (nor

[4]The extremely low entropy of the big bang, its possible dynamical explanation and its consequences are explored—among other things—in a marvelous way in [138]

[5]When we intrepret entropy as arising from a (macroscopic) coarse-graning of microstates[26, 137, 51], ‘thermal equilibrium’ corresponds to the largest—by far—collection of microstates.

[6]This initial supression of gravitational degrees of freedom is captured in the so called *Weyl conjecture*, which roughly proposes that the Weyl curvature C_{abcd} vanishes at any initial singularity[141, 142, 143].

does inflation) it does attempts to incorporate quantum-gravitational features of the early universe in an effective manner. These quantum-graviational effects fuel a inflationary phase at early times and provide a mechanism of structure formation that avoid the pitfalls of the standard treatment[47].

6 Inflation from the relaxation of the cosmological constant

This chapter overlaps with [18].

We propose a model of inflation driven by the relaxation of an initially Planckian cosmological constant due to diffusion. The model generates a (approximately) scale invariant spectrum of (adiabatic) primordial perturbations with the correct amplitudes and red tilt without an inflaton. The inhomogeneities observable in the CMB arise from those associated to the fundamental Planckian granularity that are imprinted into the standard model Higgs scalar fluctuations during the inflationary phase. The process admits a semiclassical interpretation and avoids the trans-Planckian problem of standard inflationary scenarios based on the role of vacuum fluctuations. The deviations from scale invariance observed in the CMB are controlled by the self coupling constant of the Higgs scalar the standard model of particle physics. The thermal production of primordial black holes can produce the amount of cold dark matter required by observations. Remarkably, for natural initial conditions set at the Planck scale the amplitude and tilt of the power spectrum of perturbations observed at the CMB depend only on known parameters of the standard model such as the self coupling of the Higgs scalar and its mass.

6.1 Introduction

Planck mass square, M_{Pl}^2, is the natural order of magnitude of the cosmological constant, yet its observed value is about 10^{-120} times that theoretical expectation. Such huge discrepancy, referred to as the cosmological constant problem, is perhaps the most severe hierarchy problem of modern physics. The cosmological constant problem is often separated into two (possibly independent) questions: first why is the cosmological constant essentially vanishing, and second why does it have that special value. A related natural question that could connect the two is whether dark energy (the cosmological 'constant') could actually change during the evolution of the universe. Namely, would it be possible to start with a large cosmological constant that dynamically evolves to its present value? If one writes the energy momentum tensor of regular matter and includes the dark energy component as $\mathbf{T}^{\text{TOTAL}}_{ab} = \mathbf{T}_{ab} + g_{ab}\Lambda/(8\pi G)$ then one has that Einstein's equation imply that

$$\nabla_b \Lambda = -(8\pi G)\nabla^a \mathbf{T}_{ab}. \tag{6.1}$$

In other words, the only possibility (compatible with general relativity) of having Λ change is the existence of diffusion of energy between the standard matter fields and dark energy. One could postulate such diffusion at a purely phenomenological level (for different proposals along these lines see [144] and references therein). However, being dark energy a property associated with the gravitational properties of vacuum-spacetime it is appealing to search for a more fundamental description that would presumably involve quantum gravity.

In this respect, it is important to point out that equation (6.1) arises naturally in the context of unimodular gravity in a way that, we believe, has some additional conceptual value in view of the previous discussion. Thus let us explore it in some detail as the perpective it suggests will strongly motivate the model that we introduce below. The action of unimodular gravity is

$$S = \int \left(\sqrt{g}\,\mathbf{R} + \lambda\left[\sqrt{g} - v^{(4)}\right]\right) dx^4 + S_m, \tag{6.2}$$

where S_m denotes the action of matter fields,

$$\mathbf{v}^{(4)} \equiv v^{(4)} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \tag{6.3}$$

is a background four-volume form, and λ is a Lagrange multiplier imposing that the metric volume density equals the background one. Due to the presence of the four-volume background structure the diffeomorphism symmetry of general relativity is broken down to volume-preserving diffeomorphisms, whose generators are represented by the vector fields ξ^a with vanishing expansion, namely

$$\nabla_a \xi^a = 0\,. \tag{6.4}$$

Such infinitesimal generators of volume preserving diffeomorphism are completely characterized by arbitrary 2-foms ω_{ab} via the relation $\xi^a = \epsilon^{abdc}\nabla_b \omega_{cd}$.

Invariance of the matter action under the thus reduced symmetry group relaxes the usual constraints on the divergence of the energy momentum tensor. Recall that full diffeomorphism invariance of the matter action implies energy momentum conservation (see for instance [145]). Therefore, in order to find the new constraints on energy conservation one must set to zero the variation of the matter action under volume preserving diffeomorphisms under the assumption that the matter field equations hold. Namely, the new condition reads

$$\begin{aligned} 0 &= \delta S_m = \int_M \sqrt{-g}\mathbf{T}_{ab}\nabla^a\xi^b dx^4 \\ &= -\int_M \sqrt{-g}\nabla^a\mathbf{T}_{ab}\xi^b dx^4 = \int_M \sqrt{-g}\nabla_c(\nabla^a\mathbf{T}_{ab}\epsilon^{bcde})\omega_{de}dx^4 \end{aligned} \tag{6.5}$$

where we integrated by parts twice and we have assumed that fields vanish at infinity. If we define

$$\mathbf{J}_b \equiv (8\pi G)\nabla^a\mathbf{T}_{ab}, \tag{6.6}$$

the previous condition, which should be valid for arbitrary ω_{ab}, implies

$$d\mathbf{J} = 0, \tag{6.7}$$

or locally

$$\mathbf{J}_a = \nabla_a Q. \tag{6.8}$$

Therefore, the background volume structure—that partially breaks diffeomorphims (down to volume preserving ones)—allows for violations of energy momentum conservation demanding only that the energy-momentum violation current $\mathbf{J}_b$ be closed. The gravitational field equations that follow from the previous action are simply the trace-free part of Einstein's equations, namely

$$\mathbf{R}_{ab} - \frac{1}{4}\mathbf{R}g_{ab} = 8\pi G\left(\mathbf{T}_{ab} - \frac{1}{4}\mathbf{T}g_{ab}\right), \tag{6.9}$$

which, using the integrability condition (6.7) and the Bianchi identities, can be rewritten as [25]

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R}g_{ab} + \underbrace{\left[\Lambda_0 + \int_\ell J\right]}_{\Lambda} g_{ab} = 8\pi G\mathbf{T}_{ab}, \tag{6.10}$$

where Λ_0 is a constant of integration, and ℓ is a one-dimensional path from some reference event to the spacetime point where the equation is evaluated. Thus, if not vanishing, the energy-violation current $\mathbf{J}$ is the source of a term in Einstein's equations satisfying the dark energy equation of state; while if $\mathbf{J} = 0$ we simply recover the field equations of general relativity with a cosmological constant Λ_0 (a property aready pointed out by Einstein [97] as indicating the possibly non-fundamental nature of the cosmological constant). In relation to this, another very appealing feature of unimodular gravity is that quantum field theoretic vacuum energy does not gravitate [98, 146], for vacuum fluctuations only contribute to the trace part of $\mathbf{T}_{ab}$ not entering into the field equations (6.9). Finally, and highly remarkably, aside from new physics in the dark matter sector, unimodular gravity is completely equivalent to general relativity [147], and passes all the known tests of Einstein's theory. Unimodular gravity is, therefore, a very conservative modification of general relativity.

What is the role of the background four volume structure? Why should one accept such weakening of the principle of general covariance (breaking diffeomorphism down to volume-preserving diffeomorphisms)? We are guided on this issue by the perspective that the smooth classical field description of general relativity and quantum field theory is an approximation (an effective description) of a fundamental physics expected to be discrete at the Planck scale. Compatibility with Lorentz symmetry suggests that such discreteness would have to be realized by the existence of some sort of four-volume elementary building blocks. These basic spacetime elements would naturally produce a background 4-volume structure in the long wave-length effective description and justify the use of unimodular gravity for low energies. At the more fundamental level (i.e., in terms of the quantum gravity physics describing the dynamics of such elementary notions) no background structures should be preferred, à priori, and full covariance would be reestablished.

Indications of this physical hypothesis come from different indirect sources that we now mention.

First, let us come back to the discussion of the symmetries of unimodular gravity and recall that under general diffemorphisms the metric changes as $\delta g_{ab} = 2\nabla_{(a}\xi_{b)}$ where $\nabla_{(a}\xi_{b)} = \frac{\theta}{4} g_{ab} + \sigma_{ab}$ when decomposed in its trace and trace-free parts. Unimodular gravity remains invariant under the smaller group of volume-preserving diffeomorphisms which are characterized infinitesimally by vector fields ξ^a for which $\theta = 0$. Thus, the broken diffeomorphisms in unimodular gravity are those that send the metric $g_{ab} \to (1+\frac{\theta}{4})g_{ab}$ which coincide with infinitesimal conformal transformations $g_{ab} \to \Omega^2 g_{ab}$ as far as the metric is concerned. Therefore, when the field equations hold, conformal transformations and the broken symmetries of unimodular gravity are the same in the matter sector. Thus, one would expect unimodular gravity to emerge as the natural effective description of gravity in situations where scale invariance is broken by the microscopic discreteness scale associated to quantum gravity scale and those of the fundamental probing matter [1].

[1] There is a remarkable paper by Anderson and Finkelstein [52] were a very similar conceptual path leads to unimodular gravity from the assumption of the existence of a fundamental scale breaking

This is precisely what the structure of quantum field theory on curved spacetimes suggests in the way the UV (potentially divergent) contributions to the renormalization of the energy-momentum tensor break scale invariance: consideration of the ambiguities associated with the definition of the expectation value of the energy momentum tensor in quantum field theory and their (anomalous) breaking of scale invariance [66] can be argued to indicate the preferred role of unimodular gravity in semiclassical considerations.

The previous discussion based on purely symmetry considerations can be made very concrete. Still in the context of the renormalization of the (expectation value of the) energy momentum tensor in quantum field theory on curved spacetimes, the existence of a well defined regularization can be shown via the Hadamard subtraction prescription where (for the simple case of a Klein-Gordon field $\phi(x)$) one defines $\langle T_{ab}\rangle$ by considering the coincidence limit $x \to y$ of a suitable expression depending on the two-point distribution

$$F(x,y) = \langle \phi(x)\phi(y)\rangle - H(x,y), \tag{6.11}$$

where $H(x,y)$ is a Hadamard bi-distribution constructed such that $F(x,y)$ is smooth in the coincidence limit, and such that it satisfies the field equations in its first argument. Obstructions to get $H(x,y)$ to satisfy the field equations in the second argument imply that

$$\nabla^a \langle T_{ab}(x)\rangle = \nabla_b Q \tag{6.12}$$

for Q dependent on the local curvature but not on the state of the quantum field. Therefore, the simple regularization of the UV divergences leads to the violation of energy momentum conservation of the form compatible with the symmetries of unimodular gravity (6.8).

In order to make the formalism compatible with the usual semiclassical equations one performs an additional step 'by hand' [66] and defines

$$\langle T_{ab}(x)\rangle_{\text{ren}} \equiv \langle T_{ab}(x)\rangle - Q\, g_{ab}, \tag{6.13}$$

which, in the case of conformal quantum fields, introduces an anomalous trace (see for instance [148, 66], and [149] for a very detailed presentation in 2-dimensions). Our previous discussion shows that this anomaly is more naturally interpreted as a violation of energy-momentum conservation (6.6) satisfying the unimodular restriction

conformal invariance. Although in their analysis they do not discuss the possibility of diffusion that is one of the key features of our approach. We were not aware of this paper and thank T. Jacobson for pointing it out to us.

(6.7).

Interestingly, the semiclassical gravity dynamics defined using the trace free Einsteins equations of unimodular gravity with sources given by $\langle T_{ab}\rangle$ (violating conservation as in (6.12)), and the one implied by the standard Einstein's equations with sources defined by (6.13) coincide. In this sense, the trace anomaly is equivalent to a diffeomorphism anomaly where diffeomorphisms are broken—by QFT vacuum fluctuations—down to volume preserving diffeomorphism. Quantum fields and their fluctuations around a preferred ('vacuum') state are sensitive, in this sense, to an underlying four-volume structure. Finally it is worth pointing out that—even though there are ambiguities in the definition of $\langle T_{ab}\rangle$ encoded in the possibility of adding a locally conserved tensor t_{ab} constructed from the metric variation of a Lagrangian constructed out of R^2 and $R_{ab}R^{ab}$ [66]—the relevant 'diffusion' term $\nabla_b Q$ is, to our knowledge, unambiguously defined in the present context. All this strengthen the view that unimodular gravity—with the non trivial diffusion (6.8) effects that it offers—is a natural effective description emergent from the underlying UV structure of spacetime and matter expected to be described by quantum gravity [2].

Discreteness at the Planck scale (or, more precisely, the existence of microscopic degrees of freedom not accounted for in an effective field theory description) is suggested also by the physics of black holes in the semiclassical regime [68]. Black holes behave like thermodynamical systems in quasi-thermal equilibrium with an entropy given by

$$S_{BH} = \frac{A}{4\ell_{\rm Pl}^2}, \tag{6.14}$$

where A is the corresponding black hole horizon area. This formula suggests the existence of microscopic degrees of freedom at the Planck scale, $\ell_{\rm Pl}$, responsible for such huge entropy. Arguments that take these microstates as fundamental and derive from them an effective description of gravity (as an equation of state [54]) lead—not to Einstein's equations as it is often improperly stated, but rather—to the trace free Einstein's equation (6.9) of unimodular gravity.

Unimodular gravity also arises naturally from quantum gravity approaches where spacetime is emergent from 4-dimensional discrete building blocks (which are responsible for the existence of a preferred background four-volume (6.3)). A concrete example of this is the role of unimodular gravity as the effective description of gravity in the causal set approach [53]. Noisy interaction with four volume events appears

[2]The possibility of a relaxation mechanism of a positive cosmological constant via the back reaction of infrared graviton modes (IR effects) was put forward by Tsamis and Woodard in [150] and further explored in the case of scalar contributions by Brandenberger [151].

as the natural relativistic generalization of spontaneous localization models [152] that modify quantum mechanics by introducing dynamical collapse [153, 154]. This perspective was very important in motivating the use of unimodular gravity in [25] where observational bounds on the free parameters of some of such models where constrained by cosmological observations. It is possible that these, apparently independent directions, could be connected at a more fundamental level. We will not pursue this idea here, for further reading and applications to cosmology see [155] and references therein.

The existence of microscopic degrees of freedom which are not captured in our smooth field theoretic approximations conveys the idea that diffusive effects could be present which, in unimodular gravity, can be accounted for phenomenologically in terms of a non vanishing current $\mathbf{J}_b$ (as long as (6.7) are satisfied) [3]. This perspective, which is the one we follow in this work, was already taken in [128, 127] where (with the assumption that the initial cosmological constant Λ_0 in (6.10) is vanishing in the early universe) a cosmological constant emerges from the noisy diffusion of energy from the low energy matter sector into the Planckian regime during the electroweak transition. Remarkably, the model reproduces the observed value of the cosmological constant today without fine tuning [4].

6.1.1 *Background implications:* Relaxation of the cosmological constant

Building on this, here we explore the possibility that the perspective offered by unimodular gravity (as an effective description emerging from fundamental discreteness) could help addressing the first part of the cosmological constant problem. We would like to investigate the cosmological implications of having an initial cosmological constant that starts with its natural Planckian value $\Lambda_0 \approx M_{\text{Pl}}^2$, and then relaxes to zero via diffusion into the matter sector mediated by the hypothetical granular structure at

[3] If these hidden degrees of freedom can interact with the low energy ones appearing in our effective field theory formulations then quantum correlations can be established via such interactions. This is particularly relevant in the context of black hole formation and evaporation where low energy excitations falling into the black hole are forced by the gravitational field to interact with the Planck scale a finite proper time after horizon crossing (as implied by the singularity theorems). This is particularly important in any discussion of the fate of information in black hole evaporation and offers a natural channel for purification of the Hawking radiation [15] (for a toy models in quantum cosmology illustrating the mechanism see [26, 27]).

[4] The model links the two mysteriously small scales in fundamental physics—the EW scale m_{ew} and the cosmological constant—with the gravity scale M_{Pl}: the small number $(m_{\text{ew}}/M_{\text{Pl}})^7 \approx 10^{-120}$ emerges from the calculation as a result of the diffusive physics involved [128, 127]. The results of the present paper reinforces the relationship between dark energy physics and electroweak physics due to the key role that the Higgs scalar will play in what follows.

the Planck scale associated with the emergence of the preferred four volume structure of unimodular gravity at low energies.

The analogy with usual dissipative systems suggests a natural model where Λ relaxes exponentially in time. Even if time is an elusive notion in general relativity, when it comes to applying the theory to cosmology the situation is drastically different in unimodular gravity (for a more general discussion see [99, 101]). This is so thanks to the existence of the preferred 4-volume structure that singles out a preferred (up to rescaling by a constant) notion of time: 4-volume time. Such time variable can be put in direct correspondence with a dimensionless notion associated with the counting of elementary Planckian volume elements 'created' during the cosmological evolution. All this provides a natural time notion emerging from the hypothesis of discreteness in terms of which the relaxation of Λ will be defined.

To make the previous statement precise we now focus our attention to (spatially flat) Friedmann-Lemaître-Robinson-Walker (FLRW) cosmology (homogeneous and isotropic cosmology). The assumption of spacial flatness simplifies the discussion that follows yet it is probably not essential. Thus the spacetime metric is given by

$$ds^2 = -d\tau^2 + a^2(\tau)d\vec{x}^2, \tag{6.15}$$

where τ is the proper time of co-moving observers. The rationale dictating that the diffusion is sourced by the 4-volumetric granularity of spacetime suggests the natural time for the diffusion process (and associated relaxation of Λ) to be proportional to the number of spacetime grains encountered or 'created' during the evolution of the universe identified with the elapsed four volume. More precisely, consider an initial fiducial cell of co-moving coordinate volume $\ell_{\rm Pl}^3$ expanding while the universe expands. The four volume of its world tube—divided by a reference volume scale $\ell_{\rm U}^3$ in order to get time units—is given by

$$t_p = \frac{\ell_{\rm Pl}^3}{\ell_{\rm U}^3}\int a^3 d\tau. \tag{6.16}$$

If we also take $\ell_{\rm U} = \ell_{\rm Pl}$ one gets the so-called unimodular time variable t defined as

$$dt = a^3 d\tau, \tag{6.17}$$

which turns the metric (6.15) into $ds^2 = -a^{-6}dt^2 + a^2 d\vec{x}^2$. This time choice is imposed to us in unimodular gravity by the constraint $\det|g| = 1$ derived from the variations of the action (6.2) with respect to the Lagrange multiplier λ in natural coordinates where $v^{(4)} = 1$.

The question we explore in this paper is what is the natural phenomenology that follows from the assumption that Λ decays exponentially in this (number of Planck four volume elements) time, thus we postulate that

$$\Lambda(t) = \Lambda_0 \exp(-\beta M_{\rm Pl} t) \tag{6.18}$$

with β a dimensionless constant and $\Lambda_0 \sim M_{\rm Pl}^2$. Note that $n_p \equiv M_{\rm Pl} t$ in the previous expression can be interpreted as the number of Planckian 4-volume elements 'created' during the cosmological expansion out of the primordial initial cell. Note also that the time variable as defined in (6.16) is not unique as can be modified by rescaling $\ell_{\rm Pl} \to \lambda \ell_{\rm Pl}$. The phenomenology of this paper remains the same if simultaneously we rescale $\beta \to \beta/\lambda^3$ in (6.18). This freedom can be encoded in the choice of $\ell_{\rm U}$ in (6.16). We will see that the parameter β will basically control the number of e-folds of inflation before reheating. The only requirement we will find, when comparing predictions of the model with observations, is that β has to be sufficiently small. However, its precise value does not affect the type of observable features we explore in the model. A possibility of identifying a fundamental mechanism fixing this freedom and, simultaneously, rendering the value of β more natural will be discussed in Section 6.6.

Note that (6.18) implies, due to the non trivial relation between the (four volume) time t and co-moving time τ encoded in equation (6.17), that the universe undergoes a phase of exponential expansion in cosmic time τ lasting as long as $\beta M_{\rm Pl} t < 1$ (a quasi De Sitter inflationary phase). For sufficiently small β this inflationary phase can be long enough to resolve both the horizon and the flatness problems independently of the initial conditions[5] for matter fields and the energy injection encoded in equation (6.1). We will discuss this in more detail in Section 6.2.

6.1.2 *Perturbation implications:* Inhomogeneities sourced by Planckian granularity

The conceptual framework of unimodular gravity naturally suggests the possibility for a form of diffusion between the matter degrees of freedom and the dark energy sector (representing an evolving cosmological constant). The rational behing all this is the

[5]The independence of initial conditions should be taken with the same grain of salt as when one reads similar statements in the inflationary literature. More precisely, one can only make a statement of this sort once one assumes that the FLRW approximation is a good one to describe the observable universe. This is clearly a severe restriction of the phase space of general relativity as it is often emphazized by Penrose [137], and of course a very important problem that we will leave aside of the present discussion.

existence of hidden Planckian degrees of freedom which, in the effective low energy description of unimodular gravity, are capable of storing energy in the form of dark energy to be eventually released into the degrees of freedom of matter (a mechanism driven by quantum gravity and here assumed as a phenomenological hypothesis to lead to the relaxation of Λ as in (6.18)). However, as this dark energy is freed by Plankian grains of spacetime, we can envisage the possibility that inhomogeneities would arise in the matter sector at around the fundamental scale which, during the inflationary period, is close (as shown in Section 6.2) to the Hubble rate. At present one cannot describe this process from fundamental principles. Thus we will represent by a Brownian type of process, i.e a stochastic process generating small perturbations of certain background fields with a probability distribution satisfying the only requirement of homogeneity.

This view offers an interesting possibility for a mechanism of structure formation where the nearly scale invariant scalar density fluctuations observed in the CMB will be shown to arise from the steady injection of energy from the Brownian-like diffusion of fundamental Planckian granularity into the perturbations at the Hubble scale during the De Sitter phase. We will see that the semiclassical description of such diffusion leads to stochastic inhomogeneities compatible with cosmological observations. Scale invariance follows from the self similarity of the diffusion process that is granted by the exponential expansion of the background during the De Sitter phase (due to the slow relaxation of Λ as in (6.18)).

Thus the mechanism producing inhomogeneities presented here is fundamentally different from the standard account that associates inhomogeneities to quantum fluctuations of the inflaton. Here we propose an active mechanism where the fundamental quantum granularity induces semiclassical inhomogeneities in the mean field value of the Higgs scalar. Why the Higgs scalar instead of any other field in the standard model of particle physics (which we assume to be valid up to close to the Planck scale)? To answer this question first note that inhomogeneities are expected to be intrinsically present at the Planck scale according to several approaches to the fundamental theory. However, compatibility with Lorentz invariance implies that such hypothetical granularity cannot be seen as an underlying lattice-like structure selecting a preferred frame [156]. Instead, discreteness at the Planck scale must have physical manifestations when suitable massive (hence scale invariance breaking) degrees of freedom interact with the quantum geometry (massless fields cannot be sensitive to granularity as their light-like excitations cannot define a frame, their own rest frame, with respect to which the notion of Planck scale would be meaningful). At high enough energies, the only scale invariant breaking degree of freedom in the standard model of particle physics is the Higgs scalar and this is the reason why the

Higgs is the right degree of freedom that can carry the imprints of granularity. A natural order parameter of the magnitude of the strength of this effect is naturally given by

$$\gamma_{\rm H} = \frac{m_{\rm H}}{M_{\rm Pl}} \approx 10^{-17}, \tag{6.19}$$

where $m_{\rm H}$ is the Higgs mass.

Thus in our model the fundamental inhomogeneities leave their imprint on the expectation value of the Higgs scalar (assume to have, as the cosmological constant, Planckian initial value): as a consequence the Higgs scalar is not in a homogeneous and isotropic vacuum state but rather in an inhomogeneous excited semiclassical state. The De Sitter exponential expansion during the inflationary phase dilutes standard forms of matter; however, this is not the case for the zero mode of a scalar field and the inhomogeneities produced on it (long wavelength modes in the scalar field are frozen by the rapid expansion). During the inflationary phase the UV Planckian inhomogeneities are expanded to the large cosmological scales where they become the seed for the formation of structure observable in the power spectrum of perturbations on the CMB. There is no symmetry breaking of the FLRW symmetries, no need for quantum to classical transition, inhomogeneities are present from the beginning in the microscopic quantum gravitational structure of spacetime and matter. The decaying cosmological constant and its inflationary effect, bring these up to our scales.

In our view, at the conceptual level, the new perspective is an improvement of the standard picture in two ways: On the one hand it resolves the so-called trans-Planckian problem because no assumption about the validity of standard quantum field theory as well as linearized gravity are necessary at length scales below the Planck scale are necessary (perturbations treated with such tools are born here at longer scales). On the other hand our approach eliminates the conceptual difficulties [47] associated with thinking of the perturbations as originating in vacuum fluctuations of the inflaton in relation to the measurement problem in quantum mechanics and applications of its Copenhagen interpretation applied to the universe as a whole.

Finally we study the possibility that primordial black holes could be created thermally at the end of the inflationary era during the reheating phase that in our model raises the temperature to close to the Planck temperature. A key assumption here is that there are stable primordial black holes with masses close to the Planck mass. Note that even when this is suggested by general quantum gravity considerations in various contexts, it is a very natural possibility in a quantum gravity theory where the Planck energy is the fundamental scale. We show that one can natural estimates based on dimensional analysis lead to the correct order of magnitude densities necessary to account for dark matter today without fine tuning. We explain this in detail in Section 6.4.

The paper is organized as follows. In the Section 6.2 we describe the dynamics of the background geometry driven by the relaxing cosmological constant (6.18). In Section 6.3 we present the proposed mechanism for the generation of nearly scale invariant scalar density fluctuations. We confront the predictions of the minimalistic model (that assumes the validity of the standard model of particle physics all the way to the Planck scale) with the relevant observational data coming from the CMB. In section 6.4 we analyze the possibility that primordial black holes (generated during the diffusion process or via thermal fluctuations at reheating) could account for the dark energy content of the universe. We conclude the paper with a discussion Section 6.7. Appendix A contains a proof of the so-called Weinberg theorem showing the existence of adiabatic solutions of the perturbation equations. This theorem is key in understanding the link between perturbations generated during inflation and the CMB observations. In our context the theorem is a handy shortcut specially adapted to the dynamical description of the relevant consequences our stochastic process for the generation of inhomogeneities equivalent of the (more generally used) Mukhanov-Sasaki formalism in the description of standard inflationary theory of perturbations. We believe that our proof of the Weinberg theorem (even when the same in spirit as the one found in [157] or in his well known textbook [129]) is more direct and could be helpful for interested readers. In Section 6.5 we compare our mechanism for the generation of inhomogeneities with the standard paradigm. Some of the various issues opened by our perspective are considered in Section 6.6.

6.2 Background dynamics

In this section we study the dynamics of the homogeneous and isotropic FLRW geometry (6.15) and homogeneous and isotropic matter components evolving on it. The primordial cosmological constant (or dark energy component) relaxes according to (6.18) and, we assume, that the energy released feeds (as implied by equation (6.1)) a radiation component—represented by a homogeneous and isotropic perfect fluid with equation of state $\rho = 3P$—whose initial value is ρ_0. Equation (6.1) will take the form of a continuity equation with non trivial interactions between the radiation and dark energy fluid components. Naturalness of initial conditions at the Planckian regime suggests $\rho_0 \sim M_{\rm Pl}^4$. In addition we have the Higgs scalar field that is assumed to start off in a semiclassical state with expectation value $\phi_0(0) \sim M_{\rm Pl}$. However, the Higgs in such high energy initial state decays into particles of the standard model producing further interaction terms in the continuity equation (now between the

Higgs energy-momentum tensor and the radiation). We will see that these interactions are weak in the regime of interest and that a semiclassical description is available. Thus, in spite of the apparent complexity of the situation one can actually use analytic methods to get a quantitative picture of the relevant features of the dynamics of the background fields which fits well the numerical simulations (whose results we report in Figure 6.1). We show in this section that, initially, the dynamics is dominated by the decaying cosmological constant—in a way that is independent of the other matter components and their initial conditions—producing an inflationary era of the De Sitter type that can last a sufficient number of e-folds to resolve the standard problems that the standard inflationary models resolve [158]. The e-folds of inflation are controlled by the parameter β, CMB observations require this number to be larger than a minimum value but they do not constrain it otherwise. Thus the free parameter β is degenerate in this sense.

6.2.1 Quasi De Sitter phase from the relaxing Λ

We assume that the matter content of the universe is well represented by a perfect fluid,

$$T_{ab} = \rho u_a u_b + P(g_{ab} + u_a u_b), \tag{6.20}$$

where u_a is the 4-velocity of co-moving observers, and ρ and P are the energy density and pressure in the co-moving frame. In terms of 4-volume (unimodular) time t (see equation (6.17)) the Friemann equations become

$$a^4 (a')^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda(t)}{3}, \tag{6.21}$$

where $'$ denotes derivatives with respect to unimodular time t, and $\Lambda(t) = \Lambda_0 e^{-\beta M_{\rm Pl} t}$ is the decaying cosmological constant depending on the free parameter β, and from now on we normalize the scale factor so that $a(0) = 1$. The Raychaudhuri equation is

$$a^2 \frac{d}{dt}\left(a^3 a'\right) = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda(t)}{3}, \tag{6.22}$$

and the continuity equation derived from (6.1) is

$$\rho' + 3\frac{a'}{a}(\rho + P) = -\frac{\Lambda'(t)}{8\pi G}. \tag{6.23}$$

Note that equation (6.23) encodes the diffusion of energy between the dark sector and the energy density of matter (this is the symmetry reduced form of (6.1)). The only assumption in the previous equation is that the diffusion process does not disrupt the homogeneity and isotropy[6] of the background matter and geometry configurations to leading order (perturbations will be considered but they will be small in comparison with average densities). As stated before, we assume that the relevant channel into which Λ decays is massless fields so that $\rho_{\text{rad}} = 3P_{\text{rad}}$ (this justifies the subindex 'rad' in our notation, ρ and P denoting the total energy density and pressure that will have contributions coming from the Higgs scalar).

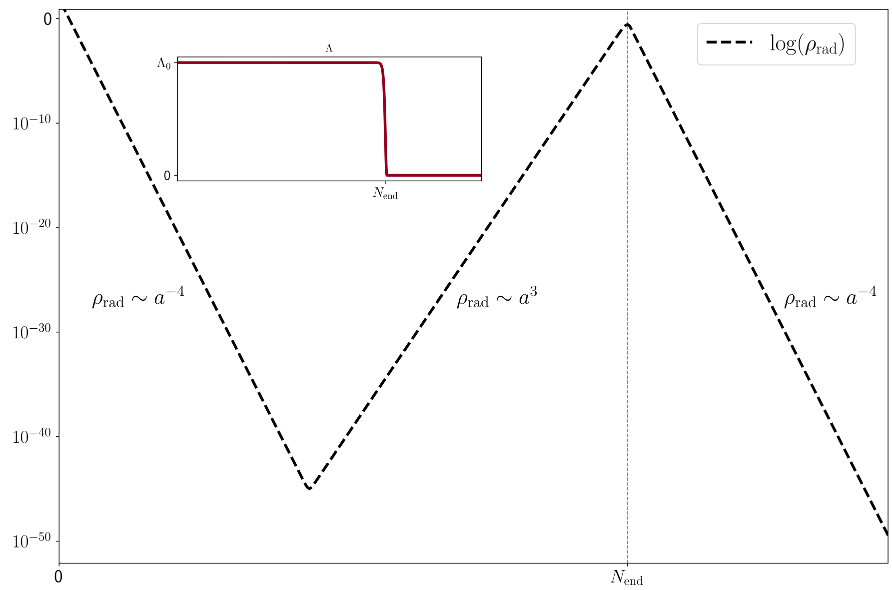

Figure 6.1 – Solution of (6.24) with $\beta = 10^{-80}$, we plot the cosmological constant Λ (in linear scale, note that the positive vertical axis is linear scale) and the radiation energy density ρ (in log scale) in terms of the number of e-folds $\log(a)$. Λ behaves effectively as a constant until about when condition (6.27) is satisfied and abruptly decays to zero thereafter. The radiation density decays exponentially from its initial Planckian value until the energy injection from the relaxation of Λ starts winning over the expansion. By the end of inflation radiation density grows back to about Planckian density again. During the inflationary phase the Hubble sphere shrinks $\sim 10^{-28}$, solving the horizon problem.

[6]Thermal equilibrium for the radiation is not a necessarily valid. We are used to this to be true in the high energy/density regime of the primordial universe. However, this is not clearly possible as we approach Planck scales and if we take seriously an extrapolation from particle physics at those scales. The reason is that the condition for thermal equilibrium $\Gamma > H$ on the interaction rate Γ (where $\Gamma \equiv n\sigma$ where $n \sim T^3$ is the number density and σ the cross section for interactions) cannot be maintained close to $T_p = M_{\text{Pl}}$ because $\sigma \sim 1/T^2$ for high energy processes and thus $\Gamma \sim T$. As T would drop dramatically if the initial ρ_0 was in thermal equilibrium, while $H \sim M_{\text{Pl}} > T$, all species decouple in the inflationary past and the radiation injection via the decaying Λ cannot achieve thermal equilibrium until later when H eventually trops below T.

In such case equations (6.21) and (6.22) can be combined to obtain

$$a'' + 4\frac{a'^2}{a} = \frac{2}{3a^5}\Lambda(t), \tag{6.24}$$

which directly relates the dynamical behaviour of $\Lambda(t)$ and the scale factor.

If the initial cosmological constant (6.18) starts at its natural Planckian value $\Lambda(0) = \Lambda_0 \sim M_{\rm Pl}^2$, the initial conditions for the matter density are not important. Indeed, the dynamics of the initial phase of the cosmic evolution is basically insensitive to the value of $\rho(0)$ in the range $0 \le \rho(0) \le M_{\rm Pl}^4$ (this is a standard aspect of the the usually emphasized robustness of inflation: matter density decays exponentially during the De Sitter phase and becomes rapidly irrelevant for the background evolution).As we show below, our model shares this property with standard inflation as long as β is sufficiently small.

Notice that the Hubble rate in terms of 4-volume time is

$$H \equiv \frac{\dot{a}}{a} = \frac{a^3 a'}{a} = \frac{1}{3}\frac{da^3}{dt}, \tag{6.25}$$

(the symbol $\cdot$ denotes derivatives with respect to comoving time τ).

During the initial phase of expansion defined by the condition $\beta M_{\rm Pl} t < 1$ the universe behaves approximately as a de Sitter universe with Hubble rate $H = H_0 = \sqrt{\Lambda_0/3}$.

During that initial phase we can integrate the previous equation (with the initial condition $a(0) = 1$) and find that $a^3 \approx 3H_0 t + 1$. Equivalently, during such period equation (6.18) can be rewritten as

$$\Lambda(a) \approx \Lambda_0 \exp(-\frac{M_{\rm Pl}}{3H_0}\beta(a^3 - 1)), \tag{6.26}$$

which yields and extremely flat curve for

$$\beta a^3 < 3\frac{H_0}{M_{\rm Pl}}, \tag{6.27}$$

with a sharp descent for $\beta a^3 \approx 3H_0/M_{\rm Pl}$ (assuming $\beta \ll 1$).

This implies that for sufficiently small β the background evolution will be very similar to that of standard inflationary cosmology.

For instance one can get the inflationary phase to last for about 60 e-folds, $N_{\rm end} \equiv \log(a_{\rm end}) \sim 60$, and thus solve the horizon and flatness problems if β is constrained by

the condition
$$\beta \leq \frac{3H_0}{M_{\rm Pl}} 10^{-80}. \tag{6.28}$$

These estimates are confirmed by the numerical solution of the previous equations illustrated in Figure 6.1.

6.2.2 Higgs Dynamics during the De Sitter phase

In the model that we are presenting the background dynamics is dominated by the relaxing cosmological constant. In such framework there is no need for the inflaton field of standard inflationary scenarios. Nevertheless, a (single) scalar field degree of freedom is still necessary for the mechanism of structure formation proposed here to work in its simplest form: this allows for the use certain conservation laws for super-Hubble modes allowing to predict the amplitude of perturbations at the CMB from initial condition during inflation (this is sometimes referred to as the Weinberg theorem [129] whose proof we revisit in the Appendix A). In addition—as the source of structure will be the hypothetical fundamental Planckian granularity, and, as mentioned in the introduction—the degree of freedom interacting with such fundamental inhomogeneities at the Planck scale must be scale-invariance-breaking in nature. Therefore, the Higgs field is the natural carrier of the inhomogeneities as, on the one hand, it is the single scalar degree of freedom in the standard model of particle physics, and, on the other hand it is the mediator of the breaking of scale invariance. Even when it is quite possible that a different realization of our scenario might exist, we will concentrate here on such minimalistic model where only the physics of the standard model enters into consideration as far as the description of matter is concerned. It is also important to point out that the necessity of a scalar field degree of freedom is rooted only in its role in the mechanism of structure formation (described in detail in the following section) as the Higgs here plays no important role in the dynamics of the background. For that reason, our model should not be confused with models of Higgs inflation [50, 159, 160].

However, the dynamics of the Higgs during the inflationary era will be central in the model so we review it in detail here. The Higgs field equation in the FLRW background is
$$\ddot{\phi}_0 + 3H_0\dot{\phi}_0 + \Gamma_{\rm Planck}\dot{\phi}_0 + \frac{dV(\phi_0)}{d\phi} = 0. \tag{6.29}$$

where the term $\Gamma_{\rm Planck}\dot{\phi}_0$ is a friction term associated to the energy loss caused the production of inhomogeneities mechanism that we will introduce in Section 6.3. There we will see that $\Gamma_{\rm Planck} \ll H_0$ and thus this term can be safely neglected from the previous equation when analyzing the dynamics of the zero mode of the Higgs[7]. We assume that ϕ_0 is in the usual 'terminal velocity' configuration where $3H_0\dot{\phi}_0 = -\partial_\phi V[\phi_0]$ which, from $V \approx (\lambda/2)\phi^4$ (where λ is the self-coupling constant of the Higgs) implies[8]

$$\dot{\phi}_0 \approx -2\lambda \frac{\phi_0^3}{3H_0}. \tag{6.30}$$

Here we are using that $\Gamma_{\text{Planck}}/H_0 \ll 1$ (as mentioned, this assumption will be shown to be valid later when we derive equation (6.89)). We will assume that the Higgs starts with a large expectation value

$$\phi_0(0) \approx M_{\text{Pl}}. \tag{6.31}$$

From (6.30) we get $|\dot{\phi}_0| \ll H_0^2$ as long as $|\lambda| \ll 1$. The requirement that the universe is dominated by the cosmological constant, namely $|V[\phi_0]| = (|\lambda|/2)\phi_0^4 \ll \Lambda m_p^2/(8\pi) = H_0^2 M_{\text{Pl}}^2/(8\pi)$ is automatically satisfied if $|\lambda| \ll 1$. For these reasons, one can neglect the effects of the potential in the dynamics of the background geometry in the De Sitter phase studied in Section 6.2.1. Finally, in the terminal velocity regime we have (from the time derivative of (6.30)) that

$$\ddot{\phi}_0 \approx \frac{12}{9}\lambda^2 H_0^3, \tag{6.32}$$

which will be neglected as a higher λ correction in the perturbation theory calculations that follow. The picture in Figure 6.1 will have to be modified when $\Lambda(t) \approx V[\phi_0]/M_{\text{Pl}}^2$. This happens after the end of inflation, and thus away from the region where the seed of structure formation are produced as discussed in Section 6.3.

From (6.30) one finds solutions

[7]Strictly speaking, another term $\Gamma_\phi\dot{\phi}_0$ encoding a standard form of diffusion representing the particles generated via the interactions of the Higgs with the rest of the fields in the standard model should be added to equation (6.29). The quantity Γ_ϕ is determined by the known interactions of the standard model (which we are assuming here to make sense all the way to close to the Planck scale). It follows from the interaction structure of the standard model that the decay rate Γ_ϕ must be quadratic in the relevant couplings times some energy scale. Taking this energy scale to be in the natural scale, i.e. H_0, we get $\Gamma_\phi \approx \alpha H_0$ for some dimensionless constant [161]. It can be argued that $\alpha \ll 1$ and thus this term is negligible in our case.

[8]One can explore this numerically and for initial 'velocities' away from (6.30) there is a transition time where (as expected) the terminal velocity approximation is not valid. However, even when starting from the (large) natural Planckian value dictated by dimensional analysis $\dot{\phi}_0 \approx -M_{\text{Pl}}^2$ the scale factor enters into the terminal velocity regime after a few e-folds when the Higgs scalar starts rolling back towards the Planck scale. We are assuming that the Higgs quartic term dominates the potential for values $m_{\text{H}} \ll \phi_0 \lesssim H_0 \approx M_{\text{Pl}}$. We also treat the Higgs as a single scalar field (for presentation simplicity) ignoring in our equations its su(2) internal indices. Our expressions make sense in a polar decomposition $\phi^A = \phi\, v^A$ with $v^A \in \text{su}(2)$ and $v^A v_A = 1$.

$$\phi_0(\tau) = \frac{M_{\rm Pl}}{\sqrt{1 + \frac{4}{3}\frac{M_{\rm Pl}^2}{H_0}\lambda\tau}} \quad \text{or} \quad \phi_0(a) = \frac{M_{\rm Pl}}{\sqrt{1 + \frac{4}{3}\frac{M_{\rm Pl}^2}{H_0^2}\lambda\log(a)}}. \tag{6.33}$$

Note that during the first e-folds, say $\mathcal{N} = \log(a) \sim 11$ (which approximately correspond to the period during which the fluctuations visible in the CMB are produced in our model), and for $\lambda \approx -10^{-2}$ (which is the correct order of magnitude value in the standard model close to the Planckian scale [162]) the Higgs changes slowly for $H_0 \approx M_{\rm Pl}$ as assumed, namely

$$\frac{\Delta\phi_0}{\phi_0} \lesssim 10^{-1}. \tag{6.34}$$

6.2.3 Radiation generated by the decaying Λ

The analysis of the background dynamics, given in Section 6.2.1, relies on neglecting the effect of the radiation emitted as Λ decays in matter field modes. In addition, there is the question of how the initial conditions for radiation affect the conclusion of Section 6.2.1. Here we show that non of these neglected aspects have an important influence and that results of the previous simplified analysis remain correct to the level of approximation considered. The physical reason is that the cosmological constant term dominates the Friedmann equation due to its slow decay in a while the radiation dilutes as a^{-4} as the energy injection (6.23) for $\beta \ll 1$ is negligible at first. Eventually, energy injection becomes comparable with the dilution rate and radiation density $\rho_{\rm rad}$ starts growing again. One can understand these features—which where first exibited by the numerical solution of the equations as plotted in Figure 6.1—semi-analytically giving a closer look at equation (6.23) which for diffusion into radiation becomes

$$\frac{d\rho_{\rm rad}}{da} + \frac{4}{a}\rho_{\rm rad} = -\frac{\Lambda'(t)}{8\pi G a'}. \tag{6.35}$$

Using that $H = \sqrt{\Lambda/3} = a^2 a' \approx H_0 = \sqrt{\Lambda_0/3}$ and also $\dot{a} \approx H_0 a$—recall equation (6.25)—we get

$$\frac{d\rho_{\rm rad}}{da} + \frac{4}{a}\rho_{\rm rad} \approx \frac{3\beta a^2}{8\pi} M_{\rm Pl}^4, \tag{6.36}$$

where we have used equation (6.30) and the fact that $\phi_0 \approx \phi_0(0) \approx H_0 \sim M_{\rm Pl}$. Integrating (6.36) we obtain

$$\rho_{\rm rad} \approx \frac{\rho_0}{a^4} + \frac{3\beta a^3}{56\pi} M_{\rm Pl}^4. \tag{6.37}$$

Therefore, our first approximation (6.23) turns out to be fine. Thus, we simply ignore that last constant contribution to the radiation density in the previous equation in order to simplify the presentation. However, a similar constant density contribution coming from diffusion will play a key role in the discussion of Section 6.4. Notice that the minimum in the radiation density observed in Figure 6.1 can be estimated from the condition $d\rho_{\rm rad}/da = 0$ and gives

$$\begin{aligned} a_{\rm min}^7 &\approx \frac{224\pi\rho_0}{9\beta M_{\rm Pl}^4} \\ \rho_{\rm min} &\approx \frac{7\rho_0}{3a_{\rm min}^4} \end{aligned} \tag{6.38}$$

—$a_{\rm min} \approx e^{27}$ and $\log(\rho_{\rm min}/M_{\rm Pl}^4) \approx -107$ for the parameters in Figure 6.1. With a bit of abuse of the approximation we can estimate the radiation at the end of the inflationary period when $\beta a_{\rm end}^3 = 3H_0/M_{\rm Pl}$, as implied by (6.26). One gets

$$\rho_{\rm end} \approx \frac{9M_{\rm Pl}^4}{56\pi} \equiv T_{\rm end}^4. \tag{6.39}$$

We see that the previous semi-analytic argument reproduces well the qualitative features of the numerical solution in Figure 6.1. Notice that the final 'reheating temperature' $\approx \rho_{\rm end}^{1/4}$ is independent of the initial conditions and of the order of Planck temperature.

6.2.4 Estimate of the lifetime of Λ after the inflationary era and number of e-folds

The numerical evolution shows that soon after we reach the end of inflation the universe becomes quickly dominated by radiation with an initial radiation density which is estimated from (6.37). The end of inflation is characterised here by the condition

$$\beta a_{\rm end}^3 \approx 3\frac{H_0}{M_{\rm Pl}}, \tag{6.40}$$

which follows from equation (6.26). The Friedmann equation (6.21) in the radiation dominated domain becomes

$$a^4 a' = \sqrt{\frac{8\pi\rho_{\rm end}}{3M_{\rm Pl}^2}}\, a_{\rm end}^2 \tag{6.41}$$

Integrating and multiplying by $\beta M_{\rm Pl}$ we get

$$\frac{1}{5}\sqrt{\frac{3M_{\rm Pl}^4}{8\pi\rho_{\rm end}}}\,\beta a_{\rm end}^3\left(\left(\frac{a}{a_{\rm end}}\right)^5 - 1\right) = \beta M_{\rm Pl}\Delta t. \tag{6.42}$$

Neglecting the -1 inside the parenthesis, using (6.40) to eliminate the β dependence, replacing for $\rho_{\rm end}$ using (6.39), and assuming that $a/a_{\rm end} \approx T_{\rm end}/T$, we obtain the following expression for the dependence of Λ on temperature after inflation

$$\Lambda = \Lambda_0 \exp\left(-\beta M_{\rm Pl} \Delta t\right) \approx \Lambda_0 \exp\left[-\frac{\sqrt{21}}{5}\left(\frac{9}{56\pi}\right)^{\frac{5}{4}}\left(\frac{H_0}{M_{\rm Pl}}\right)^{\frac{5}{2}}\left(\frac{M_{\rm Pl}}{T}\right)^5\right] \tag{6.43}$$

which implies that the cosmological constant becomes negligible in comparison to the present value $\Lambda_{\rm today} = 10^{-120} M_{\rm Pl}^2$ extremely quickly by the time when the temperature of the universe is still close to Planckian. The important point here is that this is well before the electro weak transition temperature $T_{\rm ew} \approx 10^{-17} M_{\rm Pl}$ so that the essentially vanishing cosmological constant can grow again via the mechanism presented in [128, 127] to the present observed value.

The parameter β chosen in the Figure 6.1 corresponds to an illustrative value. Here we analyze in more details observational constraints on this value. An important feature of our model is the generation of inhomogeneities in an approximately scale invariant fashion as observed on the CMB during the quasi De Sitter phase. The scale of these fluctuations range from $L_{\rm min} = 10^{-2}$Mpc to $L_{\rm max} = 10^3$Mpc today. Even when the mechanism for structure formation will be different from the inflaton 'vacuum fluctuations' of the standard paradigm, the De Sitter regime of inflation will still play a key role. In particular, one needs the scales of fluctuations visible today to correspond to the Hubble scale $H \approx H_0 \sim M_{\rm Pl}$ at the time of inflation. This demands a minimum number of e-folds from the beginning of inflation to today

$$\mathcal{N}_{\rm min}^{\rm start \to today} = \log(H_0 L_{\rm max}) = \log\left(\frac{H_0}{M_{\rm Pl}}\right) + \log(M_{\rm Pl} L_{\rm max}) = \log\left(\frac{H_0}{M_{\rm Pl}}\right) + 138. \tag{6.44}$$

On the other hand the number of e-folds since the end of inflation $N_{\rm min}^{end \to \rm today}$ is

$$\mathcal{N}_{\rm min}^{\rm end \to today} = \log(T_{\rm end} T_0^{-1}) \approx \frac{1}{4}\log\left(\frac{9H_0^2}{56 m_p^2 \pi}\right) + 74, \tag{6.45}$$

from which we get necessary minimum number of inflationary e-folds

$$\mathcal{N}_{\rm min}^{\rm start \to end} \approx \frac{1}{2}\log\left(\frac{H_0}{M_{\rm Pl}}\right) + 65. \tag{6.46}$$

6.3 Structure formation

As described in previous sections, the cosmological constant decays spontaneously due to diffusion into radiation degrees of freedom exponentially in unimodular time as a result of quantum gravity instability associated to the fundamental granularity. We have shown that this produces a quasi De Sitter dynamical evolution for the universe. We have also assumed that the Higgs potential starts in a homogeneous configuration with a natural expectation value $\phi_0 \sim M_{\rm Pl}$ and have shown how ϕ_0 is expected to evolve. Even when negligible in such a dynamics (as it will be shown at posteriory) we included a friction term controlled by $\Gamma_{\rm Planck}$ in (6.29). This term is produced, we argue, by the interaction of the Higgs scalar with the Physics at the fundamental scale: the Planckian granularity. This interaction of the homogeneous Higgs ϕ_0 and the inhomogeneous granular structure at the Planck scale—mediated by the scale-invariance-breaking of the Higgs—will generate (or excite) inhomogeneities in ϕ that are born at the Planck scale via a stochastic process described in detail in Section 6.3.1).

During the initial De Sitter phase the scalar curvature is close to Planckian so that the scale of discreteness could naturally catalyse the emergence of inhomogeneities. As argued here, and in [128], the discreteness scale should play a role in those field theoretical degrees of freedom which are not scale invariant. These are the degrees of freedom that, from a relational perspective, carry a 'ruler' or 'reference frame' with respect to which the fundamental quantum gravity scale $\ell_{\rm Pl}$ can become meaningful. In this sense it is natural to accept that as a result of such interaction inhomogeneities should be created in the Higgs scalar (which is the degree of freedom that introduces the breaking of scale invariance in the standard model). The energy flow involved in this can be parametrized (phenomenologically) as an Ohmian diffusion term in the equation of motion of ϕ_0 (assuming the back reaction process is stochastic with a probability distribution that is homogeneous and isotropic). As this effect is assumed to have a quantum geometry origin, and as the only relevant geometric scale around is the Hubble rate, dimensional analysis suggests the diffusion to be characterized by a dimensionless coefficient γ is a dimensionless coefficient as follows

$$\Gamma_{\rm Planck}\dot{\phi}_0^2 = \gamma H^5, \tag{6.47}$$

where the previous is the diffusion term in (6.29) and we assume $\gamma \ll 1$ (this assumption will be confirmed by the analysis that follows). Such friction term induces an additional steady contribution to the divergence of the Higgs energy momentum tensor component which will be absorbed by the generated inhomogeneities (quantitatively this will be described by suitable continuity equations written below). We will see below that, such steady injection of energy into the fluctuations (via the mechanism evoked in this paragraph but made mathematically precise below) produces a spectrum of scalar perturbations in the Higgs that is adiabatic and approximately scale invariant. We will also show that—using the Weinberg theorem to analyze the effect of these at CMB times—the the magnitude of the parameter γ needs to be fixed to $\gamma \approx 10^{-16}$ which is, remarkably, of the same order of magnitude as the dimensionless number $\gamma_{\rm H} \approx 10^{-17}$ charaterizing the breaking of scale invariance by the Higgs, recall (6.19).

Note that from the semiclassical perspective of quantum field theory on curved spacetimes we must also note that there are no ambiguities in the notion of particles for conformal invariant quantum fields as the FLRW background is conformally flat. As a result there is no real particle creation for such modes if thought of as test fields on the cosmological conformally flat background [148] [9]. For degrees of freedom breaking scale invariance the situation is the same as long as we concentrate on scales well within the Hubble radius. However, the notion of particle (and their number) becomes ambiguous as soon as we consider modes with super Hubble wave length. The mechanism of excitation of inhomogeneities discussed above is producing particles at around the scale where the notion of particles become ambiguous. By this we are not saying that a complete semiclassical description is at all possible (as any fundamental explanation of the role of discreteness would need to appeal to quantum gravity). Nevertheless, we are arguing here that the excitation of inhomogeneities that we postulate is taking place just exactly at around the scale where the semiclassical account allows for something peculiar to happen.

The peculiar physical aspect that we are evoking is rooted in the UV structure of our physical description of matter and geometry. In this respect it is important to recall the discussion of the renormalization of the energy momentum tensor in quantum field theory on curved spacetimes, and the fact that UV contributions lead to an

[9] The natural state for conformally invariant fields in the De Sitter phase is the Bunch-Davies vacuum (any deviations from it are exponentially diluted during inflation). This state coincides with the Gibbons-Hawking state that is perceived as a thermal state with temperature $T_{\rm gh} = H_0/(2\pi)$ by any freely falling observer [163]. However, such thermal bath should be regarded as the analog of the Unruh particles in flat spacetime. They are there but have an elusive physical reality as can be clearly seem by considering the examples of a spacetime that is initially flat, then De Sitter spacetime in an intermediate region, and finally flat again. If one starts with the Poincare vacuum state then the state will evolve into something well approximated by the Bunch-Davies vacuum in the intermediate phase (with Gibbons-Hawking temperature $T_{\rm gh}$). However, the state will emerge in the final state as the poincare vacuum again. No real particles are created by the De Sitter phase. Therefore, such 'thermal excitations' due to the presence of the De Sitter horizon in the initial phase of evolution in our model, cannot be responsible for the real fluctuations that we need to find in the future stage where the universe has gone out of the De Sitter phase and the horizon has become virtually infinite (like in Minkowski).

(anomalous from the pure quantum field theory perspective) violation of energy momentum conservation (equation (6.12)). We will see that in the case of the Higgs scalar such anomaly could be interpreted as the source of the term $\Gamma_{\rm Planck}\dot{\phi}_0$ in (6.29) (or its possible semiclassical description). We will come back to this in the discussion section once the implications of the present perspective are spelled out.

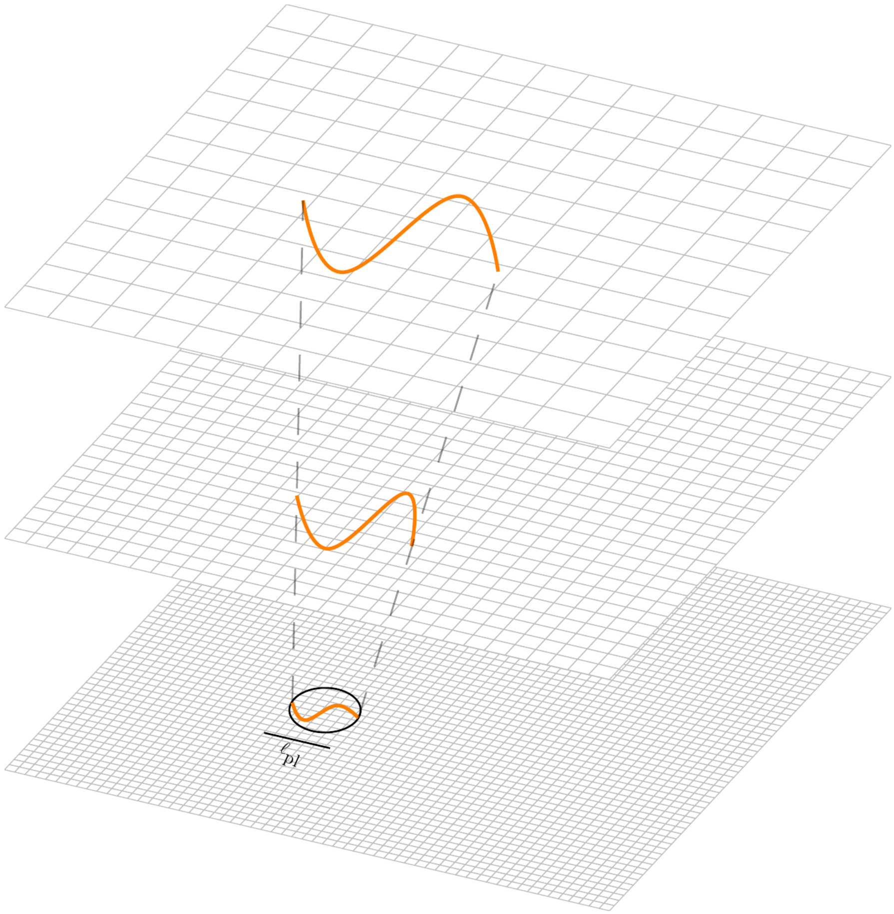

Figure 6.2 – Perturbation modes born at the Planck scale are carried to super-Hubble scales by the exponential inflation of the Universe.

6.3.1 Phenomenological analysis

Let us start from the study of the dynamical equation for the scalar field inhomogeneities. In standard treatments these perturbations are quantized and assumed to be in some (preferred) vacuum state (say the Bunch-Davies vacuum). The inhomogeneities that we see today in the CMB are assumed to arise from the quantum fluctuations which somehow become classical by the time that they leave their imprint on the visible sky. Although such perspective is largely adopted in the community, it suffers from several conceptual drawbacks ranging from the transplanckian problem to interpretational problems associated with quantum mechanics of a close system and the measurement problem (see [47] for a discussion of these issues and further references).

In contrast, inhomogeneities in the model that we propose here arise from an actual physical interaction that actively produces inhomogeneities on the background Higgs value ϕ_0. Even when these fluctuations and the background field configurations are intrinsically quantum, we will represent them by semiclassical states whose expectation values are assumed to be well approximated by classical field equations.

Consequently, we define perturbations of the zero mode $\delta\phi_k$ with wave number k for which the following field equation holds

$$\delta\ddot{\phi}_k + 3H_0\delta\dot{\phi}_k + \frac{k^2}{a^2}\delta\phi_k + \frac{d^2V(\phi_0)}{d\phi^2}\delta\phi_k = 0. \tag{6.48}$$

The last term in (6.48) is smaller that the third term when $k = aH_0$ (i.e. at horizon crossing) and it is in general suppressed by the Higgs self coupling so we will treat its influence in perturbation theory below. This follows from

$$\frac{d^2V(\phi_0)}{d\phi^2} = 6\lambda\phi_0^2 \ll H_0^2, \tag{6.49}$$

which would automatically hold for $\phi_0 \sim H_0$ as $\lambda \ll 1$ in this large field regime. Given these assumptions, and according to (6.48), super-Hubble modes (for which $k \ll aH_0$) satisfy (to zeroth order in λ)

$$\delta\ddot{\phi}_k + 3H_0\delta\dot{\phi}_k \approx 0 \quad \text{or equivalently} \quad \frac{d(a^3\delta\dot{\phi}_k)}{dt} \approx 0, \tag{6.50}$$

which implies

$$\delta\phi_k(\tau) = q_k\frac{e^{-3H_0\tau}}{3H_0} + \delta\phi_k \quad \text{or equivalently} \quad \delta\phi_k(\tau) = \delta\phi_k + \mathcal{O}(a^{-3}) \quad \delta\dot{\phi}_k = \mathcal{O}(a^{-3}), \tag{6.51}$$

for some q_k. Super-horizon modes freeze out and their time derivative $\delta\dot{\phi}_k$ decays exponential in co-moving time or as a^{-3} in terms of the scale factor [10].

[10] If we keep the contribution of the potential in equation (6.48) then one gets instead that

$$\frac{\delta\dot{\phi}_k(\infty)}{\delta\phi_k(\infty)} = -\frac{3}{2}\left(1 - \sqrt{1 - \frac{8}{3}\lambda}\right) \approx -2\lambda. \tag{6.52}$$

The fact that λ is negative (Higgs instability) introduces a growing mode that goes like $\approx \exp(-4\lambda H_0\tau)$. However, for $\lambda \approx -10^{-2}$ this growth is sufficiently slow to grant the validity of perturbation theory until

Energy fluctuations and the power spectrum

The effect of the Planckian granularity will be modelled by a Brownian diffusion process that injects energy in the Higgs scalar by leaving an imprint of the fundamental scale as inhomogeneities in the background value. More precisely, we consider a stochastic process generating density fluctuations by excitation of the Higgs scalar modes at the Planck scale which is assumed to coincide with the curvature scale $H_0 \approx M_{\rm Pl}$. As in the description of the Brownian motion, the process stochasticity is an assumption that allows for a statistical effective description of the effect of a large number of underlying independent microscopic degrees of freedom whose individual dynamics can be understood only in terms of a (more fundamental) quantum gravity analysis.

The caracterization of the stochastic process requires the analysis of the energy cost of generating the inhomogeneities in the Higgs. For that purpose let us first write the Higgs scalar as $\phi(x) = \phi_0 + \delta\phi(x)$ so that the first order perturbation of the energy density (up to second order) is

$$\begin{aligned}\delta\rho \equiv \delta T_{00}((x^\mu)) &= \frac{1}{2}\delta\dot{\phi}^2 + \frac{1}{2a^2}\delta\vec{\nabla}\phi^2 + \delta V(\phi) \\ &\approx \dot{\phi}_0\delta\dot{\phi}(x^\mu) + \frac{dV(\phi_0)}{d\phi}\delta\phi(x^\mu) + \frac{1}{2}\delta\dot{\phi}(x^\mu)^2 + \frac{1}{2a^2}(\vec{\nabla}\delta\phi(x^\mu))^2 + \frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\delta\phi(x^\mu)^2.\end{aligned} \tag{6.53}$$

At this point, it is important to point out two important features of the previous expression. First the perturbation of the energy momentum tensor in equation (6.53) is obtained by assuming that the Higgs is a test field (i.e. metric perturbations are excluded here). Second, we have expanded up to second order in perturbation while in the usual cosmological perturbation theory one only needs to go up to first order. We will see below that in all dynamical considerations involving gravity we will restrict to linear perturbations. Very importantly, during the De Sitter phase, scalar metric perturbations turn out to be trivial (see equation (6.80) below) which implies that (at least during that period) the test field energy momentum tensor and the full linearized energy momentum tensor actually coincide. This is not the case for the second order perturbations. However, the later will only be used as an interpretational devise that offers the means to talk about energy flows involved in the creation of the perturbations by the stochastic process that describes the interaction between the discreteness scale and the Higgs scale [11].

the end of inflation $H_0\tau \approx 60$.

[11] This is analogous to the discussion of energy flow in Hawking black hole radiation where back reaction is neglected and the field degrees of freedom are considered those of a test field. However, reliable physical information is captured by such notion allowing for the clear understanding of the physical consequence of particle creation ranging form negative energy flows across the horizon, to the

We now assume that $\delta\phi(x^\mu)$ is a stochastic variable with a probability distribution such that the associated linear momentum vanishes, namely

$$\langle\delta\phi(x^\mu)\rangle = 0, \tag{6.54}$$

where from now on $\langle\ \rangle$ denote ensemble averages. It follows that

$$\langle\delta T_{00}(x^\mu)\rangle \approx \frac{1}{2}\langle\delta\dot{\phi}(x^\mu)^2\rangle + \frac{1}{2a^2}\langle(\vec{\nabla}\delta\phi(x^\mu))^2\rangle + \frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\langle\delta\phi(x^\mu)^2\rangle\,, \tag{6.55}$$

where the approximate sign comes from the fact that we have truncated the expansion of $V(\phi)$ to second order in $\delta\phi(x^\mu)$. The previous equation implies that the (ensemble) average energy contribution to the field perturbations in the stochastic process is controlled by the second order terms in the expansion (to leading order). We can relate the second moments of the probability distribution to the Power spectrum of perturbations if we decompose the field in Fourier components

$$\delta\phi(t,\vec{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int dk^3\delta\phi_{\vec{k}}(t)\exp(i\vec{k}\cdot\vec{x}) \tag{6.56}$$

with reality conditions

$$\overline{\delta\phi_{\vec{k}}} = \delta\phi_{-\vec{k}}. \tag{6.57}$$

The standard definition of the 2-point correlation function (see for instance [136, 164]) is defined by

$$\begin{aligned}\xi_\phi(\vec{r}) &\equiv \langle\delta\phi(\vec{x})\delta\phi(\vec{x}+\vec{r})\rangle \\ &= \frac{1}{(2\pi)^3}\int dk^3dq^3\langle\delta\phi_{\vec{k}}\delta\phi_{\vec{q}}\rangle\exp(i(\vec{k}+\vec{q})\cdot\vec{x})\end{aligned} \tag{6.58}$$

where the second line has been expressed in terms of Fourier modes. Due to the background symmetries the stochastic process creating the perturbations the 2-point correlation function must be homogeneous (independent of $\vec{x}$) and isotropic. In terms of Fourier modes this implies that

$$\langle\delta\phi_{\vec{k}}\delta\phi_{\vec{q}}\rangle = P_{\delta\phi}(k)\delta^{(3)}(\vec{k}+\vec{q}), \tag{6.59}$$

where $P_{\delta\phi(k)}$ is the power spectrum of the perturbations $\delta\phi$. The previous equation implies the key relationship between the power spectrum and the expectation value of the square of the perturbation at the same point $\delta\phi(\vec{x})$, namely

violations of the classical area law, and the energy loss via evaporation at infinity.

$$\langle \delta\phi(\vec{x})\delta\phi(\vec{x})\rangle = \frac{1}{(2\pi)^3}\int dk^3 P_{\delta\phi}(k). \tag{6.60}$$

This relation allows, as we shall show below, to express the ensemble average of the energy momentum tensor of the perturbations in terms of their power spectra.

Energy momentum conservation (continuity equations)

In this section we study the equation of state of the Higgs perturbations when averaged in the ensemble representing the probability distribution of the stochastic process, whose general properties were introduced in the previous section. This equation of state will play a role in the continuity equations for the perturbations which will allow us to interpret the energetics of the generation of inhomogeneities. Repeating the exercise that led to (6.55), but now for the full energy momentum tensor, we obtain

$$\begin{aligned}\langle T_{ab}\rangle &= -\frac{\Lambda M_{\rm Pl}^2}{8\pi}g_{ab} + \langle \nabla_a(\phi_0+\delta\phi)\nabla_b(\phi_0+\delta\phi) - \frac{1}{2}g_{ab}\left(\nabla_c(\phi_0+\delta\phi)\nabla^c(\phi_0+\delta\phi) + 2V((\phi_0+\delta\phi))\right)\rangle \\ &= T^{(0)}_{ab} + \underbrace{\langle \nabla_a\delta\phi\nabla_b\delta\phi\rangle - \frac{1}{2}g_{ab}\left(\langle\nabla_\alpha\delta\phi\nabla^\alpha\delta\phi\rangle + \frac{d^2V(\phi_0)}{d\phi^2}\langle\delta\phi^2\rangle\right)}_{\langle\delta T_{ab}\rangle},\end{aligned} \tag{6.61}$$

where we have expanded to second order in the perturbation and the first order terms are gone due to (6.54). The second order terms are (as the zeroth order ones) of the perfect fluid form due to the (assumed) isotropy of the stochastic process generating the perturbations. Therefore, we have $\langle T_{ab}\rangle = \rho_{\rm h} u_a u_b + P_{\rm h} h_{ab}$ with

$$\begin{aligned}\rho_{\rm h} &= \frac{\dot\phi_0^2}{2} + V(\phi_0) + \langle\delta\rho^{(2)}\rangle, \\ P_{\rm h} &= \frac{\dot\phi_0^2}{2} - V(\phi_0) + \langle\delta P^{(2)}\rangle,\end{aligned} \tag{6.62}$$

where $P_{\rm h}$ and $\rho_{\rm h}$ denote the pressure and density contributions of the Higgs scalar, and the supra index (2) expresses the fact that these come from quadratic terms in the field perturbations. In order to get the explicit form of $\langle\delta\rho^{(2)}\rangle$ and $\langle\delta P^{(2)}\rangle$ we observe that

$$\langle\nabla_a\delta\phi\nabla_b\delta\phi\rangle = \langle\delta\dot\phi^2\rangle\, u_a u_b + \frac{1}{3a^2}\langle\vec\nabla\delta\phi\cdot\vec\nabla\delta\phi\rangle\, h_{ab} \tag{6.63}$$

from which we get

$$
\begin{aligned}
\langle \delta T_{ab}\rangle &= \langle \nabla_a \delta\phi \nabla_b \delta\phi\rangle - \frac{1}{2} g_{ab}\left(\langle \nabla_\alpha \delta\phi \nabla^\alpha \delta\phi\rangle + \frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle\right) \qquad (6.64)\\
&= \frac{\langle \delta\dot\phi^2\rangle + \frac{1}{a^2}\langle (\vec\nabla \delta\phi)^2\rangle + \frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle}{2} u_a u_b + \frac{\langle \delta\dot\phi^2\rangle - \frac{1}{3a^2}\langle (\vec\nabla \delta\phi)^2\rangle - \frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle}{2} h_{ab}
\end{aligned}
$$

thus

$$
\begin{aligned}
\langle \delta\rho^{(2)}\rangle &= \frac{\langle \delta\dot\phi^2\rangle + \frac{1}{a^2}\langle (\vec\nabla \delta\phi)^2\rangle + \frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle}{2} \approx \frac{1}{2a^2}\langle (\vec\nabla \delta\phi)^2\rangle + \frac{1}{2}\frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle \qquad (6.65)\\
\langle \delta P^{(2)}\rangle &= \frac{\langle \delta\dot\phi^2\rangle - \frac{1}{3a^2}\langle (\vec\nabla \delta\phi)^2\rangle - \frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle}{2} \approx -\frac{1}{6a^2}\langle (\vec\nabla \delta\phi)^2\rangle - \frac{1}{2}\frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle ,
\end{aligned}
$$

where we neglected the $\delta\dot\phi$, as justified by (6.51) in the $k \ll a H_0$ regime (which is the regime where all these equations will be used). With the previous equations at hand we can write the continuity equation that describes the amount of work that is necessary for the stochastic interaction with the granular structure to generate the perturbations. Denoting this work W^{pert} we get

$$
\begin{aligned}
\frac{dW^{\text{pert.}}}{da} &\equiv \frac{1}{\dot a}\left(\langle \delta\dot\rho^{(2)}\rangle + 3H_0\left(\langle \delta\rho^{(2)}\rangle + \langle \delta P^{(2)}\rangle\right)\right)\\
&= \frac{d\langle \delta\rho^{(2)}\rangle}{da} + \frac{3}{a}\left(\langle \delta\rho^{(2)}\rangle + \langle \delta P^{(2)}\rangle\right), \qquad (6.66)
\end{aligned}
$$

where in the second line we are using the scale factor a as time parameter. Replacing (6.65) and (6.66) the previous equation becomes

$$
\boxed{\frac{dW^{\text{pert.}}}{da} \equiv \frac{d\langle \delta\rho^{(2)}\rangle}{da} + \frac{2}{a}\langle \delta\rho^{(2)}\rangle - \frac{1}{a}\frac{d^2 V(\phi_0)}{d\phi^2}\langle \delta\phi^2\rangle}. \qquad (6.67)
$$

The assumption is that in its rolling the zero mode ϕ_0 interacts with the granularity scale and diffuses energy to the modes with (physical) wave number k/a via the discrete scale which initially (during the De Sitter phase) is close to the Planck scale $H_0 \approx M_{\text{Pl}}$. More precisely we assume that this work is extracted from the zero mode ϕ_0 while evolving in the Higgs potential in a stochastically Ohmian way so that its dynamical equation (6.29) gets a friction term $\Gamma_{\text{Planck}}\dot\phi_0$ [12]. For convenience we rewrite (6.29) here

[12] As a simple particular situation illustrating of a rational behind this modification consider a Klein-Gordon scalar field as an example. The field equation $\nabla^a \nabla_a \phi - m^2\phi^2 = 0$ is explicitly given by

$$
\frac{1}{\sqrt{|g|}}\partial_\mu\left(\sqrt{|g|} g^{\mu\nu}\partial_\nu \phi\right) - m^2\phi^2 = 0. \qquad (6.68)
$$

$$\ddot{\phi}_0 + 3H_0\dot{\phi}_0 + \frac{dV(\phi_0)}{d\phi} + \Gamma_{\text{Planck}}\dot{\phi}_0 = 0. \tag{6.71}$$

Using the definition (6.67), and including the radiation generated by the decaying of the cosmological constant and the Higgs decay in other particles of the standard model, the continuity equation (6.1) becomes

$$\begin{aligned}
&\frac{d\left(\rho_\Lambda + \rho_{\text{rad}} + \rho_{\text{h}} + \langle\delta\rho^{(2)}\rangle\right)}{d\tau} + 3H_0\left(\rho_\Lambda + \rho_{\text{rad}} + \rho_{\text{h}} + \langle\delta\rho^{(2)}\rangle + P_\Lambda + P_{\text{rad}} + P_{\text{h}} + \langle\delta P^{(2)}\rangle\right) = 0\\
&\frac{\dot{\Lambda}M_{\text{Pl}}^2}{8\pi} + \left(\dot{\rho}_{\text{rad}} + 4H_0\rho_{\text{rad}}\right) + \dot{W}^{\text{pert.}} + \dot{\phi}_0\left(\ddot{\phi}_0 + V'(\phi_0) + 3H_0\dot{\phi}_0\right) = 0\\
&\underbrace{\frac{\dot{\Lambda}M_{\text{Pl}}^2}{8\pi} + \left(\dot{\rho}_{\text{rad}} + 4H_0\rho_{\text{rad}}\right)}_{=0} + \overbrace{\dot{W}^{\text{pert.}} - \gamma H^5}^{=0} = 0,
\end{aligned} \tag{6.72}$$

where in going from the second to the last line we used the Higgs background equation (6.29), and we rearranged the terms corresponding to the continuity equation for the perturbations replacing in addition (6.47). The idea encoded in the previous equation is that the relaxation of the cosmological constant heats up radiation (which in the initial De Sitter phase dilutes exponentially and hence has negligible effect on the background dynamics) while the Brownian stochastic interaction of the Higgs rolling down the potential produces fluctuations according the balance equation (6.47), namely

$$\dot{W}^{\text{pert.}} - \gamma H^5 = 0. \tag{6.73}$$

The coefficient of friction γ will be determined later from an Einstein-like detailed balance condition that links the dissipation encoded in γ with the amplitude of the observed power spectrum of fluctuations observed the CMB [13]. To leading order, such

---

Thus we see from the previous equation that if the background is fluctuating then the equation will get a 'Brownian' modification as follows

$$\nabla^a\nabla_a\phi - m^2\phi^2 = \xi^a\nabla_a\phi, \tag{6.69}$$

where ξ^a is the contribution from the background fluctuations

$$\xi^\nu \equiv -\Delta\left(\frac{1}{\sqrt{|g|}}\partial_\mu(\sqrt{|g|}g^{\mu\nu})\right) \tag{6.70}$$

in the FLRW context the only non vanishing component of ξ^a allowed by the symmetry is $\xi^0 = 3\Delta H$—that we called $\xi^0 = \Gamma_{\text{Planck}}$—is the only possible non trivial component from which the analog of equation (6.71) follows.

[13]Before calculating the power spectrum generated from the 'detailed balance' equation (6.73) we would like to comment on the fact that first order perturbations do not contribute to the ensemble average that led to our continuity equation (6.72). What we have used (as first stated in (6.54)) is that, first order contributions to $T_{\mu\nu}$—while non-vanishing in a particular realization of the stochastic

steady injection of energy in the inhomogeneities is at the heart of the scale invariant nature of power spectrum of density perturbations produced by this means. This is what we do in the next section.

The power spectrum from diffusion

Equation (6.60) shows that the stochastic ensemble expectation value of the product of scalar field fluctuations at a single point is directly related to the power spectrum of the fluctuations. This provides a simple relation between the expectation value of the energy momentum tensor and the power spectrum of the scalar field perturbations. For instance, using (6.65), an algebraic manipulation analogous to the one leading to (6.59) implies

$$\begin{aligned}\langle\delta\rho^{(2)}\rangle \equiv \langle\delta T_{00}\rangle &= \frac{1}{2\pi^2}\int dk k^2\left(\frac{1}{2}P_{\delta\dot\phi}+\left[\frac{k^2}{2a^2}+\frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\right]P_{\delta\phi}\right)\\ &\approx \frac{1}{2\pi^2}\int dk k^2\left(\frac{k^2}{2a^2}+\frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\right)P_{\delta\phi} \qquad (6.75)\end{aligned}$$

where $P_{\delta\dot\phi}$ is defined via the analog of equation (6.59) but for the fluctuations $\langle\delta\dot\phi_{\vec k}\delta\dot\phi_{\vec q}\rangle$, we assume that the stochastic process is isotropic (so that $dk^3 \to 4\pi k^2 dk$), and we neglected $P_{\delta\dot\phi} = \mathcal{O}(a^{-6})$ due to (6.51). According to our previous discussion, we assume that the perturbations are created at horizon crossing $k = aH_0 \sim aM_{\rm Pl}$, namely $P_{\delta\phi}(k) = 0$ for $k > aH_0$. Thus, including this in the integration boundaries of (6.75) we obtain

$$\langle\delta\rho^{(2)}\rangle \equiv \langle\delta T_{00}\rangle \approx \frac{1}{2\pi^2}\int_{\mu}^{aM_{\rm Pl}} dk k^2\left(\frac{k^2}{2a^2}+\frac{1}{2}\frac{d^2V(\phi_0)}{d\phi^2}\right)P_{\delta\phi}(k), \qquad (6.76)$$

where μ is an infrared cut-off that will not have any effect in the equations describing the regime of interest. Changing time variables from τ to $a(\tau)$, equation (6.73)

process (representing the particular state of our universe)—average to zero when considering an ensemble of realizations (ensemble of universes). But how can that be relevant for our own particular universe that is one among the members of the ensemble? The answer invokes an analogy with the ergodic hypothesis: the condition $\langle\delta\phi(x)\rangle = 0$ is to be interpreted on a single realization (via this ergodicity assumption) as implying that, at a given time, the space average

$$\frac{\int_R \delta\phi(\vec x,t)dx^3}{V_R} = 0, \qquad (6.74)$$

for a sufficiently large region R (here V_R is the co-moving volume of the region). In this way, the local contribution of fluctuations to $T_{\mu\nu}$ is not vanishing in a given realization. Nevertheless, they average to zero in such the mean field sense. Similar interpretational questions arise for the ensemble average of the quadratic contributions to $T_{\mu\nu}$ when translated to our (single realization) universe. However, these are familiar issues common to conventional situations (see for instance [136]).

becomes

$$\frac{dW^{\text{pert.}}}{da} \equiv \frac{d\langle\delta\rho^{(2)}\rangle}{da} + \frac{2}{a}\langle\delta\rho^{(2)}\rangle - \frac{1}{a}\frac{d^2V(\phi_0)}{d\phi^2}\langle\delta\phi^2\rangle \quad = \quad \gamma\frac{H^4}{a}. \tag{6.77}$$

To leading order in λ, the previous equation tell us that the amount of energy that we are extracting from the Higgs zero mode to produce inhomogeneities is done in a way that is not sensitive to the size of the universe. More precisely $dW^{\text{pert.}} = W_0(da/a)$ with $W_0 = \gamma H^4$ is a self-similar process (invariant under rescaling $a \to \alpha a$) to leading order in λ (recall (6.33)).

Using equation (6.76) and (6.33) one can substitute the ansatz $P_{\delta\phi}(k) = P_0/k^3(1 + \mathcal{O}(\lambda))$ into (6.77) and check that it produces a solution of the detail balance condition to leading order in λ. Thus, in the present model the Harrison-Zeldovitch spectrum of inhomogeneities in the scalar field can be simply related to a self-similar injection of energy during the quasi-inflationary era $H_0 \approx$ constant without the need to invoke the uncertainty principle and (most importantly) the pre-existence of vacuum fluctuations as described by the extrapolation of quantum field theory to trans-Planckian scales. The solution is

$$P_{\delta\phi}(k) = \frac{P_0}{k^3} \tag{6.78}$$

with

$$P_0 = 4\pi^2\gamma\frac{H_0^3}{M_{\text{Pl}}}(1-6\lambda) \approx 4\pi^2\gamma\frac{H_0^3}{M_{\text{Pl}}}, \tag{6.79}$$

where the next to leading order correction is not relevant when comparing with observations because it does not depend on k; hence, we drop it for simplicity. However, we will see in the following section that the λ corrections will affect scale invariance when one instead analyses the effects of these perturbations in the gravitational field (which are directly related to the observed fluctuations in the CMB). In fact, the red tilt of the CMB power spectrum is linked (in this model) to the self-interaction strength of the Higgs field λ.

The Weinberg theorem and the power spectrum of density fluctuations at the CMB

The following equations concern long wavelength modes $k < aH_0$ (those added up in (6.76)). Weinberg proved a beautiful and very powerful statement concerning such modes based on the universality of free-fall. This result is know as Weinberg's theorem [129]; the proof of which is revisited and simplified in the Appendix A. One has in particular that the gravitational potential for these super Hubble modes is given by (see (A.27))

$$\Phi_k = \Psi_k = \mathscr{R}_k \left(-1 + \frac{H(\tau)}{a(\tau)} \int_{\mathscr{T}}^{\tau} a\left(\tau'\right) d\tau' \right) \approx 0, \tag{6.80}$$

where $\mathscr{R}_k$ are constants, and the right hand side approximation is valid during the De Sitter phase. This implies that scalar perturbations do not generate scalar metric perturbations during the inflationary era (they decouple gravitationally to leading order perturbation theory in λ where one has an almost exact the De Sitter phase). This justifies the omission (for presentation simplicity) of the scalar metric perturbations in the expression of the linear expansion to the energy momentum tensor (6.53). The form of the scalar density fluctuations including the gravitational potential are given below in (6.85).

Weinberg's theorem also implies the existence of adiabatic scalar perturbations as solutions of linearized gravity, namely

$$\frac{\delta \rho_k^{(1)}}{\dot{\rho}_0} = -\frac{\mathscr{R}_k}{a(\tau)} \int_{\mathscr{T}}^{\tau} a\left(\tau'\right) d\tau' \approx -\frac{\mathscr{R}_k}{H_0}, \tag{6.81}$$

where these are adiabatic in that the previous relation for each species contributing to the perturbations individually (see (A.29)). The previous two equations correspond to equations (5.4.4) and (5.4.5) in [129] and will be discussed and recovered in Appendix A. In the present context they allow us to compute $\mathscr{R}_k$ for super-Hubble scales $k < aH_0$ as

$$\mathscr{R}_k = -H_0 \frac{\delta \rho_k^{(1)}}{\dot{\rho}_0} \tag{6.82}$$

We have that (recall (6.62))

$$\rho_0 = \frac{\Lambda M_{\rm Pl}^2}{8\pi} + \frac{\dot{\phi}_0^2}{2} + V(\phi_0) + \rho_{\rm rad}, \tag{6.83}$$

Assuming that $\Lambda \approx$ constant during the De Sitter phase, and using the field equations (6.29) and the equation of state of the radiation component, the time derivative of ρ_0 gives

$$\begin{aligned} \dot{\rho}_0 &= \dot{\phi}_0 \ddot{\phi}_0 + \frac{dV(\phi_0)}{d\phi_0} \dot{\phi}_0 - 4H_0 \rho_{\rm rad} \\ &= -\dot{\phi}_0 (3H_0 \dot{\phi}_0 + \frac{dV(\phi_0)}{d\phi} + \Gamma_{\rm Planck} \dot{\phi}_0) + \frac{dV(\phi_0)}{d\phi_0} \dot{\phi}_0 - 4H_0 \rho_{\rm rad} \\ &\approx -H_0 \left[3\dot{\phi}_0^2 + 4\rho_{\rm rad} \right] \\ &\approx -\frac{4}{3H_0} \lambda^2 \phi_0^6. \end{aligned} \tag{6.84}$$

where in the last line we use that $\Gamma_{\rm Planck} \ll H_0$ (to be confirmed below), and that

$\rho_{\rm rad} \ll \lambda\phi_0^4$ (this requirement can be met if one is in the initial range of of De Sitter evolution where radiation is exponentially diluted, see Figure 6.1). From the general expression of the energy-momentum tensor we get (including the metric perturbation term)

$$\begin{aligned}\delta\rho_k^{(1)} &= \dot{\phi}_0\delta\dot{\phi}_k(x^\mu) + V'(\phi_0)\delta\phi_k(x^\mu) - \Psi_k\dot{\phi}_0^2 \\ &= V'(\phi_0)\left(-\frac{\delta\dot{\phi}(x^\mu)}{3H_0} + \delta\phi(x^\mu)\right) - \Psi_k\dot{\phi}_0^2 \\ &\approx 2\lambda\phi_0^3\delta\phi_k(x^\mu), \end{aligned} \tag{6.85}$$

where we neglect the $\delta\dot{\phi}_k$ term as it quickly dies off for super-Hubble modes according to (6.50), and we used that the long wavelength adiabatic scalar metric perturbations vanish in the De Sitter phase according to (6.80). Replacing (6.85) and (6.84) in (6.82) we get

$$\begin{aligned}\mathscr{R}_k &= \frac{3H_0^2}{2\lambda\phi_0^3}\delta\phi_k \\ &= \frac{3}{2\lambda\epsilon_\phi^3 H_0}\left[1 + 2\lambda\log\left(\frac{k}{H_0}\right)\right]\delta\phi_k, \end{aligned} \tag{6.86}$$

where we have used the expression on the right of (6.33) and used that the modes k are generated at horizon crossing when $a = k/H_0$. Squaring the previous relationship and computing its ensemble average in our stochastic process one obtains, from the definition (6.59), an equation linking the power spectrum $P_{\mathscr{R}}$ of the $\mathscr{R}_k$ and that of the scalar perturbations. Explicitly, using (6.79), we get

$$\begin{aligned}P_{\mathscr{R}} &= \frac{9}{4\lambda^2 H_0^2}\frac{P_0}{k^3}\left[1 + 4\lambda\log\left(\frac{k}{k_0}\right) - 4\lambda\log\left(\frac{H_0}{k_0}\right)\right] \\ &= \frac{9\pi^2\gamma}{k^3\lambda^2}\left[1 + 4\lambda\log\left(\frac{k}{k_0}\right) - 4\lambda\log\left(\frac{H_0}{k_0}\right)\right]. \end{aligned} \tag{6.87}$$

If we take $H_0/k_0 = 1$ which boils down to normalizing $a = 1$ at the moment the most IR mode in the CMB leaves the horizon we arrive at the final expression for the power spectrum of scalar perturbations (for $H_0 \approx M_{\rm Pl}$) we get

$$\boxed{P_{\mathscr{R}} \approx \frac{9\pi^2\gamma}{k^3\lambda^2}\left(1 + 4\lambda\log\left(\frac{k}{k_0}\right)\right).} \tag{6.88}$$

Using the customary notation where $P_{\mathscr{R}} \equiv N^2/k^3$, comparison with CMB observations (see for instance [129]) fixes the normalization factor N^2 to

$$N^2 \approx \frac{9\pi^2\gamma}{\lambda^2} \approx 1.9 \times 10^{-10}. \tag{6.89}$$

Using that $\lambda \approx -10^{-2}$ at our energy scale one needs to fix $\gamma \approx 10^{-16}$ which is remarkably close to the estimate $\gamma_{\rm H}$ given in (6.19) based on the natural measure of deviation from conformal invariance put forward in the introduction expected to control the Brownian diffusion mechanism. Deviation from scale invariance are encoded in the spectral index of scalar perturbations $n_{\rm s}$. They are controlled by the Higgs self coupling as it follows from (6.88). The result to first order in λ is

$$n_{\rm s} - 1 \equiv \frac{d\log(k^3 P_{\mathscr{R}})}{d\log k} \approx 4\lambda + \mathscr{O}\left[\lambda^2 \log\left(\frac{k_{\rm max}}{k_0}\right)\right]. \tag{6.90}$$

Observations constraint it to

$$1 - n_{\rm s} = 0.04 \pm 0.004, \tag{6.91}$$

which implies $\lambda \approx -10^{-2}$ which is compatible with the he standard model expected value of $\lambda = -(1.3 \pm 0.7) \times 10^{-2}$ at these high field values—see [162]. Notice that in our framework the spectral index is itself k dependent. Notice that the linear approximation used remains consistent inspite of the $\log(k_{\rm max}/k_0)$ in the error term as for $\lambda = -10^{-2}$ and $k_{\rm max} = 10^5 k_0$ one has $\lambda^2 \log(k_{\rm max}/k_0) \approx 10^{-3}$ which is smaller than the present observational error in $1 - n_{\rm s}$ [165]. In the same paper the deviations from a constant spectral index are reported to be given by

$$\frac{dn_{\rm s}}{d\log k} = -0.0045 \pm 0.0067. \tag{6.92}$$

One can repeat the previous analysis starting from equation (6.76) and keeping terms up to order λ^2. With this improved approximation it is possible to compute the previous quantity and the result is

$$\frac{dn_{\rm s}}{d\log k} = -0.0005 + \mathscr{O}(\lambda^3). \tag{6.93}$$

The previous is a prediction of our scheme, potentially verifiable in the future if observational data reduce the error by about 10%.

Tensor modes

So far we focused on the description of a mechanism for the generation of inhomogeneities in scalar modes only. The question of whether tensor modes are also produced is a very important one in view of future constraints on the scalar-to-tensor ratio r from CMB observations. In our model fundamental discreteness is the underlying mechanism for the active generation of the inhomogeneities. As argued in the introduction, see also [128, 127] for further discussion, such discreteness should primarily affect degrees of freedom breaking scale invariance. In the present case, with the assumption of the validity of the standard model, the breaking of scale invariance is mediated by the Higgs scalar mass. Gravitons being massless should not interact with the Planckian discrete structure according to the dimensional analysis type of rational behind our model. More precisely, as it is well known, an infinitesimal conformal transformation $\delta g_{ab} = \delta\omega g_{ab}$—here regarded as a field variation—leads to the trace-part of Einsteins equations $(R - 8\pi GT) = 0$. This clearly implies that the trace part the field equations encode conformal-invariant-breaking interactions that mediate the stochastic production of inhomogeneities in our model. Thus the Planckian granularity—imposed by the consistency with the low energy Lorentz invariance [156, 127]—cannot generate tensor modes whose sources are encoded in the tensor traceless components of the energy momentum tensor. Therefore, the expected value of the tensor-to-scalar ratio predicted by our model is basically $r \approx 0$.

6.4 Planckian black hole remnants as dark matter

The fundamental nature of dark matter remains and open question. Here we would like to stress that, if the reheating temperature at the end of inflation gets close to the Planck temperature, a model of dark matter where it is made of quantum gravity Planck mass particles (as described from our low energy perspective) that only interact gravitationally is very natural.

Little is known about the fundamental theory of quantum gravity besides the fact that it has to reproduce general relativity with massless gravitons a low energies. As emphasized in the introduction several approaches to quantum gravity propose that the smooth geometry of general relativity would be emergent from an underlying fundamental discrete structure at the Planck scale. In these approaches the fundamental energy scale $M_{\rm Pl}$ plays a central role. A point we would like to stress here is that, in addition to motivating the mechanism for generation of structure studied in this paper, such perspective naturally leads to the possibility that defect-like objects in the discrete fabric spacetime could survive the continuum limit. If so it seems likely that these would behave like particles with a mass with the natural mass scale $M_{\rm Pl}$ and would interact only gravitationally. Such defects could be thermally excited if Planckian temperatures were achieved during reheating. It is unclear how to picture such particles from our low energy perspective, for the lack of a better name we could think of them as Planckian stable primordial black holes. Unstability of tiny black holes due to Hawking radiation is often evoked to rule out such dark matter candidates. However, lacking a full quantum gravity theory, it is clear that little is certain about the properties of black holes (or such Planckian defects) of that scale. It is even unclear in what sense such objects qualify as black holes when the very notion of geometry is expected no to be available so close to the fundamental scale. The only thing that is certain in fact is that absolutely all the assumptions behind Hawkings calculation simply fail: thus the simple invocation of Hawking radiation is not a serious argument to rule out their hypothetical role in cosmology.

The possibility that dark matter is made of primordial Planckian black hole remnants (or more humbly, Planck mass purely gravitationally interacting particles) has been evoked in the literature before [166, 167, 168, 169]. Here we show that the dark matter energy density required by observations can indeed emerge naturally in a Planck scale reheating scenario as the one produced by our model. Such type of dark matter will basically behave like a dust fluid interacting with the rest of matter gravitationally only. It would be extremely hard to detect via other manifestations. Their presence would remain hard to notice locally as the Planckian size of these particles will make their gravitational cross section in interactions with usual matter extremely small (however, this form of dark matter might be directly detectable via its gravitational interaction [170]).

At the end of the inflationary era reheating raises the temperature to close to the the Planck temperature and Planck mass remnants could be created via thermal fluctuations if thermal equilibrium density is achieved. In order for this to happen one needs the remnant interaction rate $\Gamma_{\rm pbh} > H$, where the interaction rate is given by $\Gamma = n\sigma v$ with n the number density, σ the interaction cross section, and v the velocity. For remnants of mass $m_{\rm pbh}$ the interaction cross section $\sigma_{\rm pbh} \approx m_{\rm pbh}^2/M_{\rm Pl}^4$ whith their density $n \approx T^3$ while in thermal equilibrium. Using that in the radiation dominated era $H \approx (T/M_{\rm Pl})T$, we conclude that remnants decouple from thermal equilibrium when

$$T \lesssim \frac{m_p^2}{m_{\rm pbh}^2} M_{\rm Pl} \equiv T_{\rm D}. \tag{6.94}$$

If thermal equilibrium can hold up to $T_{\rm D} \lesssim T_{\rm end}$ then the thermal remnant abundance of dark matter today can be estimated to be about (see equation 4.38 in [136])

$$\frac{\rho_{\rm pbh}^{\rm thermal}(T_{\rm D})}{M_{\rm Pl}^4} \approx \left(\frac{m_{\rm pbh}}{M_{\rm Pl}}\right)^4 \left(\frac{T_{\rm today}}{T_{\rm D}}\right)^3 \left(\frac{T_{\rm D}}{m_{\rm pbh}}\right)^{\frac{3}{2}} e^{-\frac{m_{\rm pbh}}{T_{\rm D}}}. \tag{6.95}$$

One can easily check that it is possible to obtain a remnant density compatible with dark energy density today—which would correspond to evaluating the previous line to about 10^{-120}—with a $m_{\rm pbh}$ slightly larger than but of the order of $M_{\rm Pl}$. This shows that the framework provided by our model could also fit dark energy genesis from the production of stable PBHs via thermal fluctuations at the end of the De Sitter phase without extreme fine tuning where the necessary suppression is brought by the standard Gibbs factor.

6.5 What is the difference with the standard paradigm where inhomogeneities arise from vacuum fluctuations?

Here we discuss in more detail the difference of our model with the more standard (by now textbook) account where the vacuum fluctuations in the quantum state of the inflaton are the source of inhomogeneities. After all even when there is no inflaton field driving inflation, our model still has a scalar field degree of freedom which if set (asymptotically in the far past) in the Bunch-Davies vacuum would have vacuum fluctuations analogous to that of the inflaton (indeed this is the idea in models of Higgs inflation).

Some of the conceptual difficulties in interpreting such a paradigm has been discussed in [129]. Here we will simply state that, in the absence of a theory of quantum gravity, the naturally available tool is that of semiclassical gravity where one replaces Eintein's equations by

$$\mathbf{R}_{ab} - \frac{1}{2}\mathbf{R} g_{ab} = 8\pi G \langle\psi|\mathbf{T}_{ab}|\psi\rangle \tag{6.96}$$

for some quantum state $|\psi\rangle$ of the matter living on a classical geometry. One sees immediately that this approach would immediately lead (in the standard account) to no gravitational effects of vacuum fluctuations. More precisely, as cosmological perturbation theory is based on linearized gravity around the FLRW background $\langle\psi|\delta\mathbf{T}_{ab}|\psi\rangle = 0$. For that reason one is not simply doing semiclassical gravity in the standard account. There is a region of conceptual shadow around this point for which people tend to develop their own views which (not surprisingly) are intimately related to the interpretation of quantum theory in the particularly thorny context of the universe as a whole.

The first key difference introduced by our model is that in our case fluctuations are generated in the state of the Higgs itself via the interaction of the (assumed) Planckian granularity and the scalar degrees of freedom. In our case the fluctuations are present in the semiclassical state of the Higgs $|\psi\rangle$ in the sense that $\langle\psi|\delta\phi_{ab}|\psi\rangle \neq 0$ and consequently

$$\langle\psi|\delta\mathbf{T}_{ab}|\psi\rangle \neq 0. \tag{6.97}$$

Thus, our model admits a semiclassical account corresponding to the linearized version of (6.96). The state of the Higgs (via its interactions with the Planckian granularity) breaks the FLRW symmetry in contrast with the Bunch-Davies vacuum. In our case, inhomogeneities are inherent of the Planckian substratum and simply transmitted to the scalar degree of freedom during inflation.

One can ignore the previous point and compare the predictions of the two models for the power spectrum of scalar perturbations: by following our proposal, or by, instead, assuming that the Higgs is assymototically in the far past in the Bunch-Davies vacuum. In the second case (the standard approach) the result for the amplitude of the power spectrum (here we follow Chapter 10.3 in [129] which agrees with the standard treatment, for instance see [164])

$$N_{\rm vac}^2 = \frac{1}{4\pi^2|\epsilon|}\frac{H_0^2}{M_{\rm Pl}^2} \approx \frac{9}{64\pi^3}\frac{1}{\lambda^2} \tag{6.98}$$

and the spectral index

$$1 - n_{\rm S} = 2\delta + 4\epsilon \approx \frac{8}{3}\lambda + \mathcal{O}(\lambda^2) \tag{6.99}$$

where we have used the standard definitions of the slow roll parameters (evaluated in our model)

$$\epsilon = -\frac{\dot H}{H^2} \approx \frac{16\pi}{9}\lambda^2 \quad \text{and} \quad \delta = \frac{\ddot H}{2H\dot H} \approx \frac{4}{3}\lambda. \tag{6.100}$$

One easily sees that the previous results are incompatible with observations in the CMB. This puts clearly the important difference between the two different initial states for the scalar field.

In our model the perturbations of the Higgs are born at horizon crossing and hence the state differs from the Bunch-Davies vacuum state: the ‘order parameter’ revealing this difference is the expectation value $\langle\psi|\delta\phi_{ab}|\psi\rangle$ which vanishes in the Bunch-Davies vacuum but not in the present case. In the standard formulation the state of the inflaton perturbations is assumed to be given by the Bunch-Davies vacuum which is defined asymptotically in the far past introducing in this fashion the so-called trans-Planckian problem where initial conditions for the modes are given when their wavelength is well below the Planck scale. In our case the properties of the semiclassical state are defined at horizon crossing as discussed in Section 6.3. Our state would become singular in the asymptotic past if freely evolved backwards due to the De Sitter expansion. This thought exercise shows clearly the sharp difference with the standard Bunch-Davies state (a Hadamard state).

6.6 Some open questions

In this section we mention and discuss a few points that deserve further attention. We rise several questions here and propose possible tentative solutions. These open issues represent possible lines for future improvement of the ideas in this paper that we hope could be developed in the future.

On the decay of the cosmological constant after the EW transition

Among the few free dimensionless parameters entering our model there is β which needs to be extremely small ($< 10^{-80}$) to produce a sufficiently long period of inflation. Such important fine tuning is not, by itself, necessarily problematic in an effective description of a phenomenon that is emergent from the collective behaviour of tiny microscopic building blocks whose precise physics is not taken into account. Lacking such a fundamental description one can find tentative guidance in dimensional analysis. For instance one could first simply rewrite (and this is only a reparametrization) the relaxation process in terms of the rescaled time variable t_p—introduced in (6.16)—by setting the length scale $\ell_{\mathrm{U}} \gg \ell_{\mathrm{Pl}}$.

Notice that this is quite reasonable as, in addition to Planck scale, there is another natural scale in the application of the cosmological principle to the region of interest of the universe which is precisely an IR scale ℓ_{U} representing the extent of the 'patch' of the universe that is well approximated by the ansatz geometry (6.15) with homogeneous and isotropic background fields living on it. In terms of a time variable t_p defined by (6.16) with that IR scale, the relaxation is controlled by the 'bare' value β_0 given by

$$\beta = \beta_0 \left(\frac{\ell_{\mathrm{Pl}}}{\ell_{\mathrm{U}}}\right)^3. \tag{6.101}$$

Such reparametrization does not resolve the fine tuning problem and only shifts the issue of the smallness of β into that of the largeness of ℓ_{U}. However, it offers a new perspective pointing at the possibility of a physical mechanism where the size of the FLRW patch ℓ_U would stabilize the cosmological constant in essence by reducing diffusion. Such perspective suggests long range quantum coherence mechanism (like for the collective behaviour in a Bose-Einstein condensates in relation to superfluidity) and offers a prospect for future analysis.

If such would be the role of ℓ_U this would also help resolving another question that necessarily arises when considering the instability of the cosmological constant present in the late universe. That is: why is it that the present cosmological constant has not decayed yet by a similar relaxation? One key feature of our model is that it answers the question of why is the cosmological constant can start at about its natural value and be basically zero quickly after the end of inflation. The remaining issue is how can it grow back to the value that is compatible with present observations. In [128, 127] a model was proposed (motivated by the same theoretical ideas as in this work) where the cosmological constant of the correct order of magnitude is generated due to diffusion during the electroweak transition. If the two proposals are to be consistent with each other then one would need a mechanism granting that the relaxation of the newly generated cosmological constant does not completely decay away by the present time. We notice that the phenomenological proposal (6.101), characterizing the long scale coherence, would reconcile the two from the fact that the new IR (stabilizing) scale $\ell_{\rm U}^{\rm ew} = a_{\rm ew}\ell_{\rm U}$ has expanded by the time of the electroweak transition for sufficiently large $\ell_{\rm U}$. This follows from the fact that the change in unimodular time Δt from the EW transition to today goes like $\Delta t \approx H_{\rm today}^{-1} a_{today}^3$ (recall $a_0 = 1$ at the Planck initial time). A cosmological constant created at the electroweak time will last until today if

$$\beta_0 \left(\frac{\ell_{\rm Pl}}{\ell_{\rm U}}\right)^3 \left(\frac{a_{\rm today}}{a_{\rm ew}}\right)^3 \frac{M_{\rm Pl}}{H_{\rm today}} < 1, \tag{6.102}$$

i.e., we would be in a new inflationary regime for the new relaxing Λ. Taking $\beta_0 \approx 1$, the previous condition would require the initial coherence IR distance to be $\ell_U \geq 10^{35}\ell_{\rm Pl} = 1m$. This appears as a huge initial region for our original bubble inflating to the present universe; at the same time we know and it has been often emphasised on various grounds that our universe requires extremely special initial conditions to accommodate its most basic features [137].

Even when the previous scenario is simple and thus appealing to us, their could be other reasons for the relaxation process to change after the electroweak scale, rooted in some unknown quantum gravity mechanism that is no longer operational at such low energies. Such physics could be related to the role of the Higgs scalar in the whole picture. We notice that when the cosmological constant has relaxed to zero during the inflationary epoch, the Higgs scalar will settle to its $V(\phi) = 0$ configuration which certainly changes the coupling of this field with four volume in the effective action. Other possibilities seem available in order to explain a possible ‘phase transition’ that would make the relaxation stop after the electroweak scale. This is an important open question in our proposal where, we hope, future investigations can shed light.

The instability of the Higgs potential and quantum gravity

In the present model the universe starts in a special state where the cosmological constant is of the order $M_{\rm Pl}^2$ and the Higgs field is around the Hubble rate which itself is of the order of $M_{\rm Pl}$. At such high values of ϕ_0 the quartic coupling λ is negative and—as we have seen in Section 6.3.1—this is exactly what is needed to explain the red tilt of the power spectrum of scalar perturbations. However, this also implies that the Higgs field find itself exactly in the instability region and is rolling towards higher values on the way to the Planck scale and beyond.

Note that the run-away behaviour is very slow during the inflationary phase as the Hubble friction is very important due to the effect of a large cosmological constant (recall equation (6.34)). When inflation ends the Hubble rate starts decaying and the instability becomes an issue. However, such conclusion only applies if one assumes that the standard model holds true beyond the Planck scale which is of course unreasonable. Deviations from the standard model should eventually become important as the Higgs field approaches $M_{\rm Pl}$, and—even when it is hard to know what exactly that new physics would be in such regime (notice that even the standard QFT formulation on a curved background is expected to fail there)—it seems reasonable to accept that whatever that new physics is it would prevent the Higgs to roll to arbitrary high values. There are various models in the literature that try to render such conclusion more concrete (all sharing the limitation of the necessary reliable inputs from a quantum gravity theory). For instance, a non minimal coupling of the Higgs with the geometry—which are necessary in models of Higgs inflation (see [171] for a review)—is shown to help stabilizing the Higgs up to about the Planck scale [172]. Other models predict stability at around the Planck scale [173] by making assumptions on possible new physics. As an example, a repulsive barrier at the Planck scale can arrise via ϕ^6 and ϕ^8 corrections of the Higgs potential motivated by grand-unified scenarios at $M_{\rm Pl}$ [174].

Such repulsive barrier at the Planck scale would only stop the Higgs scalar from rolling to arbitrary high transplankian scales. However, this by itself would not explain how the Higgs would eventually exit from that Planckian state and evolve towards the electro-weak minimum that produces the phenomenology of the standard model in accordance with the world we see around us. This problem resonates in some respects with the 'gracefull exit' problem in models of Higgs inflation [159, 50]. Yet it is also different as, on the one hand, in our model inflation is not driven by the Higgs, and, on the other hand, the diffusion of energy from the decaying cosmological constant raises the temperature of radiation back to close to the Planck temperature at the end of the inflationary era (recall Figure 6.1 and the discussion in Section 6.2.2, equation (6.39)). When temperature reaches Planckian values, at the onset of the radiation domination (recall Figure 6.1), the Higgs can thermalize and thermal fluctuations could populate the EW region of the Higgs phase space away from the instability scale. As the temperature drops with the expansion the probability that the Higgs ends in the EW stability region and produces a universe like ours should be non vanishing (such possibility is explored in related scenarios in [172, 175]).

On the validity of the semiclassical analysis

The mechanism of generation of structure in our model is based on the interaction of the Planckian granularity of quantum gravity with the low energy degrees of freedom encoded in the Higgs scalar field of the standard model. The analysis has been performed using the classical field equations for the scalar field evolving in a classical background. This is what one can do at the moment given the limitations of present quantum gravity theories to provide reliable calculation tools in such an extreme regime. The validity of semiclassical methods is an assumption of our analysis. Nevertheless, one must keep in mind that this limitation is common and possibly more severe in standard approaches where strong assumptions about trans–Planckian physics are customarily made. Note that in contrast there is no trans Planckian issue here. In our model, perturbations are born at the length scale H_0^{-1} which can (as the estimates show, recall Section 6.4, and also the discussion of Section 6.5) be a few orders of magnitude longer that the Planck length $\ell_{\rm Pl}$, and within the regime where the semiclassical treatment could already be a reliable approximation. No assumption about the nature at shorter scale is necessary.

On a similar ground there is another issue that tis common to various approaches and it is also shared by ours. This issue is sourced in the use of stochastic methods in conjunction with Einsteins equations and the difference between stochastic averages (satisfying some form of continuity equation compatible with the Bianchi identities or with the integrability conditions of unimodular gravity in our case) and the fact that individual realisations are not subjected in any clear fashion to such constraints. This implies that a single element of our stochastic ensemble does not follow the field equations of general relativity. This problem is often overlooked but it is present even in the standard paradigm of structure formation in inflation where quantum vacuum fluctuations are interpreted as classical stochastic fluctuations of an ensemble of realisations. In our model the behaviour of the individual realisation that represents our universe follows a dynamics with should be describable via a more fundamental theory. Our mean field description is only effective and the possible conflict with the structure of Einstein's equations at the level of an individual element of the ensemble is to be resolved by quantum gravity.

On the possibility of a remnant spacial curvature $\Omega_k \neq 0$

One of the predictions of standard inflationary cosmology is that spacial curvature $\Omega_k \ll 1$. There are however some indications suggesting that observational data could favour $\Omega_k^{\text{today}} \approx \mathcal{O}(10^{-2})$. If this would be confirmed then it would constraint the duration of the inflationary period. Such constraints also apply to the present model and could in principle invalidate it due to the possible tension with the (here proposed) new mechanism of generation of inhomogeneities. The reason is that later imposes a minimum number of inflationary e-folds as discussed in Section 6.2.4. Such minimum number could be at odds with a possible remnant spacial curvature today. More precisely, in accordance with the naturalness of the initial value of the cosmological constant at around M_{Pl}^2 we could set the initial spacial curvature $K = m_p^2$. Which gives an natural initial spacial curvature

$$\Omega_k^0 \approx 1. \tag{6.103}$$

The evolution of Ω_k as a function of the scale factor is

$$\Omega_k = \frac{K}{H^2 a^2}. \tag{6.104}$$

The previous equation implies (ignoring the late recent Λ domination) that, if $\Omega_k^{\text{today}} \approx \mathcal{O}(10^{-2})$, the spacial curvature at the time of the CMB is $\Omega_k^{\text{cmb}} \approx \mathcal{O}(10^{-5})$. Using that the reheating temperature in our model is about $T_{\text{end}} \approx M_{\text{Pl}}$ we get that the spacial curvature at the end of inflation is

$$\Omega_k^{\text{end}} \approx \Omega_k^{\text{cmb}} \left(\frac{T_{\text{cmb}}}{T_{\text{end}}}\right)^2 \approx 10^{-59}. \tag{6.105}$$

From the initial condition (6.103) during the inflationary phase $\Omega_k \approx a^{-2}$ so it would reach the previous value at the end of inflation if

$$\mathcal{N} = \frac{1}{2}\log(10^{59}) \approx 68. \tag{6.106}$$

The previous value is compatible with the minimum value calculated in (6.46); however, it could run into conflict with (6.102) which would imply the universe to be basically spatially flat today. For standard inflationary models reheating involves diffusion from an oscillating inflaton which constraints the reheating temperature to $T_{\text{end}} \approx 10^{10}$GeV [158, 136]. The number of e-folds necessary for $\Omega_k^{\text{today}} \approx \mathcal{O}(10^{-2})$ is then given by just $\mathcal{N} \approx 47$.

6.7 Discussion

We have proposed a model where the cosmological constant Λ_0 starts off with its natural Planckian value and later relaxes via diffusion into the matter degrees of freedom while driving an inflationary era. We assumed that the cosmological constant decays exponentially in unimodular time which leads to the necessary number of e-folds if the parameter β is sufficiently small. However, all the observational predictions of the model are independent of the precise value of β as long as it is sufficiently small. The validity of our analysis requires only that the cosmological constant remains Planckian for a minimum number of e-folds (Section 6.2.4). The standard model of particles physics is assumed to be valid all the way to close to the Planck scale and the Higgs scalar also assumed to start with a large semiclassical value ϕ_0 close to the Planck scale. The initial conditions of the other matter components do not affect the dynamics in any important manner as long as the radiation density is not ultra-Planckian (as in standard inflation [158], the cosmological constant dominates and the expansion dilutes away any memory of these initial conditions). The relaxation mechanism is associated with the hypothesis of discreteness of quantum gravity at the Planck scale. This suggest a natural time variable proportional to the number of Planckian four volume elements created by the dynamical evolution and in terms of which the relaxation is exponential. We argue that the same underlying discreteness at about the Hubble scale H_0 should stimulate the generation of inhomogeneities in the Higgs amplitude at that very scale, and show that a stochastic model where the steady injection of energy at the Hubble scale produces (to leading order in the Higgs self coupling λ) a scale invariant spectrum of density perturbations with an amplitude that is compatible in order of magnitude with CMB observations.

More precisely, once the initial values of the Higgs background and the cosmological constant are fixed to the natural scale $M_{\rm Pl}$ the model is controlled by two parameters: the parameter β which defines the decay rate of the cosmological constant in unimodular time, and the parameter γ parametrizing the Ohmian friction term—stemming from the interaction with discreteness exciting inhomogeneities—in the field equations for the zero model of the Higgs. As mentioned above, the parameter β needs only to be sufficiently small in order to achieve a sufficient number of e-folds that makes the model compatible with observations (fixing β amount to fixing the number of e-folds of inflation). The parameter γ is a dimesionless coupling representing noisy interaction of the Higgs with the granular structure at the Planck scale which in turn is expected to be possible thanks to the breaking of scale invariance of the Higgs scalar. The natural order parameter for such breaking is $\gamma_{\rm H} \equiv m_{\rm H}/M_{\rm Pl}$. It is a remarkable fact that agreement with the observation of the perturbations at the CMB necessitates a $\gamma \approx 10^{-16}$ which coincides (in order of magnitude) with $\gamma_{\rm H}$.

Deviations from scale invariance are brought by the evolution of the Higgs on the Higgs potential and depend on λ. Remarkably, standard model physics (encoded in Λ) produces a red tilt of the spectrum that is in agreement with the data extracted from the CMB observations: the spectral n_S coincided with observations for $\lambda \approx -10^{-2}$ which is compatible with the expected value of λ at high energies in the standard model. Moreover, the model predicts a variation of the spectral index with scale that is inside the limits obtained from the analysis of latest data [165]. This correspond to a proper prediction of our analysis which could be tested in the future if observational errors are reduced by an order of magnitude.

Given the above mentioned initial conditions, Planckian temperature reheating is a robust (β-independent) prediction of our model. We observe that such feature could naturally account for the present abundance of dark matter via the thermal production of Planck mass defects if such stable particles are part of the spectrum of quantum gravity. As in the case of the so-called WIMP miracle, we notice that the decoupling temperature and mass of such hypothetical purely gravitationally interacting particles (natural objects from the perspective of quantum gravity) fall in the right range to represent a possible dark matter candidate.

We are aware of the strong assumptions in our model which stretches well established physics into the uncertain and unknown territory of quantum gravity. The speculative nature of such an enterprise is certainly very risky. Our model links naturally some of the key cosmological observations with aspects of that new physics of quantum gravity that we strive to better understand. This by itself seems to justify our adventures. We hope that these initial ideas could lead to helpful insights in the future.

Part IV

Conclusions and Outlook

In this thesis, we studied possible implications of the discrete nature of spacetime at the Planck scale. Starting from the hypothesis that spacetime is granular at the Planck scale we studied the implications in the context of black hole physics and cosmology while remaining agnostic about the precise version of the quantum theory.

Discreteness at the Planck scale is a common feature of different incarnations of quantum gravity and our approach, while inspired by Loop Quantum Gravity, does not assume a particular theory of quantum gravity. Since currently we do not have a complete theory of quantum gravity, the study of phenomenological models based on general features of the theory's expected behaviour deep in the quantum gravitational regime, provides a guide in the search of a more fundamental theory.

In the first part of this thesis we show that, by taking into account the expected discrete nature of spacetime at the Planck scale, the black hole's information loss paradox has a natural resolution. The account of the black hole formation, subsequent evaporation, and the apparent information loss in [26, 27] and Section 3 can be thought in the same lines as the burning of a piece of paper: the system is initially in a highly special state adapted to the devices of coarse-grained observers insensitive to microscopic degrees of freedom: molecular degrees of freedom in the case of the burning paper and Planckian-quantum gravitational degrees of freedom in the black hole case. The initially special state is driven into a high-entropy final state by physical processes that open up new regions of phase space for the system to explore. The physical process is the contact of the paper with a flame or, in the black hole case, the unavoidable curvature singularity that forms due to gravitational collapse makes the interaction with Planckian degrees of freedom available.. If we assume that a more fundamental theory will be singularity-free, the high-curvature region around the *would-be-singularity* ignites correlations between low-energy and Planckian degrees of freedom. For a coarse-grained observer with limited resolution, entropy grows in both cases.

This idea is made concrete in Section 4. In this Section we propose a fully quantum-gravitational model where these ideas are realized. Precisely, in the context of Loop quantum gravity, we identify the UV degrees of freedom, related to the so-called ϵ-sectors. We then show that, for low-energy observed (observers that, as in the Wheeler-de Witt quantum cosmology, given Λ, only distinguish between an expanding and a contracting universe) the universe evolves from a low-entropy state to a high-entropy state. Moreover, the entropy grows dramatically when the system goes through the bounce because correlations are establish during the high-curvature phase. More precisely, the presence of a matter degree of freedom breaks traslational invariance and different ϵ-sectors evolve differently establishing correlations between them.

Furthermore, the correlations are developped at no energy cost (*decoherence without dissipation [83]*), thus solving the usual problems associated with energy conservation in the late stages of the purification process.

The model is realized in a homogeneous isotropic Loop Quantum Cosmology instead in a Polymer Black Hole model for several reasons: the key ingredients for the entropy grow mechanism are present in both models. The interior of black holes are described via a homogeneous, non-isotropic cosmological model, the Kantowski-Sachs spacetime. While the polymerization of the non-isotropic model is technically identical to the isotropic's one, the presence of an extra degree of freedom results in technical difficulties that are still being explored. By working in isotropic and homogeneous LQC we avoid technical and conceptual difficulties present in all polymer black hole models. The realization of these ideas in a black hole polymer model will be reported elsewhere[28].

The broad scenario leading to this model [15] (detailed in Chapter 4 and Chapter 3 of Part II) can be realized in different approaches of Quantum Gravity and this implementation is important and interesting for several reason: the precise identification of the purifying degrees of freedom in each approach (the equivalent of the ε-sectors in te polymer models of Part II) and the precise nature of the coarse-grained observers together with their physical interpretation not only shed light on the evaporation process itself but also on the physical interpretation of the Quantum Theory itself (as we saw in the Loop Quantum Cosmology case, the coarse-grain defined in Section 4.6 leads to the mesoscopic Wheeler-de Witt description when observers insensitive to the ε-sectors are introduced). It would be also interesting to see which traits are shared across the realizations of the scenario in the different theories.

Our approach also highlights the role of polymer models as ideal testbeds for issues such as the continuum limit, coarse-graining and the UV structure of quantum gravity between others. Although in some cases, due to the symmetry reduced procedure, it is difficult to extract physical predictions [14] the features present in the polymer models that are expected to remain in the full quantum gravity theory, e.g. the presence of extra microscopic degrees of freedom related to discreteness, make these type of models a playground for the discussion of certain conceptual and qualitative issues in quantum gravity[176].

In the second part we show that the origin of the structures that we currently observe in the Universe can be linked to the discrete structure of the spacetime at the Planck scale.

[14] See [176], the definition of the quantum theory in the case of polymer models involves several arbitrary choices that affect the physical predictions of the theory

In particular we described two types of phenomenons associated with the Planckian discrete structure of spacetime: First, if one postulates that the cosmological constant can diffuse towards other degrees of freedom due to friction with elements of 4-volume, as described in Section 6.1, one obtains an inflationary phase fueled by the decaying cosmological constant (see Figure 6.1). If the cosmological constant starts at its natural value $\Lambda_0 \sim M_{\rm Pl}^2$ then after the decay one obtains a value compatible with current observations.

On the other hand, we proposed a mechanism which produces the right amplitude and tilt of the CMB power spectrum. That is, from quantum gravitational we obtain an amplitude N and tilt $n_{\rm S}$ compatible with observations (see (6.89)), with only one free parameter γ. The tilt $n_{\rm S}$ depends on λ, the Higgs self-coupling, and thus completely fixed by the physics of the Standard Model of Particles.

The key insight of the model is the proposal that interactions between the homogeneoous Higgs and the inhomogeneous discrite background structure will excite inhomogeneities in the Higgs that are born and the Planck scale and are modulated by the dimensionless parameter γ. This is contained in equation (6.73).

This proposal provides a framework in which study the consequences of discreteness: one can explore different ways to realize the mechanism. For example one can study this model of structure formation but with a different mechanism of an early inflationary phase: modified gravity[177], non-constant Λ [178, 179], Effective Field Theories [180] among others. One can even think of an early inflationary phase driven by an inflaton, as in the standard account of inflation, but where the primordial seeds of structure are created by the interaction with a discrete background.

This feature of our model makes the creation of the seeds of structure rather independent of the precise nature of background dynamics.

Another issue left for future exploration involves the microscopic description of the decay of the cosmological constant. In the first part of this thesis we explicitly show a model where Planckian degrees of freedom (corresponding to the same macroscopic cosmological constant) were responsible for apparent growth of entropy as measured by coarse-grained observers. A similar mechanism allowing the exchange of energy with these microscopic degrees of freedom can provide a model of a decaying cosmological constant from a model of quantum gravity. This idea could also be reproduced in others approaches to quantum gravity.

The possibility of diffusion towards microscopic degrees of freedom can too provide a clue on the future of a black hole after the high-curvature Planckian region (the region marked QG in Figure 3.4): if energy is allowed to flow towards Planckian degrees of freedom, will the bounce observed in several polymer black holes models[181, 182, 183] remain? In other terms, if a one-degree of freedom egg is dropped from some height it will bounce since the simplified description does not permit another type of behavior. Something similar is happening in the bounce scenarios? If the interaction between low-energy and ultraviolet degrees of freedom is taken into account will the final state be a highly non-trivial, degenerate state containing a myriad of Planckian defects but indistinguishable from flat spacetime for low-energy observers instead of undergoing a highly symmetrical bounce?

All this these factors point out to the paramount importance of the inclusion of the putative discrete, Planckian degrees of freedom when discussing the physics at the Planck scale.

A Revisiting the Weinberg theorem

This is a short review of cosmological perturbation theory and the proof of Weinberg conservation theorem. We follow the notation of [129]. The proof presented here is, we believe, more direct than the one in the textbook we include it for completeness. The metric is split in the usual way as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \tag{A.1}$$

where $\bar{g}_{\mu\nu}$ is the unperturbed, $K = 0$ metric and $h_{\mu\nu}$ is a perturbation. In this section

$$ds_0^2 = -d\tau^2 + a(\tau)^2 \delta_{ij} dx^i dx^j \tag{A.2}$$

The metric perturbation $h_{\mu\nu}$ can be decomposed as

$$\begin{aligned} h_{00} &= -E \\ h_{i0} &= a\left[\frac{\partial F}{\partial x^i} + G_i\right] \\ h_{ij} &= a^2\left[A\delta_{ij} + \frac{\partial^2 B}{\partial x^i \partial x^j} + \frac{\partial C_i}{\partial x^j} + \frac{\partial C_j}{\partial x^i} + D_{ij}\right] \end{aligned} \tag{A.3}$$

where (A, B, E, F), (G_i, C_i) and D_{ij} are scalar, vector and tensor degrees of freedom respectively and

$$\frac{\partial C_i}{\partial x^i} = \frac{\partial G_i}{\partial x^i} = 0, \quad \frac{\partial D_{ij}}{\partial x^i} = 0, \quad D_{ii} = 0 \tag{A.4}$$

In the same way we consider first order perturbations to the energy momentum tensor

$$\begin{aligned} \delta T_{ij} &= \bar{p} h_{ij} + a^2 \left[\delta_{ij} \delta p + \partial_i \partial_j \pi^S + \partial_i \pi_j^V + \partial_j \pi_i^V + \pi_{ij}^T \right] \\ \delta T_{i0} &= \bar{p} h_{i0} - (\bar{\rho} + \bar{p}) \left(\partial_i \delta u + \delta u_i^V \right) \\ \delta T_{00} &= -\bar{\rho} h_{00} + \delta \rho \end{aligned} \tag{A.5}$$

where

$$\partial_i \pi_i^V = \partial_i \delta u_i^V = 0, \quad \partial_i \pi_{ij}^T = 0, \quad \pi_{ii}^T = 0. \tag{A.6}$$

Due to the symmetries of the background it is possible to write the linearised field equations as a set of decoupled equations for scalar, vector and tensor modes. For argument we will only need the equations for scalar and tensor modes.

Scalar Modes

$$\begin{aligned} -4\pi G a^2 \left[\delta \rho - \delta p - \nabla^2 \pi^S \right] = &\frac{1}{2} a \dot{a} \dot{E} + \left(2\dot{a}^2 + a\ddot{a} \right) E + \frac{1}{2} \nabla^2 A - \frac{1}{2} a^2 \ddot{A} \\ &- 3 a \dot{a} \dot{A} - \frac{1}{2} a \dot{a} \nabla^2 \dot{B} + \dot{a} \nabla^2 F \end{aligned} \tag{A.7}$$

$$\partial_j \partial_k \left[16\pi G a^2 \pi^S + E + A - a^2 \ddot{B} - 3 a \dot{a} \dot{B} + 2 a \dot{F} + 4 \dot{a} F \right] = 0 \tag{A.8}$$

$$8\pi G a (\bar{\rho} + \bar{p}) \partial_j \delta u = -\dot{a} \partial_j E + a \partial_j \dot{A} \tag{A.9}$$

$$\begin{aligned} -4\pi G \left(\delta \rho + 3\delta p + \nabla^2 \pi^S \right) = &-\frac{1}{2a^2} \nabla^2 E - \frac{3\dot{a}}{2a} \dot{E} - \frac{1}{a} \nabla^2 \dot{F} - \frac{\dot{a}}{a^2} \nabla^2 F \\ &+ \frac{3}{2} \ddot{A} + \frac{3\dot{a}}{a} \dot{A} - \frac{3\ddot{a}}{a} E + \frac{1}{2} \nabla^2 \ddot{B} + \frac{\dot{a}}{a} \nabla^2 \dot{B} \end{aligned} \tag{A.10}$$

The momentum conservation equations are given by

$$\partial_j \left[\delta p + \nabla^2 \pi^S + \partial_0 [(\bar{\rho} + \bar{p}) \delta u] + \frac{3\dot{a}}{a} (\bar{\rho} + \bar{p}) \delta u + \frac{1}{2} (\bar{\rho} + \bar{p}) E \right] = 0 \tag{A.11}$$

$$\begin{aligned} \delta \dot{\rho} + \frac{3\dot{a}}{a} (\delta \rho + \delta p) + \nabla^2 &\left[-a^{-1} (\bar{\rho} + \bar{p}) F + a^{-2} (\bar{\rho} + \bar{p}) \delta u + \frac{\dot{a}}{a} \pi^S \right] \\ &+ \frac{1}{2} (\bar{\rho} + \bar{p}) \partial_0 \left[3A + \nabla^2 B \right] = 0 \end{aligned} \tag{A.12}$$

Tensor Modes For tensor modes we have only one equation

$$-16\pi G a^2 \pi^T_{ij} = \nabla^2 D_{ij} - a^2 \ddot{D}_{ij} - 3a\dot{a}\dot{D}_{ij} \tag{A.13}$$

A gauge transformation generated by an arbitrary vector field $\epsilon^\mu(x)$

$$x^\mu \to x^\mu + \epsilon^\mu(x), \tag{A.14}$$

induces a transformation on the metric perturbation given by

$$\begin{aligned} \Delta h_{ij} &= -\frac{\partial \epsilon_i}{\partial x^j} - \frac{\partial \epsilon_j}{\partial x^i} + 2a\dot{a}\delta_{ij}\epsilon_0 \\ \Delta h_{i0} &= -\dot{\epsilon}_i - \frac{\partial \epsilon_0}{\partial x^i} + 2\frac{\dot{a}}{a}\epsilon_i \\ \Delta h_{00} &= -2\dot{\epsilon}_0 \end{aligned} \tag{A.15}$$

One can decompose the spatial part of the vector field ϵ^μ into a scalar part and a divergenceless vector :

$$\epsilon_i = \partial_i \epsilon^S + \epsilon^V_i, \quad \partial_i \epsilon^V_i = 0. \tag{A.16}$$

Then the quantities defined in (A.3) transform as

$$\begin{gathered} \Delta A = \frac{2\dot{a}}{a}\epsilon_0, \quad \Delta B = -\frac{2}{a^2}\epsilon^S \\ \Delta C_i = -\frac{1}{a^2}\epsilon^V_i, \quad \Delta D_{ij} = 0, \quad \Delta E = 2\dot{\epsilon}_0 \\ \Delta F = \frac{1}{a}\left(-\epsilon_0 - \dot{\epsilon}^S + \frac{2\dot{a}}{a}\epsilon^S\right), \quad \Delta G_i = \frac{1}{a}\left(-\dot{\epsilon}^V_i + \frac{2\dot{a}}{a}\epsilon^V_i\right) \end{gathered} \tag{A.17}$$

A.0.1 The theorem

To begin, let us concentrate on the scalar mode equations only. The Newtonian gauge is defined by setting $F = B = 0$. However, we will keep the F contributions as we will actually mode away from the condition $F = 0$ in what follows. However, we will change F in a way that does not alter the scalar equations (this is the key of the proof). One renames fields according to

$$E \equiv 2\Phi, \quad A \equiv -2\Psi \tag{A.18}$$

The scalar field equations in the Newtonian gauge become (when $\pi_s = 0$)

$$-4\pi G a^2 \left(\delta\rho - \delta p\right) = a\dot{a}\dot{\Phi} + \left(4\dot{a}^2 + 2a\ddot{a}\right)\Phi - \nabla^2(\Psi - \dot{a}F) + a^2\ddot{\Psi} + 6a\dot{a}\dot{\Psi} \quad \text{(A.19)}$$

$$-8\partial_i\partial_j[\Phi - \Psi + a\dot{F} + 2\dot{a}F] = 0 \quad \text{(A.20)}$$

$$4\pi G a(\bar{\rho} + \bar{p})\partial_i \delta u = -\dot{a}\partial_i\Phi - a\partial_i\dot{\Psi} \quad \text{(A.21)}$$

$$4\pi G\left(\delta\rho + 3\delta p\right) = \frac{1}{a^2}\nabla^2(\Phi + a\dot{F} + \dot{a}F) + \frac{3\dot{a}}{a}\dot{\Phi} + 3\ddot{\Psi} + \frac{6\dot{a}}{a}\dot{\Psi} + \frac{6\ddot{a}}{a}\Phi \quad \text{(A.22)}$$

We first do a gauge transformation $\epsilon^\mu = (\epsilon_0(x^\mu), 0, 0, 0)$ on the background geometry (i.e., we have $h_{\mu\nu} = 0$ to begin with). This gauge transformation—a simple time reparametrization—maintains the Newtonian gauge condition $B = 0$ while it produces sends $F = 0 \to F = -\epsilon_0/a$. We have that

$$\begin{aligned} F &= -\frac{\epsilon_0}{a} \\ \Phi &= \dot{\epsilon}_0 \\ \Psi &= -\frac{\dot{a}}{a}\epsilon_0 \\ B &= 0 \end{aligned} \quad \text{(A.23)}$$

This implies that $\Phi + a\dot{F} + \dot{a}F = 0$ and $\Psi - \dot{a}F = 0$. Therefore, aside from the equation (A.20) where F remains, the form of the other equations is unchanged from Newtonian gauge perturbation equations. There is a way that makes equation (A.20) look like the newtonian gauge equation: it will eventually lead to a further transformation on the Ψ field that, while keeping now all the equations in the Newtonian gauge form, will not be a gauge transformation. This is how a physical solution will emerge from the process. One starts by demanding that the combination (appearing in (A.20)) $a\Delta\dot{F} + 2\dot{a}\Delta F = \mathscr{R}$ for some time independent quantity $\mathscr{R}$. This imposes the following condition on ϵ_0

$$a\frac{d}{d\tau}\left(\frac{\epsilon_0}{a}\right) + 2\frac{\dot{a}}{a}\epsilon_0 = -\mathscr{R}. \quad \text{(A.24)}$$

The time independent quantity $\mathscr{R}$ can now be absorbed in a redefinition of the potential

$$\Psi = -\frac{\dot{a}}{a}\epsilon_0 \to \Psi = -\frac{\dot{a}}{a}\epsilon_0 - \mathscr{R}. \quad \text{(A.25)}$$

Equation (A.21) is identically satisfied due to the Raychaudhuri equation fro the background before demanding that $\partial_i R = 0$ (this is obvious because only the time derivative of Ψ enters this equations). The requiring that $\mathscr{R}$ is also independent of x^i comes from (A.19) in order for $\mathscr{R}$ not to contribute in the Laplacian in (A.19).

Notice that the shift does not modify any other equation as the potential Ψ appears everywhere else inside time-derivatives. The shift of Ψ by a constant does not change the form of the Newtonian gauge perturbation equations. Notice that such shift of Ψ is not a gauge transformation. Equation A.24 is solved by

$$\epsilon_0(\tau) = -\mathscr{R}\frac{\int_T^\tau a(\tau')d\tau'}{a(\tau)} \tag{A.26}$$

We obtain for free the equality between Ψ and Φ that is imposed by equation (A.20) for general modes in the absence of stresses ($\pi_S = 0$), namely

$$\boxed{\Phi = \Psi = -\mathscr{R}\left(1 - H\frac{\int_T^\tau a(\tau')d\tau'}{a(\tau)}\right)}. \tag{A.27}$$

Notice that we did not need to impose the physicality constraint $\Phi - \Psi = 0$ imposed in [129]. Here, it just follows from the consistency of the initial gauge transformation plus the shift in the definition of Ψ. For δu one has

$$\boxed{\delta u_0 = -\epsilon_0 = \mathscr{R}\frac{\int_T^\tau a(\tau')d\tau'}{a(\tau)},} \tag{A.28}$$

and for the matter perturbations we get

$$\boxed{\delta\rho_\alpha = \dot\rho^0_\alpha \epsilon_0 = -\dot\rho^0_\alpha \mathscr{R}\frac{\int_T^\tau a(\tau')d\tau'}{a(\tau)}} \tag{A.29}$$

for any species α. Indeed for any scalar quantity the solution would look the same. The adiabatic property follows from fact that the change have been found via a special gauge transformation ϵ_0 (in fact it can be interpreted as an infinitesimal time reparametrization for scalars and hence it affects all in the same universal way. This is of course a form of equivalence principle at play). We modified the fields in two steps: first a the previous gauge transformation, and second the shift by a constant of Ψ (which restricts the time dependence of the gauge parameter $\epsilon_0(\tau)$). Because the Newtonian gauge perturbation equations are invariant under the previous action, (A.27) and (A.28) define a non-trivial[1] solution of the cosmological perturbation equations that is homogeneous. As such it must be a good approximation to solutions for modes with wavelengths much larger than H^{-1}.

Finally, it is trivial to check that D_{ij} (a traceless constant tensor) is a zero mode

[1] Very importantly, this transformation is not a gauge transformation because of the constant shift in Ψ. This is why gauge invariant observables will have non trivial values in this solution.

solution for tensor modes (A.13)

B Polymer Models and Loop Quantum Cosmology

In this appendix we recall the main ideas and methods of polymer (cosmological) models. We start by reviewing the classical theory and its Hamiltonian description, highlighting the similarities and differences between classical GR and unimodular gravity.

Then we introduce the polymer models and effective dynamics. This section mainly follow the references [96, 184, 27, 185].

B.1 Classical Hamiltonian Dynamics

The spacetime metric of an isotropic and homogeneous Universe is given by the FLRW-metric

$$\mathrm{d}s^2 = -N(t)^2\,\mathrm{d}t^2 + \frac{a(t)^2}{1-kr^2}\,\mathrm{d}r^2 + a(t)^2 r^2\,\mathrm{d}\Omega_2^2, \tag{B.1}$$

where $N(t)$ is the lapse, $a(t)$ the scale factor of the Universe and $k = -1, 0, 1$ is a constant controlling the geometry of the hypersurfaces $t =$ constant.

From now on we will set $k = 0$[1] and consider only the case where the surfaces $t =$ constant are topologically $\mathbb{R}^3$

[1]Polymer models have also been studied in the cases $k \neq 0$[110, 186] and in the case of a homogeneous but non-isotropic spacetime[187, 188, 189, 190, 191]

B.1.1 General Relativity

Now we can insert the isotropic and homogeneous metric (B.1) into the Einstein-Hilbert action with cosmological constant Λ:

$$S_{EH} = \frac{1}{2\kappa}\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,(R-2\Lambda) \tag{B.2}$$

where $\kappa = 8\pi G$. The symmetry-reduced action is then given by

$$S_{EH}[N,a] = -\frac{V_0}{2\kappa}\int \mathrm{d}t\frac{6a(t)\left(a(t)N(t)\ddot{a}(t) - a(t)\dot{a}(t)\dot{N}(t) + N(t)\dot{a}(t)^2\right)}{N(t)^2} - \frac{V_0\Lambda}{\kappa}\int dtN(t)a(t)^3 \tag{B.3}$$

where V_0 is the 3-volume of a fiducial cell.

Rewriting the first term on (B.3) as

$$\frac{6a\left(aNa - a\dot{N}\dot{N} + N\dot{a}^2\right)}{N^2} = \frac{6a\dot{a}^2}{N} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{6a^2\dot{a}}{N}\right), \tag{B.4}$$

integrating by parts and dropping the boundary terms we finally obtain

$$S_{\mathrm{EH}} = -\frac{3V_0}{\kappa}\int dt\frac{a\dot{a}^2}{N} - \frac{V_0\Lambda}{\kappa}\int dt\; N(t)a(t)^3 \tag{B.5}$$

The action for a minimally coupled (free) scalar field is given by

$$S_\phi = -\frac{1}{2}\int_M \mathrm{d}^4x\sqrt{-g}g^{\mu\nu}\left(\nabla_\mu\phi\right)\left(\nabla_\nu\phi\right) = V_0\int \mathrm{d}t\frac{a^3\dot{\phi}^2}{2N} \tag{B.6}$$

The total action of gravity coupled to a free scalar field is then

$$S = \int dt\underbrace{\left(-\frac{3V_0}{\kappa}\frac{a\dot{a}^2}{N} - \frac{V_0\Lambda}{\kappa}N(t)a(t)^3 + V_0\frac{a^3\dot{\phi}^2}{2N}\right)}_{\mathscr{L}} \tag{B.7}$$

where $\mathscr{L}$ is the gravitational Lagrangian and we define $V = V_0a^3$ the physical volume of the fiducial cell.

The symmetry-reduction freezes all gravitational degrees of freedom but the zero-mode describing the overall scale factor of the Universe a. From the Lagrangian $\mathscr{L}$ the standard Hamiltonian analysis can be carried over. The momentum conjugated to a

$$p_a = \frac{\partial L}{\partial \dot{a}} = -\frac{6V_0}{\kappa}\frac{a\dot{a}}{N} \tag{B.8}$$

while the momentum conjugated to the scalar field

$$p_\phi = \frac{\partial L_\phi}{\partial \dot{\phi}} = \frac{V_0 a^3}{N}\dot{\phi} \tag{B.9}$$

while the momentum conjugated to N vanish $p_N \approx 0$[2] as expected for a constrained system.

The total Hamiltonian is then

$$\begin{aligned} \mathscr{H} &= p_a\dot{a} + p_\phi\dot{\phi} + \lambda p_N - L \\ &= N\left(-\frac{\kappa}{12V_0}\frac{p_a^2}{a} + \frac{V_0 a^3}{2\kappa}\Lambda + \frac{p_\phi^2}{2a^3}\right) + \lambda p_N \end{aligned} \tag{B.10}$$

Consistency of $p_N \approx 0$ leads to the secondary constraint

$$\dot{p}_N = \{p_N, \mathscr{H}\} = -\frac{\kappa}{12V_0}\frac{p_a^2}{a} + \frac{V_0 a^3}{2\kappa}\Lambda + \frac{p_\phi^2}{2a^3} =: \mathscr{C} \overset{!}{\approx} 0 \tag{B.11}$$

One can check that $\{\mathscr{C}, \mathscr{H}\} \approx 0$ and thus the constraint is preserved during evolution and the Dirac algorithm concludes in this step.

As expected, N can be treated as a Lagrange multiplier whose dynamics is determined by the arbitrary function λ. This was expected from the behaviour of full GR, the freedom in choosing λ corresponds to gauge transformations, i.e, change of the time coordinate.

The phase space of the theory is spanned by the variables $(\bar{a}, p_a), (\phi, p_\phi)$ whose dy-

[2]Where $\approx$ denotes weak equality

namics is dictated by

$$\mathscr{H} = N\mathscr{C} \quad , \quad \mathscr{C} = -\frac{\kappa}{12V_0}\frac{p_a^2}{a} + \frac{V_0 a^3}{2\kappa}\Lambda + \frac{p_\phi^2}{2a^3} \approx 0. \tag{B.12}$$

The symplectic structure is given by

$$\{a, p_a\} = \{\phi, p_\phi\} = 1 \tag{B.13}$$

In Loopq Quantum Cosmology is customary to consider different pair of variables canonically conjugated to $(\bar{a}, p_a)$. For completeness we quote here the two more common change of variables considered in the literature (See for example [96])

$$\begin{aligned} c &= V_0^{1/3}\gamma\frac{\dot{a}}{N} \\ p &= V_0^{2/3}a^2, \end{aligned} \tag{B.14}$$

where γ is the Immirzi parameter.

It is easy to see[96] that c completely characterizes the gravitational spin connection A_a^i and p, its conjugated, completely characterizes the electric field A_i^aconjugated to the spin connection.

In these variables the Hamiltonian becomes

$$\mathscr{C} = -\frac{6}{\kappa\gamma^2}\sqrt{|p|}c^2 + \frac{\Lambda|p|^{3/2}}{2\kappa} + \frac{p_\phi^2}{2|p|^{3/2}} \tag{B.15}$$

Usually one can also find in the literature the pais of variables

$$\begin{aligned} V &= V_0 a^3 = |p|^{3/2} \\ b &= -\frac{\kappa\gamma}{6V_0}\frac{p_a}{a^2} = \frac{c}{|p|^{1/2}}. \end{aligned} \tag{B.16}$$

V is the 3-volume of the fiducial cell and b is related (dynamically) to the Hubble rate $H = \frac{\dot{a}}{Na}$

In these variables the Hamiltonian constraint

$$\mathscr{C} = -\frac{3}{2\kappa\gamma^2} b^2 V + \frac{1}{2\kappa} V\Lambda + V_0 \frac{p_\phi^2}{2V} \tag{B.17}$$

and the symplectic structure is given by

$$\{b, V\} = 4\pi G\gamma = \kappa\gamma \tag{B.18}$$

Now that we have already established the theory's kinematical structure, we can study the solutions of the Hamilton equations of motion.

For solving the equations of motion, we have to deal with the arbitrary function N. One strategy usually employed in the literature is to deparametrize the system using the scalar field as a physical clock.

Another strategy is to choose the gauge (and hence the physical interpretation of the time coordinate t), which amounts to determine the functional dependence of N.

This problem will not exist in unimodular cosmology: the constraint fixing a background volume element provides us with a unique notion of time: the cosmological time T.

Comoving Gauge $N = 1$

$\underline{\Lambda = 0}$The constrain becomes:

$$\mathscr{C}^1 = -\frac{3}{2\kappa\gamma^2} b^2 V + V_0 \frac{p_\phi^2}{2V}, \tag{B.19}$$

and now we may compute the equations of motion for b and V:

$$\begin{aligned} \dot{V} &= \{V, \mathscr{C}^1\} = -\kappa\gamma \frac{\partial \mathscr{C}^1}{\partial b} = \frac{3}{\gamma} bV \\ \dot{b} &= \{b, \mathscr{C}^1\} = \kappa\gamma \frac{\partial \mathscr{C}^1}{\partial V} = \kappa\gamma \left(-\frac{3}{2\kappa\gamma^2} b^2 - V_0 \frac{p_\phi^2}{2V^2} \right). \end{aligned} \tag{B.20}$$

Using the constraint we obtain the pair of evolution equations for b and V

$$\begin{aligned}\dot{V} &= \frac{3}{\gamma} b V \\ \dot{b} &= \frac{3}{\gamma} b^2 .\end{aligned} \tag{B.21}$$

We can solve for b

$$b(t) = \frac{\gamma b_0}{3 b_0 (t - t_0) + \gamma}, \tag{B.22}$$

and after solve for the volume $V(t)$

$$V(t) = V_0 \frac{3 b_0 (t - t_0) + \gamma}{\gamma}, \tag{B.23}$$

Now we solve for the scalar field[3] using the constraint (B.19)

$$\phi(t) = \phi_0 + \sqrt{\frac{3 V_0}{\kappa \gamma^2}} \frac{\gamma}{3} \log \left[\frac{3 b_0 (t - t_0) + \gamma}{\gamma} \right] \tag{B.24}$$

Everything is well defined as long as $3 b_0 (t - t_0) + \gamma > 0 \implies t > t_0 - \frac{\gamma}{3 b_0}$. Moreover in that interval ϕ is monotonically increasing and singled-valued so we can use it as a time variable. In particular the volume behaves exponentially respect to ϕ

$$V(\phi) = V_0 \exp \left[\pm \sqrt{\frac{\kappa \gamma^2}{3 V_0}} \frac{3}{\gamma} (\phi - \phi_0) \right] \tag{B.25}$$

We thus have that $\rho \propto a^{-3(1+\omega)} = a^{-6}$ and $a \propto t^{\frac{2}{3(1+\omega)}} = t^{\frac{1}{3}} \implies V \propto t$ as we derived in (B.23).

$\Lambda \neq 0$The constraint is given simply by

$$\mathscr{C}^1 = -\frac{3}{2 \kappa \gamma^2} b^2 V + \frac{1}{2 \kappa} V \Lambda + V_0 \frac{p_\phi^2}{2V}, \tag{B.26}$$

[3] A free scalar field can be shown to behave as a perfect fluid with equation of state $\rho = P$, or in another words $\omega = 1$.

and then the equation of motion become

$$\dot{V} = \frac{3}{\gamma} b V$$
$$\dot{b} = -\frac{3}{\gamma} b^2 + \gamma\Lambda \tag{B.27}$$

where in the last equation we used the scalar constraint $\mathscr{C}^1 \approx 0$.

One can solve (B.27) for $b(t)$ and $V(t)$ obtaining

$$b(t) = -\gamma\sqrt{\frac{|\Lambda|}{3}} \tan\left[\sqrt{3|\Lambda|}\left(t - \gamma b_0\right)\right] \qquad \text{if } \Lambda < 0$$
$$b(t) = \gamma\sqrt{\frac{\Lambda}{3}} \tanh\left[\sqrt{3\Lambda}\left(t - \gamma b_0\right)\right] \qquad \text{if } \Lambda > 0. \tag{B.28}$$

And respectively [4]

$$V(t) = V_0 \cos\left[\sqrt{3|\Lambda|}\left(t - \gamma b_0\right)\right] \qquad \text{if } \Lambda < 0$$
$$V(t) = V_0 \cosh\left[\sqrt{3\Lambda}\left(t - \gamma b_0\right)\right] \qquad \text{if } \Lambda > 0. \tag{B.29}$$

Pure Cosmological Constant The scalar constraint becomes simply

$$\mathscr{C}^1 = -\frac{3}{2\kappa\gamma^2} b^2 V + \frac{1}{2\kappa}\Lambda V, \tag{B.30}$$

The eom are given by

$$\dot{V} = \frac{3}{\gamma} b V$$
$$\dot{b} = 0 \implies b(t) = b_0 = \gamma\sqrt{\frac{\Lambda}{3}} \tag{B.31}$$

[4]When all the matter has already diluted by the expansion the cosmological constant dominates: if $\Lambda < 0$ then it acts in an attractive manner and the models recollapse (for every k), whereas for $\Lambda > 0$ the cosmological constant term act repulsively and makes the universe expand forever (in the $k = 0, -1$ case) whereas in the $k = 1$ case there is a fight between the matter acting attractively and the cosmological constant term acting repulsively and it depends on the amount of matter which term wins.

And then we have that $V(t) = V_0 \exp\left(\frac{3}{\gamma} b_0 t\right) = V_0 \exp\left(3\sqrt{\frac{\Lambda}{3}}t\right)$ and from it we obtain that $a(t) = \exp\left(\sqrt{\frac{\Lambda}{3}}t\right)$ which is, as expected, the de Sitter solution in comoving time. Notice also that this derivation is only valid for $\Lambda > 0$.

Harmonic Gauge $N = a^3 = \frac{V}{V_0}$

The scalar constraint becomes

$$\mathscr{C}^h = -\frac{3}{2V_0\kappa\gamma^2} b^2 V^2 + \frac{1}{2V_0\kappa} V^2 \Lambda + \frac{p_\phi^2}{2}, \tag{B.32}$$

and the eom

$$\begin{aligned} \frac{dV}{ds} &= \frac{3}{\gamma V_0} b V^2 \\ \frac{db}{ds} &= -\frac{3}{V_0\gamma} b^2 V + \frac{\gamma}{V_0} \Lambda V \end{aligned} \tag{B.33}$$

In the case where $\Lambda = 0$ the solution of the system (B.33) is given by:

$$\begin{aligned} V(s) &= \frac{c_1}{c_2} e^{\frac{3c_1 s}{\gamma V_0}} \\ b(s) &= c_2 e^{-\frac{3c_1 s}{\gamma V_0}} \end{aligned} \tag{B.34}$$

Using the scalar constraint we can see that:

$$\mathscr{C}^h \approx 0 \implies \left(\frac{d\phi}{ds}\right)^2 = \frac{3}{V_0\kappa\gamma^2} b^2 V^2 \tag{B.35}$$

The, using (B.34)

$$\frac{d\phi}{ds} = \sqrt{\frac{3}{V_0\kappa\gamma^2}} c_1 \implies \phi(s) = \pm\sqrt{\frac{3}{V_0\kappa\gamma^2}} c_1 s + \phi_0 \tag{B.36}$$

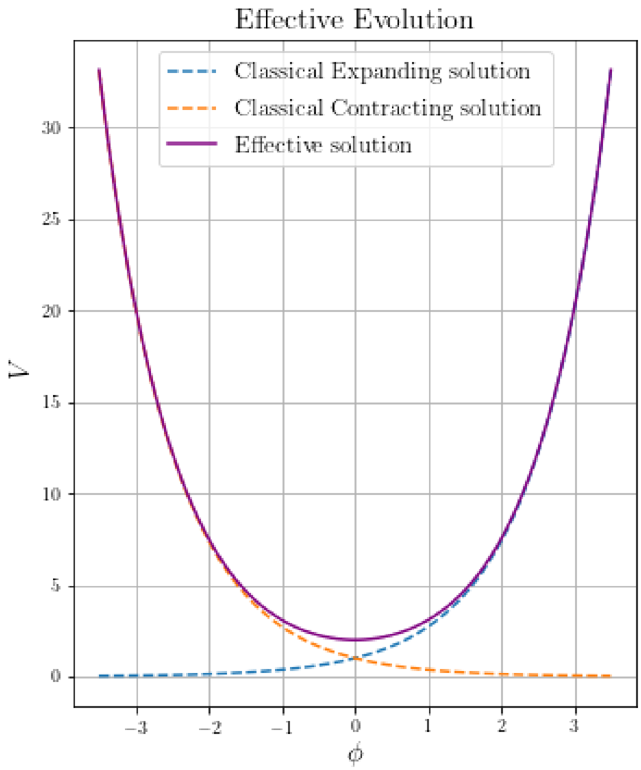

Figure B.1 – Classical and effective evolution of $V(\phi)$. We see that the Universe in the effective dynamics undergoes a bounce at some minimal volume value and it coincides with the classical solution for large V, far from the bounce.

And finally we re-obtain the deparametrized curvee

$$\phi(V) = \sqrt{\frac{V_0}{3\kappa}}\log\left(\frac{c_2}{c_1}V\right) + \phi_0 \tag{B.37}$$

Unimodular Gauge

Let us look now at the case when $N = a^{-3}$, i.e. the ***unimodular gauge***

$$\mathscr{C}^u = -\frac{3V_0}{2\kappa\gamma^2}b^2 + \frac{V_0}{2\kappa}\Lambda, \tag{B.38}$$

In this case the eom of motion become

$$\begin{aligned} \frac{dV}{d\tau} &= \frac{3V_0}{\gamma} b \\ \frac{db}{d\tau} &= 0 \implies b(\tau) = b_0 = \gamma\sqrt{\frac{\Lambda}{3}} \end{aligned} \tag{B.39}$$

And thus we can solve for $V(t)$ obtaining

$$V(\tau) = 3V_0\sqrt{\frac{\Lambda}{3}}(\tau-\tau_0) \implies a(\tau) \propto (\tau-\tau_0)^{1/3}, \tag{B.40}$$

where τ is the Unimodular time.

B.1.2 Unimodular Gravity

The Einstein-Hilbert action supplemented with the unimodular constraint become

$$S_{\text{u}} = \kappa\int\sqrt{-g}R + \lambda(\sqrt{-g}-1) = -\kappa V_0\int\left(6\frac{a\dot{a}^2}{N} - \lambda(Na^3-1)\right)dt, \tag{B.41}$$

where as in (B.7) total derivative terms have been eliminated, and the 3-volume V_0 of a fiducial cell has been introduced.

Resolving the unimodular constraint fixes the lapse $N = 1/a^3$ and thus the unimodular action becomes

$$S_{\text{u}} = -\kappa V_0\int 6a^4\dot{a}^2 dt. \tag{B.42}$$

In this case, due to the presence of a new constraint $N = \frac{V_0}{V}$, the Hamiltonian do not longer vanish but it's a constant. That is, the Hamiltonian is no longer a constraint and we recover true dynamics in the Unimodular time.

Using the variables introduced in (B.16) the gravitational action then is

$$S_{\text{u}} = -\int\frac{1}{2}m\dot{V}^2 dt. \tag{B.43}$$

with

$$m \equiv \frac{4}{3V_0}\kappa. \tag{B.44}$$

Notice this is just (modulo an overall sign) the action for a free particle. This highlights another clear advantage of working with unimodular gravity: when matter is introduced we will have a problem analogous of a particle in a potential.

Also notice that if we use comoving time $d\tau = N dt$ with $N = 1/a^3$ we have

$$p = m\dot{V} = mNV' = 3m\frac{a'}{a}, \tag{B.45}$$

where the prime denotes derivative with respect to the comoving time. We see that the momentum variable in our parametrization is just proportional to the Hubble rate in usual comoving variables! Therefore, the action including our simple matter model is

$$\boxed{S(V,\phi) = \int\left(\frac{1}{2}m\dot{V}^2 - \frac{1}{2}V^2\dot{\phi}^2 - V_0\mathscr{U}(\phi)\right)dt} \tag{B.46}$$

In this case the Hamiltonian in the (b, V) is simply given by

$$\mathscr{H} = -\frac{3V_0}{2\kappa\gamma^2}b^2 + V_0^2\frac{p_\phi^2}{2V^2} = -\frac{V_0}{2\kappa}\Lambda, \tag{B.47}$$

We use the Hamiltonian[5] $\mathscr{H}$ to obtain the equations of motion

$$\begin{aligned} \frac{dV}{d\tau} &= \{V,\mathscr{H}\} = \frac{3V_0}{\gamma}b \\ \frac{db}{d\tau} &= \{b,\mathscr{H}\} = \gamma V_0\frac{\Lambda}{V} - \frac{3V_0}{\gamma}\frac{b^2}{V} \end{aligned} \tag{B.48}$$

We can solve for $b(\tau), V(\tau)$ obtaining

$$\begin{aligned} V(\tau) &= \frac{1}{\gamma}\sqrt{\frac{3\gamma^2\Lambda^2(\tau V_0 - \gamma c_1 c_2)^2 - {c_1}^2}{\Lambda}} \\ b(\tau) &= \pm\frac{\gamma^2\Lambda^{3/2}(\tau V_0 - \gamma c_1 c_2)}{\sqrt{3\gamma^2\Lambda^2(\tau V_0 - \gamma c_1 c_2)^2 - {c_1}^2}} \end{aligned} \tag{B.49}$$

[5] Which now is not a constraint

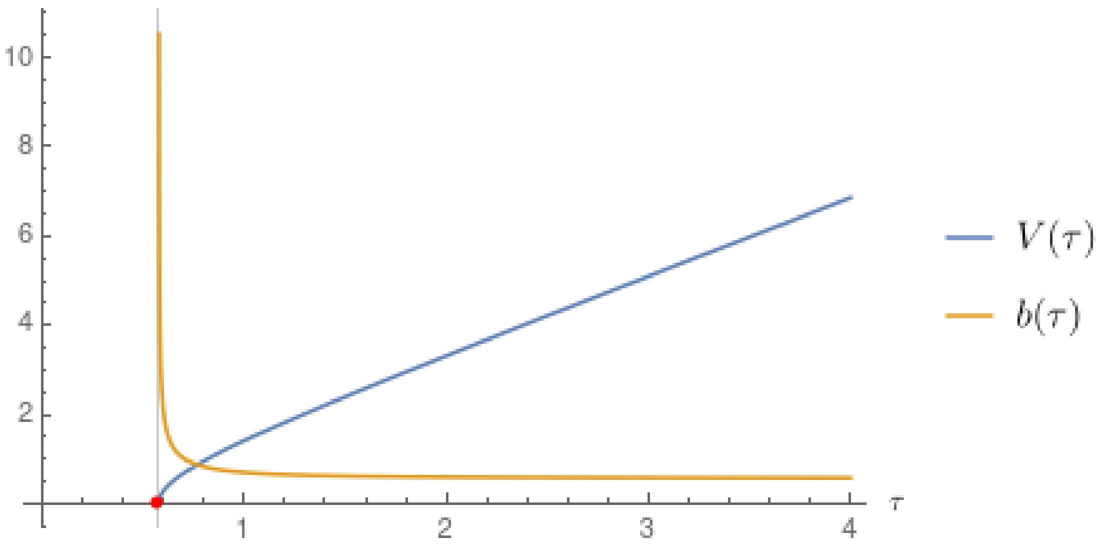

Figure B.2 – **Unimodular Gauge** - $V(\tau)$ and $b(\tau)$ for $V_0 = \gamma = \Lambda = 1$, $c_1 = 1$ $c_2 = 0$

And using (B.48) we can solve for the scalar field

$$\phi(\tau) = \sqrt{\frac{V_0}{6\kappa}} \tanh^{-1}\left(\frac{\sqrt{3}\gamma\Lambda(\tau V_0 - \gamma c_1 c_2)}{c_1}\right) + \phi_0 \tag{B.50}$$

and finally we can write $\phi = \phi(V)$

$$\phi(V) = \sqrt{\frac{V_0}{6\kappa}} \tanh^{-1}\left(\frac{\sqrt{c_1^2 + \gamma^2\Lambda^2 V^2}}{c_1}\right) + \phi_0 \tag{B.51}$$

Finally, note that the change in *Unimodular time* $\Delta t = \int_{\tau_1}^{\tau_2} d\tau\ a(\tau)^3$ (where τ is the comoving time defined by $d\tau = a^{-3} dt$) can be interpreted as the 4-volume of the world tube traced by a fiducial cell of initially Planckian volume between (comoving) time τ_1 and τ_2.

B.2 Effective Equations

In this section, we will discuss how symmetry-reduced cosmological models can be quantized motivated by the insight of LQG. We start by studying the classical effective dynamics. The idea behind it is to modify the Hamiltonian using a so-called polymerization scheme that is supposed to encode the dynamics of a semi-classical state in the quantum theory[192].

In practice, this is achieved by replacing certain variables in the Hamiltonian (e.g. in (B.17)) by *point holonomies*. In cosmology a suitable choice is to replace b by

$$b \to e^{ib}. \tag{B.52}$$

This replacement is based on a rationale borrowed from LQG: the connection is not represented in the Hilbert space of LQG, but only its holonomies are defined on the quantum theory. This idea is realized in the symmetry-reduced models by considering only functions of the type e^{ib}. The, the Hamiltonian is regularized by replacing

$$b \to f(e^{i\lambda b}) \tag{B.53}$$

where $\lambda \in \mathbb{R}_+$ is the regularization parameter and f is an arbitrary function such that $f\left(e^{i\lambda b}\right) \xrightarrow{\lambda b \ll 1} b$. This requirement is such that the correct classical behaviour is recovered in the appropiated limit. The ambiguity in the choice of f can be used to obtain an arbitrary dynamical behaviour of the scale factor a[176].

In the classical theory the variable b is related to the Hubble rate and thus to the scalar curvature. It becomes large only in high curvatures regimes. If the regularization parameter is chosen so that $\lambda \sim \ell$[6], the effective theory will depart the classical theory only in the Planckian regime, where the classical theory is expected to break down. The usual choice in the literature is to do the following replacement in the Hamiltonian

$$b \to \frac{\sin(\lambda b)}{\lambda} \tag{B.54}$$

The effective Hamiltonian is then given by

$$\mathcal{H}^{\text{eff}} = N\mathcal{C}^{\text{eff}} \quad , \quad \mathcal{C}^{\text{eff}} = -\frac{3}{2\kappa\gamma^2}\frac{\sin(\lambda b)^2}{\lambda^2}V + \frac{1}{2\kappa}V\Lambda + V_0\frac{p_\phi^2}{2V}. \tag{B.55}$$

One can now find the equations of motion generated by the effective Hamiltonian (B.55). This calculation is still classical and its meant to provide a somewhat accurate description of the dynamics of semi-classical states in the quantum theory. To assess

[6]In principle λ can be a function of the phase space variables. The case λ = constant is known as the $\bar{\mu}$-scheme[96].

whether this description is accurate or not, it is necessary to study the quantum theory. In [192] (and in [176] for a more general situation), it is shown that the effective equations are accurate for a particular class of states.

The effective equations of motion in the comoving gauge are given by

$$\begin{aligned}
\dot{v} &= -v\frac{\sin(\lambda b)}{\lambda}\cos(\lambda b) \\
\dot{b} &= \frac{1}{2}\frac{\sin(\lambda b)^2}{\lambda^2} + \frac{p_\phi^2}{2v^2} \\
\dot{\phi} &= \frac{p_\phi}{v} \\
\dot{p}_\phi = 0 &\implies p_\phi = \text{const.} \\
0 &= -\frac{v}{2}\frac{\sin(\lambda b)^2}{\lambda^2} + \frac{p_\phi^2}{2v}
\end{aligned} \tag{B.56}$$

Using the Hamiltonian constraint one can solve for the volume in comoving time

$$V(t) = \sqrt{\left(\pm p_\phi (t - t_i) + \sqrt{V_i^2 - \lambda^2 p_\phi^2}\right)^2 + \lambda^2 p_\phi^2} \tag{B.57}$$

where $V(t_i) = V_i$ is the initial condition. First, one sees that the volume is bounded by below by $V_{min} = \lambda p_\phi$ and thus the singularity is *resolved* as the curvature is bounded $\mathbf{R} \leq \frac{2}{\lambda^2}$.

Finally, it can be shown that

$$v(\phi) = \lambda p_\phi \cosh\left((\phi - \phi_i) + \cosh^{-1}\left(\frac{v_i}{\lambda p_\phi}\right)\right) \tag{B.58}$$

The plot of the two classical branches (B.25) and the effective solution (B.58) is given in Figure B.1.

C Decaying Λ models

C.1 Background Dynamics

The background dynamics of our model is characterised by the continuity equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + P\right) = -\frac{\dot{\Lambda}}{8\pi G} \tag{C.1}$$

which can be written in terms of the number of e-folds $\mathcal{N} = \log(a)$ as

$$\frac{d\rho}{d\mathcal{N}} + 3\left(\rho + P\right) = -\frac{d\Lambda}{d\mathcal{N}}\frac{1}{8\pi G} \tag{C.2}$$

In our paper the cosmological constant is modeled by a decaying exponential in Unimodular time. We want to show in these notes that the inflation mechanism that we introduced is independent of the particular details of this decay. In other words, the mechanism still produces the right amount of inhomogeneities provided that there is an early inflatonary phase fueled by an almost constant Λ followed by an abrupt decay of the cosmological constant into the matter sector.

C.1.1 A discontinuous decay

Let us start by looking at a cosmological constant that decays instantaneously at some arbitrary number of e-folds $\mathcal{N}_0$

$$\Lambda(a) = \begin{cases} \Lambda_0 & \mathcal{N} < \mathcal{N}_0 \\ 0 & \mathcal{N}_0 < \mathcal{N} \end{cases} \tag{C.3}$$

or, equivalently

$$\frac{d\Lambda}{d\mathcal{N}} = -\Lambda_0\,\delta\left(\mathcal{N}-\mathcal{N}_0\right) \tag{C.4}$$

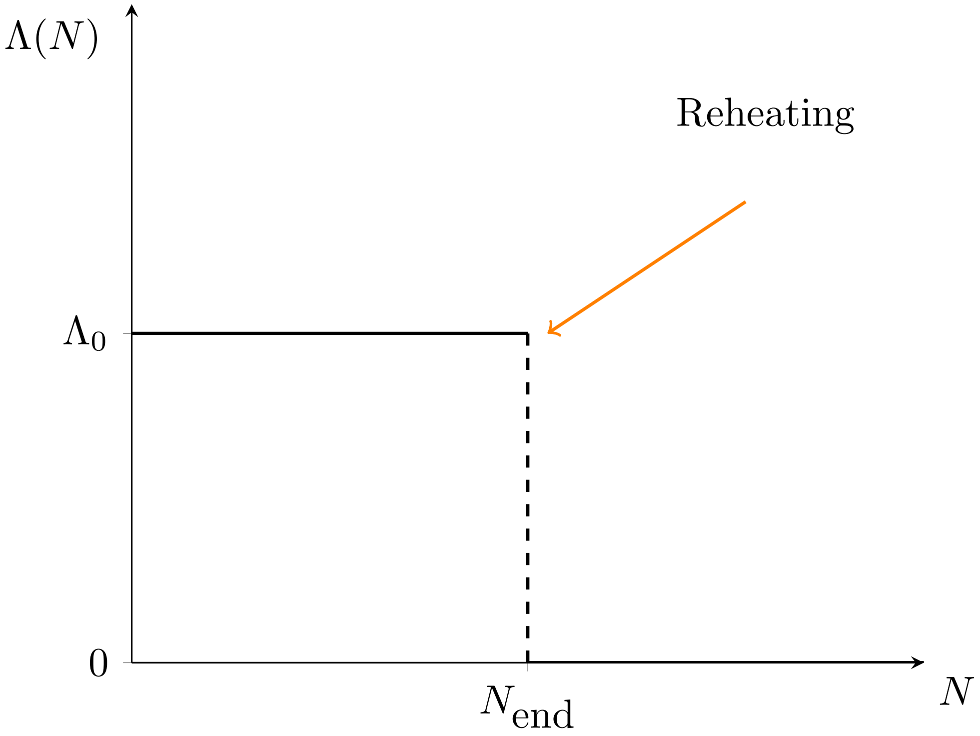

Figure C.1 – Ultra simplified model for the decay of the cosmological constant. The phenomenology remains the same as in the model presented in the first sections of the chapter. The only change is the value of the radiation density right after the reheating event which remains of the same order of magnitude of that predicted in Section 6.2. N_{end} denotes the end of inflation.

In this case we can solve the continuity equation (C.2) for ρ in the two regions $\mathcal{N} < \mathcal{N}_0$ and $\mathcal{N} > \mathcal{N}_0$ and match across the discontinuity (as is done for example for the Schroedinger equation with a delta potential).

We integrate the continuity equation in a interval around $\mathcal{N}_0$

$$\int_{\mathcal{N}_0-\epsilon}^{\mathcal{N}_0+\epsilon} d\mathcal{N}\,\frac{d\rho}{d\mathcal{N}} + 3\,(1+\omega)\int_{\mathcal{N}_0-\epsilon}^{\mathcal{N}_0+\epsilon} d\mathcal{N}\,\rho = \frac{\Lambda_0}{8\pi G}\int_{\mathcal{N}_0-\epsilon}^{\mathcal{N}_0+\epsilon} d\mathcal{N}\,\delta\left(\mathcal{N}-\mathcal{N}_0\right) \tag{C.5}$$

Taking the limit $\epsilon \to 0$ and supposing that ρ is piecewise continuos and bounded in this interval we obtain

$$\Delta\rho \equiv \rho\left(\mathcal{N}_0^+\right) - \rho\left(\mathcal{N}_0^-\right) = \frac{\Lambda_0}{8\pi G} \tag{C.6}$$

and using that $H_0^2 \sim \frac{\Lambda_0}{3}$ we recover the estimate we had in our paper

$$\rho_{\text{end}} \sim \frac{9H_0^2 m_p^2}{56\pi} \tag{C.7}$$

C.1.2 A sudden linear decay

For the non-continuos decay model we considered last section the energy density ρ is non continuous. Let us consider now a scenario where Λ is piecewise smooth and thus ρ is continuous.

$$\Lambda(a) = \begin{cases} \Lambda_0 & \mathcal{N} < \mathcal{N}_0 \\ \alpha\mathcal{N} + \beta & \mathcal{N}_0 < \mathcal{N} < \mathcal{N}_0 + \Delta\mathcal{N} \\ 0 & \mathcal{N} < \mathcal{N}_0 + \Delta\mathcal{N} < \mathcal{N} \end{cases} \tag{C.8}$$

The coefficients α and β for a continuous $\Lambda(a)$ are given by

$$\alpha = -\frac{\Lambda_0}{\Delta\mathcal{N}} \qquad \beta = \Lambda_0\left(1 + \frac{\mathcal{N}_0}{\Delta\mathcal{N}}\right) \tag{C.9}$$

As before, we solve the equation in the three different regions and we match the solutions at $\mathcal{N}_0$ and $\mathcal{N}_0 + \Delta\mathcal{N}$.

By solving the continuity equation we obtain the three solutions

$$\Lambda(a) = \begin{cases} \rho_- e^{-3(1+\omega)(\mathcal{N}-\mathcal{N}_0)} & \mathcal{N} < \mathcal{N}_0 \\ \rho_0 e^{-3(1+\omega)(\mathcal{N}-\mathcal{N}_0)} + \frac{\Lambda_0 M_{\text{p}}^2}{8\pi\Delta\mathcal{N}}\frac{1}{3(1+\omega)} & \mathcal{N}_0 < \mathcal{N} < \mathcal{N}_0 + \Delta\mathcal{N} \\ \rho_+ e^{-3(1+\omega)(\mathcal{N}-\mathcal{N}_0)} & \mathcal{N}_0 + \Delta\mathcal{N} < \mathcal{N} \end{cases} \tag{C.10}$$

Let us define also $\rho_* = \rho_- e^{3(1+\omega)\mathcal{N}_0}$, the value of the energy density at $\mathcal{N} = 0$. The matching conditions give us the pair of equations

$$\begin{aligned}\rho_- &= \rho_0 + \frac{\Lambda_0 M_{\rm P}^2}{8\pi\Delta\mathcal{N}}\frac{1}{3(1+\omega)}\\ \rho_+ &= \rho_0 + \frac{\Lambda_0 M_{\rm P}^2}{8\pi\Delta\mathcal{N}}\frac{e^{3(1+\omega)\Delta\mathcal{N}}}{3(1+\omega)}\end{aligned} \tag{C.11}$$

One can show that the energy density in the *decay* region can be written as

$$\begin{aligned}\rho(\mathcal{N}) &= \left(\rho_* e^{-3(1+\omega)\mathcal{N}_0} - \frac{\Lambda_0 M_{\rm P}^2}{8\pi\Delta\mathcal{N}}\frac{1}{3(1+\omega)}\right)e^{-3(1+\omega)(\mathcal{N}-\mathcal{N}_0)} + \frac{\Lambda_0 M_{\rm P}^2}{8\pi\Delta\mathcal{N}}\frac{1}{3(1+\omega)}\\ &= \rho_* e^{-3(1+\omega)\mathcal{N}} + \frac{\Lambda_0 M_{\rm P}^2}{8\pi\Delta\mathcal{N}}\frac{1}{3(1+\omega)}\left(1-e^{-3(1+\omega)(\mathcal{N}-\mathcal{N}_0)}\right)\end{aligned} \tag{C.12}$$

It can be shown that $\Delta\rho \equiv \rho(\mathcal{N}_0) - \mathcal{N}_0 + \Delta\mathcal{N}$, that is, the change in energy density duing the *decay* phase

$$\Delta\rho = \rho_* e^{-3(1+\omega)\mathcal{N}_0}\left(1-e^{-3(1+\omega)\Delta\mathcal{N}}\right) + \frac{\Lambda_0 M_{\rm P}^2}{8\pi}\frac{1}{3(1+\omega)}\frac{1-e^{-3(1+\omega)\Delta\mathcal{N}}}{\Delta\mathcal{N}} \tag{C.13}$$

And we see that in the limit $\Delta\mathcal{N} \to 0$, $\Delta\rho$ reduces to the *delta decay* value, as expected

$$\Delta\rho(\Delta\mathcal{N}\to 0) = \frac{\Lambda_0 M_{\rm P}^2}{8\pi} \tag{C.14}$$

where we have used that $\lim_{x\to 0}\frac{1-e^{-\alpha x}}{x} = \alpha$.

Bibliography

[1] C. M. Will, "The confrontation between general relativity and experiment," *Living Reviews in Relativity*, vol. 17, p. 4, Jun 2014.

[2] B. C. Odom, D. Hanneke, B. D'Urso, and G. Gabrielse, "New Measurement of the Electron Magnetic Moment Using a One-Electron Quantum Cyclotron," *Phys. Rev. Lett.*, vol. 97, p. 030801, 2006. [Erratum: Phys. Rev. Lett.99,039902(2007)].

[3] G. Gabrielse, D. Hanneke, T. Kinoshita, M. Nio, and B. C. Odom, "New Determination of the Fine Structure Constant from the Electron g Value and QED," *Phys. Rev. Lett.*, vol. 97, p. 030802, 2006. [Erratum: Phys. Rev. Lett.99,039902(2007)].

[4] B. P. Abbott *et al.*, "Observation of Gravitational Waves from a Binary Black Hole Merger," *Phys. Rev. Lett.*, vol. 116, no. 6, p. 061102, 2016.

[5] B. P. Abbott *et al.*, "GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence," *Phys. Rev. Lett.*, vol. 116, no. 24, p. 241103, 2016.

[6] B. P. Abbott *et al.*, "GW170814: A Three-Detector Observation of Gravitational Waves from a Binary Black Hole Coalescence," *Phys. Rev. Lett.*, vol. 119, no. 14, p. 141101, 2017.

[7] B. P. Abbott *et al.*, "GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2," *Phys. Rev. Lett.*, vol. 118, no. 22, p. 221101, 2017.

[8] B. Abbott *et al.*, "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral," *Phys. Rev. Lett.*, vol. 119, no. 16, p. 161101, 2017.

[9] M. Bronstein, "Quantum theory of weak gravitational fields," *Gen. Rel. Grav.*, vol. 44, pp. 267–283, 2012.

[10] S. Hossenfelder, "Minimal Length Scale Scenarios for Quantum Gravity," *Living Rev. Rel.*, vol. 16, p. 2, 2013.

[11] C. Rovelli and F. Vidotto, *Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory.* Cambridge Monographs on Mathematical Physics, Cambridge University Press, 11 2014.

[12] R. M. Wald, *General relativity.* University of Chicago press, 2010.

[13] T. Thiemann, "Closed formula for the matrix elements of the volume operator in canonical quantum gravity," *J. Math. Phys.*, vol. 39, pp. 3347–3371, 1998.

[14] C. Rovelli and L. Smolin, "Discreteness of area and volume in quantum gravity," *Nucl. Phys.*, vol. B442, pp. 593–622, 1995. [Erratum: Nucl. Phys.B456,753(1995)].

[15] A. Perez, "No firewalls in quantum gravity: the role of discreteness of quantum geometry in resolving the information loss paradox," *Class. Quant. Grav.*, vol. 32, no. 8, p. 084001, 2015.

[16] E. Fermi, "Tentativo di una teoria dell'emissione dei raggi beta," *Ric. Sci.*, vol. 4, pp. 491–495, 1933.

[17] E. Fermi, "An attempt of a theory of beta radiation. 1.," *Z. Phys.*, vol. 88, pp. 161–177, 1934.

[18] L. Amadei and A. Perez, "Inflation from the relaxation of the cosmological constant," 4 2021.

[19] K. C. Wong *et al.*, "H0LiCOW – XIII. A 2.4 per cent measurement of H0 from lensed quasars: 5.3σ tension between early- and late-Universe probes," *Mon. Not. Roy. Astron. Soc.*, vol. 498, no. 1, pp. 1420–1439, 2020.

[20] L. Verde, T. Treu, and A. G. Riess, "Tensions between the Early and the Late Universe," *Nature Astron.*, vol. 3, p. 891, 7 2019.

[21] E. Di Valentino, O. Mena, S. Pan, L. Visinelli, W. Yang, A. Melchiorri, D. F. Mota, A. G. Riess, and J. Silk, "In the realm of the Hubble tension—a review of solutions," *Class. Quant. Grav.*, vol. 38, no. 15, p. 153001, 2021.

[22] A. Perez and D. Sudarsky, "Dark energy from quantum gravity discreteness," *arXiv preprint arXiv:1711.05183*, 2017.

[23] A. Perez, D. Sudarsky, and E. Wilson-Ewing, "Resolving the H_0 tension with diffusion," *Gen. Rel. Grav.*, vol. 53, no. 1, p. 7, 2021.

[24] A. Perez, D. Sudarsky, and J. D. Bjorken, "A microscopic model for an emergent cosmological constant," *arXiv preprint arXiv:1804.07162*, 2018.

[25] T. Josset, A. Perez, and D. Sudarsky, "Dark energy as the weight of violating energy conservation," *Phys. Rev. Lett.*, vol. 118, no. 2, p. 021102, 2017.

[26] L. Amadei and A. Perez, "Hawking's information puzzle: a solution realized in loop quantum cosmology," 11 2019.

[27] L. Amadei, H. Liu, and A. Perez, "Unitarity and information in quantum gravity: a simple example," *Front. Astron. Space Sci.*, vol. 8, p. 46, 2021.

[28] L. Amadei and A. Perez, "Polymer black holes: A solution to the black hole information paradox." in preparation.

[29] I.-O. Stamatescu and E. Seiler, "Approaches to fundamental physics: An assessment of current theoretical ideas," *Lect. Notes Phys.*, vol. 721, pp. 1–422, 2007.

[30] L. J. Garay, "Quantum gravity and minimum length," *Int. J. Mod. Phys.*, vol. A10, pp. 145–166, 1995.

[31] B. Dittrich and T. Thiemann, "Are the spectra of geometrical operators in Loop Quantum Gravity really discrete?," *J. Math. Phys.*, vol. 50, p. 012503, 2009.

[32] A. Ashtekar, J. Baez, A. Corichi, and K. Krasnov, "Quantum geometry and black hole entropy," *Phys. Rev. Lett.*, vol. 80, pp. 904–907, 1998.

[33] C. Rovelli, "Black hole entropy from loop quantum gravity," *Phys. Rev. Lett.*, vol. 77, pp. 3288–3291, 1996.

[34] R. Penrose, "On the Gravitization of Quantum Mechanics 1: Quantum State Reduction," *Found. Phys.*, vol. 44, pp. 557–575, 2014.

[35] C. Rovelli, *Quantum gravity.* Cambridge Monographs on Mathematical Physics, Cambridge, UK: Univ. Pr., 2004.

[36] T. Thiemann, "Modern canonical quantum general relativity," 2001.

[37] D. Oriti, *Approaches to quantum gravity: Toward a new understanding of space, time and matter.* Cambridge University Press, 3 2009.

[38] C. Kiefer, *Quantum Gravity*, pp. 709–722. 2014.

[39] J. Polchinski, *String theory. Vol. 1: An introduction to the bosonic string.* Cambridge Monographs on Mathematical Physics, Cambridge University Press, 12 2007.

[40] D. Clowe, M. Bradac, A. H. Gonzalez, M. Markevitch, S. W. Randall, C. Jones, and D. Zaritsky, "A direct empirical proof of the existence of dark matter," *Astrophys. J. Lett.*, vol. 648, pp. L109–L113, 2006.

[41] A. Arbey and F. Mahmoudi, "Dark matter and the early Universe: a review," *Prog. Part. Nucl. Phys.*, vol. 119, p. 103865, 2021.

[42] M. Persic, P. Salucci, and F. Stel, "The Universal rotation curve of spiral galaxies: 1. The Dark matter connection," *Mon. Not. Roy. Astron. Soc.*, vol. 281, p. 27, 1996.

[43] V. F. Mukhanov, H. A. Feldman, and R. H. Brandenberger, "Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions," *Phys. Rept.*, vol. 215, pp. 203–333, 1992.

[44] C. Kiefer and D. Polarski, "Why do cosmological perturbations look classical to us?," *Adv. Sci. Lett.*, vol. 2, pp. 164–173, 2009.

[45] C. Kiefer, "Origin of classical structure from inflation," *Nucl. Phys. B Proc. Suppl.*, vol. 88, pp. 255–258, 2000.

[46] J. Lesgourgues, D. Polarski, and A. A. Starobinsky, "Quantum to classical transition of cosmological perturbations for nonvacuum initial states," *Nucl. Phys. B*, vol. 497, pp. 479–510, 1997.

[47] A. Perez, H. Sahlmann, and D. Sudarsky, "On the quantum origin of the seeds of cosmic structure," *Class. Quant. Grav.*, vol. 23, pp. 2317–2354, 2006.

[48] J. Berjon, E. Okon, and D. Sudarsky, "Critical review of prevailing explanations for the emergence of classicality in cosmology," *Phys. Rev. D*, vol. 103, no. 4, p. 043521, 2021.

[49] D. Sudarsky, "Shortcomings in the Understanding of Why Cosmological Perturbations Look Classical," *Int. J. Mod. Phys. D*, vol. 20, pp. 509–552, 2011.

[50] F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov, "Higgs inflation: consistency and generalisations," *JHEP*, vol. 01, p. 016, 2011.

[51] R. Penrose, *The road to reality: A complete guide to the laws of the universe.* Random house, 2005.

[52] J. L. Anderson and D. Finkelstein, "Cosmological constant and fundamental length," *Am. J. Phys.*, vol. 39, pp. 901–904, 1971.

[53] L. Bombelli, J. Lee, D. Meyer, and R. Sorkin, "Space-Time as a Causal Set," *Phys. Rev. Lett.*, vol. 59, pp. 521–524, 1987.

[54] T. Jacobson, "Thermodynamics of space-time: The Einstein equation of state," *Phys. Rev. Lett.*, vol. 75, pp. 1260–1263, 1995.

[55] S. K. Modak, L. Ortíz, I. Peña, and D. Sudarsky, "Non-Paradoxical Loss of Information in Black Hole Evaporation in a Quantum Collapse Model," *Phys. Rev. D*, vol. 91, no. 12, p. 124009, 2015.

[56] E. Okon and D. Sudarsky, "Black Holes, Information Loss and the Measurement Problem," *Found. Phys.*, vol. 47, no. 1, pp. 120–131, 2017.

[57] E. Okon and D. Sudarsky, "Losing stuff down a black hole," *Found. Phys.*, vol. 48, no. 4, pp. 411–428, 2018.

[58] J. Maldacena and L. Susskind, "Cool horizons for entangled black holes," *Fortsch. Phys.*, vol. 61, pp. 781–811, 2013.

[59] G. Penington, S. H. Shenker, D. Stanford, and Z. Yang, "Replica wormholes and the black hole interior," 11 2019.

[60] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, "Replica Wormholes and the Entropy of Hawking Radiation," *JHEP*, vol. 05, p. 013, 2020.

[61] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, "The entropy of Hawking radiation," *Rev. Mod. Phys.*, vol. 93, no. 3, p. 035002, 2021.

[62] A. Ashtekar and M. Bojowald, "Black hole evaporation: A Paradigm," *Class.Quant.Grav.*, vol. 22, pp. 3349–3362, 2005.

[63] A. Ashtekar, J. Olmedo, and P. Singh, "Quantum Transfiguration of Kruskal Black Holes," *Phys. Rev. Lett.*, vol. 121, no. 24, p. 241301, 2018.

[64] M. Bojowald, "Spherically symmetric quantum geometry: States and basic operators," *Class. Quant. Grav.*, vol. 21, pp. 3733–3753, 2004.

[65] L. Modesto, "Loop quantum black hole," *Class. Quant. Grav.*, vol. 23, pp. 5587–5602, 2006.

[66] R. M. Wald, *Quantum Field Theory in Curved Space-Time and Black Hole Thermodynamics.* Chicago Lectures in Physics, Chicago, IL: University of Chicago Press, 1995.

[67] T. Maudlin, "(Information) Paradox Lost," 5 2017.

[68] A. Perez, "Black Holes in Loop Quantum Gravity," *Rept. Prog. Phys.*, vol. 80, no. 12, p. 126901, 2017.

[69] T. Jacobson, "Black hole evaporation and ultrashort distances," *Phys. Rev. D*, vol. 44, pp. 1731–1739, 1991.

[70] T. Jacobson, "Black hole radiation in the presence of a short distance cutoff," *Phys. Rev. D*, vol. 48, pp. 728–741, 1993.

[71] S. Corley, "Computing the spectrum of black hole radiation in the presence of high frequency dispersion: An Analytical approach," *Phys. Rev. D*, vol. 57, pp. 6280–6291, 1998.

[72] W. G. Unruh, "Sonic analog of black holes and the effects of high frequencies on black hole evaporation," *Phys. Rev. D*, vol. 51, pp. 2827–2838, 1995.

[73] S. Corley and T. Jacobson, "Hawking spectrum and high frequency dispersion," *Phys. Rev. D*, vol. 54, pp. 1568–1586, 1996.

[74] W. G. Unruh and R. Schutzhold, "On the universality of the Hawking effect," *Phys. Rev. D*, vol. 71, p. 024028, 2005.

[75] K. Fredenhagen and R. Haag, "On the Derivation of Hawking Radiation Associated With the Formation of a Black Hole," *Commun. Math. Phys.*, vol. 127, p. 273, 1990.

[76] S. Hollands and R. M. Wald, "Quantum fields in curved spacetime," *Phys. Rept.*, vol. 574, pp. 1–35, 2015.

[77] J. Manchak and J. O. Weatherall, "(Information) Paradox Regained? A Brief Comment on Maudlin on Black Hole Information Loss," 1 2018.

[78] H. Kodama, "INEVITABILITY OF A NAKED SINGULARITY ASSOCIATED WITH THE BLACK HOLE EVAPORATION," *Prog. Theor. Phys.*, vol. 62, p. 1434, 1979.

[79] R. M. Wald, "BLACK HOLES, SINGULARITIES AND PREDICTABILITY," 1984.

[80] R. M. Wald, "The thermodynamics of black holes," *Living Rev.Rel.*, vol. 4, p. 6, 2001.

[81] R. P. Geroch, "The domain of dependence," *J. Math. Phys.*, vol. 11, pp. 437–439, 1970.

[82] W. G. Unruh and R. M. Wald, "On evolution laws taking pure states to mixed states in quantum field theory," *Phys. Rev. D*, vol. 52, pp. 2176–2182, 1995.

[83] W. Unruh, "Decoherence without Dissipation," *Trans.Roy.Soc.Lond.*, vol. 370, p. 4454, 2012.

[84] P. Chen, Y. C. Ong, and D.-h. Yeom, "Black Hole Remnants and the Information Loss Paradox," *Phys. Rept.*, vol. 603, pp. 1–45, 2015.

[85] M. Christodoulou and T. De Lorenzo, "Volume inside old black holes," *Phys. Rev.*, vol. D94, no. 10, p. 104002, 2016.

[86] M. Bojowald, "Absence of singularity in loop quantum cosmology," *Phys.Rev.Lett.*, vol. 86, pp. 5227–5230, 2001.

[87] A. Ashtekar, T. Pawlowski, and P. Singh, "Quantum Nature of the Big Bang: Improved dynamics," *Phys. Rev.*, vol. D74, p. 084003, 2006.

[88] L. Modesto, "Disappearance of black hole singularity in quantum gravity," *Phys. Rev.*, vol. D70, p. 124009, 2004.

[89] M. Bojowald, R. Goswami, R. Maartens, and P. Singh, "A Black hole mass threshold from non-singular quantum gravitational collapse," *Phys. Rev. Lett.*, vol. 95, p. 091302, 2005.

[90] M. Bojowald, "Nonsingular black holes and degrees of freedom in quantum gravity," *Phys. Rev. Lett.*, vol. 95, p. 061301, 2005.

[91] R. Gambini and J. Pullin, "Loop quantization of the Schwarzschild black hole," *Phys. Rev. Lett.*, vol. 110, no. 21, p. 211301, 2013.

[92] A. Corichi and P. Singh, "Loop quantization of the Schwarzschild interior revisited," *Class. Quant. Grav.*, vol. 33, no. 5, p. 055006, 2016.

[93] S. Liberati, G. Tricella, and A. Trombettoni, "The information loss problem: an analogue gravity perspective," *Entropy*, vol. 21, no. 10, p. 940, 2019.

[94] A. Ashtekar and M. Bojowald, "Quantum geometry and the Schwarzschild singularity," *Class. Quant. Grav.*, vol. 23, pp. 391–411, 2006.

[95] N. Bodendorfer, F. M. Mele, and J. Münch, "Effective Quantum Extended Spacetime of Polymer Schwarzschild Black Hole," *Class. Quant. Grav.*, vol. 36, no. 19, p. 195015, 2019.

[96] A. Ashtekar and P. Singh, "Loop Quantum Cosmology: A Status Report," *Class. Quant. Grav.*, vol. 28, p. 213001, 2011.

[97] A. Einstein, "Spielen Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?," *Sitzungsber. Preuss. Akad. Wiss. Berlin*, pp. 349–356, 1919.

[98] S. Weinberg, “The Cosmological Constant Problem,” *Rev. Mod. Phys.*, vol. 61, pp. 1–23, 1989. [,569(1988)].

[99] W. G. Unruh, “A Unimodular Theory of Canonical Quantum Gravity,” *Phys. Rev.*, vol. D40, p. 1048, 1989.

[100] D.-W. Chiou and M. Geiller, “Unimodular Loop Quantum Cosmology,” *Phys. Rev.*, vol. D82, p. 064012, 2010.

[101] L. Smolin, “Unimodular loop quantum gravity and the problems of time,” *Phys. Rev.*, vol. D84, p. 044047, 2011.

[102] L. Smolin, “The Quantization of unimodular gravity and the cosmological constant problems,” *Phys. Rev.*, vol. D80, p. 084003, 2009.

[103] A. Ashtekar, V. Taveras, and M. Varadarajan, “Information is Not Lost in the Evaporation of 2-dimensional Black Holes,” *Phys.Rev.Lett.*, vol. 100, p. 211302, 2008.

[104] C. Rovelli, “Time in quantum gravity: An hypothesis,” *Phys. Rev. D*, vol. 43, pp. 442–456, Jan 1991.

[105] M. Varadarajan. Private Communication.

[106] G. de Berredo-Peixoto, L. Freidel, I. L. Shapiro, and C. A. de Souza, “Dirac fields, torsion and Barbero-Immirzi parameter in Cosmology,” *JCAP*, vol. 1206, p. 017, 2012.

[107] M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*. Reading, USA: Addison-Wesley, 1995.

[108] E. Wilson-Ewing, “Lattice loop quantum cosmology: scalar perturbations,” *Class. Quant. Grav.*, vol. 29, p. 215013, 2012.

[109] P. Singh and E. Wilson-Ewing, “Quantization ambiguities and bounds on geometric scalars in anisotropic loop quantum cosmology,” *Class. Quant. Grav.*, vol. 31, p. 035010, 2014.

[110] D. Green and W. G. Unruh, “Difficulties with closed isotropic loop quantum cosmology,” *Phys. Rev. D*, vol. 70, p. 103502, 2004.

[111] K. Noui, A. Perez, and K. Vandersloot, “On the physical Hilbert space of loop quantum cosmology,” *Phys. Rev.*, vol. D71, p. 044025, 2005.

[112] S. Gielen, D. Oriti, and L. Sindoni, “Cosmology from Group Field Theory Formalism for Quantum Gravity,” *Phys. Rev. Lett.*, vol. 111, no. 3, p. 031301, 2013.

[113] D. Oriti, L. Sindoni, and E. Wilson-Ewing, “Emergent Friedmann dynamics with a quantum bounce from quantum gravity condensates,” *Class. Quant. Grav.*, vol. 33, no. 22, p. 224001, 2016.

[114] D. Oriti, L. Sindoni, and E. Wilson-Ewing, “Bouncing cosmologies from quantum gravity condensates,” *Class. Quant. Grav.*, vol. 34, no. 4, p. 04LT01, 2017.

[115] T. Thiemann, “Quantum spin dynamics (QSD),” *Class. Quant. Grav.*, vol. 15, pp. 839–873, 1998.

[116] A. Laddha and M. Varadarajan, “Quantum Dynamics,” in *Loop Quantum Gravity: The First 30 Years* (A. Ashtekar and J. Pullin, eds.), WSP, 2017.

[117] J. Fernando Barbero G, J. Margalef-Bentabol, and E. J. S. Villaseñor, “On the distribution of the eigenvalues of the area operator in loop quantum gravity,” *Class. Quant. Grav.*, vol. 35, no. 6, p. 065008, 2018.

[118] J. Lewandowski, A. Okolow, H. Sahlmann, and T. Thiemann, “Uniqueness of diffeomorphism invariant states on holonomy-flux algebras,” *Commun.Math.Phys.*, vol. 267, pp. 703–733, 2006.

[119] J. Ambjorn, J. Jurkiewicz, and R. Loll, “Emergence of a 4-D world from causal quantum gravity,” *Phys. Rev. Lett.*, vol. 93, p. 131301, 2004.

[120] D. Oriti, “The microscopic dynamics of quantum space as a group field theory,” in *Foundations of Space and Time: Reflections on Quantum Gravity*, pp. 257–320, 10 2011.

[121] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, “Black Holes: Complementarity or Firewalls?,” *JHEP*, vol. 1302, p. 062, 2013.

[122] S. L. Braunstein, S. Pirandola, and K. ?yczkowski, “Better Late than Never: Information Retrieval from Black Holes,” *Phys. Rev. Lett.*, vol. 110, no. 10, p. 101301, 2013.

[123] D. Marolf, “The Black Hole information problem: past, present, and future,” *Rept. Prog. Phys.*, vol. 80, no. 9, p. 092001, 2017.

[124] T. Banks, L. Susskind, and M. E. Peskin, “Difficulties for the Evolution of Pure States Into Mixed States,” *Nucl.Phys.*, vol. B244, p. 125, 1984.

[125] M. Hotta, R. Schützhold, and W. G. Unruh, "Partner particles for moving mirror radiation and black hole evaporation," *Phys. Rev. D*, vol. 91, no. 12, p. 124060, 2015.

[126] R. M. Wald, "Particle and energy cost of entanglement of Hawking radiation with the final vacuum state," *Phys. Rev. D*, vol. 100, no. 6, p. 065019, 2019.

[127] A. Perez, D. Sudarsky, and J. D. Bjorken, "A microscopic model for an emergent cosmological constant," *Int. J. Mod. Phys. D*, vol. 27, no. 14, p. 1846002, 2018.

[128] A. Perez and D. Sudarsky, "Dark energy from quantum gravity discreteness," *Phys. Rev. Lett.*, vol. 122, no. 22, p. 221302, 2019.

[129] S. Weinberg, *Cosmology.* 2008.

[130] V. Mukhanov, *Physical Foundations of Cosmology.* Oxford: Cambridge University Press, 2005.

[131] A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," *Phys. Rev. D*, vol. 23, pp. 347–356, 1981.

[132] D. Tong, "Cosmology (lecture notes)."

[133] D. Baumann, "Inflation," in *Theoretical Advanced Study Institute in Elementary Particle Physics: Physics of the Large and the Small*, pp. 523–686, 2011.

[134] N. Aghanim *et al.*, "Planck 2018 results. VI. Cosmological parameters," *Astron. Astrophys.*, vol. 641, p. A6, 2020. [Erratum: Astron.Astrophys. 652, C4 (2021)].

[135] G. F. R. Ellis and J.-P. Uzan, "Causal structures in inflation," *Comptes Rendus Physique*, vol. 16, pp. 928–947, 2015.

[136] P. Peter and J.-P. Uzan, *Primordial cosmology.* Oxford Graduate Texts, Oxford: Oxford Univ. Press, 2009.

[137] R. Penrose, "On the Second law of thermodynamics," *J. Statist. Phys.*, vol. 77, pp. 217–221, 1994.

[138] R. Penrose, *Cycles of time: an extraordinary new view of the universe.* Random House, 2010.

[139] D. Fixsen, "The temperature of the cosmic microwave background," *The Astrophysical Journal*, vol. 707, no. 2, p. 916, 2009.

[140] N. Aghanim *et al.*, "Planck 2018 results. I. Overview and the cosmological legacy of Planck," *Astron. Astrophys.*, vol. 641, p. A1, 2020.

[141] R. Penrose, "Singularities and time-asymmetry," pp. 581–638, 1979.

[142] P. C. W. Davies, "Inflation and Time Asymmetry: Or What Wound Up the Universe?," *Nature*, vol. 301, pp. 398–400, 1983.

[143] O. Gron and S. Hervik, "The Weyl curvature conjecture," 5 2002.

[144] H. Velten and T. R. P. Caramês, "To conserve, or not to conserve: A review of nonconservative theories of gravity," *Universe*, vol. 7, no. 2, p. 38, 2021.

[145] R. Wald, *General Relativity.* Chicago: University of Chicago Press, 1984.

[146] G. F. R. Ellis, H. van Elst, J. Murugan, and J.-P. Uzan, "On the Trace-Free Einstein Equations as a Viable Alternative to General Relativity," *Class. Quant. Grav.*, vol. 28, p. 225007, 2011.

[147] G. F. R. Ellis, "The Trace-Free Einstein Equations and inflation," *Gen. Rel. Grav.*, vol. 46, p. 1619, 2014.

[148] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space.* Cambridge Monographs on Mathematical Physics, Cambridge, UK: Cambridge Univ. Press, 1984.

[149] A. Fabbri and J. Navarro-Salas, "Modeling black hole evaporation," *Published in London, UK: Imp. Coll. Pr. (2005) 334 p.*

[150] N. Tsamis and R. Woodard, "Relaxing the cosmological constant," *Physics Letters B*, vol. 301, no. 4, pp. 351–357, 1993.

[151] R. H. Brandenberger, "Back reaction of cosmological perturbations and the cosmological constant problem," in *18th IAP Colloquium on the Nature of Dark Energy: Observational and Theoretical Results on the Accelerating Universe*, 10 2002.

[152] D. J. Bedingham, "Relativistic state reduction dynamics," *Found. Phys.*, vol. 41, pp. 686–704, 2011.

[153] G. C. Ghirardi, P. Pearle, and A. Rimini, "Markov processes in hilbert space and continuous spontaneous localization of systems of identical particles," *Phys. Rev. A*, vol. 42, pp. 78–89, Jul 1990.

[154] G. C. Ghirardi, A. Rimini, and T. Weber, "A Unified Dynamics for Micro and MACRO Systems," *Phys. Rev. D*, vol. 34, p. 470, 1986.

[155] P. Cañate, P. Pearle, and D. Sudarsky, "Continuous spontaneous localization wave function collapse model as a mechanism for the emergence of cosmological asymmetries in inflation," *Phys. Rev. D*, vol. 87, no. 10, p. 104024, 2013.

[156] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, "Lorentz invariance and quantum gravity: An additional fine-tuning problem?," *Phys. Rev. Lett.*, vol. 93, p. 191301, Nov 2004.

[157] S. Weinberg, "Adiabatic modes in cosmology," *Phys. Rev. D*, vol. 67, p. 123504, 2003.

[158] J. Martin, C. Ringeval, and V. Vennin, "Encyclopædia Inflationaris," *Phys. Dark Univ.*, vol. 5-6, pp. 75–235, 2014.

[159] F. L. Bezrukov and M. Shaposhnikov, "The Standard Model Higgs boson as the inflaton," *Phys. Lett. B*, vol. 659, pp. 703–706, 2008.

[160] D. Sloan and G. Ellis, "Solving the Cosmological Entropy Issue with a Higgs Dilaton," *Phys. Rev. D*, vol. 99, no. 6, p. 063518, 2019.

[161] A. Albrecht, P. J. Steinhardt, M. S. Turner, and F. Wilczek, "Reheating an Inflationary Universe," *Phys. Rev. Lett.*, vol. 48, p. 1437, 1982.

[162] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, and A. Strumia, "Higgs mass and vacuum stability in the Standard Model at NNLO," *JHEP*, vol. 08, p. 098, 2012.

[163] G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, and Particle Creation," *Phys. Rev. D*, vol. 15, pp. 2738–2751, 1977.

[164] R. H. Brandenberger, "Lectures on the theory of cosmological perturbations," *Lect. Notes Phys.*, vol. 646, pp. 127–167, 2004.

[165] Y. Akrami *et al.*, "Planck 2018 results. X. Constraints on inflation," *Astron. Astrophys.*, vol. 641, p. A10, 2020.

[166] J. H. MacGibbon, "Can planck-mass relics of evaporating black holes close the universe?," *Nature*, vol. 329, no. 6137, pp. 308–309, 1987.

[167] P. Chen and R. J. Adler, "Black hole remnants and dark matter," *Nucl. Phys. B Proc. Suppl.*, vol. 124, pp. 103–106, 2003.

[168] S. Rasanen and E. Tomberg, "Planck scale black hole dark matter from Higgs inflation," *JCAP*, vol. 01, p. 038, 2019.

[169] C. Rovelli and F. Vidotto, "Small black/white hole stability and dark matter," *Universe*, vol. 4, no. 11, p. 127, 2018.

[170] D. Carney, S. Ghosh, G. Krnjaic, and J. M. Taylor, "Proposal for gravitational direct detection of dark matter," *Phys. Rev. D*, vol. 102, no. 7, p. 072003, 2020.

[171] J. Rubio, "Higgs inflation," *Frontiers in Astronomy and Space Sciences*, vol. 5, p. 50, 2019.

[172] J. R. Espinosa, G. F. Giudice, E. Morgante, A. Riotto, L. Senatore, A. Strumia, and N. Tetradis, "The cosmological Higgstory of the vacuum instability," *JHEP*, vol. 09, p. 174, 2015.

[173] V. Branchina, E. Messina, and M. Sher, "Lifetime of the electroweak vacuum and sensitivity to Planck scale physics," *Phys. Rev. D*, vol. 91, p. 013003, 2015.

[174] G. Isidori, G. Ridolfi, and A. Strumia, "On the metastability of the standard model vacuum," *Nucl. Phys. B*, vol. 609, pp. 387–409, 2001.

[175] A. Joti, A. Katsis, D. Loupas, A. Salvio, A. Strumia, N. Tetradis, and A. Urbano, "(Higgs) vacuum decay during inflation," *JHEP*, vol. 07, p. 058, 2017.

[176] L. Amadei, A. Perez, and S. Ribisi, "The landscape of loop quantum cosmology." in preparation.

[177] S. Bahamonde, C. G. Böhmer, S. Carloni, E. J. Copeland, W. Fang, and N. Tamanini, "Dynamical systems applied to cosmology: dark energy and modified gravity," *Phys. Rept.*, vol. 775-777, pp. 1–122, 2018.

[178] M. Szydłowski and A. Stachowski, "Cosmology with decaying cosmological constant—exact solutions and model testing," *JCAP*, vol. 10, p. 066, 2015.

[179] Y. Fujii and T. Nishioka, "Model of a Decaying Cosmological Constant," *Phys. Rev. D*, vol. 42, pp. 361–370, 1990.

[180] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan, and L. Senatore, "The Effective Field Theory of Inflation," *JHEP*, vol. 03, p. 014, 2008.

[181] F. M. Mele, J. Münch, and S. Pateloudis, "Quantum Corrected Polymer Black Hole Thermodynamics: Mass Relations and Logarithmic Entropy Correction," 2 2021.

[182] P. Martin-Dussaud and C. Rovelli, "Evaporating black-to-white hole," *Class. Quant. Grav.*, vol. 36, no. 24, p. 245002, 2019.

[183] E. Bianchi, M. Christodoulou, F. D'Ambrosio, H. M. Haggard, and C. Rovelli, "White Holes as Remnants: A Surprising Scenario for the End of a Black Hole," *Class. Quant. Grav.*, vol. 35, no. 22, p. 225003, 2018.

[184] M. Bojowald, *Canonical Gravity and ApplicationsCosmology, Black Holes, and Quantum Gravity.* Cambridge University Press, 12 2010.

[185] I. Agullo and P. Singh, "Loop Quantum Cosmology: A brief review," 2016.

[186] A. Ashtekar, T. Pawlowski, P. Singh, and K. Vandersloot, "Loop quantum cosmology of k=1 FRW models," *Phys. Rev. D*, vol. 75, p. 024035, 2007.

[187] A. Ashtekar and E. Wilson-Ewing, "Loop quantum cosmology of Bianchi I models," *Phys. Rev. D*, vol. 79, p. 083535, 2009.

[188] A. Ashtekar and E. Wilson-Ewing, "Loop quantum cosmology of Bianchi type II models," *Phys. Rev. D*, vol. 80, p. 123532, 2009.

[189] E. Wilson-Ewing, "Loop quantum cosmology of Bianchi type IX models," *Phys. Rev. D*, vol. 82, p. 043508, 2010.

[190] M. Martin-Benito, G. A. Mena Marugan, and T. Pawlowski, "Loop Quantization of Vacuum Bianchi I Cosmology," *Phys. Rev. D*, vol. 78, p. 064008, 2008.

[191] D.-W. Chiou, "Loop Quantum Cosmology in Bianchi Type I Models: Analytical Investigation," *Phys. Rev. D*, vol. 75, p. 024029, 2007.

[192] V. Taveras, "Corrections to the Friedmann Equations from LQG for a Universe with a Free Scalar Field," *Phys. Rev. D*, vol. 78, p. 064072, 2008.