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## Résumé

Les insectes fascinent depuis longtemps une grande communauté interdisciplinaire d'ingénieurs, de biologistes, de physiciens et de mathématiciens par leurs extraordinaires capacités de vol en battant des ailes. Le vol des insectes a été largement étudié dans le passé en supposant que les insectes volent avec des ailes rigides dans des conditions d'écoulement au repos. Dans le monde réel, cependant, la plupart des ailes d'insectes sont des structures complexes, qui consistent en une fine membrane flexible soutenue par un réseau de veines. L'objectif de cette thèse est d'étudier l'influence de la flexibilité des ailes sur les performances aérodynamiques des insectes. Pour cela, un modèle d'aile a été développé en utilisant un système masse-ressort où l'aile est discrétisée par des points de masse reliés par des ressorts. Sur la base de différents comportements mécaniques, les veines sont modélisées comme une tige en utilisant des ressorts d'extension et de flexion, tandis que les membranes sont modélisées comme une feuille mince en utilisant uniquement des ressorts d'extension. Cette approche nous permet d'imiter la structure et la dynamique particulières des ailes d'insectes. Le modèle d'aile est ensuite couplé à un solveur fluide, qui est basé sur une discrétisation spectrale des équations de Navier-Stokes pénalisées en trois dimensions. Le code est conçu pour fonctionner sur des supercalculateurs massivement parallèles pour des calculs à haute résolution. Après avoir été validé par rapport aux travaux précédents, le code est d'abord utilisé pour simuler un bourdon attaché avec des ailes flexibles. Afin d'analyser l'effet de la flexibilité des ailes, le module de Young de la cuticule de l'aile est modifié pour faire une comparaison entre deux modèles d'ailes différents que nous appelons flexibles et très flexibles. Nous examinons ensuite une deuxième espèce, qui est Calliphora vomitoria (mouche à viande) dans un contexte de vol attaché. En utilisant la stratégie d'évolution de l'adaptation de la matrice de covariance, la rigidité de l'aile est optimisée en comparant le modèle d'aile avec un ensemble de données expérimentales de déformation de l'aile en réponse à des forces ponctuelles statiques. Nos études montrent que la flexibilité de l'aile joue un rôle
important dans l'économie des coûts énergétiques du vol. De plus, l'inertie de l'aile a également contribué à amortir la fluctuation de la force aérodynamique et a donc aidé l'insecte à se stabiliser en vol.

Mots clés: vol des insectes, flexibilité des ailes, modèle de masse-ressort, intéraction fluide-structure, méthode spectrale, méthode de pénalisation volumique, stratégies d'évolution avec adaptation de matrice de covariance (CMA-ES)

## Abstract

Insects have fascinated a large, interdisciplinary community of engineers, biologists, physicists and mathematicians for a long time with their extraordinary capabilities of flying by flapping their wings. Insect flight has been extensively studied in the past assuming that insects fly with rigid wings in quiescent flow conditions. In the real world, however, most insect wings are complex structures that consist of a thin, flexible membrane supported by a network of veins. The aim of this thesis is to investigate the influence of wing flexibility on the aerodynamic performance of insects. For this purpose, a wing model has been developed using a mass-spring system where the wing is discretized by mass points connected by springs. Based on different mechanical behaviors, veins are modeled as a rod using extension and bending springs while membranes are modeled as a thin sheet using extension springs only. This functional approach allows us to mimic the distinctive structure and dynamics of insect wings. The wing model is then coupled with a fluid solver which is based on a spectral discretization of the three-dimensional penalized Navier-Stokes equations. The code is designed to run on massively parallel supercomputers for high resolution computations. After being validated with respect to previous works, the code is firstly employed to simulate a tethered bumblebee with flexible wings. In order to analyze the effect of wing flexibility, the Young's modulus of wing cuticle is varied to make a comparison between two different wing models that we refer to as flexible and highly flexible. We then examine a second species which is Calliphora vomitoria (blowfly) in a tethered flight context. Using covariance matrix adaptation evolution strategy, the wing stiffness is optimized by comparing the wing model with a set of experimental data of wing deformation in response to static point forces. Our studies show that wing flexibility plays an important role in minimizing flight energetic cost. Moreover, the wing inertia also helped to damp out the fluctuation of the aerodynamic force and thus stabilized the insect during flight.

Keywords: insect flight, wing elasticity, mass-spring model, fluid-structure interaction, spectral method, volume penalization method, Covariance Matrix Adaptation Evolution Strategies (CMA-ES)

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## Chapter 1

## Introduction

Since time immemorial, man's dream of flying has begun by our ancestor's observation of bird flight. This led to a multi-millennia journey of man learning to fly and nature has been a prestigious school for this subject. The early Greek legend of Daedalus and his son Icarus has always been told as a story about the first attempts of man for directly emulating birds. Eventually, after centuries of slow development, some investors began to discover and define some of the basics of aerodynamics. Most notable of these was Leonardo da Vinci in the Renaissance period. He wrote about and sketched many designs for flying machines based on his studies of bird and bat flights. Later, over the past 100 years has witnessed a big breakthrough of human kind in aeronautical technology, beginning with the groundbreaking work of the Wright Brothers in 1903. Nevertheless, nature's flying animals with over 150 million years of evolution are still a great inspiration for mankind.

Insects fly by flapping their wings with high wingbeat frequency at high angle of attack [86]. Compared to conventional airplanes with fixed wings, flapping wings have several aerodynamic advantages with the ability to create lift even at high angles of attack due to the delayed stall of the leading edge vortex (LEV) [33]. These extraordinary capabilities of hovering and manoeuvrability have made bio-inspired flapping wings a strong candidate for developing human-engineered micro-air-vehicles (MAVs) with possible applications in environmental monitoring, surveillance and security. For more than a century, insect flight has fascinated biologists and physicists with their sophisticated flight mechanism. Traditional works trying to understand insect flight were usually based on observation and experiments. Ellington [32] attempted to capture free-flight wing kinematics of a hover fly using single-image high-speed cine. Even more challenging than capturing wing motion in 3D is measuring aerodynamic
forces acting on the body of the insect [17, 107, 111]. Mountcastle and Combes [70] showed that passive deformations enhance lift production in bumblebees by artificially stiffening their wings using a micro-splint. Campos et al. [14] and Fu et al. [44] used experimental methods and found that highly flexible wings show significant tip-root lag which weakened vortices and reduce the force production.

However, the rapid advances in computational fluid dynamics and computer power during the last decades has offered an alternative approach to study insect flight. Although numerical studies still have to rely on experimental data for relevant inputs and validations, they allow researchers to look at the phenomenon from different perspectives. For example, numerical simulations can provide the instantaneous 3D structure of the flow field around the flapping insect, which is difficult to obtain from experiments. Yet, the application of the approach is not trivial due to the high complexity of the problem. First, the complicated geometries of insects make it difficult to impose the no-slip boundary conditions on the fluid-structure interface. Furthermore, the geometry is time-dependent due to the movement of the studied insect. As a result, both surrounding air flow and the insect dynamics need to be solved simultaneously as well as their interactions.

In previous studies on flapping flight, the wings were usually considered as rigid to simplify the problem. Earlier work on insect flight focused on two-dimensional configurations, as briefly discussed in [102]. Then, Liu and Kawachi [64, 65] were among the first to use the finite volume method and moving, body-fitted grids for computations in 3D. The approach requires lower memory, at least for small enough Reynolds number, compared to non-body fitted grids approaches. However, their code was not designed for CPU parallelism and introduced large numerical dissipation of the small scale turbulent structures in the wake. Ramamurti and Sandberg [81, 82] simulated hovering fruit flies by using Arbitrary Lagrangian Eulerian formulation to solve the incompressible Navier-Stokes equations. Dickinson, Lehmann et al. [27] investigated numerically the timing of stroke reversal. However, the need for more efficient approaches has motivated the development of methods that allow for a geometry-independent discretization such as the volume penalization method and the immersed boundary method. While the former was based on the work of Angot et al. [2], the latter was established by Peskin [79, 80]. Although the boundary conditions of the Navier-Stokes equations are imposed only approximately, these methods are simpler to implement with higher resolutions and performance. Yokoyama et al. [109]
used the immersed boundary method to simulate a rigid-winged butterfly in free forward flight with a state of the art resolution of 1 billion grid points. They showed that the abdominal motion plays an important role in flight stabilization.

The volume penalization method is on the other hand employed by Engels et al. [38] for the development of a solver called FLUSI (FLUid Structure Interaction) ${ }^{1}$. The solver, which is the starting point of this thesis, is a fully parallel software dedicated for modeling three-dimensional flapping flight in viscous flows. The heart of this software is the Fourier pseudospectral method with adaptive time stepping used for the discretization of the 3D incompressible Navier-Stokes equations. In [58], the code was used to perform numerical simulations of rotating triangular rigid wings at Reynolds number $R e=250$ to investigate the Leading-Edge Vortices (LEVs) as a function of the wing aspect ratio and the angle of attack. High resolution direct numerical simulations of rotating and flapping bumblebee wings were presented in [35] using likewise the FLUSI code with rigid wings focusing again on the role of LEVs and the associated helicity production. The interaction of flapping bumblebees with turbulent inflow in free and tethered flight was investigated in [36] using once again massively parallel computations with FLUSI. The study showed that the fluctuations of aerodynamic observables significantly grow with increasing turbulence intensity, even if the mean values are almost unaffected. Changing the length scale of the turbulent inflow, while keeping the turbulence intensity fixed, showed that the fluctuation level of forces and moments can be significantly reduced. Nevertheless, a solver dedicated for the simulation of the structure deformation was not fully developed in this software FLUSI. Hence, all aforementioned simulations of insect flight have been performed with the essential assumption that the insect is composed of linked rigid parts including the wings.

In the current work, the goal is to move from rigid to flexible wings and to present a fluid-structure interaction solver including a wing model for flapping flight. Wing's dynamic shape change is essential for developing a comprehensive understanding of insect flight because instantaneous wing shape determines the direction and magnitude of fluid dynamic forces during wing flapping [18, 112]. However, the complex interaction between the anisotropic wing structures and the surrounding unsteady flow makes the analysis of flapping flexible wings challenging, but at the same time

[^0]intriguing. In the last two decades, with the dramatic improvement of computing power, many numerical studies have investigated the effect of wing flexibility and drawn contradictory conclusions [83]. Computational studies with flexible wings can be divided in "active" and "passive" flexibility, depending on whether the deformation is actively computed from fluid forces or measured externally and then imposed in the simulation $[110,113]$. For active fluid-structure interaction, due to complexity of the problem, pioneering works were mostly done in simplified two-dimensional frameworks [59, 69], considering chord-wise flexibility. Only recently, the aerodynamic performance of flexible flapping wings is studied numerically in a 3D context. Nakata and Liu presented three-dimensional computations of hawkmoth with flexible wings simulated by finite element method [74, 75]. Du and Sun [28] solved the Navier-Stokes equations coupled with measured wing deformation data and compared with the rigid counterparts. They obtained a $10 \%$ increase in lift caused by the camber deformation and a $5 \%$ reduction in required power. Walker et al. [99] studied on deformable wings of free-flying hoverflies and demonstrated a recoil effect as the wings change abruptly at stroke reversal, improving flight control by elasticity of the wing's alula.

In this context, the current thesis aims at investigating the role of wing flexibility on flight performance of insects. Consequently, a wing model is required for simulating the deformation of the solid part under the impact from the fluid. There are various models based on continuum mechanics theory, which are used in many well-known solid solvers. Among these, Finite Element Methods [22] (FEM) are mostly used in both research and industrial fields due to their reliability and effectiveness. Despite this dominance, the requirement for faster and more efficient methods motivates the development of alternative models. One of these is the mass-spring system, which is known for its computational efficiency and straightforward implementation. The approach is widely used in the fields of medical modeling [54] and computer graphics [85]. As part of this work, a solver using a network of masses and springs is developed to model a flexible insect wing and coupled with FLUSI. The motivation as well as the development of the solver will be presented in chapter 2 . We start first with the mathematical description of mass-spring systems. The main aspect making the solver suitable for modeling insect wings is the functional approach where we distinguish between the venation system and the membrane. The veins are viewed as rods and they are modeled by using extension and bending springs. In a different manner, the membrane part is simulated as a stretching sheet using only extension
springs. The two models are then validated numerically using some classical test cases.
Next, the application of mass-spring systems for modeling flexible bumblebee and blowfly wings is introduced in chapter 3 . The deformations of insect wings during flight are mostly passive without muscular control past the wing base. Moreover, the structure and material properties of the wing are mainly responsible for its dynamic behavior. Consequently, we explain in this chapter the approaches used for estimating and assigning these properties. They are based on acquired experimental data and nonlinear optimization algorithm. For bumblebees, since we do not have sufficient data for determining the wing stiffness for these simulations, we choose to vary the wing flexibility to get wings with different stiffness. The bumblebee wing model is then coupled with the fluid solver for simulations of flexible bumblebee wings. This study, presented in chapter 4, will provide us some insights about the influence of wing flexibility on aerodynamic performance of flapping wings.

On the other hand, the stiffness for the blowfly wing model is optimized in a more rigorous way by using an evolution strategy. The objective of the optimization is defined as a least square fit between the solutions of the numerical model and the experimental static data of blowfly wings. The experiments are conducted by our colleagues, biologists from team of Prof. Lehmann in Rostock University (https ://www.tierphysiologie.uni-rostock.de/). The outcome of chapter 5 is expected to contribute to our understanding about insect wing dynamics.

Finally, conclusions on the results obtained from the developed solver are given in chapter 6, along with the discussion about some perspectives for future work.

## Chapter 2

## Mass-spring system and its application for modeling flexible insect wings

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Insect wings are considered as the most crucial parts that enable insects to fly. Yet,
in most recent research, elasticity properties of insect wings are not fully taken into account. The flexibility of insect wings is ignored due to its extremely complicated structure comprising of networks of veins and membranes with various material and mechanical properties. This assumption is important for simplifying the problem in order to model insect flight and to obtain practical results. However, this simplification is not necessarily true since insect wings are observed to have large and significant bending and twisting deformation during flight and these changes in form may influence many aspects of flight performance [74, 110]. This chapter is dedicated to the development of a solid solver that allows to model insect wing dynamics under inertial and external aerodynamic forces. The content of this chapter is adapted from [91].

### 2.1 Motivation for using mass-spring systems

The story of mass-spring models started back at the end of the $20^{t h}$ century when people were dealing with observable deformations of flexible objects in computer graphics applications, such as soft tissue, skin, hair, ball, cloth, textiles, etc. Being considered as one of the pioneers on this field, Terzopoulos et al. [88] proposed using elasticity theory for modeling deformable objects instead of previously-used kinematic models, where the motion and the deformation of materials were prescribed. During this period, most of simulations of flexible objects had been calculated using finite element methods until the requirement of faster models was claimed by Eischen et al. [31]. Consequently, the development of physically based deformable models started to grow, especially in the field of computer graphics [13] and biomedical engineering [54]. While classic solvers, based on finite elements or finite difference methods, are generally employed in the static case for computing stress and strain in a structure, new solvers for deformable objects must have the ability to deal with large deformations and large displacements, i.e. the nonlinear regime. Furthermore, these models need to be fast enough since they are usually employed for real-time applications or coupled with other models which are already time-consuming. Among all the deformable models proposed, mass-spring networks stand out as the most intuitive and simplest due to their computational efficiency [76]. Mass-spring systems have already been employed in many fields such as medical applications [54] (muscle, red blood cells and virtual surgery), computer graphics, fluid-structure interaction and
insect flight. Miller and Peskin [69] used mass-spring networks to model insect wings in two-dimensional numerical simulations. A mass-spring network was used by Yeh and Alexeev [108] to model a flexible plate swimmer and performed fluid-structure interaction simulation with Lattice-Boltzmann methods in 3D. The development of our solid solver is motivated by their mass-spring network approach, aiming to model the flexibility of insect wings in the three-dimensional case.

Mass-spring models allow to solve for the dynamics of an object simply by solving a system of coupled nonlinear ordinary differential equations, instead of partially differential equations like in other methods. With all these advantages, mass-spring networks appear to be an ideal choice for the simulations of flexible objects. Nevertheless, the application of ordinary mass-spring systems into the field of physical modeling is still limited because of several drawbacks. Primarily, instead of starting from the governing equations of continuum mechanics, mass-spring models try to mimic the dynamic behaviors of the original system by replacing it with a discrete model composed of point masses connected together by a network of massless springs. This approach makes mass-spring systems less accurate than other alternatives based on continuum mechanics theory. As a result, the simulation depends strongly on the topology and the resolution of the mesh. The numerical solutions do not necessarily converge to the analytical solutions given by continuum models when the mesh is refined. Moreover, the parameter setting also plays an essential role to the accuracy of the method. The masses and the spring parameters need to be chosen carefully in order to obtain the same mechanical properties with the ones of the original object. Otherwise, the mesh and the spring stiffnesses will generate undesired anisotropic behavior.

Considering a homogeneous material (i.e. a wing membrane), the masses can be computed by using the Voronoi region associated with each mass point [25]. However, choosing the spring parameters is not trivial. There are two main approaches proposed by researchers to tackle these issues. The first method is to correlate the mass-spring model with the actual physical model by examining analytically the spring functions and the mesh configuration. Ng-Thow-Hing [89] and Miller [68] aligned some of the springs with the fiber directions to manually create the anisotropy property of the material. Gelder [47] showed that assigning the same stiffness to all springs will generate undesired distortions of a uniform elastic membrane and proposed a formula to calculate these stiffnesses by triangle area over edge length squared. Delingette [23]
proposed new kinds of springs called tensile biquadratic and quadratic springs which are capable of producing isotropic deformations but are $60 \%$ more expensive than the original mass-spring model. Bourguignon et al. [12] presented a new way to deal with the isotropy or anisotropy of elastic material, independently of the mesh structure used for mass discretization, by making the direction of restoring forces acting along the axes of interest (i.e. fiber orientations of the modeled material) instead of acting along the spring directions. Omori et al. [77] derive the analytical relationships between the spring constant and continuum mechanical properties for different mesh structures in both small and large deformation cases.

The second approach is based on optimization methods to minimize the difference between the results solved by the mass-spring model and the reference data. These reference data can come from the measurements, the visual appearance of real objects [67] or numerical solutions using validated methods such as finite element methods [7, 8]. In general, this approach cannot be applied if the system has too many degrees of freedom with many unknown spring constants or the mesh topology varies in time since one set of parameters works for solely one mesh structure. Otherwise, tuning the spring stiffness by using optimization can give satisfying results with reasonable computational cost.

These studies have shown promising results for the application of mass-spring models into scientific fields where sufficient accuracy is required. By carefully controlling the spring parameters and the mesh geometry, one can still model an flexible object with an acceptable precision using the mass-spring model. In fact, mass-spring systems have already been employed in medical fields for modeling muscle, red blood cells and virtual surgery [54]. Above all, the basic idea of using mass-spring networks to model insect wings was carried out by Peskin et al. [69] in two-dimensional numerical simulations. We are now using this approach for our solid solver, aiming to model the flexibility of insect wings in the three-dimensional case.

### 2.2 Mathematical model for mass-spring systems

### 2.2.1 Governing equations

The very basic idea of the mass-spring model is to discretize an object using mass points $m_{i}(i=1, \ldots, n)$. These points are then connected by springs. There exist many
kinds of springs for different purposes but in the limit of our work, we have used only linear extension and bending springs to model insect wings. The dynamic behavior of the mass-spring system, at a given time $t$, is defined by the position $\mathbf{x}_{i}$ and the velocity $\mathbf{v}_{i}$ of the mass point $i$. For this, we need to solve the dynamic equations of the system, which govern their motion in time under certain external forces. This is one of the elegant advantages of the mass-spring network where these governing equations are simply the corresponding classical well-known Newton's second law, given in eqns. (2.1).

$$
\begin{align*}
\mathbf{F}_{i} & =m_{i} \mathbf{a}_{i} \\
\mathbf{F}_{i} & =\mathbf{F}_{i}^{i n t}+\mathbf{F}_{i}^{\text {ext }} \quad \text { for } i=1 \ldots n \\
\mathbf{v}_{i}(t=0) & =\mathbf{v}_{0, i}  \tag{2.1}\\
\mathbf{x}_{i}(t=0) & =\mathbf{x}_{0, i}
\end{align*}
$$

where $n$ is the number of mass points, $\mathbf{F}_{i}$ is the total force (internal force $\mathbf{F}_{i}^{i n t}$ and external force $\mathbf{F}_{i}^{\text {ext }}$ ) acting on the $i^{t h}$ mass point, $m_{i}, \mathbf{a}_{i}$ are mass and acceleration of the $i^{\text {th }}$ mass point, respectively.

Here, all the terms are quite simple to derive except for the forces. The external forces come from outside of the system and depend on the surrounding field and the problem we are dealing with. On the other hand, the internal forces represent the restoring forces caused by the springs. The complicated properties of these forces make the system (2.1) become nonlinear. Hence, we have a nonlinear system of $3 n$ second order ordinary differential equations (ODEs) corresponding to three dimensions $x, y, z$ and $n$ mass points. In the general case, this system (2.1) needs to be solved numerically since its analytical solution cannot be derived. In practice, it is more convenient to convert a system of $3 n$ second order equations into a system of $6 n$ first order equations by using the relations $\mathbf{a}_{i}=d \mathbf{v}_{i} / d t$ and $\mathbf{v}_{i}=d \mathbf{x}_{i} / d t$. Eqns. (2.1) can then be rewritten as below :

$$
\begin{align*}
\frac{d \mathbf{x}_{i}}{d t} & =\mathbf{v}_{i} \\
m_{i} \frac{d \mathbf{v}_{i}}{d t} & =\mathbf{F}_{i}^{i n t}+\mathbf{F}_{i}^{e x t} \quad \text { for } i=1 \ldots n  \tag{2.2}\\
\mathbf{v}_{i}(t=0) & =\mathbf{v}_{0, i} \\
\mathbf{x}_{i}(t=0) & =\mathbf{x}_{0, i}
\end{align*}
$$

Let us call $\mathbf{q}=\left[\mathbf{x}_{i}, \mathbf{v}_{i}\right]^{\top}$ the phase vector containing positions and velocities of all mass points and $\mathbf{f}(\mathbf{q})=\left[\mathbf{v}_{i}, m_{i}^{-1}\left(\mathbf{F}_{i}^{\text {int }}+\mathbf{F}_{i}^{\text {ext }}\right)\right]^{\top}$ the right hand side function, eqns.(2.2) can be rewritten again under the familiar form of a system of first order ODEs as follows :

$$
\begin{align*}
\frac{d \mathbf{q}}{d t} & =\mathbf{f}(\mathbf{q}, \mathbf{t})  \tag{2.3}\\
\mathbf{q}(t=0) & =\mathbf{q}_{\mathbf{0}}
\end{align*}
$$

### 2.2.2 Time integration

In order to obtain the dynamic behaviors of the wing, eqns. (2.3) need to be advanced in time employing a numerical discretization scheme. The choice for this scheme is not straight forward since it depends on many factors. Due to the size of the wings, the mass-spring network contains a lot of tiny springs which make the system very stiff and we need an implicit scheme for time marching. For this reason, a centered scheme or a backward scheme can be used. Although centered schemes are usually in favor for their conservation property without numerical diffusion, a centered scheme, for example the trapezoidal scheme, can lead to numerical instability at some points because the eigenvalues of the operator lie exactly on the imaginary axis, the borderline of the stability zone. Furthermore, the coupling between the fluid and the solid solver will require an adaptive time stepping scheme. For all these reasons, a second order backward differentiation scheme with variable time steps [5] is used to discretize eqns. (2.3) in time.

$$
\begin{equation*}
\mathbf{q}_{i}^{n+1}-\frac{(1+\xi)^{2}}{1+2 \xi} \mathbf{q}_{i}^{n}+\frac{\xi^{2}}{1+2 \xi} \mathbf{q}_{i}^{n-1}=\frac{1+\xi}{1+2 \xi} \Delta t^{n} \mathbf{f}\left(\mathbf{q}_{i}^{n+1}\right) \tag{2.4}
\end{equation*}
$$

where $\xi=\Delta t^{n} / \Delta t^{n-1}$ is the ratio between the current $\Delta t^{n}$ and the previous time step $\Delta t^{n-1}$. Eqn. (2.4) is a system of nonlinear equations with the variable $\mathbf{q}^{n+1}$, the phase vector of the system at the current time step, which needs to be solved. The NewtonRaphson method, a powerful iterative method, is employed to solve this nonlinear system of equations. With an initial guess, which is reasonably close to the true root of the equations, Newton-Raphson helps to find approximations of the root with the rate of convergence estimated to be quadratic. For our mass-spring solver, the initial guess chosen is the phase vector $\mathbf{q}^{n}$ solved at the previous time step; this allows the Newton-Raphson method to converge quickly since the structure is advanced slowly
and smoothly in time, which assures that $\mathbf{q}^{n+1}$ remains close to $\mathbf{q}^{n}$. In most cases, the Newton-Raphson method in the solver needs three to four iterations to converge within a relative or absolute $L_{2}$ norm error of $10^{-6}$ as the stopping criterion. In practice, eqn. (2.4) leads to a sparse linear system of size $6000 \times 6000$ which is too large to be solved efficiently by standard algorithms. Consequently, we used SuperLU library for this task. SuperLU is an open-source library ${ }^{1}$ which contains a set of subroutines to solve a sparse linear system $A \cdot X=B$. It uses Gaussian elimination with partial pivoting (GEPP) and takes into account the sparsity of the system. By implementing SuperLU, we managed to speed up the solid solver with a factor of 10 .

### 2.2.3 Forces

The nonlinear properties of eqn. (2.3) come from the forces acting on the mass-spring system. These forces are composed of external forces and internal forces.

### 2.2.3.1 External forces

The external forces consist of all forces coming from outside of the mass-spring system. These forces, normally in an insect flight context, include gravity, air friction and pressure. The last two forces are generated due to the interaction between the wing and the surrounding air and they will be derived in detail later in chapter 4 when the solid solver is coupled with the fluid solver.

In addition, the wing dynamics is solved in the wing reference frame which is noninertial. At time $t$, the angular velocity $\boldsymbol{\Omega}$ and the angular acceleration $\dot{\boldsymbol{\Omega}}$. The fictitious forces acting on a mass point $m_{i}$ are given by

$$
\begin{equation*}
\mathbf{F}_{\text {fictitious }}=-2 m_{i} \boldsymbol{\Omega} \times \mathbf{v}_{i}-m_{i} \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{i}-m_{i} \boldsymbol{\Omega} \times\left(\boldsymbol{\Omega} \times \mathbf{r}_{i}\right) \tag{2.5}
\end{equation*}
$$

where $\mathbf{r}_{i}$ is the position vector of the mass point $m_{i}$

### 2.2.3.2 Internal forces

The internal forces represent the interaction among the mass points through the connecting springs. Extension springs (figure 2.1a) and bending springs (figure 2.1b)

[^1]are common mechanical devices, which resist against the external forces to get back to their resting positions. The former is designed to operate with axial forces, while the latter is used for torques. The relations between the displacement and the restoring force are given by :

(A) extension spring

(B) bending spring

FIGURE 2.1 : Illustration of the restoring forces corresponding to the deformation of extension and bending springs.

- Linear extension spring

$$
\begin{align*}
& \mathbf{F}_{i}=k_{i j}^{e}\left(\left\|\mathbf{x}_{i j}\right\|-\left\|\mathbf{x}_{0, i j}\right\|\right) \mathbf{e}_{i j}  \tag{2.6}\\
& \mathbf{F}_{j}=-\mathbf{F}_{i}
\end{align*}
$$

where $k_{i j}^{e}$ is the extension stiffness, $\mathbf{e}_{i j}=\left(\mathbf{x}_{j}-\mathbf{x}_{i}\right) /\left\|\mathbf{x}_{j}-\mathbf{x}_{i}\right\|$ is the unit position vector and $\mathbf{F}_{i}$ and $\mathbf{F}_{j}$ are the restoring forces of the extension spring acting on two points $i$ and $j$, respectively;

- Linear bending spring

$$
\begin{equation*}
\mathbf{M}_{i j k}=-k_{i j k}^{b}\left(\theta_{i j k}-\theta_{0, i j k}\right) \tag{2.7}
\end{equation*}
$$

or in terms of forces

$$
\begin{align*}
\mathbf{F}_{i} & =k_{i j k}^{b}\left(\theta_{i j k}-\theta_{0, i j k}\right)\left(\mathbf{e}_{i j} \times \mathbf{e}_{j k}\right) \times \mathbf{e}_{i j} \\
\mathbf{F}_{k} & =k_{i j k}^{b}\left(\theta_{i j k}-\theta_{0, i j k}\right)\left(\mathbf{e}_{i j} \times \mathbf{e}_{j k}\right) \times \mathbf{e}_{j k}  \tag{2.8}\\
\mathbf{F}_{j} & =-\mathbf{F}_{i}-\mathbf{F}_{k}
\end{align*}
$$

where $\mathbf{M}_{i j k}$ is the restoring moment, $k_{i j k}^{b}$ is the bending stiffness, $\theta_{0, i j k}$ is the initial bending angle among three points $i, j$ and $k, \theta_{i j k}$ the current bending angle and $\mathbf{F}_{i}, \mathbf{F}_{j}, \mathbf{F}_{k}$ are the restoring force vectors (as shown in figure 2.1) of the bending spring acting on three points $i, j$ and $k$, respectively.

However, the calculation of $\theta_{i j k}$ in eqn.(2.8) is not trivial since it involves the geometrical definition of the angle in 3D space with respect to a fixed coordinate system. Firstly, we consider a simpler case when the three points are in the same plane, a 2 D coordinate system $O x y$. This leads to $\mathbf{x}_{i}=\left(x_{i}, y_{i}\right)^{T}, \mathbf{x}_{j}=\left(x_{j}, y_{j}\right)^{T}$ and $\mathbf{x}_{k}=\left(x_{k}, y_{k}\right)^{T}$. The angle is now determined by :

$$
\begin{array}{r}
\theta_{i j k}=\operatorname{atan} 2\left[\left(y_{k}-y_{j}\right)\left(x_{j}-x_{i}\right)-\left(y_{j}-y_{i}\right)\left(x_{k}-x_{j}\right)\right. \\
\left.\left(x_{k}-x_{j}\right)\left(x_{j}-x_{i}\right)+\left(y_{k}-y_{j}\right)\left(y_{j}-y_{i}\right)\right] \tag{2.9}
\end{array}
$$

Here, atan2 (usually known as two-argument arctangent) is a special function first introduced in computer programming languages to give a correct and unambiguous value for the angle by taking into account the sign of both arguments. This function helps us to calculate on the whole space when the angle can vary in the range of $(-\pi, \pi]$, instead of the range of $(-\pi / 2, \pi / 2)$ when using the standard arctangent function.

For a problem in 3D space, only one bending angle will not be sufficient. This can be easily seen by considering a simple case of one bending spring. At an instant $t$, the spring is deformed and has a bending angle $\theta_{i j k}$. Nevertheless, corresponding to this $\theta_{i j k}$, there is an infinite number of solutions $\mathbf{x}_{i}, \mathbf{x}_{j}$ and $\mathbf{x}_{k}$ that can satisfy this deformation and the set of all these solutions forms a cone, like shown in figure 2.2. Consequently, one more angle is needed to obtain a unique solution. To define these


Figure 2.2 : All possible shapes of a bending spring corresponding to a nonzero bending angle $\theta_{i j k}$.
two bending angles, the bending spring is projected on the $O x y$ and the $O x z$ planes (cf. figure 2.3) which gives us two 2D bending angles $\theta_{i j k}^{y}$ and $\theta_{i j k}^{z}$ on the $O x y$ and the $O x z$ planes, respectively. Then, like in the 2D problem, these two bending angles are calculated as below :

$$
\begin{array}{r}
\theta_{i j k}^{y}=\operatorname{atan} 2\left[\left(x_{j}-x_{i}\right)\left(y_{k}-y_{j}\right)-\left(x_{k}-x_{j}\right)\left(y_{j}-y_{i}\right),\right. \\
\left.\left(x_{j}-x_{i}\right)\left(x_{k}-x_{j}\right)+\left(y_{j}-y_{i}\right)\left(y_{k}-y_{j}\right)\right]  \tag{2.10}\\
\theta_{i j k}^{z}=\operatorname{atan} 2\left[\left(x_{j}-x_{i}\right)\left(z_{k}-z_{j}\right)-\left(x_{k}-x_{j}\right)\left(z_{j}-z_{i}\right),\right. \\
\left.\left(x_{j}-x_{i}\right)\left(x_{k}-x_{j}\right)+\left(z_{j}-z_{i}\right)\left(z_{k}-z_{j}\right)\right]
\end{array}
$$



Figure 2.3 : Illustration of the two bending angles $\theta_{i j k}^{y}$ and $\theta_{i j k}^{z}$ that are used for calculating the forces of the bending spring $i j k$.

### 2.3 Insect wing structure and functional approach

Modeling insect wings is challenging due to the fact that these wings have complex structures composed of a network of veins, partly connected through hinges, with thin membranes spanned in between and their elasticity properties are still poorly understood. Certain studies have shown that the vein arrangements in insect wings have strong impact on their mechanical properties [18, 19]. Thus, it will be inaccurate to consider a wing as a homogeneous structure; the vein pattern as well as the difference in terms of mechanical behaviors between vein and membrane need to be taken into account. However this is not an easy task, since insects are the most diverse group of animals living on Earth with more than one million known species with varying wing sizes and wing shapes. As a result, in this study, we want to limit ourself to a specific case when we examine only bumblebee (Bombus ignitus) and blowfly (Calliphora vomitoria). Insect wings are mainly composed of veins and membranes 2.4 . The longitudinal veins extending along the wing in the spanwise direction are big, hollow and providing conduits for nerves while the cross veins are smaller, solid and connecting the longitudinal veins to form a truss structure. In the space between these veins, we find a thin layer called membrane.


Figure 2.4: Calliphora vicina wing structure [61] which is mainly composed of veins and membranes. The big longitudinal veins extending along the wing in the spanwise direction connected by the smaller cross veins, forming a truss structure. In the space between these veins, we find a thin layer called membrane.

From a mechanical point of view, veins can be considered as rods which resist mainly the torsion and bending deformation. On the other hand, a membrane is fabric-like, it behaves like a piece of cloth which resists again the extension deforma-
tion. Consequently, instead of considering the wing as a homogeneous structure, a functional approach is used to distinguish veins and membranes. We then propose two models using mass-spring networks to imitate the mechanical behavior of the vein and the membrane.

A vein is considered as a rod whose length is much greater than its height and width. The total effect of all the external loads applied on a mechanical structure results in deformation which can usually be classified into three main types : bending, twisting and stretching. Although the whole wing is observed to be twisted significantly in many studies using high speed cameras or the digital particle image velocimetry [10, 71, 98-100], it is not entirely clear that torsion happens locally at veins or the unsynchronized bending deformations between veins cause the whole wing to twist. To simplify the model, we study only the latter in which we ignore the local torsion of veins and model solely the bending deformation of veins by using extension and bending springs. Thus, we model a vein by a curve line which is discretized by $n$ mass points as shown in figure 2.5. Two neighboring points are connected with each other by an extension spring (e.g. the mass points $i$ and $i+1$ are connected by the extension spring $k_{i}^{e}$ ) and three neighboring points are connected with each other by a bending spring (e.g. the mass points $i-1, i$ and $i+1$ are connected by the bending spring $k_{i-1}^{b}$ ).


Figure 2.5 : Illustration of a vein modeled by mass points, extension springs and bending springs. White circles represent mass points, solid lines represent extension springs, black circles represent both mass points and bending springs.

When flapping, most of external forces will act in the direction perpendicular to the wing surface. As a result, the stretching deformation is negligible comparing to the bending deformation. Thus, the role of the extension springs in the model is solely to conserve the length of the vein. The stiffness of these extension springs is artificial and they do not need to reflect the mechanical property of the vein itself. They should be chosen stiff enough to make the rod unstretched but not too stiff to avoid problems
2.4 - Correlation between mass-spring network models and continuum constitutive
with numerical stability when integrating the dynamical system in time.
Compared to veins, membranes are totally different in terms of geometrical and mechanical properties. Geometrically, a membrane is an object whose thickness is much smaller than its extent. Consequently, a membrane is usually considered as a planar two-dimensional sheet or a set of planar sheets in the case of non-planar three-dimensional membranes [42, 106]. On the mechanical side, a membrane is a special kind of structure compared to other structural elements, i.e. a rod, a bar, a plate or a beam. It behaves like a piece of cloth which is much easier to be bent than to be stretched or compressed. Keeping these in mind, the membrane part of the wing is modeled by a 2D sheet which is discretized by a system of mass points and extension springs. There are several ways to discretize a 2D sheet (as shown in figure 2.6) but an unstructured triangular mesh needs to be employed for our problem due to the complicated geometry of insect wing. Moreover, an unstructured mesh is preferred for modeling isotropic membranes [16] since the random orientations of the springs will average out the forces.


Figure 2.6: Different mesh structures for 2D discretization [77].

### 2.4 Correlation between mass-spring network models and continuum constitutive laws

Besides the mesh topology, the parameter setting is another challenge that one has to solve in order to model correctly the material of which the object is made. There are two main parameters needed to be assigned for a mass-spring model : the masses and the spring stiffness. Although a Voronoi diagram can be used to find the masses
in a proper way [25], selecting spring stiffness is still an open question and there are two common solutions to overcome this [66]. The first approach is based on optimization methods to minimize the difference between the results solved by the mass-spring model and the reference data. These reference data can come from the measurements, the visual appearance of real objects [67] or numerical solutions using validated methods such as finite element methods [7, 8]. In general, this approach cannot be applied if the system has too many degrees of freedom with many unknown spring constants or the mesh topology varies in time since one set of parameters works for solely one mesh structure. Otherwise, tuning the spring stiffness by using optimization can give satisfying results with reasonable computational cost.

The second way is about deriving a relation between spring stiffness and other continuum mechanic properties, such as Young modulus, the Poisson ratio and the flexural rigidity. In contrast to the discrete models, the elasticity parameters have been obtained for many materials and can be used to calculate the corresponding spring stiffness. Omori et al. [77] succeeded in doing this for a planar membrane by considering a 2D sheet under small uniaxial deformation. The relation between spring networks and continuum models for three types of meshes is shown in figure 2.7.

| Mesh Type | $k$ as a function of $E_{s}$ | $v_{s}$ |
| :---: | :---: | :---: |
|  | $k=\frac{L_{0}+2 w_{0} \sin \theta_{0}}{L_{0}+2\left(w_{0} \sin \theta_{0}+l_{0} \cos \theta_{0}\right)} \frac{w_{0}}{l_{0}} E_{s}$ <br> Isotropic mesh: $k=2 / 3 E_{s}$ | $v_{s}=\frac{2 l_{0} \sin \theta_{0}}{L_{0}+2 w_{0} \sin \theta_{0}} \frac{l_{0}}{w_{0}}$ <br> Isotropic mesh: $v_{s}=1 / 2$ |
|  | $\begin{aligned} & k=\frac{L_{0}+w_{0} \sin \theta_{0}}{L_{0}+w_{0} \sin \theta_{0}+l_{0} \cos \theta_{0}} \frac{w_{0}}{l_{0}} E_{s} \\ & \text { Isotropic mesh: } k=3 / 4 E_{s} \end{aligned}$ | $v_{s}=\frac{l_{0} \sin \theta_{0}}{L_{0}+w_{0} \sin \theta_{0}} \frac{l_{0}}{w_{0}}$ <br> Isotropic mesh: $v_{s}=1 / 3$ |
|  | Isotropic mesh: $k=\frac{\sqrt{3}}{2} E_{s}$ | Isotropic mesh: $v_{s}=1 / 3$ |

Figure 2.7 : Relationship between the spring constant $k$, Young modulus $E_{s}$, and Poisson ratio $v_{s}$ in the small deformation limit for a 2D membrane under uniaxial deformation. The membrane is discretized by three types of meshes and subjected to homogeneous deformation. Figure adapted from [77].

For the vein model, a relation between the flexural rigidity $E I$ and the stiffness of the
2.4 - Correlation between mass-spring network models and continuum constitutive
bending springs $k_{i}^{b}$ is needed. To derive this relation, we consider a classical problem of a cantilever beam length $l_{b}$, under a point force $\mathbf{F}$ at the free end (figure 2.8). In the limit of small displacement, the principle of energy yields the value of the bending spring stiffness $k^{b}$ as a function of the flexural rigidity $E I$. The energy stored in this beam at the static state can be calculated easily using the Euler-Bernoulli beam theory as shown in eqn. (2.11).

$$
\begin{equation*}
E_{\text {beam }}=\frac{F^{2} l_{b}^{3}}{6 E I} \tag{2.11}
\end{equation*}
$$

The mass-spring network is called an equivalent model of the continuous beam if under the same external loads, its mechanical behavior (in this case, it is the energy stored in the system) is the same as the one of the beam. Let us now study a massspring network discretized into $n+2$ mass points connected by bending and extension springs as shown in figure 2.8. All the bending and extension springs are the same with a stiffness $k^{b}$ and $k^{e}$, respectively and $k^{e} \gg k^{b}$. The first two points are totally fixed to represent the boundary condition of the fixed end of the beam. Writing the equilibrium equations for the remaining $n$ points, we have :

$$
F \frac{l_{b}}{(n+1)}(n+1-i)=k^{b}\left(\theta_{i+1}-\theta_{i}\right) \text { for } i=1 \ldots n
$$

Considering the deformations of extension springs are very small, the total potential energy of all the bending springs of the system is

$$
\begin{equation*}
E_{\text {mass-spring }}=\frac{1}{2} k^{b} \sum_{i=1}^{n}\left(\theta_{i+1}-\theta_{i}\right)^{2} \tag{2.13}
\end{equation*}
$$

With eqn. (2.12) and eqn. (2.13) we obtain :

$$
\begin{align*}
E_{\text {mass-spring }} & =\frac{F^{2} l_{b}^{2}}{2 k^{b}(n+1)^{2}} \sum_{i=1}^{n} i^{2} \\
& =\frac{F^{2} l_{b}^{2}}{2 k^{b}(n+1)^{2}} \frac{n(n+1)(2 n+1)}{6}  \tag{2.14}\\
& =\frac{F^{2} l_{b}^{2}}{12 k^{b}} \frac{n(2 n+1)}{n+1}
\end{align*}
$$

By comparing eqn. (2.11) and eqn. (2.14), we can derive an analytical relation bet-
ween $k_{b}$ and $E I$ as following:

$$
\begin{equation*}
k^{b}=\frac{E I}{l_{b}} \frac{n(2 n+1)}{2(n+1)} \tag{2.15}
\end{equation*}
$$

Since eqn. (2.15) is derived based on the assumption of small displacement, we still have here a linear problem thus the principle of superposition can be applied. During the flight, the aerodynamic loads acting on insect wings can be considered to be equivalent to distributed loads on the surface of the wings. These distributed loads can be discretized into many point forces using a work-equivalent method [22] and then the superposition principle can be applied. Thus, it is sufficient to analyze only one point force case to find the relation between $E I$ and $k^{b}$, since it does not depend on the point force $\mathbf{F}$.

However, as mentioned at the beginning of this section, insect wings are deformed significantly to create lift for flying. Here, we are dealing with a large displacement problem and the question is if eqn. (2.15) still remains valid. The technique used to derive eqn. (2.15) is no longer applicable since the solution for large deflection of a cantilever beam cannot be obtained analytically [4]. This problem involves calculating elliptical integrals of the second kind [9] and needs to be solved numerically. Consequently, the relation between $E I$ and $k^{b}$ is put into a large displacement, nonlinear test case to check if we still get the same mechanical behaviors between the continuous beam and the mass-spring model. The results are presented in the next section.

### 2.5 Validations of the mass-spring model

### 2.5.1 Vein model - Cantilever beam under gravitational force

Static case Firstly, we consider a static case of a cantilever beam with length $L=0.3$, flexural rigidity $E I=0.24$ and loaded by a point force $F$ at the free end. The force $F$ varies from 0.39 to 11.76 and it must be strong enough to cause a large deflection. All the parameters here are dimensionless. The vertical displacement $\delta y$ and the horizontal displacement $\delta x$ of the free end of the beam at equilibrium state can be calculated by using the fundamental Bernoulli-Euler theorem [101] and the massspring network as given in table 2.1. The static state of the vein model (discretized by $n=64$ mass points) is obtained by solving the dynamic equations of the system with


FIGURE 2.8 : Illustration of deformation corresponding to forces applied on extension and bending springs.
artificial damping forces to make the system go quickly towards its balanced position. Despite the small displacement assumption for deriving the relation between $E I$ and $k^{b}$, the relation in eqn. (2.15) is still valid even in very large deflection problem. For the case $F=3.92$, the vertical displacement of the free end is already more than $30 \%$ of the total length of the beam and we still get very good agreements between both models with the relative error being smaller than $1 \%$. The mapping from $E I$ to $k^{b}$ can then be generalized for nonlinear, large deflection cases with good agreement between the continuum theory and the discrete mass-spring network.

Dynamic case The vein model will now be compared with another solid solver developed by Engels et al. [39]. It is based on the classical nonlinear beam equation, the Euler-Bernoulli theory. All details about this solver can be found in [34]. We study the case when we have a 2D cantilever beam (figure 2.9) of length $l_{b}=1$, density $\rho=0.0571$, flexural rigidity $E I=0.0259$. The beam is in vacuum, subjected to a gravity

Table 2.1 : Comparison between the continuum theory and the discrete mass-spring network in the static large deflection case.

| Point force | Nonlinear beam [9] |  | Mass-spring network |  | Relative error |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $F$ | $\delta x_{r e f}\left[10^{-2}\right]$ | $\delta y_{r e f}\left[10^{-2}\right]$ | $\delta x\left[10^{-2}\right]$ | $\delta y\left[10^{-2}\right]$ | $e r r_{x}[\%]$ | $e r r_{y}[\%]$ |
| 0.39 | 29.96 | -1.46 | 29.96 | -1.46 | 0 | 0 |
| 1.96 | 29.02 | -6.93 | 29.01 | -6.92 | 0.03 | 0.14 |
| 3.92 | 26.87 | -12.14 | 26.85 | -12.1 | 0.07 | 0.33 |
| 7.84 | 22.53 | -17.93 | 22.53 | -17.85 | 0 | 0.45 |
| 11.76 | 19.37 | -20.69 | 19.40 | -20.57 | 0.15 | 0.58 |

field $g=0.7$ strong enough to cause large deflection. All the parameters here are dimensionless. Both computations are performed for the same numerical parameters with the time step $d t=10^{-2}$ and $n=64$ grid points. Although both solvers require the same amount of CPU time for the same resolution, the mass-spring network is still more efficient since it is designed to deal with 3D problems. For the computation, the mass-spring solver handles a system of $3 n$ degrees of freedom, corresponding to 3 dimensions, while the nonlinear-beam solver only solves for $2 n$ variables.

The deflection line of the two models at a given time $t$ and the oscillation of the trailing edge $y_{t e}(t)$ are shown in figure 2.10. The dashed blue line calculated by the nonlinear beam theory and the solid red line calculated by the mass-spring network are almost coincident with each other. We have an excellent agreement between these two models with a relative error smaller than $1 \%$.

### 2.5.2 Membrane model - Uniaxial and isotropic deformations of a two-dimensional sheet

We consider here the same test case proposed by Omori et al. [77] where a square 2D sheet with an initial side length $l_{0}=1$ is extended by a uniaxial tension $T=0.005$ and has a final length $l$ in the $x$-direction at the equilibrium state, as shown in figure 2.6. This tension must be small enough to cause small deformation on the sheet. The Young modulus $E_{S}$ is defined by :

$$
\begin{equation*}
E_{s}=\frac{T}{\epsilon} \tag{2.16}
\end{equation*}
$$




Figure 2.9 : Cantilever beam subjected to gravity field modeled by a continuous nonlinear beam and a mass-spring network.
where $\epsilon$ is the strain.
The sheet is then discretized by using three types of meshes, illustrated in figure 2.6. The grid size $\Delta l$ is the side length of one triangle element of the mesh and inversely proportional to the number of grid points $n$. The grid size is varied to refine the mesh resolution. Since we are only interested in the equilibrium state of the model, the masses will not have any effect on the result and they are chosen properly for the numerical convergence. Instead of solving the static equation of the system, we still solve here the dynamic equations of the system with artificial damping forces to make the system go quickly towards its balanced position. Last but not least, all the spring stiffnesses are set to the same value $k=1$. Figure 2.12 shows the results we get for all three mesh structures. First, for the cross-center structure, we are able to reproduce the result of Omori et al. [77]. When the mesh is refined, the ratio $k / E_{s}$ converges to the analytical value $3 / 4$ with the relative error being smaller than $1 \%$. For the regular triangle, due to the shape of the square, we have some minor flaws of the mesh at the border. But these can be neglected when the mesh is fine enough and we can


Figure 2.10 : (A) The oscillation of the trailing edge $y_{t e}(t)$ and (B) the deflection lines at $t=2$ calculated by the nonlinear beam theory [34] (dashed blue line) and the mass-spring network (solid red line).
consider it as a regular triangular mesh. Indeed, for high resolution, we find again an excellent agreement with the analytical ratio $k / E_{s}=\sqrt{3} / 2$ derived by Omori et al. The relative error is also smaller than $1 \%$. However, for the unstructured mesh, the convergent value of $k / E_{s}$ is larger than the one of Omori et al., but identical to the analytical solution for the regular triangle. This finding is in fact expected by Omori et al. since these two meshes are both constructed with triangles, each node being connected to six springs. Yet, the random structure of the unstructured mesh makes it difficult to explain the difference.

Using our mass-spring model, we are capable of reproducing the results from the references, which indicates the reliability of the solver. These results for both small and large deformation cases allow us to have the same conclusions as in the literature about the mass-spring system. Even though the mechanical properties of spring networks are strongly dependent on the mesh topology, a correlation between the discrete model and the continuum model can still be obtained if the mesh is fine enough. However, this needs to be compromised with the computational efficiency which is the main reason that we choose mass-spring network in the first place.


Figure 2.11 : Deformation of a 2D sheet along the $x$-axis under the uniaxial tension $T$.

### 2.6 Conclusion

A solid solver dedicated for modeling flexible insect wings has been developed by using mass-spring systems. The method is chosen due to its efficiency and simplicity. Nevertheless, ordinary mass-spring systems suffer from several intrinsic limitations that restrict their use in physical modeling. The most troubling of these limitations are listed here :

- Compared with models based on continuum mechanics theories, such as the finite element or the finite differences methods, ordinary mass-spring system are conditionally accurate. They does not necessarily converge to the analytical solution of the continuous model as the mesh is refined;
- The behavior of these systems depends strongly on the resolution and the topology of the mesh;
- Finding the right spring functions and parameters to obtain an accurate model is a very difficult and application dependent process;
- The systems can generate undesired isotropic or anisotropic effects;
- It is difficult to enforce a constant-volume constraint.

Despite of these drawbacks, the results obtained from the validations indicate that the mechanical behavior of spring-network models are conditionally equivalent to that of continuum models. The discrete mass-spring model can still be used to mimic


Figure 2.12 : Effect of mesh topology on the relation between the spring stiffness $k$ and the corresponding Young modulus $E_{s}$
the behaviors of continuum models depending on the mesh structure and the spring coefficients that we use.

The structure of wings is taken into account to correctly represent the mechanical behavior of the wings themselves. The vein and the membrane parts are modeled differently by employing various kinds of springs and mesh structures. These models have been tested separately and have shown excellent agreement with the literature. However, the parameters for properly simulating real insect wings is still an open question. Consequently, the two approaches presented here will be used in the next chapter for finding these parameters.

## Chapter 3

## Flexible wing model

## Content

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The simulation of insects with flexible wings is demanding not only because it involves solving for both fluid and solid dynamics, but also due to the fact that insect wings are sophisticated structures. In our work, we want to take into account the mechanical properties of the insect wing as much as possible, in order to correctly model its dynamical behavior. In this chapter, the modeling of flexible wings by coupling the vein model and the membrane model introduced in chapter 2 will be presented along with its application to bumblebees and blowflies. The content of this chapter is adapted from [91-93].

### 3.1 Bumblebee wing

A bumblebee (Bombus ignitus) wing model is developed based on the measurement data from [60]. There are three main factors introduced, which are considered to have the most impact on wing deformation during flight : venation pattern, mass distribution and vein stiffness.

### 3.1.1 Wing venation

The venation architecture is claimed to affect the anisotropy of the wing. Throughout measurements from different insects, Combes et al. $[18,19]$ suggest that wing flexural stiffness declines exponentially towards the tip and trailing edge. This is explained by the common venation patterns of insect wings : most insect wings have thick, stiff veins at the leading edge while cross veins are thinner as they expand toward the wing tip. This structure allows insect wings to resist against strong bending deformation in the spanwise direction, while creating camber for lift generation in the chordwise direction [19]. Nakata and Liu [74] modeled the anisotropy caused by wing veins. To this end, they took into account the variation of wing thickness and introduced a "rule of mixture" of composite materials.

In our model, the functional approach is used to take into consideration the venation pattern. The vein structure, as well as the wing contour, are adapted from the data from [60] and encoded into the mass-spring network, as shown in figure 3.1. Compared to the reference data, two more veins are added (vein 21 in the forewing and vein 7 in the hindwing) and two forewing veins 19 and 20 are extended toward the tip of the wing. These modifications are made to add bending stiffness to the tip of the wing and thus to obtain a more realistic behavior during the simulation. The meshing is done by identifying firstly the contour of the wing and all the veins (green, red and blue curves in figure 3.1). The membrane is then discretized by a triangular mesh using SALOME ${ }^{1}$, an open-source integration platform for mesh generation.

To study the influence of wing flexibility on the aerodynamics performance, the flexural rigidity of veins will be changed to alter the bending stiffness of the wing. Consequently, only the Young's modulus $E$ will be varied. Insect cuticles are reported to have a Young's modulus about $1 k P a$ to 20GPa [96]. For our study, the value of $E$

[^2]

Figure 3.1 : Illustration of the mass-spring model which is meshed based on measured data of real bumblebee wings [60]. The black and white markers represent mass centers. Color codes (red, green and blue) are used for identifying different veins and the membranes are represented by cyan circles.
is varied in this range to have wings with different bending stiffness. This allows us to have some insights about the influence of wing flexibility on their aerodynamic performance. Then, the flexural rigidity $E I$ of each vein will be calculated using the second moment of area $I$ of circular-section veins with diameters given in table 3.1.

### 3.1.2 Mass distribution

Another property which plays an essential role on the aerodynamics of the wing, is the mass distribution. It represents the inertia of the system and the position of the mass center has a strong connection with the stability of the wing during flight. The mass distribution is calculated based on the measured wing mass data from [60]. For our numerical simulations, the total wing mass is chosen as the same used by Kolomenskiy et al. [60], $m_{w}=0.791 \mathrm{mg}$. The mass is then distributed into vein and membrane parts based on their geometry and material. Each vein is considered as a rod composed of cuticle, $\rho_{c}=1300 \mathrm{~kg} / \mathrm{m}^{3}$ [60], with a circular cross section of constant diameter $d_{v}[60]$ and length $l_{v}$, calculated directly from the model. The mass of each vein is then calculated and the results are shown in table 3.1. Both diameter and mass are dimensionless quantities, normalized by wing length $R$ and air density
$\rho_{a i r} R^{3}$, respectively.
The mass distribution for the membrane is more tricky since we do not have the material properties of bumblebee membranes. A bi-linear regression is employed due to the fact that the membrane is heavier near the wing root and the leading edge [60]. A simple mathematical optimization is employed to find the parameters for the regression using the mass center from the measured data as an objective function. For a mass point $m_{i}$ belonging to the membrane at position $\left[x_{i}, y_{i}\right]$ (the $z$ coordinate is neglected here because we assume that the membrane is a planar sheet), we get:

$$
\begin{equation*}
m_{i}=1.75 \cdot 10^{-4}-2.83 \cdot 10^{-4} x_{i}+4.91 \cdot 10^{-4} y_{i} \tag{3.1}
\end{equation*}
$$

This yields a difference, between two mass centers, of 0.0013 in the $x$-direction and 0.0085 in the $y$-direction which are negligible compared to the reference wing length $R=1$.

### 3.2 Fly wing

Using the same methodology with some adjustments, another model of blowfly (Calliphora vomitoria) wings is built based on the experimental data from [104].

### 3.2.1 Wing shape and venation pattern

The wing shape along with the venation structure is adapted from the data obtained by Wehmann et al. [104] using profilometer measurements and a geometrical analysis. The wing length, averaged over 11 tested individuals, is $9.29 \pm 0.20 \mathrm{~mm}$. Figure 3.2a shows the schematic of blowfly wings used for the meshing task and figure 3.2 b illustrates its corresponding mass-spring model. The wing length used here for the model is $R=9 \mathrm{~mm}$. As in the previous case of the bumblebee wing, the color codes (red, green and blue) are for vein identification purpose while the black circles represent mass points belonging to the membrane. Both length and mass are dimensionless quantities, normalized by the fly wing length $R$ and the air density $\rho_{\text {air }} R^{3}$, respectively.

Instead of assuming that each vein has a constant diameter, the change of the diameter along each vein is now taken into account. Each vein is considered as a conical frustum. The diameters at two ends of each conical frustum are determined

| Forewing |  |  | Hindwing |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| \# | Nominal diameter | $\begin{gathered} \text { Nominal } \\ \text { mass } \end{gathered}$ | \# | Nominal diameter | Nominal mass |
| 1 | 0.0070 | 0.0209 | 1 | 0.0065 | 0.0180 |
| 2 | 0.0074 | 0.0237 | 2 | 0.0043 | 0.0071 |
| 3 | 0.0055 | 0.0076 | 3 | 0.0046 | 0.0024 |
| 4 | 0.0070 | 0.0063 | 4 | 0.0011 | 0.0001 |
| 5 | 0.0040 | 0.0031 | 5 | 0.0038 | 0.0043 |
| 6 | 0.0048 | 0.0094 | 6 | 0.0037 | 0.0005 |
| 7 | 0.0040 | 0.0019 | 7 | 0.0020 | 0.0012 |
| 8 | 0.0038 | 0.0009 |  |  |  |
| 9 | 0.0041 | 0.0023 |  |  |  |
| 10 | 0.0048 | 0.0064 |  |  |  |
| 11 | 0.0045 | 0.0017 |  |  |  |
| 12 | 0.0038 | 0.0018 |  |  |  |
| 13 | 0.0042 | 0.0010 |  |  |  |
| 14 | 0.0038 | 0.0020 |  |  |  |
| 15 | 0.0034 | 0.0008 |  |  |  |
| 16 | 0.0032 | 0.0005 |  |  |  |
| 17 | 0.0032 | 0.0004 |  |  |  |
| 18 | 0.0044 | 0.0009 |  |  |  |
| 19 | 0.0015 | 0.0001 |  |  |  |
| 20 | 0.0018 | 0.0001 |  |  |  |
| 21 | 0.0020 | 0.0009 |  |  |  |

Table 3.1 : Dimensionless vein diameters $d_{v}$ (adapted from [60]) and their corresponding dimensionless mass $m_{\nu}$.
from pictures of three female flies' wings. The photos were taken using a camera (EOS-750D, Canon) attached to a stereomicroscope (Stemi 508, Zeiss) with a magnification of $2170 \mathrm{pixel} / \mathrm{mm}$. This assumption allows for the estimation of the wing mass distribution and the veins' second moment of area $I$.

### 3.2.2 Mass distribution

The distribution of mass along the wing is estimated using the same technique as in section 3.1.2. Ganguli et al. [45] measured 10 wing pairs and found their weights varied from a minimum of $200.3 \mu \mathrm{~g}$ to a maximum of $272.3 \mu \mathrm{~g}$ with a standard deviation of $22.94 \mu \mathrm{~g}$. For our model, the wing mass is set at $250 \mu \mathrm{~g}$. In reality, wing veins are hollow tubes filled with hemolymph which supplies nutrients and other factors to


Figure 3.2 : Illustration of the mass-spring model (B) which is meshed based on measured data (A) of real blowfly wings [104]. The black and white markers represent mass centers. Color codes (red, green and blue) are used for identifying veins and the membrane is represented by the black triangular mesh. The center of wing mass is shown by the black and white marker. The length is normalized by the fly wing length. The dimensionless vein diameters are displayed by real ratios in the figure.
the wing's living tissues [78]. However, this structure is too complex to be taken into account in our model. Consequently, each vein is simply considered as a solid rod made of cuticle with $\rho_{c}=1300 \mathrm{~kg} / \mathrm{m}^{3}$. This assumption will have the tendency to overestimate the mass of the vein system.

Since veins are treated as conical frustums, the distribution of the mass into the mass points is not trivial. We consider one segment $i$ of one arbitrary vein as shown in figure 3.3. The segment is a conical frustum with the left $r_{i}$ and the right $r_{i+1}$ radii $\left(r_{i}>r_{i+1}\right)$. The mass of this segment $m_{i}^{v}$ is distributed into the two mass points at the left $m_{i}^{M S M}$ and the right $m_{i+1}^{M S M}$ in the way where the two systems have the same center of gravity $C G$. This condition is given by the relation :

$$
\begin{equation*}
h_{i} m_{i}^{v}=m_{i+1}^{M S M}\left(h_{i}+h_{i+1}\right)=m_{i+1}^{M S M} l_{i}^{v} \tag{3.2}
\end{equation*}
$$

where $h_{i}$ is the centroid of the conical frustum :

$$
\begin{equation*}
h_{i}=\frac{l_{i}^{\nu}}{4} \frac{\left(r_{i}^{2}+2 r_{i} r_{i+1}+3 r_{i+1}^{2}\right)}{\left(r_{i}^{2}+r_{i} r_{i+1}+r_{i+1}^{2}\right)} \tag{3.3}
\end{equation*}
$$

From eqns. (3.2) and (3.3), we have :

$$
\begin{align*}
& m_{i+1}^{M S M}=m_{i}^{v} \frac{l_{i}^{v}}{4} \frac{\left(r_{i}^{2}+2 r_{i} r_{i+1}+3 r_{i+1}^{2}\right)}{\left(r_{i}^{2}+r_{i} r_{i+1}+r_{i+1}^{2}\right)}  \tag{3.4}\\
& m_{i}^{M S M}=m_{i}^{v}-m_{i+1}^{M S M}
\end{align*}
$$

Eqn. (3.4) is applied for all veins to distribute the mass of the vein network into the mass points. The vein system accounts for more than $65 \%$ of the wing mass. The remaining $35 \%$ is assumed to belong to the membrane of the wing model.


FIGURE 3.3 : Illustration of the distribution of the mass of one vein segment $m_{i}^{\nu}$ into the two mass points $m_{i}^{M S M}$ and $m_{i+1}^{M S M}$ at the two ends of the segment.

For the membrane part, the method which optimizes the mass center of the wing can no longer be used. Unlike for bumblebee wings, to our knowledge, data about mass center positions of Calliphora wings are unclear. The mass of the membrane is distributed into the mass points by using the bi-linear relation in eqn. (3.1) and adjusted by the wing length and wing mass of Calliphora :

$$
\begin{equation*}
m_{i}=2.02 \cdot 10^{-4}-1.65 \cdot 10^{-4} x_{i}+1.13 \cdot 10^{-4} y_{i} \tag{3.5}
\end{equation*}
$$

where $m_{i}$ is the $i^{t h}$ mass point belonging to the membrane whose position is [ $x_{i}, y_{i}$ ].

### 3.3 Vein stiffness identification using genetic optimization

One of the challenges when using the mass-spring system is setting the modeling parameters (masses, spring stiffness values and damping coefficients). As the vein system is responsible for the bending stiffness of the wing, the first critical property is the flexural rigidity $E I$ of the wing veins. While the second moment of area $I$ can
be estimated from the vein geometries, the Young's modulus $E$ remains unknown and it needs to be determined. In addition, the vein joints also play an important role in wing bending behavior. The stiffness of the joints is however difficult to measure since it depends on various factors, such as the existence of vein spikes, the vein shape or the distribution of resilin. For these reasons, the Young's modulus and the joints' stiffness are considered as the parameters which characterize the bending stiffness of the wing in our study and they need to be determined.

As presented in chapter 2, there are two main approaches arising in the literature for assigning these parameters. The deriving analytical expression method, as used in chapter 2 for deriving eqn. (2.15), appears to be rigorous because it is physically-based. However the deriving process is usually complicated and leads to long, sophisticated formulae. It is also based on certain assumptions, e.g. small deformation in linear regime [66, 77]. These limit the advantages of using mass-spring systems which are their simplification and their capability of handling large deformation.

The data-driven strategy, on the other hand, uses reference data for estimating the parameters. The method focuses mainly on using optimization methods where the objective functions are defined as the differences between the reference data and the optimizing data calculated from the model. Yet, the acquisition of the reference data is not straightforward. In some simple cases, solutions from other validated physically-based methods (finite difference or finite element methods) can be employed to calculate the objective function. Unluckily, in other real applications where the studying object is too complex (e.g. blood cells, soft tissue, insect wings), a validated reference model is not always available. Consequently, data from experimental works are usually preferred for this purpose.

In the domain of insect flight, measuring dynamic wing deformation at high temporal frequencies during flight usually requires high-tech equipment which makes the process costly and delicate. Thus, most of qualitative results of wing deformation, found in the literature, are based on high quality stationary photographs instead of descriptions in real time [20]. Even though some studies succeed in recording the wing surface deformation during wing flapping motion [57,71], the information about external inertial and aerodynamic forces acting on the wing in real time is inaccessible. Another alternative is to measure the wing deformation at the static equilibrium as a reference data. Although this approach does not measure the wing dynamics, it facilitates the acquisition of the reference data and still allows us to quantify wing bending
and twisting under known external forces. Consequently, the static measurement is chosen as the reference data for optimizing Young's modulus $E$ and the joints' stiffness. However, due to the cost of the optimization process, we decided to optimize only 10 joints. Based on the structure of the wing, only the joints near the wing leading edge and the wing root are chosen for the optimization. These joints are numbered for identification and shown in figure 3.4.


Figure 3.4 : Calliphora wing diagram with the numbered joints that are needed to be optimized.

### 3.3.1 Experimental and numerical setup

For studying insect wing flexibility, an experimental setup was designed by Wehmann et al. [104] to measure local deformation of blowfly wings under external loads. A blowfly was mounted to a holder below an optical profilometer which projected an Eulerian grid on the entire wing surface and recorded local vertical height at each of the grid nodes. Point forces were applied either at one location with different magnitudes or at different locations with different magnitudes. The locations are shown in figure 3.5b. The force magnitude was measured by a small, cantilever force sensor. During the measurement, the wings were kept attached to their living body to limit the dry-out effect. Otherwise, insect wings will quickly stiffen and become brittle [78]. More details of the measurements can be found in [104].

The data from these measurements are used as the reference for the stiffness identification problem. To be compared with the reference wing, the modeled wing needs

(A)

(B)

Figure 3.5 : (A) Illustration of the experimental setup and (B) the locations of the point forces applied to the wing for measuring the deformation. Figure adapted from [104].
to be put in a similar configuration, numerically. This is done by applying a simple 3D regression method to find the plane fitting all the measurement data points as shown in figure 3.6. The numerical wing is then placed at the same position of the experiment wing and undergoes the same point load for calculating the equilibrium position.

After obtaining both the static equilibrium of the real wing and the mass-spring model wing, the basic idea is to define and optimize an objective function which determines, for any set of model parameters $\kappa$ (spring stiffness values), the difference between the two wings. One possibility is to use the Euclidean distance between the corresponding points of the two models [66]. Such a general objective function $g(\kappa)$ is defined as :

$$
\begin{equation*}
g(\kappa)=\sqrt{\frac{\sum_{i=1}^{N}\left(h_{i}^{r}(\kappa)-h_{i}^{o}(\kappa)\right)^{2}}{N}} \tag{3.6}
\end{equation*}
$$

where $N$ is the number of Eulerian points, $h_{i}^{r}$ and $h_{i}^{o}$ are the local vertical heights


Figure 3.6 : Experimental data showing the normalized surface height $z$, represented by the color map, of one blowfly wing (A) and the corresponding modeled wing (B), represented by the blue surface, which is placed at the same position using 3D linear regression.
of these points belonging to the reference model $\mathbf{x}_{i}^{r}$ and the optimizing model $\mathbf{x}_{i}^{r}$, respectively.

Because the mass points are defined using a Lagrangian grid composed of triangular elements, the vertical heights at grid nodes of the Eulerian grid, projected on the modeled wing, need to be interpolated. We consider an Eulerian grid node $\mathbf{x}^{(E)}$ and a triangular element with three vertices $\mathbf{x}_{i}^{(L)}, \mathbf{x}_{j}^{(L)}$ and $\mathbf{x}_{k}^{(L)}$ whose heights are $h_{i}^{(L)}, h_{j}^{(L)}$ and $h_{k}^{(L)}$, respectively. When the Eulerian point is projected onto the triangle, there are two possibilities :

- The projection of $\mathbf{x}^{(E)}$ is outside the triangle and the interpolation cannot be done;
- The projection of $\mathbf{x}^{(E)}$ is inside the triangle or right on one of the three edges of the triangle. In both cases, the height of the projection of $\mathbf{x}^{(E)}$ is calculated by using barycentric interpolation :

$$
h^{(E)}=\frac{A_{i}}{A} h_{i}^{(L)}+\frac{A_{j}}{A} h_{j}^{(L)}+\frac{A_{k}}{A} h_{k}^{(L)}
$$

where
$A$ is the area of the triangle composed of $\mathbf{x}_{i}^{(L)}, \mathbf{x}_{j}^{(L)}$ and $\mathbf{x}_{k}^{(L)}$, $A_{i}$ is the area of the triangle composed of the projection of $\mathbf{x}^{(E)}, \mathbf{x}_{j}^{(L)}$ and
$\mathbf{x}_{k}^{(L)}$,
$A_{j}$ is the area of the triangle composed of the projection of $\mathbf{x}^{(E)}, \mathbf{x}_{k}^{(L)}$ and $\mathbf{x}_{i}^{(L)}$,
$A_{k}$ is the area of the triangle composed of the projection of $\mathbf{x}^{(E)}, \mathbf{x}_{i}^{(L)}$ and $\mathbf{x}_{j}^{(L)}$.

However, this way of defining the objective function based on the vertical height can cause some errors. Unlike the wing model which is flat, the real blowfly wing has corrugation which leads to some errors even at the initial state when both wings are at rest, as seen in figure 3.6b. To limit the error caused by this effect, another objective function $f(\kappa)$ is employed as below :

$$
\begin{equation*}
f(\kappa)=\sqrt{\frac{\sum_{i=1}^{N}\left(d_{i}^{r}(\kappa)-d_{i}^{o}(\kappa)\right)^{2}}{N}} \tag{3.7}
\end{equation*}
$$

where $N$ is the number of Eulerian points, $d_{i}^{r}$ and $d_{i}^{o}$ are the deformation of these points belonging to the reference model $\mathbf{x}_{i}^{r}$ and the optimizing model $\mathbf{x}_{i}^{r}$, respectively.

Moreover, the deformation of each wing was measured several times with different force locations (cf. figure 3.5b). As a result, the final objective function is not calculated for one but all these measurements :

$$
\begin{equation*}
f(\kappa)=\sqrt{\sum_{j=1}^{n} \frac{\sum_{i=1}^{N}\left(d_{i, j}^{r}(\kappa)-d_{i, j}^{o}(\kappa)\right)^{2}}{N_{j}}} \tag{3.8}
\end{equation*}
$$

where $n$ is the number of measurements conducted on one wing.
Based on the nature of the problem, two penalized zones are specified as shown in figure 3.7. A point force is applied to the wing from below at the unload position. Thus, any solution giving an equilibrium position below the dashed line will be non-physical. On the other hand, if the horizontal distance from the root to the tip of the wing is smaller than half of the wing length at initial position, the wing is considered as too flexible.


Figure 3.7 : Demonstration of the calculation of the objective function and the two penalized zones. The cross section of the wing along the spanwise direction and the point force $\mathbf{F}$ are shown. The difference between the deformation of the reference wing $d^{r}$ and the deformation of the optimizing wing $d^{0}$ is used for the calculation of the objective function in eqn. (3.7). The penalized zone below the unloaded wing (the dashed line) corresponds to the non-physical solutions while the penalized zone above the dotted line corresponds to the too flexible case.

After the objective function is defined in a rigorous way based on the physical properties of the problem and the experimental setup, it is then optimized using the Covariance Matrix Adaptation Evolution Strategy method.

### 3.3.2 Covariance Matrix Adaptation Evolution Strategy

The CMA-ES (Covariance Matrix Adaptation Evolution Strategy) is an optimization algorithm based on the process of natural selection where the best individuals are selected for reproduction of the next generation. The method is developed for difficult
non-linear non-convex black-box optimisation problems in continuous domain [48, 49], especially in cases where an analytical formulation of the objective function is not easily derived. In other words, function values at search points are the only accessible information on $f$.

A standard CMA-ES, as described in detail in [48, 49], is used with weighted intermediate recombination, step size adaptation, and a combination of rank- $\mu$ update and rank-one update. The algorithm addresses the following optimization problem : minimize a nonlinear multivariable objective function from search space $\mathscr{S} \subseteq \mathbb{R}_{+}^{n}$ to $\mathbb{R}_{+}$. Let $x_{k}^{(g)}$ be the $k^{t h}$ offspring (solution candidate) at the $g$ generation (iteration). The new offspring at the next generation $g+1$ are given by :

$$
\begin{equation*}
\mathbf{x}_{k}^{(g+1)}=\mathbf{m}^{(g)}+\boldsymbol{\sigma}^{(g)} \mathscr{N}\left(\mathbf{0}, \mathbf{C}^{(g)}\right) \text { for } k=1 \ldots \lambda \tag{3.9}
\end{equation*}
$$

where $\boldsymbol{\sigma}^{(g)}$ is the overall standard deviation (step size), $\mathscr{N}$ denotes the normal distribution with zero mean and $\mathbf{C}^{(g)}$ is the covariance matrix. After each iteration, the offsprings are evaluated on the objective function $f$ and sorted in decreasing order as :

$$
\left\{x_{i: \lambda} \mid i=1 \ldots \lambda\right\}=\left\{x_{i} \mid i=1 \ldots \lambda\right\} \text { and } f\left(x_{1: \lambda}\right) \leq \cdots \leq f\left(x_{\mu: \lambda}\right) \leq f\left(x_{\mu+1: \lambda}\right) \leq \cdots,
$$

Only the best $\mu$ candidates are chosen as the parents for the reproduction of the next generation. Here, $\mathbf{m}^{(g)}$ is the mean of the sampling distribution which is the weighted intermediate recombination of the $\mu$ best candidates from the previous generation :

$$
\begin{equation*}
\mathbf{m}^{(g)}=\sum_{i=1}^{\mu} w_{i} x_{i}^{(g)} \tag{3.10}
\end{equation*}
$$

A super-linear relation is used for the recombination, given by :

$$
\begin{equation*}
w_{i}=\frac{\ln (\mu+1)-\ln (i)}{\sum_{i=1}^{\mu}(\ln (\mu+1)-\ln (i))} \tag{3.11}
\end{equation*}
$$

The second term on the right hand side of eqn. (3.9) is a normally distributed random vector which represents the mutation of the evolutionary strategy. It is obvious to see that the parameters of the normal distribution play an important role in the performance of the optimization. At each iteration, the step size $\boldsymbol{\sigma}^{(g)}$ and the covariance
matrix $\mathbf{C}^{(g)}$ are updated in a way that will increase the probability of producing the best offspring for the next generation. In short, the CMA-ES algorithm implements a principle component analysis of the previously selected mutation steps to determine the new mutation distribution. Due to long and complicated formulae, we refer readers to $[48,49]$ for more details as well as the mathematical derivation of the covariance matrix C. The CMA-ES does not require a tedious parameter tuning for its application. In fact, the choice of strategy internal parameters is not left to the user (arguably with the exception of population size $\lambda$ ). Finding good (default) strategy parameters is considered as part of the algorithm design, and not part of its application.


Figure 3.8 : The flowchart illustrates the CMA-ES algorithm along with the MPI structure that was used for the wing stiffness optimization. The code was run in parallel on $\lambda$ CPUs to evaluate $\lambda$ objective functions $f(x)$ for each iteration.

Since the evaluation of the objective function is expensive, the code is run in parallel using the Message Passing Interface (MPI) where the process is distributed to multiple processors as shown in figure 3.8.

### 3.3.2.1 Validation

The algorithm is first tested using the same setup as described in section 3.3.1 where the reference is the deformation of the modeled wing with a fixed set of parameters. The purpose is to check if the CMA-ES algorithm can converge toward the known parameters of the reference.

By default, the population size recommended by the algorithm [72] is $\ln (n)+4$ where $n$ is the number of parameters needed to be optimized. This give us a population of 12 . However, to have a faster convergence and taking into account the number of CPUs available, a slightly bigger population 16 was chosen for the computations. Consequently, the code was run on 16 CPUs for 2500 CPU hours to optimize the wing stiffness and the result is shown in table 3.2.

The optimized Young's modulus value $E$ is very close to the reference value with a relative error smaller than $10^{-6}$. This is an expected outcome due to the fact that the stiffness of the venation system, except for the joints needed to be optimized, is proportional to the Young's modulus $E$ as explained in section 3.2.1 and this parameter should govern the overall deformation of the entire wing. On the other hand, the joints' stiffness values are more difficult to find since each joint solely has effects on the regional deformation. By definition, the objective function does not reflect the local deformation of the wing because it averages the Euclidean distance between all the nodes of the reference and the optimizing wings.

Figure 3.9 displays the deformation of the reference wing (top) and the optimizing wing (bottom) at the equilibrium state. Although the difference between the optimizing joint stiffness and the reference is noticeable, the behavior of the optimizing wing under point forces is similar to the reference wing. Under the same point load, the both wings exhibit almost the same deformation. By this criteria, the algorithm is reliable enough to optimize a wing that has the same mechanical behaviors as the reference data.

|  | Objective function | $\mathrm{E}(\mathrm{GPa})$ | k 1 | k 2 | k 3 | k 4 | k 5 | k 6 | k 7 | k 8 | k 9 | k 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Reference | 0 | 11.14 | 549.3 | 901.5 | 996.9 | 748.2 | 688.5 | 130.8 | 792.8 | 846.0 | 774.0 | 426.0 |
| Optimizing | $4.45 \times 10^{-5}$ | 11.14 | 719.5 | 841.00 | 883.1 | 760.7 | 472.7 | 303.1 | 626.8 | 777.4 | 542.8 | 704.4 |

Table 3.2 : The Young's modulus and the joints' stiffness values of the reference model and the optimized model.


Deformation $\Delta z(\mathrm{~cm})$

Figure 3.9 : Wing deformation under point loads at different location (from left to right : point force number 1, 11, 2 and 10) calculated by using the wing model with the reference stiffness (top) and the optimizing stiffness (bottom). The colorbar shows the wing deformation in vertical direction $\delta z$ in units of centimeters.

### 3.3.3 Results and discussion

The CMA-ES algorithm is used for the optimization of 9 individuals' wing stiffness. To speed up the convergence, each optimization was run with a population of 64 . Two stopping criteria were set to determine when to end the searching process. The first one was when the objective function smaller than a fixed value. By construction, the objective function represented the difference between the optimizing wing and the reference wing. The best scenario should give us a null objective function where we had exactly the same deformation of both wings. This was however not probable in practice and we determined the stopping criterion for the objective function based on the validation test. When the objective function given by eqn. (3.8) was smaller than $10^{-4}$, the difference between two wings could be considered negligible. The
second criterion was the maximum iteration performed by the algorithm. Because the calculation of the wing equilibrium state was numerically expensive, the maximum iteration was set at 100 corresponding to more than 4250 CPU hours for each run.

In addition, the search space for our problem was restricted using the two penalized zones shown in figure 3.7. Some tests had been performed quickly, before the CMA-ES algorithm was employed, to determine which value of the Young's modulus $E$ would give us an equilibrium position in these two zones. The upper bound for the Young's modulus was 100 GPa because when $E$ greater than this value, the wing was too stiff and remained almost undeformed under the applied forces. On the other hand, the lower bound for the Young's modulus was set at 0.1 GPa since smaller value of $E$ resulting an equilibrium in the too-flexible zone. Then based on the relation between the Young's modulus and the bending spring stiffness presented in chapter 2, the bounds for the joint stiffness were estimated at 0.1 N.cm.rad ${ }^{-1}$ for the lower bound and 1000 N.cm.rad ${ }^{-1}$ for the upper bound. These constraints then allow us to have more realistic results and to speed up the searching process.

With this setup, the optimization was run on 64 CPUs with each of 9 individuals by employing the parallel MPI structure as explained in the previous section 3.3.2. The results are assembled in table 3.3. As seen from the table, there is not any optimization run succeeding at finding a set of parameters which gives an objective function smaller than $10^{-4}$. All the runs were stopped by exceeding the maximum number of iterations which is 100 . The average value of Young's modulus is 12.58 GPa with a standard deviation 3.03 GPa . This is close to the Young's modulus of insect wings which greatly vary from tens to hundreds of MPa around the mean value 5 GPa [90, 96]. Nevertheless, the joints' stiffness varies significantly among individuals. Although the stiffness of vein joints depends on various factors such as the distribution of resilin, the shape of veins or the existence of vein spikes [105], these large deviations can also be explained by non-biological reasons. By definition, the objective function does not reflect the local deformation of the wing because it averages the Euclidean distance between all the nodes of the reference and the optimizing wings. On the other hand, the vein joints solely have effects on the regional deformation.

The deformations of individual number 8, one of the best solutions with the second smallest objective function, are shown in figure 3.10. The left figures represent the deformation measured from experiment while the right figures show the deformation calculated by the wing model. All data is presented in centimeters. The 10
measurements (A-J) corresponding to 10 force locations are plotted.

### 3.4 Conclusion

Two flexible wing models of two species, the bumblebee (Bombus ignitus) and the blowfly (Calliphora vomitoria), have been developed based on the experimental data. The sophisticated structures of the wings were taken into account by distinguishing the vein and the membrane during the meshing procedure. The membrane was modeled as a 2D planar sheet whose tensile strength was much larger than its bending stiffness and the veins were modeled as rods whose bending stiffness values were calculated based on their flexural rigidity $E I$. While the second moment of area $I$ can be estimated using the vein diameters, the Young's modulus $E$ remains somewhat limited due to the vast range of known cuticle's property [96]. There were two approaches proposed for overcoming this challenge.

For the bumblebee wing model, the Young's modulus was simply varied to make a comparison between two different wing models that we referred to as flexible and highly flexible.

For the blowfly wing model, the mathematical optimization tool CMA-ES [72] was employed for determining the right elastic properties of the wing model. We obtained here 9 sets of stiffness parameters for the Calliphora wing model. The wing model was compared with static experimental measurements in order to optimize the Young's modulus of cuticle and the joints' stiffness. The method allowed us to find appropriate stiffness values for approximating the static deformation behavior of real insect wings under the external point forces.

In the next part of this study, the wing model with these stiffness parameters are then put in a tethered flight context for investigating the influence of wing flexibility on the aerodynamic performance of flapping wings.

| Indivilual | $\begin{gathered} \mathrm{E} \\ (\mathrm{Gpa}) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 1 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 2 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \end{gathered}$ | $\begin{gathered} \mathrm{k} 3 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 4 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \text { k5 } \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 6 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 7 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \end{gathered}$ | $\begin{gathered} \mathrm{k} 8 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 9 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{k} 10 \\ \left(\text { N.cm.rad }{ }^{-1}\right) \\ \hline \end{gathered}$ | Objective function |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 11.26 | 701.40 | 477.00 | 760.50 | 0.90 | 107.40 | 742.20 | 0.10 | 636.10 | 849.60 | 385.90 | $2.19 \mathrm{E}-02$ |
| 2 | 10.48 | 847.20 | 976.90 | 985.40 | 787.80 | 740.60 | 757.00 | 823.80 | 0.10 | 7.43 | 358.50 | $1.52 \mathrm{E}-02$ |
| 3 | 11.15 | 274.00 | 683.90 | 368.60 | 910.50 | 795.50 | 425.90 | 0.10 | 0.10 | 310.00 | 799.60 | $2.37 \mathrm{E}-02$ |
| 4 | 10.26 | 584.60 | 595.60 | 183.20 | 1.55 | 551.20 | 429.50 | 572.50 | 543.50 | 462.90 | 823.40 | $2.07 \mathrm{E}-02$ |
| 5 | 9.92 | 452.60 | 523.20 | 0.20 | 706.40 | 436.30 | 427.70 | 786.50 | 729.00 | 380.10 | 770.90 | $1.98 \mathrm{E}-02$ |
| 6 | 18.01 | 762.20 | 0.10 | 0.10 | 822.20 | 918.80 | 601.70 | 928.70 | 62.40 | 607.90 | 962.10 | $2.36 \mathrm{E}-02$ |
| 7 | 17.23 | 518.10 | 836.40 | 0.32 | 707.60 | 634.30 | 822.80 | 707.30 | 0.10 | 589.60 | 372.30 | $1.73 \mathrm{E}-02$ |
| 8 | 14.78 | 1000.00 | 1000.00 | 1000.00 | 1000.00 | 655.10 | 0.10 | 194.00 | 0.10 | 108.20 | 0.10 | $1.69 \mathrm{E}-02$ |
| 9 | 10.1 | 331.00 | 741.00 | 465.10 | 442.00 | 556.70 | 16.85 | 422.30 | 0.10 | 594.50 | 535.20 | $1.97 \mathrm{E}-02$ |

Table 3.3 : The Young's modulus and the joints' stiffness values optimized by the CMA-ES algorithm. Among 9 individuals used for the optimization, individual number 2 is considered as the best solution. It has the smallest objective function which represents the difference in $L_{2}$ norm between the optimized wing and the experimental wing.


Figure 3.10 : Wing deformation under point loads at different location (from left to right : point force number $1,11,2$ and 10) calculated by using the wing model with the reference stiffness (top) and the optimizing stiffness (bottom). The colorbar shows the wing deformation in vertical direction $\Delta z$ in centimeters. Noise from experimental measurements cause some dots appearing on the edge of the reference wing.

## Chapter 4

## Fluid-structure interaction for insect flight

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#### Abstract

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During flight, insect wings passively deform due to both inertial-elastic forces and aerodynamic forces. While inertial forces depend only on wings' motion and material structure, aerodynamic forces rely strongly on the solid-fluid interactions between the wing surface and its surrounding air flow. It is thus indispensable to couple the solid solver with the fluid solver for the investigation of aerodynamic effects of wing flexibility.

The coupling between the solid solver and the fluid solver will be dealt with in this chapter. The numerical method employed for simulations of moving obstacles as well as the time stepping of the coupled system will be firstly introduced. The integrated code is validated with respect to classical benchmarks and our previous works. Finally, we present simulations of fully coupled fluid-structure interaction where a tethered bumblebee with flexible wings will be studied. The content of this chapter is adapted from [91-93].


### 4.1 Numerical method for fluid-structure interactions

The active fluid-structure interaction of flapping wing is one of current challenges that have attracted attention from the biological and computational modeling communities. It consists of solving for the wing deformation in an unprescribed way due to the resulting fluid forces acting on the wing. On the other way around, the wing deformation is significant enough to change the fluid flow itself which also needs to be resolved. Although the last decades have witnessed an accelerated development in the fields of computational fluid dynamics and computational structural dynamics, the coupling between a structure and the surrounding flow is still a major challenge and requires advanced numerical techniques. One of the problems typically encountered when dealing with the coupling is how to impose the no-slip boundary condition on the fluid-structure interface. Various strategies and approaches have been proposed so far to handle this issue. Body fitted coordinate systems [43], allow a well adapted
discretization for a given geometry and they have been used successfully for a wide range of active FSI problems [94, 95]. However, for time-dependent geometries, these methods require a large computational effort spent on the grid generation. Moreover, arbitrarily complex geometry of insects, including legs and antennae, make the methods even less suitable in the field of insect flight.

To overcome the limitation of grid generation, researchers have started to look for alternative approaches whose aim is to avoid body-fitted unstructured meshes in order to take advantage of fast and efficient finite differences, finite volumes or spectral methods on Cartesian meshes. Peskin [79] was one of the pioneers who proposed an immersed boundary method to solve this fully coupled FSI problem with the application in modeling flexible wings. Miller and Peskin [69] showed that flexibility can reduce the lift-to-drag ratio in 2D configuration. Immersed boundary methods base on the idea that the Navier-Stokes equations can be discretized on a equidistant grid and the complex geometry is then immersed into the computational domain. The boundary conditions are imposed by adding terms to the equations. This approach however presents a drawback to low order approximation, i.e., first or second order, along with ill-conditioned linear systems or restrictive small time steps caused by their computational stiffness.

On the other hand, the volume penalization approach is physically motivated by modeling obstacles as porous media of which permeability tends to zero [3] and mathematically proven in [1, 15]. In the context of biolocomotion, the penalization method was employed to study the C-start of a fish at rest by Gazzolla et al. [46]. Bergmann and Iollo [6] investigated fish-like locomotion by simulating the flow past bioinspired swimmers. In the current work, the volume penalization approach is applied for the fluid-structure interaction.

### 4.1.1 Volume penalization method

Due to their small sizes and elevated flapping frequencies, insect flight is normally categorized in the Reynolds number regime between $\mathscr{O}\left(10^{1}\right)$ and $\mathscr{O}\left(10^{4}\right)$. For example, for hawkmoth we have $R e=6000$, bumblebee $R e=2000$, fruit fly $R e=100$ or thrips $R e=10[36,87]$. The flow can be considered as incompressible and governed by :

$$
\begin{align*}
\partial_{t} \mathbf{u}+\boldsymbol{\omega} \times \mathbf{u} & =-\nabla \Pi+\nu \nabla^{2} \mathbf{u}-\underbrace{\frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right)}_{\text {penalization term }}-\underbrace{\frac{1}{C_{\mathrm{sp}}} \nabla \times \frac{\left(\chi_{s p} \boldsymbol{\omega}\right)}{\nabla^{2}}}_{\text {sponge term }}  \tag{4.1}\\
\nabla \cdot \mathbf{u} & =0  \tag{4.2}\\
\left.\mathbf{u}\right|_{\partial \Omega_{s}} & =\mathbf{u}_{s}  \tag{4.3}\\
\mathbf{u}(\mathbf{x}, t=0) & =\mathbf{u}_{0}(\mathbf{x}) \tag{4.4}
\end{align*}
$$

where $\mathbf{u}$ is the fluid velocity, $\mathbf{u}_{s}$ is the solid velocity, $\nabla$ is the nabla operator $\nabla=$ $\left(\partial_{x}, \partial_{y}, \partial_{z}\right)^{T}, \omega=\nabla \times \mathbf{u}$ is the vorticity, $\Pi=p+\frac{1}{2} \mathbf{u} \cdot \mathbf{u}$ is the total pressure, $v$ is the kinematic viscosity. We find here again all the familiar terms of the classical NavierStokes equations except for the sponge and the penalization terms. The sponge term is a vorticity damping term used to gradually damp vortices and alleviate the periodicity inherent to the Fourier discretization. The notation $1 / \nabla^{2}$ corresponds to the Green's function of the Laplace operator. On the other hand, the former is used to impose the no-slip boundary conditions on the fluid-solid interface [38]. It was shown that the solution of the penalized Navier--Stokes equations tends towards the exact solution of Navier-Stokes imposing no-slip boundary conditions for $C_{\eta} \rightarrow 0$, where $C_{\eta}$ denotes the penalization parameter [2]. The parameter $C_{\eta}$ should thus be chosen as small as possible. However, because the penalized eqns. (4.2-4.4) are discretized and solved numerically, $C_{\eta}$ is constrained by the maximum time step $\Delta t=O\left(C_{\eta}\right)$ [38]. Consequently, a good choice for $C_{\eta}$ ) is not only small enough, but also as large as possible. In practice, the value of $C_{\eta}$ is usually taken in the range of $10^{-1}-10^{-4}$, depends on the applications. Besides, the sponge penalization parameter $C_{s p}$ is usually chosen to a larger value than $C_{\eta}$, normally $C_{s p}=10^{-1}$ [38].

Eqns. (4.1-4.4) are then discretized in space using a Fourier pseudo-spectral discretization [38]. The general idea consists of representing quantities $q$ (velocity, pressure, vorticity) as truncated Fourier series :

$$
\begin{equation*}
q(\mathbf{x}, t)=\sum_{k_{x}=0}^{N_{x}-1} \sum_{k_{y}=0}^{N_{y}-1} \sum_{k_{z}=0}^{N_{z}-1} \widehat{q}(\mathbf{k}, t) \exp (i \mathbf{k} \cdot \mathbf{x}) \quad \mathbf{x} \in[0 ; 2 \pi)^{3} \tag{4.5}
\end{equation*}
$$

where $\mathbf{k}=\left[k_{x}, k_{y}, k_{z}\right]^{\mathbf{T}}$ is the wavevector, $i=\sqrt{-1}$ and $\widehat{q}$ are the discrete complex Fourier coefficients of $q$. The Fourier coefficients can be computed with the fast


Figure 4.1 : Illustration of a computational domain where a solid obstacle $\Omega_{s}$ is immersed in a fluid domain $\Omega_{f}$. Here, $\partial \Omega_{s}$ is the fluid-structure interface and $V$ is an arbitrary control volume enclosing the obstacle.

Fourier transform (FFT) using the P3DFFT library. The main motivation of using a Fourier discretization is the simplicity of inverting a diagonal Laplace operator and the high numerical precision reflected in the absence of numerical diffusion and dissipation in the discretization. The gradient of a scalar can, for instance, be obtained by multiplying with the wavevector and the complex unit, $\widehat{\nabla q}=i \mathbf{k} \hat{q}$. The Laplace operator becomes a simple multiplication by $-|\mathbf{k}|^{2}$, it is thus diagonal in Fourier space. For further details, we refer the reader to the reference article on the FLUSI solver [38].

All the geometry information of the solid is encoded in the mask function $\chi$, which is usually taken as one inside the solid and zero otherwise. However, we are dealing with a moving flexible obstacle and the discontinuous mask function $\chi$ need to be replaced by a smooth one, eqn. (4.6), to avoid oscillations in the hydrodynamic forces [34]. Thus, we employ a mask function $\chi$ as shown below :

$$
\chi(\delta)= \begin{cases}1 & \delta \leq t_{w}-h  \tag{4.6}\\ \frac{1}{2}\left[1+\cos \left(\pi \frac{\left(\delta-t_{w}+h\right)}{2 h}\right)\right] & t_{w}-h<\delta<t_{w}+h \\ 0 & \delta \geq t_{w}+h\end{cases}
$$

where $h$ is the semi-thickness of the smoothing layer, $t_{w}$ is the semi-thickness of the wing and $\delta$ is the distance function which gives us the distance from Eulerian fluid nodes to the center surface of the wing. As presented in chapter 2, an unstructured triangular mesh is employed for our wing model. Thus, the discretized wing surface is composed of triangles constructed by three vertices (e.g. $\mathbf{x}_{s, i}, \mathbf{x}_{s, j}$ and $\mathbf{x}_{s, k}$ ). The distance function $\delta$ is computed by cycling over all these triangles. Since we are only interested in the fluid nodes near the fluid-solid interface, a bounding box is used
to save computing time. For each triangle, the distance from it to all the fluid nodes belonging to its bounding box will be computed by using the algorithm from [30]. The distance function at one fluid node is finally the minimum distance from this fluid node to all the triangles nearby.

$$
\begin{equation*}
\delta(\mathbf{x}, t)=\min \left(\left\|\mathbf{x}-\operatorname{triangle}\left(\mathbf{x}_{s, i}, \mathbf{x}_{s, j}, \mathbf{x}_{s, k}\right)\right\|_{2}\right) \tag{4.7}
\end{equation*}
$$

The solid velocity field $\mathbf{u}_{s}$ is calculated in the same way as the distance function $\delta$. If the triangle $\left(\mathbf{x}_{s, i}, \mathbf{x}_{s, j}, \mathbf{x}_{s, k}\right)$ is the one closest to the fluid node $\mathbf{x}, \mathbf{x}$ will be projected onto the plane of the triangle and the solid velocity of the projected point is interpolated from the velocities of the three vertices by using barycentric interpolation. Because we do not consider the flexibility of the wing in the direction perpendicular to the wing surface, the velocity of the projected point should be the same as the one of the fluid node. Nevertheless, the solid velocity field is defined in the global reference frame for the fluid solver while the velocity solved by the solid solver is in the local wing reference frame. These velocities are needed to be transformed back to the global reference frame using eqn. (4.8) where $\mathbf{V}_{O^{(w)}}$ and $\boldsymbol{\omega}$ are the translating and rotating velocity of the wing reference frame, $\mathbf{v}^{(w)}$ and $\mathbf{x}^{(w)}$ are the velocity and the position computed by the solid solver in the wing reference frame, respectively.

$$
\begin{equation*}
\mathbf{u}_{s}=\mathbf{V}_{O^{(w)}}+\mathbf{v}^{(w)}+\boldsymbol{\omega} \times \mathbf{x}^{(w)} \tag{4.8}
\end{equation*}
$$

### 4.1.2 Aerodynamic forces and power requirement

One aspect that is interested to study when simulating insect flight is the aerodynamic forces (e.g., drag and lift) and the required aerodynamic power. A rigorous derivation of the formulae for the force and power was presented by Engels in [34]. The force $\mathbf{F}$ on an obstacle is defined as the surface integral of the stress tensor over the obstacle's surface :

$$
\begin{equation*}
\mathbf{F}=\oint_{\partial \Omega_{S}} \boldsymbol{\sigma} \cdot \mathbf{n} d s \tag{4.9}
\end{equation*}
$$

where the stress tensor $\boldsymbol{\sigma}=-\nabla \Pi+v \nabla^{2} \mathbf{u}$. By applying the Gauss theorem, we can rewrite eqn. (4.9) in the volume integral form :

$$
\begin{equation*}
\mathbf{F}=\int_{\Omega_{S}} \nabla \cdot \boldsymbol{\sigma} d V \tag{4.10}
\end{equation*}
$$

On the other hand, eqn. (4.1) can also be rewritten with the stress tensor and the material derivative as :

$$
\begin{equation*}
\frac{\mathbf{D u}}{\mathbf{D} t}=\nabla \cdot \boldsymbol{\sigma}-\frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right)-\frac{1}{C_{\mathrm{sp}}} \nabla \times \frac{\left(\chi_{s p} \boldsymbol{\omega}\right)}{\nabla^{2}} \tag{4.11}
\end{equation*}
$$

We consider a control volume $V$ enclosing the obstacle, as illustrated in figure 4.1. In the control volume $V$, the sponge term can be left out because the sponge is put at the boundary of the domain. Integrating eqn. (4.11) over the control volume and rearranging the terms give us:

$$
\begin{equation*}
\int_{V} \nabla \cdot \boldsymbol{\sigma} d V=\int_{V} \frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right) d V+\int_{V} \frac{D \mathbf{u}}{D t} d V \tag{4.12}
\end{equation*}
$$

By considering $V=\Omega_{S}$, eqns. (4.10) and (4.12) allow us to calculate the total aerodynamic force acting on the object :

$$
\begin{equation*}
\mathbf{F}=\int_{\Omega_{S}} \frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right) d V+\int_{\Omega_{S}} \frac{D \mathbf{u}}{D t} d V \tag{4.13}
\end{equation*}
$$

Inside the obstacle, the velocity $\mathbf{u}$ approaches the solid velocity $\mathbf{u}_{s}$. Thus, the final formula used in this work for calculating the aerodynamic force is given by :

$$
\begin{equation*}
\mathbf{F}=\int_{\Omega_{S}} \frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right) d V+\frac{d}{d t} \int_{\Omega_{S}} \mathbf{u}_{s} d V \tag{4.14}
\end{equation*}
$$

Similarly, the total torque $\mathbf{m}$ acting on the obstacle can be computed by :

$$
\begin{equation*}
\mathbf{m}=\oint_{\Omega_{S}} \mathbf{r} \times(\boldsymbol{\sigma} \cdot \mathbf{n}) d V=\int_{\Omega_{S}} \mathbf{r} \times \frac{\chi}{C_{\eta}}\left(\mathbf{u}-\mathbf{u}_{s}\right) d V+\int_{\Omega_{S}} \mathbf{r} \times \frac{D \mathbf{u}_{s}}{D t} d V \tag{4.15}
\end{equation*}
$$

where $\mathbf{r}=\mathbf{x}-\mathbf{x}_{c}$ is the position vector relative to the point of reference. In the case of a flapping wing, the point of reference is the pivot point $\mathbf{x}_{p}^{b}$ about which the wing rotates. Then, the corresponding aerodynamic power requirement for actuating the wing is given in [34] :

$$
\begin{equation*}
P_{\text {aero }}=-\mathbf{m} \cdot\left(\boldsymbol{\Omega}_{w}^{(g)}-\boldsymbol{\Omega}_{b}^{(g)}\right) \tag{4.16}
\end{equation*}
$$

where $\boldsymbol{\Omega}_{w}^{(g)}$ and $\boldsymbol{\Omega}_{b}^{(g)}$ are the relative angular velocity of the wing and the body in the global reference frame, respectively. The minus sign in eqn. (4.16) represents that the power calculated here is a power consumption. A positive power indicates that the insect has to consume power to actuate its wings while power with an opposite sign means that the energy is stored in the insect's muscle.

### 4.1.3 Coupling fluid and solid

For time-stepping, the coupled fluid-solid system is advanced by employing a semiimplicit staggered scheme, as proposed in [34]. On the one hand, we advance the fluid by using the Adams-Bashforth second order (AB2) scheme. On the other hand, the Back Differentiation Forward second order (BDF2) scheme is used for the time discretization of the solid solver. The two modules are then coupled by the algorithm presented in the flow chart in figure 4.2. The scheme is called a weak coupling method since the static pressure is computed from the previous state of the solid model. This makes the system conditionally stable only if the structure is heavy enough with respect to the fluid density. However, the scheme is efficient because the fluid and the solid need to be advanced only one time at the current time level. Full details of the fluid-structure interaction (FSI) framework as well as detailed validation of the results can be found in our previous work [91].

For time-stepping, the coupled fluid-solid system is advanced by employing a semiimplicit staggered scheme as proposed in [34]. At time step $t^{n}$, the fluid velocity field is advanced to new time level $\mathbf{u}^{n} \rightarrow \mathbf{u}^{n+1}$ from the old mask function $\chi^{n}$ and the old solid velocity field $\mathbf{u}_{s}^{n}$ by using the Adams-Bashforth scheme. Then, the pressure field at the new time step $p^{n+1}$ is calculated from the fluid velocity field $\mathbf{u}^{n+1}$. Finally, the solid is advanced to the new time step $\chi^{n+1}$ and $\mathbf{u}_{s}^{n+1}$ and the whole process is repeated until the final time.


Figure 4.2 : Semi-implicit staggered scheme of the time advancement for the fluidstructure interaction problem.

### 4.2 Numerical validations

### 4.2.1 Turek benchmark - test case FSI3

The ultimate goal of our work is the fluid-structure interaction simulation of insects with flexible wings. For this reason, we move on to a classical but more intricate validation, usually known as the Turek benchmark test case FSI3 [94, 95]. This involves a flexible appendage of length $l=0.35$ and thickness $h=0.02$ placed right behind a circle cylinder of radius $R=0.05$; the whole obstacle is immersed in a channel of size $H \times L=0.41 \times 2.5$ with a Poiseuille inflow of meanflow $\bar{U}=2$, as shown in figure 4.3. The center of the cylinder is placed at $(0.2,0.2)$ to trigger the instability and to make the appendage oscillate. The non-dimensional parameters characterizing the appendage are the density $\rho_{s}=1 \times 10^{3}$, the Young's modulus $E_{s}=5.6 \times 10^{6}$ and the Poisson ratio $n_{s}=0.5$. The Reynolds number is defined as $R e=\bar{U} l / v_{f}=700$ where $v_{f}=10^{-3}$ the fluid kinematic viscosity.

The setup remains the same as the test case FSI3 with a resolution of $5200 \times 1152$ whose details can be found in [34]. Only the solid solver based on the nonlinear beam equation, which is used for validation in section 2.5.1, chapter 2, is now replaced by the new solver using the mass-spring network for validation. The results of this simulation are presented in table 4.1 for the comparison with the reference solutions presented in the literature [34, 94, 95].


Figure 4.3 : Computational domain of the FSI3 Turek benchmark and dimensions of the solid part [95]. The obstacle consists of a rigid circle cylinder and a flexible appendage (gray part).


Figure 4.4: The oscillation of the trailing edge $y_{t e}(t)$ (a), the drag (b) and the lift (c) from the references [95] (blue dashed line), [34] (black dash-dotted line) and the mass-spring network (red continuous line).

For the oscillation of the trailing edge $y_{t e}$, the result is in excellent agreement with all three references when the maximum relative error, for both maximum and minimum values of $y_{t e}$, is only $1.76 \%$ and the relative error for the frequency of oscillation is $1.65 \%$. The vertical displacement of the trailing edge with respect to time in the periodic state is also plotted in figure 4.4 to compare with the reference [95]. The two lines are almost superposed on each other. Nevertheless, the computed drag is less accurate with a relative error which can go up to $4.57 \%$ comparing the maximum value with the reference [34], but only $1 \%$ comparing with [94]. From figure 4.4, the curves of the two solutions appear to have the same shape but have some offset. This offset is explained, in [34], to exist due to the smoothing layer in the mask function which plays a role as surface roughness. This leads to the over-prediction of the drag

| References | $y_{t e}\left[10^{-3}\right]$ |  | Drag |  | Lift |  | $f_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\max$ | $\min$ | $\max$ | $\min$ | $\max$ | $\min$ |  |
| Mass-spring network | 36.22 | -32.93 | 503.02 | 442.12 | 189.94 | -186.23 | 5.56 |
| (1) T. Engels [34] | 35.63 | -32.71 | 481.20 | 432.50 | 188.52 | -181.30 | 5.44 |
| (2) S. Turek [94] | 36.37 | -33.45 | 487.81 | 432.79 | 156.13 | -151.31 | 5.47 |
| (3) S. Turek [95] | 36.46 | -33.52 | 488.24 | 432.76 | 156.40 | -151.40 | 5.47 |

Table 4.1 : Results of the FSI3 benchmark. The results solved by the current solver are compared to various results from Engels [34] and Turek [94, 95].
force. Concerning the lift force, the mass-spring model yields result very close to the one calculated by T. Engels with the error of $2.76 \%$, and the difference is around $20 \%$ with respect to [94, 95] for both max and min values. Like in Engels [34], the amplitude of the lift force is over-predicted by coupling FLUSI and the mass-spring solver. In conclusion we find satisfying agreement with the results from the literature, for the solid solver alone, as well as for the FSI algorithm coupling the solid solver with the fluid solver.

### 4.2.2 Rigid revolving wing

Prior to studying the flexibility of the wing, a common test case of a rigid revolving wing is considered to validate the coupling between the fluid and the solid solver in $3 D$, i.e. the mask function generation and the velocity field of the solid. The setup is taken the same as the one used by Engels et al. [35] as shown in figure 4.5. The angle of attack is fixed at $\alpha=45^{\circ}$ while the rotation angle $\phi(t)$ is given by

$$
\begin{equation*}
\phi(t)=\tau e^{-t / \tau}+t-\tau \tag{4.17}
\end{equation*}
$$

The wing is rotated around the center of the computational domain of size $4 \times 4 \times 2$, which is discretized using a mesh of $1024 \times 1024 \times 512$ grid points. To be consistent with the reference simulation [35], the wing shape is not the one presented in chapter 2 but adapted from the wing planform taken from the reference. The wing shape is then discretized by a triangular mesh as shown in figure 4.5 b. However, the vein structure will not be taken into account in this model because we are considering a rigid wing. The triangular mesh is solely exploited for the creation of the mask function and the
solid velocity field by using the algorithm presented at the beginning of this section. Here, all the quantities are normalized. The wing length is chosen as the length scale $L=13.2 \mathrm{~mm}$; the mass scale is based on the air density $M=\rho_{\text {air }} \times L^{3}=2.817 \mathrm{mg}$ and the time scale is chosen in the way that wing tip velocity is unity, thus $T=1 \mathrm{~s}$ . The Reynolds number is then defined as in [35] $R e=\bar{u}_{t i p} c_{m} / v$ where the mean wingtip velocity $\bar{u}_{t i p}=1\left[L T^{-1}\right]$ by definition from eqn. (4.17), the fluid viscosity $v=1.477 \cdot 10^{-4}\left[L^{2} T^{-1}\right]$ and the mean chord, the ratio between the wing surface area $A$ and the wing length $R, c_{m}=A / R=0.304$ [ $L$ ]. This yields $R e=2060$. Additionally, the lift and drag coefficient are defined as below

$$
\begin{equation*}
C_{L}=\frac{F_{L}}{M L T^{-2}} ; C_{D}=\frac{F_{D}}{M L T^{-2}} \tag{4.18}
\end{equation*}
$$

where the lift $F_{L}$ is the force in the vertical direction $O z$ and the drag $F_{D}$ is the force perpendicular to the plane formed by the vertical and the wing spanwise axes, as shown in figure 4.5 a .

(A) Scheme of the revolving wing, adapted(B) Corresponding discrete mass-spring from [35].
 network.

Figure 4.5 : (A) Schematic diagram of the revolving wing simulation. (B) The wing mass-spring model adapted from [35]. The wing is rotated around the hinge point with an angle of attack $\alpha=45^{\circ}$.

The computed lift and drag coefficients are shown in figure 4.6 along with the reference values from [35]. To evaluate quantitatively the error, the average lift and drag during the steady state (for rotation angles $\phi$ varying from $160^{\circ}$ to $320^{\circ}$ ) are computed and compared with the reference. A very good agreement is obtained with the relative error of $1.3 \%$ for the drag and $1.6 \%$ for the lift.

From the results obtained from these two test cases, the satisfactory agreements can


Figure 4.6 : Comparison of lift and drag coefficients for a rigid wing, calculated using the coupling between FLUSI and the developed mass-spring solver, with the reference data from [35].
give us the confidence about the solid solver, based on mass-spring system, as well as the coupling with the flow solver FLUSI. Any difference between all the numerical studies carried out can be explained by the difference between the continuum model and the discrete model together with the way of generating the mask function.

### 4.3 Fluid-structure interaction of flexible bumblebee wings

The goal of this paper is to move from rigid to flexible wings and to present the fully coupled fluid-structure interaction solver for flapping flight, based on the open access software FLUSI, where we integrated the solid model presented in chapter 2. However, the wing kinematics of insects is very difficult to obtain, since the measurements usually require high tech equipment to capture all the dynamic motion at small time scale and length scale. Instead, a revolving wing model is usually employed to study the aerodynamics of flapping wings thanks to its simplicity. Accordingly, the flow fields and force generation aspects of revolving wings have been analyzed for a wide range of parameters, as reported in the literature [53, 55, 63]. Di et al. [26] studied the role of forewings in generating LEVs of three revolving insect wing models : hawkmoth (Manduca sexta), bumblebee (Bombus ignitus) and fruitfly (Drosophila melanogaster). Van de Meerendonk et al. [24] investigated experimentally the flow field and fluid-
dynamic loads of a flexible revolving wing to quantify the influence of flexibility on the force generation performance of the wing. In our study, we also consider revolving flexible bumblebee wings and assess the influence of the wing deformation

### 4.3.1 Flexible revolving bumblebee wing

In the following we present results of high resolution computations of revolving bumblebee wings which are either rigid, flexible or highly flexible. First we perform computations for different resolutions to check the mesh convergence for both fluid and solid solvers. Then a comparison of the flexible wings with the rigid case allows to assess the influence of the wing deformation on the aerodynamic forces. The setup is exactly the same as described in the revolving wing test case of the previous section. The only difference is the wing shape which is now changed back to the one presented in section 3.1. As a result, the length scale and the mass scale are changed as follows : $L=15 \mathrm{~mm}$ and $M=\rho_{a i r} \times L^{3}=4.13 \mathrm{mg}$ while the time scale remains the same $T=1 \mathrm{~s}$. The corresponding Reynolds number is 1800 where the fluid viscosity is assumed to be $v=1.477 \cdot 10^{-4}\left[L^{2} T^{-1}\right]$, the wing tip velocity $u_{t i p}=1\left[L T^{-1}\right]$ and the mean chord calculated from the new wing surface area is $c_{m}=A / R=0.266[L]$.

### 4.3.2 Study of mesh convergence

### 4.3.2.1 Fluid mesh

The following mesh convergence study for the fluid solver is performed considering five different resolutions: $128 \times 128 \times 64,256 \times 256 \times 128,512 \times 512 \times 256,768 \times 768 \times 384$ and $1024 \times 1024 \times 512$.

The mean drag generated during the second half cycle of the rotation is chosen for the evaluation of the mesh convergence $\left(160^{\circ} \leq \phi \leq 320^{\circ}\right)$. Because it is impossible to obtain the exact values for the mean drag in this case, we use here the result obtained with the finest mesh as a reference value. The relative error of the mean drag with respect to the reference drag for the different mesh sizes is shown in figure 4.8. In all the simulations, the penalization parameter $C_{\eta}$ is chosen to satisfy that the number of points per thickness of the penalization boundary layer $K_{\eta}=\sqrt{v C_{\eta}} / \Delta x$ is always constant (as recommended in [34]) and equal to 0.052 . The drag obtained for each simulation (figure 4.7) shows the convergence to the finest resolution solution when


Figure 4.7: Drag coefficient generated by a flexible wing, calculated at different resolutions : $128^{2} \times 64,256^{2} \times 128,512^{2} \times 256,768^{2} \times 384$ and $1024^{2} \times 512$
we refine the mesh. The spatial convergence exhibits a first to second order behavior when we plot the error versus the mesh size.

### 4.3.2.2 Solid mesh

As mentioned above, the dynamics of the mass-spring system depends strongly on the mesh size. Thus another convergence test on the number of mass points is performed. Two simulations of a revolving flexible wing at resolution $768^{2} \times 384$ are run to compare between a medium-mesh and a fine-mesh wing which are discretized by 465 and 1065 mass points, respectively. As shown here in figure 4.9, although the number of mass points is more than doubled, the forces remain almost unchanged with an increase of $1.1 \%$ and $0.8 \%$ in average lift and drag coefficients during the steady state, respectively. Since the fluid solver is itself already costly in term of CPU time, the medium-mesh wing with 465 mass points is sufficient and can be chosen for the following study in section 4.3.3.

### 4.3.3 Influence of wing flexibility

To examine the influence of vein stiffness on the aerodynamic performance of the wing, the flexural rigidity of veins will be varied by changing the Young modulus $E$. Two values of the Young modulus are used : $E=1.25 \cdot 10^{8}\left[M L^{-1} T^{-2}\right]$ and $E=1.25 \cdot$


Figure 4.8 : The error of the mean drag versus mesh size. The dashed lines represent first and second order convergence.
$10^{7}\left[M L^{-1} T^{-2}\right]$, corresponding to the flexible and highly flexible cases, respectively.
Lift and drag coefficients at resolution $1024^{2} \times 512$ for the rigid, flexible and highly flexible cases are presented in figure 4.10.

During the transition phase (rotation angle $\phi \leq 40^{\circ}$ ), the lift generated by the rigid wing increases instantly and then decreases before going up again. The drag follows the same trend as the lift, but is larger in magnitude. When the flexibility of the wing is taken into account, the rapid rise at the beginning of the forces for both flexible and highly flexible wings disappear. Instead, the forces increase gradually and the more flexible the wing is, the lower the lift and the higher the drag are.

At steady state, similar behaviors between the rigid and the flexible wings can be observed. When the rotation angle reaches $160^{\circ}$, the forces generated by these two wings are stabilized. This can be explained by the fact that no dynamic deformation of the wings takes place and just the shape plays a role.

We also find that the lift-to-drag ratio at the steady state of the flexible wing is $1.2,14.5 \%$ higher than the one of the rigid case, which is only 1.05 (figure 4.11). This finding is consistent with conclusions found in literature [24, 112]. A flexible wing generates less lift and drag than a rigid one. However, due to the flexibility of the wing, the bending in the chordwise direction makes the effective geometric angle of attack


Figure 4.9 : Lift and drag coefficients generated by a revolving flexible wing discretized by 465 and 1065 mass points at $R e=1800$.
decrease and alters the direction of the total resultant force upward [24]. This makes the lift-to-drag ratio raise and allows better flight performance.

On the contrary, the highly flexible wing acts differently. Both the lift and the drag increase gradually to attain their maximum values at the rotation angle $\phi=120^{\circ}$ and then decline instead of being stabilized as in the other simulations. The lift-to-drag ratio is surprisingly much less than the one of the rigid case at the beginning of the steady state but then increases and keeps up with the rigid wing. This can be explained by the fact that the bending of the wing in the spanwise direction (figure 4.12) prevents the development of the LEV growing further toward the wing tip and makes the LEV burst sooner at mid-span of the wing.

The change of aerodynamic forces compared to the rigid case is linked to the deformation of the wing, which is modeled by the mass-spring solver. The wing deformation for all three cases is shown in the same figure 4.12 for comparison at three time instants $t=2, t=4$ and $t=6$. By applying the functional approach, the difference between the vein and the membrane is visible in the visualization.

At the finest resolution of the mesh $\left(1024^{2} \times 512\right)$, the flows generated by the flexible wing are shown (figure 4.13) by plotting their vorticity magnitude at four time instants of the simulation. The formations of the LEV as well as the tip vortex can be observed clearly at the beginning of the rotation ( $t=1.0$ and $t=2.0$ ). Then, the vortex burst happens and a region of inhomogeneous vorticity forms at the wing tip. However, the

LEV remains attached to the wing surface and this results in constant lift and drag.

### 4.4 Bumblebee with flexible flapping wings

To study the influence of wing flexibility on the aerodynamic forces, we compare the flexible wings with rigid ones using the same numerical set up as in [37].

### 4.4.1 Flow configuration

Figure 4.14 shows the computational domain of size $6 R \times 4 R \times 4 R$ where $R$ is the bumblebee wing length. The space discretization uses $1152 \times 768 \times 768$ grid points. Both, the translational and rotational motion of the insect body are disabled and the bumblebee is tethered at $\mathbf{x}_{\mathrm{cntr}}=(2 R, 2 R, 2 R)^{T}$. The periodicity of the Fourier spectral method requires a thin vorticity sponge outlet, covering the last 4 grid points in $x$-direction. This damps out the wake and avoids the upstream influence of the computational domain. The value of the sponge penalization parameter is set to $C_{s p}=10^{-1}$, which is larger than the permeability $C_{\eta}=2.55 \times 10^{-4}$. The sponge term is divergence-free and thus avoids the non local influence on the pressure field. Details on the influence of the vorticity sponge are given in [38]. A head wind with a mean flow accounting for the forward velocity of the insect, $\mathbf{u}_{\infty}=(1.246 R f, 0,0)^{T}$, where $f$ is the wingbeat frequency, is imposed in the entire computational domain by simply fixing the zeroth Fourier mode of the velocity $\mathbf{u}$ [34].

In nature, insects do not always fly in a calm, quiescent environment. Instead, they face, most of the time, many kinds of aerial perturbations such as gusty wind, vortices or turbulent flow generated by surrounding obstacles. Taking this into account, both laminar and turbulent flows are investigated here to study the role of wing flexibility under these two circumstances. For the laminar case, in the entire computational domain a mean flow $\mathbf{u}_{\infty}$ is imposed by simply setting the zeroth Fourier mode of the velocity $\mathbf{u}$ [34]. On the other hand, information on turbulent flow conditions, which are experienced by flying insects in nature, remains an open question with limited data [21]. However, for indoor wind tunnel experiments, isotropic or near-isotropic turbulence generated by a grid has been used as inflow condition to study the impact of turbulence on insect flight performance. Consequently, a homogeneous isotropic turbulence (HIT) is chosen as turbulent inflow in our present work in order to com-
pare with the results obtained for rigid wings in [36]. For this purpose, in the inlet region containing the first 48 grid points along the axial direction, a precomputed HIT velocity field $\mathbf{u}^{\prime}$ is added into the mean flow as velocity fluctuations $\mathbf{u}_{\text {in }}=\mathbf{u}_{\infty}+\mathbf{u}^{\prime}$. The HIT field is then transported downstream by the mean flow and evolves dynamically like grid turbulence. In order to compare with the results from [37], we use here a HIT field characterized by the same parameters which are the turbulent intensity $T u=u_{R M S}^{\prime} / u_{\infty}=0.33$, the integral length scale $\Lambda=0.77 R$ and the turbulent Reynolds number $R e_{\lambda}=u_{R M S} \lambda / v=129$, based on the Taylor-micro scale $\lambda=0.18$. More technical details on this approach can be found in [34, 36, 37].

### 4.4.2 Bumblebee model

The bumblebee model here is the same as the one used in [36] and derived from case BB01 in [29].

The bumblebee is composed of rigid components including the head, thorax, abdomen, all legs, proboscis and antennae. These parts are circular elliptical or cylindrical sections joined by spheres, and bilateral symmetry is assumed. The animal's body mass, $M$, is 175 mg , the gravitational acceleration $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$. Based on the measured data from [60], the wing length $R=15 \mathrm{~mm}$ which is slightly bigger than the rigid wing (with $R=13.2 \mathrm{~mm}$ ) used in [36]. The Reynolds number is defined by $R e=U_{t i p} c_{m} / v_{\text {air }}=2685$, where $U_{\text {tip }}=2 \Phi R f=9.15 \mathrm{~m} / \mathrm{s}$ is the mean wingtip velocity, $c_{m}=4.6 \mathrm{~mm}$ the mean chord length, $v_{\text {air }}=1.568 \cdot 10^{-5} \mathrm{~m}^{2} / \mathrm{s}$ is the kinematic viscosity of air, $f=152 \mathrm{~Hz}(T=1 / f=6.6 \mathrm{~ms})$ is the wingbeat frequency ( $T$ is duration) and $\phi=115^{\circ}$ is the wingbeat amplitude. The wingbeat kinematics are prescribed based on the measured data of Dudley and Ellington [29].

### 4.4.3 Effect of turbulence

We first study the influence of an isotropic turbulence on the aerodynamic performance of a bumblebee by putting it in a turbulent flow. The simulation is computed for 4 strokes with 29000 time steps using 3 Intel Cascade Lake 6248 nodes (with 120 cores at 2.5 GHz ) and consumed 9000 CPU hours. Figure 4.16 presents the flow structure of the bumblebee flying in a turbulent flow visualized by the normalized vorticity isosurfaces at two levels $\|\boldsymbol{\omega}\|=15$ and $\|\boldsymbol{\omega}\|=100$. The aerodynamic forces and the corresponding power in this turbulent condition are shown in figure 4.15. The results
demonstrate insignificant differences between turbulent and laminar flow conditions. The aerodynamic forces generated by the bumblebee are almost identical to those derived during unperturbed, laminar inflow, with the same required energetic cost. For $R e>100$, the aerodynamic forces are mainly produced by the differential dynamics pressure across the wing [83].

Figure 4.17 shows the normalized pressure distribution on top and bottom wing surfaces of the two cases just before the stroke reversal $t=0.45 \mathrm{~T}$. The effect of turbulence can hardly be seen here which explains the negligible change of aerodynamic forces. The outcome here is consistent with the one observed in the rigid case in [37].

### 4.4.4 Effect of wing flexibility

### 4.4.4.1 Bending stiffness of the membrane

So far, a mass-spring system composed of mass points and extension springs with an unstructured triangular mesh has been used to model the membrane. By construction, every four neighboring points $i, j, k, l$ form two adjacent triangles as shown in figure 4.18a. The limitation of this model is that the bending stiffness of the wing membrane is not taken into account. As a result, when the wing undergoes large forces, e.g. during wing rotation at the end of the downstroke, folds and wrinkles are observed at the trailing edge and the tip of the wing (cf. figure 4.19a). These undesired deformations make the wing look unrealistic and affect the force production. To overcome this, the bending stiffness is added to the current model to prevent such folds and wrinkles.

(a) Old model without crossover springs

(b) New model with crossover springs

Figure 4.19 : Wing deformation during the stroke reversal at the end of the downstroke.

Unlike the tensile stiffness, which involves computing in-plane deformations and forces, the modeling of the bending stiffness of a sheet is rather more complicated
because it requires computing out-of-plane forces normal to the deformed surface. There are two main methods have been proposed in the literature to deal with this problem. The first one is to model precisely the bending stiffness of the sheet by applying bending momentum which is inversely proportional to the angle between two adjacent mesh elements. Although this approach can result in a good accuracy, the evaluation of bending forces is costly and it can reduce the numerical performance of the mass-spring system [97]. On the other hand, the second approach is much simpler where a crossover spring is added between the two triangles from vertex $i$ to vertex $j$ as shown by the dashed line in figure 4.18 b . When a bending deformation occurs, the added spring will apply forces to these two vertices to resist the bending. This method is claimed to be inaccurate in some cases when the bending is too small or too large [97]. However for our problem, the method is confirmed to work effectively. The folds and the wrinkles are no longer observed during wing rotation while the numerical efficiency of the model remains almost as effective as the old model. These changes of wing shape in turn affect the force production of the wing, especially during stroke reversals as shown in figure 4.20 .

### 4.4.5 Tethered flight in laminar flow

The aerodynamic forces and the corresponding power consumption of the bumblebee model with flexible wings and highly flexible wings will be presented in this section. Moreover, with a view to the influence of wing flexibility on the insect aerodynamic performance, the results are compared with the ones obtained in [37] where the same bumblebee with rigid wings was studied. Figures 4.21A-C shows the vertical and horizontal forces as well as the aerodynamic power computed using rigid, flexible and highly flexible wings. For the purpose of comparison, the forces are normalised by $F=\rho_{a i r} R^{4} f^{2}$ and the aerodynamic power by $P=\rho_{\text {air }} R^{5} f^{3}$. By the definition of the coordinate system, the positive of the vertical (horizontal) forces correspond to the lift (drag). The simulations are computed for 4 strokes with approximately 28800 time steps using 3 Intel Cascade Lake 6248 nodes (with 120 cores at 2.5 GHz ) and consumed about 8200 CPU hours for each simulation.

Overall, the forces generated by the flexible wings tend to converge toward the forces generated by the rigid wings when the wing stiffness gets larger. Moreover, during one stroke, the more flexible the wing is, the weaker the peaks and the valleys of
the forces at the ends of upstroke and downstroke get. These peaks and valleys can be explained by the sudden rotation of the wings during stroke reversals. Based on standard Kutta-Joukowski theory, Sane and Dickinson [83, 84] showed that the net force generated by a rotating wing is proportional to the angular velocity of the wing. Since the motion of the rigid wing is imposed, every point on the wing rotates with the same angular velocity. This is no longer the case with the flexible wing. By taking the wing flexibility into account, the influence of the inertial and aerodynamic forces can now be observed. These forces resist against the movement of the wing, make the trailing edge bend in the opposite direction and create a negatively cambered wing compared to the flat rigid wing. This kind of deformation is predicted to weaken LEVs [112] and causes the wing to generate less force. Another way of looking at the problem is that the ability of adapting the wing shape helps to mitigate the large pressure jump between upper and lower surfaces, especially at the trailing edge [56] and provides a smoother flight [50, 51].

For each stroke, the cycle-average values of the thrust, the lift and the required aerodynamic power are integrated and shown in table 4.2. The effect on the average thrust of the wing flexibility can be considered negligible since these changes are quite small comparing to the fluctuations of the thrusts. On the other hand, the average lifts generated by the flexible and highly flexible wings are $12.44 \%$ and $25.68 \%$ smaller then the one of rigid wings, respectively. The decrease of the effective angle of attack caused by wing deformation can be understood as the reason for the declines of the average lift. The instantaneous angle of attack, which is claimed to play a significant role in the force generation [44], is altered by the shape adaptation of the wing during the flapping motion. However, it will be premature to conclude that that the rigid wings outperform aerodynamically their flexible counterparts based on these negative impacts on lift. Although the flexible wings generate smaller forces, they consume much less energy, with almost $18 \%$ and $29 \%$ required aerodynamic power are reduced for the flexible and highly flexible wings, respectively. Both flexible and highly flexible wings have better cycle-averaged lift-to-power ratios than the rigid wings (cf. table 4.2). Nevertheless, that does not necessarily mean that the more flexible the wing is, the higher lift-to-power ratio the wing attains since the flexible wings have a greater lift-to-power ratio than the highly flexible wings.

| Wing model | Thrust | Lift | Aerodynamic <br> power | Lift-to-power <br> ratio |
| :---: | :---: | :---: | :---: | :---: |
| Rigid | 0.17 | 2.09 | 8.84 | 0.236 |
| Flexible | 0.03 | 1.83 | 7.25 | 0.252 |
| Highly flexible | 0.11 | 1.55 | 6.28 | 0.247 |

Table 4.2 : Cycle-averaged forces and power calculated with three wing models : rigid [37], flexible and highly flexible.

### 4.5 Conclusion

In this chapter, we presented a numerical approach for fluid-structure interaction in the open source framework FLUSI, which is based on a mass-spring model describing the structure of the insect wings and a pseudospectral method for solving the incompressible Navier-Stokes equations. For imposing no-slip boundary conditions in the complex time-changing geometry we used the volume penalization technique. The solver has been implemented on massively parallel supercomputers using MPI and allows high resolution computations, here with more than half a billion grid points. Code validation for two classical benchmarks, a flow past a cylinder with a flexible appendage at $R e=700$ and the flow generated by a rigid revolving wing, is likewise presented.

The impact of wing flexibility was investigated firstly using a simple revolving motion. In order to assess the influence of wing flexibility on the aerodynamics, we vary the Young's modulus $E=3.5 \mathrm{kPa}$ and $E=35 \mathrm{kPa}$ referred to the flexible and the highly flexible wing models, respectively. The flexibility reduces the buildup of the aerodynamic force during the beginning of motion. Nevertheless, after the start-up phase, the wing yields a steady state configuration, and no significant oscillation nor unsteady deformation of the wing are observed. A better aerodynamic performance of the flexible wing, characterized by the increase of the lift-to-drag ratio during the steady state, is explained by the decrease of the effective angle of attack caused by the deformation of the flexible wing. On the other hand, the highly flexible wing appears to be less efficient than the rigid wing.

Next, the flexible wings was studied in the context of tethered flight using flapping wing kinematics measured in experiments by Dudley and Ellington [29]. Both lami-
nar and turbulent inflows were considered to investigate diverse flight conditions of insects. The preliminary results obtained in this work allow us to have some understanding about the role of wing flexibility in flapping flight. In a laminar flow, a comparison of the aerodynamic forces and the power requirements between highly flexible, flexible and rigid wings showed that flexibility allows reducing the energetic cost of flapping flight characterized by the lift-to-power ratio. However, the highly flexible wing appears to be less efficient than the flexible wing. This can be interpreted that there is an optimized zone of wing flexibility, which is ideal for flying. Furthermore, the wing inertia contributes damping fluctuations in the aerodynamic forces and hence stabilizes the insect during flight.

Under turbulent conditions, although the ability of shape adaptation of flexible wings makes them more sensitive to fluctuations of the flow structure than their rigid counterparts, the impact of turbulence is still negligible under the considered flight conditions. Nevertheless, due to costly computational time, the statistical property of the turbulent flow is not considered because only one simulation has been done to obtain the results for the turbulent case.

Despite of these findings, we have to keep in mind that the wing kinematics has an essential effect on the aerodynamic performance of wings and we have considered only one set of wing motion in this study. In perspective, these limitations can be overcome by examining other species with different wing kinematics or including flight control in our model. This is planned for our work in the future where we will study Calliphora with its wing kinematics measured from experiments.

Finally, although the wing flexibility was calculated based on the geometrical property of the veins, the estimation of veins' Young's modulus remains somewhat limited due to the vast range of known cuticle's property. This can be improved by using mathematical optimization for determining the right elastic properties of the wing model. To this end the equilibrium state of the wing model under external static force as a function of wing stiffness will be calculated and compared with data acquired from experimental measurements.


FIGURE 4.10 : Influence of wing flexibility on lift and drag coefficients of a revolving wing at $R e=1800$.


FIGURE 4.11 : Lift-to-drag ratio for the three wings during the steady state, rotation angle $120^{\circ} \leq \phi \leq 320^{\circ}$.


Figure 4.12 : Wing deformation corresponding to rigid (dark blue), flexible (blue) and highly flexible (light blue) wings at three time instants, $t=2, t=4$ and $t=6$. The more the wing is flexible, the more it deforms.


Figure 4.13 : Flow generated by a flexible revolving wing, visualized by their vorticity magnitude $|\boldsymbol{\omega}|$ at four time instants $t=1,2,4$ and 6 . The simulation is performed with resolution $1024^{2} \times 512$.


FIGURE 4.14 : Computational domain of size $6 R \times 4 R \times 4 R$ used in all simulations. A bumblebee model with flexible wings is tethered at $\mathbf{x}_{\text {cntr }}=(2 R, 2 R, 2 R)^{T}$ in a flow with the mean flow velocity $\mathbf{u}_{\infty}=(1.246 R f, 0,0)^{T}$. A vorticity sponge region is placed at the outlet to damp out vortices. Periodic boundary conditions are set for the four other sides of the domain.


Figure 4.15 : Normalized vertical force, horizontal force and aerodynamic power generated by a bumblebee with rigid wings (black) [36] and flexible wings in laminar flow (red) and turbulent flow (blue). Circles represent the cycle-averaged value of forces and power.


FIGURE 4.16 : Visualization of flow generated by a tethered flapping bumblebee with flexible wings in turbulence $T u=0.33$ showing normalized absolute vorticity isosurfaces at two levels $\|\omega\|=15$ (light blue) and $\|\omega\|=100$ (dark blue). The flow fields are plotted at time $t=0.45 / T$ and the weaker vortices are only shown in the region $3.7 R \leq y \leq 4 R$.

turbulent - top surface
turbulent - bottom surface

FIGURE 4.17 : Normalized pressure distribution on the wing for top and bottom surfaces, plotted at time $t=0.45 / T$ just before the stroke reversal, for the laminar (top) and turbulent cases (bottom).

(A) 2D triangular mesh

(B) 2D triangular mesh with crossover spring

(C) 3D out-of-plane bending

FIGURE 4.18 : Illustration of two adjacent triangles of the unstructured triangular mesh. The mass points are represented by the circles while the extension springs are represented by the continuous lines and the dashed line (crossover spring). The upper figures are the previous triangular mesh (A) without crossover spring and the currently used triangular mesh (B) with crossover spring in 2D. The lower figure illustrates the role of crossover spring for creating bending stiffness of the surface.


Figure 4.20 : Vertical force and horizontal force, normalised by $F=\rho_{a i r} R^{4} f^{2}$, generated by the old wing model without crossover springs (continuous red line) and the new wing model with crossover springs (dash-dot blue line). The new wing generates more force than the old one during stroke reversal.


FIGURE 4.21 : Vertical force, horizontal force (normalised by $F=\rho_{\text {air }} R^{4} f^{2}$ ) and aerodynamic power (normalised by $P=\rho_{\text {air }} R^{5} f^{3}$ ) generated by a bumblebee with rigid wings (continuous black) [36], flexible wing (dash-dot blue) and highly flexible (dash red). Circles represent the cycle-averaged value of forces and power.


FIGURE 4.22 : Visualization of flow generated by a tethered flapping bumblebee with highly flexible wings in laminar flow at $R e=2685$ showing normalized absolute vorticity isosurfaces $\|\omega\|=100$ (light blue). The flow field is plotted at time $t=0.45 / T$. The vortices are only shown in the region $2 R \leq y \leq 4 R$ for the purpose of visualizing the deformation of the left wing.

## Chapter 5

## Blowfly with flexible flapping wings

## Content

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The bumblebee with flexible wings previously studied gave us some insights about the influence of wing flexibility on the aerodynamic efficiency of insects. Nevertheless, one limitation of the model was the missing information about the wing stiffness. Although the wing geometry (venation pattern and vein diameters) was taken into account, the choice of Young's modulus for calculating flexural rigidity of veins as well as the estimation of joints' stiffness were not based on experimental data. The blowfly (Calliphora vomitoria) wing, on the other hand, is modeled with the optimizing stiffness parameters presented in chapter 3. Consequently, the blowfly wing model is expected to behave more like the real wings seen in nature.

This chapter is the outcome of all the preceding works where the mass-spring model developed in chapter 2, the wing geometry and its stiffness parameters optimized in chapter 3 and the fluid-structure solver validated in chapter 4 are combined together. Firstly, the numerical setup for tethered flapping wings will be described. Then, all
nine sets of stiffness parameters optimized from nine individuals will be simulated. The purpose is to study the aerodynamic forces generated by different individuals of the same species with distinct wing stiffness. Finally, the ultimate goal of this chapter is to reproduce the dynamic behaviors of real blowfly wings. These wings are then compared with their rigid counterparts in order to investigate the impact of wing flexibility on the aerodynamic force production of insects.

### 5.1 Numerical setup

We begin with the description of the setup along with the numerical parameters that are used for the study of aerodynamic performance of flexible blowfly wings. From an aerodynamic point of view, the insect body solely acts as a source of drag in a tethered flight context and we are only interested in the role of wing flexibility. Thus, the wings are studied alone without their body.

### 5.1.1 Computational domain

The computational domain, shown in figure 5.1, is $4 R \times 4 R \times 2 R$ large, where $R=9 \mathrm{~mm}$ is the blowfly wing length. The domain is discretized by $1024 \times 1024 \times 512$ grid points, yielding a total of 537 million grid points. This results an equidistant grid of which grid size is $\Delta x=35.19 \mu m$. Due to the constraint of the volume penalization method, the wing thickness must be at least 4 grid points. Hence, the wing thickness in our numerical study is constant and equals to $h_{w}=4 \Delta x=140.76 \mu \mathrm{~m}$. The wings are put as if the fly body is tethered at the center of the computational domain. As a result, the left wing is rotating about a fixed pivot point $\mathbf{x}_{\text {pivot,l }}=(2 R, 2.5 R, 1 R)^{T}$ and the right wing is rotating about a fixed pivot point at $\mathbf{x}_{\text {pivot,r }}=(2 R, 1.5 R, 1 R)^{T}$. They are exposed to a head wind with a mean flow accounting for the insect's forward velocity $\mathbf{u}_{\infty}=(-1.35,0,0)^{T} m s^{-1}$, this corresponds to a cruising speed of freely flying blowflies [11]. Based on the global coordinate system, the positive vertical force corresponds to the lift while the negative horizontal force represents the drag.

A thin vorticity sponge outlet, covering the last 20 grid points in $x$-direction, is used to minimize the upstream influence of the computational domain due to the periodicity inherent to the spectral method. The sponge penalization parameter is set at $C_{s p}=0.05$ [38], larger than the permeability $C_{\eta}=2.476 \times 10^{-4}$. By construction, the
sponge term is divergence-free to avoid the influence on the pressure field.


Figure 5.1 : Illustration of the computational domain used for the present study. The size of the domain is $4 R \times 4 R \times 2 R$ large, where $R$ is the Calliphora wing length. The wings, without the fly body, are placed as if the fly is tethered at the center of the domain. The mean flow corresponds the blowfly's cruising speed of $1.35 \mathrm{~ms}^{-1}$. A vorticity sponge outlet is used to prevent vortices from re-entering the domain. Periodic condition is imposed on the other 4 sides of the bounding box.

### 5.1.2 Wing kinematics

Although stroke kinematics patterns in flies vary during manoeuvring flight, a generic wing kinematics is used for the study [103]. In a tethered flight condition, mean stroke frequency of Calliphora varies between 127 and 180 Hz with a mean value $f=158 \mathrm{~Hz}$ [73]. This value is chosen for our study.

The Reynolds number is $R e=U_{\text {tip }} c_{m} / v_{\text {air }}=1542$, where $U_{\text {tip }}=2 \Phi R f=6.7 \mathrm{~ms}^{-1}$ is the mean wingtip velocity, $c_{m}=3 \mathrm{~mm}$ the mean chord length, $v_{\text {air }}=1.568 \times$ $10^{-5} \mathrm{~m}^{2} \mathrm{~s}^{-1}$ is the kinematic viscosity of air, $f=158 \mathrm{~Hz}\left(T=1 / f=6.33 \times 10^{-3} \mathrm{~s}\right)$ is the wingbeat frequency ( $T$ is duration) and $\phi=135^{\circ}$ is the wingbeat amplitude.

A


B

C

$t=2.5$

$t=2.6$
$\mathrm{t}=2.2$
$t=2.3$
$t=2.4$



$\mathrm{t}=2.7$
$\mathrm{t}=2.9$




Figure 5.2 : Generic wing kinematics of the studied blowfly wing. (A) Time evolution of the positional angle $\phi$ (blue), the feathering angle $\alpha$ (red) and the deviation angle $\theta$ (green) [103]. (B) The geometrical definition [38] of the wing angles with respect to the stroke plane. The mean stroke plane was inclined by $-20^{\circ}$ with respect to the horizontal plane. (C) The snapshot of flexible wings for a complete flapping cycle.

### 5.2 Variance of wing stiffness among blowfly individuals

In this section, the aerodynamic performance of different individuals with different wing stiffness will be investigated. We have in total nine sets of stiffness parameters that were optimized in chapter 3.

For better comparison, all nine wing models must share geometric similarity, kinematic similarity and dynamic similarity. The first two conditions are clearly satisfied since they have the same wing shape and wing kinematics. The third condition is

| Individual | Wing length <br> $R(m m)$ | Mean chord <br> $c_{m}(m m)$ | $u_{t i p}$ <br> $\left(m s^{-1}\right)$ | $u_{\infty}$ <br> $\left(m s^{-1}\right)$ | Kinematic <br> viscosity <br> $\left(m^{2} s^{-1}\right)$ | Reynolds <br> number <br> $R e$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 9.1 | 3.037 | 6.775 | 1.365 | $1.603 \times 10^{-5}$ |  |
| 2 | 9.1 | 3.037 | 6.775 | 1.365 | $1.603 \times 10^{-5}$ |  |
| 3 | 8.7 | 2.903 | 6.478 | 1.305 | $1.465 \times 10^{-5}$ |  |
| 4 | 9.1 | 3.037 | 6.775 | 1.365 | $1.603 \times 10^{-5}$ | 1542 |
| 5 | 8.7 | 2.903 | 6.478 | 1.305 | $1.465 \times 10^{-5}$ |  |
| 6 | 9.2 | 3.070 | 6.850 | 1.380 | $1.638 \times 10^{-5}$ |  |
| 7 | 9.1 | 3.037 | 6.775 | 1.365 | $1.603 \times 10^{-5}$ |  |
| 8 | 9 | 3.004 | 6.701 | 1.350 | $1.568 \times 10^{-5}$ |  |
| 9 | 9.1 | 3.037 | 6.775 | 1.365 | $1.603 \times 10^{-5}$ |  |

Table 5.1 : The wing lengths and the mean chords of nine individuals and the corresponding numerical parameters used for the comparison. Individual number 8 is chosen as the reference. The kinematic viscosity $v_{a i r}=$ $1.568 \times 10^{-5} \mathrm{~m}^{2} \mathrm{~s}^{-1}$ and the cruising speed $u_{\infty}=1.5 \mathrm{~ms}^{-1}$ of this individual are real values observed nature. For other individuals, these parameters are scaled based on their wing lengths in order to have the same Reynolds number $R e=1542$.
satisfied if and only if all the simulations have the same Reynolds number Re. As the models were tested at forward speed, the Reynolds number Re is calculated based on both cruising speed $u_{\infty}$ and mean wing tip velocity $u_{t i p}$ as :

$$
\begin{equation*}
R e=\frac{\left(u_{t i p}+u_{\infty}\right) c_{m}}{v_{a i r}} \tag{5.1}
\end{equation*}
$$

where $c_{m}$ is the mean chord length and $v_{a i r}$ is the kinematic viscosity of surrounding flow. Among all the studied individuals, number 8 is chosen as the reference whose cruising speed $u_{\infty}=1.35 \mathrm{~ms}^{-1}$ and the kinematic viscosity of air $v_{\text {air }}=1.568 \times 10^{-5} m^{2} s^{-1}$ is the real value observed in the nature. Because each individual has different wing length, the fluid viscosity as well as the cruising speed of other individuals must be adjusted to match the common Reynolds number. These values are presented in table 5.1. By doing this, any difference in aerodynamic properties between individuals can be explained by the differences in their wings' flexibility.

The dimensionless aerodynamic forces and the aerodynamic power generated by nine individuals in time are presented in figure 5.3. Since the wings started at rest and they need some time to stabilize, the data are shown for time starting from the third flapping cycle. In general, the aerodynamic performance of all individuals is almost identical. For each individual, most of the lift (cf. figure 5.3A) is generated during

| Individual | Lift | Drag | Aerodynamic <br> power |
| :---: | :---: | :---: | :---: |
| 1 | 2.630 | 0.267 | 11.076 |
| 2 | 2.642 | 0.265 | 11.110 |
| 3 | 2.597 | 0.279 | 11.089 |
| 4 | 2.667 | 0.255 | 11.166 |
| 5 | 2.733 | 0.310 | 11.550 |
| 6 | 2.710 | 0.381 | 11.798 |
| 7 | 2.604 | 0.343 | 11.293 |
| 8 | 2.597 | 0.279 | 11.089 |
| 9 | 2.656 | 0.270 | 11.164 |
| Mean | 2.648 | 0.294 | 11.259 |
| Standard deviation | 0.046 | 0.040 | 0.237 |

Table 5.2 : Cycle-averaged over the last two cycles of forces and power calculated for nine individuals. All nine individuals exhibit almost the same aerodynamic behavior, characterized by the small standard deviations for the three quantities. They are accounting for $1.7 \%, 13.6 \%$ and $2.1 \%$ of the mean values of lift, drag and aerodynamic power, respectively.
the downstroke, followed by a peak and a valley caused by the wing reversal from a downstroke to an upstroke, or the supination. Almost no lift is produced during the upstroke and the wing reversal from an upstroke to a downstroke, or the pronation, produces another peak and valley. The whole pattern is then repeated again. The same pattern for the drag and the aerodynamic power can be seen in figure 5.3B-C. The difference between these individuals is noticeable at the mid-downstroke where the maximum lift values generated by individuals 5 and 6 reach $8.8\left[M L T^{-2}\right]$ while the others' maximum lift values are around $8.0\left[M L T^{-2}\right]$. This gives us a difference of $10 \%$. The corresponding gap between the aerodynamic power at this instant goes up to $22 \%$.

Another way to compare among the individuals is to look at the cycle-averaged values of these quantities. Table 5.2 shows the averaged values calculated for the last
two cycles. The standard deviation values of the lift, the drag and the aerodynamic power are low compared to their mean values. This can allow us to confirm about the aerodynamic similarity of these individuals. Although the stiffness values optimized for these individuals in chapter 3 vary significantly among them, their dynamic behaviors under inertial and aerodynamic forces are similar.

In conclusion, from this study, no major difference of aerodynamic performance among individuals due to different wing flexibility was found. However, it is necessary to point out that our findings are restricted to the selected kinematics pattern.

### 5.3 Influence of wing flexibility on aerodynamic performance of blowflies

As shown in the previous section, since the aerial performance does not change much among individuals, only individual number 8 is chosen for the next study. Therefore, for this section, flexible wings refer to the ones of individual number 8 . The flexible wings will be compared with rigid wings for studying the influence of wing flexibility on aerodynamic performance of Calliphora.

### 5.3.1 Aerodynamic performance

The time history of the vertical and horizontal forces as well as the required aerodynamic power generated by the rigid and the flexible wings are shown in figure 5.4. Compared to the rigid wings, the stroke reversal, characterized by a peak and a valley of forces, is delayed in the case of flexible wings due to their inertia. Nevertheless, both rigid and flexible wings produce most of lift during the downward movement and the maximum lift occurs at the middle of the downstroke. While the two rigid wings produce a maximum lift of 2.49 mN , the lift generated by the flexible wings only reach a maximum value of $1.58 \mathrm{mN}, 36.55 \%$ less than their rigid counterparts. This decline is due to the wing flexibility and can be explained by looking at the wing deformation at the mid-downstroke ( 0.25 stroke cycle). Figure $5.5 \mathrm{~A}-\mathrm{C}$ show the cross-section of the right rigid wing (blue) superimposed on the right flexible wing (gray) at $0.25,0.5$ and 0.75 wing length. At this instant, the wings move almost in parallel with the oncoming flow and the angle of attack can be approximated as the angle between the mean flow velocity $\mathbf{u}_{\infty}$ and the the wing. As can be seen from the figure, at the proximal (A)

| Wing model | Lift | Drag | Aerodynamic <br> power | Lift-to-drag <br> ratio | Lift-to-power <br> ratio |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(m N)$ | $(m N)$ | $(m W)$ |  |  |
| Rigid | 0.754 | 0.167 | 5.228 | 4.513 | 0.144 |
| Flexible | 0.513 | 0.062 | 3.128 | 8.294 | 0.164 |

Table 5.3 : Cycle-averaged forces and power calculated with the rigid wing model and the flexible wing model. Overall, the rigid wings generate more forces than their flexible counterparts. The flexible wings have however better performance with higher lift-to-drag and lift-to-power ratios.
and the middle part (B), the leading edge of the flexible wing remains undeformed compared to the one of the rigid wing. The trailing edge is however much more flexible because there are fewer veins to support the membrane. As a result, the wing trailing edge is pushed upward and adapts its shape to align with the mean flow. This mechanism is caused by the flexibility of the membrane part. It lowers the effective angle of attack, but at the same time reduces the projected area of the wing with respect to the oncoming airflow. Figure 5.5D-F show the spanwise vorticity around the flexible wing while figure 5.5 G -I show the same kind of data for the rigid wing. From the vorticity distribution in these figures, the development of the LEV along the spanwise direction of the wing can be seen. For both wing models, the LEV expands gradually toward the wing tip and appears to burst at $75 \%$ of the wing length. The size and strength of the LEV generated by the rigid wing are, however, larger than the one created by the flexible wing.

On the other hand, during the upstroke, the dynamic behaviors of both flexible and rigid wings are very similar with little forces generated. Figure 5.6 shows the wing deformation as well as the spanwise vorticity of the left flexible wing and the corresponding rigid wing at the mid-upstroke. The flexible wing almost aligns with the rigid one and the intensities of spanwise vorticity generated by the two models are small.

Similar to bumblebee wings, flexible blowfly wings exhibit the same behavior. The flexible wings generate less forces than the rigid wings but they consume less power. This is characterized by the lift-to-power ratio, 0.144 of the rigid wings compared to 0.164 of the flexible wings, as presented in table 5.3.

### 5.3.2 Pressure distribution and wing deformation

Figure 5.7 presents the pressure distribution on the ventral and dorsal wing side. The time and the pressure are normalized by the flapping frequency $f$, the wing length $R$ and the fluid density $v_{\text {air }}$. As explained earlier, the data is shown for the third cycle after the wings stabilize from the onset.

During the mid-downstroke ( $t=2.2-2.3$ ), the pressure difference between the two sides is high and it is the origin of the force production. A large part of the dorsal side becomes a suction zone with the dimensionless pressure $P \leq-10\left[M L^{-1} T^{-2}\right]$. This low-pressure area is located at the leading edge and expands from the root to the tip. The suction footprint is consistent with the development of the conical leading-edge vortex (LEV) observed on the wing surface in figure 5.9. This finding is anticipated since the existence of LEV on insect wings is associated with the improvement in overall lift production. Figure 5.8 shows the wing deformation in both chordwise and spanwise directions under the inertial and aerodynamic loading. The deformation in the chordwise direction is understandable as explained in the previous part. There is no vein to support the membrane belonging to the trailing edge of the wing. The spanwise deformation is, on the other hand, thought-provoking. Combes and Daniels [18] reported that spanwise flexural stiffness is 1-2 orders of magnitude larger than chordwise flexural stiffness when measuring the forewings of 16 insect species. However, the external forces acting on the wing are strong enough to make the wing deform in both chordwise and spanwise direction. The maximum deflection of the wing leading edge occurs at the wing tip and corresponds to $10 \%$ of the wing length. On the other hand, during the downstroke and upstroke reversals, the wing is deformed only in chordwise direction due to strong inertial force caused by the wing rotation. The dominant contribution of inertial effect to wing torsion at the stroke reversals was earlier pointed out by Ennos [40, 41] for Diptera. Finally, during the upstroke, the pressure difference is weakened since the wings move in the same direction with the mean flow, resulting a low relative oncoming airspeed.

From a 3D point of view, the flow generated by the flexible wings is presented in figure 5.9 for one cycle. The flow structure is visualized by the iso-surfaces of vorticity magnitude $|\boldsymbol{\omega}|$. The development of LEV during the downstroke is considered as the basic aerodynamic mechanism behind the lift production of flapping wings. This spiral structure of the LEV starts to form at the beginning of the downstroke and remains
stable until the end. The centrifugal force creates a spanwise flow going from the root to the tip which has been explained as the main mechanism helping to stabilize the LEV. However, at approximately three-quarters of the wing length, the LEV starts to detach from the wing surface and forms a wing-tip vortex. During the upstroke, these vortices are weakened and hard to be seen.

### 5.4 Conclusion

The numerical simulations at high-resolution of a Calliphora wing model, flapping in a moving airflow, allow us to get some insights into the dynamic behavior of insect wings as well as the influence of wing flexibility on the aerodynamic performance of insects. Firstly, we performed the numerical experiments with all set of stiffness parameters optimized based on the measurements conducted on nine different individuals. We found that even though wing stiffness can vary among individuals, their aerodynamic properties are very similar by comparing dimensionless parameters at the same Reynolds number $R e$. Next, the flexible wings were compared with their rigid counterpart to investigating the influence of wing flexibility. By imposing the same kinematics for rigid and flexible wings, wing flexibility does not enhance lift production due to the decline of the effective angle of attack caused by the shape adaptation of flexible wings. However, flexible wings still perform better than rigid wings with respect to aerodynamic and energy efficiency characterized by a $83.78 \%$ larger lift-to-drag ratio and a $13.88 \%$ larger lift-to-power ratio.

Moreover, the study also reveals the dynamic behavior of a flexible membrane wing model, inspired by Calliphora wings. By using the same wing planform and venation pattern along with the stiffness parameters optimized from static measurements, we expect to recreate what happens to insect wings in nature under inertial force and aerodynamic loading. We observed a strong deformation of the wing model in both chordwise and spanwise direction, especially during the mid-downstroke, caused entirely by the inertial and aerodynamic forces. This agrees with the common finding that insect wing deformations during flight are predominantly passive.


Figure 5.3 : Dimensionless vertical force, horizontal force and aerodynamic power generated by nine individuals with distinct wing flexibility. The time is normalized by the wingbeat period $T=1 / f$. Circles represent the cycleaveraged value of forces and power.


Figure 5.4 : Time evolution of vertical force (A) and horizontal force (B) generated by the rigid wings (blue) and the flexible wings (red). (C) shows the corresponding aerodynamic power requirement. The time is normalized by the wingbeat period $T=1 / f$. Circles represent the cycle-averaged value of forces and power.


Figure 5.5 : Wing deformation and spanwise vorticity at the middle of the downstroke ( 0.25 stroke cycle). The data show the cross-section of the left rigid wing (blue) superimposed on the left flexible wing (gray) at 0.25 (A,D,G), 0.5 (B,E,H) and 0.75 (C,FI) wing length.


Figure 5.6 : Wing deformation and spanwise vorticity at the middle of the upstroke ( 0.75 stroke cycle). The data show the cross-section of the left rigid wing (blue) superimposed on the left flexible wing (gray) at 0.25 (A,D,G), 0.5 (B,E,H) and 0.75 (C,FI) wing length.


FIGURE 5.7 : Dimensionless pressure distribution on the ventral and dorsal wing side during one flapping cycle. During the downstroke, the difference between the two surfaces is high and generates most of lift. During the upstroke, the pressure difference is weakened.


Figure 5.8 : Wing deformation during one flapping cycle viewed from the side and the top. The data are shown in the wing system. The wing is deformed significantly in the chordwise direction during the stroke reversals and in the spanwise direction at the middle of the downstroke. The maximum spanwise deflection occurs at the tip of the wing going up to $10^{\circ}$.


Figure 5.9 : Flow structure generated by Calliphora wings during flapping. Data show the iso-surface of vorticity magnitude $|\boldsymbol{\omega}|$ of 100 (light blue), 150 (dark blue) and 200 (red). The view point is from the top : the downstroke corresponds to a movement of the wings away from the observer while the upstroke describe the wings moving toward the observer.

## Chapter 6

## Conclusion and perspectives

## Conclusion

Insect wings play an essential role in the aerial performance of flying insects. In the recent years, there has been a rise in research activity about the influence of wing flexibility on insect flight aerodynamics with contradictory results [14, 28, 59, 70, 74, 99]. The aim of this thesis is to improve our understanding about the fascinating mechanisms underlying insect flight. This work is placed in an interdisciplinary field that bridges the fields of biology, fluid dynamics, solid mechanics and applied mathematics. The research track employed here is based on computational methods combined with acquired experimental data of real insects.

Numerical analyses of active fluid-structure interaction with flexible wings are generally demanding and computationally expensive because one needs to solve for both fluid dynamics and structural response. At high resolution, the fluid solver itself can already be costly. Therefore, the solid solver needs to be efficient and the same time, be able to cope with large deformation of insect wings during flight. Within this context, a solid solver written in FORTRAN 95 has been developed based on mass-spring systems due to their simplicity and numerical efficiency. The dynamics of the system is governed by Newton's second law which leads to a system of first order ordinary differential equations. The implicit discretization of this system in time gives us a system of nonlinear equations which is in turn linearized and solved using Newton-Raphson method. Thus, the solid solver spends most of its time solving a sparse linear system whose the size is approximately $6000 \times 6000$. Obviously, the performance of the solver depends strongly on how efficiently this system is solved. In order to speed up the code, the SuperLU, an open-source solver using Gaussian
elimination with partial pivoting, was coupled with the solid solver via a C-Fortran interface. The integration helped us to speed up the solid solver with a factor of 10 . As a result, the solid solver is efficient enough to be used for simulations of active fluid-structure interaction. The drawback of the method is that it is not based on continuum mechanics theories like the finite element or the finite difference methods. The behavior of mass-spring systems relies critically on the mesh topology and the spring coefficients. Consequently, the parameters for the model need to be determined carefully in order to closely mimicking actual wings.

On the modeling side, the solid solver was designed specifically for modeling membrane wings where the supporting vein structure is distinguished from the flexible membrane part. From the literature [18, 19] and the experience that I gained from my two week internship measuring Calliphora vein stiffness in Rostock, the venation structure of insect wings plays an essential role in wing stiffness distribution. Consequently, two models were proposed to mimic two different mechanic behaviors of membranes and veins. Since the membrane was considered as a 2D sheet of extensible material with insignificant bending stiffness, it was modeled only by extension springs. The membrane had been firstly discretized using an unstructured triangular mesh but later, a crossover spring between two adjacent triangles was added to remove undesired artifacts. On the other hand, the veins predominantly determine flexural stiffness of the wing $E I$ (the product between the Young's modulus $E$ and the second moment of area $I$ ) and they were modeled using extension and bending springs. Unlike common bending springs used by others [54], we came up with bending springs with two bending angles to fully describe geometrically a curved vein in 3D space. After the vein and membrane models had been validated, the complete wing model was applied for studying aerodynamic performance of two species : bumblebee and blowfly.

Beside the mathematical, topological and mechanical representations of the wing model, the parameter setting is another crucial step that determines whether or not the model can correctly reproduce the dynamic behavior of real wings. Two methods for identifying the parameter values have emerged in the literature : deriving analytical expressions and optimization. Both these two approaches were used in our work. We started by deriving a relationship between the bending spring coefficients and the flexural stiffness $E I$ of a rod. The formula was then applied to set the bending spring coefficients for the veins of the wing model, based on its corresponding
flexural stiffness $E I$ estimated for real wings. For the bumblebee wing model, while the second moment of area $I$ can be estimated from the vein diameters, the value of Young's modulus $E$ needs to be determined. As a result, the Young's modulus is varied according on practical experience to make a comparison between two different wings that we refer to as flexible and highly flexible. For the blowfly wing model, the Young's modulus $E$ as well as selected joint stiffness coefficients are optimized based on experimental data of real blowfly wings. The data measure vertical deflection in response to static point forces at different characteristic locations. An optimization algorithm, called Covariance Matrix Adaptation - Evolution Strategy, was employed where the objective function is the difference between the deformations measured on the real wings and those calculated by the wing model. The scope is to find spring parameters that allow us to imitate mechanical behaviors of insect wing under static loads. For this task, a parallel FORTRAN 90 library for the evolution strategy with covariance matrix adaptation named pCMALib was used. The library was coupled with the solid solver for the objective function evaluation. Since the calculation of the objective function was expensive and it needed to be evaluated thousands of time for the optimization, the code was modified for running in parallel using the Message Passing Interface (MPI), where the evaluation was distributed to multiple processors. We obtained from the optimization nine sets of stiffness parameters corresponding to nine measured individuals. The average Young's modulus is 12.58 GPa with a standard deviation of 3.03 GPa , accounting for $24 \%$ of the mean value, albeit the optimized stiffness of joints varies greatly from one to four orders of magnitude. The variance can come from either biological reasons or the non-locality property of the objective function.

The ultimate goal of the thesis is to investigate the influence of wing flexibility on aerodynamic performance of insects. The fluid and structure interaction is of critical importance for the study. The solid solver is coupled with FLUSI, a fully parallel open source software for pseudo-spectral simulations of three-dimensional flapping flight in viscous flows. The code was designed to run on massively parallel supercomputers for high-resolution simulations.

The flexible and highly flexible bumblebee wings were firstly studied in a tethered flight context. They were then compared with their rigid counterpart to analyze the effect of wing flexibility. Based on the aerodynamic forces and the power requirements calculated for the three wing models, the comparison showed that flexibility allows
reducing the energetic cost of flapping flight characterized by the lift-to-power ratio. However, the highly flexible wing appears to be less efficient than the flexible wing. This suggests that there exists an optimized zone of wing flexibility, which is ideal for flying. Furthermore, the wing compliance damps fluctuations in the aerodynamic forces and hence stabilizes the insect during flight. In turbulent flow, although the ability of shape adaptation of flexible wings makes them more sensitive to fluctuation of the flow structure than their rigid counterparts, the impact of turbulence is still negligible under the considered flight conditions. The limit of this study was that the statistical property of the turbulent flow is not considered because only one simulation has been done to obtain the results for the turbulent case. Nevertheless, with a cost of 9000 CPU hours on 3 Intel Cascade Lake 6248 nodes (with 120 cores at 2.5 GHz ), the simulation demonstrates the feasibility of applying the code to study insects with flexible wings in turbulent flow, yielding insight into the triadic interaction of turbulence, wing flexibility and flapping flight. To our knowledge, numerical studies on this subject are still limited. Finally, simulations of tethered blowfly wings were performed. With different wing stiffness and wing kinematics, the blowfly model shows the same aerodynamic behavior like the bumblebee model. Although the wing flexibility does not enhance lift production, it reduces the form drag of the wing and the consumed aerodynamic power. Their greater lift-to-drag and lift-to-power ratios result in better performance compared with rigid wings. In addition, the model also reveals the wing deformation caused by aerodynamic and inertial forces during flight. Maximum chordwise deformations occur at supination and pronation due to wing rotation. In the spanwise direction, a maximum wing tip deflection up to nearly $10^{\circ}$ is observed at the middle of the downstroke.

Overall, we found that wing flexibility hardly contributed to lift enhancement effect. However, the significant reduction of the drag and the required power suggested that wing flexibility plays an important role in saving flight energetic cost. Moreover, the wing inertia also helped to damp out the fluctuation of the aerodynamic force and helped thus the insect to stabilize during flight. Despite of these findings, we have to keep in mind that the wing kinematics has an essential effect on the aerodynamic performance. However, these findings are limited for the studied wing motion. In addition, flexibility of insect wings also serves for different purposes, such as preventing mechanical damage or cleaning, rather than just for flight.

The limitations of the current approach are the resolution and CPU time require-
ments imposed by the use of a uniform grid. Hence large scale simulations become prohibitively expensive. Moreover, the thickness of bumblebee wing is much smaller than the spatial mesh size in our present simulations. Consequently, the virtual thickness of wings studied here is set to four times the mesh size, necessary for the usage of the volume penalization method.

An adaptive version of the FLUSI code, likewise fully parallel, using wavelet-based grid refinement is currently being developed to be able to reduce memory and CPU time requirements. High resolution numerical simulation of flapping flight for larger species and large Reynolds numbers will thus become possible. Implementing the solid solver presented in the current thesis into the adaptive Navier-Stokes solver will allow to perform fluid-structure interaction on adaptive grids at reduced computational cost.

## Perspectives

For the near-term, it is planned to complete the stiffness parameter identification for two other flies : fruit flies (Drosophila melanogaster) and house flies (Musca). The aim is to compare not only within each species but also between species, in order to get a bigger picture about insect wing structures along with their stiffness distribution. Another interesting aspect is the energetics of insect flight. Other numerical models employ rigid wing simplifications are consequently unable to identify the influence of flexibility on flight energetics. There have been studies $[52,62,110]$ showing that insect wings have the ability to store energy under elastic potential form of wing deformation. However, these studies relied mainly on experimental measurements and cannot easily be extended to examine various flapping kinematics. The elastic potential energy of the mass-spring system can be calculated easily for this task. Therefore, we intent to examine how much potential energy stored during wing deformation may contribute to energy recycling.

For a longer-term, with available geometrical data of a complete house fly (body, head, legs) from micro-computed tomographic ( $\mu \mathrm{CT}$ ) scans, a house fly with flexible wings will be investigated. The ultimate goal are simulations of house flies in turbulent airflow. For this purpose, due to the tremendous computational complexity of insect
flight in turbulent flow conditions, a wavelet-based adaptive solver WABBIT ${ }^{1}$ (Wavelet Adaptive Block-Based solver for Interactions with Turbulence) will be used for the fluid-structure interaction. The software is designed for wavelet-based adaptive simulations of complex, time-dependent geometries in turbulent flows on massively parallel supercomputers. The coupling between the flexible wing model and WABBIT is expected to be efficient enough to study insects with flexible wings in turbulence. Under turbulent conditions, there are some possible mechanisms by which flexible wings can improve flight stability of insects. As shown earlier in this work, wing deformations reduce the projected area of the wing with respect to the oncoming airflow, thereby reducing drastically its drag comparing to rigid wings. If unpredictable flow asymmetries between the left and right wings, provoked by a turbulent gust, decrease the stability of the insect, then smaller net drag caused by these aerodynamic asymmetries might make it easier for the insect to maneuver. It is in our best interest to investigate these kinds of hypotheses and to broaden our understanding about the influence of insect wing flexibility in a turbulent world.

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[^0]:    ${ }^{1}$ FLUSI : freely available for noncommercial use from GitHub (https ://github.com/pseudospectators/FLUSI).

[^1]:    ${ }^{1}$ https ://github.com/xiaoyeli/superlu

[^2]:    ${ }^{1}$ https ://www.salome-platform.org/

[^3]:    ${ }^{1}$ freely available for noncommercial use from GitHub (https ://github.com/adaptive-cfd/WABBIT).

