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Credit Rating Agencies

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Résumé

1 Introduction :

Les agences de notations ont récemment été l'objet d'une grande attention. Leur responsabilité dans la crise des subprimes a été questionné. Les médias ont mis en avant les notes trop généreuses qui avaient été attribué à certains produits complexe, avant de s'interroger sur leur comportement quand elles ont dégradé les notes des dettes souveraines. x Dans cette thèse, après avoir revu une partie de la littérature sur le sujet, je m'interroge sur deux aspects spécifique de leur activité:

Quels sont les conséquences de confier une information publique (comme une note de crédit) à une entité privée ?

Les agences de notations disent avoir pour principal actif la réputation. Cette dernière peut expliquer pour on observe des périodes de sur- notations et d'autres de sous-notations ?

2 Revue de littérature:

Les agences de notations sont apparues à la fin du XIXème siècle. Leur fonction économique consiste à réduire l'asymétrie d'information entre emprunteurs et prêteurs. Elles ont commencé par évaluer la dette des compagnies ferroviaires lors de l'expansion américaine vers l'ouest. Leur activité se concentrait sur les dettes d'entreprises. L'information étant transmise sous forme de caractère alphanumérique: Le XXème a vu deux changements majeurs concernant leur activité :

La gamme de produits notés à évoluer pour couvrir une grande part des différents produits financiers comme les dettes de pays ou encore les produits structurés ;

Le modèle économique des agences de notations a changé passant d'un modèle où les agences de notations étaient payés par les investisseurs à un modèle où ce sont les émetteurs de dette qui supportent les frais de notation.

Grade	Type
AAA	Investment Grade
AA+	
AA	
AA-	
A+	
A	
A-	
BBB+	
BBB	
BBB-	
BB+	Speculative Grade
BB	
BB-	
B+	
B	
B-	
CCC	
CC	
C	
RD	
SD	
D	

Figure 1: S&P ratings

Ces changements ne sont pas sans conséquences. En effet la littérature académique s'est penché sur la question des agences de notations, sur la qualité de leur travail à travers les spécificités de cette industrie. Parmi ces spécificités on relèvera :

- Un modèle économique qui pose des questions : En effet, l'agence étant payé par l'émetteur de dette, et les frais de notations étant corrélés à la hauteur de la note, ce système crée un conflit d'intérêt. Les agences de notation affirment que la réputation étant leur principal actif, elles ne peuvent s'engager dans un comportement déviant. Nous discuterons cette question.

- L'oligopoly, en effet, s'il existe plusieurs agences de notations, une très grande partie de l'activité de notation se concentre sur les trois grandes, à savoir S& P, Moody's et Fitch. Le manque de compétition peut il expliquer la mauvaise qualité des notes observés ?

- Les cycles économiques peuvent ils expliquer le comportement des agences de notation, en effet les notes semblent suivre une certaine dynamique.

- Le rôle de la régulation qui fait référence aux agences de notations, et augmente ainsi leur poids dans l'économie. Tout comme le fait que les notes soient une information publique.

- L'activité d'expert : en exprimant leur avis, les agences de notations sont soumises au même incitations que d'autres experts, cela peut il expliquer leur comportement ?

Le modèle économique :

Les agences de notations affirment que leur modèle économique ne saurait être à l'origine de sur notation car cela entamerait leur réputation. Cet argument a été questionné par Mathis et al. (2009). Les auteurs considèrent un modèle avec une agence de notation dont les revenues dépendent à la fois de sa réputation et du nombre de produits bien notés. Les auteurs montrent qu'en présence d'asymétrie d'information, l'agence peut donner une bonne note à un mauvais produit, sa réputation ne saura pas trop entamer car l'agence peut se cacher derrière des erreurs potentiellement honnêtes.

Le manque de compétition :

Le manque de compétition a également été très présent dans les critiques à l'encontre des agences de notations. Mais peut elle expliquer la faible qualité des notes ? Dans Skreta and Veldkamp (2009), les auteurs montrent que la compétition peut conduire à une "course vers le bas", où les agences seront en compétition pour attirer les émetteurs de dettes les moins sérieux. Leur approche théorique a également été confortés par des analyses empiriques.

Le rôle de la régulation financière :

Dans la réglementation bâloise, les notes émanant des agences de notations peuvent être utilisé pour estimer le capital réglementaire nécessaire aux banques pour conduire leur activité. D'autres références existent par ailleurs, par exemple, les fonds de pension vont user de référence aux notes dans leur brochure pour à la fois garantir la nature des investissements réalisés, et également communiquer facilement les notions de risque avec leurs clients. Cela a pour conséquence d'offrir aux agences de notations une audience garantie comme souligné par Opp et al. (2013).

Les cycles économiques :

La qualité des notes émises semblent varier au cours des cycles économiques. Ferri (1999) montrent que le comportement des agences de notations a aggravé la crise d'Asie du sud-est. En analysant la stratégie de notation des agences, les auteurs montrent que ces dernières ont été généreuses avec le crash avant

d'être excessivement sévère après. Dans une autre approche, Bar-Isaac and Shapiro (2013) montrent que les agences de notations ont tendance à être plus performantes dans une récession que dans un boom. Ce résultat repose sur le fait qu'il coûte moins cher d'embaucher un bon analyste en récession que dans un boom où ces derniers sont plus recherchés.

Le comportement des experts :

En tant qu'expert les agences de notations sont soumises à une incitation de conformité. Par exemple, si l'opinion commune considère que les prochains chiffres seront bons, il est difficile pour un expert d'aller contre cet avis. Ainsi Mariano (2012) montre que les agences de notations ont tendance à suivre les avis déjà présent sur les marchés. Cela peut expliquer en quoi elles semblent parfois en retard.

3 Public Information from a private entity: the case of Credit Rating Agencies

Dans ce chapitre, je questionne l'interaction entre les investisseurs et une agence de notation en terme de choix d'information. En effet, l'agence de notation produit une information publique qui réduit l'incitation des autres agents à produire de l'information par eux-mêmes. Cela soulève d'ailleurs un paradoxe, car les investisseurs sont également supposés contrôlé le bon travail de l'agence.

Pour ce faire je m'appuie sur les modèles de Morris and Shin (2002) et Colombo and Femminis (2008), je considère une économie composé d'investisseurs ayant la fonction d'utilité suivante :

$$u_i(\mathbf{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}) - p\beta_i \quad \text{avec, } p > 0 \text{ and } 0 < r < 1$$

La fonction d'utilité est faite de trois termes :

Le premier terme représente l'intérêt des investisseurs d'être proche du fondamental θ (la qualité de la dette) ;

Le second terme représente l'intérêt des investisseurs à se coordonner, il s'agit d'une représentation du concours de beauté de Keynes. Le paramètre r définissant l'intensité de ce concours de beauté.

Enfin, le dernier terme consiste en une fonction de coût pour acquérir de l'information. Cette fonction est croissante avec la précision de cette information β. En effet, l'information qu'ils recoivent est de

la forme: $x_i = \theta + \varepsilon_i$ and $\beta_i = \frac{1}{\sigma^2_{\varepsilon_i}}$;

En effet, sur les marchés financiers, les investisseurs ont deux incitations différents. Ils se doivent d'agir en fonction de la valeur fondamentale d'un actif, l'estimé à sa "vraie" valeur. Mais dans le même temps, il ne sert à rien d'avoir raison seul contre tous, il faut également anticiper les actions d'autres individus. Il s'agit du "concours de beauté" comme définit par Keynes.

Il existe également une agence de notation, qui a la fonction de revenue suivante:

$$\Pi_{CRA}(\alpha) = -\int_0^1 (a_i - y)^2 \mathrm{d}i - k\alpha$$

Où $y = \theta + \eta$ représente le signal publique émis par l'agence avec $\alpha = \frac{1}{\sigma^2_\eta}$. L'agence de notation tire son profit de l'utilisation de son signal publique. Plus celui-ci sera utilisé, plus l'agence de notation sera "indispensable et le plus elle pourra facturer les émetteurs de dette.

Le déroulement du jeu est le suivant:

1 - θ est tiré par la nature;

2 - L'agence de notation choisi la qualité de son signal publique, α (connaissance commune);

3 - L'agence de notation communique son signal, y;

4 - Les investisseurs choisissent la qualité de leur signal privée, β_i et le recoivent x_i;

5 - Les investisseurs choisissent leur action a_i en fonction des informations qu'ils possèdent;

6 - Le fondamental (θ) est révélé et les paiements sont réalisés.

Le but du jeu consiste donc pour l'agence de notation à définir la qualité de son signal publique, tandis que les investisseurs définissent leur choix d'information privée et leurs actions.

Résultats : Je reprends en premier lieu les résultats de Morris and Shin (2002) et Colombo and Femminis (2008). A savoir que les investisseurs vont surpondérer le signal publique par rapport à leur information privée :

$$a_i = (1 - r)E_i(\theta) + rE_i(\int_0^1 a_j \mathrm{d}j) \tag{1}$$

Enfin, les investisseurs, à l'équilibre symétrique vont acquérir une information de la qualité suivante:

$$\beta_i = \beta = \frac{1}{\sqrt{p}} - \frac{\alpha}{1-r} \quad \text{si} \quad \beta > 0 \quad \text{otherwise} \quad \beta = 0. \tag{2}$$

L'originalité de mon modèle consiste en la présence de l'agence de notation qui cherche à maximiser son profit. A l'équilibre, on obtient: $\alpha^{sup} = \frac{1-r}{\sqrt{p}}$ qui consiste en un maximum de qualité au-delà duquel l'agence de notation n'ira jamais.

- Lorsque $p \geq k(1-r)$, les investisseurs se reposent uniquement sur l'information publique $\beta_i = \beta^* = 0$;

- Lorsque $p < k(1-r)$, L'agence de notation envoie un signal publique mais les investisseurs s'informent également: $\beta_i = \beta^* = \frac{1}{\sqrt{p}} - \frac{\alpha}{1-r}$.

On peut faire les observations suivantes:

Dès que l'agence de notation possède une meilleure technologie que les investisseurs elle est la seule productrice d'information;

Lorsque la technologie des investisseurs est supérieure ou égale à celle de l'agence de notation, le niveau d'information privée dépend de l'intensité du concours de beauté. Lorsque r est grande les investisseurs ne s'informent plus car ils sont principalement motivés à se coordonner.

Enfin on remarquera qu'un planificateur social ferait des choix différents:

Il demanderait à l'agence de notation de produire une information de plus grande qualité;

Il changerait l'action des investisseurs, cependant cette dernière entrainerait toujours une sur-pondération du signal publique. Mais si les investisseurs le font pour se coordonner, le planificateur social le ferait pour récompenser l'agence de notation.

A travers ce modèle, j'ai voulu souligner le jeu que pratique une agence de notation en terme de qualité d'information. En particulier, je voulais faire apparaître le concept de relativité de la qualité. En effet, l'information transmise est perçue en fonction des connaissances des récepteurs. Ainsi, plus l'asymétrie d'information sera grande, moins l'agence aura l'incitation de produire une information de qualité. Il est à noter que dans ce papier je ne discute pas de comportement stratégique visant à biaiser les notes (sur-notation par exemple).

Il est important de discuter du rôle des agences de notation et de leurs incitations dans ce contexte. L'intérêt d'avoir des agences de notations dans une économie, consiste à faire des économies dans la production d'information. En effet cela permet aux investisseurs de se consacrer à la bonne allocation du capital sans perdre trop de ressources dans l'analyse crédit, cela évite une redondance. Cependant, c'est les connaissances, et la capacité à s'informer qui va permettre aux investisseurs de pousser les agences de notation à faire un travail de qualité. Ce papier avait pour but de mettre en avant ce paradox.

4 Credit Rating Agencies, Reputation, Business Cycles and Uncertainty

Dans ce chapitre, je reviens sur le conflit d'intérêt lié au modèle économique des agences de notations. L'argument utilisé par les agences de notations consiste à dire qu'elles sont disciplinées par les incitations de réputation. Cependant les dernières crises financières nous ont montré que la qualité de leur travail n'était pas toujours au rendez vous.

Pour ce faire je considère une agence de notation dont les revenues dépendent du nombre de bonne notes que l'agence délivre. Si l'on se situe dans une modèle statique, il n'y a pas d'incitations de réputation, et l'agence va délivrer une bonne note. La première observation consiste donc que la réputation fonctionne tant qu'il y a un futur, exhibant incitant l'importance du facteur d'actualisation des agences de notation. L'agence de notation reçoit à chaque période: $\pi = x_0 q_0 - c$ où:

- x_0 est la proportion de projet bien noté;

- q_0 correspond au capital de réputation de l'agence;

- c répresente le coût de l'agence pour acquérir de l'information.

Si l'on se place dans un modèle à plusieurs périodes, tant que l'agence de notation observe parfaitement les différents projets qu'elle doit noter, on peut trouver une fonction de réputation qui dépend du rendement espéré ex-ante qui permet de s'assurer que l'agence de notation fera bien son travail. L'idée étant également que l'agence de notation conserve son capital de réputation au fil du tempts, je cherche une fonction qui disciline l'agence de notation en respectant les conditions suivantes:

(C1) $F^\theta(0) = F^\theta(Rp(\bar{x}) - 1) = q_0$;

(C2) $F^{\theta}(x)$ est croissante et dérivable deux fois (plus l'action de l'agence doit assurer un rendement élevé, plus sa réputation est haute);

(C3) L'agence de notation donner une bonne note à tous et uniquement les projets qui ont une valeur actualisée positive $x_0 = \bar{x}_{\theta}$.

On peut ensuite ajouter la conjoncture. On considère alors une économique qui peut se trouver dans deux états, soit dans un boom soit en récession. Il y a bien évidemment plus de bons projets dans un boom que dans une récession. Si l'agence de notation observe parfaitement l'état du monde lorsqu'elle doit noter, on peut alors définir une règle disciplinant l'agence de notation. Il faut cependant deux règles différentes (mais parallèle), une pour chaque état du monde.

Le problème apparaît alors lorsqu'il y a incertitude. L'agence de notation est incertaine quant à savoir la proportion de bons projets (qui méritent une bonne note) et elle sait qu'elle sera noté en absence d'incertitude. En effet, à la période suivante les investisseurs sauront si à la période précédente nous étions dans un boom ou une récession. Ils vont donc juger en cela l'agence. A travers ce modèle je souhaite souligner que parfois des actions à priori, sont jugés à posteriori, quand l'information disponible est plus riche.

Face à l'incertitude l'agence de notation fera donc toujours des erreurs. Soit elle aura surnoté en pariant sur un boom, ou sous noté craignant une récession. Ainsi son capital dé réputation va évoluer. Or un capital de réputation est fait pour être utilisé. Si ce dernier est haut ; alors l'agence pratiquera la sur-notation à la période suivante, tandis qu'en cas de chute du capital de réputation, l'agence de notation aura intérêt à être excessivement sévère pour s'assurer des revenues futurs.

A travers ce papier, j'ai voulu montrer l'impossibilité de parfaitement discipliner un agent via des incitations de réputation en présence d'incertitude. Il faut avoir en tête que lorsque la politique de notation d'une agence nous surprend, cela peut provenir de deux origines :

- Une erreur honnête, l'agence a été surprise par la conjoncture ;

- Un choix stratégique, dans le but d'user ou de reconstruire son capital de réputation.

5 Conclusion :

A travers cette thèse j'ai voulu en premier lieu étudié les différentes problématiques posé par l'activité des agences de notation. J'ai pu décrire comment la littérature répondait aux différentes spécificités de leur activité. Enfin à travers mes deux papiers, j'ai souhaité souligné dans un premier temps des relations entre investisseurs et agence de notations. Si les agences de notations ont mal fait leur travail lors de la dernière crise financière, on peut en dire autant des investisseurs. Si l'on confie une mission à un agent (produire une information publique par exemple), il est intéressant de bien regarder si les incitations produites par son environnement ne varient pas au cours du temps par exemple. La façon dont les investisseurs utilisent les notes de crédit, suivant que l'on évolue dans un marché financier très liquide (grande intensité du concours de beauté) ou de niche avec peu d'échanges, ne sera pas la même.

Enfin je me suis penché sur la question de la réputation, si la littérature récente s'est concentrée à montrer les situation avec sur-notation. J'ai cherché à mettre en avant, que l'incitation de réputation et l'utilisation d'un capital de réputation mènent forcément à des moments de sur-notation et également de sous-notations en présence d'incertitude.

Les agences de notations ont soulevé beaucoup de critiques suite à la crise des subprimes. Certaines étaient justifiés, d'autres un peu moins. Ce travail de thèse m'a permis de bien aborder la question. La question du manque de compétition dans cette industrie par exemple, fût mise en avant. On observe cependant qu'il s'agit d'un point peu pertinent pour expliquer la faible qualité des notes, et des phénomènes de sur-notation. Les questions d'asymétrie d'information et de certification ne sont pas uniquement réservés aux seules agences de notations, les outils présent dans cette littérature permettent de discuter également de situation similaire dans d'autres industries.

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Contents

1 Introduction **4**

2 Literature Review **5**

2.1 Credit Rating Agencies and information transmission 5

2.2 Reputation, Asset Complexity, Business Cycles and Rating's Quality 13

2.3 Expert's Decision, Regulation and Public Information 24

3 Public Information from a Private Entity: the Case of Credit Rating Agencies **32**

3.1 Introduction 32

3.1.1 Literature associated 34

3.2 The Model 35

3.2.1 Equilibrium: 37

3.3 The Model with Explicit Cost Functions 39

3.4 Conclusion 42

4 Credit Rating Agencies, Reputation, Business Cycles and Uncertainty **43**

4.1 Introduction 43

4.1.1 Related Literature 44

4.2 The Basic Model 45

4.2.1 The two period model: 46

4.3 The Dynamic Model with Uncertainty: 49

4.4 Conclusion 51

5 Conclusion: **52**

A Appendix Title **54**

A.1 Appendix - Public Information from a Private Entity: 54

A.2 Appendix - Credit Rating Agencies, Business Cycles and Reputation: 66

Chapter 1

Introduction

Credit rating agencies (hereafter CRA) have recently been under a lot of scrutiny. Their responsibility in the last financial crisis has been questioned. They received much attention from the media, and a growing literature has been developing over their activities. Their business model and the quality of their rating over time are studied. After a review of this recent literature, I look at two specific issues related to their business: (i) What issues arise when public information is released by a private entity on financial markets? (ii) Can reputation explains why a credit rating agency can be caught underrating (respectively overrating)?

The second chapter is entitled: Public information from a private entity: the case of credit rating agencies. It looks at the following paradox: a Credit Rating Agency delivers public information about the quality of a financial product as investors will not need to proceed the analysis by themselves (acquire costly information), however CRA's work should be monitored by those investors which in turn would need some private information to execute their monitoring function. Actually, what matter for an issuer is not exactly the reputation of the agency but rather the rating's impact on the market. Indeed, ratings represent a focal point for investors as they are a public signal. As shown in Morris and Shin (2002), the social value of a public signal may be ambiguous as investors, playing also a beauty contest in the sense of Keynes, tend to overreact over the public information. The CRA's case is specific as it represents a private entity sending a public signal. It should be seen as an expert which has more incentives to enhance the quality of its work when the audience is speaking to is well informed. Through its public signal, I consider that the CRA has the possibility of crushing down investors' incentives to get informed. The CRA being the only agent informed, the impact of its rating is critical even though its accuracy may be low.

The last chapter is entitled: Credit Rating Agencies, Reputation and Business Cycles. CRAs claim that they rate through the cycle, and reputation being their main asset, they cannot engage themselves in any devious rating policy. However, reputation's incentives may not be sufficient to discipline a CRA as shown in recent papers. I show that when evolving in uncertainty, a CRA cannot perfectly answer to reputation incentives. It will always be caught underrating or overrating. Thus movement of reputation are inevitable, and as reputation is considered as an asset it is made to be used. A CRA with a low reputation will act in a cautious manner. Whereas a CRA which has enjoyed good surprises and witness its reputational capital increase, will have incentives to overrate.

Chapter 2

Literature Review

The Credit rating agencies (hereafter CRAs) are an important feature of financial markets. By evaluating the creditworthiness of a debt, they reduce the asymmetry of information, and help the functioning of those financial markets. They have received much attention after the last financial crisis. Their function and the quality of their work have been questioned and a growing literature has studied their activity. This chapter provides an overlook of this literature.

(i) Credit Rating Agencies and Information transmission - After a description of CRAs' history and particularities. I will present papers that will help us to understand why a CRA can be seen as a positive agent for the economy;

(ii) Reputation, Asset Complexity, Business Cycles and Ratings' Quality - Over the last decades, ratings' quality has been criticized: in this section, I will present some of the models which show how their business model and asset complexity can explain the low quality of ratings. I will then discuss the influence of competition and of the business cycles on ratings' quality;

(iii) Expert's Decision, Regulation and Public Information - Ratings are an expert's decision. As any expert's message, the information sent might be biased for some reason. I will review some of the literature which investigates this issue. Moreover, those experts' decision have an important weight on financial markets as a rating is a public information. We will discuss how regulation drives CRAs behavior and what issues are raised with public information.

2.1 Credit Rating Agencies and information transmission

A. A brief history of Credit Rating Agencies

Reducing asymmetry of information is a central issue to ensure the well-functioning of financial markets. The idea of providing information about the credit worthiness of a firm appeared by the end of the nineteenth century. It came along with the development of the railroad industry in the United States. This industry was in need of a large amount of capital. Some firms were

reliable, others were not. However available information was poor and no records were kept about the financial situation of each firm, creating a need for an intermediary. In 1868, Henry Varnum Poor published the *Manual of the railroads of the United States*. Inside that book, you would find economic and financial information about railroad firms. It was then updated every year from this date. The focus on the railroad industry came for two reasons:

- It was a growing sector which was in need of capital with firms of different risk profile;
- For geographical reasons (financial centre and industrial activity were not in the same area) information was not easily available.

Today this information is provided through grades (AAA for example), but this structure was designed later. It was in 1909, that John Moody's decided to use alphanumeric symbols to transmit an aggregated version of the creditworthiness of a firm after he had published in 1900 *Manual of industrial and miscellaneous securities*. The opinions of these books became prevalent years after years. The main credit rating agencies were also created at the beginning of the nineteenth century:
- Moody's in 1909
- Poor's Publishing Company in 1916
- the Standard Statistics Company in 1922
- Fitch Publishing Company in 1924

Moody's is a listed company, Standard and Poor's have merged. Those two are American whereas Fitch is part of a French conglomerate (FIMALAC).

The "AAA" means an excellent creditworthiness, whereas bond rated "RD","SD","D" indicate a default over the debt.

After the 1929 crisis, American authorities looked for a scheme to prevent banks from taking too risky positions in order to protect depositors. In 1936, ratings made their entry into regulation for the first time. The Federal Reserve constrained banks to use ratings. The notions of "investment grades" and "speculative grades" were defined to distinguish between "safe" and "risky" investment. Banks were limited to invest in investment grades. The latter means a rating at least equal to $BBB-$ (S&P scale). As White (2010) underlies it, financial regulation gave ratings a crucial role. Banking regulation was followed by pension funds regulation in the 1970's and a few years later the broker dealer's activity was also bound to rely on "recognized ratings manuals".

The question of what manuals are "recognized" was raised, and in 1975 the SEC (Security and Exchange Commission) created a label "nationally recognized statistical rating organization" (NRSRO). It is a licence granted to the "recognized" credit rating agencies. This licence set a barrier to entry in the rating industry. A few credit rating agencies were given that licence, and the merger over the last decades have contributed to the oligopoly we observe today: Standard and Poor's, Moody's and Fitch are the three main companies leading this business[1].

This period witnessed another major change. After selling publications to investors, the CRAs turned towards an issuer-pay business model. This change came from regulations which increased

[1] If there is currently around 140 agencies, the big three, as they're called, accounts for 90% of the market

Grade	Type
AAA	Investment Grade
AA+	
AA	
AA-	
A+	
A	
A-	
BBB+	
BBB	
BBB-	
BB+	Speculative Grade
BB	
BB-	
B+	
B	
B-	
CCC	
CC	
C	
RD	
SD	
D	

Figure 2.1: S&P ratings

the value of a rating for an issuer[2], but also because of the development of Information Technology. As it became easier to copy books, it created an opportunity for free-riding. Ratings became public information and if, before that time, most ratings were unsolicited, solicited ratings became the norm. Since then the ratings industry has not really changed in terms of business model, nature of ratings and actors[3]. Obviously this business model raised concerns about a potential conflict of interest. CRAs argue that the conflict of interest is eliminated by reputation incentives. We will discuss this issue in the next sections.

Beside their business model and the regulation, their activity has evolved. If they started by rating the debt of a company, they now evaluate a wider range of financial products. We can separate their activities into three main areas:

- Governments (and assimilated) bonds;

- Corporate debts;

- Structured finance.

[2]As regulations asked banks to invest mostly on well rated products, it became necessary to obtain a good rating to be "eligible"

[3]some small credit rating agencies still rely on publications issued, but they focus on very specific products

Each activity has its particularities and raises different questions. Regarding Governments bonds, we should notice the different practice in play. For the most developed countries, the CRA is not paid to rate the country, it does so for many reasons (e.g publicity, the need to have a reference at a larger level before rating companies inside the country,...) but they don't receive revenues for that work and the country only provides information to the CRA. For less developed countries, the stakes are different. For those countries, ratings are crucial to attract foreign investment. To evaluate the credit worthiness of a country's debt, CRAs take into account economic and financial records but also social and political risk.

Corporate debt ratings was the CRAs' first activity but in recent years the boom in structured finance has also been an important part of their activity. It gained public attention with the subprime crisis. Those products are especially complex and opaque, as the MBS (mortgage back securities) for example. It is thus more difficult for investors to monitor the quality of the information provided. Moreover, CRA's activity consists not only on rating those products but they were also involved in the design of those securities as well. We will see how the literature looks at this activity.

When studying issues related to the CRA, we should specify which segment is under analysis. For example, Government bonds are evaluated following a methodology which include specificities not applicable to a firm's debt such as political risk.

We will also look at other specificities of the ratings' industry:

- Self-fulfilling prophecy: For a project in need of capital, the quality of the ratings will influence its probability of success as it has an influence on financial costs;

- Unsolicited ratings: Ratings can be delivered without being requested. CRA argues that this is to provide a better information to the markets, however we will see that this practice can also be used as a threat. We should notice that since 2009, The EU imposed that unsolicited ratings should be identified as such;

- The oligopoly structure and their responsibility in crisis and ratings' quality over the business cycles; the lack of competition in this industry has raised questions, as well as their behavior during booms where they seem to be more lax than in other periods.

The three main companies have a crucial influence over financial markets, which drives the interest of economists. Actually, their behavior in the last financial crisis has been criticized. They were blamed for fostering bubbles through too generous grades, not anticipating crashes and also being too severe sometimes. We need to take into consideration all details of their activity to understand which ones could explain any movement in ratings' quality. It should also be noted that there is a paradox about CRA as underlined in Partnoy (2001): their influence has been growing over the last decades, and every announcement about a new rating or an update is big news whereas they can lag the market and the informational content of ratings is questioned. Do they provide new information or do they only aggregate public information? Still, as expressed by the concerns of any government about ratings' announcement, Credit rating agencies represent a crucial element on financial markets and as Thomas Friedmann said in 1996:

> *There are two superpowers in the world today in my opinion. There's the United States and there's Moody's Bond Rating Service. The United States can destroy you by*

dropping bombs, and Moody's can destroy you by downgrading your bonds. And believe me, it's not clear sometimes who's more powerful.

We should also remember that financial markets deal with asymmetric information which can lead to bad equilibrium. As shown in the seminal paper of Akerlof (1970), asymmetry of information creates conditions in an inefficient market where only the worst asset would be traded. As mentioned above, CRA represents a way to impede a lemon's problem. Sellers will not be trusted by buyers when revealing the quality of their asset, because of the presence of bad sellers. Thus appears a need for certification, which is where CRAs' function comes in play. In the next section, I illustrate through Gorton and Ordonez (2014) and Boot et al. (2006) how CRAs can be seen as positive to the economy when multiple equilibria coexist.

B. What do credit rating agencies bring to financial markets?

As seen in Akerlof (1970), in presence of asymmetry of information, markets may collapse. On the financial markets, lenders need to screen borrowers to distinguish between safe and risky investment. Financial markets present the specificity of having low transaction costs and a large number of participants. Thus, in equilibrium, prices should reflect all information owned by every participant, as prices are selected vectors of information and should convey all available information.

Grossman and Stiglitz (1980) have shown the paradox arising from the information obtained through prices. If all information is already integrated in prices, then there is no need no get informed. Agents make an arbitration between the benefits of acquiring costly private information or relying only on free public information (i.e. prices here).To be efficient, a market would need no information costs nor transaction costs. As information is costly, it makes sense to entrust one (or few[4]) central agency to provide this information. That is the role of an "expert". Those experts can produce information at a lower cost than other agents would have (through a better technology). Usually, CRAs provide information to all agents, offering a scale economy.

The role of information was crucial in the last financial crisis. Systemic crises can arise from a small shock. Actually, in its 2011 report, the Financial Crisis Inquiry Commission revealed that "overall, for 2005 to 2007 vintage tranches of mortgage-backed securities originally rated triple-A, despite the mass of downgrades, only about 10% of Alt-A and 4% of subprimes securities had been materially impaired - meaning that losses were imminent or had already suffered-by the end of 2009 (But the crisis was large). As Bernanke noted, 12 of the most important financial institutions in the United States were at risk of failure within a period of a week or two".

One of the questions raised is how a small shock can lead to a systemic crisis. One element of the answer lies in the amount of credit in circulation. Financial crises are preceded by a credit boom. A second item lies in the asymmetry of information. At some point, risk taking incentives lead agents to trade despite the lack of information. When a small shock appears, credit worthiness of some institutions, quality of financial assets is "again" uncertain. Investors cannot distinguish good from bad borrowers. Trust in everything and in everyone is questioned. Information becomes anew crucial for trade, and as it has been missing for a period, the number of transactions plunged. The liquidity crisis can also become a solvency crisis for some fragile institutions.

[4]competition might be need to ensure a better quality and a race towards better technology, we will discuss this aspect in a later section.

Gorton and Ordonez (2014) provide a model which studies the production of information. The key element in their model is the perceived quality of the collateral. As information is costly, financial actors may rely on the average quality of commonly used collateral without more enquiries. After all, they are "all the same", there is no information that allows investors to distinguish between good and bad collateral. Thus, bad collateral can be used to raise funds leading to a credit boom and an increase of financial fragility. In other words, The lack of information drives up production and consumption but increases systemic risk by allowing firms that should not have access to funding to raise capital. In this equilibrium, a small shock can lead to a trust crisis. After a small shock, agents would like to know the quality of the collateral used. Firms with bad collateral cannot raise funds anymore, and some of them will fail, while others will reduce their activity's level to avoid revealing the nature of the collateral which is not as good as expected. The crisis has begun.

Through this model, Gorton and Ordonez (2014) explain the role played by an asset's complexity. From a firm's point of view, it is convenient to use collateral with a good reputation but complex enough so that other agents will not produce the effort needed to check its quality, which leads to an increase in information cost. This can explain the proliferation of MBS -mortgage back securities- used as collateral during the 2000's.

To illustrate their statement, the authors consider the case of firms which own, as collateral, land in a region reputed rich in oil. If the field is good (i.e. contains oil), it offers a return of C unit of capital, otherwise the return is null. It costs γ to verify the lands' quality. Thus agents have interest in acquiring information if and only if information costs are at least equal to the expected return:

$$p(qR_I + (1-q)x_I C - K) = \gamma$$

Where p is the probability than the field contains oil, and it can be extracted; q the project's probability of success; R_I the return in case of succes, x_I the share of collateral used in case of failure and K the capital raised.

However, there is a situation where a borrower can propose a return R_a and a share of collateral x_A such that investors do not have the incentive to check the quality of the collateral, and invest "blindly":

$$p(qR_a + (1-q)x_a C - K) < \gamma$$

In this case, the expected return must only cover the initial investment:

$$qR_a + (1-q)px_a C = K$$

Investment is made without guarantee about the quality of the collateral and the debt is risk-free for $R_a = x_a pC$. Leading to the following condition:

$$x_a = \frac{K}{pC} \leq 1$$

Investors have no incentives to acquire costly information when:

$$K < \frac{\gamma}{(1-p)(1-q)}$$

The authors exhibit two situations where investors will not check the quality of the collateral:

- The perceived probability of a good collateral, p, is high enough to lead investors to be confident about the collateral's quality as it seems unlikely to get a bad field. In this situation, firms raised important amounts and collateral's quality isn't verified;
- The perceived probability of a good collateral is this time very low. It seems unlikely to find oil in those fields. Firms adapt their behavior in this context by raising small amounts of capital. As returns are sufficiently low, there is no incentive to acquire costly information to evaluate the quality of the collateral. The amount that firms can raise is an increasing function of γ, as higher information costs allows them to ask for more capital.

Between these two extreme situations, there exist a range of probabilities about the perceived quality of the collateral where information is produced and collateral are checked. However, in a dynamic setting, the risk is to start from a situation where companies use good collateral, then move towards collateral of lowest quality along a higher level of complexity which increases information costs.

Financial fragility increases with the share of unchecked collateral in circulation. After a few macroeconomic shocks, a trust crisis appears (p decrease), a large share of capital will then be used to produce information. This information might be more negative than expected, reducing the number of firms able to raise capital and the systemic crisis arises. Otherwise, firms can choose to reduce their credit demands, thus no information is produced. In the short run, the crisis seems to be less severe, however investors stay in a situation where perceived quality of collateral is low. We witness a form of credit overhang where firms cannot recover.

The authors remind us that they agree with Rancière et al. (2008) about the fact that growth needs a certain level of risk taking and the occurrence of small crises. To obtain a high level of consumption, it is not wise to check the quality of every collateral just to gain investors' confidence. Actually, the authors show that a social planner will not necessary eliminate any form of financial fragility. It would be too costly to do so, even though the social planner would produce more information than private agents do. It makes a link with Credit rating agencies. The government can have incentives to push for the use of an entity spreading public information for all agents. Credit rating agencies seem to be a perfect match, however we will see in the next sections that information produce by those CRAs may not always meet the expected quality.

Gorton and Ordonez (2014) have illustrated how asymmetry of information can lead to different equilibriums and how precious information can be. Boot et al. (2006) study the impact of a credit rating agency on a financial markets where multiple equilibria would coexist without one. As rating announcement are considered as major news, a CRA, through its announcements, can help to coordinate agents and discipline a borrower and then obtain one equilibrium. The CRA's announcement represents here a focal point for the investors and the borrowers. It arises from two channels:

- The "Credit Watch procedure": It consists in a surveillance procedure after the grade has been published. The CRA can induce the firm's behavior through a threat of downgrading. The CRA plays here a disciplining role towards the borrower.
- Regulatory constraints: Major financial actors such as banks and pension funds must rely on ratings in their investment decision. Debts need to have a certain grade to be "eligible" for institutional investors, then ratings play a role of focal point.

To study this interaction, the authors consider the following model: A firm needs 1$ of capital

to finance a project. It can then choose between two types of projects, one called VP viable, and the other HR risky. The two types of projects have a net positive value (NPV) such that $NPV(VP) > NPV(HR) > 0$.

The credit worthiness of a project VP is defined through $\tilde{p}$ which represents its probability of success. In case of success, it offers a return X_VP. The ex-ante expected value of $\tilde{p}$ is p_0.

The probability of success evolves over time; at date $t = 1$, the firm receives a signal which affects its credit worthiness. The new quality can be good ($p = p_G$) or bad ($p = p_B$) with $p_B < p_0 < p_G$.

If the firm receives a bad signal, it can engage itself in a costly effort to increase the project's probability of success. With a probability β, the effort e is successful and it brings back the credit quality to p_0. Thus at date $t = 1$, the probability of success lies in $\tilde{p} \in (p_B, p_0, p_G)$.

The probability of success of an HR project is $q \in [0; 1]$ and offers a return of $X_{HR} > X_{VP}$. However, a priori, it is still better to undertake the safe project $p_B X_{VP} > qX_{HR}$.

The authors consider a competitive capital market. The zero-profit condition leads investors to expect a return of $F^{VP(\tilde{p})} = \frac{1}{\tilde{p}}$ when a project VP is undertaken, and $F^{HR} = \frac{1}{q}$ for the risky one. However, information about the type of project undertaken is not available to the investors, and they cannot take any actions to force the firm to choose the project they would like to. At date $t = 2$, cash flow are realized.

This setting can lead to 3 situations:

- When $\tilde{p} < \underline{p}$, the firm chooses the project HR;
- When $\underline{p} < \tilde{p} < \bar{p}$, the project's type chosen depends on investors' expectations, in this region there are multiple equilibria;
- When $\bar{p} < \tilde{p}$, the firm chooses the project VP.

We observe in the second region ($\underline{p} < \tilde{p} < \bar{p}$) that many equilibria coexist. To obtain a unique equilibrium, Investors should coordinate. Actually, if a sufficiently large proportion of investors (noted $\alpha(\tilde{p})$) expect the firm to undertake the viable project for a given credit quality ($\tilde{p}$), the firm will follow their expectation.

If we consider that a proportion α of investors expect the firm to select the project VP (they will ask for a return of F^{VP}) and the other investors expect the firm to select the risky project (asking for a return of F^{HR})). The firm should offer a payment of:

$$F^{\alpha} = \alpha F^{VP} + (1 - \alpha) F^{HR}$$

It is a decreasing function of α. When looking at the expected net return for each type of project: $q(X_{HR} - F^{\alpha}$ for the risky project and $\tilde{p}(X_{VP} - F^{\alpha}$ otherwise. The authors notice that when $\alpha > \alpha^{\star} = \alpha(\tilde{p})$ the firm always choose the VP project. The coordination of a sufficiently large proportion of investors can lead the firm to chooses the safe project, which leads us to the role of the credit rating agency.

In this paper, the authors consider the CRA not only through the information brought to the market, but rather through the focal point that ratings represent and the contractual relationship

that exists between a CRA and a borrower. Actually, after funds have been raised and debt is traded on the secondary market, the CRA can engage in a watching procedure which aims at potentially bringing new information to the market in a smooth way but also incite borrowers to undertake effort to enhance their credit quality.

The authors consider C the effort's cost, they also make the assumption that this one is profitable:

$$p_0 X_{VP} - c > p_B X_{HR}$$

The credit rating agency gives a rating that reflect the credit quality of the project undertaken $\rho(\tilde{p})$. Moreover, the credit watch procedure allows the CRA to confirm that an effort (or not) has been undertaken to enhance credit quality. The rating transmits this information to the investors. The signal send by the CRA (rating) creates a coordination among investors because of regulatory constraints. The CRA's action are positive in this economy as:

- The credit watch procedure drives more firms to undertake the effort than without such a procedure;
- The CRA gives a good rating to firms with a good credit quality and to the ones which have resorted to the effort when it was successful.

This paper presents CRA under a positive view, as its actions help to resolve a multiple equilibria issue and are beneficial from a social point of view. However, we will see that in the recent literature, ratings can also be biased signals and questions are raised about ratings' quality over the different type of products rated and over time.

2.2 Reputation, Asset Complexity, Business Cycles and Rating's Quality

In this section, we will discuss the different critiques raised over ratings' quality. After the subprime crisis, the CRA's business model (issuer-pay) has been questioned. CRAs defend it with the following argument: as their influence over the financial market is based on their reputation, they cannot engage themselves in any devious behavior. We will study how reputation can or cannot discipline a CRA through Mathis et al. (2009). Moreover, the question of reputation is even richer than expected with the CRAs. Because of repeated interactions with a few issuers, we will discuss the possibility for a CRA to develop a double reputation: one with the issuers, and one with the investors, with Frenkel (2015).

In order to address those questions, we will need the tools developed by the information transmission literature. The CRAs play the role of an expert which advise an investor or the whole market[5]. The communication between an expert and its audience brings up questions about the reliability of the sender. In Crawford and Sobel (1982), the authors propose an illustration of communication between an informed adviser and a decision taker. They study the situation where a receiver is not sure of the sender's preferences. In a setting where the actions taken by the receiver affect

[5]Depending on the business model, the information can be private or public.

the welfare of both, they show that communication in equilibrium leads to a greater utility for each agent when preferences are similar. Sobel (1985) proposes a similar framework but raises the question of credibility. In this paper, the author shows that long-term relationship and reputation can act as binding contracts. Reputation plays the role of a discipline device in this setting, leading the sender to send accurate signals. This framework would be helpful to understand CRAs' behavior and question their business model, as we will see in Mathis et al. (2009).

Conflict of interest through information owned on financial markets is not only reserved to Credit rating agencies. In Benabou and Laroque (1992), the authors, through a cheap-talk game framework, analyse the behavior of an insider who can exploit private information to manipulate a financial market. They show that reputation incentives are not sufficient to ensure a honest behavior. An informed agent who can invest is tempted to send signals opposite to the actions he would undertake (a journalist with great influence could advise to sell when he will buy). We will first discuss how this could be applied to CRAs.

To go further, we will also take into account an asset's complexity, as that has played a role in the subprime crisis. Bolton et al. (2012) takes into consideration this aspect. They also study the question of competition. As the rating industry is lead by an oligopoly, criticism has been raised about the lack of competition. With their paper, we will get to understand the role played by those elements in ratings' quality.

Then, if we look at ratings' quality over the last decades, we witnessed some quality changes over time. It seems that rating's accuracy vary over the business cycles. Bar-Isaac and Shapiro (2013) provide a model where a CRA must choose the quality of its ratings in a boom and in a recession. We will see how economic conditions can affect its behavior.

A. Reputation and Business Model

1- Conflict of Interest and Reputation

At first, CRAs' work was paid by investors. Investors were used to subscribe to a publication in order to obtain precious information about a set of companies. As noted earlier, the development of information technology created a free-riding opportunity, and along with the evolution of financial regulation, it drove the CRAs to change their business model. During the 70's, CRAs turned towards an issuer-pay business model. An issuer who wishes to obtain a rating must pay the CRA although the grade can be bad. However, the CRA's payment is somehow contingent upon the rating's quality, as the rating's fees will increase with the amount raised (which in turn depend on the grade) and the CRA receives yearly payment as long as it updates its ratings[6].

Mathis et al. (2009) analyse in a cheap-talk game framework the strategic behavior of a monopolistic CRA. It allows us to understand in which situation reputation might not be sufficient to ensure honest transmission of information from a CRA.

The model is built as follows: at each period, a firm aims at raising 1\$ of capital to finance a project. The project can be either good (with probability λ) or bad (with probability $(1-\lambda)$). Good projects offer a return of X when successful, whereas bad projects always fail, thus only

[6]The first payment consists at covering analysts and audit fees, whereas a second part is contingent with the issue and the amount raised, and the last part accounts for surveillance

good project should be financed. They consider that without information no financing takes place $(1-\lambda)\times 0+\lambda\times X=\lambda X<1$. Thus the firm (the issuer) must solicit a CRA to certify the quality of its project. The CRA sends this information to the investors through a good or a bad rating[7].

The CRA contributes to the efficiency of financial markets by sending information to the investors. However the CRA is a profit-maximizing entity and the goal of the article is to study the strategic behavior of an opportunistic CRA. As in most models of reputation, the CRA can be of two types, it can be honest and transmit its true information, or opportunistic and may distort information (by giving a rating different from its private signal) in order to maximize its profit. The belief about the CRA's type represents the reputation of the CRA, it is denoted q_t for the period t. q_t evolves through time. Investors observe ratings, then the success or failure of the project (if financed, that is if well rated) and then update CRA's reputation following Bayes' rules. An opportunistic CRA may give a good rating to a bad project, this strategy would be noted $x(q_t)$, $x(q_t)$ being the probability of assigning a good rating to a bad project at date t when the reputation is q_t.

Then the probability of a successful project given a good rating is:

$$p(q_t)=\frac{\lambda}{\lambda+(1-\lambda)(1-q_t)x(q_t)}$$

When $x(q_t)=0$ Information is perfectly revealed. All CRA gives a rating that corresponds to their perfect private information.

The decision tree is the following:

The zero-profit condition leads Investors to asks for an expected return such that:

$$p(q_t)R(q_t)=1$$

Which gives us the expected return:

$$R(q_t)=\frac{\lambda+(1-\lambda)(1-q_t)x(q_t)}{\lambda}$$

As long as $R(q_t)<R$, the rating provides information, and although there are doubts about the CRAs' type, in average it is an informative signal. When q_t falls to zero, the agency no longer exists.

The payment of the CRA represents the conflict of interest discussed above. Payments are contingent upon a positive rating; in case of a bad rating, the project is not financed and rating's fees are not paid. Moreover, revenues depends on reputation. The more credible are its announcements, the more a CRA can bill its clients. Those assumptions relfect the practice, especially concerning structured finance. Such a financial product would be issue only if a CRA certificates its quality.

The payment takes this form: $\Pi(q_t)=\mathbb{1}_{q_t>q_l}i_0+I(q_t)$ Where $I(q_t)$ is a convex function which increase with the CRA's reputation and i_0 represents the revenues the CRA gets from other sources as long as it is alive ($q_t>q_l$).

[7]This is a simplification, obviously there are more than 2 possible grades without taking account the outlook published however it can be seen as a representation of the distinction between investment and speculative grades

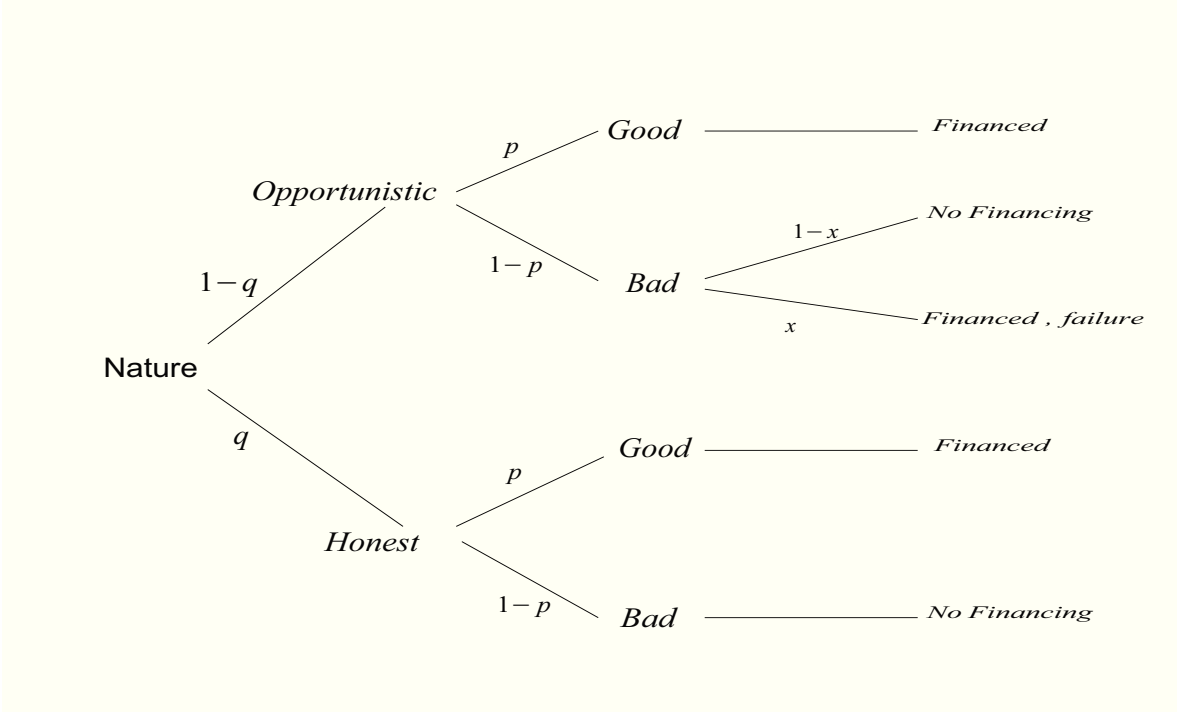

Figure 2.2: Decision's tree

Reputation evolves following Bayes' rules:

$$q_{t+1} = \begin{cases} q_t & \text{When a project with a good rating succeed} \\ 0 & \text{If a project with a good rating fails} \\ \frac{q_t}{1-(1-q_t)x(q_t)} > q_t & \text{In case of a bad rating} \end{cases}$$

The opportunistic agency must maximize its intertemporal profit. It must define its strategy for each type of project it may encounter:

- With a probability λ, the project is good, so the CRA has interest in giving a good rating to enjoy payments;
- With a probability $1 - \lambda$ the project is bad, so the opportunistic CRA faces a choice:
  - If it gives a bad rating, it will not receive the payment of a good rating $I(q)$
  - With a good rating, it receives a short term profit, but as the project will fail, the CRA will loose all future revenues.

The Results:
In equilibrium, if the agency is sufficiently patient or if its revenues from other sources are important then the opportunistic CRA would never lie. However when its reputation is high enough and its preference for short term revenue is strong (its discount factor is low), incentives to lie increase.

Moreover, if we consider a setting with an other level of imperfect information, results can be even stronger: if we consider that bad project may also succeed and good project may fail, this will give more room for an opportunistic CRA to lie. Investors cannot distinguish lies from honest mistakes, and the CRA enjoy this situation. This fits with the behavior observed in the last financial crisis. The first subprimes ratings were accurate, however as time goes, ratings' quality decreased as shown in Ashcraft et al. (2010). Those results are supported by empiric studies such as Han et al. (2012) and Jeffery D'Amato (2003).

2- Double Reputation

As shown by Mathis et al. (2009), reputation is not sufficient to discipline a CRA. The cheap-talk literature has shown that in such communication games, a strategic agent has sometimes incentives to lie. In this literature, the interaction is between a sender and a receiver. For the credit rating agencies, there are two different audiences: the investors and the issuers. As the informational set of those two types of agents are different, reputation can be different for each point of view. Moreover one of the specificities in the CRA's activity is that is has repeated interaction with the same issuer. The issuers have the tendency to request a grade from the same CRA when they first got a good rating. This repeated interaction was noticed in the MBS market, where 80% of MBS were issued by 12 financial institutions.

Frenkel (2015) studies this situation. The author considers a setting with repeated interactions between one issuer and a CRA. It allows the CRA to develop a "double reputation", one with the issuer, one with the investors. The model is the following. He considers an economy endowed with: an investor (b), an issuer (s) and one CRA. At each period, the issuer aims at selling its financial product to the investor. However, the quality of the product is unknown to the investors and if the seller knows it, he cannot communicate it to the investors for obvious credibility reason. But, he can hire a CRA to certify its product. The financial product can be either good ($a = G$) or bad ($a = B$) with probability $\frac{1}{2}$. It gives a return of R with certainty for the good products and with probability π for the bad ones which also gives a negative return $-l < -1$ in case of failure with probability $1 - \pi$. Without certification, the expected return of the product is: $E(R) = 0,5(1 - l) < 0$.

A CRA can be hired and fees w are paid upfront. Here, they are not contingent upon a good rating. The CRA can be of two types $\theta \in (C, S)$. It can be corrupted ($\theta = C$) which only gives good ratings. It can also be strategic ($\theta = S$), in this case it may also assign a good rating to a bad product in order to maximize fees in the next period. This strategy is defined by $x = Pr(r = G|a = B)$. CRA's reputation is defined as $\mu = Pr(\theta = S)$, the probability of being of the strategic type.

At the beginning of the game, the CRA's type θ is drawn by nature as well as its reputation μ_0. The author considers a two period model. During a period, the sequence of actions is the following:

1- Nature draws the quality of the financial product;

2- Issuer and investors agree on ratings' fee ω ;

3- Issuer and the CRA perfectly observe the product's quality;

4- CRA publishes a rating $r \in (B, G)$;

5- The investors buy the products at price $p > 0$ or decline;

6- Pay-offs are realized and observed by all agents;

7- CRA's reputation is updated.

The author considers that the CRA can extract all the surplus from the information delivered through its rating, thus $w = E(p)$ which depends on the expected rating. He also considers in this game that the issuer is the same over the two periods (in order to reflect the situation of the MBS market). It should be noted that the issuer has access to more information than the investor regarding the CRA's type as a bad project can succeed. Thus we consider μ^b the CRA's public reputation (from the investor's point of view) and μ^s the reputation from the issuer's point of view (the "hidden" reputation).

Obviously, at the start of the game $\mu = \mu^s = \mu^b$. But after the first period, the two reputations may differ. Actually, if the CRA gives a good rating to a bad project, two outcomes are possible:

If the bad project fails, the CRA's reputation is common for the investors and the issuer;$\hat{\mu}^b = \hat{\mu}^s = \dfrac{\mu x}{1-\mu(1-x)} < \mu$;

If the bad project succeed, the CRA's reputations are different :$\hat{\mu}^s = \dfrac{\mu x}{1-\mu(1-x)} < \mu$ and $\hat{\mu}^b = \dfrac{\mu(1+\pi x}{1+\pi(1-\mu(1-x))}$.

We see that if a bad project succeeds, the CRA develops a "double reputation".

Equilibrium

In the last period, reputation (any of the two) plays no role in CRA's action. The author considers, that the CRA always gives a rating that reflects the true quality of the financial product. p^r is the product's price for a rating r. A bad rating means a bad project thus $p^B = 0$. At the second stage of the game we obtain the following price:

$$p^G = \begin{cases} \dfrac{1-(1-\mu^b)l}{2-\mu^b} \\ \\ 0 \end{cases}$$

$\hat{w}$ being the rating fees, the issuer expects a pay-off $p - \hat{w}$. The CRA extracts all the surplus and can charge at this period $\hat{w} = E(p)$. Issuer expect a good rating with probability $1 - 0.5\hat{\mu}^s$ then:

$$\hat{w} \equiv E(p|\hat{\mu}^s, \hat{\mu}^b) = \frac{2-\hat{\mu}^s}{2-\hat{\mu}^b}\frac{1-(1-\mu^b)l}{2}$$

We notice that fees are an increasing function of the hidden reputation (issuer's point of view) and decreasing function of the public reputation.

The Results:

- With a different issuer at each date, the CRA cannot develop a double reputation. In this setting, reputation is always the same for both audiences, and the CRA has no incentive to lie. There is no ratings' inflation;

- With repeated interaction, and if the prior reputation of the CRA is high enough as well as the bad projects' probability of success; The CRA has an incentive to give a good rating to a bad project in order to develop a double reputation.

The question of double reputation has also been treated by Bouvard and Levy (2013). This paper has highlighted the issue raised when a CRA and an issuer develop a close business relationship through repeated interactions. When this happens, the public reputation is not powerful enough to discipline the CRA. The MBS markets fits well with this model, however different features of the rating industry may have also play a role in the dynamic of ratings' quality in the last financial crisis. The next section will study the impact of complexity and competition on a CRA's behavior.

B. Competition, Complexity and Rating's Quality

Critics of CRAs have also pointed out: (i) the lack of competition; (ii) the poor performance of structured finance ratings. We will discuss those aspects in this section. In the previous section, we have considered a monopolistic CRA. This assumption seems understandable as the rating industry is run by an oligopoly. Thus, the question of competition was also raised after the subprime crisis. Common thoughts lead one to think that the lack of competition could explain the poor quality of ratings. However, it has been observed that competition can lead to a race to the bottom, as expressed in an empiric paper by Becker and Milbourn (2008). The authors have shown that the entry of Fitch on the corporate markets has led towards a deterioration of the average ratings' quality. More competition leads to more good ratings and a lower average quality than in a monopoly. Besides the lack of competition, the naive behavior of investors which have been too trustful with ratings, has also been highlighted. To understand this phenomenon, Bolton et al. (2012) study the consequence of competition in a financial market with naive investors. Their model relies mainly on the following specificities:

- The issuer-pay business model and its conflict of interest;

- The possibility for an issuer to ask a different CRA for a rating, in order to get only the most favorable rating published. This practice is known by the name "rating shopping";

- A share of investors are naive.

The goal of this paper is to identify the conditions leading to rating inflation and to study the impact of competition on ratings.

Monopoly vs Duopoly: The authors consider an economy made of three types of risk-neutral agents: issuers, CRAs and a continuum of investors. A financial project can be good $\omega = g$, or bad $\omega = b$. Bad projects are characterized by their default probability $p > 0$, whereas good projects never default. When successful, those investment produce a cash-flow of $R > 0$ and 0 in case of failure.

A priori, a project is good with probability $\frac{1}{2}$. This uncertainty creates a need for information, a CRA owns a technology that allows it to identify the project's type. Its private signal $\theta \in \{g, b\}$ is imperfect as:

$$P(\theta = g|\omega = g) = P(\theta = b|\omega = b) = e$$

e represents the quality of the CRA's private information. The authors make the assumption that $e > \frac{1}{2}$ (the signal is informative), e is common knowledge. Rating's fees are ϕ; A rating is a public signal, it can be good $m = G$ or bad $m = B$. The issuer can decide to get the rating published and pay the fees or he can refuse. After the publication of the rating or its absence, the issuer defines the price of its product T.

The authors consider a financial market with two types of investors; a proportion $1-\alpha$ is *sophisticated* when the other share is *confident*. The latter consider that a good rating is always associated with a good project. Without a rating, *confident* investors keep their prior whereas the *sophisticated* ones consider the absence of a rating as information.

CRA cares about reputation, thus when a good rated project defaults, the "lie" is discovered and the CRA is punished; its future messages are ignored. The cost of reputation is noted ρ.

The model relies on the following assumptions:

- There is uncertainty about the CRA's punishment when a lie is observed $\rho \in [\tilde{\rho} - \varepsilon, \tilde{\rho} + \varepsilon]$; from a technical point of view, it leads to restrict the CRA's action to pure strategy.
- Investors can buy one or two units of a project. Their reservation utility grows with the level of investment which leads them to expect a higher return if they buy a second unit of the same project. Their reservation utility is noted u for the first unit, and U for the second with$u < U$.
- Returns follow the following rules:
  - $(1-p)R > u$, when an investor knows an investment is bad, he will buy one unit;
  - $(1-(1-e)p)R > U$, if the available information is trustful, and a good project is identified, the investor is willing to buy two units of the project;
  - $(1-\frac{p}{2})R < U$, when the investor is unsure about the quality of the project, he will not buy two units.

The marginal value of a project rated G, B or not rated (O) when the CRA is honest from the sophisticated investors point of view (and in any circumstances for the naive ones) are $V^G = (1-(1-e)p)R - U$, $V^B = (1-(1-e)p)R - U$, $V^0 = (1-(1-e)p)R - U$.

With a single CRA:

The timing of actions is the following:

1- Ratings fees are defined ϕ;

2- The CRA receives its private information and gives a rating $m \in (G, B)$;

3 - The issuer observes the rating and decides to pay for publication or to decline the offer; he put a price T for one unit of the project;

4- Investors observe the price T of the project, and its grade m, if published, and choose the number of units they want to buy;

5- The Return is realized.

The arbitration between reputation incentives and ratings fees play a central role as revenues depend upon good ratings.

In equilibrium, fees obey the following rules:

- If $\alpha 2V^G - V^0 > ep\rho$ There is overrating and ratings' fees are equal to $\phi = \alpha 2V^G - V^0$. The CRA's profit is: $\alpha 2V^G - V^0 + \frac{(1-ep)}{2}\rho$

- If $\alpha 2V^G - V^0 < ep\rho$ the CRA gives a rating corresponding to its private information and charges:$\phi = min[2V^G - max[\alpha V^0, V^B] - 2V^0, ep\rho]$. The CRA gets a pay-off: $\frac{1}{2}min[2V^G - max[\alpha V^0, V^B] - 2V^0, ep\rho] + \rho$.

When $\phi > ep\rho$ there is over-rating, The CRA only gives good rating; Otherwise, the CRA transmits truthfully its private information through the grade. The CRA still continues to compare profit from rating's inflation ($\alpha 2V^G - V^0$) and reputation costs $ep\rho$. We notice that inflated ratings appear when reputation costs(ρ) are low and the proportion of *confident* investors is high. It appears clearly in $\alpha 2V^G - V^0$.

Duopoly To test the effect of competition, the authors introduce a second CRA, timing of action is similar to the monopoly situation:

1- Each CRA define its ratings' fees ϕ_k, with $k = 1, 2$;

2- Each CRA receive a private signal and gives a rating, $m = G$ ou $m = B$;

3- The issuer observes both ratings, and can decide to reject them, or to pay for the publication of one or of the two; he put a price T for one unit of the project;

4- Investors observe the price T of the financial products, and ratings if published. They choose how many units of the financial products they want to buy;

5- The return is realized.

The author defines V^{GG} et V^{BB} the value of the financial product when both CRAs report honestly their private signal from the point of view of the sophisticated investors. As in the monopoly situation, it is same valuation for the naive investors, whatever is the behavior of the CRA. $V^{GG} = (1 - \frac{(1-e)^2}{(1-e)^2 + e^2}p)R - U$

$V^{BB} = (1 - \frac{(1-e)^2}{(1-e)^2 + e^2}p)R - U$

By assumption, The marginal value of a second rating is lower than the first one. It appears in the following equation: $\alpha 2V^G - V^0 > 2(V^{GG} - V^G)$

When $\phi_k > ep\rho^D$ there is rating inflation, otherwise the CRA gives honestly its ratings. It shows

the importance of the "punishment" to discipline a CRA.

Ratings' fee obey to the following rule:

- If $\alpha 2(V^{GG} - VG) > ep\rho^D$; the two CRAs only report good ratings, and fees are equal to: $\phi_k = \alpha 2(V^{GG} - VG)$ their pay-off is $\alpha 2(V^{GG} - VG) + (1 - \frac{ep}{2})\rho^D)$;

- If $\alpha 2(V^{GG} - VG) < ep\rho^D$, The two CRA transmits their true private information through ratings;

  (a) If $\alpha \in (\frac{\frac{3}{2}V^0}{2V^G}, 1)$, the issuer solicit both CRA, each one charging $\phi > 0$

  (b) IF $\alpha \in (\frac{V^0}{2V^G}, \frac{\frac{3}{2}V^0}{2V^G})$, the issuer solicit only one CRA and fees are null.

Once again, the CRAs' strategy represent an arbitration between gains from rating's inflation and reputational cost. We also notice that "rating shopping" occurs more often when the CRA's private signal is noisy. The less accurate CRA's private information is, the more an issuer will ask different CRAs hoping for a good rating.

This model allows us to analyse four situations:

- A monopoly with truth-telling;

- A monopoly with inflated ratings;

- A duopoly with truth-telling;

- A duopoly with inflated ratings;

The question here is what situation is better in terms of social welfare. We observe that the situation in *monopoly with truth-telling* dominates the *duopoly with truth-telling*. This comes from rating shopping; as information is imperfect an issuer will challenge the two CRAs hoping for a good rating. This leads us to question the critics over competition in the rating industry. The model shows that competition is not a key element to enhance ratings' quality. The model highlights two elements that seem to have a stronger impact:

- The asset's complexity can lead firms to practice ratings shopping;

- To discipline a CRA through reputation, one needs to have a strong punishment.

C. Ratings and Business Cycles

The last financial crisis enlighted the varying quality of ratings. If we consider the subprime crisis, ratings were accurate at first, before a degradation appeared. A similar path was also witnessed before the East Asian crisis, when CRAs were blamed for having given too generous ratings before the baht's crisis and to have been excessively severe afterwards. Explaining the difference of rating's quality over time is one of the goal of Bar-Isaac and Shapiro (2013). They consider an "honest"

CRA, they show that because of the economic conditions, its ratings would be more accurate in a recession than in a boom. The issue raised here is about the means and not about incentives to lie as seen in the previous paper.

The quality of ratings seemed to follow a pattern during the last financial crisis. At the peak of the bubble, ratings were of poor quality concerning MBS. However before 2004, such products were given accurate ratings while throughout the boom we've witness a deterioration, as seen in Ashcraft et al. (2010).

Throughout this approach, the question isn't about honesty of ratings but rather about the effort put in place (i.e. the number of products rated by an analyst over a time lapse). To study this question, The authors consider an economy which can be in two states (in a boom or in a recession). In a boom, analysts are more expensive than in a recession. As the labour market is under much more tension, and attracting a good analyst can be difficult. To attract those talents, the CRA must propose the best salary, knowing that:

- When considering profit maximizing, competence is expensive;
- Rating quality follows analysts' competence. The risk of giving a good rating to a bad product decreases;
- Better analysts reduce the turn-over as they do not give good ratings to bad products leading to a decline in CRA's profit.

The CRA must define its strategy in each state (boom and recession) and the corresponding probabilities of transition. It aims at maximizing its intertemporal profit. Hiring good analysts leads to save CRA's reputation and also ensure future revenues by surviving.

In a boom period, wages increase, and hiring a good analyst is more costly than in a recession. Moreover, the risk of being wrong (to give a good rating to a bad product) is lower than in a recession as it is more likely for any type of project to succeed. During a boom, there are few chances that the lack of competence of a CRA would be revealed. In equilibrium, the CRA has incentive to be more accurate in a recession than in a boom when the probability of being in a boom or in a recession in the next period are equal. It should be noted that the level of ratings' quality depends on the transition's probabilities from one state to another. A longer boom period tends to discipline a CRA as future revenues play this role. But lasting recession will not give incentives for a CRA to hire good analysts.

Ratings quality is contra-cyclic: CRA tends to be accurate in a recession and lax in a boom. It explains the difficulty for CRA to identify turning points. If we consider that ratings have an influence on transitions's probabilities through self-fulfilling prophecy for example, we could say that CRAs amplify the business cycles.

Actually, those findings are supported by empiric papers. Jeffery D'Amato (2003) questions if ratings are procyclical. He finds that ratings seem to amplify economic cycles as ratings are more generous in a boom than in a recession. His analysis focuses on the corporate market in the US over two decades. When looking at the East Asian crisis, we find similar ratings quality patterns for countries' grade, as shown by Ferri (1999).

2.3 Expert's Decision, Regulation and Public Information

In this section, we will study three aspects of the ratings' activity:

- As experts' decisions, ratings can also be "honestly" biased;
- Regulation gave a central role to ratings; how this affect CRA's behavior;
- By turning towards an issuer-pay business model, ratings became public information. What are the consequences of this new "nature" of ratings?

A. Expert's Behavior and Reputation

Mariano (2012) studies the question of reputation not through the conflict of interest arising from the issuer-pay business model, but rather through the question of an expert's reputation and his behavior. As for every expert, his reputation and the perceived accuracy of his ratings depend on public expectations and the amount of available public information. The author considers first a model where a CRA aims at looking competent in a monopoly and then with potential entrants.

In a monopoly :

We consider a two-period model, with dates $t = 0, 1$, and a risk-neutral CRA. At date O, there is one project which can be either good (G) or bad (B). When it is a good project, the return is positive, it is negative otherwise. A priori, project's type is unknown but there is a public belief θ, the probability that the project is good.

The CRA must give a rating to the project, and its objective is to maximize its reputation at period 1. It receives an imperfect private signal $s \in s_G, s_B$; s_G for a good project, s_B for a bad one. As in standard reputation games, the CRA can be of two types: it can be an high type $a = H$ or a low type $a = L$. A H type CRA identifies perfectly a project's type whereas an L type received imperfect information. The CRA knows its type, the other players have a prior $\alpha_0 \in (\frac{1}{2}; 1)$ which represents the probability of facing an H type. This reputation is updated after the first period and is then α_1.

A L CRA owns the following technology: $Pr(s_G|G, L) = Pr(s_G|B, L) = Pr(s_B|G, L) = Pr(s_B|B, L) = \frac{1}{2}$.

The CRA defines its rating policy by taking into account its type a, its reputation α, the prior θ and its private signal : $Pr(r_G|s_B, \theta, a, \alpha)$ and $Pr(r_B|s_g, \theta, a, \alpha)$. It should be noted that ratings have an impact on financing costs. Thus a bad rating to a good project lowers its probability of success. The author considers that a good project with a bad rating gives a positive return with probability $1 - \beta$. This goes along with the fact that, when a good project fails, the investors cannot identify with certainty the project's type.

Posterior belief:

The outcome of a project is observable (success S or failure F) and leads to an update in CRA's reputation. When a well rated project succeeds (fails), reputation will be noted $\alpha_1(r_G, S)$ ($\alpha_1(r_G, F)$). When a bad rated project succeeds (fails), reputation will be noted $\alpha_1(r_B, S)$ ($\alpha_1(r_B, F)$).

Equilibrium:

The question raised in this section is to understand when a CRA has incentives to contradict its private signal. At date 0, if the CRA follows its private signal and gives a good rating, its expected reputation is: $Pr(S|s_g,a)\alpha(S,r_G)+Pr(F|s_g,a)\alpha(F,r_G)$. From this equation, we see the arbitration the CRA must do: It must maximize its reputation knowing that the public's prior will influence its update. When the CRA receives a good private signal about the project's type, it will publish the rating that maximizes its revenue.

- When $Pr(S|s_G,a)\alpha(S,r_G)+Pr(F|s_G,a)\alpha(F,r_G)-Pr(S|s_G,a)\alpha(S,r_B)+Pr(F|s_G,a)\alpha(F,r_B)>0$, the CRA has incentives to follow its private signal, thus it will give a good rating.
- When $Pr(S|s_G,a)\alpha(S,r_G)+Pr(F|s_G,a)\alpha(F,r_G)-Pr(S|s_G,a)\alpha(S,r_B)+Pr(F|s_G,a)\alpha(F,r_B)<0$, the CRA will give a bad rating although it has received a positive private signal.

When the CRA receives a bad signal:

- When $Pr(S|s_g,a)\alpha(S,r_G)+Pr(F|s_g,a)\alpha(F,r_G)-Pr(S|s_g,a)\alpha(S,r_B)+Pr(F|s_B,a)\alpha(F,r_B)>0$, the CRA has incentives to follow its private signal, it will give a bad rating.
- When $Pr(S|s_g,a)\alpha(S,r_G)+Pr(F|s_g,a)\alpha(F,r_G)-Pr(S|s_g,a)\alpha(S,r_B)+Pr(F|s_B,a)\alpha(F,r_B)>0$, the CRA will give a good rating although it has received a negative private signal.

The probabilities for a L type to contradict its private signal in equilibrium are: $Pr(r_G|s_B,\theta,L)\equiv\overline{\gamma}$ and $Pr(r_B|s_G,\theta,L)\equiv\underline{\gamma}$

We can then exhibit the behavior of a CRA in this economy. First of all, an H type CRA will always follow its private signal, because its information is perfect. The L type CRA might receive a wrong signal, it aims at maximizing its reputation and thus its behavior would be partly driven by the public's expectations. In equilibrium, there are two threshold $\overline{\theta}$ and $\underline{\theta}$:

- When $\theta\in(\overline{\theta},1)$, The L type CRA gives always a good rating;
- When $\theta\in(0,\underline{\theta})$, The L type CRA always gives a bad rating
- When $\theta\in(\underline{\theta},\overline{\theta})$, then :
  - With a positive private signal, it gives a bad rating with probability $\underline{\gamma}$ and a good rating otherwise;
  - With a negative private signal, it gives a good rating with probability $\overline{\gamma}$ and a bad rating otherwise;

Competition

Now, the author considers a similar economy with potential entrants. A CRA i is already active and another group of new CRAs (j) may enter into competition. There is no information about the new entrant's type: $\alpha_{0,j}=\dfrac{1}{2}$.

Reputation is central in this model. The incumbent aims at maximizing its reputation to protect its share. Whereas potential entrants wish also to maximize their reputation to get in and distinguish themselves.

Two situations must be considered:

- **With one rating only:** In this case, entrants might play only once the incumbent's CRA has failed.

- **With two ratings:** In this case, once the CRA i has given a bad rating, a type j CRA is drawn and can propose an other rating, even though, because of its low reputation, its ratings are not very influential.

We observed the following results:

- **With one rating only:** as in the last section, the type H CRA always follows its private signal. When considering an incumbent CRA of type L, it must take into account the threat of a potential entrant. It will follow its private signal more often than in a monopoly. The fear of an entrant leads the CRA to follow its private signal.

- **With two ratings:** First, a i CRA of type H never fails, thus a project with a bad rating succeeding indicates a type L CRA. As after a bad grade, a potential entrant can give another rating to the project, the incumbent CRA has new incentives, leading it to give more good rating than in an equilibrium.

To conclude, public information tends to lead a CRA not to rely on its private information because of the reputation incentive. The public expectation drives the CRA's behavior. Competition does not seem to have a beneficial impact for this situation as it could lead to ratings inflation. A similar approach was made by Ottaviani and Sorensen (2002). The authors propose a study of an expert's behavior. They show how one expert's message can influence another expert's decision. It is relevant to see CRA as expert, as they suffer the same "wrong" incentives than any other. Their ratings might be influenced by the ratings of other agencies or even by other disclosure of public information.

B. Unsolicited Ratings and Regulation

In this section, we will study the practice of unsolicited ratings. Fulghieri et al. (2014) studies this practice where financial regulation is also considered. Unsolicited ratings can be a useful tool for a CRA: those ratings can be excessively severe towards an issuer in order to (i) incite issuers to ask for a rating (ii) show investors that it can deliver bad ratings. CRAs argue that this practice is needed to ensure a broader coverage of the financial markets.

CRAs do not receive any payment for unsolicited ratings. This practice raises questions. Econometric studies have shown than on average, these ratings are lower than the solicited ones. They argue that they do so in an effort to provide better information to the market. Without remuneration, they will evaluate the creditworthiness of debt only through available information, usually public information. Fulghieri et al. (2014) define a dynamic setting with rational expectations which encompass three specificities of CRAs' activity:

- They can lie: give a good rating to a bad project;

- They can give a grade without any request;

- Reputation incentives are present.

The economy is composed of three risk neutral players: firms, investors and one CRA. The game is made of two periods. At the beginning of each period, the firm can have access to a project with a probability β. This project needs I unit of capital to be engaged. The firms cannot finance the project by itself and must raise capital from the market. The project offers a return of R if successful ($\omega = S$) otherwise it gives zero ($\omega = F$). The outcome of each project is public.

Projects can be good ($\theta = G$) or bad ($\theta = B$), a good project succeeds with probability q, a bad project always fails. If a firm does not undertake a project, it looses the opportunity and becomes worthless. Firms without a project are defined by $\theta = N$. Without a project a firm does not raise capital and is worth $\bar{V} > 0$. On average, firms have access to good projects and their value exceeds the one of firms without a project: $\alpha qR - I > \bar{V} > 0$.

Each firm knows the quality of its project but cannot communicate it to the market. The asymmetry of information is reduced by the presence of a CRA. The CRA owns a technology that allow itself to perfectly identify the project's type if solicited by the firm, if not the information is imperfect and it identifies the project's type with probability $\delta \in (1/2; 1)$. This difference reflects the fact that unsolicited ratings are based on public information, whereas in the case of solicited rating the CRA can rely on public and private information.

From this information, the CRA can propose a good $r = H$ or a bad $r = L$ rating for a fee ϕ^r. $\mathcal{F}$ defines the information available to the credit rating agency. The rating policy of a credit rating agency is defined as $\{r, \phi^r_{\mathcal{F}}\}$. If the firm accepts the proposal, fees are paid and the rating is published. If not, fees are not paid, and the CRA can choose to publish or not an unsolicited rating (which can be different than the one from the proposal). The rating has an impact on the firm's valuation, however the magnitude of this impact depends on the reputation of the CRA.

The CRA can be of two types:

- $\tau = e$, The CRA is ethical, its signal represents its true information;

- $\tau = o$, The CRA is opportunistic, it defines a strategy to maximise profit.

At the beginning of the first period, investors have a prior μ_1 that the CRA is of type e (μ represents the reputation of the CRA). A firm's objective is to maximize its value, whereas an opportunistic CRA's objective is to maximize its profit.

1- A firm, with a project, chooses or not to solicit the CRA;

2- Next to the firm's choice, the CRA gather available information;

3- The CRA makes an offer $\{r, \phi^r_{\mathcal{F}}\}$;

4- If the proposal is accepted, the rating is published, if not the CRA can decide wether or not to publish any rating;

5- CRA's reputation is updated next to the publication (or the absence) of a rating, firm's valuation is updated too;

6- If the project is financed, investors observe the outcome and update the CRA's reputation.

Strategy of a CRA with Unsolicited Ratings:

By assumption, the ethical CRA does not publish any unsolicited rating. We consider the case of an opportunistic CRA. The rating policy of the CRA is defined by $\{\hat{p}^r_{F,t}, \hat{\phi}^r_{F,t}, \hat{u}^r_{F,t}\}$, where $\hat{p}^r_{F,t}$ denotes the probability of giving a rating $r \in \{H, L\}$, $\hat{\phi}^r_{F,t}$ the rating' fees and $\hat{u}^r_{F,t}$ the probability of giving an unsolicited rating.

We consider only the situation where firms never pay for a bad rating. All firms with a good project ask for a rating, and pay $\hat{\phi}^H_{F,t} = \hat{\phi}_t$, they get a good rating $\hat{p}^H_{F,t} = 1$ (thus $\hat{u}^H = 0$). An ethical CRA($\tau = e$), when the firms refuses the rating's proposal, does not published the unsolicited rating. The difference with the situation without unsolicited rating is that firms with a bad rating cannot always group themselves with unrated firms, there is a credible threat of a bad rating.

After a bad rating ($r = L$) CRA's reputation is updated:

$$\hat{\mu}^l_t \equiv prob[\tau = e | r_t = l] = \frac{\mu_t \Lambda^B_t}{\mu_t \Lambda^B_t + (1-\mu_t)\Lambda^B_t(1-\tilde{p}_t)\tilde{u}^B_t}$$

Where $(\tilde{p}_t, \tilde{u}_t)$ represents the investors' conjecture over the CRA's strategy$(\hat{p}_t, \hat{u}_t)$ and $\Lambda^B_t = (1-\alpha)\beta(\tilde{\lambda}_t + (1-\tilde{\lambda}_t\delta)$ represents the proportion of firms identified as type $\theta = B$ by the CRA:

$$\hat{\mu}^{\emptyset}_t \equiv prob[\tau = e | r_t = \emptyset] = \frac{\mu_t \Lambda^N_t}{\mu_t \Lambda^N_t + (1-\mu_t)\Lambda^N_t + \Lambda^B_t(1-\tilde{p}_t)(1-\tilde{u}^B_t))}$$

Where $\Lambda^N_t = 1 - \beta + (1-\alpha)\beta(1 - \tilde{lambda_t})(1-\delta)$ represents the share of firms not having a project from the CRA's point of view. The CRA's reputation increases when no rating or a bad rating is released. When a good rating is published, its reputation is updated twice, first after the publication:

$$\hat{\mu}^H_t \equiv prob[\tau = e | r_t = H] = \frac{\mu_t \alpha}{\mu_t \alpha + (1-\mu_t)(\alpha + (1-\alpha)(\tilde{\lambda}_t + (1-\tilde{\lambda})_t\delta)\tilde{p}_t)}$$

The valuation of a firm with a good rating is: $\hat{V}^H_t = \hat{\alpha}^H_t qR$ where

$$\alpha_H \equiv prob[\theta = G | r_t = H] = \mu^H_t + (1-\mu^H_t)\frac{\alpha}{\alpha + (1-\alpha)\tilde{p}_t} \tag{2.1}$$

And then after the outcome of the project:

$$\hat{\mu}^{H,S}_t \equiv prob[\tau = e | r_t = H, \omega_t = S] = \frac{\mu_t \alpha q}{\mu_t \alpha q + (1-\mu_t)\alpha q}$$

$$\hat{\mu}^{H,F}_t \equiv prob[\tau = e | r_t = H, \omega_t = F] = \frac{\mu_t \alpha(1-q)}{\mu_t \alpha(1-q) + (1-\mu_t)(\alpha(1-q) + (1-\alpha)\tilde{p}_t)}$$

We consider that the CRA gets a share γ of the surplus coming from a good rating through fees charged. The opportunistic CRA maximizes its profit over the two periods taking into account reputation effect:

$$\hat{\pi}_1 = \alpha\beta\Big(\hat{\phi}_1 + q\hat{\pi}_2(\hat{\mu}_1^{H,S}) + (1-q)\hat{\pi}_2(\hat{\mu}_1^{H,F})\Big)$$
$$+ \Lambda_1^B\Big(\hat{p}_1(\hat{\phi}_1 + \hat{\pi}_2(\hat{\mu}_1^{H,F})) + (1-\hat{p_1})(\hat{u}_1\hat{\pi}_2(\hat{\mu}_1^L) + (1-\hat{\mu_1})\hat{\pi}_2(\hat{\mu}_1^{\emptyset}))\Big)$$
$$+ \Lambda_1^N\hat{\pi}_2(\hat{\mu}_1^{\emptyset})$$

With unsolicited ratings, the equilibria has the following characteristics:

- Firms with a good project always ask for a rating, firms with a bad one ask for a rating with probability $\hat{\lambda} \in [0,1]$ and pay a fee $\hat{V}_t^H - I$ (the CRA gets all the surplus). Firms with an H rating always raise funds and invest in the project.
- At period 1, the CRA charges the publication of a good rating for a fee $\hat{\phi} = \hat{V}_1^H - I$. Good firms soliciting a rating will get a good one. Type B and firms which did not ask for a rating will receive a good one with probability $\tilde{p}_1$. If the firms refuse the CRA's offer, then the CRA publishes an unsolicited L rating.
- At period 2, the opportunistic CRA gives a good rating to any firm asking for a fee $\hat{\phi} = \hat{V}_2^H - I$, as reputational incentives are no longer present.

CRAs find an interest in using unsolicited ratings, as it allows them to increase their reputation. Actually, we should notice that $\hat{\mu}_1^H < \hat{\mu}_1^{\emptyset} < \hat{\mu}_1^l$. Bad ratings appear more often when CRA can published unsolicited ratings, it allows the opportunistic CRA to increase its reputation.

With solicited ratings only: When considering a CRA that cannot issue unsolicited ratings, we find the same results as Mathis et al. (2009). As an ethical CRA will never give a good rating to a bad project (firm)), if the opportunistic CRA chooses a similar strategy, failure would be associated to "honest mistakes". Thus, reputation would not play any role. It creates a room for this opportunistic CRA to give a good rating to a bad project with a strictly positive probability. This is why in equilibrium, bad firms request a rating with a positive probability and may obtain a good one.

We compare the equilibrium with and without unsolicited ratings:

- The use of unsolicited ratings allows a CRA to increase its fees. Actually, the threat of a bad ratings increases the value of a good rating.
- In terms of social welfare, the impact of unsolicited ratings depends on the valuation of a firm without a project $\bar{V}$. When $\bar{V}$ is low (we could see this as the economy being in a recession, the possibility of unsolicited ratings lead to more stringent ratings. Whereas when $\bar{V}$ is high (in a boom), the opportunistic CRA will be more lax than in a system with only solicited ratings.

This paper offers an insight about the consequence of unsolicited ratings. These ratings concerns only firms of bad quality, but it also affects indirectly good issuer. In this model, unsolicited ratings are low but not really as a punishment or a threat but because it fits with the firms' profile. In this setting, unsolicited ratings are not downward biased as it is sometimes supposed. However even if it increases the value of a good rating through the enhancement of CRA's reputation, this increase vanishes in the ratings' fee.

C. Public information

Credit ratings are public information, that help investors in their decision-making process. Morris and Shin (2002) study a situation where investors can rely on public and private signals. The idea is to study how investors will use the public information, to see if an increase in the public signal accuracy is always beneficial in terms of social welfare. If private information were not available, obviously, welfare will increase with the quality of the public signal. But when investors have also access to private information the results are more ambiguous. Actually, investors, at least partly, act as if they were competing in a beauty contest ,as expressed by Keynes. Investors follow a coordinative motive; they have to guess the true value of an asset, but also anticipate the action of other agents.

To study this situation, the author consider the following setting; the economy is made of a continuum of agents i, $i \in [0;1]$, with agent choosing an action $a_i \in \mathbb{R}$. We define $\mathbf{a}$ as the action of all agents. The utility function for each agent is:

$$u_i(\mathbf{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}) \qquad \text{where r is a constant, } 0 < r < 1$$

$$\text{et} \qquad L_i \equiv \int_0^1 (a_i - a_j)^2 \mathrm{d}j \quad \text{and} \quad \bar{L} \equiv \int_0^1 L_j \mathrm{d}j$$

The loss function of the agent is made of two components:

- The first is the quadratic loss with the distance between the true fundamental θ and the action chosen a_i;
- The second part is the "beauty contest", Investors aim to have an action as close as possible to other agents' actions; The intensity of this coordinative motive is represented by r. In this setting, the "beauty contest" is a zero-sum game where the winners earn at the expense of the losers.

When considering social welfare, as expressed by: $W(\mathbf{a}, \theta) \equiv \frac{1}{1-r} \int_0^1 u_i(\mathbf{a}, \theta) \mathrm{d}i = -\int_0^1 (a_i - \theta)^2 \mathrm{d}i$
The second component of the agents' utility function does not appear and what matters is only the distance between agents' actions and the true value of θ (it comes from the fact that the coordinative motive is defined as a zero-sum game).

The first order condition gives us the action chosen by agent i in equilibrium:

$a_i = (1-r)E_i(\theta) + rE_i(\bar{a})$ where $\bar{a}$ is the average action of all agents.

The public signal is defined as $y = \theta + \eta$ where η follows a normal distribution with mean O and variance σ_η^2. If no private information is available, agents will rely only on the public signal and $a_i(y) = y$ for all i. Therefore, social welfare increases with the public signal accuracy.

Now, we consider that agents have access to a private signal $x_i = \theta + \varepsilon_i$, where ε_i follows a normal distribution with mean O, and variance σ_ε^2, i.i.d. $E(\varepsilon_i \varepsilon_j) = 0$. We note $a_i(I_i)$ agent i's action, with I_i its informational set made of its private signal and the public signal. Accuracy of each signal is noted:

$$\alpha = \frac{1}{\sigma_\eta^2} \qquad \beta = \frac{1}{\sigma_\varepsilon^2}$$

From all information available, agents can estimate the value of θ : $E_i(\theta) = \frac{\alpha y + \beta x_i}{\alpha + \beta}$.

The authors exhibit the only equilibrium:

$$a_i(I_i) = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$$

We should notice that agents overweight the public signal. Due to the coordinative motive, their actions deviate from their expectation of θ. Actually $E[\theta|y, x_i] = \frac{\alpha y + \beta x_i}{\alpha + \beta}$ Social welfare is then equal to:

$E[W(\mathbf{a}, \theta)|\theta] = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}$ We can then look at the impact of the public signal's accuracy over social welfare: $\frac{\partial E[W|\theta]}{\partial \alpha} \geq 0$ if and only if $\frac{\beta}{\alpha} \leq \frac{1}{(2r-1)(1-r)}$

When the coordinative motive is high ("the beauty contest"; $r > 0,5$), an increase in the public signal precision can be detrimental in terms of social welfare. The quality of public information matters when agents are not informed or their private information is very noisy, otherwise it could be beneficial to avoid sending a public signal.

Agents overreact to the public signal. It appears when we consider expectation at different order:

$$\bar{E}(\theta) \neq \bar{E}(\bar{E}(\theta)) \qquad E_i(\bar{E}(\theta)) \neq E_i(\theta)$$

When considering the first order condition: $a_i = (1-r)E_i(\theta) + rE_i(\bar{a})$, knowing $E_i(\theta)$, we have:

$$a_i = (1-r)E_i(\theta) + (1-r)rE_i(\bar{E}(\theta)) + (1-r)r^2E_i(\bar{E}^2(\theta))$$

If we had $E_i(\bar{E}(\theta)) \neq E_i(\theta)$ then the chosen action would have been $a_i = E_i(\theta)$.

The overreaction towards the public signal comes from the failure of the law of iterated expectations. The incidence of a public signal has been long associated with central banks and government bodies. In this paper, the authors mention the case of Australia. The country went from releasing monthly figure to quarterly figures because the noise in the information delivered created too much volatility. We see that the very nature of public information brings its own troubles, and it also helps us to understand why ratings' announcements are such a big news. If investors intend to coordinate themselves, they will use ratings to do so, even if they own better private information. If Boot et al. (2006) shows the positive aspect of a focal point, this paper illustrates different implications. Moreover, What would be the situation when this public information is sent by a profit maximizing entity? This will be the subject of the next chapter

The next chapter will discuss the consequences of a public signal sent by a private entity in a similar setting, after which, we will discuss how to discipline a CRA over business cycles in presence of uncertainty.

Chapter 3

Public Information from a Private Entity: the Case of Credit Rating Agencies

What are the equilibrium features of an economy where public information is released by a private company ? The impact of public information over financial markets has been regularly questioned. A seminal paper by Morris and Shin (2002), discuss the impact of the public signal's accuracy in terms of welfare. They consider a financial market where investors have access to a private and a public information. As they show, because of a coordinative motive, a better public information's accuracy does not always induce a better welfare. But what issues are raised when considering a costly public information sent by a profit maximizing entity ? In this paper, I propose another look at the credit rating activity. By publishing grades, credit rating agencies act as a public entity. I found that in equilibrium ratings' quality depend upon the information costs of the investors. A social planner would induce a different behavior from the investors even though he would also overweight public information in order to reduce overall information costs and also retribute the CRA.
Keywords: Credit Rating Agencies, Public information.

3.1 Introduction

The last decade has seen a growing interest about credit rating agencies (hereafter CRAs). The subprime crisis has emphasized their influence over the financial markets. CRAs aim at providing valuable information about the creditworthiness of a borrower. They reduce the asymmetry of information between issuers and investors. With the publication of ratings, they provide investors information which can be used in their decision making process. However, after the last financial crisis, critics have raised concerned about the quality of their work. The poor quality of ratings has been blamed: before the subprime crisis, a large quantity of complex financial products received better grade than they should have, as seen in Ashcraft et al. (2010). Those too generous grades have fostered the pouring of credit in the real estate sector, which fed the bubble in the housing market. This event has led the recent economic literature to investigate the industrial organization,

and the behavior of CRAs.

The rating industry has it own specific features. This activity is handled by an oligopoly. And most of the CRAs rely on a issuer-pay business model that has raised questions over potential conflict of interests. Also, we should highlight the role played by regulation which has been fostering ratings' influence over financial markets. Actually, the Basel rules demands banks to rely on ratings to evaluate the regulatory capital needed. That type of rules can also come from individual actors such as an investment fund which would promote their investment strategies by relying on ratings in their leaflet. It also concerns other actors which have built their own analysis about a financial product but must also rely on ratings, because of their influence for other actors (such as the ones mentioned previously). Considering also the media coverage of ratings (countries' ratings are big news), it makes ratings a public signal nature. Moreover, investors tends to play a double game. On one side, they are looking at the true "value" of a financial product, on another side, they must also anticipate the market's reaction, this principle is often associated to the "beauty contest" of Keynes.

Relying on few actors who will produce information for all the markets' participants seems logical from an economic point of view. Instead of having each investors producing its private information, it should be more efficient to entrust a few entity to provide an accurate information to all actors. If this activity was handled by any government body or public institution, conflict of interest would arise[1]. However, relying on private entities might also brings other troubles. This paper provides a study of this situation.

To do so, I will rely on Morris and Shin (2002). They study the effect of public information on welfare. They built a model where a continuum of agents play a game where a fundamental (a data needed in their investment strategies such as a growth rate) is unknown. Each investor can rely on a private and a public information (both imperfect) about the fundamental. Their utility function is made of two components; the first one consists in being close to the true value of the fundamental through a standard quadratic distance, and the second represents the beauty contest through a zero-sum game. Because of the coordination motive in their utility function, in equilibrium, investors associate a weight to the public signal higher than it would be if they were only trying to guess the true value of the fundamental. Therefore, an increase in the public signal's accuracy is not always profitable from a social point of view, as investors will tend to ignore their private information. From this setting, I consider a CRA that has access to a costly information technology. The CRA chooses the quality of its signal, knowing that the more it will be used by the investors the higher its revenue will be.Then, the CRA sends a public signal (a rating) about the fundamental (the creditworthiness of a financial product). Investors have also access to a private costly information technology. Knowing the quality of the public information, they decide whether or not to acquire a private information. They then take actions according to their informational set, considering also their coordinative motive (the beauty contest). I show that the quality of the information provided by the CRA depends upon the technology owned by both actors. The more costly is information production for investors, the less accurate would be the information provided by the CRA. This situation leads the CRA to provide a lower quality of information than a social planner would. Although a social planner would also invite agents to overweight the public signal in order to reward the CRA.

[1]When rating its own debt, or the debt of firms critical for a country for example

3.1.1 Literature associated

The recent literature has been focus over different axes too explain the poor quality of ratings and question the particularities of the ratings activity. One approach consider a strategic behavior where good ratings were on purpose given to bad products, despite reputational incentives. This is the approach used to consider the reputation's argument used by CRAs when they defend their business model.

The conflict of interest arising from their business model has been studied, and despite CRAs' argument, reputation appears not to be sufficient to avoid overrating. This can be seen in Mathis et al. (2009), where the asymmetry of information offers a room for an opportunistic CRA to give good ratings to bad products and hide its errors behind potential honest mistakes. The question of competition was also raised. The lack of competition seems not to be a factor of poor quality in ratings. Actually, more competition could lead to a practice called "rating shopping" where an issuer will ask different CRAs until it obtains the desired ratings as shown in Bolton et al. (2012). It could also results in a race to the bottom which could decrease rating's quality according to Becker and Milbourn (2008). Those papers have also emphasized the role played by complexity. More opaque products generates poorer ratings, as information asymmetry is large, the possibility of honest mistakes increases the room for unethical behavior.

When considering reputation as a disciplining device, the CRA's reputation is updated by the investors. This approach highlights the mission incumbent to the investors. They should monitor the ratings quality to update CRA's reputation and frame the behavior of a CRA. The recent financial crisis, where CRA were involved, came from a specific financial area (countries' debt in East Asia and MBS for the subprime crisis). The first ratings concerning this sector were accurate. Then, the number of products to be rated increased (alongside with the amount of capital raised). Investors seem to be less prudent, knowing that those products were "liquid". The quality of ratings decreased.

In White (2001), the author avoids the term "agency" because it makes think of CRAs as a government body. However the CRAs are private companies aiming at maximizing their profit, not the social welfare. Moreover, for a CRA, the reputation is a proxy of what really matters for them: the market power of a rating, i.e. the weight associated to a rating when investors made their decision. The more investors rely on ratings, the more the CRA can charge its services. In a certain way, Risk department inside institutional funds are also their competitor. In this paper, I do not consider the conflict of interest between the issuers and the investors but the conflict of incentive for investors (find the true value of an asset, but also anticipate and coordinate with the action of other agents, rely on public information and acquire a private one) and how the CRAs could enjoy this situation. Moreover, the regulation has led financial entities to rely on ratings, fostering the influence of ratings. As underline by Dasgupta and Prat (2008), financial institutions and their employees may be tempted to excessive risk taking. This could be done through excessive risky positions or by a drop in risk analysis costs "The endogenous limits on the informativeness of trades and prices that we derive have important implications for the liquidity and volatility of assets traded by institutions".

In another literature, Ottaviani and Sorensen (2002) shows how experts' decision are influenced by other experts. In my paper, the influence is not exercised by other CRA but rather indirectly by the analysts' work, which are not directly competitor for the CRA but still exercise an influence on their work. This brings us to Fong et al. (2014), they show how the relationship between analysts can influence ratings accuracy. Studying ratings and analysts coverage over the period 1986- 2005,

they show that a better coverage leads to better ratings. Actually, when analysts' coverage dropped due to mergers in this industry, ratings became less conservative. Quality of ratings is measured through default rate, downgrades and mispricing.

Morris and Shin (2002) study the social value of public information in a setting where the agents have two incentives, the first one is to guess the fundamental the other is to be as close as possible with the actions of others agents. This strategic complement leads agents to overweight the public signal in their decision. The authors also notice that an increase in the public signal accuracy is ambiguous from a social welfare point of view. This paper goes also along with the experimental study Alfarano et al. (2015), they study in a laboratory the behavior of agents who have access to private and public information. Their results show that agents overweight the public signal.

Colombo and Femminis (2008) consider also a beauty contest framework, where the public authorities takes into account the costly information acquisition process of investors. They show that when the marginal cost of public information is cheaper than the one of investors, an increase in public information accuracy is always welfare enhancing. Actually, it induces investors not to get informed and then save the use of private resources. Ui (2014) provides a similar study by considering convex information costs. Actually, welfare is increasing in public information accuracy when information choices are endogenous and information costs are linear. They consider an alternative setting with non linear information costs, they show that the convex function mitigate the crowd-out effect of the public signal, thus welfare can decrease with a public information of better quality.

My paper provides a new approach to study CRAs by looking at the consequences of public information released by a private entity. Indeed this private entity should be monitored by the other private agents (here the investors). I find that: (i) investors will never acquire private information if their acquisition costs are higher than the CRA's and even if they own a better technology than the CRA if the coordinative motive is strong they will only rely on ratings; (ii) ratings' quality depends upon both cost functions (investors and the CRA) but also on the intensity of the beauty contest. The rest of the paper is built as follows: section 2 presents the model and discuss the property of the static equilibrium, whereas section 3 studies this economy with explicit cost functions and compare the behavior of a CRAs and the choices that a social planner would make.

3.2 The Model

I consider an economy made of a continuum of agents (investors) and one credit rating agency. The continuum of investors is indexed by $i \in (0;1)$. θ is a random variable which parameterizes the economy's fundamental. θ is uniformly distributed over $\mathbb{R}$, such that there is no prior. θ should be seen as a parameter needed by the investors in their decision making process. In this situation, it can represents the credit worthiness of a financial product.

- The Investors:

Each agent chooses an action $a_i \in \mathbb{R}$ and $\mathbf{a}$ is defined as the actions undertaken by all agents. Each agent can acquire a costly private signal: $x_i = \theta + \varepsilon_i$, where ε_i is a noise normally distributed with mean zero and variance $\sigma^2_{\varepsilon_i}$. Private signals are not correlated, then for $i \neq j$ $E(\varepsilon_i \varepsilon_j) = 0$. Signal's

accuracy is defined as: $\beta_i = \frac{1}{\sigma^2_{\varepsilon_i}}$. The cost function $C_{inv}(\beta_i)$ is the same for all agents i. It is twice differentiable and satisfies $C'_{inv}(\beta_i) \geq 0$, $C''_{inv}(\beta_i) \geq 0$, $C'_{inv}(0) = 0$ and $\lim_{\beta \to +\infty} C_{inv}(\beta_i) = +\infty$.

$$u_i(\mathbf{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(L_i - \bar{L}) - C_{inv}(\beta_i) \qquad \text{where r is a constant, } 0 < r < 1 \quad (3.1)$$

$$\text{and} \qquad L_i \equiv \int_0^1 (a_i - a_j)^2 \mathrm{d}j \quad \text{et} \quad \bar{L} \equiv \int_0^1 L_j \mathrm{d}j$$

The first component of the utility function is a quadratic loss in the distance between the investor's action and the fundamental. Whereas the second one reflects the beauty contest as the loss depends on the average distance between the agent i's action and the actions taken by all other agents. The intensity of the beauty contest is defined by r. One should notice that the beauty contest is a zero-sum game. Regarding the beauty contest component, the winners earn at the expense of the losers. As individuals are atomic, $\bar{L}$ is seen as a constant from the point of view of an agent.

- **The CRA:** The CRA aims at maximizing its profit function:

$$\Pi_{CRA}(\alpha) = -\int_0^1 (a_i - y)^2 \mathrm{d}i - C_{CRA}(\alpha) \quad (3.2)$$

The CRA sends a public signal $y = \theta + \eta$, where η is a noise normally distributed with mean zero and variance σ^2_η. I define $\alpha = \frac{1}{\sigma^2_\eta}$ as the public signal's accuracy. $\int_0^1 (a_i - y)^2 \mathrm{d}i$ represents the distance between investors' actions and the public signal sent by the CRA. To acquire information about the fundamental, the CRA owns a costly technology. $C_{CRA}(\alpha)$ is a cost function increasing in the CRA's signal accuracy. The cost function is a differentiable function satisfying $C'_{CRA}(\alpha) \geq 0$, $C''_{CRA}(\alpha) \geq 0$, $C'_{CRA}(0) = 0$ and $\lim_{\alpha \to +\infty} C_{CRA}(\alpha) = +\infty$.

The CRA earns its revenue from its influence over the financial market. An issuer will pay higher fees if he knows that ratings will drive, at least partly, the market's response. In this model, the more investors' actions rely on the rating, the more the CRA will charge its services. In this setting, I consider that the CRA reveals truthfully its information, the strategy of the CRA consists only in choosing the quality of the information it delivers.

Welfare is defined as the sum of investor's utilities and the CRA's profit:

$$W(\alpha, \beta, \theta, \mathbf{a}) \equiv \int_0^1 u_i \mathrm{d}i + \Pi_{CRA} = -(1-r)\int_0^1 (a_i - \theta)^2 \mathrm{d}i - C_{inv}(\beta_i) - \int_0^1 (a_i - y)^2 \mathrm{d}i - C_{CRA}(\alpha) \quad (3.3)$$

As mentioned earlier, when aggregating investors' utility the beauty contest component vanishes. However welfare depends on r, thus when r goes to 1, investors' utility does not matter anymore. Actually when r is high, the model mimics the situation of a bubble where only the coordinative

motive would be present. In this situation, the distance of the actions undertaken by the investors with the fundamental does not matter anymore. When r is close to 0, we witness a different situation. Investors want to be as close as possible to the fundamental.

The timing of actions is the following: first the CRA choose the accuracy of its signal, then release the public information. Knowing y and α, investors choose if they acquire or not a costly private signal x_i. From their information set, they choose an action a_i, then θ is revealed and pay-off are realized.

- Timing of Actions:

1 - θ is drawn by nature;

2 - The CRA chooses the accuracy of the public signal, α (common knowledge);

3 - The CRA sends its public signal, y;

4 - Investors choose the accuracy of their private signal, β_i and received it x_i;

5 - They choose their actions a_i according to their information set (the public signal and their private information);

6 - The fundamental (θ) is revealed and pay-off are realized.

It is considered to be common knowledge: the technology owned by the CRA and the investors, the choice of accuracy made by the CRA. Actually, CRA released public information about the evolution of their ratings over the years. This information appears in the transition matrix (which are also used to measure credit risk).

- Table of notations:

symbol	Notes
θ	Fundamental
a_i	action taken by agent i
y	public signal sent by the CRA
x_i	private signal received by agent i
α	public signal accuracy
β_i	private signal accuracy for agent i
r	beauty contest intensity
W	social welfare

3.2.1 Equilibrium:

I seek a linear symmetric equilibrium (i.e. a linear equilibrium where all agents choose the same $\beta_i = \beta$). The equilibrium is solve by backward induction. I do this in two steps: first I look for the best action for agent i given his information set, then I look at its best choice of β. For the rest of the paper, I define $E_i()$ as $E(.|I_i)$, where I_i denotes the information set of agent i.

Proposition 3.1 *In equilibrium, the best reply for agent i is the following action:*

$$a_i = (1-r)E_i(\theta) + rE_i(\int_0^1 a_j \mathrm{d}j) \tag{3.4}$$

Proof: In appendix.

As in Morris and Shin (2002) and Colombo and Femminis (2008), I seek a linear equilibrium where $a_i^* = \gamma y + (1-\gamma)x_i$ where γ is a parameter to be found in equilibrium.

Proposition 3.2 *As shown in Colombo and Femminis (2008), there exists only one linear symmetric equilibrium (where all investors choose the same $\beta_i = \beta$):*

$$\beta_i = \beta = \frac{1}{\sqrt{C'_{inv}(\beta)}} - \frac{\alpha}{1-r} \quad \text{if} \quad \beta > 0 \quad \text{otherwise} \quad \beta = 0. \tag{3.5}$$

And the action taken in equilibrium by agent i is: $a_i = \dfrac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$.

Proof: In appendix.

I find back the results of Morris and Shin (2002). The action undertaken by the investors in equilibrium is different than the expectation of the fundamental: $E_i[\theta] = \dfrac{\alpha y + \beta x_i}{\alpha + \beta} \neq \dfrac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$ (as $0 < r < 1$). The coordinative motive leads investors to rely more on the public signal than they would do if they were only trying to guess the fundamental θ: $a_i = y\dfrac{\alpha}{\alpha + \beta(1-r)} + x_i\dfrac{\beta(1-r)}{\alpha + \beta(1-r)}$, whereas $E_i[\theta] = y\dfrac{\alpha}{\alpha+\beta} + x_i\dfrac{\beta}{\alpha+\beta}$. The action chosen in equilibrium highlights the balance between the fundamental's expectation and the actions undertaken by other agents. When r is high, the financial market can be seen as being in a bubble. In this situation the liquidity of a product matters as much as (or more) than its true value.

Proposition 3.3 *In equilibrium, the CRA never chooses a level of accuracy greater than* $\alpha^{sup} = \dfrac{1-r}{\sqrt{C'(\beta)}}$

Proof: This result comes from the profit function of the CRA. Beyond this level of public signal's accuracy, investors won't acquire any private information so any increase in the accuracy of the public signal would only cost the CRA without increasing its profit. End of proof.□

In this model, the CRA will never provide a signal with an accuracy higher than $\alpha^{sup} = \dfrac{1-r}{\sqrt{C'(\beta)}}$. This threshold is a decreasing function of r and $C'(\beta)$. Obviously, the more the investors are focus on the coordinative motive, the less they will care about the true value of the fundamental. The CRA also benefits from higher information costs for the investors. As it is costly for them to acquire information, even a noisy signal would be sufficient to them to drive their actions. It is another illustration of the impact of asset's complexity on CRAs behavior. The more opaque are the product rated, the more ratings will be considered although they are less accurate than ratings for simpler products.

3.3 The Model with Explicit Cost Functions

For the rest of the paper, I consider that the investors and the CRA own the following technology:

$C_{inv}(\beta) = p\beta$,with $p > 0$;

$C_{CRA}(\alpha) = k\alpha$,with $k > 0$.

In this section, I will discuss the different situations that may arise from this economy when:

- When the CRA owns a better technology than the investors (i.e. $p > k$). It could be seen as the usual situation. Through its experience, and as credit ratings is its core activity, CRAs should be more efficient than investors to perform financial analysis. It is also aligned with the idea of relying on one central entity rather than asking each actors to perform an analysis in its corner;
- When the investors own a better technology than the investors (i.e. $k > p$). It could reflect the situation for very specific markets where some products are produced and exchanged between few actors. Those actors may have developed a better knowledge than a CRA.

Proposition 3.4 *In equilibrium, the CRA chooses as its signal's accuracy:*

$$\alpha^* = min(\frac{1-r}{\sqrt{rp + k(1-r)^2}}, \frac{1-r}{\sqrt{p}}) \tag{3.6}$$

And the investors acquire a private information, if and only if $\frac{1-r}{\sqrt{rp + k(1-r)^2}} < \frac{1-r}{\sqrt{p}}$ *thus:*

- *When* $p \geq k(1-r)$*, the investors will rely only on the public information* $\beta_i = \beta^* = 0$*;*
- *When* $p < k(1-r)$*, the credit rating agency provide a public signal but the investors also rely on a private information* $\beta_i = \beta^* = \frac{1}{\sqrt{p}} - \frac{\alpha}{1-r}$.

Proof: In appendix.

When the CRA owns the best technology, it is the only agent to produce information. But the condition does not rely only on the two cost functions but also on the intensity of the coordinative motive. Even when the investors own a better technology than the CRA, if r is high enough there will not acquire a costly information. The coordinative motive leads them to be more focus over liquidity than over the fundamental value. We also notice that the coordinative motive leads to a lower level of accuracy for both agents ($alpha^*(r)$ and $\beta^*(r)$ are both decreasing functions). This situation makes the CRA to enjoy a bubble, as it maximize its profit with lower information costs, it can be seen in the CRA's profit function. If $p \geq k(1-r)$:

$$\Pi_{CRA}(\alpha) = -p\frac{1-r}{\sqrt{p}} \tag{3.7}$$

The social planner's choices: We have seen the equilibrium but what would be the actions and the level of accuracy chosen by a social planner. I consider a social planner that can choose the value of $\beta_i = \beta \ \forall i \in (0,1)$, α and the action a_i undertaken by any agents considering its information set I_i. I will investigate what choice such a social planner would make to maximize the social welfare.

Proposition 3.5 *The social planner would choose the following action for agent i:*

$$a_i^{FB} = \frac{y(\alpha(1-r) + (\alpha+\beta)) + x_i\beta(1-r)}{(\alpha+\beta)(2-r)} \tag{3.8}$$

Then, the social planner would make the following choice in terms of production of information:

- *When* $p > (2-r)k$, *then* $(\alpha^{FB}, \beta^{FB}) = (\sqrt{\frac{1-r}{k}}, 0)$;

- *When* $p < (2-r)k$, *then* $(\alpha^{FB}, \beta^{FB}) = (\sqrt{\frac{1-r}{(2-r)(k-p)}}, \frac{1-r}{\sqrt{p(2-r)}} - \sqrt{\frac{(1-r)}{(2-r)(k-p)}})$.

Proof: In appendix.

We should notice the action chosen by the social planner does not fit with the expectations of the fundamentals $a_i^{FB} \neq E_i[\theta]$. In order to retribute the central organisation delivering the information, the social planner would like investors to overweight its signal. This could help us to look at financial regulation from another eye. The banking regulation have define references towards ratings. If we look at the basel rules, we will see that banks can rely on their own ratings (if their methodology has been approved) or on CRA's ratings. Obviously, the goal was twofold:

- To define a common sound practice;
- Allow the "small" institutions to have a better risk management without the development of heavy methodology, and allow the bigger institutions to develop their own technology.

Boot et al. (2006) also underlines why some coordination might be helpful. In an economy with multiple equilibria, a focal point may help agents to coordinate themselves in order to obtain a better equilibrium. We can see this through the model, actually the reference to CRA's ratings is beneficial if the job is well done (better one provider of information than many in their corner), whereas the incitation towards a proper technology put pressure on ratings' quality. If I should critic the reference towards ratings in the financial regulation or in leaflet from pension funds or equivalent, I would highlight the missing incentives for CRAs to provide an accurate information.

Proposition 3.6 *The social planner would always choose a public signal with a better accuracy than the one chosen by the CRA:* $\alpha^{FB} > \alpha^*$. *Trough this choice, the social planner reduce the profit of the CRA but the gain in the investors' utility overweight the loss in the CRA's profit.*

Proof: In appendix.

About this graphic, I make the following remarks:

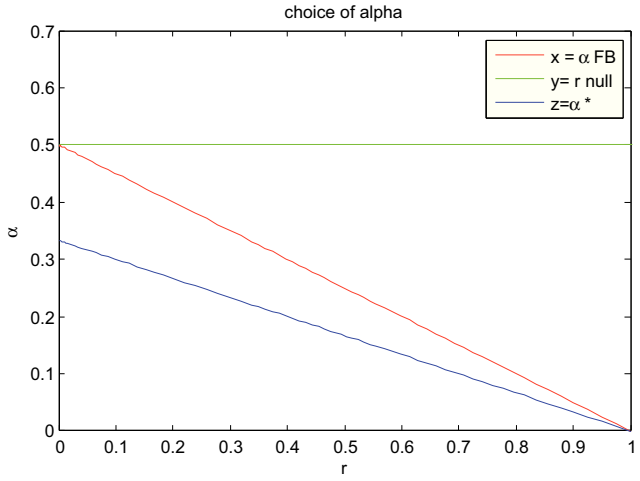

Figure 3.1: Choice of alpha in equilibrium

- The social planner always would provide a better public signal, but the difference decrease with the beauty contest intensity r. It comes from the utility function of the investors, when r is close to one, being close to the fundamentals brings them nothing, there are only motivated by the coordinative motive, thus any signal ,whatever its quality, would lead investors to coordinate over it.

- If we consider that the social planner focus on the distance over the fundamental and does not take into account the beauty contest (as it is done in Morris and Shin (2002), it would always provide $\alpha = \frac{1}{\sqrt{k}}$. In this case, the difference in ratings quality with the CRA's choice increase with the intensity of the beauty contest.

Actually, the difference between CRA's action and the one chosen by the social planner rely on the cost function of each entity. This helps us to understand where we have witnessed some ratings' failure:

- The Subprime crisis;

- The Worldcom failure and the internet crisis;

- The East Asian crisis.

The asymmetry of information could also be seen as an asymmetry of technology. I have highlighted the role played by investors in disciplining a CRA. Instead of talking about reputation and close monitoring, one should also focus on easying the production of information from other actors. When an advisor meet an agent with potential good information, he might be tempted to enhance the quality of its work. The disclosure rules put in place by the financial regulation are working towards this direction.

3.4 Conclusion

In this paper, I have aimed at providing a new approach about credit rating agencies. It allows me to exhibit another aspect of the rating industry. When considering incentives about ratings quality, we should consider the ability of other financial actors to produce information. The quality of a signal of average quality might appear to be good when its costly for other agents to perform an analysis.

When talking about poor performance about CRA, we could also talk about poor monitoring from the market. For many reasons, the recent financial crises have shown that investors do not perform all the analysis they should do. A similar situation is presented in Rochet and Tirole (1996), the authors present the issue of interbank lending and peer monitoring. They show that peer monitoring is weak because of its costs and also because of the possibility of a public bail-out in case of the failure of a major financial institution. Here the issue should be seen as similar. Investors should monitor the quality of rating before investing, however it is a costly process and in case of failure, they will not fall alone. One must notice the progress made in financial regulation. As mentioned in Basel 2 (in its third pillar), it was supposed to reduce the analyst's costs by making easier comparison between banks. However, as the liquidity panic of 2008 indicates, this was not enough to ensure full transmission of information. It also helps us to understand the role played by innovation in the financial crises. When new products are designed, most of financial actors do not have the sufficient knowledge to evaluate their creditworthiness. As the "common" knowledge is weak, the intermediary can be satisfied even with signals of poor quality. When the question of competition was raised regarding CRAs, the question has to be address both in terms of competition between CRAs but also between the different actors playing a role on financial markets.

Chapter 4

Credit Rating Agencies, Reputation, Business Cycles and Uncertainty

The question of reputation as a disciplining device has been raised after the subprime crisis. Credit Rating Agencies (hereafter CRAs) claimed that reputation being their main asset, they cannot engage themselves in any devious behavior. The recent literature has shown that it is not sufficient to avoid overrating. In this paper, I consider the question through another lens: I consider that reputation can perfectly induce the CRA to give good ratings only to products that deserve such a rating. However, if the CRAs faces uncertainty, it cannot perfectly anticipate how its reputational capital will evolve. This uncertainty lead the CRA to adopt a different rating policy. Thus in this paper, I show that overrating and underrating can be consequences of uncertainty. In effects, CRA rates a priori when their work will be judged a posteriori Moreover, a reputational rules must be made to ensure a steady level of capital. Uncertainty leads to movement of the CRA's reputational capital. The CRA will act in a manner to rebuilt or enjoy its new reputational level. This can explain why sometimes CRA are more lax or more severe, especially after a surprise (which can be good or bad).
Keywords: Credit Rating Agencies, reputation and business cycles.

4.1 Introduction

Credit Rating Agencies have recently been under the spotlight. After the subprime crisis, they were accused of bearing a strong responsibility next to the default of too many well-rated products. Few years later, during the European debt crisis, they were blamed for being too severe when downgrading countries' grades. Those events have shown how influent CRAs can be and how judgements about their behavior evolve. Credit Rating Agencies first appeared by the end of the nineteenth century. They were providing investors infirmations about the ability of a firm to honor its debt through a grade (making capital flows closely linked to ratings). At the beginning, CRAs were selling books, and were paid by the investors, who needed this information to select in which company to invest. This business model changed in the second part of the twentieth century, when the progress of information technologies created an opportunity of free-riding. This situation led the CRAs to adapt their business model; they changed to an issuer-pay business model. This was

also fostered because ratings received a stronger role to play in the economy through regulation. Financial regulation has increased the ratings' value for issuers (ratings are needed to evaluate the riskiness of an investment and are used through financial regulation to compute regulatory capital needs for banks and pension funds). Today, this activity is handled by an oligopoly. Standard and Poor's, Moody's and Fitch are the three main companies leading this business[1]. Their influence on the economy through the financial sphere is major, especially in an economy where financial disintermediation is important (United States), or in an economy sensitive to capital flows which are linked to the grade of the country (emerging countries).

The subprime crisis has raised questions about the CRAs' responsibility and their business model. CRAs' argumentation rejects critics over their issuer-pay business model: reputation being their main asset, they can't engage themselves in any devious rating policy. However as shown in Mathis et al. (2009), reputation may not be sufficient to ensure truthfulness from a CRA. When looking at ratings over the last decades, we find period of rating's inflation (the subprimes, before the east asian crisis) and period of, if not rating deflation, rating stickiness (after the east asian crisis). For some investors, ratings represents a lagged indicator. This shouldn't be the case if we listen to the CRAs: they claim that their ratings are independent of the business cycle[2]. Recent papers aims at explaining how ratings' inflation appears despite the disciplining role of reputation. In most of the recent literature, reputation is defined as the probability of being an honest CRA.

I consider reputation over another angle. In my paper, reputation is not updated according to Bayes' rules, but rather through a function based on the actual return observed by investors. In a two period model, I consider that the CRA is endowed with a certain reputational capital for the first period, and its reputational capital in the second period will depend on the realized return. The goal here is to find a disciplinary function that would ensure the perfect transmission of information and maintain a reputational capital over time. I show that such a scheme perfectly discipline a CRA when there is no uncertainty about the current state of the economy, there is a rule to discipline the CRA in each state. But when the CRA faces uncertainty, it does not know exactly how its reputational capital will evolve. This lead the CRA to choose between two strategies: (i) If the probability of a recession is low but the reputational rule associated to this state is severe, the CRA will give less weight to its future revenues leading to overrate (ii) otherwise its rating policy will be to give a good rating to a proportion of projects superior as the one it is used to do in a recession and lower than in a boom. As a result, the CRA will always be caught underrating or overrating.

4.1.1 Related Literature

The recent literature has studied the role of CRAs in the last financial crisis. Ashcraft et al. (2010) looks at credit ratings on subprime and MBS securities between 2001 and 2007, with ratings from S&P, Moodys, Fish and DBRS. They find that ratings contains useful information, however when studying performance of those financial products they show that ratings became less conservative close to the peak of the bubble. And this results holds for different specification of performance (Default rate, losses, downgrading). This illustrates a common pattern across the last financial

[1] If there is currently around 140 agencies, the big three, as they're called, represent 90% of the market

[2] Moody's rating policy:"Since Moody's ratings are intended to measure long-term risk, our analytical focus is on fundamental factors that will drive each issuer's long-term ability to meet debt payments, such as a change in management strategy or regulatory trends. As a rule of thumb, we are looking through the next economic cycle or longer"

crisis, where a promising financial sector shows up, with accurate ratings at first. Then as the "bubble" grows, ratings become less conservative until the crash, we can think of the subprime crisis, but also of the East Asian one.

Actually, Ferri (1999) argues that the CRAs' behavior amplified the magnitude and the duration of the crisis. To show this shift in the behavior of rating agencies, the authors uses macroeconomic fundamentals of East Asian countries and extrapolate the methodologies used before and after the crisis. It appears that the rating contains a larger part of qualitative judgment after the shock. The author pointed out that this judgement may be motivated by reputation incentives. Their reputation being hit by the crisis, they acted in a cautious manner to rebuilt it. If the procyclical behavior of CRA appears when ratings concerns countries, Jeffery D'Amato (2003) finds the same patterns for corporate bonds. He studies ratings and ratings' transition of US firms between 1981 and 2001. The ratings come from the S&P database. If looking only at initial ratings provides no evidence of ratings linked to the business cycle, when considering ratings' updates, he finds evidence of an excess sensitivity towards the business cycles. Mora (2006) provides a similar study: looking at the East Asian financial crisis, the author finds that if ratings are not linked to the business cycles, they are at least sticky.

In another paper, Mariano (2012) studies the interaction between public information and the incentives for a CRA to reveal its private information and how those incentives are influenced by competition. She studies a situation where a CRA has access to public and private information, the CRA can choose to contradict public information to maximize reputation or when its private information is noisy, the CRA may have interest to follow public information with a bias towards bad ratings. The author shows that to deter potential entrant, and to look competent an incumbent CRA has interest in contradicting public information more often than in monopoly.

A theoretical approach was provided by Bar-Isaac and Shapiro (2013). They study the quality of ratings over the business cycles as a function of the labor market conditions and they show that ratings are of better quality in a recession than in a boom. Their results rely mainly on the labor conditions; it is easier and cheaper to hire good analysts in a recession than in a boom where competition is stronger and incentives to hire better analyst are lower because of a better average quality of the financial products in the economy. They conclude that ratings' quality are linked with the business cycles.

In the next section, I will present the basic model and show that there exists a disciplining function that will lead the CRA to give a good rating only to projects with a positive net present value. I show that there exists such a function for each state of the economy(a boom or a recession). In the third section, I introduce uncertainty, the CRA does not know if today the economy is in recession or in a boom, however it knows that its reputation will evolve according to the state observed.

4.2 The Basic Model

In this section, I consider an economy made of:

- A continuum of risky projects $x_0 \in (0,1)$, $p(x)$ being the probability of success of the project. The distribution $p(x)$ is such that $p(0) = 1$, $p(1) = 0$ and $p(x)$ is a decreasing strictly concave function ($p'(x) < 0$ and $p"(x) < 0$). A project needs 1 unit of capital to be financed and offer

a return R in case of success and 0 otherwise;

- A competitive capital market where investors are willing to invest in all projects with a positive net present value (hereafter NPV).

- Without information investors cannot order the different projects, the expected return of a project is negative: $E[p(x)] < \frac{1}{R}$

- A Benchmark With Perfect Information: When investors can identify and order all projects. They will finance all project with a positive NPV, that is the first $\bar{x}^*$ projects such that:

$$p(\bar{x}^*) = \frac{1}{R} \tag{4.1}$$

In this situation the first $\bar{x}^*$ are financed, whereas the $1 - \bar{x}^*$ projects are not. In presence of asymmetry of information, the investors cannot distinguish projects with a positive NPV from others. In average, a project has a negative present value: $E[p(x)] < \frac{I}{R}$.

To solve the problem of asymmetry of information, there is a need for an intermediary. There exists one CRA which, with a cost c can perfectly identify and order all projects. I consider a CRA which gets its revenue from the issuer, reflecting the conflict of interest caused by the business model of CRAs. The goal of this paper is to look for a reputational scheme that will discipline this intermediary. As the CRA is disciplined by reputation, investors will trust its ratings.

- The CRA: The CRA owns a technology that allows itself to perfectly identify and order the projects from the safer to the riskier. The CRA has a profit function:

$$\pi = x_0 q_0 - c \tag{4.2}$$

Where $q_0 > 0$ is the CRA's reputational capital, x_0 represents the quantity of well rated projects and c the information costs. In this paper, at the beginning of the game, the CRA is endowed with a reputational capital of q_0.

- The one period model: I consider first a one period model. The CRA inherits a reputational capital q_0 and observe the distribution $p(x)$.

Proposition 4.1 *Without any reputation incentives, the CRA will give a good rating to all projects.*

Proof: In appendix.□

Obviously, reputational incentives work only in a dynamic environment. When the CRA does not consider future revenues, it will give a good rating to all the projects. To discipline an agent through reputation, we need to take into consideration a second period.

4.2.1 The two period model:

The idea of this section is to find a disciplining function that will ensure that all projects with a positive NPV receive a good rating and that its reputational capital remains stable. The goal in

this paper is to see if such a function exists. Investors will update CRA's reputation after observing the realized return of period 0. I do not consider reputation as the probability of being honest or opportunistic. Here reputation evolves through time with the return experienced by the investors.

I look for a reputation's update rule such that $q_1(x_0) = F(Rp(x_0) - 1)$ is: - Twice differentiable over $x_0 \in (0,1)$ [3]; - Is an increasing function of the return realized; reputation should be higher when returns are positive (resp. lower when $Rp(x_{CRA}) - I < 0$); - Built in such a way that there is a level of reputation that remains stable and ensure the true transmission of information that is $q_1 = q_0$ if and only if $x_0 = \bar{x}^*$.

The CRA is endowed with a reputational capital q_0 at the beginning of the game and has for discount factor δ.

The CRA aims at maximizing its profit over two periods:

$$\pi(x_0, x_1) = x_0 q_0 - c + \delta[x_1(F(Rp(x_0) - 1) - c] \qquad for x_0, x_1 \in (0,1) \tag{4.3}$$

Where:

- x_0 and x_1 are the proportions of projects that receive a good rating at date 0 and 1;

- q_0 is the reputational capital given to the CRA at the beginning of the game;

- $q_1 = F((Rp(x_0) - 1)$ is the reputation of the CRA at date 1;

Assumption 1 *I consider a CRA sufficiently patient:* $\delta > \dfrac{1}{R}$

As we will see, this condition is needed to ensure that future revenues would matter enough for the CRA to ensure a truthful rating in the first period. If δ was lower, the CRA might have incentives to give a good rating to all projects in the first period and ignore future revenues.

The CRA finds itself back in a similar situation than in the one period model. It has no more reputation incentives to consider. It will thus grant a good grade to all projects expect if its reputational capital is negative. The goal of this section is not about looking at the CRA's behavior in the second period but rather in the first one to see if there is an incitation scheme could discipline the CRA in the first period.

I look for a "reputational" function $F(Rp(x) - 1)$such that:

(C1) $F(0) = F(Rp(\bar{x}) - 1) = q_0$;

(C2) $F(x)$ is increasing and twice differentiable (the higher is the return realized, the higher will be its reputation);

(C3) The CRA will choose $x_0 = \bar{x}$ (I remind that $Rp(\bar{x}) - 1 = 0$).

Proposition 4.2 *There exists a "reputational" function* $F(Rp(x) - 1$ *that it respects conditions (C1),(C2) and (C3). It ensures that all and only the good projects receive a good rating, whereas*

[3] I exclude the cases $x_0 = 0, 1$ as those should never be chosen

the projects with a negative net present value are not financed. At period $t = 1$, the CRA will give a good rating to all projects $x_1 = 1$.

Moreover, there exists only one linear function that disciplines the CRA:

$$F(Rp(x) - 1) = q_0 - \frac{q_0}{\delta p'(\bar{x}^*)}(Rp(x) - 1) \tag{4.4}$$

Proof: In appendix.□

To obtain the uniqueness, I need to restrict the analysis to linear function. However, there are other functions that can discipline the CRA, I will discuss this point in the next section when considering uncertainty over business cycles.

In effects, finding a good reputational function is about balancing between the valuation of today's revenues, and also punishment to ensure that future revenues matters. This can be seen in the following equation when we consider that the CRA will give a good rating to all projects in the second period:

$$\frac{\partial \pi(x_0, 1)}{\partial x_0} = q_0 + \delta[Rp'(x_0)F'(Rp(x) - 1)] \tag{4.5}$$

The second component represents the disciplining role played by future revenues.
$\delta[Rp'(x)F'(Rp(x) - 1)]$ is a decreasing function of x_0. It represents the trade off between today's revenues and those of tomorrow. Any deviation towards higher revenue today will have a negative impact on tomorrow's revenues.

Proposition 4.3 *If the CRA starts the game with a reputational capital superior than $\bar{q_0}$ (resp. inferior), the CRA will over rate (resp. underrate).*

Proof: In appendix. □

This is the principle of the disciplining function. As said earlier, it makes the CRA cautious in the first period. When its reputation is higher (respectively lower) than q_0, the future revenue counts relatively less (respectively more). This leads the CRA to focus over today's revenue by giving an $x_{CRA} > \bar{x}$ and ignore. It also reflects the fact that a reputational capital is made to be used.

- With Business Cycles:

I consider now that the economy can be in two states. We can either be in a boom or in a recession. Investors do not know in which state they evolve, the CRA observes θ_0. The difference between the two states appears through different probability distribution functions. When in a Boom, projects' probability of success is defined by $p_B(x)$ ($p_R(x)$ in a recession). I consider that there are a greater proportion of good projects in a boom than in a recession: $p_B(x) > p_R(x) \forall x \in (0, 1)$. At the end of the first period $t = 0$, returns are realized and θ_0 is revealed.I ignore the nature of the state in the second period, as it will not matter for the CRA.

The CRA aims at maximizing its profit over two period:

$$\pi^\theta = x_0 q_0 - c + \delta[x_1 q_1 (F^\theta(Rp_\theta(x_0) - 1) - c] \qquad (x_0, x_1) \in (0, 1)^2 \qquad \theta \in R, B \tag{4.6}$$

Proposition 4.4 *There exists a unique linear reputational function for each state ($F^B(Rp_B(x)-1)$, $F^R(Rp(x) - 1)$) which discipline the CRA.*

- $F^B(Rp_B(x_0)-1) = q_0 - \frac{q_0}{\delta p'(\bar{x}^B)}(Rp_B(x)-1)$

- $F^R(Rp(x_0)-1) = q_0 - \frac{q_0}{\delta p'(\bar{x}^R)}(Rp(x)-1)$

Proof: In appendix. □

With two different state, the CRA needs to be judged according to two disciplinary rule. Each one would be adapted to the probability of success associated to each state.

4.3 The Dynamic Model with Uncertainty:

I consider a situation where there is uncertainty about the current state of the economy. Investors do not know in which state they are and the CRA just knows the probability of being in a boom or in a recession ($\theta \in R, B$. But at the end of the period, returns are realized, and the nature of the state is revealed. Investors will evaluate the new reputation of the CRA knowing what was the state of the period. In this section, I will discuss the consequences of this uncertainty.

I consider the CRA knows the transition probabilities: $\tau_B = prob[\theta = B]$ and $\tau_R = prob[\theta = R] = 1 - \theta_B$. The CRA is still endowed with q_0, its reputational capital at the beginning of the game. I still consider that the CRA's reputation will evolve according to: $F^\theta(Rp_\theta(x_0)-1)$

The CRA must maximizes:

$$\pi^\theta = x_0 q_0 - c + \delta[\tau_B(x_1 q_1(F^R(Rp_R(x_0)-1)) + (1-\tau_B)(x_1 F^B(Rp_B(x_0)-1)) - c] \tag{4.7}$$

The CRA faces uncertainty at two levels:

- About which probability of projects it should refer to rate, $p^B(x)$ or $p^R(x)$.

- About which reputational rule it will face, $F^R(Rp^R(x_0)-1)$ or $F^B(Rp^R(x_0)-1)$.

I remind that $p^B(x) > p^R(x) \forall x \in (0,1)$. I define x^θ_{max} such that $F(Rp(x^\theta_m ax)-1) = 0$, obviously, $x^\theta_{max} > \bar{x}^\theta$

Proposition 4.5 *As in the previous section, the CRA will give a good rating to all projects in the second period. But in the first one, with uncertainty about the current state, the CRA will choose an $x_{CRA} \in (\bar{x}^R, x^B_{max})$. The consequence is that the CRA will always be caught over rating or under rating, and its reputational capital at date 1 will always be different than q_0:*

- *If $x^R_{max} > x^B_{max}$ then the CRA will choose an x_0 such that $\bar{x}^R < x_0 < \bar{x}^B$;*

- *If $x^R_{max} < 1$ and $x^R_{max} < x^B_{max}$ then the CRA will choose an x_0 such that $\bar{x}^R < x_0 < x^B_{max}$).*

- *If $x_0 > \bar{x}^\theta_0$, then $q_1 < q_0$;*

- *If $x_0 < \bar{x}^\theta_0$, then $q_1 > q_0$;*

Proof: In appendix.

This result deserves some comments:

- When $x^R_{max} > x^B_{max}$, the CRA will not consider the option of having a reputation equal to q_O in the second period. Thus it will give a good rating to a proportion of projects that is superior than the one it usually give in a recession and inferior to its uses in a boom;
- When $x^R_{max} < x^B_{max}$, it means that the punishment is more severe than in the previous situations. If revenues in a boom are much higher than in a recession (i.e. the larger is $\bar{x}^B - \bar{x}^R$) the CRA may be tempted to give good ratings to the projects as if it was in a boom, it might ignore the possibility of being in a recession. And as it might loose all its reputational capital if there is a crisis, its future revenues are less important; this leads the CRA to give a good rating to more products than it would do if it knows that the economy was in a boom.

The uncertainty is not the only parameter to explain the bad quality of ratings, the difference in the probability functions matters as well. When evolving in any state, the CRA knows how its reputational capital will evolve. But, with uncertainty it has doubts about which disciplinary rule would be applied to itself. The wider upturn or downturn are, the higher will be the movement of its reputational capital.

Consequences: As the CRA may experience shocks over the curse of economic activity, its reputation might not be always stuck in q_0. The uncertainty faced by the CRA, leads it to make some "mistakes". And those mistakes will be considered as the CRA is judged after investors have observed the true state of the economy. This influences in turn, its reputational capital. As said previously, the reputational capital owned by the CRA dictates its behavior. A reputational capital is made to be used. Too low, it "increases" the value of future revenues, which will lead the CRA to be cautious. This is why with a low reputation a CRA will be caught underrating.

When observing overrating (or under), we should notice that the CRA acts in that manner:

- Not intentionally, as the CRA faces uncertainty and is caught by surprise;
- On purpose, as the CRA may face a strong punishment tomorrow if today is finally a recession. It minors its future revenues;
- After its reputational capital has moved, it has new incentives:
    - If its reputation is above its usual level, it will overrate;
    - If its reputation is below its usual level, it will underrate;

When an agent is ruled by reputation, the punishment rules can be tricky. In normal times, it works through the threat of loosing future revenues. But in period of uncertainty, a too strong reputational rule could lead the CRA to ignore future revenues if a recession is already there. This situation arises when there are different degrees of uncertainty. There is uncertainty about which projects are good, and the CRA asks itself what ratings' policy it should use to rate as it does not know if we are in a boom or in a recession.

Moreover through this setting, I have aimed at emphasized the difficulty to, a posteriori, identify the origin of a mistake in ratings. Strategic behavior might be the cause, but luck (good or bad) are also factors that should not be ignored.

4.4 Conclusion

Reputation is an interesting device, it helps to discipline an agent which faces conflict of interests. But those schemes are put in place in context of asymmetry of information and thus uncertainty. Then reputation cannot be the only device we should rely on. Unless we accept a part of flexibility. This is what happen in most periods, some ratings are too high or too low. But most of them are accurate, and CRAs correct their mistakes. However, when there is a strong downturn or upturn, especially when it is unlikely, mistakes are inevitable. And after, judgements will induce a different behavior as the reputational capital has changed in a greater proportion than in usual times.

The CRAs have often been criticized, but those blames came after a downturn or an upturn that was not expected. Throughout normal times, there behavior is less pointed out. In this paper, I have emphasized, that when dealing with uncertainty, the CRA cannot perfectly respond to reputational incentives.

During the East Asian crisis, the CRA downgraded severely some countries (South Korea per example) more than they should have, as shown in Ferri (1999). This crisis was a shock which led most CRAs to face a severe loss in reputational capital. As the reputation is their main asset, as CRAs claimed, the logical response was to act in a cautious manner to rebuilt this reputational capital.

A bubble could be considered as a sequence of good signals. Thus an agent dealing with uncertainty and accounting for its reputation will see that its reputational capital evolves in a positive manner. As said previously, a reputational capital is made to be used. It seems then coherent to witness overrating at the peak of a bubble. This is in line with Ashcraft et al. (2010). The authors show that ratings became less conservative with time, the first ones were accurate whereas the last ones before the crisis occurred were not.

Chapter 5

Conclusion:

Throughout my work, I have been given the opportunity to get an overview of CRAs activities. The recent literature provided me a set of tools that are helpful to study CRAs but also the behavior of other types of agents dealing with asymmetry of information. Information, beliefs and expectations play a critical role in the financial markets today. One of my goal when starting this PhD was to obtain a better understanding of financial markets and especially what factors plays a role in a crisis. I will not say that now-I get it all- but I have improved my comprehension of those markets. Moreover it has provided me an overlook of the difficulty to see all mechanisms in play that can explain a financial crisis. If the role played by CRA appears clear after this work, I have no doubt that other actors carry also a responsibility.

The specificities of the subprime crisis have been pointed out, as the role played by asset complexity, the repeated interactions between some issuers and CRAs. Here, I have aimed at looking at more general factors that can explain the change in ratings quality.

In my first paper, I have focus my work over the interactions between investors and issuers. In effects, investors needed to witness the crisis to realize that ratings had been of poor quality for a long period (several months at least). It highlights the lack of monitoring from investors. Moreover, when talking about good information, we should remind us that quality is of relative nature, I run fast when others are slow. In a world with poor information, even an information of "average" quality might appear as good. If I do not deny all blames express to the CRA, I have intended to illustrate the role played by investors in the production of information.

I think that the production of information by few actors to the whole market is a good thing, and that this principle should not be given up. The recent literature has provided insights to understand failure, which should be used, not only to blame CRA but also the whole structure of financial markets. I think regulators could enhance the performance of CRAs. The distinction between the different categories of rated products is a first step. I am not sure about the disappearance of ratings reference is a good thing. It could also have been seen as a potential threat to evolve in this direction.

My last work illustrates also the limits of reputation as a disciplining device. As mentioned previously, actors providing information act in an uncertain environment and are a judged a posteriori when all (almost) information is finally available. Moreover, the reputation might also have unexpected consequences, as a too strong punishment could lead an agent to give up future revenues.

Finance is anything but future revenues, this can work only when agents are sufficiently patient. We should also remind ourselves that perfect information is not the reality. When evolving with uncertainty, I wanted to emphasized the role played by luck.

With those two articles, I have aimed at bringing a new look over CRA's reputation. Reputation can be seen and computed in different manners. I wanted to emphasize some issues that appears when information cannot be perfect and the economy rely on reputational rules. It comes from the interactions between a sender and the receiver. The two mains points I have raised in this work is that:

- When the receiver has a low knowledge of the information he will receives, it will be difficult for him to judge the quality of the sender's signal. And it also gives low incitation for the sender to provide a work of good quality;
- With uncertainty, the sender acts a priori, whereas he will be judged a posteriori.

I think that those two items are part of the factors that can explain the CRAs behavior observed over the last decades and also the reaction of the market towards CRA.

Appendix A

Appendix Title

A.1 Appendix - Public Information from a Private Entity:

Proof of proposition 3.1: Agent i's objective is:

$$\underset{a_i \in \mathbb{R}}{\text{maximize}} \quad E_i(u_i(a_i))$$

Or, $E_i(u_i) = E_i\big(-(1-r)(a_i-\theta)^2 - r(L_i - \bar{L}) - C_{inv}(\beta_i)\big)$

$= -(1-r)E_i(a_i^2 - 2a_i\theta + \theta^2) - rE_i(\int_0^1 (a_i - a_j)^2 \mathrm{d}j) + r\bar{L} - C_{inv}(\beta_i)$

$= -(1-r)(a_i^2 - 2a_iE_i(\theta) + E_i(\theta^2)) - r(a_i^2 - 2a_iE_i(\int_0^1 a_j \mathrm{d}j) + E_i(\int_0^1 a_j^2 \mathrm{d}j) + r\bar{L}) - C_{inv}(\beta_i).$

We should notice that, as the agent is atomic, he will consider $\bar{L}$ as a constant. As $E_i(u_i(a_i))$ is concave in a_i, the first order condition gives us the action a_i^* in equilibrium:

$$a_i^* = (1-r)E_i(\theta) + rE_i(\int_0^1 a_j \mathrm{d}j)$$

End of proof.□

Proof of proposition 3.2: This proof comes from Colombo and Femminis (2008).

- First step, a_i and a_j : We construct a linear equilibrium in which $a_j = \gamma_j y + (1-\gamma_j)x_j$. Assuming that all agents other than i follow a similar strategy and choose the same precision of information $\beta = \beta_j$, implying $\gamma_j = \gamma \ \forall j \neq i$, we have: $\int_0^1 a_j \mathrm{d}_j = \gamma y + (1-\gamma)\theta$.

Then agent i's best response reduces to:

$$a_i = (1-r)E_i(\theta) + r(\gamma y + (1-\gamma)E_i(\theta)) = (1 - r\gamma)E_i(\theta) + r\gamma y$$

Where, $E_i(\theta) = \delta_i y + (1-\delta_i)x_i$ with $\delta_i = \dfrac{\alpha}{\alpha + \beta_i}$, thus $a_i = \gamma_i y + (1-\gamma_i)x_i$ where $\gamma_i = (1-r\gamma)\delta_i + r\gamma$

Because agent i has zero-measure, the equilibrium value of γ when all agents j other than i choose a precision $\beta_j = \beta$ is given by: $\gamma = \dfrac{\delta}{1 - r(1-\delta)}$ with $\delta = \dfrac{\alpha}{\alpha + \beta}$.

With $\gamma = \dfrac{\delta}{1-r(1-\delta)}$ and $\gamma_i = (1-r\gamma)\delta_i + r\gamma$, I obtain: $\gamma_i = \dfrac{(1-r)\delta_i + r\delta}{1-r(1-\delta)}$.

- Second step, β_i:

I get back to the expected utility of agent i:

$$E_i[u_i(\mathbf{a},\theta,\beta_i)] = -(1-r)E_i[(a_i-\theta)^2] - rE_i\big[\int_0^1 (a_i-a_j)^2 \mathrm{d}j\big] + rE\big[\int_0^1\int_0^1 (a_h-a_j)^2 \mathrm{d}h\mathrm{d}j\big] - C_{inv}(\beta_i) \tag{A0}$$

I look at the first addendum of equation $(A0)$:

$$-(1-r)E_i\big[(a_i-\theta)^2\big] = -(1-r)E_i\big[(\gamma_i y + (1-\gamma_i)x_i - \theta)^2\big] \tag{A1}$$

Recalling that $y = \theta + \eta$, $x_i = \theta + \varepsilon_i$ and that signals are not correlated:

$$-(1-r)E_i[(a_i-\theta)^2] = -(1-r)\big[\frac{\gamma_i^2}{\alpha} + \frac{(1-\gamma_i)^2}{\beta_i}\big]$$

Recalling that $\gamma_i = \dfrac{(1-r)\delta_i + r\delta}{1-r(1-\delta)}$, $\delta_i = \dfrac{\alpha}{\alpha+\beta_i}$ and $\delta = \dfrac{\alpha}{\alpha+\beta}$,

$$-(1-r)E_i[(a_i-\theta)^2] = -(1-r)\frac{\dfrac{[(1-r)\delta_i + r\delta]^2}{\alpha} + \dfrac{[(1-r)(1-\delta_i)]^2}{\beta_i}}{[1-r(1-\delta)]^2}$$

Which can be rearranged:

$$-(1-r)E_i[(a_i-\theta)^2] = \frac{-(1-r)}{[1-r(1-\delta)]^2}\Big[(1-r)^2[\frac{\delta_i^2}{\alpha} + \frac{(1-\delta_i)^2}{\beta_i}] + \frac{2r(1-r)\delta\delta_i}{\alpha} + \frac{r^2\delta^2}{\alpha}\Big]$$

Because $\delta_i = \dfrac{\alpha}{\alpha+\beta_i}$, $\dfrac{\delta_i^2}{\alpha} + \dfrac{(1-\delta_i)^2}{\beta_i} = \dfrac{1}{\alpha+\beta_i} = \dfrac{\delta_i}{\alpha}$

I obtain:

$$-(1-r)E_i[(a_i-\theta)^2] = -\frac{(1-r)}{[1-r(1-\delta)]^2}[\frac{(1-r)^2\delta_i}{\alpha} + \frac{2r(1-r)\delta\delta_i}{\alpha} + \frac{r^2\delta^2}{\alpha}] \tag{A2}$$

That's it for the first addendum. Knowing that $a_i = \gamma_i y + (1-\gamma_i)x_i$ $a_j = \gamma y + (1-\gamma)x_j$, the second addendum of $(A0)$ gives us:

$$-rE_i[\int_0^1 (a_i-a_j)^2 \mathrm{d}_j] = -rE_i[\int_0^1 \big((\gamma-\gamma_i)y + (1-\gamma)x_j - (1-\gamma_i)x_i\big)^2 \mathrm{d}_j] \tag{A3}$$

Which yields:

$$-r[\frac{(\gamma-\gamma_i)^2}{\alpha} + \frac{(1-\gamma)^2}{\beta} + \frac{(1-\gamma_i)^2}{\beta_i}] \tag{A4}$$

This is it for the second addendum. I look at the third one now:

$$rE_i\big[\int_0^1\int_0^1(a_h-a_j)^2\mathrm{d}j\mathrm{d}h\big]=\frac{2r(1-\gamma)^2}{\beta} \tag{A5}$$

When summing the last two addendum ((*A*4) and (*A*5)), I get:

$$-rE_i\big[\int_0^1(a_i-a_j)^2\mathrm{d}j\big]-rE_i\big[\int_0^1(a_i-a_j)^2\mathrm{d}j\big]=-r\big[\frac{(\gamma-\gamma_i)^2}{\alpha}+\frac{(1-\gamma_i)^2}{\beta_i}-\frac{(1-\gamma)^2}{\beta}\big]$$

When substituting γ and γ_i:

$$-\frac{r}{[1-r(1-\delta)]^2}\big[\frac{(1-r)^2(\delta-\delta_i)^2}{\alpha}+\frac{(1-r)^2(1-\delta_i)^2}{\beta_i}-\frac{(1-r)^2(1-\delta)^2}{\beta}\big]$$

I can rearrange this equation:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\big[\delta^2(\frac{\beta-\alpha}{\alpha\beta})+\delta_i^2(\frac{\beta_i+\alpha}{\alpha\beta_i})-\frac{2\delta\delta_i}{\alpha}+\frac{(1-2\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\big]$$

From the definition of δ_i, I have: $\delta_i^2\frac{(\alpha+\beta_i)}{\alpha\beta_i}=\frac{\delta_i}{\beta_i}$. Hence:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\big[\delta^2(\frac{\beta-\alpha)}{\alpha\beta})-\frac{2\delta\delta_i}{\alpha}+\frac{(1-\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\big]$$

Substituting δ_i and δ for the two last addendum in the square brackets:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\big[\delta^2(\frac{\beta-\alpha}{\alpha\beta})-\frac{2\delta\delta_i}{\alpha}+\frac{\delta_i}{\alpha}-\delta\frac{\beta-\alpha}{\alpha\beta}\big]$$

And rearranging:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\big[\delta(\delta-1)(\frac{\beta-\alpha}{\alpha\beta})+\delta_i\frac{(1-2\delta)}{\alpha}\big]$$

Recall that $(1-2\delta)=\frac{\beta-\alpha}{\alpha+\beta}$, then $\frac{1-2\delta}{\alpha}=\frac{\beta-\alpha}{\alpha\beta}\frac{\beta}{\alpha+\beta}=\frac{\beta-\alpha}{\alpha\beta}(1-\delta)$. We get:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}(\frac{\beta-\alpha}{\alpha\beta})(1-\delta)(\delta_i-\delta)$$

When summing with the first addendum (*A*2):

$$-\frac{(1-r)}{[1-r(1-\delta)]^2}\big[\frac{(1-r)^2\delta_i}{\alpha}+\frac{2(1-r)r\delta\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}+r(1-r)(\frac{\beta-\alpha}{\alpha\beta})(1-\delta)(\delta_i-\delta)\big]$$

This yields:

$$-\frac{(1-r)}{[1-r(1-\delta)]^2}\Big[\big[(1-r)^2+2(1-r)r\delta+r(1-r)(1-\delta)\big]\frac{\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}-r(1-r)(1-\delta)\frac{\delta_i}{\beta}-r(1-r)(1-\delta)\delta(\frac{\beta-\alpha}{\alpha\beta})\Big]$$

Collecting $r\delta$ for the second ant the fourth addendum of the brackets, I obtain: $r\delta[r\delta\alpha^{-1}-(1-r)(1-\delta)(\frac{1}{\alpha}-\frac{1}{\beta}]$ That is :$r\delta[(-1+r+\delta)\frac{1}{\alpha}+(1-r)(1-\delta)\frac{1}{\beta}]$ And because $\frac{(1-\delta)}{\beta}=\frac{\delta}{\alpha}$ the last expression can be rewritten: $r\delta[\delta-(1-r)(1-\delta)]\frac{1}{\alpha}$ using again $\frac{(1-\delta)}{\beta}=\frac{\delta}{\alpha}$ the third addendum gives $-r(1-r)\delta\delta_i\frac{1}{\alpha}$ then the equation can be rewritten:

$$-\frac{(1-r)}{\alpha[1-r(1-\delta)]^2}\big[(1-r)\delta_i+r\delta[\delta-(1-r)(1-\delta)]\big]$$

Substituting for $\delta=\frac{\alpha}{\alpha+\beta}$ and $\delta_i=\frac{\alpha}{\alpha+\beta_i}$:

$$-\frac{(1-r)}{[\alpha(1-r(1-\frac{\alpha}{\alpha+\beta})]^2}\big[(1-r)\frac{\alpha}{\alpha+\beta_i}+r\frac{\alpha}{\alpha+\beta}[\frac{\alpha}{\alpha+\beta}-(1-r)\frac{\beta}{\alpha+\beta}]\big] \tag{A.1}$$

and simplifying:

$$-\frac{(1-r)(\alpha+\beta)^2}{[\alpha+(1-r)\beta]^2}\big[\frac{1-r}{\alpha+\beta_i}+\frac{r}{(\alpha+\beta)^2}(\alpha-(1-r))\beta\big] \tag{A.2}$$

And then we get:

$$E[u_i(\mathbf{a},\theta,\beta_i)]=-\frac{(1-r)(\alpha+\beta)^2}{(\alpha+(1-r)\beta)^2)}(\frac{1-r}{\alpha+\beta_i}+\frac{r(\alpha+(r-1)\beta)}{(\alpha+\beta)^2})-C_{inv}(\beta_i) \tag{A.3}$$

Agent i chooses β_i, because $E[u_i(\mathbf{a},\theta,\beta_i)]$ is concave in β_i the first order condition:

$$\frac{(1-r)^2(\alpha+\beta)^2}{(\alpha+(1-r)\beta)^2(\alpha+\beta_i)^2}-C'_{inv}(\beta_i)-\mu=0 \tag{A.4}$$

Where $\mu\geq 0$ it the Lagrange multiplier associated to the constraint $\beta_i\geq 0$ We need to consider two cases:

- When $C'_{inv}(0)\geq(\frac{(1-r)}{\alpha})^2$ the unique symmetric equilibrium is given by $\beta_i=0$.
- When $C'_{inv}(0)<(\frac{(1-r)}{\alpha})^2$ we have: $\beta_i=\beta=\frac{1}{\sqrt{C'_{inv}(\beta_i)}}-\frac{\alpha}{1-r}$

End of proof.□

Proof of proposition 3.4:

The CRA must find the accuracy of its public signal that would maximize its profit.

Recalling that $\Pi_{CRA}(\alpha) = -\int_0^1 (a_i - y)^2 \mathrm{d}i - k\alpha$, $\beta = \frac{1}{\sqrt{p}} - \frac{\alpha}{1-r}$ and $a_i = \frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)}$.

$$E(\Pi_{CRA}(\alpha)|\mathbf{a}, \beta) = E(-\int_0^1 (\frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)} - y)^2 \mathrm{d}i - k\alpha)$$

$$= -E(\int_0^1 (\frac{\alpha y + \beta(1-r)x_i}{\alpha + \beta(1-r)} - y)^2 \mathrm{d}i) - k\alpha$$

$$= -\int_0^1 (\frac{\sqrt{p}\beta(1-r)}{1-r})^2 E((x_i - y)^2) \mathrm{d}i - k\alpha$$

$$= -p\beta^2(\frac{1}{\beta} + \frac{1}{\alpha}) - k\alpha$$

$$= -\sqrt{p} + \frac{p}{1-r}\alpha - \frac{1}{\alpha} - \frac{p}{(1-r)^2}\alpha + 2\frac{\sqrt{p}}{1-r} - k\alpha$$

$$E(\Pi_{CRA}(\alpha)|\mathbf{a}, \beta) = -\sqrt{p} + 2\frac{\sqrt{p}}{1-r} - \frac{1}{\alpha} - \alpha(\frac{rp}{(1-r)^2}) - k\alpha$$

$E(\Pi_{CRA}(\alpha)|\mathbf{a}, \beta)$ is concave in α

The program considered is the following:

$$\begin{aligned} &\underset{\alpha \in \mathbb{R}}{\text{Maximize}} \quad E(\Pi_{CRA}(\alpha)|\mathbf{a}, \beta) \\ &\text{subject to} \quad 0 \leq \alpha \leq \frac{1-r}{\sqrt{p}} \end{aligned} \tag{A.5}$$

To solve this problem, I consider the following lagrangian:

$$\mathcal{L}[\pi(\alpha), \mu_1, \mu_2] = -\sqrt{p} + 2\frac{\sqrt{p}}{1-r} - \frac{1}{\alpha} - \alpha(\frac{rp}{(1-r)^2}) - k\alpha + \mu_1\alpha - \mu_2\frac{1-r}{\sqrt{p}}\alpha \tag{A.6}$$

The first order condition gives us:

$$\frac{\partial E(\pi)}{\partial \alpha} = \frac{1}{\alpha^2} - \frac{rp}{(1-r)^2} - k - \mu_1 - \mu_2\frac{1-r}{\sqrt{p}} = 0$$

Where μ_1 and μ_2 are the lagrangian multiplier associated to the constraints $\alpha \geq 0$ and $\alpha \leq \frac{1-r}{\sqrt{p}}$.

It should be noted that $\lim\limits_{\substack{\alpha \to 0 \\ \alpha > 0}} \frac{\partial E(\pi)}{\partial \alpha} = +\infty$, thus $\alpha = 0$ is not a solution.

Only two cases are left: $\alpha_1^* = \frac{1-r}{\sqrt{rp + k(1-r)^2}}$ or, $\alpha_2^* = \frac{1-r}{\sqrt{p}}$ with $\mu_2 > 0$.

From the first order condition, I obtain: $\mu_2 = (\frac{1}{\alpha^2} - \frac{rp}{(1-r)^2} - k)\frac{\sqrt{p}}{1-r}$ $\mu_2 = (\frac{p}{1-r} - k)\frac{\sqrt{p}}{1-r}$ thus $\mu_2 > 0$ if and only if $p > k(1-r)$.

When $p > k(1-r)$, I obtain $\alpha_1^* > \alpha_2^*$ and $\beta^* = 0$. The only solution is then α^* when $p < k(1-r)$ the only solution is $\alpha_1^* = \dfrac{1-r}{\sqrt{rp + k(1-r)^2}}$.

It can be summarize by $\alpha^* = min(\dfrac{1-r}{\sqrt{rp + k(1-r)^2}}, \dfrac{1-r}{\sqrt{p}})$ End of Proof.□

Proof of proposition 3.5:
I proceed in two steps, I first seek the best investors' action from a social planner point of view (a_i^{FB}), then I solve the maximization problem to find the best choice of public and private information's accuracy: $(\alpha^{FB}, \beta^{FB})$.

Calcul of a_i^{FB}:

We need to evaluate the best action a social planner which could define investors' behavior. The social planner aims at maximizing welfare, according to α,β and $a_i \forall i \in (0,1)$.

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = \int_0^1 u_i \mathrm{d}i + \Pi_{CRA} = -(1-r)\int_0^1 (a_i - \theta)^2 \mathrm{d}i - C_{inv}(\beta) - \int_0^1 (a_i - y)^2 \mathrm{d}i - C_{CRA}(\alpha)$$

I look at the best investor's action chosen from the point of view of the social planner. I seek a linear solution where $a_i = \gamma y + (1-\gamma)x_i$ where γ is a parameter to be found in equilibrium. Then,

$$\begin{aligned} \underset{a_i \in \mathbb{R}}{\text{Maximize}} \quad & E(W(\mathbf{a},\alpha,\beta)) \\ \text{subject to} \quad & a_i = \gamma y + (1-\gamma)x_i \end{aligned} \tag{A.7}$$

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = E[\int_0^1 -(1-r)(a_i - \theta)^2 - (a_i - y)^2 \mathrm{d}i - p\beta - k\alpha]$$

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = \int_0^1 E[-(1-r)(a_i - \theta)^2 - (a_i - y)^2]\mathrm{d}i - p\beta - k\alpha$$

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = -(1-r)\int_0^1 E[(\gamma y + (1-\gamma)x_i - \theta)^2]\mathrm{d}i - \int_0^1 E[(\gamma y + (1-\gamma)x_i - y)^2]\mathrm{d}i - p\beta - k\alpha$$

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = -(1-r)\int_0^1 E[(\gamma\eta + (1-\gamma)\varepsilon_i)^2]\mathrm{d}i - \int_0^1 E[(1-\gamma)^2(\varepsilon_i - \eta)^2]\mathrm{d}i - p\beta - k\alpha$$

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = -(1-r)[\frac{\gamma^2}{\alpha} + \frac{(1-\gamma)^2}{\beta}] - (1-\gamma)^2[\frac{1}{\beta} + \frac{1}{\alpha}] - p\beta - k\alpha$$

$$= -(1-r)\big[\frac{\gamma^2}{\alpha} + \frac{\gamma^2 - 2\gamma + 1}{\beta}\big] - (\gamma^2 - 2\gamma + 1)[\frac{1}{\beta} + \frac{1}{\alpha}] - p\beta - k\alpha$$

$$= \gamma^2\big[\frac{-(2-r)}{\alpha} + \frac{-(2-r)}{\beta}\big] + 2\gamma[\frac{1}{\alpha} + \frac{(2-r)}{\beta}] - \frac{2-r}{\beta} - p\beta - k\alpha$$

This expression is concave in γ, thus the γ that will maximise the social welfare is given by the first order condition:

$$2\gamma\big[\frac{-(2-r)}{\alpha} + \frac{-(2-r)}{\beta}\big] + 2[\frac{1}{\alpha} + \frac{(2-r)}{\beta}] = 0 \tag{A.8}$$

Then I obtain the value of $\gamma = \dfrac{\alpha(2-r)+\beta}{(\alpha+\beta)(2-r)}$, and $a_i^{FB} = \dfrac{y(\alpha(2-r)+\beta) + x_i\beta(1-r)}{(\alpha+\beta)(2-r))}$.

Looking for α^{FB}, β^{FB}:

Knowing a_i^{FB} we can seek the information choice made by the social planner, i.e. α^{FB}, β^{FB}:

$$\begin{aligned} &\underset{(\alpha,\beta)\in(\mathbb{R}_+^2)}{\text{maximize}} && E(W(\mathbf{a},\alpha,\beta)) \\ &\text{subject to} && \alpha,\beta \geq 0. \end{aligned} \tag{A.9}$$

I recall that

- $a_i = \dfrac{y(\alpha(2-r)+\beta) + x_i\beta(1-r)}{(\alpha+\beta)(2-r))} = \theta + \dfrac{\eta(\alpha(2-r)+\beta) + \varepsilon_i\beta(1-r)}{(\alpha+\beta)(2-r)}$
- $E(\eta\varepsilon_i) = 0$, $\forall i$
- $E(\eta^2) = \dfrac{1}{\alpha}$
- $E(\varepsilon_i^2) = \dfrac{1}{\beta}$.

$$E[W(\alpha,\beta,\theta,\mathbf{a})] = E[\int_0^1 -(1-r)(a_i-\theta)^2 - (a_i-y)^2 \mathrm{d}i - p\beta - k\alpha]$$

$$E[W(\mathbf{a},\alpha,\beta)] = -(1-r)\left[\frac{\frac{1}{\alpha}\big[(\alpha+\beta)+\alpha(1-r)\big]^2 + \frac{1}{\beta}\big[\beta(1-r)\big]^2}{(\alpha+\beta)^2(2-r)^2}\right] - \frac{\beta^2(1-r)^2}{(\alpha+\beta)^2(2-r)^2}[\frac{1}{\alpha}+\frac{1}{\beta}] - p\beta - k\alpha$$

. Which leads to:

$$E[W(\mathbf{a},\alpha,\beta)] = -\frac{(1-r)}{(\alpha+\beta)^2(2-r)^2}\left[\frac{(\alpha+\beta)^2}{\alpha}+\alpha(1-r)^2+2(\alpha+\beta)(1-r)+\beta(1-r)^2\right] - \frac{\beta(1-r)^2}{(\alpha+\beta)^2(2-r)^2} - \frac{\beta^2(1-r)^2}{\alpha(\alpha+\beta)^2(2-r)^2} - p\beta - k\alpha$$

$$E[W(\mathbf{a},\alpha,\beta)] = -\frac{(1-r)}{(\alpha+\beta)^2(2-r)^2}\left[\alpha+2\beta+\frac{\beta^2}{\alpha}+(\alpha+\beta)(1-r)^2+2(\alpha+\beta)(1-r)\right] - \frac{\beta(1-r)^2}{(\alpha+\beta)^2(2-r)^2} - \frac{\beta^2(1-r)^2}{\alpha(\alpha+\beta)^2(2-r)^2} - p\beta - k\alpha$$

$$E[W(\mathbf{a},\alpha,\beta)] = -\frac{(1-r)}{(\alpha+\beta)^2(2-r)^2}\left[\alpha+2\beta+(\alpha+\beta)(1-r)^2+2(\alpha+\beta)(1-r)+\beta(1-r)\right] - \frac{\beta^2(1-r)(2-r)}{\alpha(\alpha+\beta)^2(2-r)^2} - p\beta - k\alpha$$

$$E[W(\mathbf{a},\alpha,\beta)] = \frac{-(1-r)^3-2(1-r)^2-(1-r)}{(\alpha+\beta)(2-r)^2} - \frac{(1-r)(2-r)\beta}{(\alpha+\beta)^2(2-r)^2} - \frac{\beta^2(1-r)}{\alpha(\alpha+\beta)^2(2-r)} - p\beta - k\alpha$$

$$E[W(\mathbf{a},\alpha,\beta)] = \frac{-(1-r)[(1-r)^2+2(1-r)+1]}{(\alpha+\beta)(2-r)^2} - \frac{(1-r)\beta}{(\alpha+\beta)^2(2-r)} - \frac{\beta^2(1-r)}{\alpha(\alpha+\beta)^2(2-r)} - p\beta - k\alpha$$

$$E[W(\mathbf{a},\alpha,\beta)] = -\frac{1-r}{(\alpha+\beta)} - \frac{(1-r)\beta}{(\alpha+\beta)^2(2-r)} - \frac{\beta^2(1-r)}{\alpha(\alpha+\beta)^2(2-r)} - p\beta - k\alpha$$

First, I show the concavity of the social welfare function. To do so, I proceed in two steps:

- I first show that $\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha} < 0$;
- Then I show that $\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha^2}\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta^2} - \frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha}\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} > 0$.

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha} = \frac{1-r}{(\alpha+\beta)^2} + 2\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^3} + \frac{\beta^2(1-r)[(\alpha+\beta)^2+2\alpha(\alpha+\beta)]}{\alpha^2(\alpha+\beta)^4(2-r)} - k \quad \text{(A.10)}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha} = \frac{1-r}{(\alpha+\beta)^2} + 2\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^3} + \frac{\beta^2(1-r)(3\alpha+\beta)}{\alpha^2(\alpha+\beta)^3(2-r)} - k \quad \text{(A.11)}$$

$$\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha^2} = -2\frac{1-r}{(\alpha+\beta)^3} - 6\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^4} + \frac{\beta^2(1-r)[3\alpha^2(\alpha+\beta)^3 - (3\alpha+\beta)(3\alpha^2(\alpha+\beta)^2 + 2\alpha(\alpha+\beta)^3)]}{\alpha^4(\alpha+\beta)^6(2-r)} \tag{A.12}$$

$$\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha^2} = -2\frac{1-r}{(\alpha+\beta)^3} - 6\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^4} + \frac{\beta^2(1-r)[3\alpha(\alpha+\beta) - (3\alpha+\beta)(3\alpha+2(\alpha+\beta))]}{\alpha^3(\alpha+\beta)^4(2-r)} \tag{A.13}$$

$$\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha^2} = -2\frac{1-r}{(\alpha+\beta)^3} - 6\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^4} - \frac{\beta^2(1-r)[6\alpha^2 + 2(3\alpha+\beta)(\alpha+\beta)]}{\alpha^3(\alpha+\beta)^4(2-r)} \tag{A.14}$$

α and β being positive: $\dfrac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha^2} < 0$.

Now I look at $\dfrac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha^2}\dfrac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta^2} - \dfrac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta \partial \alpha}\dfrac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha \partial \beta}$.

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta} = \frac{1-r}{(\alpha+\beta)^2} - \frac{(1-r)}{(2-r)}\frac{[(\alpha+\beta)^2 - 2\beta(\alpha+\beta)]}{(\alpha+\beta)^4} - \frac{(1-r)}{\alpha(2-r)}\frac{[2\beta(\alpha+\beta)^2 - 2\beta^2(\alpha+\beta)]}{(\alpha+\beta)^4} - p \tag{A.15}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta} = \frac{1-r}{(\alpha+\beta)^2} - \frac{(1-r)}{(2-r)}\frac{(\alpha-\beta)}{(\alpha+\beta)^3} - \frac{(1-r)}{\alpha(2-r)}\frac{2\beta\alpha}{(\alpha+\beta)^3} - p \tag{A.16}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta} = \frac{1-r}{(\alpha+\beta)^2} - \frac{(1-r)}{(2-r)}\frac{(\alpha-\beta)+2\beta}{(\alpha+\beta)^3} - p \tag{A.17}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta} = \frac{(1-r)^2}{(2-r)(\alpha+\beta)^2} - p \tag{A.18}$$

$$\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial \beta^2} = -2\frac{(1-r)^2}{(2-r)(\alpha+\beta)^3}$$

Then I look at $\dfrac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha \partial \beta}$, recalling that:

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha} = \frac{1-r}{(\alpha+\beta)^2} + 2\frac{(1-r)\beta}{(2-r)(\alpha+\beta)^3} + \frac{\beta^2(1-r)(3\alpha+\beta)}{\alpha^2(\alpha+\beta)^3(2-r)} - k \tag{A.19}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha \partial \beta} = -2\frac{1-r}{(\alpha+\beta)^3} + 2\frac{(1-r)}{(2-r)}\frac{[(\alpha+\beta)^3 - 3\beta(\alpha+\beta)^2]}{(\alpha+\beta)^6} + \frac{(1-r)}{\alpha^2(2-r)}\frac{\left[(2\beta(3\alpha+\beta)+\beta^2)(\alpha+\beta)^3 - 3\beta^2(3\alpha+\beta)(\alpha+\beta)^2\right]}{(\alpha+\beta)^6} \tag{A.20}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} = -2\frac{1-r}{(\alpha+\beta)^3} + 2\frac{(1-r)}{(2-r)}\frac{(\alpha-2\beta)}{(\alpha+\beta)^4} + \frac{(1-r)}{\alpha^2(2-r)}\frac{\left[(2\beta(3\alpha+\beta)+\beta^2)(\alpha+\beta)-3\beta^2(3\alpha+\beta)\right]}{(\alpha+\beta)^4} \tag{A.21}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} = -2\frac{1-r}{(\alpha+\beta)^3} + 2\frac{(1-r)}{(2-r)}\frac{(\alpha-2\beta)}{(\alpha+\beta)^4} + \frac{(1-r)}{(2-r)}\frac{6\beta}{(\alpha+\beta)^4} \tag{A.22}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} = -2\frac{(1-r)^2}{(2-r)(\alpha+\beta)^3} \tag{A.23}$$

Then I look at $\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha}$, recalling that:

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta} = \frac{(1-r)^2}{(2-r)(\alpha+\beta)^2} - p \tag{A.24}$$

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha} = -2\frac{(1-r)^2}{(2-r)(\alpha+\beta)^3} \tag{A.25}$$

We should note that:

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha} = \frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta^2} = \frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} \tag{A.26}$$

and as $\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha^2} < \frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta^2}$ Thus it follows that :

$$\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha^2}\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta^2} - \frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha}\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} > 0 \tag{A.27}$$

As $\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha^2} < 0$
and $\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha^2}\frac{\partial^2 E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta^2} - \frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\beta\partial\alpha}\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial\alpha\partial\beta} > 0$, the social welfare function is concave.

I can know consider the following Lagrangian :

$$\mathcal{L}[\alpha,\beta,\lambda_\alpha,\lambda_\beta] = E(W(\mathbf{a},\alpha,\beta)) + \lambda_\alpha\alpha + \lambda_\beta\beta \tag{A.28}$$

All constraints are linear thus qualified, and the welfare function is concave. I use the Kuhn and Tucker conditions to find the solution to this program.

The first order conditions give us :

1. $\dfrac{\partial \mathcal{L}[\alpha, \beta, \lambda_\alpha, \lambda_\beta]}{\partial \alpha} = 0$

2. $\dfrac{\partial \mathcal{L}[\alpha, \beta, \lambda_\alpha, \lambda_\beta]}{\partial \beta} = 0$

3. $\lambda_\alpha \geq 0$, $\alpha \geq 0$ and $\lambda_\alpha \alpha = 0$

4. $\lambda_\beta \geq 0$, $\beta \geq 0$ and $\lambda_\beta \beta = 0$

We have four cases to consider:

a. $\alpha = 0$, $\beta = 0$, $\lambda_\alpha > 0$ and $\lambda_\beta > 0$

b. $\alpha = 0$, $\beta > 0$, $\lambda_\alpha > 0$ and $\lambda_\beta = 0$

c. $\alpha > 0$, $\beta > 0$, $\lambda_\alpha = 0$ and $\lambda_\beta = 0$

d. $\alpha > 0$, $\beta = 0$, $\lambda_\alpha = 0$ and $\lambda_\beta > 0$

The two first cases ([a],[b]) are quickly excluded, actually:
$\dfrac{\partial E[W(\mathbf{a}, \alpha, \beta)]}{\partial \alpha} = \dfrac{\beta(1-r)^3}{(\alpha+\beta)^2(2-r)^2} + \dfrac{2(1-r)^2}{(\alpha+\beta)^2(2-r)^2} + \dfrac{(1-r)}{\alpha^2(2-r)^2} + \dfrac{\beta(1-r)^2[2\alpha+\beta]}{\alpha^2(\alpha+\beta)^2(2-r)^2} - k$
Then,

$\lim\limits_{\substack{\alpha \to 0 \\ \alpha > 0}} \dfrac{\partial E[W(\mathbf{a}, \alpha, \beta)]}{\partial \alpha} = +\infty.$

As $\dfrac{\partial \mathcal{L}[\alpha, \beta, \lambda_\alpha, \lambda_\beta]}{\partial \alpha} = \dfrac{\partial E[W(\mathbf{a}, \alpha, \beta)]}{\partial \alpha} + \lambda_\alpha = 0$ The condition $\lambda_\alpha > 0$ can't be met. Thus, there can't be any solution with $\alpha = 0$ and $\lambda_\alpha > 0$.

Then I look for an interior solution, the case [c], i.e. where $\alpha > 0$, $\beta > 0$ and $\lambda_\alpha = \lambda_\beta = 0$.

$$\frac{\partial \mathcal{L}[\alpha, \beta, \lambda_\alpha, \lambda_\beta]}{\partial \beta} = \frac{(1-r)^3 + 2(1-r)^2}{(\alpha+\beta)^2(2-r)^2} - \frac{\alpha(1-r)^2}{\alpha(\alpha+\beta)^2(2-r)^2} - p = 0$$

Thus,

$$\frac{(1-r)^3 + (1-r)^2}{(\alpha+\beta)^2(2-r)^2} - p = 0$$

$$(\alpha+\beta)^2 = \frac{(1-r)^2}{p(2-r)}$$

α and β being both positive, we have:

$$\beta = \frac{1-r}{\sqrt{p(2-r)}} - \alpha$$

With [1], we have, $\frac{\partial \mathcal{L}[\alpha,\beta,\lambda_\alpha,\lambda_\beta]}{\partial \alpha} = 0$:

$$\frac{(1-r)^3}{(\alpha+\beta)^2(2-r)^2} + \frac{2(1-r)^2}{(\alpha+\beta)^2(2-r)^2} + \frac{(1-r)}{\alpha^2(2-r)^2} + \frac{\beta(1-r)^2[2\alpha+\beta]}{\alpha^2(\alpha+\beta)^2(2-r)^2} - k = 0$$

With $(\alpha+\beta)^2 = \frac{(1-r)^2}{p(2-r)}$, we find:

$$\frac{(1-r)p}{2-r} + \frac{2p}{2-r} + \frac{(1-r)}{\alpha^2(2-r)^2} + \frac{p\beta(2\alpha+\beta)}{\alpha^2(2-r)} - k = 0$$

knowing that $\beta = \frac{1-r}{\sqrt{p(2-r)}} - \alpha$

$$\frac{(1-r)}{\alpha^2(2-r)^2} + \frac{p(\frac{1-r}{\sqrt{p(2-r)}} - \alpha)(\frac{1-r}{\sqrt{p(2-r)}} + \alpha)}{\alpha^2(2-r)} = k - \frac{(1-r)p}{2-r} - \frac{2p}{2-r}$$

$$\frac{(1-r)}{\alpha^2(2-r)^2} + \frac{p(\frac{(1-r)^2}{p(2-r)} - \alpha^2)}{\alpha^2(2-r)} = k - \frac{(1-r)p+2p}{2-r}$$

$$\frac{(1-r)+(1-r)^2}{\alpha^2(2-r)^2} = k + \frac{p}{2-r} - \frac{(1-r)p+2p}{2-r}$$

$$\frac{(1-r)}{\alpha^2(2-r)} = k + \frac{p}{2-r} - \frac{(1-r)p+2p}{2-r}$$

$$\frac{(1-r)}{\alpha^2(2-r)} = k - p$$

If $k < p$, thus there isn't a solution where $\alpha > 0$ and $\beta > 0$. Let's consider the case where $k > p$.

$$\alpha = \sqrt{\frac{1-r}{(2-r)(k-p)}} \tag{A.29}$$

Knowing that $\beta = \frac{1-r}{\sqrt{p(2-r)}} - \alpha$. It follows that $\beta = \frac{1-r}{\sqrt{p(2-r)}} - \sqrt{\frac{(1-r)}{(2-r)(k-p)}}$. I need to check that β is positive. $\beta = \frac{1-r}{\sqrt{p(2-r)}} - \sqrt{\frac{(1-r)}{(2-r)(k-p)}} > 0$ if and only if $k > (2-r)p$

If $k < (2-r)p$ I consider the only case left [d], for $\beta = 0$ and $\lambda_\alpha = 0$

We must consider $\frac{\partial \mathcal{L}[\alpha,\beta,\lambda_\alpha,\lambda_\beta]}{\partial \alpha} = \frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha} = 0$.

$$\frac{\partial E[W(\mathbf{a},\alpha,\beta)]}{\partial \alpha} = \frac{(1-r)^3}{\alpha^2(2-r)^2} + \frac{2(1-r)^2}{\alpha^2(2-r)^2} + \frac{(1-r)}{\alpha^2(2-r)^2} - k = 0$$

$$\frac{(1-r)^3 + 2(1-r)^2 + (1-r)}{\alpha^2(2-r)^2} = k$$

Then $\alpha^{FB} = \sqrt{\frac{1-r}{k}}$

With $\alpha^{FB} = \sqrt{\frac{1-r}{k}}$ I verify that $\frac{\partial E[W(\mathbf{a},\alpha^{FB},\beta)]}{\partial \beta} = -\lambda_\beta$

$$\frac{\partial \mathcal{L}[\alpha,\beta,\lambda_\alpha,\lambda_\beta]}{\partial \beta} = \frac{(1-r)^3 + 2(1-r)^2}{(\alpha+\beta)^2(2-r)^2} - \frac{\alpha(1-r)^2}{\alpha(\alpha+\beta)^2(2-r)^2} - p + \lambda_\beta = 0$$

Then,

$$\lambda_\beta = p - \frac{(1-r)^3 + 2(1-r)^2}{(\alpha+\beta)^2(2-r)^2} - \frac{\alpha(1-r)^2}{\alpha(\alpha+\beta)^2(2-r)^2}$$

Then,

$$\lambda_\beta = p - [\frac{(1-r)^3 + 2(1-r)^2}{\alpha^2(2-r)^2} + \frac{(1-r)^2}{\alpha^2(2-r)^2}]$$

Knowing that $\alpha = \sqrt{\frac{1-r}{k}}$

$$\lambda_\beta = p - k$$

Thus, $\lambda_\beta = p - k > 0$, the only solution is $(\alpha^{FB}, \beta^{FB}) = (\sqrt{\frac{1-r}{k}}, 0)$

The results can be sum-up:

- The action in equilibrium undertaken by each agent is $a_i^{FB} = \frac{y(\alpha(2-r)+\beta) + x_i\beta(1-r)}{(\alpha+\beta)(2-r))}$

- If $k < (2-r)p$, then $\alpha_i^{FB} = \sqrt{\frac{1-r}{k}}$ and $\beta_i^{FB} = 0$;

- If $k > (2-r)p$, then $\alpha_i^{FB} = \sqrt{\frac{1-r}{(2-r)(k-p)}}$ and $\beta_i^{FB} = \frac{1-r}{\sqrt{p(2-r)}} - \sqrt{\frac{(1-r)}{(2-r)(k-p)}}$;

End of proof □.

A.2 Appendix - Credit Rating Agencies, Business Cycles and Reputation:

Proof of proposition 4.1: The CRA must maximise its profit: $\pi(x_0) = x_0 q_0 - c$ for $x_0 \in [0,1]$. As q_0 is positive, the only solution is $x_0 = 1$.

Proof of proposition 4.2: I recall the profit function of the CRA:

$$\pi(x_0, x_1) = x_0 q_0 - c + \delta[x_1(F(Rp(x_0) - 1) - c] \qquad for x_0, x_1 \in (0,1) \tag{A.30}$$

First I suppose there exists a function $F(Rp(x) - 1)$ such that $F(0) = q_0$ and
$\bar{x} = \underset{x_0 \in (0,1)}{\arg\max} \pi(x_0, 1) = x_0 q_0 - c + \delta[F(Rp(x_0) - 1) - c]$ (I remind that $p(\bar{(}x)) = \frac{1}{R}$)

The CRA has two options, it could also give a good rating to all projects in the first period: $\pi(1,0) = q_0 - c - \delta c$ whereas $\pi(\bar{x}, 1) = q_0 \bar{(}x) - c + \delta(q_0 - c)$.

But $p(x)$ is stricly concave and such that $p(0) = 1$, $p(1) = 0$. Thus $\forall x \in (0,1) p(x) \geq 1 - x$. Thus $\bar{x} > 1 - \frac{1}{R}$, which leads that for any $p(x)$, $\bar{x} \geq 1 - \frac{1}{R}$

I remind that $\delta > \frac{1}{R}$

$$\pi(\bar{x}, 1) - \pi(1,0) = q_0(\bar{x} + \delta) - q_0 \geq q_0(\frac{1}{R} + \delta) \geq 0 \tag{A.31}$$

The CRA would then choose to abide by the disciplining rules as it ensures higher revenues. Now I need to verify that such an $F(.)$ function exists.

To obtain a unique solution, we need to find a F function such that $\pi(x_0, 1)$ is concave:

$$\frac{\partial^2 \pi(x_0)}{\partial x_0^2}(x_0) < 0 \qquad \forall x_0 \in (0,1) \tag{A.32}$$

$$\frac{\partial^2 \pi(x)}{\partial x^2}(x) = \delta[F"(Rp(x) - 1]R^2(p'(x))^2 + Rp"(x)F'(Rp(x) - 1)] \tag{A.33}$$

I consider $F(Rp(x) - 1)$ as an increasing function of the realized return: $F'(Rp(x) - 1) > 0$. $p(x)$ being strictly concave, we have $p"(x) < 0$. Thus if F is a concave function, it guarantees the existence of a maximum (obviously we could also find other functions but the goal here is just to verify if there is one).

$$\frac{\partial^2 \pi(x)}{\partial x^2}(x) < 0 \qquad \forall x \in (0,1) \tag{A.34}$$

Then we must ensure that the maximum will be obtain for $x = \bar{x}$:

$$\frac{\partial \pi(\bar{x_0}}{\partial x_0} = 0 \tag{A.35}$$

The reputational function must also give a constant reputational capital, that is $F(0) = q_0$. Any function that is concave and satisfies the previous equations fits. If I restrict the analysis to a linear function of the return realized, there exists only one such function. I need a linear function $F(Rp(x) - 1) = \alpha + \beta(Rp(x) - 1)$ where $\alpha, \beta \in \mathbb{R}^2$). I find:

$$F(Rp(x) - 1) = q_0 - \frac{q_0}{\delta p'(\bar{x}^*)}(Rp(x) - 1) \tag{A.36}$$

At least there exists also other non linear function such as:

$$F(Rp(x)-1) = q_0 - \frac{q_0}{\delta p'(\bar{x}^*)} \ln(Rp(x)) \tag{A.37}$$

The different solutions are characterized by different steeps. The punishment rule can be more or less tougher. I will discuss the implication of the choice of $F(Rp(x)$ when considering uncertainty.

End of proof.□

Proof of proposition 4.3:

The proof here is the same than without business cycles, as $p^B(x)$ and $p^R(x)$ are both strictly concave. End of proof.□

Proof of proposition 4.4: Now the CRA considers the following profit function:

$$\pi^{\theta}(x_0,1) = x_0 q_0 - c + \delta[\tau_B(x_1 F^R(Rp^R(x_0)-1)) + (1-\tau_B)(x_1 F^B(Rp^B(x_0)-1)) - c] \tag{A.38}$$

First I show that the CRA has no incentives to give a good rating to all products in the first period and abandon about future revenues:

First I check that the CRA has no incentives to give a good rating to all projects in the first period because of the uncertainty.

As it appears, it could also only give good rating considering the recession and obtain better revenues than by forgiving about the second period revenues. I show first that by giving a good grade as if the current state was a recession, $\bar{x}^R$ leads to higher revenues than by giving a good rating to all projects in the first period. If the actual state is a recession:

$$\pi^R(\bar{x}^R,1) = \bar{x}^R q_0 - c + \delta(q_0 - c) \tag{A.39}$$

If it is a boom

$$\pi^B(\bar{x}^R,1) = \bar{x}^R q_0 - c + \delta(F^B(Rp^B(\bar{x}^R-1) - c) \tag{A.40}$$

When giving all projects a good rating in the first period, the CRA obtains:

$$\pi^R(1,0) = \pi^B(1,0) = \bar{x}^R q_0 - c + \delta(q_0 - c) \tag{A.41}$$

It should be noticed that:

$$\pi^B(\bar{x}^R,1) \geq \pi^R(\bar{x}^R,1) \geq \pi^R(1,0) = \pi^B(1,0) = \bar{x}^R q_0 - c + \delta(q_0 - c) \tag{A.42}$$

The strategy consisting in $(x_0,x_1) = (1,0)$ is strictly dominated by $(x_0,x_1) = (\bar{x}^R,1)\ \forall \tau_B \in (0,1)$.

Now that I have shown that the CRA will not choose $(x_0,x_1) = (1,0)$ I need to consider two different situations:

When $x^R_{max} > x^B_{max}$:

It should be noted that $barx^R < barx^B$, thus $x^B_{max} > barx^R$

I first consider the CRA's problem for $x_0 \in (0, min(1, x^B_{max}))$.

The CRA maximizes, for $x_0 \in (0, x^B_{max})$:

$$\pi^\theta(x_0, 1) = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] \quad (A.43)$$

This function admits a maximum in $(0, x^B_{max})$, as $\pi^\theta(x_0, 1)$ is concave:

$$\frac{\partial^2 \pi^\theta(x_0, 1)}{\partial x_0^2} = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] \quad (A.44)$$

I show that the only solution x_0^* is such that $\bar{x}^R \leq x_0^* \leq \bar{x}^B$. In effects:

$$\frac{\partial E[\pi(\bar{x}^R, 1)]}{\partial x_0} = q_0 + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] > 0 \quad (A.45)$$

positive

$$E[\pi(\bar{x}^R, 1)] = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] < O \quad (A.46)$$

Thus x_0^* lies in $(\bar{x}^R, \bar{x}^B)$.

If $(x^B_{max} < 1$ I consider the CRA's problem for $x_0 \in (x^B_{max}, x^R_{max})$. In this situation, the CRA aims at maximizing:

$$\pi^\theta(x_0, 1) = x_0 q_0 - c + \delta[\tau_R(F^R(Rp^R(x_0) - 1)) - c] \quad (A.47)$$

Obviously any $\forall x_0 \in (x^B_{max}, x^R_{max})$, $\pi^\theta(x_0, 1) < \pi^R(\bar{x}^R, 1) = \bar{x}^R q_0 - c + \delta(q_0 - c)$

as $\bar{x}^R = \underset{x_0 \in (0,1)}{\arg\max} \pi(x_0, 1) = x_0 q_0 - c + \delta[(F^R(Rp^R(x_0) - 1)) - c] > x_0 q_0 - c + \delta[\tau_R(F^R(Rp^R(x_0) - 1)) - c]$.

And finally, I consider the CRA's problem for $x_0 \in (x^R_{max}, 1)$. Here the only solution to maximize its profit is to choose $x_0 = 1$ which leads not to an optimum as shown above. Thus the only solution is x_0^* is such that $\bar{x}^R \leq x_0^* \leq \bar{x}^B$.

When $x^R_{max} < 1$ and $x^R_{max} < \bar{x}^B$:

I first consider the CRA's problem for $x_0 \in (0, min(1, x^B_{max}))$.

The CRA maximizes, for $x_0 \in (0, min(1, x^B_{max}))$:

$$\pi^\theta(x_0, 1) = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] \quad (A.48)$$

This function admits a maximum in $(0, x^B_{max})$, as $\pi^\theta(x_0, 1)$ is concave:

$$\frac{\partial^2 \pi^\theta(x_0, 1)}{\partial x_0^2} = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] \quad (A.49)$$

I show that the only solution x_0^* is such that $\bar{x}^R \leq x_0^* \leq \bar{x}^B$. In effects:

$$\frac{\partial E[\pi(\bar{x}^R, 1)]}{\partial x_0} = q_0 + \delta[\tau_B(F^R(Rp^R(x_0) - 1)) + (1 - \tau_B)(F^B(Rp^B(x_0) - 1)) - c] > 0 \quad (A.50)$$

positive

$$E[\pi(\bar{x}^R,1)] = x_0 q_0 - c + \delta[\tau_B(F^R(Rp^R(x_0)-1)) + (1-\tau_B)(F^B(Rp^B(x_0)-1)) - c] < O \quad \text{(A.51)}$$

Thus x_0^* lies in $(\bar{x}^R, \bar{x}^B)$.

If $(x_{max}^R < 1$ I consider the CRA's problem for $x_0 \in (x_{max}^R, min(x_{max}^B, 1)$. In this region, the CRA knows that if the actual state of the economy is a recession, its reputational capital will be null, thus second period revenues are considered only if today's state is a boom. In this situation, the CRA aims at maximizing:

$$E[\pi^\theta(x_0,1)] = x_0 q_0 - c + \delta[\tau_B(F^B(Rp^B(x_0)-1)) - c] \quad \text{(A.52)}$$

The function is concave, we should notice that compare to the CRA's problem in the first period, future revenues account less than without uncertainty as the CRA might loose all its future revenues. So there exists an $x_0^{**} = \underset{x_0 \in (0,1)}{\arg\max} E[\pi(x_0,1)] = x_0 q_0 - c + \delta[\tau_B(F^B(Rp^B(x_0)-1)) - c]$

The CRA will obtain for revenues $\pi(x_0^{*1},1) = x_0^{*2} q_0 + \delta[\tau_B F_B(Rp_B(x_0^{*2})-1)]$:

Then I compare the revenues obtain in the two region considered: I define:

$$\pi_A = E[\pi(x_0^{*1},1)] = x_0^{*1} q_0 + \delta[\tau_B F_B(Rp_B(x_0^{*1})-1) + (1-\tau_B)F_R(Rp_R(x_0^{*1})-1) \quad \text{(A.53)}$$

and

$$\pi_B = E[\pi(x_0^{*1},1)] = x_0^{*2} q_0 + \delta[\tau_B F_B(Rp_B(x_0^{*2})-1)] \quad \text{(A.54)}$$

$$\pi_A - \pi_B = x_0^{*1} q_0 + \delta[\tau_B F_B(Rp_B(x_0^{*1})-1) + (1-\tau_B)F_R(Rp_R(x_0^{*1})-1) = q_0(x_0^{*1} - x_0^{*2}) + \delta[\tau_B(F_B(Rp_B(x_0^{*1})-1) - F \quad \text{(A.55)}$$

Thus

- If $q_0(x_0^{*2} - x_0^{*1} > \delta[\tau_B(F_B(Rp_B(x_0^{*1})-1) - F_B(Rp_B(x_0^{*2})-1)) + (1-\tau_B)F_R(Rp_R(x_0^{*1})-1)$ then $x^* = x_0^{*2}$
- Otherwise $x^* = x_0^{*1}$

End of proof.

Bibliography

Akerlof, G. A. (1970). The market for "lemons": Quality uncertainty and the market mechanism. *The Quarterly Journal of Economics*, 84(3):488–500.

Alfarano, S., Camacho, E., and Morone, A. (2015). Do investors rely too much on public information to be justified by its accuracy? An experimental study.

Ashcraft, A., Goldsmith-Pinkham, P., and Vickery, J. (2010). Mbs ratings and the mortgage credit boom. (449).

Bar-Isaac, H. and Shapiro, J. (2013). Ratings quality over the business cycle. *Journal of Financial Economics*, 108(1):62–78.

Becker, B. and Milbourn, T. (2008). Reputation and competition: evidence from the credit rating industry. (09-051).

Benabou, R. and Laroque, G. (1992). Using privileged information to manipulate markets insiders, gurus, and credibility. *The Quarterly Journal of Economics*, 107(3):921–58.

Bolton, P., Freixas, X., and Shapiro, J. (2012). The credit ratings game. *Journal of Finance*, 67(1):85–112.

Boot, A. W. A., Milbourn, T. T., and Schmeits, A. (2006). Credit Ratings as Coordination Mechanisms. *Review of Financial Studies*, 19(1):81–118.

Bouvard, M. and Levy, R. (2013). Two-sided reputation in certification markets. (446).

Colombo, L. and Femminis, G. (2008). The social value of public information with costly information acquisition. *Economics Letters*, 100(2):196–199.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–51.

Dasgupta, A. and Prat, A. (2008). Information aggregation in financial markets with career concerns. *Journal of Economic Theory*, 143(1):83–113.

Ferri, Liu, S. (1999). The procyclical role of rating agencies: Evidence from the east asian crisis. *Economic Notes by Banca Monte*, 28(3):335–355.

Fong, K. Y. L., Hong, H. G., Kacperczyk, M. T., and Kubik, J. D. (2014). Do Security Analysts Discipline Credit Rating Agencies? *AFA 2013 San Diego Meetings Paper*, pages 484–518.

Frenkel, S. (2015). Repeated Interaction and Rating Inflation: A Model of Double Reputation. *American Economic Journal: Microeconomics*, 7(1):250–80.

Fulghieri, P., Strobl, G., and Xia, H. (2014). The Economics of Solicited and Unsolicited Credit Ratings. *Review of Financial Studies*, 27(2):484–518.

Gorton, G. and Ordonez, G. (2014). Collateral Crises. *American Economic Review*, 104(2):343–78.

Grossman, S. J. and Stiglitz, J. E. (1980). On the Impossibility of Informationally Efficient Markets. *American Economic Review*, 70(3):393–408.

Han, S. H., Pagano, M. S., and Shin, Y. S. (2012). Rating Agency Reputation, the Global Financial Crisis, and the Cost of Debt. *Financial Management*, 41(4):849–884.

Jeffery D'Amato, C. H. F. (2003). Are credit ratings procyclical. *BIS Working paper*.

Mariano, B. (2012). Market power and reputational concerns in the ratings industry. *Journal of Banking & Finance*, 36(6):1616–1626.

Mathis, J., McAndrews, J., and Rochet, J.-C. (2009). Rating the raters: Are reputation concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics*, 56(5):657–674.

Mora, N. (2006). Sovereign credit ratings: Guilty beyond reasonable doubt? *Journal of Banking & Finance*, 30(7):2041–2062.

Morris, S. and Shin, H. S. (2002). Social Value of Public Information. *American Economic Review*, 92(5):1521–1534.

Ottaviani, M. and Sorensen, P. N. (2002). Professional advice: The theory of reputational cheap talk. (02-05).

Partnoy, F. (2001). The paradox of Credit Rating Agencies. *U San Diego Law & Econ Research*, 20.

Rancière, R., Tornell, A., and Westermann, F. (2008). Systemic Crises and Growth. (halshs-00754308).

Rochet, J.-C. and Tirole, J. (1996). Interbank Lending and Systemic Risk. *Journal of Money, Credit and Banking*, 28(4):733–762.

Sobel, J. (1985). A theory of credibility. *Review of Economic Studies*, 52(4):557–73.

Ui, T. (2014). The social value of public information with convex costs of information acquisition. *Economics Letters*, 125(2):249–252.

White, L. J. (2001). The Credit Rating Industry: An Industrial Organization Analysis.

White, L. J. (2010). The credit rating agencies. *Journal of Economic Perspectives*, 24(2):211–226.

Résumé

Les agences de notations ont récemment été l'objet d'une grande attention. Leur responsabilité dans la crise des subprimes a été questionné. Les médias ont mis en avant les notes trop généreuses qui avaient été attribué à certains produits complexe, avant de s'interroger sur leur comportement quand elles ont dégradé les notes des dettes souveraines.

Dans cette thèse, après avoir revu une partie de la littérature sur le sujet, je m'interroge sur deux aspects spécifique de leur activité:
(i) Quels sont les conséquences de confier une information publique (comme une note de crédit) à une entité privée ?
(ii) Les agences de notations disent avoir pour principal actif la réputation. Cette dernière peut expliquer pour on observe des périodes de sur-notations et d'autres de sous-notations ?

Abstract

Credit rating agencies have recently been under a lot of scrutiny. Their responsibility in the last financial crisis has been questioned. They received much attention from the media. The credit rating agencies have been blamed for their too generous ratings before the crisis and also for being too severe during the European debt crisis.

In this thesis, after an overlook of the recent literature, I look at two specific issues related to their activity:
(i) What issues arise when public information is released by a private entity on financial markets?
(ii) Can reputation explains why a credit rating agency can be caught underrating (respectively overrating)?

Mots Clés

Agences de notations
Réputation
Information transmission

Keywords

Credit Rating Agencies
Reputation
Information transmission