UNIVERSITÉ PARIS DAUPHINE
ÉCOLE DOCTORALE DE DAUPHINE

ILLIQUIDITÉ, CONTAGION ET RISQUE SYSTEMIQUE

THÈSE

pour l'obtention du titre de

DOCTEUR EN SCIENCES DE GESTION

(Arrêté du 7 août 2006)

présentée et soutenue publiquement par

Jérémy DUDEK

10 Décembre 2013

JURY

Directeurs de thèse	**Gaëlle Le Fol** Professeure, Université Paris-Dauphine
Rapporteurs	**René Garcia** Professeur, EDHEC Business School **Valérie Mignon** Professeure, Université Paris Ouest - Nanterre La Défense
Examinateurs	**Michel Crouhy** Directeur Recherche et Développement, Natixis **Serge Darolles** Professeur associé, Université Paris-Dauphine **Christophe Hurlin** Professeur, Université d'Orléans

L'Université n'entend donner aucune approbation ni improbation aux opinions émises dans les thèses : ces opinions doivent être considérées comme propres à leurs auteurs.

Remerciements

Je tiens tout d'abord à remercier Gaëlle Le Fol d'avoir accepté d'encadrer cette thèse et de m'avoir fait confiance tout au long de ce travail. Ses conseils, ses critiques et ses encouragements ont été indispensables à la réalisation de ce travail.

Je remercie également René Garcia et Valérie Mignon d'avoir accepté de rapporter cette thèse, Michel Crouhy, Serge Darolles et Christophe Hurlin d'avoir bien voulu faire partie du jury.

Je tiens à exprimer ma reconnaissance envers Serge Darolles pour sa disponibilité, son écoute et ses précieux conseils. Travailler à ses côtés fut un réel plaisir.

Je remercie également Christian Gouriéroux et Jean-Michel Zakoïan de m'avoir accueilli au sein du Laboratoire de Finance-Assurance du Crest et d'avoir manifesté un intérêt constant pour mes travaux.

Je remercie Nadine Guedj et Fanda Traoré d'avoir toujours trouvé une solution à tous mes problèmes sans jamais perdre patience mais aussi Sally qui savait nous rappeler que la journée était bientôt finie.

Un remerciement à l'ensemble des personnes ayant eu l'incroyable chance de s'asseoir plus d'une journée dans le bureau 1104. Bien que coincé au fond du couloir, il reste le principal facteur de réussite de ce travail. Merci à Guillaume et Giuseppe avec qui l'élaboration de nouvelles distractions revêt à elle seule un caractère de recherche théorique. Merci à eux pour l'ensemble des moments partagés également en dehors de ce bureau et pour leur persévérance face à mon incroyable talent une raquette à la main. Je remercie également Gulten et Mathilde qui ont su s'adapter à nos enfantillages. Mes remerciements vont à l'ensemble des membres du laboratoire, Erwan, Jean-Cyprien, Guillaume, les orléanais et plus particulièrement Sylvain et tout ceux que je n'aurai la place de citer dans ces quelques lignes.

Je remercie ma famille pour le soutien sans faille qu'elle a su m'apporter et sans qui je n'aurai pu réaliser ce travail ainsi que la famille Lallemant qui m'a accueilli et m'a soutenu inconditionnellement. Une pensée à tous mes amis "*pseudo-gamers*" avec qui je passe certaines de mes soirées et qui ont su forger l'être immature que je suis aujourd'hui.

Pour finir, une attention particulière pour Pauline, le "coeur" du bureau 1104, à qui revient tout le mérite de m'avoir supporté durant ces années mais également d'avoir fait le pari que je serai plus facile à vivre une fois que je serai docteur.

Résumé.

Cette thèse est articulée autour de trois risques financiers que sont : la liquidité, la contagion et le risque systémique. Ces derniers sont au centre de toutes les attentions depuis la crise de 2007-08 et resteront d'actualité à la vue des évènements que rencontrent les marchés financiers. Le premier chapitre de cette thèse présente un facteur de liquidité de financement obtenu par l'interprétation d'un phénomène de contagion en termes de risque de liquidité de marché. Nous proposons dans le second chapitre, une méta-mesure de cette liquidité de marché. Cette dernière tient compte de l'ensemble des dimensions présentes dans la définition de la liquidité en s'intéressant à la dynamique de plusieurs mesures de liquidité simultanément. L'objectif du troisième chapitre est de présenter une modélisation des rendements du marché permettant la prise en compte de la liquidité de financement dans l'estimation de la DCoVaR. Ainsi, ce travail propose une nouvelle mesure du risque systémique ayant un comportement contracyclique. Pour finir, nous nous intéressons à l'hypothèse de non-linéarité de la structure de dépendance entre les rendements de marché et ceux des institutions financières. Au coeur de la mesure du risque systémique, cette hypothèse apparait contraignante puisqu'elle n'a que peu d'impact sur l'identification des firmes les plus risquées mais peut compliquer considérablement l'estimation de ces mesures.

Mots clés : Liquidité, Contagion financière, Risque Systémique, Mesures de Liquidité.

Abstract.

The aim of this thesis is to improve the management of financial risks through the employment of econometric methods. We focus on liquidity (market and funding), contagion and systemic risk, which have attracted a particularly large interest in the last years of financial turmoil. Firstly, we construct a funding liquidity factor based on the contagion effects that market liquidity risks encounter. This procedure can be useful to provide a better management of the liquidity mismatch among the assets and liabilities of a fund. Secondly, we propose a meta-measure of liquidity which incorporates multiple liquidity measures through the use of a conditional correlation model. As a result, we are able to detect drastic liquidity problems by using a single measure. Thirdly, we propose a new modeling framework for financial returns by adding an extra component related to funding liquidity to the standard DCoVaR model. In this way we obtain a countercyclical measure of systemic risk. Finally, we study to which extent a change in the estimation method affects the identification of systemically relevant Financial Institutions. In particular, the most popular measures aim at capturing the nonlinearity of the dependence structure between financial firms and market returns. We show, however, that similar results can be obtained by simply assuming a linear dependence, which can also largely simplify the estimation.

Keywords: Liquidity, Financial Contagion, Systemic Risk, Liquidity measures.

Contents

Introduction **1**

1 Liquidity risk and contagion for liquid funds **23**

1.1 Introduction 25

1.2 Data 28

1.2.1 Emerging Sovereign Debt Markets 28

1.2.2 The description of the Sovereign Debt Markets 31

1.3 The Emerging Sovereign Debt Market Liquidity measure 32

1.3.1 The CDS Bond Spread basis 33

1.3.2 Basis trade with multiple currencies 37

1.4 Liquidity contagion and funding liquidity 39

1.4.1 Filtering a common liquidity factor 40

1.4.2 Identifying as a funding liquidity factor 42

1.5 Robustness Checks 45

1.5.1 The heteroscedasticity problem 45

1.5.2 Pure contagion in terms of prices 46

1.6 Conclusion 49

Appendices **51**

A.1 The CDS Bond Spread basis 51

A.2 Computing P&L 54

A.3 RSDC model 56

A.4 Estimation of RSDC 58

A.4.1 First Step 59

A.4.2 Second Step . . . 60
A.5 Correlations Matrices . . . 64
A.6 Robustness Check . . . 65

2 MLiq toward a Meta-measure of liquidity 67
2.1 Introduction . . . 69
2.2 MLiq measurement . . . 72
2.3 Data, liquidity measures and empirical evidences . . . 76
2.4 MLiq results . . . 82
2.4.1 Long/Short Portfolios . . . 84
2.4.2 Two-ways Portfolio Sorts . . . 86
2.4.3 The liquidity as a factor . . . 90
2.4.4 Leaking illiquid assets . . . 91
2.5 Conclusion . . . 96

Appendices 97
B.1 Estimation of RSDC . . . 97
B.1.1 First Step . . . 97
B.1.2 Second Step . . . 98
B.2 Figures . . . 102

3 Measuring Systemic Risk under liquidity constraints 107
3.1 Introduction . . . 109
3.2 Problematic . . . 112
3.2.1 Market Liquidity versus Funding Liquidity . . . 113
3.2.2 Measuring systemic risk . . . 114
3.2.3 $\Delta CoVaR$ and a threshold model . . . 116
3.3 Data and Summary Statistics . . . 119
3.3.1 Summary statistics . . . 119
3.3.2 The TED spread as a funding liquidity indicator . . . 120
3.4 Empirical Results and Discussion . . . 122
3.4.1 Estimation parameters and dynamic . . . 122

3.4.2 Impact on the identification of SIFIs . . . 126
3.4.3 Discussion . . . 129
3.5 Conclusion . . . 131

Appendices **133**
C.1 $\Delta CoVaR$ estimation . . . 133
C.2 Threshold Estimation . . . 135
C.3 Dataset . . . 136

4 Identifying SIFIs: Toward a Simpler Approach **137**
4.1 Introduction . . . 139
4.2 Estimation of Systemic Risk Measures . . . 141
4.2.1 Definitions . . . 142
4.2.2 Nonlinear Estimation . . . 144
4.2.3 Linear Estimation . . . 146
4.3 Data and Estimation Results . . . 147
4.4 Comparison of Systemic Risk Rankings . . . 149
4.4.1 Nonlinearity versus Linearity . . . 149
4.4.2 Impact of More Extreme Events . . . 154
4.5 Conclusion . . . 155

Appendices **157**
D.1 ΔCoVaR Framework and Estimation . . . 157
D.2 MES Framework and Estimation . . . 159
D.3 SRISK Framework and Estimation . . . 160
D.4 Dataset . . . 161

Conclusion **177**

Introduction

Contexte

Les années 1990 ont connu de nombreuses crises financières qui ont notamment, eu un impact majeur sur les économies des pays émergents. Que ce soit entre les marchés d'un même pays ou entre différents pays, ces crises sont caractérisées par l'importance des effets de contagion financière qu'elles ont rencontrée. La crise du Mexique de 1994-95, aussi appelée *Tequila crisis* ou *el error de diciembre*, en est la première illustration. La dévaluation de la monnaie mexicaine, le Peso, est à l'origine de cette crise qui touchera dans un premier temps l'économie réelle du pays avant de s'étendre à l'ensemble du monde. Quelques années plus tard, la crise asiatique de 1997 met également en avant des phénomènes de contagion financière internationaux. En effet, ces phénomènes ne se cantonnent plus uniquement aux marchés d'un même pays mais très rapidement, ont un impact sur les pays alentours se trouvant dans la même région géographique. Partie de Thaïlande par l'éclatement de la bulle financière qui entraine une chute des cours boursiers, la crise s'étend tout d'abord aux *Tigres* (Malaisie, Indonésie, Vietnam et Philippines) avant de se propager à Hong Kong, la Corée du Sud et finalement, fait novateur de cette crise, aux pays d'Amérique du Sud, démontrant que la contagion n'avait plus de limites régionales. Enfin, en 1998, cette crise ira jusqu'à s'étendre à la Russie. A ce moment là, le *Wall Street Journal* faisait état le 18 novembre 1998 de ces phénomènes de contagion et écrivait :

Earlier this year, so many families living in the fashionable suburb of San Peddro Garza Garcia invested in Russian bonds that it became known as San Pedroburgo. Now this wealthy enclave feels more like Stalingrad...

Cette crise russe sera marquée notamment par la faillite du *hedge fund* LTCM (pour *Long Term Capital Management*) qui entraine une augmentation du risque systémique à travers le monde. Par la suite, les crises Brésilienne (1999), Turque (2001) et Argentine (2002) sont aussi caractérisées par l'importance des phénomènes de contagion qu'elles ont entrainées avant bien sûr, la crise des *subprimes* en 2007-2008. Les économistes sont, de manière unanime, en mesure de définir un ensemble de crises financières ayant connus des phénomènes de contagion significatifs. Il est cependant plus difficile de se mettre d'accord sur la définition et la manière de mesurer ces évènements.

La contagion financière est au centre de toutes les attentions depuis le déclenchement imprévisible de la crise de 2007. En effet, comme le décrivent Adrian and Shin (2008), les prêts hypothécaires (*subprimes*), décrits comme étant à l'origine de cette crise, ne représentaient qu'une partie minime du système financier. Mais les auteurs ajoutent que l'abondante titrisation de l'époque dissimulait la majeure partie des expositions. On peut alors imaginer la contagion financière comme étant le principal facteur des effets qu'a eu ce marché sur l'ensemble des marchés de crédit et par la suite, sur le système financier de manière générale. Aujourd'hui, les phénomènes de contagion financière restent préoccupants aussi bien pour les régulateurs que pour les opérationnels, si bien que l'étude de ces phénomènes et leur modélisation sont toujours des sujets d'actualité et le resteront sans aucun doute encore de nombreuses années.

Le décloisonnement des marchés financiers à la fin des années 80 organisé conjointement par le FMI, la Banque Mondiale et la communauté européenne a mené à la globalisation du système financier [Batisdon et al. (2010)]. De ce fait, les marchés étant de plus en plus interconnectés, ils sont plus fragiles face aux phénomènes de contagion qu'ils

peuvent être amenés à subir. Jusqu'ici, la contagion était usuellement définie par un effet de domino entre les institutions financières. L'institution financière A connaissait des problèmes la menant à faire faillite ce qui entrainait des problèmes pour la firme B qui empruntait auprès de A. Ce schéma peut être reproduit avec un nombre d'institutions financières suffisamment grand pour représenter le marché. Cependant, comme le font remarquer Adrian and Shin (2008), les canaux de propagation de la contagion ne sont visiblement plus les mêmes depuis 2008. Les auteurs décrivent trois canaux qui sont : (i) les variations de prix, (ii) les risques mesurés et (iii) le capital, valorisé en valeur de marché. Il est donc d'autant plus important d'établir une définition consensuelle de la contagion permettant de la mesurer.

Néanmoins, bien que la définition des périodes pour lesquelles les marchés ont connu des phénomènes de contagion soit communément acceptée, les économistes ne parviennent pas à être unanimes quant à la définition de la contagion ou la manière de la mesurer [Rigobon (2001), Dungey et al. (2005)]. De ce fait, il est aujourd'hui particulièrement difficile d'évaluer l'impact de la contagion lors d'une crise financière, mais plus généralement, il reste compliqué de modéliser les phénomènes de contagion pour en prédire les conséquences et les risques qu'ils induisent. En effet, de manière générale, ils induisent une amplification des chocs pouvant mener à l'apparition de risques extremes. Ces derniers sont particulièrement redoutés par les intervenants dès lors qu'ils conduisent à une paralysie des activités souvent associée à de lourdes pertes.

Différentes définitions de la contagion

Il parait évident de faire l'analogie entre la contagion financière et la médecine. En effet, la contagion est dans un premier temps associée à la transmission des maladies. Elle est dans ce cadre définie par le fait que deux sujets rencontrent les mêmes symptômes alors qu'à la période précédente, seul l'un des deux les avait. Toutefois, pour conserver

le parallèle avec le monde médical, connaître le moyen de transmission est aussi important que de détecter le phénomène de contagion. Il en est de même en économie en ce qui concerne la mise en place d'une politique optimale. Une partie de la littérature s'intéresse donc aux canaux de transmission des chocs et propose une approche structurelle. Parmi les canaux les plus régulièrement cités, on retrouve le commerce international. Glick and Rose (1999) montrent que c'est un acteur majeur lors de la propagation des crises de change des années 1970 et 1990. La similitude des économies et la coordination régionale des politiques économiques sont également deux principaux facteurs comme il est décrit respectivement dans Eichengreen et al. (1996) et Fazio et al. (2003). Enfin, le système financier est bien entendu l'un des principaux canal dans la transmission des crises. Van Rijckeghem and Weder (2001) montrent empiriquement l'importance de ce canal dans la propagation des crises de change mexicaine, thaïlandaise et Russe. Bien que plus difficile à appréhender, les marchés financiers sont les plus étudiés lorsqu'il s'agit de mesurer les phénomènes de contagion. Toutefois, les différents canaux sont décrits comme particulièrement instables, ce qui ne permet pas de mettre en place une mesure des phénomènes de contagion basée sur ces derniers. Ainsi, la contagion financière est plus généralement étudiée en se concentrant sur la transmission des chocs sans pour autant décrire le moyen par lequel ils sont transmis.

De nombreuses définitions des phénomènes de contagion sont aujourd'hui proposées dans différents papiers académiques mais aucune d'entre elles ne fait l'unanimité. Pour autant, cette définition a une importance particulière lorsqu'il s'agit de mettre en place une mesure de la contagion. Parmi les plus citées, la plus large consiste à définir la contagion comme un processus de transmission des chocs entre les pays, ou les firmes. Généralement, cette définition est appliquée aux institutions financières ou, au niveau macro économiques, aux pays eux-mêmes. Bekaert and Harvey (2003) font référence au fait que, lors d'épisodes de contagion financière, les marchés se mettent à évoluer dans le même sens. La contagion est alors définie comme la propagation entre pays, d'un

choc, en excès de ce à quoi nous pouvions nous attendre compte tenu des fondamentaux économiques et considérant la transmission d'un choc usuel. Cette notion, bien que d'apparence évidente, ne permet toutefois pas de mesurer les phénomènes de contagion sans ambiguïté puisque notamment, la définition des fondamentaux peut mener à différentes interprétations.

Alors, la Banque Mondiale propose trois définitions de la contagion[1], allant de la plus générale à la plus opérationnelle. Nous verrons toutefois, que seule la plus opérationnelle d'entre elles permet une modélisation efficace des phénomènes de contagion. La première définition est la plus générale. La contagion est la transmission d'un choc au sens large. Ce phénomène peut se passer entre pays ou entre marchés domestiques. En d'autres termes, la contagion n'est autre que l'impact sur un pays ou un marché, d'un choc ayant eu lieu dans un autre pays ou sur un autre marché. Cette définition nécessite un ensemble d'informations particulièrement étendu et ne peut que difficilement être mise en place dans le but de mesurer les phénomènes de contagion. La seconde définition reprend les bases de la précédente en s'intéressant à la propagation des risques. Cependant, elle ne se concentre que sur une partie réduite de cette diffusion. En effet, elle se limite à la corrélation entre les pays au delà des liens fondamentaux établis entre les pays. Nous remarquons le lien avec la définition utilisée par Bekaert and Harvey (2003). Cette dernière fait donc référence aux co-mouvements au delà de ceux communément expliqués par les fondamentaux en période normale ou par le comportement moutonnier des pays. Enfin, la plus opérationnelle des définitions proposées par la Banque Mondiale propose de caractériser la contagion comme l'augmentation des corrélations entre les pays ou les firmes durant les périodes de crises relativement aux corrélations prévalant pendant les périodes calmes. Cette troisième définition permet de mesurer et de modéliser les phénomènes de contagion comme il est décrit par exemple dans Forbes and Rigobon (2001). Nous pouvons déjà remarquer le lien très fort qu'il y a entre contagion et corrélation. En effet, comme nous allons pouvoir

[1]Voir http://go.worldbank.org/JIBDRK3YC0.

le voir dans la section suivante, la corrélation et plus particulièrement ses variations, vont se retrouver au coeur de nombreuses études empiriques.

King and Wadhwani (1990) ont réalisé ce qui apparaît comme étant la première étude empirique des phénomènes de contagion. Ils ont ainsi montré qu'une augmentation de la volatilité des prix sur les marchés américains entraîne une augmentation de la corrélation des rendements des différents marchés. Cependant, le courant principal dans la littérature concernant les mesures de contagion a été initié par Forbes and Rigobon (2001). Comme nous avons pu le constater, la troisième définition, sous toutes ses formes, permet aisément de mesurer les effets de contagion, comme c'est le cas dans les papiers suivants : King and Wadhwani (1990), Calvo and Reinhart (1996), Forbes and Rigobon (2001) et Forbes and Rigobon (2002) parmi d'autres. On retrouve également une revue de l'ensemble des méthodes permettant de mesurer les effets de contagion dans les travaux de Dungey et al. (2005). Ces papiers ont pour point commun de s'intéresser au changement de corrélations durant les périodes de crise financière. Ainsi, pour détecter et mesurer les effets de contagion, il suffit d'estimer les sauts en termes de corrélation entre différentes séries temporelles représentant des institutions financières. Cela peut être ramené à vérifier la stabilité des paramètres d'un modèle économétrique lorsqu'une crise apparaît. En d'autres termes, dans le cas d'un phénomène de contagion, le modèle permettant de représenter les rendements des institutions doit voir ses paramètres changer en période de crise. En effet, comme la contagion est définie comme une augmentation significative des liens entre les marchés après un choc sur un pays ou un groupe de pays, il suffit de tester la significativité du changement de lien après ce choc. Le modèle étudié et alors définit comme :

$$y_t = \beta x_t + \epsilon_t \tag{1}$$

où y_t et x_t sont les variables d'intérêts des pays. Elles peuvent aussi bien représenter des

taux d'intérêt, des rendements de marchés actions, des taux de changes ou autres. ϵ_t est le vecteur d'innovations idiosyncrasiques, c'est à dire, propres au pays représenté par y_t. On remarque que le paramêtre β est celui qui centralise l'attention puisque c'est lui qui, par sa variation statistiquement significative ou non, va permettre d'identifier les phénomènes de contagion. Cependant son estimation peut être biaisée. Au delà de l'implication de l'hétéroscédasticité, cette méthodologie fait face à différents problèmes. L'hypothèse des termes d'erreur ϵ_t indépendamment et identiquement distribués, de moyenne nulle, de variance unitaire et indépendants de x_t [soit $E(x_t\epsilon_t) = 0$] peut être remise en cause ce qui implique un biais lors de l'estimation du paramètre β. Ces problèmes peuvent être liés à des variables omises ou à l'endogénéité de la variable x_t. Rigobon (2001) propose une revue de littérature concernant les méthodes capables de tester la stabilité des paramètres. Elles sont principalement basées sur les Moindres Carrés Ordinaires, l'Analyse en Composante Principale, les modèles Logit/Probit et l'analyse des coefficients de corrélations.

Par la suite, Boyer et al. (1999), Rigobon (2000), Rigobon (2001), Rigobon (2003a) et Rigobon (2003b) proposent d'étudier la différence en termes de covariance selon que l'on soit en période calme ou en période de crise. Cependant, le problème le plus important et ayant l'impact le plus grand se trouve être le problème d'hétéroscedasticité.

Problème d'hétéroscedasticité

Il est empiriquement établi que la volatilité des rendements financiers n'est pas constante et qu'elle augmente en période de crise. L'identification même des crises financières est généralement faite en observant l'augmentation de cette volatilité. De ce fait, les rendements de marché par exemple n'ont pas un comportement homoscedastique. Cependant, de nombreux papiers mettent en avant le lien qu'il existe entre une augmentation de la volatilité et celle des corrélations (comme par exemple Forbes and Chinn (2004)). Différentes méthodes ont alors été mises en place, comme dans Knif et al. (2005) qui

étudient la relation entre les corrélations et les volatilités conditionnelles pour montrer qu'une augmentation de la volatilité des rendements des actions entraine une augmentation des corrélations. Ce constat est le même à une échelle internationale comme le montrent Solnik et al. (1996) ou Chesnay and Jondeau (2001a) parmi d'autres. Enfin, Ramchand and Susmel (1998) estiment que les corrélations des actions américaines sont 2 à 3.5 fois plus élevées lorsque ce marché est dans un état de haute volatilité. Ainsi, dans le cadre d'étude de la contagion financière, Rigobon (2001) et Forbes and Rigobon (2002) reprenant l'approche proposée par Boyer et al. (1999) montrent que les coefficients de corrélations, jusqu'ici utilisés, sont sur-estimés. En effet, durant les périodes de crises, la volatilité du marché augmente et peut ainsi avoir un impact à la hausse sur l'estimation de ces coefficients de corrélation. Par conséquent, cela peut mener à accepter l'hypothèse d'un changement en termes de corrélations alors que ce n'est pas le cas. Les auteurs proposent alors un coefficient de corrélation qu'ils ajustent pour corriger le biais que peut causer une augmentation de la volatilité. Mais plus important encore, ils distinguent les phénomènes de contagion pure, des phénomènes d'interdépendance.

Les phénomènes de contagion pure sont caractérisés par une augmentation significative des co-mouvements. En d'autres termes, la structure de dépendance est modifiée qu'il y ait ou non une augmentation de la volatilité. En effet, par définition, l'hétéroscedasticité est prise en compte lorsque l'on étudie un phénomène de contagion pure. Contrairement à ce dernier, le phénomène d'interdépendance est caractérisé par un accroissement des co-mouvements dû aux fondamentaux entre les pays. Autrement dit, l'augmentation des coefficients de corrélation peut être attribuée à un choc exogène commun à l'ensemble des acteurs, ce qui correspond à une dépendance fondamentale. Comme les effets de contagion pure révèlent un changement dans la structure de dÃĭpendance, ils intéressent d'avantage praticiens et régulateurs. En effet, c'est bien le changement de structure de dépendance qui peut entrainer des contraintes supplémentaires ajoutées à celles induites par l'augmentation de la volatilité caractéristique des périodes de crise. Il est donc pri-

mordial de corriger de la volatilité les résultats obtenus afin d'éviter les erreurs et ainsi se focaliser sur la contagion pure.

C'est pour cela que de nombreuses études économétriques ont cherché à modéliser de manière efficace la contagion pure et donc à éviter le problème d'hétéroscedasticité. Parmi les méthodes les plus utilisées, on retrouve les modèles ARCH et GARCH (pour *Generalized AutoRegressive Conditional Heteroskedasticity*). Hamao et al. (1990) estiment la variance conditionnelle par un modèle GARCH et testent la corrélation entre trois marchés différents. Edwards and Susmel (2001) quant à eux, incluent un changement de régime dans une modélisation ARCH. Cette méthode met en avant une forte augmentation des corrélations durant les périodes pour lesquelles la volatilité du marché est élevée, confirmant ainsi l'existence d'effets de pure contagion. Cependant, les modèles GARCH multivariés proposent une alternative plus performante pour analyser les co-mouvements et les impacts d'une variation de la volatilité entre différents actifs. Wang and Nguyen Thi (2007), Chiang et al. (2007) ou encore Naoui et al. (2010) estiment les phénomènes de contagion se servant du modèle *Dynamic Conditional Correlation* de Engle (2002). Ainsi, il est facile de voir l'évolution de cette corrélation impliquant ou non, des phénomènes de contagion. Kenourgios et al. (2010) ajoutent un coefficient d'asymétrie dans le modèle précédent et étudient les corrélations entre les quatre pays formant le groupe BRIC (Brésil, Russie, Inde et Chine), et les marchés américains et britanniques. L'ajout du coefficient d'asymétrie permet de discriminer entre l'impact d'un rendement négatif et l'impact d'un rendement positif sur les corrélations. On remarque alors que la littérature académique a bien intégré la nécessité de prendre en compte l'hétéroscedasticité.

Définition des périodes de crise

Boyer et al. (1999) déjà, mettent en avant que le seul problème n'est pas l'hétéroscedasticité. Ils signalent en effet que la définition exogène des périodes de crises peut également mener

à des résultats erronés. Il apparaît évident que la modification de la date de commencement de la crise peut avoir un impact sur les résultats. L'ensemble d'informations étant modifié, un biais peut être créé si le moment du choc n'est pas convenablement défini. Par conséquent, il est préférable d'utiliser un modèle à espace/état ayant une spécification de la volatilité conditionnelle au temps afin de résoudre ce problème. Billio and Caporin (2005) proposent alors une approche mêlant un modèle DCC et un modèle à changement de régime[2]. Dans cette classe de modèles, le mécanisme de propagation est discontinu. Une chaîne de Markov est introduite pour décrire cette discontinuité et ainsi permettre une définition endogène des périodes de crise. D'autres auteurs s'intéressent à cette modélisation à changement de régime dans le cadre de l'étude des phénomènes de contagion comme par exemple Ramchand and Susmel (1998), Chesnay and Jondeau (2001b) ou Ang and Bekaert (2002). Cette approche implique de définir la contagion comme une rupture produisant des non-linéarités dans les liens entre les différents marchés financiers. De plus, comme le font remarquer ces auteurs, cette méthode corrige à la fois le problème causé par l'hétéroscedasticité mais également celui causé par une définition endogène de la date à laquelle le choc a eu lieu. Pour finir, Dungey et al. (2012) s'intéressent à un modèle GARCH avec des transitions lisses (*smooth transition structural GARCH*) permettant de définir les dates de début mais aussi de fin des périodes de crises. Ils montrent ainsi d'importants phénomènes de contagion entre 2001 et 2010, avec notamment une période de crise définie entre Octobre 2007 et Juin 2009. De plus, ils ajoutent que si la structure de dépendance a bien été modifiée lors de la crise, elle ne revient pas à son état initial ensuite.

Par conséquent, on remarque que pour convenablement mesurer et modéliser la contagion, il est nécessaire d'utiliser un modèle à volatilité conditionnelle et changement de régime. En effet, ce dernier indique une variation dans la structure de dépendance (matrice de corrélations) et les volatilités conditionnelles au temps permettent de traiter les

[2]Le modèle est un *multivariate Markov Switching Dynamic Conditional Correlation GARCH model*.

variations de volatilité dont les rendements financiers sont la cible. Néanmoins, les approches citées précédemment ne permettent que l'étude des corrélations deux à deux et n'autorisent pas une interprétation évidente, dès lors que l'on s'intéresse à la matrice de corrélation dans son ensemble. Le modèle proposé par Pelletier (2006) possède tous les pré-requis précédemment cités et permet une interprétation économique aisée puisque le changement de régime se situe sur la matrice de corrélation dans son ensemble. Ainsi, le modèle donne pour principal résultat la probabilité de se situer dans un régime pour lequel les corrélations sont plus élevées que dans l'autre régime. Par conséquent, une augmentation de cette probabilité indique directement une augmentation de l'ensemble des corrélations simultanément. Ce modèle est décrit comme particulièrement performant dans le papier de Bauwens and Otranto (2013) qui le comparent notamment à d'autres modélisations comme le DCC (pour *Dynamic Conditional Correlations*) de Engle (2002).

Le modèle RSDC

Le modèle RSDC pour *Regime Switching Dynamic Correlations* de Pelletier (2006) qui, comme nous le verrons dans les chapitres suivant, nous permet de modéliser de manière adéquate les phénomènes de contagion a également été utilisé dans le cadre d'allocation d'actifs dans le papier de Giamouridis and Vrontos (2007). Contrairement aux autres modèles présentés précédemment et basés sur le modèle DCC [Engle (2002)], le RSDC permet d'estimer un nombre de paramètres bien inférieur. En effet, si dans l'approche DCC, la matrice de corrélation est estimée pour chaque date de l'échantillon, ce n'est pas le cas du modèle RSDC pour lequel, seulement n matrices de corrélations sont estimées. n correspond au nombre de régimes souhaités. Ce dernier est souvent égal à 2 pour étudier les phénomènes de contagion puisqu'il suffit dans ce cas d'identifier un changement de régime dans la structure de dépendance.

Le modèle est alors décrit de la manière suivante. On définit les rendements des K

actifs comme :

$$r_t = H_t^{1/2} U_t, \tag{2}$$

avec $U_t|\phi_{t-1} \sim iid(0, I_k)$, U_t est le vecteur d'innovation de dimension $T \times K$ et ϕ_{t-1} est l'ensemble d'information disponible à la date $t-1$.

Tout comme dans la modélisation proposée par Engle (2002), la matrice de covariance conditionnelle est décomposée entre la matrice de corrélation (Γ_t) et une matrice ayant sur sa diagonale les écart-types conditionnels (S_t):

$$H_t = S_t \Gamma_t S_t. \tag{3}$$

Les deux matrices sont conditionnelles au temps t. Cependant, elles n'ont pas le même type de dynamique. En effet, si S_t change à chaque période, Γ_t correspond seulement à n matrices de corrélations, et ne change que par périodes.

La modélisation, telle qu'elle est présentée ci-dessus, implique d'estimer les variances conditionnelles univariées. Ainsi, pour chaque actifs, l'écart-type à la date t est estimé tel que :

$$\sigma_{i,t} = \omega_i + \alpha_i^- min(r_{i,t-1}, 0) + \alpha_i^+ max(r_{i,t-1}, 0) + \beta_i \sigma_{i,t-1}, \tag{4}$$

avec ω_i, α_i^-, α_i^+ et β_i qui sont des nombres réels.

Cette modélisation TGARCH(1,1) pour *Threshold GARCH* [Zakoian (1994)] permet de tenir compte de l'asymétrie présente dans les rendements des actifs financiers. Après avoir défini la modélisation de la matrice S_t, il reste à décrire le comportement de la matrice de corrélation, au centre de notre attention.

Considérant Γ_n comme étant la matrice de corrélation dans l'état n, et Δ_t une chaine de Markov inobservable et indépendante de U_t, la matrice de corrélation à la date t s'écrit:

$$\Gamma_t = \sum_{n=1}^{N} \mathbb{1}_{(\Delta_t=n)}\Gamma_n. \tag{5}$$

Le changement de régime est alors gouverné par la matrice de probabilité de transition $\Pi = (\pi_{i,j})$ telle que:

$$Pr(\Delta_t = j|\Delta_{t-1} = i) = \pi_{i,j} \quad \forall i, j = 1 \cdots N. \tag{6}$$

Cette matrice définit la probabilité de rester dans le même régime ou au contraire, de passer dans l'autre.

Cette brève description du modèle RSDC nous permettant de modéliser les phénomènes de contagion rappelle ce que nous avons décrit précédemment. Les problèmes causés par l'hétéroscedasticité des rendements d'actifs financiers sont traités par le modèle, dans la modélisation de la matrice S_t à l'aide d'un modèle TGARCH. D'autre part, la définition des périodes de crise est endogène. En effet, le modèle lui même définit la date à laquelle le changement de régime s'effectue. De cette façon, nous évitons les problèmes liés à une mauvaise détermination de la date du choc, entrainant par la suite, les phénomènes de contagion. Comme nous pourrons le constater dans le reste de cette thèse, les applications de ce modèle sont diverses. En effet, plus encore que la notion de contagion, ce modèle décrit une re-corrélation de différentes séries temporelles. Autrement dit, ce modèle permet de détecter lorsque ces dernières adoptent un comportement de plus en plus proche. L'intérêt est d'autant plus grand, que de cette manière, le modèle peut être appliqué à n'importe quel type d'actifs ou d'indicateurs de risque, sous certaines contraintes propres à leur distribution statistique.

Lien entre liquidité, contagion et risque systémique

Depuis 2007-08, il apparaît primordial de lier à l'étude des phénomènes de contagion, l'analyse de la liquidité et la mesure du risque systémique. Pour cela, la première étape est une nouvelle fois la définition de chacun de ces risques afin de pouvoir établir le cadre de travail dans lequel les mesurer. Alors, la liquidité se définit dans un premier temps par la capacité du marché à effectuer des échanges. En effet, un marché parfaitement liquide permet de convertir en monnaie n'importe quel montant d'actif et d'effectuer la transaction inverse instantanément et sans coût. En comparaison, un marché est dit liquide lorsque ces transactions s'effectuent à un coût minimal. Nous devons cette définition à Harris (1990). Or, Brunnermeier and Pedersen (2009) ont introduit la notion de liquidité de financement. En effet, la liquidité n'est plus uniquement envisagée comme la liquidité du marché. Cette dernière se définit comme la facilité qu'un actif a à être échangé alors que la liquidité de financement représente l'aisance que l'intervenant de marché a pour financer ses échanges. Brunnermeier and Pedersen (2009) montrent qu'il existe un lien entre les deux types de liquidité, notamment par la création de spirales de liquidité renforçant tour à tour les deux types de liquidité. Ainsi, nous avons pu constater que les problèmes de liquidité ont un impact particulièrement important à la fois sur le déclenchement de la crise mais également en ce qui concerne son maintien voir même son amplification. De ce fait, l'étude des problèmes de liquidité comme simple facteur de risque idiosyncratique ne correspond plus aux problèmes que rencontrent les intervenants de marché. En effet, il n'est plus suffisant d'étudier la liquidité des actifs des institutions financières. Il devient indispensable de se concentrer sur une dimension plus large de la liquidité et non plus seulement sur un point de vue micro-économique abordé jusqu'alors.

Cette nécessité d'appréhender le risque financier avec une vision plus large concerne également le risque de marché. La faillite de Lehman Brothers le 15 septembre 2008 a plongé l'ensemble de l'économie mondiale dans un profond marasme. Les répercutions

dues à l'interconnexion entre les établissements bancaires sont telles qu'il devient indispensable de mettre en place un politique de régulation macro-prudentielle. Cette dernière ne se concentre plus uniquement sur le risque individuel de la firme, mais également sur son impact sur le marché et les autres firmes. Le risque systémique ne pouvant être assimilé à la chute du système dans son ensemble dès lors qu'un tel évènement n'a jamais été recensé, nous faisons l'hypothèse simplificatrice qu'il peut être assimilé aux retombées économiques faisant suite à la faillite d'une institution financière. De plus, afin de le rendre mesurable, une part de la littérature se focalise uniquement sur les pertes recensées par le système financier dès lors que l'impact sur l'économie réelle est très difficilement estimable. Bisias et al. (2012) présentent un nombre important de mesures du risque systémique. Parmi elles, les mesures basées sur les données de marché et nommées par les auteurs *"Cross-Sectional measures"* sont particulièrement utilisées. Ces dernières sont rendues accessibles par le fait qu'elles ne nécessitent pour être calculé, qu'un ensemble de données disponible publiquement. Ces mesures étudient différentes relations entre les rendements de firmes financières et les rendements du marché. Par conséquent, elles permettent d'estimer l'importance qu'une firme a dans le système et son impact en cas de crise. La régulation doit maintenant se concentrer sur ce type de mesures obligeant ainsi les firmes à internaliser leurs externalités.

La réflexion conduite dans le cadre de ce travail met en avant le lien qu'il y a entre la liquidité, la contagion et le risque systémique. En effet, il est usuel de rencontrer des problèmes de liquidité ralentissant ou paralysant l'activité des institutions financières. Comme nous avons pu le décrire précédemment, les phénomènes de contagion peuvent en théorie étendre à l'ensemble du système cette contrainte de liquidité jusqu'à la paralysie du système. Cette thèse s'est alors principalement concentrée sur les méthodes quantitatives qui permettent de mesurer l'ensemble de ces phénomènes. Néanmoins, nous mettons l'accent sur l'importance de la définition du phénomène étudié. En effet, selon cette dernière, les méthodes et mesures peuvent être diamétralement opposées. Enfin, dans le

cadre d'étude du risque systémique, nous attachons une attention particulière à conserver des méthodes relativement simples mais aussi, ne nécessitant que des données disponibles publiquement.

Nous revenons dans le premier chapitre de cette thèse sur la prise en compte de la liquidité de marché de la dette souveraine. Nous focalisant sur les marchés émergents, l'étude de la relation entre les primes de CDS (pour *Credit Default Swap*) et les taux obligataires nous permet de mesurer le risque de liquidité de ce marché. De plus, utilisant cet indicateur, nous remarquons que les phénomènes de contagion ne concernent plus uniquement les variations de prix mais également les problèmes de liquidité de marché. Ainsi, partant des indicateurs de la liquidité de marché de chaque dette souveraine, l'application d'un modèle RSDC permet de filtrer un indicateur de la liquidité de financement. En effet, pour un gérant de portefeuille, il n'apparait pas suffisant de se concentrer sur le risque de liquidité de marché. Nous montrons que dans cette situation, l'ensemble des intervenants fait face à des difficultés pour financer ses activités, dues à un problème généralisé de liquidité de marché. Ce premier chapitre a donc différentes contributions. Dans un premier temps, nous utilisons un modèle présenté par Levy (2009) afin de considérer la relation d'arbitrage entre les primes de CDS et les taux obligataires (du sous-jacent) comme mesure de liquidité de marché. Cette dernière est particulièrement adaptée au cas de la dette souveraine. Cependant, l'étude de la dette émise en monnaie locale implique de tenir compte de la différence des monnaies dans lesquelles sont émis la prime de CDS et le taux obligataire. Par conséquent, nous décrivons une manière d'étudier cette mesure de liquidité tout en tenant compte de ce problème. La contribution de ce travail est également empirique puisque nous étudions une base de données de 9 pays émergents et montrons que les phénomènes de contagion ne sont plus uniquement concentrés sur les variations de primes de CDS ou de taux mais concernent aussi le risque de liquidité de marché. Ainsi, ils dénotent un problème de liquidité de financement mettant en avant le lien qu'il y a entre liquidité de marché et liquidité de financement.

Malgré le lien établi entre les deux, il est important de distinguer la manière de les mesurer. En effet, la liquidité de financement se définit comme la facilité qu'a le gérant à financer ses échanges. La liquidité de marché quant à elle, fait référence à la facilité d'échanger l'actif rapidement et à moindre coût. C'est cette seconde définition que nous étudions dans le deuxième chapitre de cette thèse. En effet, il apparait primordial de pouvoir évaluer la liquidité d'un actif. Or, malgré le grand nombre de mesures proposées dans la littérature il n'est toujours pas évident de définir un évènement d'illiquidité. En effet, la liquidité est liée à différents concepts. Comme sa définition fait appel à 4 dimensions distinctes, il est particulièrement difficile de parvenir à l'évaluer. Ainsi, un grand nombre de mesures ont vu le jour dans la littérature sans toutefois parvenir à capturer l'ensemble des dimensions de la liquidité. Par conséquent, des méthodologies ont été explorées impliquant plusieurs mesures simultanément et permettant d'extraire un facteur de liquidité tirant son information d'un ensemble de mesures censé représenter les différentes dimensions de la liquidité de marché. Cependant, ces méthodes ne permettent pas d'obtenir les résultats escomptés car elles traitent une vision moyenne et statique de cette liquidité. Dans ce travail, nous montrons que, comme leurs définitions le suggèrent, les mesures de liquidité ne capturent pas la même prime de risque. Le choix de la mesure a donc un impact particulièrement important dans le cadre de l'allocation d'actifs. Mais plus important encore, ces mesures de liquidité de marché ne détectent pas un problème de liquidité aux mêmes dates ce qui indique cette fois que le choix de la mesure aura un impact sur la gestion des risques. Les méthodes proposées pour extraire un facteur dit de "pure liquidité" ne permettent pas d'évaluer correctement le moment où les mesures de liquidité indiquent un problème. Elles ont tendance à lisser l'impact d'un évènement d'illiquidité. Dans ce chapitre, nous nous intéressons à la corrélations entre les variations de ces mesures. Nous détectons les dates pour lesquelles, l'ensemble de ces mesures indiquent un problème de liquidité. Ainsi, la méta-mesure proposée dans ce chapitre évalue la probabilité que l'actif rencontre un évènement d'illiquidité drastique. Empiriquement, les résultats de l'étude des rendements de stratégies d'arbitrage indiquent qu'une prime

de liquidité est effectivement extraite grâce à cette mesure. De plus, une des contributions de ce travail est d'étudier le comportement d'une stratégie basée sur une mesure de liquidité standard mais pour laquelle nous évitons d'investir dans les titres particulièrement illiquides détectés par notre méta-mesure. Ainsi, nous voyons que cette stratégie capturent toujours la même prime de risque mais améliore son couple rendement-risque. Ce chapitre présente donc une manière efficace de mesurer la liquidité d'un actif et plus particulièrement de tenir compte de l'ensemble des dimensions que sa définition implique.

Le troisième chapitre de cette thèse est consacré à l'estimation du risque systémique tel qu'il est définit dans la littérature récente. Cependant, la contribution de ce papier se situe dans le fait que l'on propose une modélisation des rendements du marché permettant de considérer un facteur de liquidité de financement. En d'autres termes, le risque systémique est évalué compte-tenu de la facilité avec laquelle les institutions financières peuvent réaliser leurs échanges ou plus particulièrement dans ce cadre de travail, satisfaire les besoins de capitaux réglementaires. Pour se faire, nous conservons la modélisation proposée par Adrian and Brunnermeier (2011) et nous étudions leur mesure de risque systémique, la $\Delta CoVaR$. En effet, depuis 2008, la littérature s'est attachée à proposer de nombreuses solutions pour évaluer le risque qu'une banque ou plus généralement qu'une institution financière faisait encourir à l'ensemble du système. Ainsi nous avons vu dans un premier temps que deux mesures se sont démarquées parmi un nombre important de méthodes proposées. Tout d'abord, la $\Delta CoVaR$ de Adrian and Brunnermeier (2011) que nous étudions, mais également le MES de Acharya et al. (2010) et sa modélisation dynamique proposée par Brownlees and Engle (2012). Bien que basées toutes les deux sur des données publiques, la seconde adopte une modélisation ne permettant pas aisément de tenir compte de la liquidité. À l'inverse, la $\Delta CoVaR$ adopte une modélisation simple du rendement de marché conditionnellement au rendement de la firme. Il est alors possible d'ajouter un facteur dépendant de la liquidité de financement. Dans notre cas, ce facteur est une variable binaire tenant compte du niveau de la liquidité de financement.

Ainsi, les paramètres permettant de calculer la $\Delta CoVaR$ sont estimés conditionnellement à l'indicateur de liquidité de financement. Ce dernier n'est autre que le *TED Spread*, fréquemment utilisé dans la littérature comme indicateur de la qualité du crédit mais également pour témoigner de la facilité des échanges sur le marché interbancaire correspondant donc à la liquidité de financement [Goyenko (2012), Boyson et al. (2010), Teo (2011)]. La contribution de ce travail se situe dans la méthodologie appliquée. C'est la première mesure du risque systémique conditionnelle à un niveau de liquidité de financement. Empiriquement, nous pouvons constater que les deux mesures $\Delta CoVaR$ standard et $\Delta CoVaR$ tenant compte de la liquidité n'aboutissent pas aux mêmes résultats. Tout d'abord en termes de valeurs, nous remarquons que tenir compte de la liquidité induit une $\Delta CoVaR$ plus élevée en moyenne sur notre échantillon durant les périodes calmes et légèrement inférieure durant les périodes de crise. Ce comportement est particulièrement adapté à la régulation du risque systémique puisqu'il est totalement contracyclique. De plus, si l'on regarde le classement obtenu par les institutions financières tenant compte de ces deux mesures, nous remarquons des écarts importants, confirmant que les deux mesures sont bien différentes et n'apportent pas la même information. Ce travail se place donc dans deux perspectives. La première est de permettre aux firmes d'évaluer plus précisément les capitaux requis qui leur permettraient de traverser une nouvelle crise tout en restant dans des conditions normales. La seconde se fait du point de vue du régulateur qui peut ainsi classer et taxer les firmes compte tenu de leur capacité à financer leurs opérations, adoptant ainsi une régulation contracyclique.

Pour finir, nous revenons sur les mesures de risque systémique dans un dernier chapitre. Dans le cadre de la mise en place d'une régulation, nous étudions les deux mesures précédemment citées et la *SRISK* proposée par Acharya et al. (2012). Elles sont unanimement reconnues comme mesures de risque systémique et servent d'indicateur dans le cadre de certaines régulations. Toutes ces mesures sont uniquement basées sur des données accessibles au public. Cependant, il est très souvent souligné la difficulté qu'ont les régulateurs

à mesurer le risque systémique, laissant ainsi place à de nombreux modèles internes aussi différents qu'inappropriés. Les mesures de risque systémique présentées ci-dessus, aussi simples soient-elles dans leurs définitions, nécessitent toutefois des méthodes d'estimation plus ou moins compliquées. Au premier plan, la modélisation de la dépendance entre les rendements du marché et les rendements des institutions financières fait l'objet de nombreuses recherches. La structure de cette dépendance est établie comme étant non linéaire. Empiriquement, nous montrons que c'est effectivement le cas sur un échantillon de firmes américaines. Mais modéliser la non-linéarité de cette relation a amené la littérature à proposer des méthodes de plus en plus compliquées. De ce fait, nous avons voulu constater si cette course à l'armement était nécessaire. En d'autres termes, nous avons choisi d'aller dans le sens opposé et notamment déterminer si accepter une erreur de modélisation en supposant une structure de dépendance linéaire avait un réel impact sur les résultats. Cela passe tout d'abord par les mesures, mais surtout par les méthodes permettant leur estimation. Ainsi, nous proposons de modéliser l'ensemble de ces trois mesures en faisant cette hypothèse de linéarité de la structure de dépendance entre les rendements. De ce fait, les méthodes d'estimation sont simplifiées. Les résultats obtenus montrent que cette hypothèse n'a que très peu d'impact dans le cadre usuel de régulation. La valeur des mesures de risque n'est que très peu modifiée tout comme le classement des firmes qui se voit seulement marginalement influencé. Par conséquent, ce dernier travail avance vers une régulation du risque systémique simplifiée allant à l'encontre de ce qui est communément proposé dans la littérature.

Cette thèse permet donc d'établir le lien entre les problèmes de liquidité, les phénomènes de contagion et le risque systémique. Que ce soit théoriquement ou empiriquement, ce travail, au travers de ses quatre chapitres, répond à différentes questions fréquemment posées aussi bien dans un cadre académique qu'opérationnel. La première est de savoir comment mesurer la liquidité. Aussi bien la liquidité de marché que la liquidité de financement, toutes deux sont passées en revue et de nouvelles méthodes sont proposées

pour les évaluer. La seconde question est la définition et la détection des phénomènes de contagion. Au travers de la première partie de cette introduction, nous avons pu définir la contagion financière et nous la modélisons au sein des deux premiers chapitres de ce travail. Ainsi, nous proposons différentes applications de cette modélisation. Pour finir, ce travail revient sur le risque systémique et une nouvelle fois, sur la méthodologie employée pour le mesurer. Que ce soit au travers d'une nouvelle approche, comme c'est le cas dans le troisième chapitre, ou encore par une méthode d'estimation différente, comme dans le quatrième chapitre, nous répondons aux questions que les opérationnels et les régulateurs se posent. Parmi elles, la prise en compte de la liquidité en est une des plus importantes, mais nous retrouvons aussi la manipulation des mesures de risque systémique qui est au centre de nombreuses attentions. Ainsi, ce travail propose de nombreuses contributions et a un intérêt particulier aussi bien pour la recherche académique que pour son application dans un cadre opérationnel.

Cette thèse est organisée en quatre chapitres. Le premier a pour objectif de présenter une modélisation des phénomènes de contagion en termes de prix mais surtout en termes de risque de liquidité. L'application est faite aux pays émergents et il en est extrait un facteur de liquidité de financement. Le second chapitre propose une nouvelle mesure du risque de liquidité ou plus précisément une méta-mesure de la liquidité de marché. Cette dernière s'appuie sur une modélisation des phénomènes de contagion et tient compte de l'ensemble des dimensions présentes dans la définition de la liquidité. L'objectif du troisième chapitre est de présenter une modélisation des rendements du marché permettant la prise en compte de la liquidité de financement dans l'estimation de la $\Delta CoVaR$. Ainsi, ce travail propose une nouvelle mesure du risque systémique ayant un comportement contracyclique. Pour finir, le quatrième chapitre revient sur l'hypothèse faite concernant la non-linéarité de la structure de dépendance des rendements du marché et des institutions financières. Il en résulte une méthodologie plus simple n'ayant pas d'impact sur les résultats dans le cadre d'une régulation standard.

Chapter 1

Liquidity risk and contagion for liquid funds

Ce chapitre est issu d'un travail commun avec Serge Darolles et Gaëlle Le Fol.

La gestion des risques financiers s'est historiquement focalisée sur le risque de marché. Or, depuis quelques années, chercheurs et opérationnels ont mis en avant l'importance grandissante de tenir compte des risques engendrés par un problème de liquidité. Toutefois, bien que la régulation commence à tenir compte de ce type de risque, elle se focalise sur la liquidité de marché omettant les problèmes éventuels de liquidité de financement. Or, cette dernière apparaît tout aussi problématique et nécessite un intérêt particulier.

La liquidité peut être séparée en deux parties distinctes : la liquidité de marché et la liquidité de financement. La première correspond principalement à l'aisance avec laquelle le gérant peut échanger un actif. Quant à la seconde, elle représente la facilité que le gérant a pour financer ses activités. Le risque de liquidité de financement se concentre alors autour de trois composantes : (i) le risque d'appel de marges dans le cas de la gestion de produits dérivés, (ii) le risque de roulement concernant la mise en place d'un effet de levier et (iii) le risque de sortie de capitaux pour les gérants exposés aux retraits des clients.

Dans le but d'étudier le risque de liquidité de financement, il est primordial d'isoler une de ces trois composantes. L'étude des fonds indiciels en est un exemple puisque dans ce cas, les gérants ne se servent ni de levier, ni de produits dérivés dans leur gestion. En revanche, ils sont particulièrement exposés au risque de fuite des capitaux de leurs clients.

Dans ce chapitre de thèse, nous nous intéressons dans un premier temps à la liquidité de marché de la dette de différents pays émergents. Nous nous concentrons sur la dette émise en monnaie locale impliquant de tenir compte des différentes monnaies considérées. Dans ce travail, la liquidité de marché est mesurée par l'intermédiaire de la relation d'arbitrage qui lie la prime de CDS (ou *Credit Default Swap*) et le taux obligataire en surplus du taux sans risque. Les CDS sont des dérivés de crédit reflétant la prime de risque associée au risque de défaut de l'actif sous-jacent (ici, la dette souveraine). Le risque de défaut du pays est également représenté par le taux de l'obligation d'état auquel on soustrait le taux sans risque. De ce fait, les deux sont supposés avoir le même prix, ce qui n'est pas toujours le cas. En effet, lorsque cette relation n'est pas vérifiée, elle reflète un problème de liquidité sur l'un des deux marchés. Ainsi, nous obtenons un indicateur de la liquidité de marché pour chaque dette souveraine.

La contribution principale de ce papier réside alors dans l'extraction d'un facteur de liquidité de financement à partir des données de marché. Très proche de la notion de contagion financière, la liquidité de financement est ici décrite comme un problème de liquidité de marché commun à tous les pays simultanément. Ainsi, la probabilité d'être dans un état pour lequel l'ensemble des pays rencontre un problème de liquidité de marché est un indicateur de liquidité de financement. Nous montrons dans ce travail que ce fut le cas notamment en 2008 mais également, que le retour à un régime normal, identique à celui précédent la crise, n'est toujours pas d'actualité.

1.1 Introduction

The main difference between traders and fund managers come from their sources of funding. Indeed, traders are funded by banks while fund managers are funded directly by investors. Moreover, we are able to distinguish the liability part of the fund which consists of the investor's inflows and the asset part which contains the fund's holdings. Consequently, the fund managers face a liquidity mismatch between the asset and the liability sides. On the one hand, investors want more and more liquid exposures. As a result, the liquidity of the liabilities is contractually defined and usually very high. On the other hand, for the asset side of the fund, the liquidity is determined by the nature of investments and usually lower than that of the liabilities. As the behavior of these funding providers can largely differ, the fund managers need to monitor this liquidity mismatch. Increasing the cash balance of the fund is one way to minimize this problem. However, if the amount of cash is too large, it will be idle and not producing. Conversely, if it is too small, the fund will still be exposed to the liquidity risk so that, it would be useless. In addition to the mismatch of liquidity that the fund managers suffer, we know that investors need funding to trade securities. When the funding liquidity conditions are bad, they cannot easily access to capitals which impair their trade capacities. If many investors are concerned by such a funding liquidity problem, trading is slowing down and market liquidity reduces[1]. As investors' funding also depends on assets' market liquidity, these problems can be mutually reinforced leading to liquidity spirals [see e.g., Gromb and Vayanos (2002), Morris and Shin (2004), Brunnermeier and Pedersen (2009), Menkveld and Wang (2011)]. Indeed, during financial turmoil, like in 2008, crises can spread across assets and markets as many investors were seeking for liquidity creating a contagion effects.

In this paper, we compute a market liquidity indicator and we define a funding liquidity

[1] Brunnermeier and Pedersen (2009) distinguish funding liquidity from market liquidity. The former characterizes the possibility for traders to find funds while the second characterizes the ease to trade an asset on the market. Traders provide market liquidity and their ability to do so depends on their capacity of funding. Funding liquidity is binding market liquidity as traders can only provide liquidity if they can access to fundings.

problem based on a simultaneous constraint of market liquidity for every markets. In order to control for the liquidity risk, we have to define how to measure it but despite the large number of liquidity measures available[2], measuring liquidity remains a difficult task. In fact, many liquidity measures require the use of high-frequency transactions and quotes data, which may not be available for some markets and even more so for emerging markets. Goyenko et al. (2009) compare the performances of several liquidity measures relatively to the effective or realized bid-ask spread. However, the poor availability of data encourages us to focus only on liquidity measures based on price data. There exist few measures based on daily price data. Roll (1984) proposes an estimation of the effective bid-ask spread based on the serial covariance of daily price changes. Hasbrouck (2004) uses a Bayesian estimation approach to estimate the Roll model and proposes a Gibbs measure of liquidity. Lesmond et al. (1999) use the proportion of zero return days as a proxy for liquidity. In the line of Levy (2009), we use the Credit Default Swap (CDS hereafter) Bond Spread basis as a liquidity indicator. We use an arbitrage relation to extract a liquidity measure of the sovereign debt market that solely relies on price data. From Garleanu and Pedersen (2009), Fontana and Scheicher (2010) and Bai and Collin-Dufresne (2011), we know that the basis is related to the credit risk of a bond. In other words, a larger deviation from parity is found for lower rated bonds because it is more costly to finance the arbitrage trade. In this paper, we tackle the problem of different currencies into the CDS Bond spread basis measurement and we focus on its liquidity component. Then, we use a Regime Switching Dynamic Correlation model (or RSDC) in order to define whether contagion effects occur. In this model, both heteroscedasticity problem and exogenous definition of crisis dates are tackled using a GARCH model and a regime switching governed by a Markov chain, respectively.

The contribution of this paper is to propose a methodology able to extract a funding liquidity factor from the market liquidity indicators. We establish the occurrence of pure

[2] Aitken and Winn (1997) report more than 68 measures for market liquidity.

contagion effects on the CDS Bond Spread basis and define them as liquidity contagion. Indeed, as Adrian and Shin (2008) described, contagion is not anymore only modeled as a domino's fall. Effectively, the contagion refers to the transmission of shocks. In this case, the channel of transmission is usually the price of the assets and the financial contagion is often represented as multiple sequential bankruptcies of financial institutions. But the financial crisis of 2007-08 exhibits the fact that financial shocks are also transmitted through liquidity problems. This channel represents a new kind of contagion that we define as the liquidity contagion and that us represented by an increase of correlations between market liquidity risks. Then, we propose to associate these phenomena of liquidity contagion with the identification of a funding liquidity problem. Indeed, although Brunnermeier and Pedersen (2009) determine a link between market and funding liquidity, it remains difficult to model it. First of all, in order to study funding liquidity, we need to isolate one of its three components: (i) margin risk, (ii) roll over risk and (iii) redemption risk. Considering the case of index funds, we only focus on the last one; the redemption risk implied by the behavior of fund clients. We focus on index funds tracking the performance of emerging sovereign debt markets. They do not use neither leverage nor derivatives leading to only have redemption risk by clients who want a very liquid exposure to particularly illiquid sovereign debt.

Until 2008, the financial sector almost only controls for market risk. The fund managers being constrained to build a portfolio to benefit from the diversification principle have as main fear the re-correlation of their assets. However, in addition to this risk, fund managers face particular liquidity constraints defined in the characteristics of the fund. These funding liquidity problems alter the ability of the manager to finance their trades. Indeed, the flow of clients is a key driver of the funding liquidity of a fund manager. Nonetheless the funding liquidity is also very worrying for both clients and regulators. The first wants to know the exposition of the fund and determine whether this risk is priced into the returns of the fund. The second has to improve the regulation taking into

account funding liquidity and consequently, the regulator looks for a relevant indicator. Finally, the funding liquidity is at the very center of the preoccupations of many people and has strong implications both in terms of asset management and regulation.

The remainder of the paper is organized as follows. Section 2 presents the data and the motivation of focusing on the Emerging Markets. Section 3 introduces the CDS Bond Spread Basis, the market liquidity measure and the methodology for applying it in the case of multiple currencies. Section 4 describes the results about the liquidity contagion and its implications. Finally, section 5 proposes robustness check and additional results before the conclusion of the paper in section 6.

1.2 Data

In this section, we present the interest of focusing on the Emerging Markets and especially the sovereign debts when we study funding liquidity problems. Then, we introduce the database needed in order to compute our liquidity indicator and the sample of countries for which we explore the possible presence of contagion effects.

1.2.1 Emerging Sovereign Debt Markets

The term of *Emerging Markets* (EM hereafter) appears for the first time in 1981. Since then the World Bank classifies as EM any markets meeting at least one of the following criteria: (i) being located in a low or middle-income economy as defined by the World Bank, (ii) not exhibiting financial depth; the ratio of the country's market capitalization to its Gross Domestic Product (GDP) is low[3], (iii) existence of broad based discriminatory controls for non-domiciled investors, or (iv) being characterized by a lack of transparency, depth, market regulation, and operational efficiency. The creation of emerging markets

[3]World Bank define *low GDP* as less than 755 USD per capita.

is motivated by the need of developing countries to raise capital to finance their growth. Before the 2000's developing countries borrowed either from commercial banks or from foreign governments multilateral lenders (International Monetary Fund or World Bank). Capital flows to emerging markets increased dramatically and commercial bank debt that was the dominant source of foreign capital has been replaced by portfolio flows[4] or foreign direct investment [Bekaert and Harvey (2003)].

EM are today considered as an asset class *per se* by many investors. Emerging economies have passed an important stress test during the period 2008-2009 and are now the key drivers for global growth of the world economy. As pointed out by the JP Morgan recent study[5], "*Potential growth rates for emerging economies of 5.8% now overshadow potential growth of only 1.6% for advanced economies*". This explains why these markets are associated with very interesting investment opportunities for any investor seeking both returns enhancement and diversification. Inflows into EM have reached a record of US$70 billion in 2010 and will continue to grow as EM yields stay attractive in the context of current global bond markets. Also interesting to notice, the proportion of EM sovereign debt in local currency now accounts for around 80% of the total EM sovereign debt. As a consequence, any simple mean-variance portfolio optimization suggests a high allocation to EM debt. Different client surveys made by banks show an increase in EM debt allocation from around 20% in 2009 to around 25% one year later. Therefore, EM investments appear as really interesting but they suffer from additional risks, such as liquidity risk and, in some cases, contagion effects, that are not taken into account in the basic mean-variance approach or more generally, by asset pricing models. For example, Brandon and Wang (2012) show that the performances of hedge funds are strongly impacted by the consideration of liquidity risk. This is especially the case for hedge funds invested in the asset class of EM. In this paper, we consider the example of

[4]Essentially composed of fixed income and equity.

[5]JP Morgan Securities, Emerging Markets Research, EM Moves into the Mainstream as an Asset Class, November 23, 2010.

index funds invested on Emerging Sovereign Debt Markets.

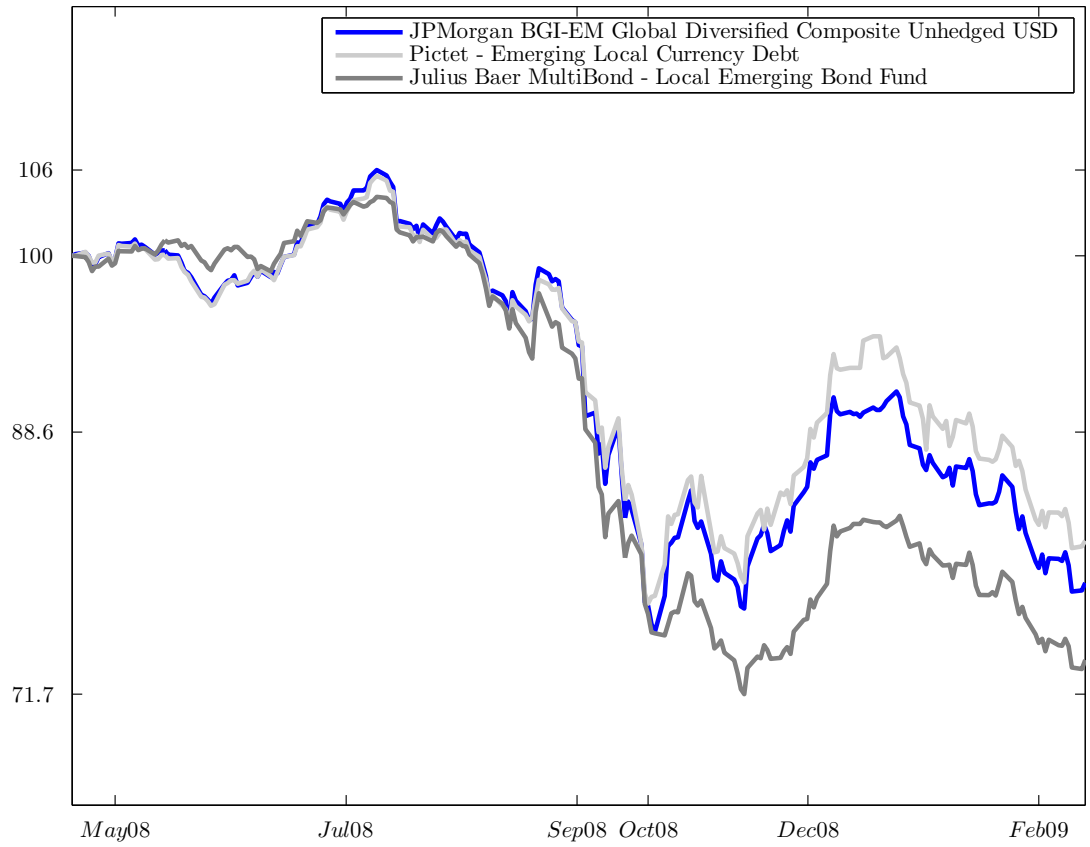


Figure 1.1 – Both index and funds are presented in USD at a daily frequency from May 2008 to February 2009.

Taking the example of a fund manager tracking the performances of EM, Figure 1.1 shows that a funding liquidity problem can largely impact the returns of a fund. Indeed, we see that the performances of two different index funds tracking the JP Morgan BGI-EM Index largely differ in October 2008. Pictet (light gray line) has experienced a 800 millions cash outflow corresponding to 10% of the Assets Under Management (AUM) while Julius Baer (dark gray line) did acknowledged a 1400 millions cash outflow representing more than 30% of the AUM. As we see, the latter exposes the difficulty for the managers tracking the index while exposed to liquidity problems.

1.2.2 The description of the Sovereign Debt Markets

We use data on sovereign bond yield spreads, sovereign CDS, interest rates and foreign exchange rates[6] for 9 emerging markets: Brazil, Chile, Hungary, Mexico, Poland, Russia, South Africa, Thailand and Turkey. Our sample period is ranging from 01/01/2007 to 26/03/2012. The database is downloaded from Bloomberg.

The time series cover many of the recent crisis and allows us to explore emerging markets behavior during economic disturbances. We are interested to know if it is still possible to benefit from diversification principle and when these benefits could be higher. The CDS premiums are based on 5-year U.S. dollar contracts, for senior claims, and they assume a recovery rate of 25%. We use as risk-free rate the US Swap rate 30/360 paid semiannually.

	Mean	Std	Min	Med	Max	Skewness	Kurtosis
Brazil	148.53	77.94	61.50	123.00	586.41	2.185	7.761
Chile	86.93	59.28	12.50	74.79	322.96	1.369	4.904
Hungary	261.24	172.27	17.34	250.58	738.60	0.314	2.265
Mexico	141.38	90.31	28.17	119.92	601.21	1.755	6.466
Poland	129.56	88.83	7.67	133.50	415.00	0.507	2.809
Russia	209.91	181.84	36.88	153.60	1113.38	2.229	7.758
South Africa	164.00	103.12	24.57	147.80	654.96	1.538	5.891
Thailand	123.30	66.17	31.84	113.01	489.56	1.267	5.242
Turkey	230.46	94.12	118.61	195.72	824.61	2.006	8.447

Table 1.1 – Summary statistics for CDS premiums from 1-1-2007 to 3-26-2012. Results are expressed in basis points and percentage.

Table 1.1 provides descriptive statistics for the sovereign CDS premiums, in other words, for the sovereign default risk. The wide range of averages highlights the high degree of heterogeneity among countries with a minimum of 86.93 for Chile and a maximum of 356.95 for Thailand. This is confirmed by the relative STD, where its value for Russia (86.63%) is more than twice that of Turkey (40.84%). For example, the cost of credit protection for Russia increases from 36.88 to 1,113.38 basis points while it reaches only

[6]Exchange rates are only used to deal with the problem of different currency issuance among CDS and bonds.

	Mean	Std	Min	Med	Max	Skewness	Kurtosis
Brazil	1242.57	125.59	1000.20	1228.70	1900.40	1.774	8.115
Chile	575.67	80.68	349.96	570.59	822.91	0.373	3.654
Hungary	793.59	134.02	580.50	739.75	1300.03	1.394	4.647
Mexico	704.26	101.70	489.81	731.01	1013.84	-0.228	2.484
Poland	557.46	40.86	474.12	554.89	737.21	0.585	3.383
Russia	777.86	196.81	596.54	712.10	1350.85	1.512	4.126
South Africa	816.56	77.10	679.46	802.22	1109.09	1.001	4.320
Thailand	356.95	65.38	210.32	348.46	571.08	0.515	3.494
Turkey	542.31	146.12	256.20	556.91	1303.43	1.098	6.012

Table 1.2 – Summary statistics for Bond yields from 1-1-2007 to 3-26-2012. Results are expressed in basis points and percentage.

from 12.50 to 322.96 for Chile. We have similar results for the sovereign debt market as shown in Table 1.2. Brazil has the higher mean with 1242.57 while Thailand exhibits the lowest average bond yield. The return on an investment into the sovereign debt of one of these countries allowed the investor to earn at least 2.56% annually, investing in Turkey and a maximum of 19% investing in Brazil. Comparing Table 1.1 and Table 1.2, we see that the ranking of countries widely differs. Brazil has the biggest bond yield while it is only the sixth country in terms of CDS premium. We have similar results for some other countries that exhibit different behavior of their bond yield relatively to their CDS premium. These tables show that the sovereign debt market is less volatile than the CDS market with a maximum of 26.94% for the relative standard deviation while the minimum for the CDS market is 40.84%.

1.3 The Emerging Sovereign Debt Market Liquidity measure

In this section, we explore the CDS Bond Spread Basis and discuss its ability to accurately measure the liquidity of the sovereign debt market. Although some other factors in addition to liquidity contribute to the level of the Basis, we explain why they do not have

an impact on the dynamic of the liquidity indicator and especially in the case of contagion study. We also present a way to compute the Profit and Loss distribution (P&L hereafter) of CDS and Bonds in order to get them comparable. As they are issued in two different currencies, we can not directly subtract the Bond yield issued in local currency from the CDS Premium expressed in US dollar.

1.3.1 The CDS Bond Spread basis

CDS were created in 1994 by J.P Morgan & CO. Since its creation the CDS market rose until 2008 and has stagnated since. CDS became in a few years a standardized financial product used by most of the market major participants (banks, hedge funds, mutual funds...). Nowadays, it is one of the most popular tool for transferring credit risk. The CDS contract is defined as a bilateral contract that provides protection on the par value of a specified reference asset. The protection buyer pays a periodic fixed fee or a one-off premium to a protection seller. In return, the seller will make a payment on the occurrence of a specified credit event [Choudhry (2006), Mengle (2007)]. Then, CDS provides to buyer a protection against the risk of default by borrowers, named the entities. The default, also named *credit event* is contractually defined by the two parties and could be bankruptcy, failure to make a schedule payment, obligation default, debt moratorium, financial or debt restructuring and credit downgrade[7]. This is important to precise that rating agencies have not influence in triggering CDS. Their actions may, but not need, taken into account. The protection buyer has to pay an amount of fees (also named CDS premium or CDS spread) to protection seller and receives a payoff if the underlying bond experiences a credit event. At the deal inception, the two parts define which kind of settlement they want. The CDS contract could be settled in one of two ways: cash or physical settlement. Most of the time, contracts are physically settled (about 75-85%).

[7]The main part of CDS are documented using the 2003 ISDA Credit Derivatives Definitions, as supplemented by the July 2009 Supplement.

Although the CDS contract has a given maturity, it may terminate earlier if a credit event occurs. In this case, the protection seller has to pay an amount called the protection leg.

The basis is nothing else but correcting the CDS from the sovereign bond (CDS bond spread basis). This is a way to cancel out the global macro effects when analyzing the commonality of sovereign risk. In other words, we focus on the long term liquidity. The basis is defined as the difference between the asset and its synthetic version. The no arbitrage theory of pricing CDS implies that the basis should be zero. As both of these two assets should price the same default risk of the country, from the law of one price, they should be equal. In practice, this situation almost never occurs. The breaking case highlights a liquidity problem on one or the other market. In addition to the liquidity, the level of the basis could fluctuate for many reasons that could be split into two categories: technical and market factors. We mainly find in the technical factors the delivery option and counterparty risk. To characterize the first, we have to define what *deliverable options* means. CDS contracts usually allow buyer and seller to agree on a panel of alternative assets that the buyer can deliver in case of a credit event. It allows to the buyer to deliver the cheapest obligation that he possesses in his eligible basket of assets. This option does not add value systematically even in the case of sovereign debt market. As we see in Ammer and Cai (2007), the Cheapest-to-Deliver (CtD) option could be valuable for the emerging sovereign debt market. However, our model is based on the existence of frictions interfering with exact arbitrage between CDS and bonds. One of these frictions we are particularly interested in is the liquidity of the sovereign debt market. In this context, it becomes really difficult to model and evaluate the CtD option. Indeed, Ammer and Cai (2007) propose to measure the spread part that could be attributed to the CtD option. Their model requires two strong assumptions allowing to measure the CtD option: the recovery rate is independent of time-to-default and the CtD option is the only friction. This second assumption is not realistic in our case and this is empirically proved that market liquidity is one of the main frictions interfering in the arbitrage relation between

the CDS premium and the bond yield spread over the risk free rate. As the CtD option, although valuable, is sometimes null we neglect it in our model to focus on the market liquidity. The second is the counterparty risk. On the one hand, the protection seller can default and do not settle the protection buyer in case of a credit event. On the other hand, the buyer can also default and stop paying the CDS premium to the seller. However, some mechanisms like the counterparty clearing system allow to reduce these risks (almost half of CDS are treated by clearing). Moreover, as showing in Levy (2009), if the default probability of the underlying bond and the default probability of the counterparty are not correlated, the two effects may cancel each other out. Furthermore, counterparty risk is a joint event of two defaults. Thus, the excess premium associated is weighted by a product of two probabilities and should be really small, or negligible. Our aim being the analysis of the dynamic of the emerging market liquidity, we consider that the main part of CDS is issued by companies in countries which are outside our sample of EM. As a consequence, if the counterparty risk changes, its impact is approximately the same for all countries and does not alter the dynamic of correlations that we study. Based on a demonstration proposed by Levy (2009), we focus on the liquidity premium induced by the movements of the basis on emerging markets.

CDS includes two legs corresponding to the premium payments and the default payment. The pricing of a CDS depends, among others, on the recovery amount (a recovery rate of par value and accrued interest). Duffie (1999) or Hull and White (2000) expose two approaches for the pricing of CDS premium. The first, that we call "no arbitrage" approach, follows the idea that an investor can buy a CDS and the underlying bond to replicate the risk free rate. The second is based on a reduced-form model with random stopping time. In order to demonstrate the impact of liquidity on the CDS Bond spread basis, we use the first one. As a result, buying a risky bond and its CDS with the same maturity allow to the investor to eliminate the default risk associated with the bond. Assuming that there is no arbitrage opportunity, this portfolio should be equal to the value

of the risk free bond with the same maturity. As in Zhu (2006), we price CDS premiums and Bonds separately. We construct a portfolio that replicate the CDS contract and we obtain the CDS Spread Basis. In this context, we assume a risk neutral world with three assets: a risk-free bond, a risky bond and a CDS contract.

Following Levy (2009), under the risk neutral valuation, we express the value of a CDS premium, a risky bond and a risk-free bond. Then, we construct a portfolio that shorts the risky bond and buys the risk free bond subtracting (4) to (5). We propose details of computations in appendix and finally, assuming that the risky bond is traded at par, thus we have:

$$b - (y - r) = 0 \tag{1.1}$$

As a consequence, the CDS Bond Spread Basis is equal to zero, theoretically. However, this relation changes assuming that there are two types of traders: one trading with high liquidity and no holding costs (h) and the other one trading with low liquidity and having holding cost of d (l).

In this case, both the CDS premium and the Bond yield have an additionnal component due to the search cost of liquidity. As a result, we have:

$$\tilde{b} = \tilde{y} - r - (S_{bond} - S_{CDS}). \tag{1.2}$$

Thus, the parity between CDS and risky bond should hold only for the pure risk component that is priced into the two assets. As a consequence, we can expect a non zero basis when liquidity differences exist.

1.3.2 Basis trade with multiple currencies

The CDS Bond Spread basis that compares CDS premium denominated in dollar to the local currency denominated sovereign debt is biased. To tackle this problem, we compute and correct the P&L of an investment strategy corresponding to the basis. In other words, we buy/sell both instruments when the basis is negative/positive. The computation of the P&L is the way that traders refer to the daily change of the value of their trading positions. The P&L is generally defined as the difference between the value at time $t+1$ and t. In other words, the P&L of an asset is the profit or the loss that this asset makes between two dates. In this sense, we can split the P&L between two parts: the Mark-to-Market (MtM hereafter) part and the Carry part. The former is the gain (or the loss) realized when selling the asset. The latter, called the carry, is the gain (or the loss) i.e., the income you earn on the asset during the period you own it (in our case, one day). Schematically, we can express the P&L of an asset as the sum of the MtM P&L and the Carry P&L.

Both CDS and Generic bonds P&L are computed at a daily frequency, and thus, ignoring the carry. This component is very close to zero due to the daily investment horizon. As a consequence, the annual return of the asset has to be divided by 250, but could be neglected in the P&L computation. We detail computation of P&L for both CDS and Generic Bond in appendix A.2. Once we compute these time series of P&L, we can easily calculate the CDS Bond Spread Basis. Indeed, when the basis is negative, we buy both the CDS protection and the bond and conversely when the basis is positive. As a result, in the first case, we add the two P&L of the CDS and the bond while we make the sum of their opposite value in the second case.

Our empirical study of the market liquidity indicators confirms some stylized facts and the collapse of Lehman Brothers just as the 2007-2008 financial crisis are strongly highlighted. Figure 1.2 presents, for each of the 9 emerging countries of our sample, the

level of CDS Bond spread basis. Firstly, we can see that the basis is almost never equal to zero indicating that the arbitrage relation is not verified. We see that all countries experienced almost simultaneously a liquidity problem. Indeed, the graphics reveal a large increase of the basis for every country at the end of September 2008. Even if the close relation between all the basis is obvious during this period, the increase of volatility for all the markets may be the only source of contagion. If this is true, the fund manager does not have to change his portfolio allocation. However, if the correlations increase despite a control for volatility, corresponding to pure contagion effects, the fund manager is exposed to a new risk and the allocation of his portfolio is not efficient anymore.

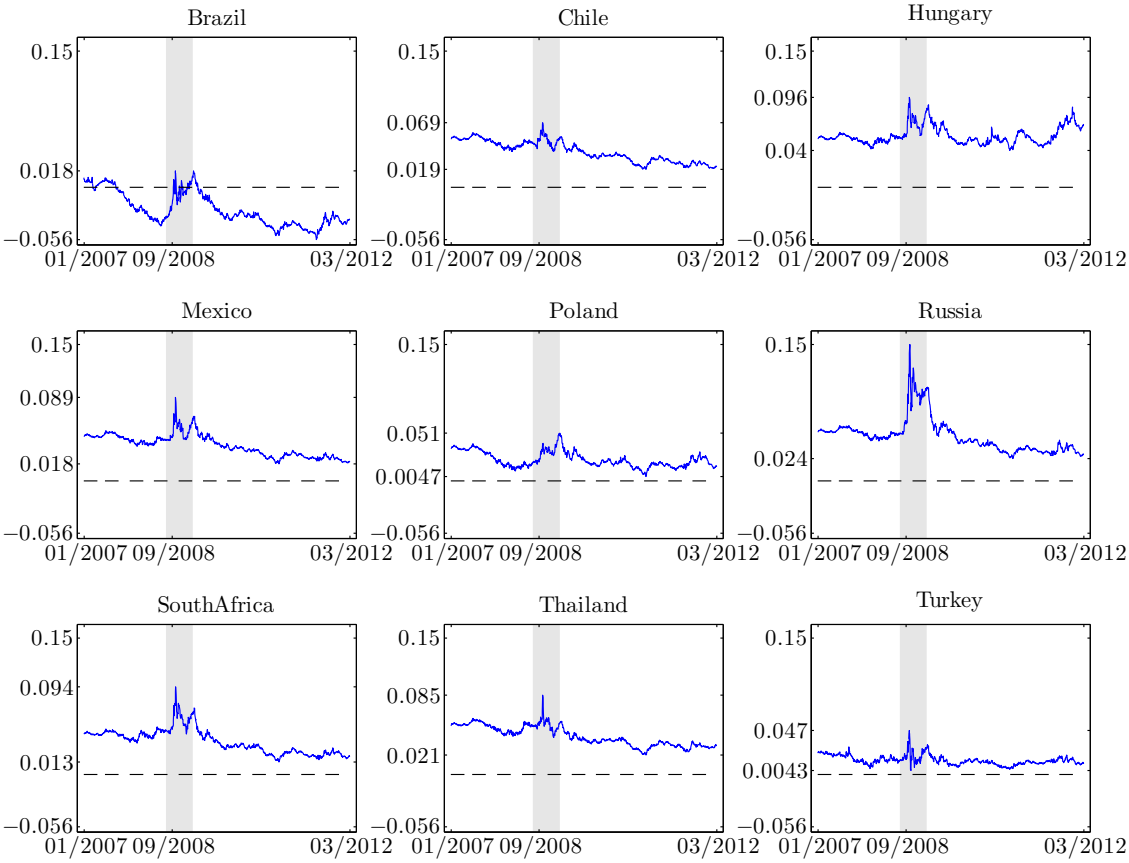


Figure 1.2 – CDS Bond spread basis for each of the 9 emerging countries from the 01/01/2007 to the 03/26/2012. The results are computed with daily observations, expressed or corrected in US dollars. The grey band represents a period with explicit higher correlations between all the basis.

Finally, coming back to liquidity, we know that the contagion between markets drives their in and outflows, and liquidity moves consequently. In the line of the above approach,

we can link contagion and liquidity moves by comparing the commonalities between the liquidity indicators introduced in the previous section. If the commonality is between liquidity and volatility, there is no contagion effect but only interdependence. On the contrary, if the liquidity shock has an impact on the correlation matrix, liquidity can be considered as a contagion channel.

1.4 Liquidity contagion and funding liquidity

Now, we focus on the dynamic of the link between market liquidity risks of sovereign debt markets. In order to detect financial contagion effects and following Pelletier (2006), we apply the RSDC model. This latter is a dynamic model allowing a regime switching of the correlation matrix and a conditional modeling of volatility. These two characteristics are very relevant when we focus on financial contagion effects. Firstly, we have to make the distinction between interdependence and pure contagion. On the one hand, the first is represented by common exogenous shocks able to increase links between countries or markets without a significant change of dependence. On the other hand, the pure contagion is described as a significant increase of correlations after a shock and considering the previous link between markets or countries. We find a large literature that focuses on the link between an increase of the volatility and an increase of correlations. For example, Forbes and Chinn (2004) show that correlations between countries go up in crash times. But, several papers demonstrate that a positive evolution of the volatility leads to an increase of correlations (e.g. Hamilton and Susmel (1994), Solnik et al. (1996) or Chesnay and Jondeau (2001a)). Secondly, in order to detect and identify pure contagion effects, the definition of crisis periods has to be endogenous. The RSDC is a state/space model allowing to specify the dates of crisis in an endogenous way. As a result, we are able to detect if there exists a shift in terms of correlations and we also know the period when this contagion effects occurs. The RSDC model is detailed in as one of the most

performing to model the behavior of correlations compared to other multivariate conditional correlation models. Finally, we choose a two-regimes RSDC since we only need to detect a shift in terms of correlations, represented here as low or high.

1.4.1 Filtering a common liquidity factor

Usually, we test for financial contagion considering the price as the main channel. However, in this paper, we focus on the contagion in terms of liquidity. As a result, we filter a common liquidity factor from several market liquidity indicators. Indeed, we study the smoothed probabilities obtained using the RSDC model. This latter is nothing but the probability to be in a state for which the correlations are high relatively to other periods. Indeed, as we focus on detecting shift in terms of correlations, we only consider two states for the RSDC model. The first is a state of calm periods and the second is a state of crisis periods characterized by higher correlations. Figure 1.3 displays the smoothed probability obtained based on the CDS Bond Spread basis of 9 emerging countries. We see that the probability to be highly correlated is very low, almost null; with some peaks from the start of our sample period to October 2008. Thus, we can conclude that the Lehman Brother collapse has a great influence on the re-correlation phenomenon. The important fact is that the probability stays high after the end of 2008 meaning that we do not come back to a normal state even after some years.

To confirm the results of pure contagion effects, we have to determine whether the two correlation matrices are significantly different. In other words, we test the number of regimes. However, the Markov switching approach does not allow us to apply standard tests methods. Under the null hypothesis, a nuisance parameter is not identified. Garcia (1998) shows that asymptotic theory works for Markov switching only assuming the validity of the score distribution. Nevertheless, the asymptotic distribution is not so far from the standard Chi-square distribution while our likelihood ratio statistic is much greater

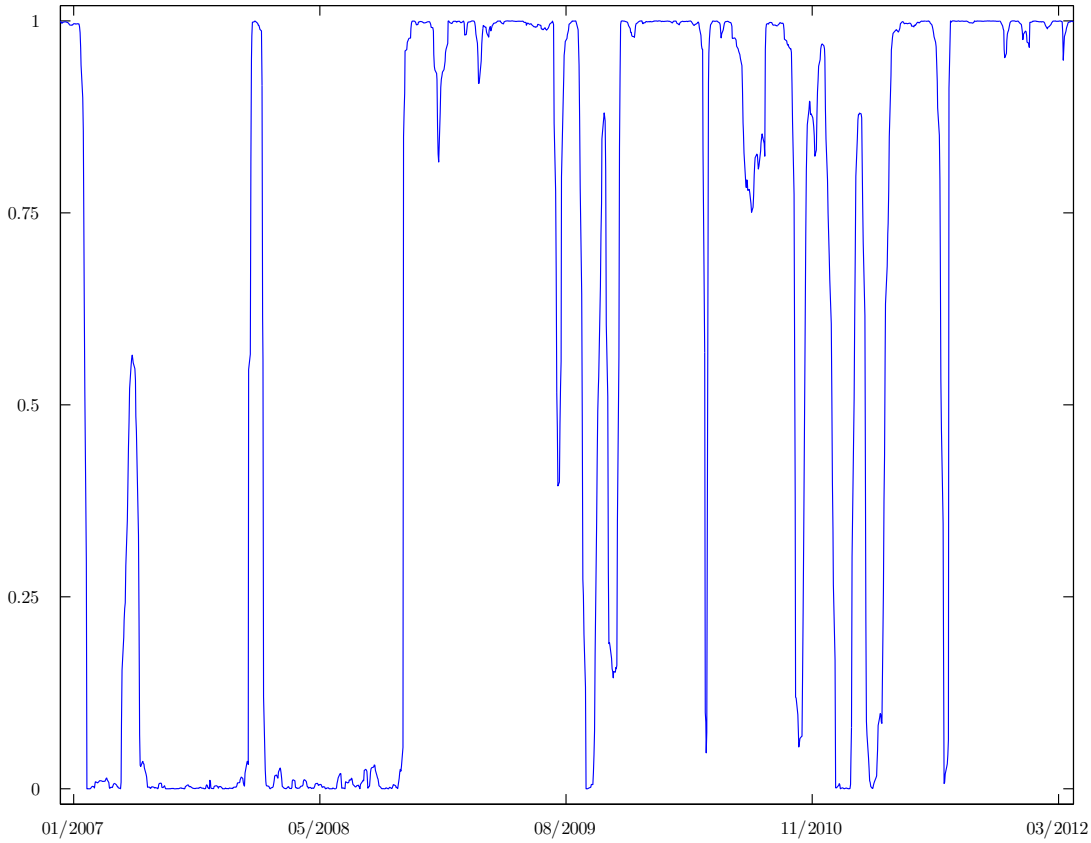


Figure 1.3 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the basis between 01/01/2007 and 26/03/2012.

than the critical value of this distribution. We conclude that a two regimes model offers greater results and confirm the significance of the difference between the two correlation matrices. To compute this statistic, we have to compare the likelihood of our model, the RSDC with two regimes and the CCC model (for Constant Conditional Correlations) which is assimilated to the RSDC with only one regime.

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		-0.015	-0.092	-0.049	0.317	-0.059	0.259	-0.009	0.290
Chile	-0.087		0.284	0.393	0.043	0.345	0.207	0.314	-0.178
Hungary	-0.052	0.422		0.364	-0.043	0.439	0.301	0.244	-0.291
Mexico	0.067	0.612	0.484		-0.063	0.647	0.222	0.289	-0.277
Poland	0.337	-0.063	0.099	-0.024		-0.134	0.268	-0.025	0.382
Russia	-0.087	0.493	0.753	0.515	0.033		0.331	0.376	-0.371
South Africa	0.148	0.274	0.569	0.352	0.332	0.641		0.157	0.070
Thailand	-0.021	0.317	0.446	0.361	0.090	0.486	0.384		-0.239
Turkey	0.381	-0.192	-0.052	-0.133	0.512	-0.085	0.228	-0.076	

Table 1.3 – Correlations matrices of the two regimes for the basis considered in terms of profits and losses generated by a such strategy. The upper part of the matrix corresponds to the regime 1 while the lower part corresponds to the regime 2.

Table 1.3 presents the values of correlations both for state 1 and state 2 based on the CDS Bond Spread basis. The RSDC model has the main advantage to propose an aggregate result for a multivariate model. We see that almost all correlation pairs increase going from regime 1 to regime 2. As a consequence, in a general way, we can conclude that the links between countries become stronger in the state considered as defining turmoil periods.

1.4.2 Identifying as a funding liquidity factor

The funding liquidity is still difficult to be measured. However, Goyenko (2012) among others[8] shows that the TED spread and the VIX could be considered as funding liquidity indicators. In this subsection, we study the link between our Funding Liquidity Indicator (or FLI hereafter) and the TED spread or the VIX.

The TED spread is the difference between the three-month LIBOR and the three-month T-bill interest rate. It reflects the credit risk of the financial system as a whole. The VIX is a measure of the implied volatility of S&P 500 index options. It denotes the predicted volatility over the next month and it is usually defined as a measure of fear on financial markets. Figure 1.4 displays the behavior of the TED spread and the VIX compared to the dynamic of the FLI. We show some similarities between the TED spread and the VIX with a stronger volatility in the case of the VIX. However, both of them have the same behavior at the end of 2008. All the three indicators experience a large increase during October 2008. However, their dynamic largely differ after this event since both the TED spread and the VIX indicate a come back to a normal state while the FLI remains high indicating that funding liquidity problems still occurs.

[8] Brunnermeier and Pedersen (2009), Boyson et al. (2010) or Teo (2011) describe the TED spread and the VIX as an indicator of speculator's capital availability in the economy.

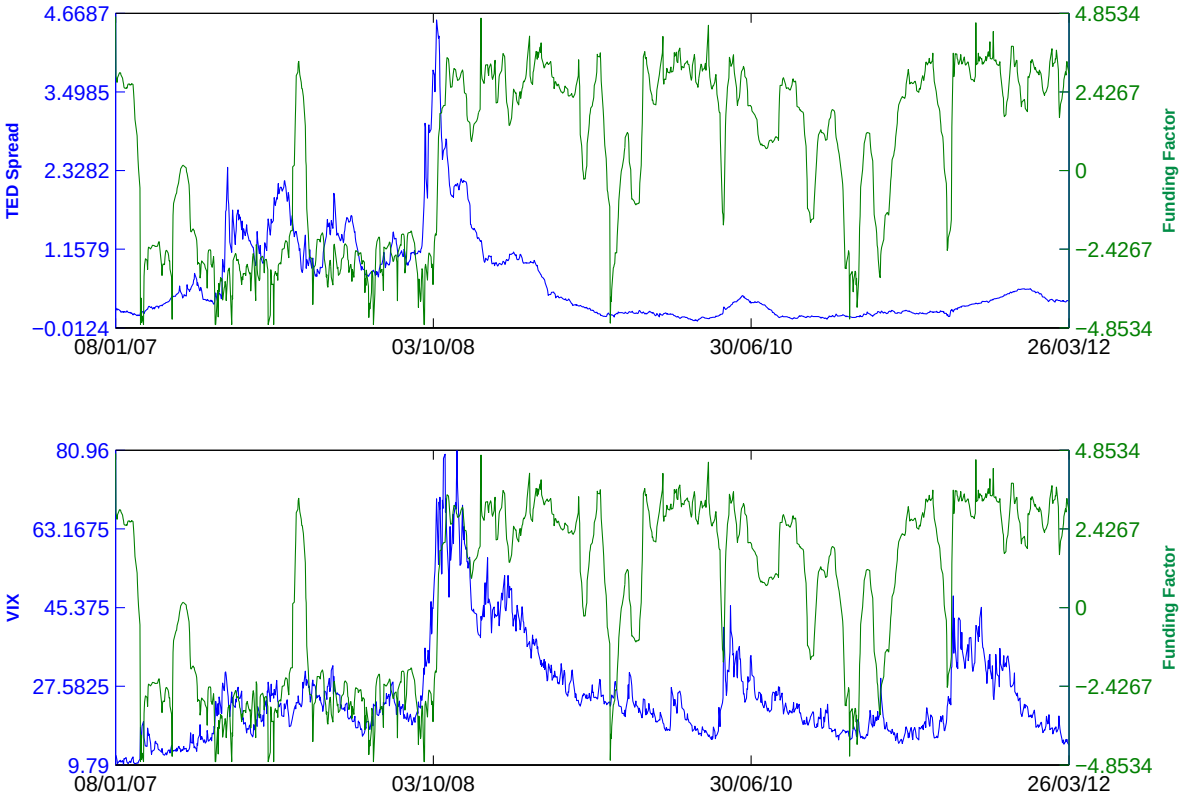


Figure 1.4 – The top graph presents TED spread and our funding liquidity indicator (FLI) while the bottom graph displays the VIX and the FLI. Both of them are at a daily frequency ranging from 01/01/2007 to 26/03/2012.

	Before 2009			After 2009			Whole sample		
Intercept	0.158**	-0.077**	-0.044	0.701**	0.602**	0.682**	0.737**	0.310**	0.212**
	(4.976)	(-2.396)	(-1.492)	(39.039)	(17.595)	(18.597)	(42.902)	(10.198)	(8.508)
TED	0.057**		-0.275	0.373**		0.345**	-0.205**		-0.474**
	(2.557)		(-1.492)	(9.045)		(5.466)	(-11.218)		(-25.979)
VIX		0.012**	0.024**		0.009**	0.001		0.011**	0.028**
		(10.844)	(14.764)		(7.096)	(0.578)		(10.419)	(25.512)
R^2	1.33%	19.45%	31.88%	9.37%	5.99%	9.41%	8.95%	7.82%	39.66%

Table 1.4 – This table presents results of time series regressions from 01/01/2007 to 31/12/2012. We split the sample in two parts, before and after 2009. The betas are estimated as the slope coefficients of our funding liquidity indicator (or FLI) on intercept, TED spread and VIX. The t-statistics are in parentheses.
* Denotes parameter estimates significant at 10% level.
** Denotes parameter estimates significant at 5% level.

Table 1.4 presents the regression results of the FLI on the TED spread and the VIX. We see the link between them is statistically significant whatever the sample studied. Indeed, except for the TED spread before 2009 and the VIX after 2009 that are not significant at 10% when we consider it simultaneously with the other indicator, all the estimated betas are significant at a 5% threshold. As we perceive on Figure 1.4, the relation between the FLI and the VIX is stronger before 2009 than after. Moreover, the volatility indicator has a positive impact on the funding liquidity indicator in all cases while the TED spread has an opposite impact after 2009 and over the whole sample. Finally, we see that the R^2 is largely greater when we consider the VIX rather than the TED spread before 2009. Moreover, it raises from 19% to 32% when considering both of them. Nevertheless, when we focus on the results after 2009, we see that the explanation capacity of the two indicators is really limited. The R^2 remains under 10% even considering both TED and VIX. All these results indicate that the FLI, a common liquidity factor is able to measure funding liquidity problems. Moreover, while the usual indicators appear driven by other factors in addition to funding liquidity, the FLI adopts a different behavior still indicating funding liquidity problems until the end of the sample.

1.5 Robustness Checks

1.5.1 The heteroscedasticity problem

All the results are based on the RSDC model of Pelletier (2006) and its capacity to distinguish interdependence from pure contagion. We consider a two-step estimation and assume that the first one is able to correct for heteroscedasticity. In this robustness check, we want to control whether the volatility is removed accurately allowing to concentrate on pure contagion phenomena. In other words, the univariate conditional volatility model where the matrix S_t in equation (28) comes from all the dynamic of the univariate variance. We check for ARCH effects in the standardized residuals and determine the best model for the first step in the estimation. We start with the well known GARCH(1,1) model, and when we reject the hypothesis of independence of the squared residuals, we estimate a TGARCH(1,1) taking into account the asymmetry of P&L time series. When there is still some remaining heteroscedasticity in all time series, we propose an alternative model [TGARCH(2,2)].

We sum up the results in Table 7. First, we see that the GARCH(1,1) model captures all heteroscedasticity effects for all countries except for Hungary in the case of the CDS and the Basis. However, in a second step, we see that even a TGARCH(1,1) cannot capture the remaining ARCH effects. In fact, we have to apply a TGARCH(2,2) on our time series to get homoscedastic standardized residuals. To test if the univariate volatility estimation fully captures the heteroscedasticity, we use the LM test [Engle (1982)] that checks for the autocorrelation of squared residuals. We can conclude about the presence of ARCH effects in the residuals of our model.

However, whatever the univariate volatility model, we detect pure contagion effects at the very same date. Indeed, the smoothed probabilities have a similar dynamic no matter which model is applied. As a result, the RSDC model is robust to the volatility

specification and allows us to concentrate on pure contagion effects resulting from a shift in the correlation structure.

1.5.2 Pure contagion in terms of prices

Usually, the contagion is studied in terms of prices but we show in this paper that the liquidity also experiences pure contagion effects. It appears relevant to determine whether there exists an additional information focusing on the liquidity contagion. Thus, we propose to apply our financial contagion model to the CDS premium and the Bond yield.

We give in Figure 1.5 the smoothed probabilities of being in the regime of high correlation for the bond market. We see that the correlations appear to be dynamic and switch between regimes. At each date, there is an uncertainty about the regime of correlations. On the one hand, the process spends more time in regime two and spells in regime one are shorter on average. The probability of being in regime two at time t, conditional on being in regime two at time $t-1$, $p_{2,2}$ is 0.9998. Such a high probability means a very high level of persistence in the Markov chain. In comparison, for regime one this probability fells around 0.9040. Despite the close proximity of these probabilities, as Pelletier (2006) shows, it results in large differences. Indeed, after 5 periods, these probabilities are respectively approximately equal to 0.95 and 0.55[9]. On the other hand, we show that the magnitudes of almost all the correlations in regime one are smaller than in regime two. Correlation matrices are presented in appendix A.5 page 64. Moreover, the smoothed probability to be in state two largely increases at the end of September 2008. As a consequence, we can consider that there exists a re-correlation phenomenon on the sovereign debt market and this phenomenon occurs almost simultaneously with the Lehman Brother collapse.

The results of the CDS market are similar (see Figure 1.6). Indeed, the shift in correlations appears at the same date as the bond market. However, the smoothed probability

[9]Indeed, $0.99^5 = 0.95$ and $0.90^5 = 0.55$

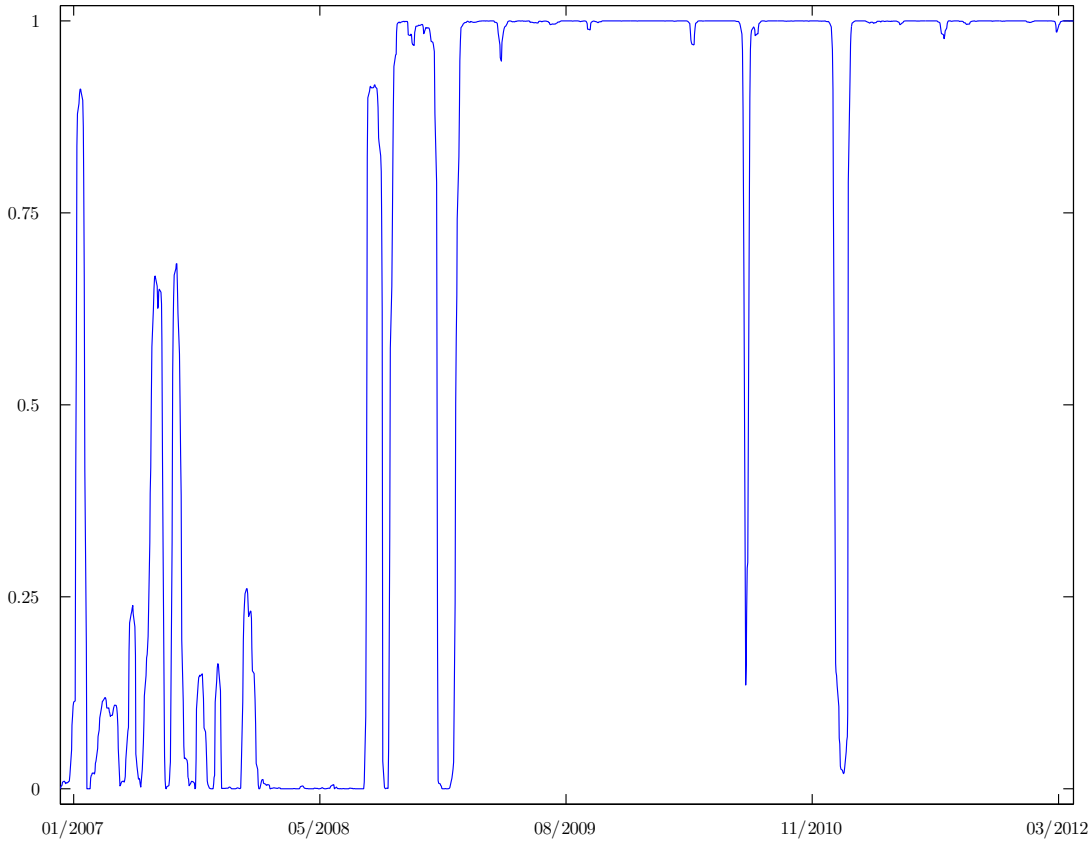


Figure 1.5 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the bond market between 01/01/2007 and 26/03/2012.

to be in the state of high correlations for the CDS market is more volatile than for the bond market. The probability to be in the state of low correlations at time t and $t+1$ is equal to 0.9414 which is much greater than in the case of the bond market. Thus, as both CDS and bond markets exhibit shifts in terms of correlations at the end of 2008, the benefits from the diversification principle have plummet and the risk of the portfolios dramatically increased. Furthermore, we have to know if this phenomenon also comes from a liquidity problem since it would represent an additional risk for the fund manager. Moreover, we state that even if the assets behaviors come back to normal after some months, the correlations between them stay high. In that case, it remains particularly difficult to benefit from the diversification principle on the sovereign debt markets.

Finally, the only common event is the shift that appears at the end of 2008. However, we show that focusing on market liquidity indicators leads to additional information about

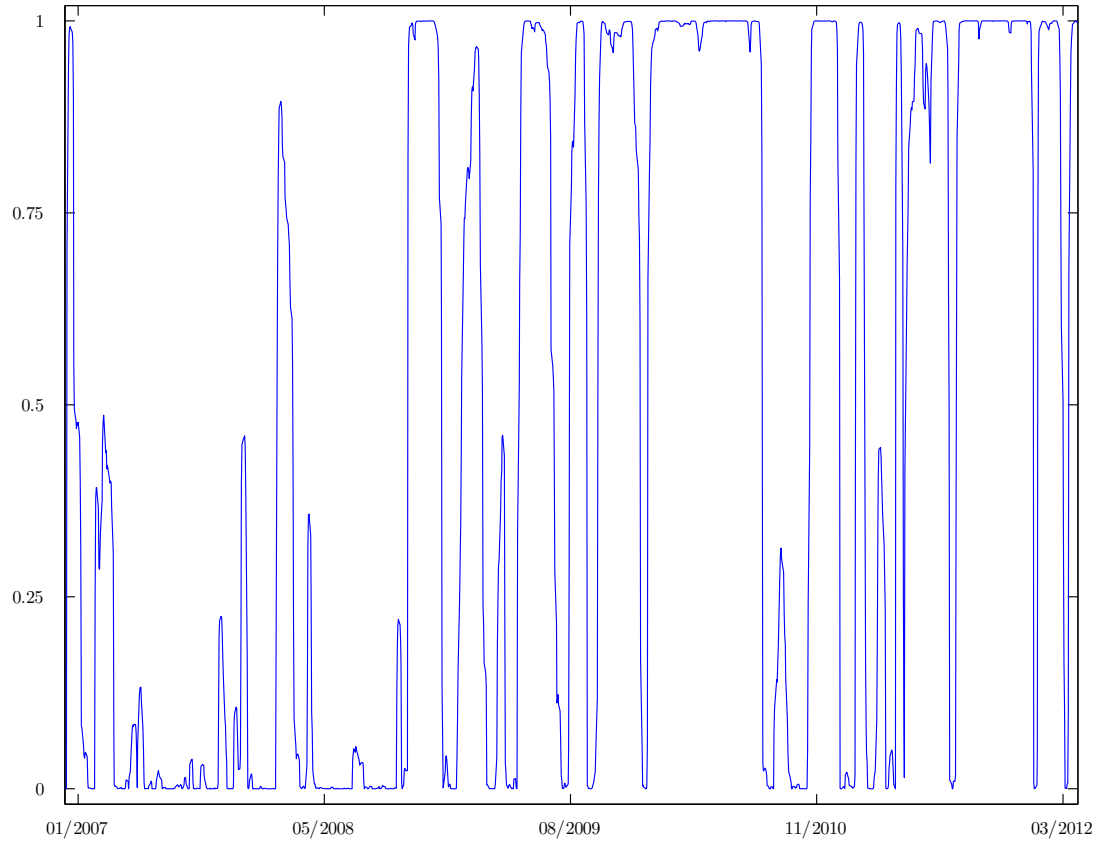


Figure 1.6 – Smoothed Probabilities to be in the state of high correlations at a daily frequency. The results concern the CDS market between 01/01/2007 and 26/03/2012.

the behavior of emerging sovereign debt markets and allows a better understanding of the funding liquidity risk.

1.6 Conclusion

EM have experienced many financial crisis with contagion problems but, they are today the key drivers for global growth of the world economy. They propose very attractive investment opportunities for asset managers who consider them as an asset class. Nevertheless, the main risk for an asset manager is to loose the diversification benefits of his/her portfolio.

Firstly in this paper, we study the liquidity of the sovereign debt market showing the ability of the CDS Bond spread basis to accurately measuring liquidity. Secondly, we use a non linear model to detect contagion effects both in terms of prices and liquidity. Whereas interdependence is not a main concern for a fund manager, pure contagion phenomena may be problematic. The RSDC model is able to separate interdependence from pure contagion in order to focus on the second one. Such a phenomenon is highlighted by a shift in the probability to be in the state where the assets are more correlated.

Our main focus is the pure contagion phenomena in terms of market liquidity problems that we detect during the 2007-2008 financial crisis for a sample of 9 emerging countries. Indeed, the contagion model allows us to extract a funding liquidity indicator based on the market liquidity of the sovereign debt of several countries. As a result, we show there still exists a funding liquidity problem on the emerging sovereign debt market while standard indicators let appear a come back to a pre crisis state.

A.1 The CDS Bond Spread basis

The CDS premium b satisfies:

$$\sum_{t=1}^{T} be^{-rt}F(t) = \int_0^T (100 - RV_t)e^{-rt}f(t)dt, \tag{3}$$

where T is the number of times till maturity or default, r is the risk-free rate, RV_t is the recovery value at time t, $f(t)$ is the probability of default at time t and $F(t)$ is the survival probability[10]. The left hand side is called the Premium leg and the right hand side is called the Insurance leg.

The value of the risky bond is expressed as:

$$Y = \sum_{t=1}^{T} Ce^{-rt}F(t) + 100e^{-rT}F(T) + \int_0^T RV_t e^{-rt}f(t)dt, \tag{4}$$

where C is the fixed coupon paid for each period.

And the value of a risk-free bond at risk-free rate r is expressed as:

$$100 = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT}. \tag{5}$$

We construct a portfolio that shorts the risky bond and buys the risk free bond subtracting (4) to (5). We obtain:

$$100 - Y = \sum_{t=1}^{T} re^{-rt} + 100e^{-rT} - \sum_{t=1}^{T} Ce^{-rt}F(t) - 100e^{-rT}F(T) - \int_0^T RV_t e^{-rt}f(t)dt. \tag{6}$$

[10] $F(t) = 1 - \int_0^t f(x)dx$

If we modify the risk-free bond equation to include the default probability, we get:

$$100 = \sum_{t=1}^{T} re^{-rt}F(t) + 100e^{-rT}F(T) + \int_0^T 100e^{-rt}f(t)dt. \tag{7}$$

The value of our portfolio becomes:

$$100 - Y = \sum_{t=1}^{T}(r - C)e^{-rt}F(t) + \int_0^T (100 - RV_t)e^{-rt}f(t)dt. \tag{8}$$

Combining equation (3), we can write:

$$100 - Y = \sum_{t=1}^{T}(b + r - C)e^{-rt}F(t) \tag{9}$$

Finally, the CDS Bond Spread Basis is expressed as:

$$b + (r - C) = \frac{100 - Y}{\sum_{t=1}^{T} e^{-rt}F(t)} \tag{10}$$

The CDS Spread Basis has to be equal to zero since the risky bond is traded at par, i.e. $Y = 100$. Moreover, the fixed coupon of a par bond is equal to the bond's yield to maturity ($y = C$) and we have:

$$b - (y - r) = 0 \tag{11}$$

Furthermore, assuming that there are two types of traders: one trading with high liquidity and no holding costs (h) and the other one trading with low liquidity and having holding cost of d (l). We denote by b_i the CDS premium fair price for trader i, $i = l, h$, $\tilde{S}$, the market price for this CDS and p_i the probability to immediately find a trader of type i. We know that a trader, who has liquidity problems, should pay an additional holding

cost. Then, from equation (3) we obtain:

$$\sum_{t=1}^{T} b_h e^{-rt} F(t) = \int_0^T (100 - RV_t) e^{-rt} f(t) dt \qquad \text{for high liquidity traders,} \quad (12)$$

$$\sum_{t=1}^{T} b_l e^{-(r+d)t} F(t) = \int_0^T (100 - RV_t) e^{-(r+d)t} f(t) dt \qquad \text{for low liquidity traders,} \quad (13)$$

where d is the additional holding cost.

From these two equations we can extract the CDS premium for each type of traders as:

$$b_h = \frac{\int_0^T (100 - RV_t) e^{-rt} f(t) dt}{\sum_{t=1}^{T} e^{-rt} F(t)} \qquad \text{for high liquidity traders,} \quad (14)$$

$$b_l = \frac{\int_0^T (100 - RV_t) e^{-(r+d)t} f(t) dt}{\sum_{t=1}^{T} e^{-(r+d)t} F(t)} \qquad \text{for low liquidity traders.} \quad (15)$$

Obviously, trade occurs only if $b_h < \tilde{b} < b_l$. Introducing the value of search process V, the trader has to be indifferent between searching alone or buying to a market maker. We get:

$$V = p_h b_h + p_l (V + C) = \frac{p_h b_h + C p_l}{1 - p_l}, \quad (16)$$

where C is the search cost.

The market price $\tilde{b}$ is equal to:

$$\tilde{b} = V = b_h + \frac{C p_l}{1 - p_l}, \quad (17)$$

where $\frac{Cp_l}{1-p_l}$ is the additional spread for the asset (CDS and bond that we note respectively S_{CDS} and S_{bond}).

$\tilde{b}$ is the market price for the CDS and is such that $\tilde{b} = b + S_{CDS}$. $\tilde{y}$ is the market price for the bond and is equal to $y + S_{bond}$. Taking into account liquidity, equation (11) becomes:

$$\tilde{b} = \tilde{y} - r - (S_{bond} - S_{CDS}). \tag{18}$$

A.2 Computing P&L

Generic Bond P&L

The MtM component of the P&L of an asset is the same whatever the asset, i.e. the difference between the prices at two distinct dates. In the case of a bond, it can be expressed as the variation of the yield-to-maturity (YtM hereafter) multiplied by the sensitivity of a one unit variation. Thus, we note:

$$P\&L_t^{bond} = [YtM_t - YtM_{t-1}] \times sensi_t^{bond}. \tag{19}$$

In order to define the sensitivity, we have to specify what the duration is. This latter is the weighted average maturity of cash flows, expressed as:

$$D = \sum_{t_i=1}^{N} \frac{t_i \times \frac{CF_i}{(1+YtM)^{t_i}}}{PB}, \tag{20}$$

where t_i is the time in year until the next i^{th} payment, CF_i is the ith cash flow, YtM is the Yield-to-Maturity and PB is the present value of the bond.

The sensitivity, $sensi_t^{bond}$ is defined as the opposite of the modified duration, that is, in the case of periodically compounded yields, the duration over the Yield-to-Maturity:

$$sensi_t^{bond} = \frac{D}{(1 + YtM_{\%})}. \tag{21}$$

In the case where the CDS and the Bond are not expressed in the same currencies, we need to correct the P&L of the generic bond by the corresponding exchange rate. We call PB_t the present value of the bond in local currency and X_t the exchange rate at time t. We know that the dollar price's variation is expressed as:

$$[PB_t - PB_{t-1}]_{\$} = \frac{PB_t}{X_t} - \frac{PB_{t-1}}{X_{t-1}}. \tag{22}$$

Linearizing this expression, we can separate the MtM component of the bond's P&L in two parts:

$$[PB_t - PB_{t-1}]_{\$} \simeq \frac{1}{X_{t-1}}(PB_t - PB_{t-1}) - \frac{PB_{t-1}}{X_{t-1}}\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right). \tag{23}$$

The first term represents the P&L of the Bond and the second, the gain (or loss) due to the variation of the exchange rate. Crossing the expression of the P&L given in equation (19), we obtain:

$$[PB_t - PB_{t-1}]_{\$} \simeq \frac{1}{X_{t-1}}\left[(YtM_t - YtM_{t-1}) \times sensi_t^{bond}\right] - \frac{PB_{t-1}}{X_{t-1}}\left(\frac{X_t - X_{t-1}}{X_{t-1}}\right). \tag{24}$$

CDS P&L

To compute the P&L of the CDS, two informations need to be recalled: the trading horizon is one day and the price of a CDS strategy at the issuance is equal to zero. Consequently, the P&L of a CDS, assuming that we neglect the carry part, equals the selling price. In the case of a CDS, we are able to express the price at time t as the product of the premium's variation between t and the issuance date, and the sensitivity to a change of 1bp of the CDS premium. Summing up, the P&L of the CDS can be expressed as:

$$[PC_t - PC_{t-1}] = PC_t = [S_t - S_{t-1}] \times sensi_t^{CDS}, \tag{25}$$

where PC_t is the price of the CDS contract at time t. As we consider $t-1$ as the date when we start the contract, PC_{t-1} is null because the value of a CDS at the opening is equal to zero.

Using a continuous time Poisson model, the sensitivity to a 1bp premium variation is equal to:

$$sensi_t^{CDS} = \int_0^T e^{-(r+\lambda)\theta} d\theta = \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}, \tag{26}$$

where $\lambda = \frac{S_t}{1-RR}$ with RR is the recovery rate.

A.3 RSDC model

According to these results, our approach is in the line of Pelletier (2006) that has been used in the context of portfolio allocation [see Giamouridis and Vrontos (2007)]. It allows in particular to decrease the number of variance parameters to consider. Our model is a combination of a mixture model for the correlation matrix and a Threshold GARCH

model [or TGARCH, Zakoian (1994)] to take into account asymmetric volatility dynamics. However, our estimation method imposes to assume that the heteroscedasticity is asset specific and not common across assets.

Note the K asset returns are defined by:

$$r_t = H_t^{1/2} U_t, \tag{27}$$

where $U_t | \, \Phi_{t-1} \sim iid(0, I_K)$, U_t is the $T \times K$ innovation vector, and Φ_t is the information available up to time t.

The conditional covariance matrix H_t is decomposed into [Bollerslev (1990) or Engle (2002)]:

$$H_t \equiv S_t \Gamma_t S_t, \tag{28}$$

where S_t is a diagonal matrix composed of the standard deviation $\sigma_{k,t},\ k = 1, \cdots, K$ and Γ_t is the $(K \times K)$ correlation matrix. Both matrices are time varying.

The conditional variance may follow a TGARCH(1,1) such that:

$$\sigma_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \sigma_{i,t-1}, \tag{29}$$

where ω_i, α_i^-, α_i^+ and β_i are real numbers.

Under assumptions of: $\omega_i > 0$, $\alpha_i^- \geq 0$, $\alpha_i^+ \geq 0$ and $\beta_i \geq 0$, $\sigma_{i,t}$ is positive and could be interpreted as the conditional standard deviation of $r_{i,t}$. However, it is not necessary to impose the positivity of the parameters and the conditional standard deviation is the absolute value of $\sigma_{i,t}$.

The correlation matrix is defined as:

$$\Gamma_t = \sum_{n=1}^{N} \mathbb{1}_{(\Delta_t = n)} \Gamma_n, \tag{30}$$

where $\mathbb{1}$ is the indicator function, Δ_t is an unobserved Markov chain process independent from U_t which can take N possible values ($\Delta_t = 1 \cdots, N$) and Γ_n are correlation matrices. Regime switches are assumed to be governed by a transition probability matrix $\Pi = (\pi_{i,j})$, where

$$Pr\left(\Delta_t = j | \, \Delta_{t-1} = i\right) = \pi_{i,j}, \; i, j = 1, \cdots, N.$$

This approach allows to discriminate between on the one hand the volatility dynamics through S_t and on the other one the correlation dynamics through the state variable Δ_t.

A.4 Estimation of RSDC

The estimation of this model is made using a two-step procedure: (i) the univariate estimation of standardized residuals with GARCH or TGARCH models and maximum likelihood and, (ii) the estimation of correlation matrices and probabilities to be in state n ($n = 1, \cdots, N$) with an EM algorithm (Dempster et al. (1977)). Using this method is more tractable when the number of observed series is more than a few. Indeed, the number of parameters could become very large and the one-step likelihood maximisation becomes intractable.

We should introduce θ, the complete parameter space that we split in two parts with: θ_1 that corresponds to the parameter space of the univariate volatility model and θ_2 that corresponds to the parameter space of the correlation model. We compute the log-likelihood taking a correlation matrix equal to the identity matrix. In other words, we

estimate univariate TGARCH model for each asset.

A.4.1 First Step

To model the full covariance matrix, we estimate the standard deviations and the correlations separately. This first step focus on the estimation of standard deviations.

The parameters of univariate TGARCH model are estimated with maximum likelihood, taking the case of a TGARCH(1,1), as presented in section 1. We have to specify the distribution of U_t in order to estimate the likelihood function that we want to maximize. In our case, U_t are iid and normally distributed $[U_t \sim \mathcal{N}(0,1)]$ allowing to consider Gaussian likelihood. However, we don't make the assumption that is the true law of U_t.

Note $\theta_1 = (\omega, \alpha^-, \alpha^+, \beta)$. As a result, the Gaussian likelihood is:

$$L(\theta_1) = L(\theta_1; r_0, \cdots, r_T) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2}} exp\left(-\frac{r_t^2}{2\tilde{\sigma}_t^2}\right) \tag{31}$$

with $\tilde{\sigma}_t$ are obtained recursively $(\forall t \geq 1)$ as:

$$\tilde{\sigma}_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \tilde{\sigma}_{i,t-1}$$

Taking the logarithm, we have to minimize the log-likelihood $\tilde{l}_t(\theta_1)$ defined as:

$$\tilde{l}_t = \tilde{l}_t(\theta_1) = \frac{r_t^2}{\tilde{\sigma}_t^2} + log(\tilde{\sigma}_t^2)$$

Thus, $\hat{\theta_1}$ is the solution of:

$$\hat{\theta_1} = \arg\min_{\theta_1} \frac{1}{T} \sum_{t=1}^{T} \tilde{l}_t(\theta_1) \tag{32}$$

After the estimation of parameters, we get the standardized residuals, noted $\tilde{U}_t$ as:

$$\tilde{U}_{i,t} = \frac{r_{i,t}}{\tilde{\sigma}_{i,t}}$$

In the next step, we use it to estimate the correlation matrices. We introduce a regime switching adding dynamic correlations. It measures the probability to be in the state n (in our case $n = 0, 1$ corresponding respectively to liquid and illiquid states).

A.4.2 Second Step

In this second part of estimation of our model, we use the Expectation Maximization algorithm (EM thereafter). The main advantage is the ability to take into account high number of parameters coming from each Γ_n.

EM Algorithm

This algorithm is presented in Hamilton (1994, chapter 22). We have to estimate the vector of parameters θ_2:

$$\tilde{\theta}_2 = \arg\min_{\theta_2} \left[\frac{1}{2} \sum_{t=1}^{T} K log(2\pi) + log(|\Gamma_t|) + \tilde{U}_t' \Gamma_T^{-1} \tilde{U}_t \right] \tag{33}$$

The number of parameters increases at a quadratic rate with the number of asset returns. As a consequence, to realize these estimation, we use EM algorithm that has no restrictions on the number of parameters.

Then, Hamilton (1994, chapter 22) expose that Maximum Likelihood estimates of the transition probabilities (i) and the correlation matrices (ii):

$$(i) \quad \tilde{\pi}_{i,j} = \frac{\sum_{t=2}^{T} P\left[\Delta_t = j, \Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=2}^{T} P\left[\Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \tag{34}$$

$$(ii) \quad \tilde{\Gamma}_n = \frac{\sum_{t=1}^{T} (\tilde{U}_t \tilde{U}_t') P\left[\Delta_t = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=1}^{T} P\left[\Delta_{t-1} = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \quad \text{for } n = 1, 2 \tag{35}$$

Estimates of transition probabilities are based on the smoothed probabilities. We could see that $\tilde{\Gamma}_t$ is not directly a correlation matrix. It must be rescaled because their diagonal elements are not constrained to be equal to one. Off-diagonal elements are between -1 and 1. This step is needed because the product of standardized residuals is not constrained to have elements between -1 and 1. Then we rescale Γ_t at each iteration as:

$$\Gamma_t = D_t^{-1} \tilde{\Gamma}_t D_t^{-1} \tag{36}$$

where D_t is a diagonal matrix with $\sqrt{\tilde{\Gamma}_{i,i,t}}$ on row n and column n.

The algorithm starts with initial values $\tilde{\theta}_2^{(0)}$ for the vector θ_2. With $\tilde{\theta}_2^{(0)}$ we can compute a new vector $\tilde{\theta}_2^{(1)}$ based on equations (34) and (35). The algorithm works until the difference between $\tilde{\theta}_2^{(m)}$ and $\tilde{\theta}_2^{(m+1)}$ is less than a defined threshold.

Computation

We develop in this subsection the method to compute the EM algorithm. The elements of the transition probabilities matrix, $\tilde{\pi}_{i,j}$ are defined as the ratio of consecutive probabilities ($P[\Delta_t = j, \Delta_{t-1} = i | \tilde{U}_t, \theta_2]$) and the probabilities to be in state j at time t. They are obtained iteratively from $t = 1$ to T.

Note that, conditional probability is defined by [see Hamilton, (22.3.7)]:

$$P[\Delta_t = j|\tilde{U}_t, \theta_2] = \frac{\pi_j \times f(\tilde{U}_t|\Delta_t = j, \theta_2)}{f(\tilde{U}_t|\theta_2)} \tag{37}$$

where $f(\tilde{U}|\Delta_t = j, \theta_2)$ is the probability density of the multivariate normal distribution with zero mean and Γ_j as covariance matrix, evaluated for $\tilde{U}_t$.

With equation (37), we compute probabilities at time $t = 1$. Then, we compute consecutive probabilities recursively:

$$P\left[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}, \theta_2\right] = P\left[\Delta_{t-1} = i|\tilde{U}, \theta_2\right] \times P\left[\Delta_t = j|\tilde{U}, \theta_2\right] \times \pi_{i,j} \tag{38}$$

where $P\left[\Delta_t = j|\tilde{U}, \theta_2\right] = f(\tilde{U}|\Delta_t = j, \theta_2)$.

Then, conditional probabilities to be in state j at time t are obtained making the ratio of the sum of the two consecutive probabilities of being in state j at time t and the sum of all consecutive probabilities.

Introduce the notation $\xi_{t|\tau}$, the $(N \times 1)$ vector whose j^{th} element is $P[\Delta_t = j|\tilde{U}_\tau, \theta_2]$. This notation allows to present two cases of $\xi_{t|\tau}$: (i) for $t > \tau$ it represents a forecast about the regime and (ii), for $t < \tau$ it represents the smoothed inference (about the regime in date t based on data obtained through some later date τ). We focus on smoothed probabilities that is defined by:

$$\tilde{\xi}_{t|\tau} = \tilde{\xi}_{t|t} \odot \{\Pi' \cdot [\tilde{\xi}_{t+1|T}(\div)\tilde{\xi}_{t+1|t}]\} \tag{39}$$

Smoothed probabilities are obtained iterating on backward for $t = T, T-1, T-2, \cdots, 1$. We come back from equation (38) to compute consecutive probabilities with smoothed probabilities. Then, we compute $\theta_2^{(m)}$ with equation (34) and (35) rescaling at each iteration the correlation matrix with equation (36).

The breaking rule of the algorithm is defined by the fact that the correlation matrix computed by the last iteration is almost equal to the previous correlation matrix. We have to define a threshold under which, we consider that matrices are equal.

Initialisation of the Algorithm

To start the algorithm, we have to choose the space of initial parameters, $\theta_2^{(0)}$. In this space, we input correlation matrices for each state of our model (in our case, two). The algorithm starts with one correlation matrix of the state (1) equal to identity matrix. For the second state, we use the Gramian matrix method (Holmes (1991)) to generate random correlation matrix. Note that a correlation matrix has to be defined semi-positive with diagonal elements that are equal to one and off-diagonal elements that are between -1 and 1. We use the Gramian matrix $T'T$ where $T := (t_1, \cdots, t_K)$ and t_i is the ith column. Then, we normalize the matrix as: $t_i = \tau_i / ||\tau_i||$.

For a K-variate process, we generate K independent pseudo-random vectors normally distributed, τ_i.

A.5 Correlations Matrices

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		0.452	0.364	0.877	0.313	0.608	0.522	0.304	0.673
Chile	0.608		0.312	0.490	0.287	0.381	0.345	0.310	0.350
Hungary	0.535	0.440		0.410	0.588	0.537	0.570	0.317	0.503
Mexico	0.940	0.634	0.551		0.338	0.621	0.534	0.318	0.662
Poland	0.579	0.464	0.773	0.570		0.456	0.489	0.357	0.427
Russia	0.634	0.520	0.759	0.637	0.743		0.747	0.436	0.795
South Africa	0.619	0.532	0.746	0.614	0.744	0.883		0.390	0.716
Thailand	0.400	0.332	0.425	0.388	0.412	0.479	0.459		0.402
Turkey	0.675	0.534	0.745	0.654	0.742	0.904	0.890	0.479	

Table 5 – Correlations matrices of the two regimes for the CDS market. The upper part of the matrix corresponds to the regime 1 while the lower part corresponds to the regime 2.

	Brazil	Chile	Hungary	Mexico	Poland	Russia	South Africa	Thailand	Turkey
Brazil		0.270	0.396	0.482	0.293	0.188	0.432	0.010	0.378
Chile	0.329		0.285	0.244	0.335	0.216	0.313	0.062	0.296
Hungary	0.560	0.396		0.331	0.657	0.528	0.509	0.005	0.472
Mexico	0.582	0.404	0.586		0.217	0.096	0.435	0.044	0.326
Poland	0.603	0.412	0.869	0.646		0.614	0.348	0.092	0.407
Russia	0.385	0.381	0.558	0.412	0.542		0.219	0.078	0.269
South Africa	0.596	0.393	0.688	0.636	0.718	0.497		0.036	0.513
Thailand	0.224	0.209	0.277	0.250	0.285	0.268	0.280		0.046
Turkey	0.591	0.413	0.700	0.612	0.712	0.499	0.694	0.312	

Table 6 – Correlations matrices of the two regimes for the sovereign debt market. The upper part of the matrix corresponds to the regime 1 and the lower part corresponds to the regime 2.

A.6 Robustness Check

		GARCH(1,1)	TGARCH(1,1)	TGARCH(2,2)
	Brazil	13.29		
		0.35		
	Chile	3.14		
		0.99		
	Hungary	**34.41**	**29.19**	18.89
		0.00	**0.00**	*0.09*
	Mexico	11.44		
		0.49		
CDS	Poland	6.62		
		0.88		
	Russia	9.18		
		0.69		
	South Africa	13.87		
		0.31		
	Thailand	15.93		
		0.19		
	Turkey	3.27		
		0.99		
	Brazil	12.40		
		0.41		
	Chile	10.16		
		0.60		
	Hungary	12.49		
		0.41		
	Mexico	18.48		
		0.10		
Bond	Poland	8.47		
		0.75		
	Russia	15.55		
		0.21		
	South Africa	13.19		
		0.36		
	Thailand	18.51		
		0.10		
	Turkey	9.01		
		0.70		
	Brazil	14.31		
		0.28		
	Chile	3.25		
		0.99		
	Hungary	**35.09**	**30.26**	19.58
		0.00	**0.00**	*0.08*
	Mexico	3.31		
		0.99		
Basis	Poland	13.95		
		0.30		
	Russia	9.47		
		0.66		
	South Africa	16.82		
		0.16		
	Thailand	12.32		
		0.42		
	Turkey	9.54		
		0.66		

Table 7 – ARCH effect test on standardized residuals from GARCH(1,1) model and alternative models in case of heteroscedasticity. The test is realized for a risk threshold of 5% and a number of lags equal to 12. In other words, we test whether all ρ are equal to 0 in the following equation: $u_t^2 = \alpha + \sum_{p=1}^{P} \rho^p u_{t-p}^2 + \epsilon_t$, $\forall p = 1, \cdots, 12$.

Chapter 2

MLiq toward a Meta-measure of liquidity

Ce chapitre est issu d'un travail commun avec Serge Darolles et Gaëlle Le Fol.

Comment mesurer la liquidité d'un actif ? Cette question, qui est encore régulièrement posée aujourd'hui, ne semble pas encore avoir trouvé de réponse évidente. En effet, bien que de nombreux chercheurs se soient intéressés à la liquidité, le nombre important de mesures (68 déjà répertoriées en 2008) nous indique qu'aucune d'entre elles ne parvient à fournir un résultat satisfaisant, ou communément accepté.

De prime abord, la définition de la liquidité de marché soulève déjà quelques difficultés. En effet, cette dernière relie quatre dimensions différentes. Tout d'abord, la liquidité de marché est assimilée à l'immédiateté. Cette notion fait référence à la vitesse d'exécution des échanges, en d'autres termes, au temps séparant deux échanges consécutifs de l'actif concerné. La seconde dimension est la profondeur. Induite par le carnet d'ordre, elle reflète la possibilité d'échanger un certain volume de cet actif sans avoir d'impact sur les prix. La notion de coût de transaction est également très largement débattue dans la littérature et exprime une troisième dimension de la liquidité de marché. En effet, un

actif nécessitant un coût de transaction important sera plus difficile à échanger. Pour finir la description détaillée de la définition de liquidité de marché, la résilience souligne la capacité que le prix de l'actif a à revenir vers son prix d'équilibre. En d'autre termes, elle peut être assimilée à l'impact sur les prix qui n'est ni capturé par les phénomènes de volatilité ni par la fourchette des prix.

Ces quatre dimensions apparaissent particulièrement difficile à capturer au travers d'une unique mesure. L'ensemble des nombreuses mesures de liquidité s'attache donc à capturer au mieux une des dimensions. Cependant, différents papiers académiques s'intéressent à définir un facteur dit de "pure liquidité" puisqu'il est extrait de différentes mesures chacune reflétant une dimension différente de la définition de la liquidité. Pour se faire, ils utilisent la méthode de l'Analyse en Composante Principale. Ce chapitre met l'accent sur les limites de cette méthodologie comme par exemple, avoir une vision moyenne et statique de la liquidité. Alors, nous proposons une méta-mesure de liquidité, basée sur les corrélations entre différentes mesures. En effet, la probabilité qu'un évènement drastique de liquidité survienne, c'est à dire, pouvant avoir un impact significatif sur les performances d'un actif, est définie comme la probabilité que toutes les mesures de liquidité étudiées détectent simultanément un problème de liquidité.

La contribution de ce papier est de proposer un indicateur de liquidité de marché tenant compte de l'ensemble des dimensions que l'on retrouve dans sa définition. De plus, cet indicateur ne s'expose pas aux problèmes rencontrés usuellement dans les méthodologies plus courantes d'Analyse en Composante Principale. Ainsi, au travers d'une étude des rendements de portefeuilles, nous montrons que cette mesure permet de capturer une prime de liquidité, entre 1964 et 2012, significativement positive. De plus, la construction de portefeuilles selon deux critères différents comprenant notre mesure de liquidité démontre que cette dernière n'est pas impactée par la taille des titres ou leur exposition au risque de marché. Pour finir, nous étudions le comportement de différents portefeuilles lorsque nous utilisons notre mesure non plus pour capturer une prime de liquidité, mais

pour fuir et donc, ne pas investir dans les titres sujets à un problème drastique de liquidité. Il en résulte une augmentation des performances associées à un risque équivalent. Cette mesure se veut donc particulièrement utile, aussi bien pour contrôler le risque de liquidité de marché que dans le cadre de la gestion d'actifs et la construction de portefeuilles.

2.1 Introduction

Financial markets are subject to many risks. Among them, the 2007-2008 financial crisis shows us that is crucial to control for the liquidity risk. Indeed, the role of liquidity is twofold during the crisis. Firstly, we note that liquidity participate to the maintenance of the crisis level. But, secondly, we observe that liquidity is a factor of the crisis propagation. Actually, we know that a liquidity problem may lead to liquidity spirals [see e.g., Gromb and Vayanos (2002), Morris and Shin (2004), Brunnermeier and Pedersen (2009), Menkveld and Wang (2011)]. When the market liquidity decreases, we show a "fly-to-liquidity" phenomena corresponding to the wish of managers to sell their assets before they become illiquid. In that special case, the price doesn't matter for the manager and it usually falls down. This decrease of price leads to an increase of illiquidity and so on. Historically, it was the case in 1998 with the LTCM hedge fund crisis and more recently, in 2008 with the Lehman Brother collapse and the huge fear of asset managers to get stuck with unwanted toxic assets. As a consequence, both managers and regulators should watch liquidity but there is still no consensus about how to measure it. Indeed, the definition of liquidity can be extended to the degree with which an asset or security can be bought or sold without affecting its price neither losing money, or, the ability to trade large volumes fast with the smallest price impact. As a consequence, the definition of liquidity is usually linked to: (i) immediacy, the time between trades or quotes, (ii) depth, the ability to absorb trade volume without price impact, (iii) tightness, the transaction costs and (iv) resiliency, the price impact that can not be captured by volatility or bid-ask

spread. We note that we are not able to consider simultaneously the different dimensions with only one measure [Chollete et al. (2007)].

In this paper, we propose a new measure of liquidity. Indeed, the contribution of this paper is to describe a way to measure liquidity without focus on only one measure, avoiding any specification or definition problems. As a consequence, this measure is able to consider all the dimensions of liquidity in a non-linear framework. In that sense, this is a Meta-Measure of liquidity, or $\mathcal{M}Liq$. This latter is defined as the probability to be in the illiquid state. Indeed, liquidity is not a continuous phenomena. Thus, we model liquidity as a two states model. The illiquid state is defined as the period during which the correlations between the liquidity measures are higher. Indeed, that is not exactly relevant to think that an illiquid event occurs just because the level of one measure, representing one dimension of the liquidity, is high. Thus, we consider different liquidity measures and we study the probability that all the measures detect a trouble at the same time. In this way, we encompass all the advantage of each measure without neglect any dimension of the liquidity. Indeed, we study the dynamic of correlations with the Regime Switching Dynamic Correlation model [Pelletier (2006)]. This latter allows a definition of correlations conditionally on time and moreover, using a state-space model, the turmoil periods are endogenously defined. As a result, the probability to be in the state of high correlations in terms of liquidity problems sheds light on a market liquidity problem for the underlying asset.

More than a decade ago, Aitken and Winn (1997) already present almost 70 liquidity measures that are able to be computed with public data. More recently, Goyenko et al. (2009) describe and compare some of the most used measures making horseraces relative to the Bid-Ask spread. However, choosing a benchmark is not relevant since we can not observe liquidity. Only few papers propose to compare liquidity measures but Holl and Winn (1995) study 25 liquidity measures and show that measures with similar design are correlated. Moreover, they conclude that each liquidity measure captures

different characteristics of the asset. Later, Aitken and Comerton-Forde (2003) divide liquidity into two categories: order-based and traded-based and find a little correlation between them. However, the latent characteristic of liquidity does not allow to ensure that information added by a liquidity measure effectively corresponds to the market liquidity. As a consequence, some studies try to consider more information than those provided by only one liquidity measure. For example, Korajczyk and Sadka (2008) consider several liquidity measures using a Principal Component Analysis, or Menkveld and Wang (2011) introduce the time dimension into the market liquidity measurement. As we describe, our methodology has the advantage to consider several liquidity measures and it proposes a dynamic liquidity measure that take into account extreme liquidity problems.

In order to check whether the market liquidity component detected using our new indicator *MLiq* is priced as a liquidity premium, we perform a battery of usual tests. Firstly, we explore a standard pricing analysis on stock returns. As a result, a trading strategy that is long in high *MLiq* stocks and short in low *MLiq* stocks yields a significant positive return. The average annual excess return earned by such a strategy based on our market liquidity indicator is equal to 1.94% over the whole sample. Then, we study the liquidity premium through a two-ways portfolio sorts analysis. We show that there exists a strong positive liquidity premium whatever we control for the β, the volume or the size. We also perform a factor analysis showing the α generated by a low liquidity portfolio or a long/short portfolio, is always positive and statistically significant. Finally, the main assumption of the previous studies is, even for the low liquid assets, that we are able to trade them. However, in the case of liquidity, this is a very restrictive assumption and we focus on the returns of a portfolio leaking the less liquid stocks in order to remove this assumption. As a result, we see positive excess returns and a better performance/risk ratio.

The remainder of the paper is organized as follows. Section 2 introduces our econometric model to estimate the smoothed probabilities to be in a state of high correlations

between liquidity measures and identify liquidity problems. Section 3 presents the data and the liquidity measures used. Section 4 shows our results and robustness checks while section 5 concludes.

2.2 MLiq measurement

Recently, some papers propose some ways to avoid the misspecification of the liquidity measure. One of those is to consider simultaneously different measures of liquidity. Korajczyk and Sadka (2008) propose to make a Principal Component Analysis based on 8 liquidity measures. They extract the first factor assuming that it corresponds to the pure liquidity factor. However, the linear combination of liquidity measures may have poor results in case of strong movements from one of the proxies. As a consequence, the results are accurate only in some cases. Indeed, this method is valid mostly if we consider that the liquidity measures experience peaks simultaneously. However, empirical results show some evidences proving that measures do not detect a liquidity problem at a same time. For example, when one is over its 80th percentile, there exist few dates for which the main part of liquidity measures experiences the same problem. Consequently, we should focus on the second order studying the covariance matrix of liquidity measures rather than the first order leading to smooth results. One other way studied in the literature is to focus on the time dimension of liquidity, i.e. looking at the time that an asset spend experiencing a liquidity problem. Menkveld and Wang (2011) propose to consider an illiquid event only if it occurs during 5 consecutive days or more. They estimate the probability of such an event and they define it as a liquidity measure. This methodology has a practical implication since the trader is really in trouble when his portfolio experiences a liquidity problem during 5 consecutive days or more. Indeed, they assume that he can manage a short term liquidity problem during a few days but he has some trouble to manage his portfolio with frozen positions during a long period. However, this method takes into

account only one liquidity measure, the Amihud ratio that induces a concentration on one dimension of liquidity and may lead to a misspecification of the liquidity problem.

MLiq follows the idea that we can not consider the liquidity as a continuous variable. A stock can basically experience two states (or more) of liquidity. In our case, we define the state of liquidity based on the correlations between several liquidity measures. As a consequence, we choose these measures in order to take into account the maximum number of dimensions of the liquidity definition.

Definition:

MLiq is the probability to be in a state for which, liquidity risks measured by several indicators, are highly correlated.

As the definition of *MLiq* suggests, the natural way to model such a specification is the estimation of a Markov regime switching model. In the case of *MLiq* estimation, we focus on the dynamic of the correlation matrix. The switches may occur at the same frequency than the data but we see a persistence to stay in the same regime than the previous day. Within each state, the liquidity measures may experienced different dynamics and their level may move strongly. As a consequence, *MLiq* captures the fact that all the measures shed light on a liquidity problem simultaneously. In other words, when all the pairwise correlations increase at the same time. To model the dynamic of correlations, we use the RSDC model (or Regime Switching Dynamic Correlation) from Pelletier (2006). This model is between the CCC model [or Constant Conditional Correlations, Bollerslev (1990)] and the DCC model [or Dynamic Conditional Correlations, Engle (2002)]. Between these three models, the main difference lies in the frequency with which the matrix of correlations can change. In the CCC model, the matrix is constant over time while it can switch between N matrices of correlations in the RSDC model (N being the number of regimes) and it can change at every periods in the case of DCC model. The definition of *MLiq* implies that we choose a two regimes model corresponding

to the binary characteristic[1] of the liquidity.

This approach has already been used in the context of portfolio allocation [Giamouridis and Vrontos (2007)] but also for detecting the presence of liquidity pure contagion effects [Darolles et al. (2013)]. It allows in particular to decrease the number of variance parameters to consider. Thus, this model is a combination of a mixture model for the correlation matrix and a conditional volatility model. According to data, we may be constrained to use different models in order to fully capture the dynamic of the volatility in the covariance matrix. However, we use Threshold GARCH model [or TGARCH, Zakoian (1994)] to take into account asymmetric volatility dynamics. Moreover, our estimation method imposes to assume that the heteroscedasticity is specific to liquidity measures and not common across them.

Note the K liquidity measures are defined by:

$$liq_t = H_t^{1/2} U_t, \tag{2.1}$$

where $U_t | \, \Phi_{t-1} \sim iid(0, I_K)$, U_t is the $T \times K$ innovation vector, and Φ_t is the information available up to time t.

The decomposition of the conditional covariance matrix H_t is the same as into Bollerslev (1990) or Engle (2002) and it can be presented as:

$$H_t \equiv S_t \Gamma_t S_t, \tag{2.2}$$

where S_t is a diagonal matrix composed of the standard deviation $\sigma_{k,t}$, $k = 1, \cdots, K$ and Γ_t is the $(K \times K)$ correlation matrix. Both matrices are time varying but S_t is dynamic at the same frequency than the data while Γ_t can only move by periods. Following this

[1] The liquidity of an asset is defined for being "high" or "low".

idea, it allows to model separately the univariate conditional variance from the correlation matrix. Thus, the estimation is simplified because of the smaller number of parameter. Indeed, we only have N matrices of correlation corresponding to the number of regimes and not, one matrix for each date as it is the case for the DCC model. Thus, the number of liquidity measures studied may increase without problem of over-identification.

Assuming that the conditional variance follows a TGARCH(1,1), we have:

$$\sigma_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \sigma_{i,t-1}, \tag{2.3}$$

where ω_i, α_i^-, α_i^+ and β_i are real numbers.

Under assumptions of: $\omega_i > 0$, $\alpha_i^- \geq 0$, $\alpha_i^+ \geq 0$ and $\beta_i \geq 0$, $\sigma_{i,t}$ is positive and could be interpreted as the conditional standard deviation of $r_{i,t}$. However, it is not necessary to impose the positivity of the parameters and the conditional standard deviation is the absolute value of $\sigma_{i,t}$.

The correlation matrix is defined as:

$$\Gamma_t = \sum_{n=1}^{N} \mathbb{1}_{(\Delta_t = n)} \Gamma_n, \tag{2.4}$$

where $\mathbb{1}$ is the indicator function, Δ_t is an unobserved Markov chain process independent from U_t which can take N possible values ($\Delta_t = 1 \cdots, N$) and Γ_n are correlation matrices. Regime switches are assumed to be governed by a transition probability matrix $\Pi = (\pi_{i,j})$, where

$$Pr\left(\Delta_t = j \middle| \, \Delta_{t-1} = i\right) = \pi_{i,j}, \; i, j = 1, \cdots, N.$$

This approach allows to discriminate between on the one hand the volatility dynamics

through S_t and on the other hand the correlation dynamics through the state variable Δ_t.

It is usual in this case to study the unconditional probability, that is, here, the probability for the asset to have been illiquid. In some sense, this is a measure of historical liquidity over the past. That may allow to rank assets and make a discrimination between more or less liquid stocks. However, a stock that is considered as a liquid asset over the past may be in the illiquid state at the last period that is a better information for an asset manager that look for investing. As a consequence, in order to propose a measure able to be used in an asset allocation context, we study the conditional probability at time t, to be in the illiquid state.

2.3 Data, liquidity measures and empirical evidences

Data. The data are obtained from the Center for Research in Security Prices (CRSP). We study a sample of 5937 stocks from 01/01/1964 to 31/12/2012. The database is composed from daily data of common stocks listed on the NYSE or AMEX. The sample has 12341 time observations. The CRSP data include stock prices, returns, volume and shares outstanding. The empirical analysis covers two stages. In the first one, we study separately the behaviour of each liquidity measure focusing on the stock ranking and the inability for standard liquidity measures for efficiently ranking stocks from one time to the other. In this first descriptive part, we only focus on data available between 01/01/2005 and 31/12/2012. For each stock, liquidity measures are winsorized at the 1% and the 99% quantile. In the second one, we study *MLiq* and we do as standard pricing analysis in order to study whether an illiquid event, as defined according to *MLiq*, leads to higher returns. In order to conserve the largest sample, we compute only daily liquidity measures. As a result, we are able to do the asset pricing analysis based on 47 years of data.

Liquidity measures. We use in this study 4 liquidity measures. We choose them in order to cover the maximum of dimensions of the liquidity definition without selecting

too many measures avoiding to add noise in our results.

1. The first measure and possibly the most used, is the Amihud Ratio or *Illiq* [Amihud (2002)]. Note the daily stock return as $r_{i,t}$ and the daily volume as $V_{i,t}$, this ratio is defined as:

$$Illiq_{i,m} = \frac{|r_{i,t}|}{V_{i,t}}, \tag{2.5}$$

   Described like that, this is a rough measure of price impact and this measure focuses on the resiliency dimension of liquidity. This measure increases with the level of illiquidity of the market.

2. The Relative Bid-Ask spread RS, (or Quoted percentage Bid-Ask spread) is defined as the difference between Ask and Bid prices, divided by mid-quote price.

$$RS_{i,m} = \frac{(Ask_{i,t} - Bid_{i,t})}{mq_{i,t}}, \tag{2.6}$$

   with $mq_{i,t} = \frac{(Ask_{i,t}+Bid_{i,t})}{2}$, $Ask_{i,t}$ and $Bid_{i,t}$ are respectively the ask and bid quotes of asset i prevailing at the end of the day t.

3. The effective spread, ES, is the absolute value of the difference between the end price of a day and the mid-quote at the end of this day:

$$ES_{i,m} = |p_{i,t} - mq_{i,t}|, \tag{2.7}$$

   where $p_{i,t}$ is the price of asset i at the end of the day t.

   The Bid-Ask spread and its derivatives as the relative Relative Bid-Ask spread or the effective spread focus on two dimensions of the liquidity definition: the tightness and the immediacy. Both of them express the illiquidity level of the asset.

4. The turnover of a trading day is considered as a proxy of liquidity since it sheds light on the exchanged volume of an asset relatively to the number of shares outstanding. Considering the daily volume $vol_{i,t}$, we have a turnover expressed as:

$$TO_{i,t} = \frac{vol_{i,t}}{SO_{i,t}} \tag{2.8}$$

where $SO_{i,t}$ is the number of shares outstanding at time t.

Unlike the first three measures, the turnover is a proxy of the market liquidity. As a consequence, we take the opposite of the turnover as a measure of illiquidity allowing us to compare it with the others.

We focus on the dynamic of these 4 liquidity measures and a first look on the results shows they almost never give us the same information. Indeed, our results show that all these measures do not flag the same dates when it comes to detecting illiquid dates. There exists a huge asynchronism considering different liquidity measures. Second, we rank the stocks for each date and each liquidity measure for observing that the ranking is highly volatile but even more interesting, the liquidity measures are not able to provide the same result. One stock may be considered as liquid for one measure and, at the same time, illiquid for another one.

Summary Statistics. The table 2.1 presents summary statistics on the variables and liquidity measures used in the empirical part. Summary statistics are presented for the rough sample. The panel dataset is unbalanced over the whole sample but balanced among sub periods. We divide the sample into 9 subsamples of approximately 5 years.

We see that the average price over all the sample is equal to 24.96\$ with a minimum smaller than one cent and a maximum equal to 35474\$. The average return is equal to $5.126bps$ and the overall volatility is equal to $313.40bps$. As we perceive, the median of returns is null, showing the asymmetry of the return's distribution over all the sample.

The next section extends the analysis focusing on each liquidity measure and especially the ranking of stocks according to theses measures.

Table 2.1 – Summary Statistics

	Mean	Std	Min	5th	Median	95th	Max
ret(bps)	5.126	313.403	-9166.670	-433.740	0.000	476.190	17900.000
prc($)	24.958	141.807	0.000	1.938	18.063	61.240	35474.000
volume($ mln)	0.358	3.901	0.000	0.000	0.011	1.334	1247.364
sdret(%)	10.929	77.521	0.000	1.105	7.996	23.164	5875.750
number of stocks	3864	231	3454	3520	3850	4358	5936

This table presents summary statistics for the database as a whole. It consists of NYSE/AMEX stocks from January 1, 1964 trough December 31, 2012. In this table, we study the daily stock return (ret) expressed in basis points, the daily end-of-day transaction price (prc), the daily dollar volume expressed in million of dollars, the standard deviation of daily return (sdret) and the number of stock available at each date.

Empirical evidences. We define for a specific liquidity measure, an illiquid date when the liquidity measures experiences a value greater than its 80th percentile. In other words, we assume that it detects a liquidity problems as it has been defined in (Menkveld and Wang, 2011). This is an arbitrary definition but allowing to show that illiquid events do not occurs at the same time for different liquidity measures but we add results based on different threshold, 90th and 95th percentiles in order to check for extreme illiquid events.

Table 2.2 – Percentage of hits according to n liquidity measures.

Panel A					Panel B					Panel C				
n	Mean	Std	Min	Max	n	Mean	Std	Min	Max	n	Mean	Std	Min	Max
1	23.57	2.06	18.52	34.46	1	16.07	1.57	12.16	25.17	1	9.95	1.04	7.05	13.70
2	10.74	0.90	7.85	14.90	2	5.74	0.55	4.12	7.85	2	2.99	0.37	1.74	4.17
3	8.31	0.68	5.36	10.77	3	3.71	0.44	1.59	5.11	3	1.56	0.28	0.45	2.48
4	1.80	0.66	0.10	3.97	4	0.38	0.24	0	1.69	4	0.08	0.09	0	0.50

This table presents summary statistics about the percentage of hits. This latter is defined as the liquidity measure is greater than a defined threshold. Panel A presents results based on the 80th percentile and Panels B and C, based on 90th and 95th, respectively. We compute these results in rolling window over 100 days. This table presents results for $n = 1, 2, 3$ and 4 liquidity measures, i.e. when n liquidity measures experience a hit simultaneously. These results are computed based on data available from January 2005 and December 2012.

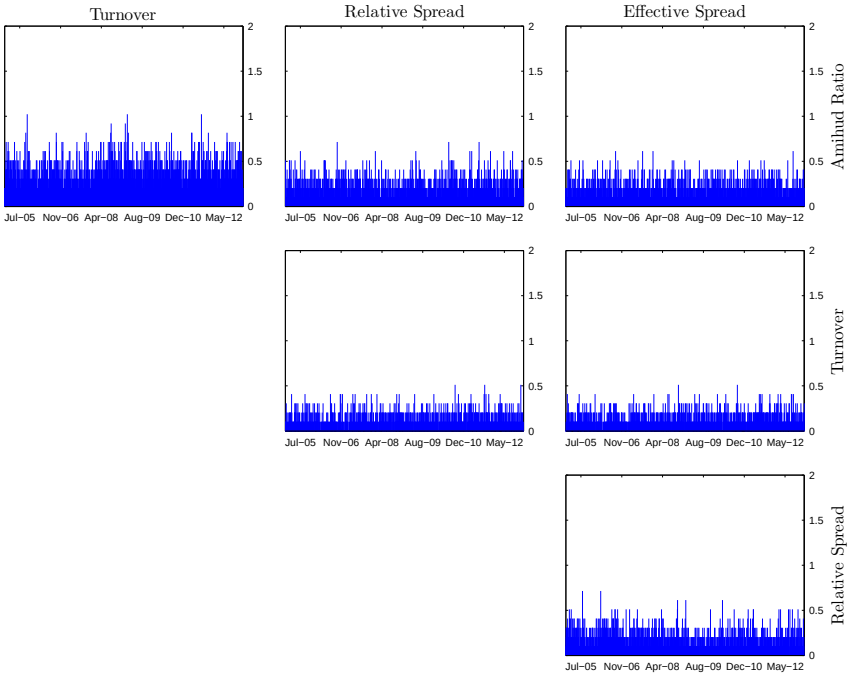
Panel A of Table 2.2 presents percentage of hits according to 1, 2, 3 and 4 liquidity measures and based on a threshold equal to the 80th percentile. In that sense, we study when the liquidity measure is greater than the threshold. As a result, we show an average

percentage of hits equal to 23.57% considering the dates for which only one liquidity measure detects a liquidity problem. This result decreases until 10.74% when we look for two measures experiencing a problem simultaneously. The percentage falls down to 8.31% and 1.80% considering respectively 3 and 4 measures simultaneously. In a perfect world where liquidity measures detect the same illiquid event at the same time, we should have null percentage for the first three sub samples and a constant 20% for the fourth as our illiquidity problem is defined. However, this is absolutely not the case empirically. We show that the percentage of illiquid events detected by one liquidity measures is around 25% meaning that the information given by liquidity estimates is not really relevant since they do not shed light on the same dates. As a consequence, that proves the importance of the choice of a liquidity measure leading to different results. Indeed, across firms, only 0.10% to 3.97% of dates are detected by the four liquidity measures simultaneously. These results may have a huge impact on trading strategy. According to the liquidity measure, the ranking of stocks selected may largely differs. These characteristics of liquidity measures are even more highlighted since we focus on higher percentiles as we can see in Panels B and C.

In order to study the ranking of stocks according liquidity measures, we focus on the percentage of concordant pairs between liquidity measures. In other words, we study the number of times for which two different liquidity measures give the same rank for one stock. This information is particularly relevant in the case of a trading strategy based on sorting the stocks according to their level of liquidity. Assuming that every liquidity measures reveal the same information, this percentage should be equal to 100. Obviously, this is not the case due to the different dimensions of liquidity captured by the measures.

Figure 2.1 presents the pairwise percentage of concordant pairs between the liquidity measure in row and the other one in column. We show that the percentage of concordant pairs, i.e. ranks exactly similar is very small ranging from 0 to 20 percents. As a consequence, whether the asset manager is willing to construct a portfolio controlling for

Figure 2.1 – Percentage of concordant pairs between measures.



This figure presents pairwise percentage of concordant pairs. At each date, we compute the ranking of stocks according to two different liquidity measures. Then, the number of concordant pairs corresponds to how many assets have the same rank. These results are computed based on daily data.

liquidity, the choice of the measure is preponderant in his allocation and the results may be consequently impacted by this choice.

To quantify the impact of the measure chosen, we compute an identical trading strategy built at a daily investment horizon. It corresponds to be long on low liquidity stocks and short on high liquidity stocks. This kind of strategy is supposed to be able to capture the liquidity premium according to the measure used to rank the stocks. Figure 2.2 presents results on the 9 sub samples. As a result, we see that for each sub samples, the results of strategies largely differ. Moreover, we observe that the differences between results change according the sub sample. But, for both measures and sub samples, the main part of portfolios obtained positive results. These strategies earn money in almost all cases. For example, between 1964 and 1969, a long-short strategy based on liquidity whatever the measure used, could earn at least 100% over the five years. But choosing the Turnover or the Relative Spread allows to get a portfolio value largely bigger. Even,

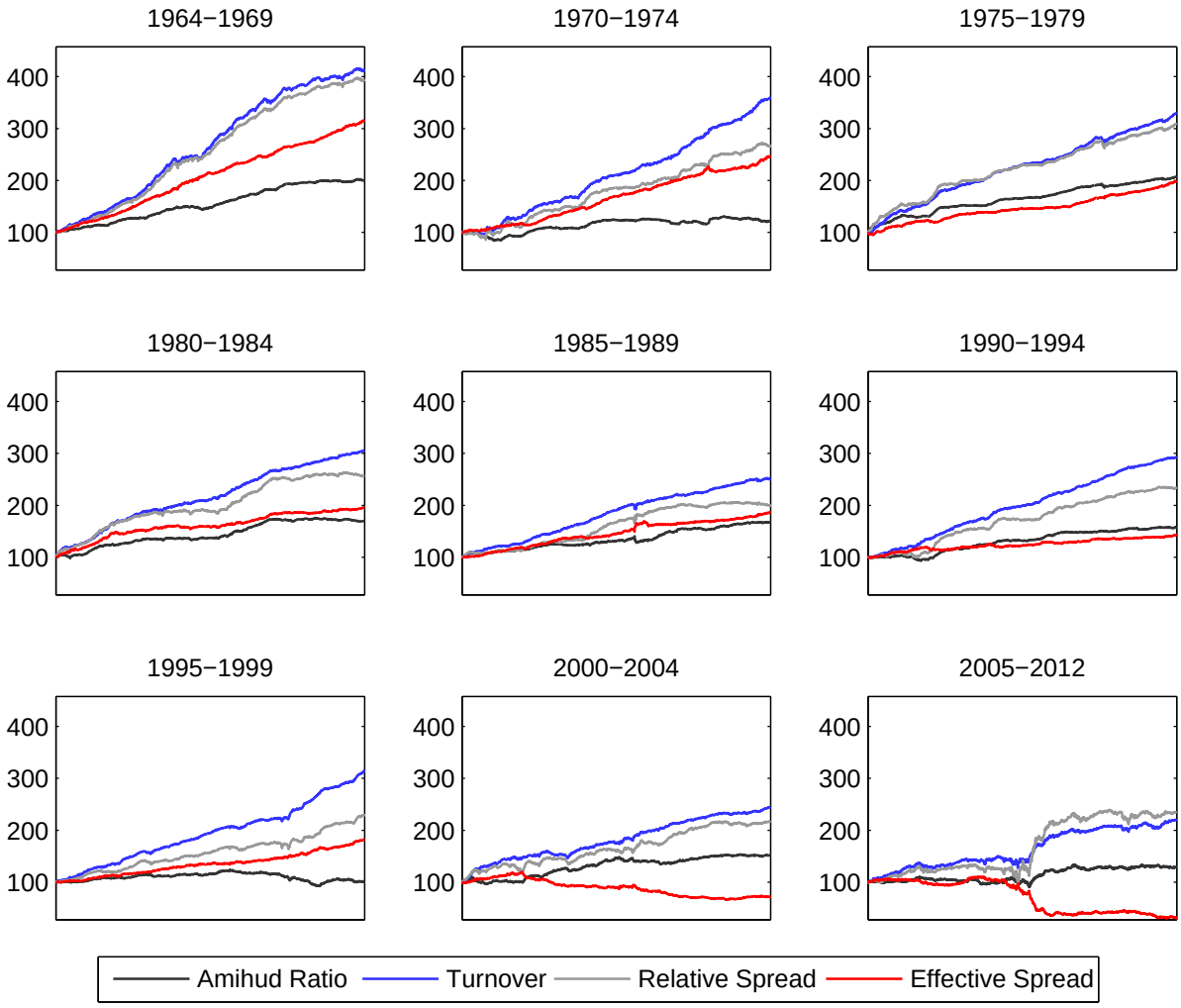
if this result is similar for almost all the other sub samples, we see that the difference with the other measures is not always so big. Indeed, from 1985 to 1989, the 4 long-short portfolios have close results and especially concerning the strategies based on Amihud ratio, Relative spread and Effective spread. During this period, we are able to identify the crash of 1987 that is reported on the 4 portfolio performances. However, in this case, we see that the Effective spread strategy increases after this crisis while the Amihud ratio decrease before to recover. This is exactly the opposite case concerning the 2008 crisis and especially the following the Lehman Brother collapse. At this time, we perceive the impact of the bankruptcy and during the next weeks, the strategies base don Amihud ratio, Turnover and Relative spread are able to extract the liquidity premium induced by strong liquidity problems during this period. The last strategy, focusing on the Effective spread, experiences a large decrease over the same time period. As a consequence, these results are telling us that capturing the liquidity premium strongly depends on the measure used. Indeed, whatever the sub sample, we show that the performances of long-short strategies are largely impacted by the liquidity indicator chosen.

All these results prove that considering liquidity depends on the proxy with which we choose to measure it. As a consequence, we propose a meta-measure of liquidity allowing to take into account all the dimensions that are captured by the different standard market liquidity measures in order to cancel the problems of mispecification.

2.4 MLiq results

Remember that *MLiq* is defined to detect whether the stock will experience a drastic liquidity problem. Thus, we perform some empirical tests in order to define if this kind of liquidity problem is priced into the stock returns. First, we compute sorted portfolios according to the past *MLiq*, i.e. according to the past-year level of liquidity. That allows to show whether there exists a premium due to liquidity risk. Second, the two-ways

Figure 2.2 – Cumulative returns from 1990 to 2011.



This figure presents cumulative returns resulting from 4 different portfolios. These portfolios are built according different liquidity measures. The positions are long for the 20% more illiquid assets and short for the 20% more for liquid assets. We consider the liquidity of the previous day for investing and the investment horizon is a day.

portfolio sorts allow to determine if other factor capture the liquidity premium rather than our liquidity indicator. Then, we propose to study the liquidity as a factor studying the monthly returns of the long-short portfolio and the long-only portfolio through the Fama-French three factor model for example. However, this liquidity premium remains theoretical since an illiquid stock may have difficulties to be traded. As a consequence, we study the ability of *MLiq* to avoid the asset manager to get stuck with an unwanted position due to a strong liquidity problem exploring the behavior of a long-short portfolio

for which the manager avoid to invest into the most illiquid stocks into both long and short part.

2.4.1 Long/Short Portfolios

We propose to study the returns of a portfolio built based on the level of liquidity risk defined by *MLiq*. In other words, we sort available stocks at each dates by the relative number of times that they experienced a liquidity problem during the past month. To define a liquidity problem, we start from the definition of the *MLiq*, our liquidity measure. Usually, as *MLiq* is defined as the probability to be in a regime of high correlations between liquidity indicators, the natural threshold to separate the two states is 0.5. As a consequence, a liquidity problem is defined as a day for which the *MLiq* is greater than 0.5. In other words, if the probability to experience a drastic liquidity problem is greater than 50%, we consider the stock as illiquid during this day. However, we show in section 2.4.4 the impact of changing the threshold concerning the detection of drastic illiquid events. We are in line with the idea that if *MLiq* is priced, in the case of a portfolio that is long high *MLiq* stocks and short low *MLiq* stocks earns a significant and positive return.

Table 2.3 – Excess returns Long/Short portfolios

	All months		1964-1985		1986-2012		1988-2007	
	$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat
1 (low MLiq)	0.69**	2.95	0.60*	1.80	0.76**	2.35	0.70**	2.59
2	0.79**	2.89	0.76*	1.75	0.80**	2.37	0.76**	2.59
3	0.72**	2.76	0.70*	1.79	0.74**	2.11	0.71**	2.40
4	0.69**	2.99	0.61*	1.82	0.76**	2.39	0.77**	2.98
5 (high MLiq)	0.85**	3.07	0.90**	2.04	0.81**	2.30	0.85**	2.99
differential (5-1)	0.16**	2.04	0.30**	2.11	0.05	0.71	0.15**	2.15

This table presents excess returns for portfolios of stocks that are ranked according to the previous year number of days that they spend into an illiquid state i.e. with a *MLiq* greater than 0.5. These portfolios are rebalanced monthly. They are equally weighted. The excess return ($r_t - rf$ (%)) is the time-series mean of weekly portfolio returns. We compute robust t-statistics (based on Newey and West (1987)). The results are presented for the full data sample and three sub-periods.

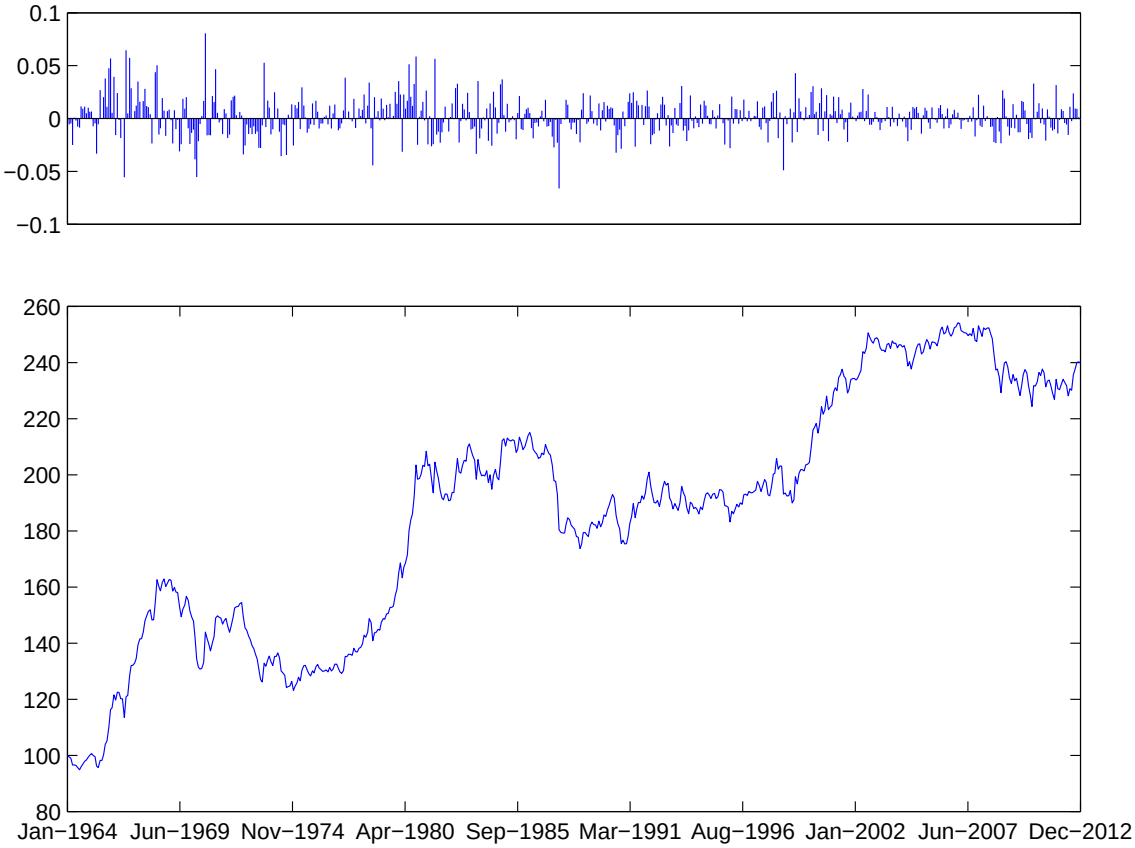
Table 2.3 presents the results for long-short portfolios with a monthly investment horizon. These portfolios are monthly rebalanced and among each of them, the stocks are

equally weighted. The results exposed in this table are particularly meaningful. Firstly, considering the whole sample, we see that each of the five portfolios earn a significant and positive return. In the case of the most liquid stocks, this return is equal to 0.69% (or 8.60% annually) while the return of the most illiquid portfolio is equal to 0.85%. The difference between them or, in other words, the return of the dollar-neutral position is equal to 0.16%. This significant liquidity premium of 1.94% annually is the proof that liquidity, as defined by *MLiq*, is priced into stock returns. Moreover, we split the sample into two sub samples, the first from 1964 to 1985 and the second from 1986 to 2012. Concerning the former, we see that the distinction between liquid and illiquid stocks is stronger than for the whole sample. Indeed, the liquidity premium is, in this case, equal to 3.61% annually and is statistically significant. However, from 1986 to 2012, although each portfolio earns a significant positive return, the long-short portfolio has a non significant return equal to 0.05%. But, if we focus on the sub sample going from 1986 to 2007, avoiding the two big events of 1987 and 2008, we find a positive and significant liquidity premium. All these results confirm that *MLiq* is able to distinguish liquid and illiquid stocks.

Figures 2.3 and 2.4 show the evolution of the price started from 100 in 1964. The first highlights some of the well known financial crashes as in 1987 and in 2008 but this portfolio has a positive trend from 1964. The second presents separately the cumulative returns of the low-liquid stocks portfolio and the high-liquid stocks portfolio. We distinguish the difference between them and that we observe on figure 2.3. We see that the impact of the 2007-08 financial crisis is largely stronger for these portfolios.

Although these results show us that *MLiq* is able to capture a liquidity premium in a long-short strategy, we have to precise that this premium is theoretical. Indeed, *MLiq* is designed to detect extreme illiquid events. In that sense, if such an event occurs, it could be difficult to trade the stock due to the lack of liquidity. As a consequence, measuring the liquidity premium in this context remains theoretical and may be far from real results.

Figure 2.3 – Excess returns of Long-Short portfolio based on *MLiq*



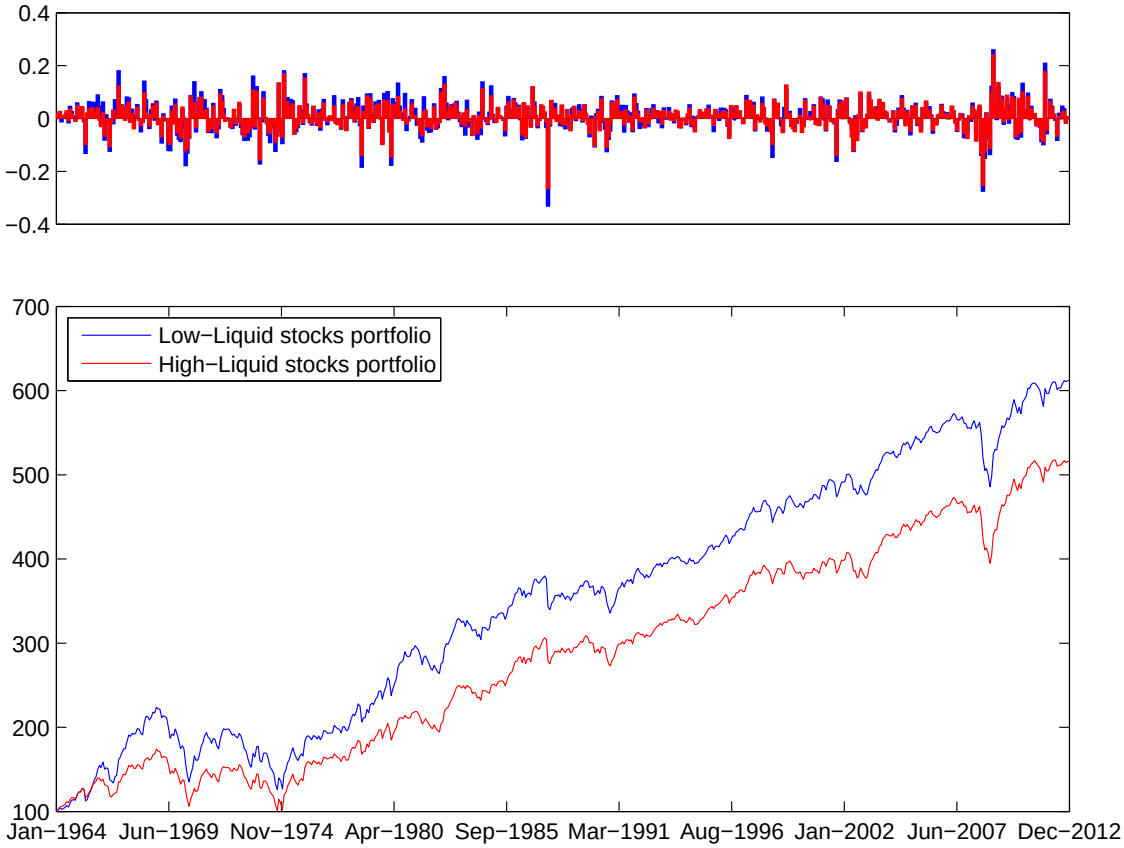
This figure presents the excess returns and compound returns on the whole sample for a Long/Short strategy based on the *MLiq*. This portfolio is built according to the ranking of stocks based on the previous month average number of days for which *MLiq* was greater than 0.50. They are monthly rebalanced and the results presents excess returns from January 1965 to December 2012.

2.4.2 Two-ways Portfolio Sorts

Focusing on the distinction between *MLiq* and other factors, we construct double-quartile portfolios combining our liquidity indicator with those factors. Indeed, we are looking to determine if investing in a low-liquidity portfolio is equivalent to investing in portfolios constrained by other factors.

Firstly, we oppose *MLiq* to the β of the market extracted from the CAPM model. Table 2.4 presents the results of post *MLiq* and β. We report geometric mean return, arithmetic mean return, standard deviation of returns and the average number of stocks in each intersection portfolios. As a result, we see across the low-β quartile, the low-liquidity

Figure 2.4 – Low-Liquid stocks and High-Liquid stocks portfolio returns



This figure presents the excess returns and cumulative returns on the whole sample for two distinct portfolios invested in most liquid and less liquid stocks based on the *MLiq*. These portfolios are built according to the ranking of stocks based on the previous month average number of days for which *MLiq* was greater than 0.50. They are monthly rebalanced and the results presents excess returns from January 1965 to December 2012.

portfolio earns a geometric mean return of 0.559% while the high liquidity portfolio only receives a geometric mean return of 0.127%. Across the high-β quartile, the difference between the high- and low-liquidity portfolio, still largely positive, is smaller than in the previous case. Indeed, the low-liquidity portfolio obtains a geometric mean return equal to 0.473% while the high-liquidity portfolio only earns 0.278%. The liquidity effect is equal to 0.195% while it was equal to 0.432% in the case of the low-β portfolio.

Similarly, we want to know how our liquidity measure differs from volume and size. We produce identical tables with equally weighted double sorted portfolios on *MLiq* and volume. Table 2.5 reports the results for which, we see, the liquidity effects os still strongly

Table 2.4 – Beta

Quartiles		High Liquidity	Mid-High	Mid-Low	Low Liquidity
High Beta	Geom. Mean	0.127%	0.408%	0.267%	0.559%
	Arithm. Mean	0.245%	0.509%	0.384%	0.668%
	Std. Dev.	0.451%	0.503%	0.489%	0.465%
Mid-High	Geom. Mean	0.182%	0.361%	0.274%	0.427%
	Arithm. Mean	0.310%	0.470%	0.387%	0.524%
	Std. Dev.	0.519%	0.506%	0.462%	0.501%
Mid-Low	Geom. Mean	0.114%	0.483%	0.279%	0.433%
	Arithm. Mean	0.228%	0.590%	0.391%	0.540%
	Std. Dev.	0.483%	0.556%	0.488%	0.429%
Low Beta	Geom. Mean	0.278%	0.341%	0.193%	0.473%
	Arithm. Mean	0.386%	0.455%	0.306%	0.585%
	Std. Dev.	0.520%	0.451%	0.401%	0.496%

This table presents portfolios monthly returns. Stocks are sorted into four quartiles based on *MLiq* and β from 1964 to 2012. Portfolios are equally weighted. The tables report the geometric mean (compound) return, arithmetic mean return, return standard deviation in each cell.

positive. Indeed, among the big-volume quartile, the low-liquidity portfolio earns 0.593% while the high-liquidity portfolio perceives a geometric average return equal to 0.110%. We have similar results according to the small-volume quartile. As a result, this table shows the volume does not capture drastic liquidity as defined by the *MLiq*. In other words, we see a strong positive liquidity premium regardless of the volume group.

Finally, we study the relation between our liquidity indicator and the size of the firm. Using the same methodology, we compute returns of portfolios ranked by *MLiq* and the size of the firm. Thus, table 2.6 shows almost identical results that table 2.5. Indeed, the liquidity effect is still very relevant whatever the size group. As a consequence, that means the size does not capture the liquidity and especially drastic illiquid events considered with *MLiq*. Indeed, we see that the geometric mean return of the low-liquidity portfolio for small firms is equal to 0.574% while the high-liquidity portfolio of the same quartile earns 0.107%. The liquidity effect is 0.467%, much bigger thant 0.248% of the liquidity effect for big firms quartile. However, both of them are strongly positive, proving that liquidity is not fully captured by the size.

Table 2.5 – Volume

Quartiles		High Liquidity	Mid-High	Mid-Low	Low Liquidity
High Volume	Geom. Mean	0.110%	0.406%	0.250%	0.593%
	Arithm. Mean	0.226%	0.515%	0.363%	0.692%
	Std. Dev.	0.447%	0.500%	0.488%	0.443%
Mid-High	Geom. Mean	0.187%	0.370%	0.263%	0.414%
	Arithm. Mean	0.307%	0.474%	0.379%	0.514%
	Std. Dev.	0.519%	0.504%	0.460%	0.506%
Mig-Low	Geom. Mean	0.119%	0.460%	0.288%	0.442%
	Arithm. Mean	0.243%	0.575%	0.395%	0.541%
	Std. Dev.	0.491%	0.565%	0.487%	0.428%
Low Volume	Geom. Mean	0.231%	0.344%	0.208%	0.496%
	Arithm. Mean	0.360%	0.454%	0.317%	0.595%
	Std. Dev.	0.522%	0.452%	0.405%	0.490%

This table presents portfolios monthly returns. Stocks are sorted into four quartiles based on *MLiq* and volume from 1964 to 2012. Portfolios are equally weighted. The tables report the geometric mean (compound) return, arithmetic mean return, return standard deviation in each cell.

Table 2.6 – Size

Quartiles		High Liquidity	Mid-High	Mid-Low	Low Liquidity
Big firms	Geom. Mean	0.107%	0.423%	0.259%	0.574%
	Arithm. Mean	0.222%	0.531%	0.368%	0.679%
	Std. Dev.	0.447%	0.496%	0.488%	0.454%
Mid-Big	Geom. Mean	0.196%	0.367%	0.257%	0.425%
	Arithm. Mean	0.308%	0.488%	0.370%	0.525%
	Std. Dev.	0.519%	0.498%	0.461%	0.500%
Mig-Small	Geom. Mean	0.116%	0.479%	0.272%	0.446%
	Arithm. Mean	0.228%	0.584%	0.392%	0.551%
	Std. Dev.	0.483%	0.559%	0.488%	0.422%
Small firms	Geom. Mean	0.250%	0.355%	0.180%	0.498%
	Arithm. Mean	0.370%	0.475%	0.289%	0.595%
	Std. Dev.	0.520%	0.443%	0.401%	0.490%

This table presents portfolios monthly returns. Stocks are sorted into four quartiles based on *MLiq* and size from 1964 to 2012. Portfolios are equally weighted. The tables report the geometric mean (compound) return, arithmetic mean return, return standard deviation in each cell.

2.4.3 The liquidity as a factor

In this subsection, we study the liquidity defined by the *MLiq* as a factor, or in other words, as a series of dollar-neutral returns. Indeed, we are looking for the impact of standard factors on the returns obtained capturing the liquidity premium.

In order to do so, we study the monthly returns of the long-short strategy that we previously explored and the returns of the least liquid quintile of stocks. We regress both of these series upon the CAPM, Fama-French and four factors models. Thus, the factors used are the market, size, value and momentum.

The first framework is the so-called CAPM. In this case, the long-short time series of monthly returns is regressed upon the excess return of the market portfolio as presented here:

$$r_{LS,t} = \alpha + \beta_1(r_{m,t} - r_{rf,t}) + \epsilon_{LS,t} \tag{2.9}$$

where $r_{LS,t}$ is the return of the long-short portfolio, $r_{m,t}$ is the return of the market and $r_{rf,t}$ is the risk free rate at month t.

Then, we add two other factors proposed by Fama and MacBeth (1973). The Fama-French three factors model study the impact of the size and the value in addition to the market of the CAPM.

$$r_{LS,t} = \alpha + \beta_1(r_{m,t} - r_{rf,t}) + \beta_2 SmB_t + \beta_3 HmL_t + \epsilon_{LS,t} \tag{2.10}$$

where $r_{LS,t}$ is the return of the long-short portfolio, $r_{m,t}$ is the return of the market, $r_{rf,t}$ is the risk free rate, SmB_t is the size factor (Small minus Big) and HmL_t is the value factor (High minus Low) at month t.

Finally, we propose to focus on a four factor model adding the momentum factor:

$$r_{LS,t} = \alpha + \beta_1(r_{m,t} - r_{rf,t}) + \beta_2 SmB_t + \beta_3 HmL_t + \beta_4 WmL_t + \epsilon_{LS,t} \tag{2.11}$$

where $r_{LS,t}$ is the return of the long-short portfolio, $r_{m,t}$ is the return of the market, $r_{rf,t}$ is the risk free rate, SmB_t is the size factor (Small minus Big), HmL_t is the value factor (High minus Low) and WmL_t is the momentum factor (Winners minus Losers) at month t.

Note that we replicate the regressions on the long-only portfolio and the results are presented in table 2.7. We can see that for each of the 6 regressions, the α is positive and statistically significant no matter whether we adjust for the market, size, value or momentum. All these results are telling us that the *MLiq* is able to extract a positive liquidity premium. Moreover, we show that the returns of the Long-Short portfolio are not impacted by the market unlike the returns of the Long-only portfolio. This result is similar concerning the momentum factor while it is opposite when we focus on the Value factor.

2.4.4 Leaking illiquid assets

As we previously expressed, the liquidity premium observed in our studies is purely theoretical. Indeed, if the stock encounters a liquidity problem as it is the case for the portfolio of the less liquid stocks, we won't be able to trade it. In this scenario, the return is likely to be smaller than this we obtain in our studies. Thus, in regards to the previous results, we can consider the *MLiq* able to detect drastic illiquidity events. In that sense, we want to use it in a long-short strategy framework in order to extract from the investment portfolios, the stocks that are likely to experience such an event. We propose a long-short strategy based on the Amihud ratio. In line of Menkveld and Wang (2011), we are long for high Amihud ratio and short for low Amihud ratio. However, we disregard the most

Table 2.7 – Regression Analysis

	Liquidity Factor			Low Liquidity Long		
	CAPM	Fama-French	Four factor	CAPM	Fama-French	Four factor
Monthly α	0.0015**	0.0019**	0.0019**	0.0074**	0.0079**	0.0104**
	2.19	2.73	2.60	2.97	3.11	4.12
Market β	0.0009	0.0006	0.0006	0.0097**	0.0099**	0.0101**
	1.46	0.96	0.96	4.32	4.24	4.43
Size		0.0028**	0.0028**		0.0086*	0.0092**
		2.22	2.20		1.89	2.07
Value		-0.0037**	-0.0037**		-0.0022	-0.0061
		-2.67	-2.58		-0.43	-1.22
Momentum			0.0001			-0.0032**
			0.46			-5.57
R^2	0.36%	2.50%	2.54%	3.10%	3.73%	8.62%
Number of months	587	587	587	587	587	587

This table presents coefficients and t-statistics of regressions. The liquidity factor is a Long/Short portfolio obtained having a long position on low-liquidity stocks and a short position on high-liquidity stocks (20% highest for each case). The factors used are Fama-French factors and Momentum (Winners minus Losers), all available on the Kenneth R. French's website.

illiquid stocks according *MLiq*. As a consequence, the portfolio earns from the liquidity premium as long as the liquidity risk is not too large and avoids assets that we consider as preventing a major liquidity problem.

Table 2.8 presents the results for 3 distinct long-short strategies. All these portfolios are built using the Amihud ratio. Indeed, stocks are ranked according to their Amihud ratio, but, in addition, we extract 10% or 20% of the most illiquid stocks according to the *MLiq*. In that sense we avoid to consider the stocks that are not available to trade. In regards to the results, we see that being long high Amihud ratio assets and short low Amihud ratio assets leads to significant positive returns. Indeed, no matter if we control or not for big illiquid events, the annual return of such a strategy ranges from 1.09% to 9.25% according to the period and is significantly different from zero. Moreover, we perceive a little decrease of the returns, for the long-short strategy sometimes and especially for the portfolios 5 that correspond to the most illiquid stocks. This framework has not for objective to increase the performance of the long-short strategies even if we see on figure (2.5) that, over the whole sample, leaking drastic liquid events appear to

Table 2.8 – Excess returns of Long/Short portfolio based on Amihud ratio

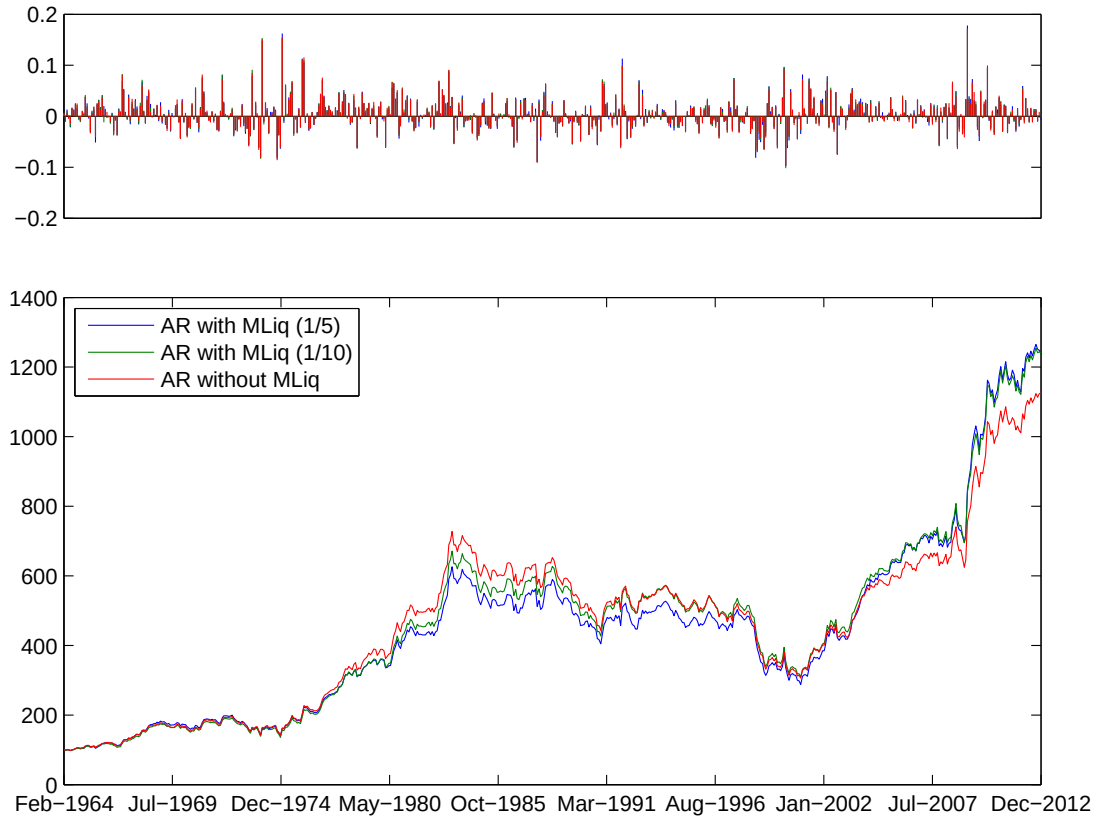
		All months		1964-1985		1986-2012		1988-2007	
		$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat	$rt - rf(\%)$	t-stat
Without control of MLiq	1 (low Amihud ratio)	0.54**	2.53	0.39	1.25	0.67**	2.29	0.72**	2.95
	2	0.67**	2.81	0.57	1.60	0.75**	2.35	0.76**	2.86
	3	0.67**	2.58	0.66*	1.67	0.68**	1.97	0.65**	2.29
	4	0.86**	3.09	0.83**	1.96	0.88**	2.40	0.83**	2.69
	5 (high Amihud ratio)	1.01**	3.38	1.13**	2.46	0.91**	2.34	0.81**	2.46
	differential (5-1)	0.46**	3.20	0.74**	3.38	0.24	1.25	0.09	0.40
Except top decile MLiq	1 (low Amihud ratio)	0.53**	2.46	0.39	1.25	0.64**	2.19	0.70**	2.85
	2	0.67**	2.79	0.57	1.61	0.74**	2.31	0.75**	2.80
	3	0.66**	2.55	0.65*	1.64	0.67**	1.97	0.64**	2.28
	4	0.85**	3.07	0.81*	1.92	0.88**	2.41	0.83**	2.69
	5 (high Amihud ratio)	1.01**	3.40	1.10**	2.40	0.93**	2.42	0.85**	2.55
	differential (5-1)	0.48**	3.31	0.71**	3.27	0.30	1.51	0.15	0.64
Except top quintile MLiq	1 (low Amihud ratio)	0.50**	2.32	0.35	1.13	0.61**	2.11	0.67**	2.71
	2	0.66**	2.76	0.57	1.59	0.73**	2.29	0.72**	2.69
	3	0.66**	2.55	0.65*	1.65	0.67**	1.96	0.63**	2.24
	4	0.84**	3.01	0.80*	1.88	0.88**	2.36	0.81**	2.59
	5 (high Amihud ratio)	0.98**	3.31	1.03**	2.28	0.93**	2.41	0.84**	2.49
	differential (5-1)	0.48**	3.26	0.68**	3.16	0.32	1.57	0.17	0.72

This table presents excess returns and associated t-statistics according to Long/Short portfolio based on Amihud ratio. The first defines the strategy only based on the Amihud ratio while the second and the third do not invest on assets with a *MLiq* over a threshold (0.99 for the second and 0.5 for the third). These results are presented on the sample as a whole and two sub periods. ** means significant at a 5% level, * means significant at a 10% level.

allow an extra gain. Indeed, in average, the annual return is equal to 5.66% while it rises to 5.91% when we take care of drastic illiquidity events. Focusing on the first sub sample from 1964 to 1985, we see that portfolios 1 and 2 earn a non significant return while the others have a return statistically different from zero. However, the most important is the difference between portfolios of low- and high-liquid stocks that is positive and significant. The results show that the Amihud ratio has a better ability to discriminate high- and low-liquid stocks during this period. Indeed, the long-short liquidity premium is not significantly different from zero between 1986 and 2012. This is the case for all the 3 strategies, no matter we control for drastic illiquidity events or not. However, unlike the results presented for the *MLiq* long-short strategy in table 2.3, the excess return of the same strategy based on the Amihud ratio is not significant between 1988 and 2007. This result is similar when we control for drastic illiquidity problems.

Figure 2.5 shows the cumulative returns. The out-performance of the portfolio taking into account drastic illiquidity events, i.e. not investing in assets with higher *MLiq*, is

Figure 2.5 – Excess returns of controlling for drastic liquidity events



This figure presents the excess returns and cumulative returns on the whole sample for two portfolios: (1) without control for drastic illiquidity events and (2) controlling for such an event. These portfolios are built according to the ranking of stocks based on the previous year average Amihud ratio. In the first case, we consider the differential between the highly illiquid and the highly liquid stocks. In the second case, we extract from these portfolios the stocks that have a previous day *MLiq* greater than 0.50. They are weekly rebalanced and the results presents excess returns from January 1965 to December 2012.

large at the end of our sample. The out-performance is mostly concentrated during the end of 70's, the post 9/11 crisis and finally, at the end of 2008. The stocks on which we focus have experienced illiquidity events that *MLiq* is able to avoid. During the first part of our sample, the performances are similar. We see that strategies start to differ at the end of 70's but are again very close in 1997. After the start of 2002, we see that the two portfolios controlling for illiquid events exhibit the same behavior having a better performance than the long-short portfolio only based on the Amihud ratio. Although the trend of our portfolio controlling for liquidity accidents is a bit greater, the dynamic is very close. Indeed, the extra gain observed for extracting some stocks according the

MLiq is very ponctual and the rest of the time, the performances are very similar. This is consistent with the definition of drastic illiquidity problems conversely with small liquidity troubles that stocks may experience every days.

Table 2.9 presents summary statistics and performance measures for the 3 long-short strategies and 3 sub samples. Looking at the average excess returns, over the whole sample, we see the small increase of performances that we show on the cumulative returns figure. This is especially the case for the whole sample and the second sub sample while we see a decrease of performances regarding the first sub sample. Indeed, the average annual return goes from 8.46% without control, to 8.81% and 9.21% when we extract respectively 10% and 20% of the highest illiquid stocks. We see the α are comparable accross strategies. We also present in this table the results of performance measures for both of the two portfolios and the three samples. As a consequence, we are able to see the impact of controlling for drastic illiquidity events on the portfolio risk.

Table 2.9 – Summary statistics and performance measures of 3 portfolios

	Without MLiq control			Except top decile Mliq			Except top quintile Mliq		
	All	1964-1985	1986-2012	All	1964-1985	1986-2012	All	1964-1985	1986-2012
Mean (%)	0.465	0.737	0.243	0.482	0.706	0.299	0.483	0.679	0.322
Std.Dev	0.032	0.033	0.031	0.032	0.033	0.031	0.033	0.033	0.032
Min (%)	-9.775	-8.609	-9.775	-10.194	-8.311	-10.194	-9.776	-8.328	-9.776
Max (%)	17.060	15.349	17.060	17.187	15.350	17.187	17.814	16.215	17.814
Range	0.268	0.240	0.268	0.274	0.237	0.274	0.276	0.245	0.276
α	0.005	0.008	0.003	0.005	0.007	0.003	0.005	0.007	0.003
β	-0.303	-0.421	-0.184	-0.298	-0.421	-0.175	-0.285	-0.430	-0.139
Sharpe ratio	0.145	0.223	0.078	0.149	0.213	0.095	0.148	0.206	0.100
Sortino ratio	0.251	0.415	0.127	0.262	0.408	0.156	0.261	0.393	0.167
Treynor ratio	-0.015	-0.017	-0.013	-0.016	-0.017	-0.017	-0.017	-0.016	-0.023
Historical VaR (5%)	-0.042	-0.038	-0.046	-0.043	-0.039	-0.047	-0.046	-0.039	-0.048

This table presents summary statistics and Standard Performance Measures on excess returns for 3 different strategies: (1) Long/Short based on Amihud ratio, (2) and (3) Long/Short based on Amihud ratio with a control for drastic illiquidity events respectively at a 99% and a 50%. These portfolios are already presented in table 2.8 and figure 2.5.

Although we see the performances increase, we expect a decline of the risk of the portfolio since we put out the most risky stocks considering the liquidity. But, in regards to the standard deviations, the risk almost does not change since the values are equal whatever the strategies are. As a consequence, we see the increase of the Sharpe ratio

only due to the increase of performances. The Sortino ratio also increase going from 0.251 to 0.262 and 0.261 according to the strategy over the whole sample, but the impact appears greater showing that the downside risk declines when we leak the illiquid stocks. However, we see that the minimum return is even smaller for strategies taking into account drastic illiquidity events. In other words, the larger lost is even bigger especially when we extract the top decile *MLiq* stocks. Thus, the extreme events appears to be smoothed by the performances of stocks experiencing illiquidity problems. Removing these stocks leads to accentuate extrem events. This result also appears through the historical *VaR* at 5% since the value from the whole sample goes from -0.042 to -0.046. Table 2.9 is the demonstration that standard risk measures do not incorporate the risk of drastic illiquidity problem. In order to take into account this kind of risk, we have to consider and use other tools and add them in asset allocation methods.

2.5 Conclusion

We propose in this paper a new indicator of market liquidity for a stock experiencing a problem that concerns all the dimensions of the liquidity's definition. Indeed, this kind of event may have huge consequences since a drastic illiquidity problem may lead to large losses and fire sales. As a consequence, we focus on the increase of correlations between different liquidity measures using a state-space model determining the probability for the stock to be in a state of high correlations between liquidity measures. We perform a standard asset pricing analysis and show that this kind of liquidity risk leads to higher returns. Moreover, we prove *MLiq* is able to extract highest illiquid stocks allowing to capture liquidity premium but avoiding to get stuck with very illiquid stocks. It results better performances in the same class of risk.

B.1 Estimation of RSDC

The estimation of this model is made using a two-step procedure: (i) the univariate estimation of standardized residuals with TGARCH model and maximum likelihood and, (ii) the estimation of correlation matrices and probabilities to be in state n ($n = 1, \cdots, N$) with an EM algorithm (Dempster et al. (1977)). Using this method is preferable when the number of observed series is more than a few. Indeed, the number of parameters could become very large and the one-step likelihood maximisation becomes untractable.

We should introduce θ, the complete parameter space, that we split in two parts with: θ_1 that corresponds to the parameter space of the univariate volatility model and θ_2 that corresponds to the parameter space of the correlation model. Firstly, we compute the log-likelihood taking a correlation matrix equal to the identity matrix. In other words, we estimate univariate TGARCH model for each asset.

B.1.1 First Step

To model the full covariance matrix, we estimate the standard deviations and the correlations separately. This first step focus on the estimation of standard deviations.

The parameters of univariate TGARCH model are estimated with maximum likelihood, taking the case of a TGARCH(1,1), as presented in section 1. We have to specify the distribution of U_t in order to estimate the likelihood function that we want to maximize. In our case, U_t are *iid* and normally distributed [$U_t \sim \mathcal{N}(0,1)$] allowing to consider gaussian likelihood. However, we don't make the assumption that is the true law of U_t.

Note $\theta_1 = (\omega, \alpha^-, \alpha^+, \beta)$. Thus, the gaussian likelihood is:

$$L(\theta_1) = L(\theta_1; r_0, \cdots, r_T) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\tilde{\sigma}_t^2}} exp\left(-\frac{r_t^2}{2\tilde{\sigma}_t^2}\right) \tag{12}$$

with $\tilde{\sigma}_t$ are obtained recursively ($\forall t \geq 1$) as:

$$\tilde{\sigma}_{i,t} = \omega_i + \alpha_i^- min(\epsilon_{i,t-1}, 0) + \alpha_i^+ max(\epsilon_{i,t-1}, 0) + \beta_i \tilde{\sigma}_{i,t-1}$$

Taking the logarithm and simplifying the expression, we have to minimize the log-likelihood $\tilde{l}_t(\theta_1)$ that is defined by:

$$\tilde{l}_t = \tilde{l}_t(\theta_1) = \frac{r_t^2}{\tilde{\sigma}_t^2} + log(\tilde{\sigma}_t^2)$$

Thus, $\hat{\theta_1}$ is the solution of:

$$\hat{\theta_1} = \arg\min_{\theta_1} \frac{1}{T} \sum_{t=1}^{T} \tilde{l}_t(\theta_1) \tag{13}$$

After the estimation of parameters, we get the standardized residuals, noted $\tilde{U}_t$ as:

$$\tilde{U}_{i,t} = \frac{r_{i,t}}{\tilde{\sigma}_{i,t}}$$

In the next step, we use it to estimate the correlation matrices. We introduce a regime switching to add dynamic in correlations. It measures the probability to be in the state n (in our case $n = 0, 1$ corresponding respectively to liquid and illiquid states).

B.1.2 Second Step

In this second part of estimation of our model, we use the Expectation Maximization algorithm (EM thereafter). The main advantage is is the possibility to taking into account high number of parameters coming from each Γ_n.

EM Algorithm

This algorithm is presented in Hamilton (1994, chapter 22). We have to estimate the vector of parameters θ_2:

$$\tilde{\theta}_2 = \arg\min_{\theta_2} \left[\frac{1}{2} \sum_{t=1}^{T} K log(2\pi) + log(|\Gamma_t|) + \tilde{U}_t' \Gamma_T^{-1} \tilde{U}_t \right] \tag{14}$$

Unlike the first step, we have to use Hamilton filter because in this part of the estimation, Δ_t is unobserved. Moreover, the number of parameters increases at a quadratic rate with the number of asset returns. Thus, to realize these estimation, we use EM algorithm that has no restrictions on the number of parameters.

Then, Hamilton (1994, chapter 22) expose that Maximum Likelihood estimates of the transition probabilities (i) and the correlation matrices (ii) satisfy:

$$(i) \quad \tilde{\pi}_{i,j} = \frac{\sum_{t=2}^{T} P\left[\Delta_t = j, \Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=2}^{T} P\left[\Delta_{t-1} = i | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \tag{15}$$

$$(ii) \quad \tilde{\Gamma}_n = \frac{\sum_{t=1}^{T} (\tilde{U}_t \tilde{U}_t') P\left[\Delta_t = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]}{\sum_{t=1}^{T} P\left[\Delta_{t-1} = n | \underline{\tilde{U}_T}; \tilde{\theta}_2\right]} \quad \text{for } n = 1, 2 \tag{16}$$

Estimates of transition probabilities are based on the smoothed probabilities. We could see that $\tilde{\Gamma}_n$ is not directly a correlation matrix. It must be rescaled because their diagonal elements are not constrained to be equal to one. Off-diagonal elements are between -1 and 1. This step is needed because the product of standardized residuals is not constrained to have elements between -1 and 1. Then we rescale Γ_t at each iteration as:

$$\Gamma_t = D_t^{-1}\tilde{\Gamma}_t D_t^{-1} \tag{17}$$

where D_t is a diagonal matrix with $\sqrt{\tilde{\Gamma}_{n,n,t}}$ on row n and column n.

The algorithm starts with initial values $\tilde{\theta}_2^{(0)}$ for the vector θ_2. With $\tilde{\theta}_2^{(0)}$ we can compute a new vector $\tilde{\theta}_2^{(1)}$ based on equations (15) and (16). The algorithm works until the difference between $\tilde{\theta}_2^{(m)}$ and $\tilde{\theta}_2^{(m+1)}$ is less than a defined threshold.

Computation

We develop in this subsection the method to compute the EM algorithm. The elements of the transition probabilities matrix, $\tilde{\pi}_{i,j}$ are defined as the ratio of consecutive probabilities ($P[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}_t, \theta_2]$) and the probabilities to be in state j at time t. They are obtained iteratively from $t = 1$ to T.

Note that, conditional probability is defined by [see Hamilton, (22.3.7)]:

$$P[\Delta_t = j|\tilde{U}_t, \theta_2] = \frac{\pi_j \times f(\tilde{U}_t|\Delta_t = j, \theta_2)}{f(\tilde{U}_t|\theta_2)} \tag{18}$$

where $f(\tilde{U}|\Delta_t = j, \theta_2)$ is the probability density of the multivariate normal distribution with zero mean and Γ_j as covariance matrix, evaluated for $\tilde{U}_t$.

With equation (18), we compute probabilities at time $t = 1$. Then, we compute consecutive probabilities recursively:

$$P\left[\Delta_t = j, \Delta_{t-1} = i|\tilde{U}, \theta_2\right] = P\left[\Delta_{t-1} = i|\tilde{U}, \theta_2\right] \times P\left[\Delta_t = j|\tilde{U}, \theta_2\right] \times \pi_{i,j} \tag{19}$$

where $P\left[\Delta_t = j|\tilde{U}, \theta_2\right] = f(\tilde{U}|\Delta_t = j, \theta_2)$.

Then, conditional probabilities to be in state j at time t are obtained making the ratio of the sum of the two consecutive probabilities of being in state j at time t and the sum of all consecutive probabilities.

Introduce the notation $\xi_{t|\tau}$, the $(N \times 1)$ vector whose j^{th} element is $P[\Delta_t = j|\tilde{U}_\tau, \theta_2]$. This notation allows to present two cases of $\xi_{t|\tau}$: (i) for $t > \tau$ it represents a forecast about the regime and (ii), for $t < \tau$ it represents the smoothed inference (about the regime in date t based on data obtained through some later date τ). We focus on smoothed probabilities that is defined by:

$$\tilde{\xi}_{t|\tau} = \tilde{\xi}_{t|t} \odot \{\Pi' \cdot [\tilde{\xi}_{t+1|T}(\div)\tilde{\xi}_{t+1|t}]\} \tag{20}$$

Smoothed probabilities are obtained iterating on backward for $t = T, T-1, T-2, \cdots, 1$. We come back from equation (19) to compute consecutive probabilities with smoothed probabilities. Then, we compute $\theta_2^{(m)}$ with equation (15) and (16) rescaling at each iteration the correlation matrix with equation (17).

The breaking rule of the algorithm is defined by the fact that the correlation matrix computed by the last iteration is almost equal to the previous correlation matrix. We have to define a threshold under which, we consider that matrices are equal.

Initialisation of the Algorithm

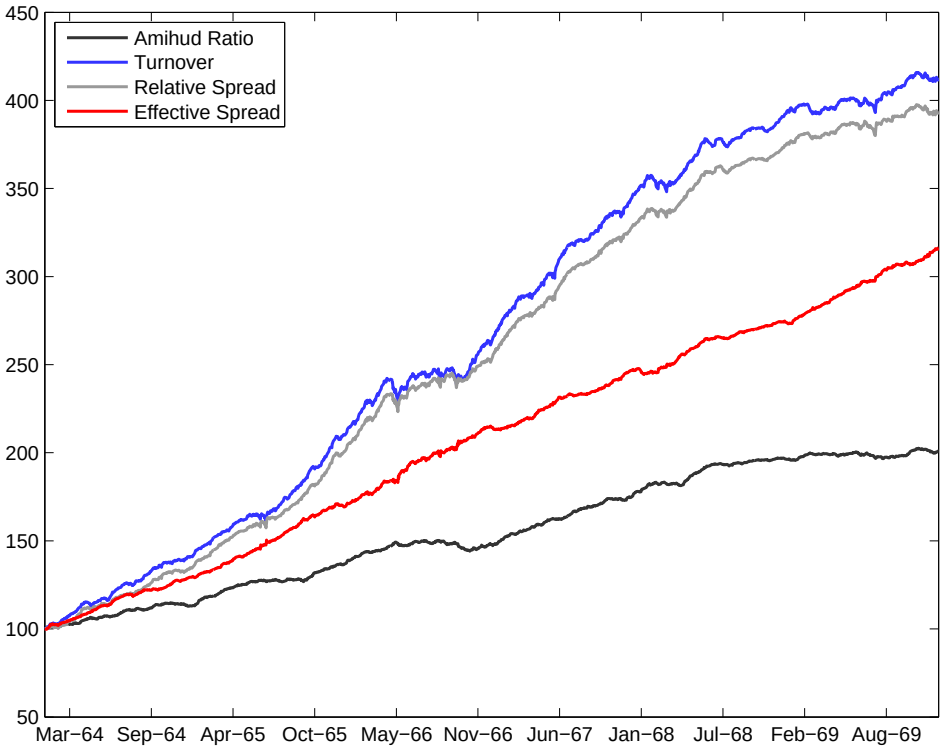
To start the algorithm, we have to choose the space of initial parameters, $\theta_2^{(0)}$. In this space, we input correlation matrices for each state of our model (in our case, two). The algorithm starts with one matrix of correlations of the state (1) equal to identity matrix. For the second state, we use the Gramian matrix method (Holmes (1991)) to generate random correlation matrix. Note that a correlation matrix has to be defined semi-positive

with diagonal elements that are equal to one and off-diagonal elements that are between -1 and 1. We use the Gramian matrix $T'T$ where $T := (t_1, \cdots, t_K)$ and t_i is the ith column. Then, we normalize the matrix as: $t_i = \tau_i / ||\tau_i||$.

For a K-variate process, we generate K independent pseudo-random vectors normally distributed, τ_i.

B.2 Figures

Figure 6 – Excess returns of controlling for drastic liquidity events



This figure presents cumulative returns resulting from 4 different portfolios. These portfolios are built according different liquidity measures. The positions are long for the 20% more illiquid assets and short for the 20% more for liquid assets. We consider the liquidity of the previous day for investing and the investment horizon is a day.

Figure 7 – Excess returns of controlling for drastic liquidity events

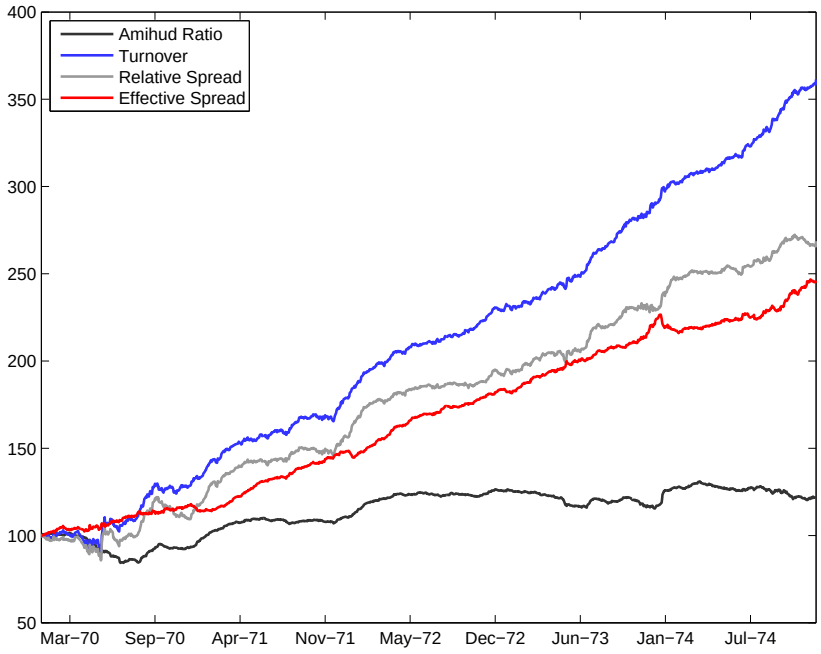


Figure 8 – Excess returns of controlling for drastic liquidity events

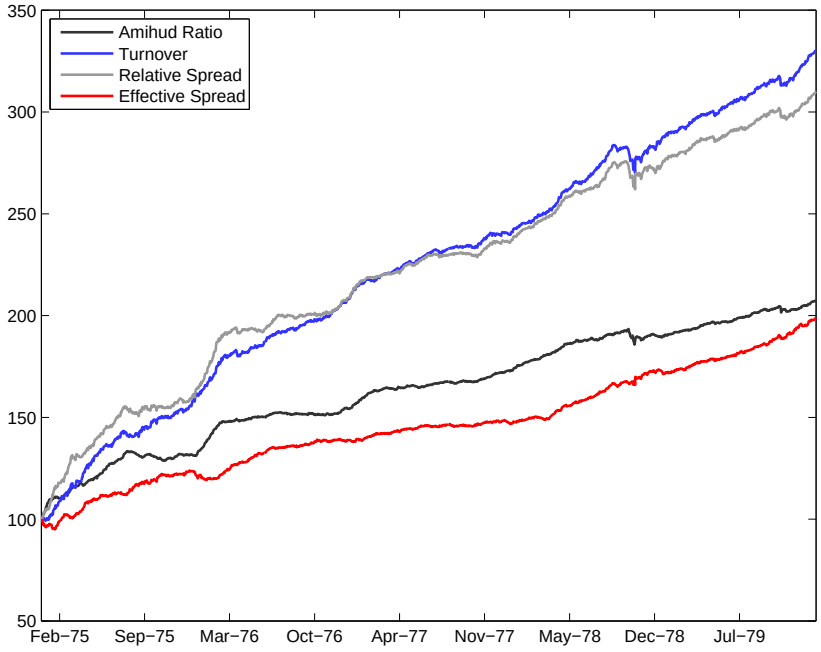


Figure 9 – Excess returns of controlling for drastic liquidity events

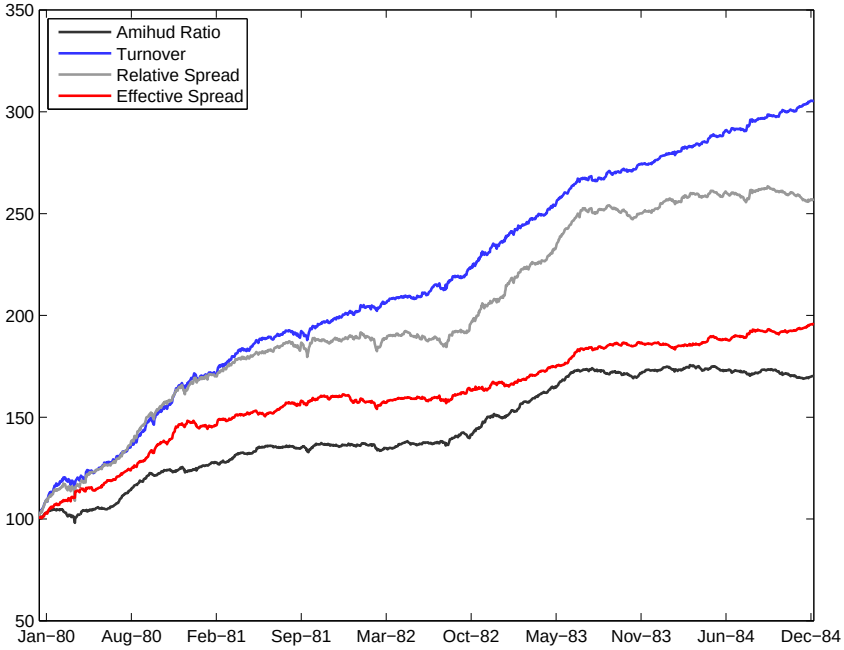


Figure 10 – Excess returns of controlling for drastic liquidity events

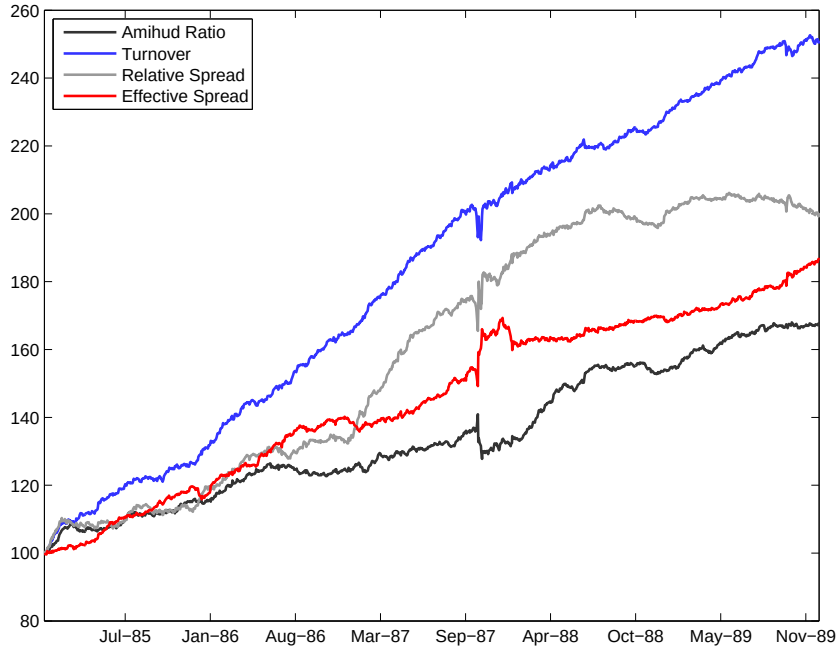


Figure 11 – Excess returns of controlling for drastic liquidity events

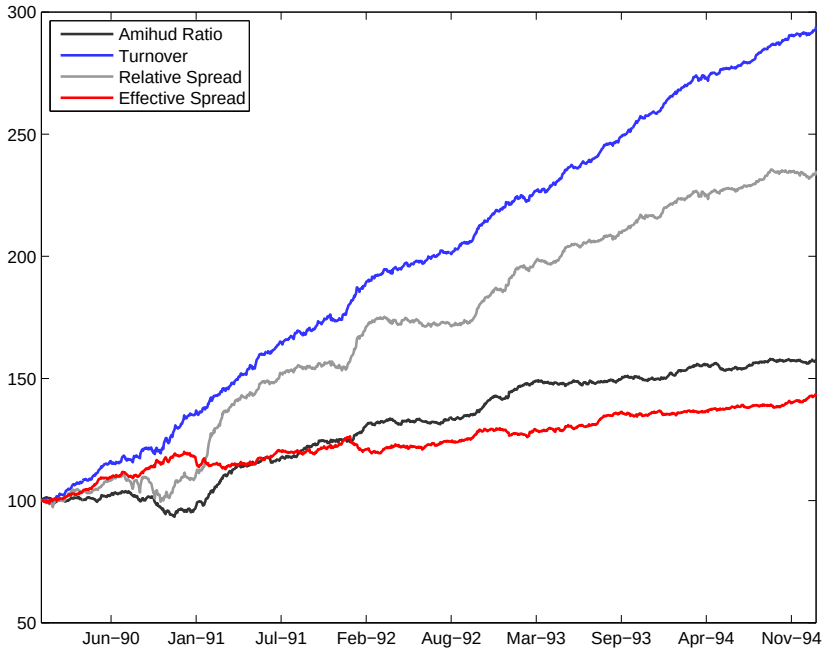


Figure 12 – Excess returns of controlling for drastic liquidity events

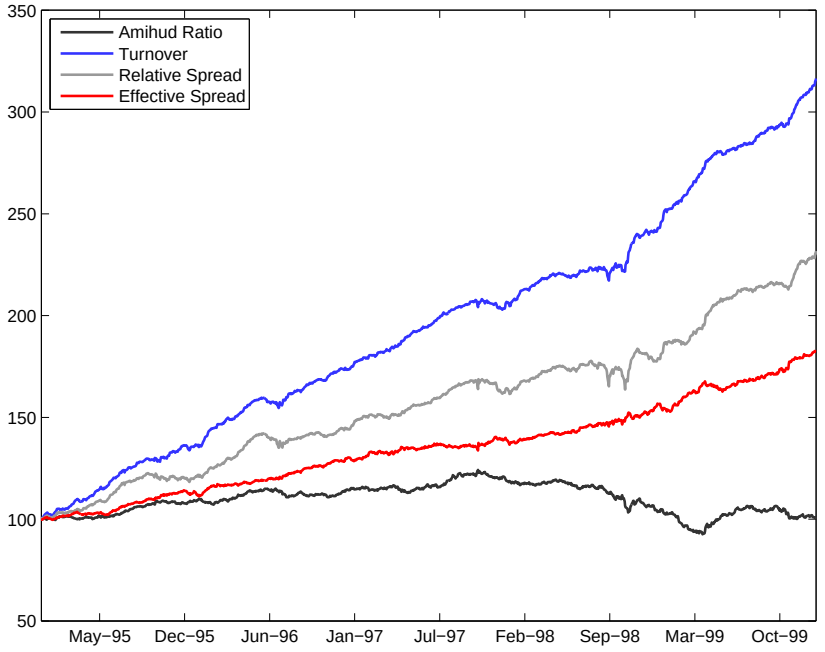


Figure 13 – Excess returns of controlling for drastic liquidity events

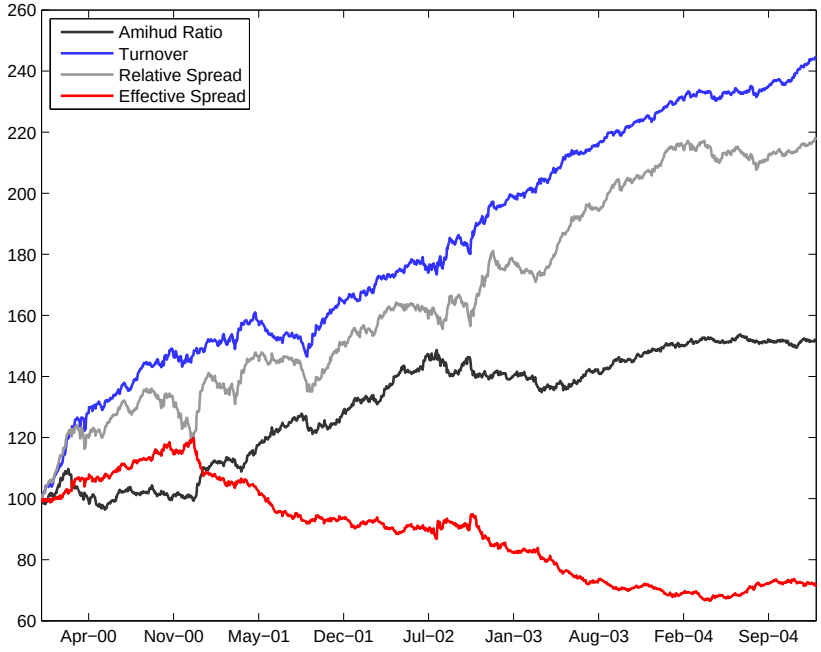
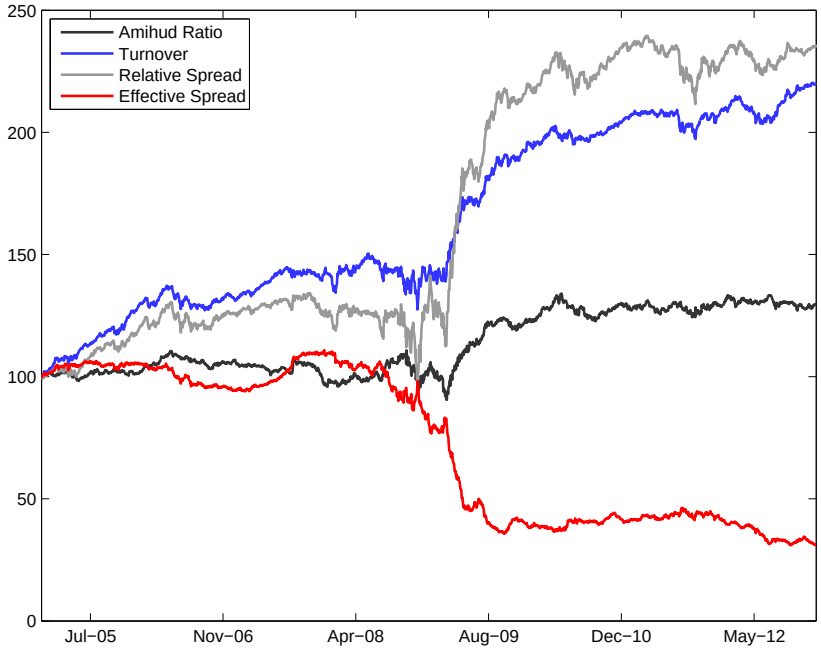


Figure 14 – Excess returns of controlling for drastic liquidity events



Chapter 3

Measuring Systemic Risk under liquidity constraints

La régulation des institutions financières était jusqu'à la crise de 2007-2008, uniquement focalisée sur leur risque individuel. Néanmoins, depuis la faillite de *Lehman Brother* et l'impact que cela a eu sur les marchés, les régulateurs souhaitent mettre en place une supervision visant à définir le capital requis de chaque institution en fonction de sa contribution au risque du système. En effet, les inter-connections entre les établissements financiers font qu'une institution ayant un risque individuel modéré peut avoir une contribution majeure au risque qu'encourt le système. Par conséquent, le régulateur cherche à internaliser les externalités de chacune des institutions financières, ayant pour objectif d'éviter la propagation des dommages liés à la faillite de l'une d'entre elles.

Comme nous l'avons vu précédemment, à ceci s'ajoute le besoin de contrôler le risque de liquidité. En effet, les banques et plus généralement les institutions financières sont exposées à des problèmes de liquidité de financement. Contrairement à la liquidité de marché présentée dans le chapitre *MLiq*, la liquidité de financement s'apparente à la facilité qu'a le gérant ou la banque, à financer ses activités. Or, la régulation financière se base sur un montant de capital requis afin de pouvoir traverser une crise financière tout

en conservant un comportement normal. Il est alors évident que la firme aura plus de difficultés à satisfaire cette demande de capitaux si elle est contrainte par la liquidité.

Une mesure de risque standard, basée sur le risque individuel de la firme est très largement procyclique. En effet, ce type de mesure à tendance à demander plus de capitaux requis lors des périodes de crises, c'est à dire, lorsque la firme est le moins capable de satisfaire cette demande. Il est donc important de proposer une mesure permettant une régulation contracyclique qui impose aux institutions financières de capitaliser lorsqu'elles le peuvent en vue d'une possible crise financière. Il serait alors préférable que les firmes actuellement classées comme systémiques selon les mesures actuelles, se voient identifiées comme risquées durant les périodes calmes pour lesquelles elles ne sont pas contraintes par la liquidité.

Dans ce chapitre, nous présentons une modélisation des rendements de marché à l'aide d'un modèle à seuil permettant de définir deux régimes basés sur la liquidité de financement. En d'autres termes, la modélisation du rendement diffère conditionnellement au fait que la liquidité de financement soit bonne ou mauvaise. De ce fait, il est possible d'intégrer la liquidité de financement dans le calcul de la $\Delta CoVaR$ et ainsi considérer les contraintes de liquidité dans la mesure du risque systémique. Cette dernière représente l'impact sur les rendements du marché d'une faillite de l'institution financière concernée.

Les résultats sont particulièrement intéressant dès lors qu'ils mettent en avant un changement significatif du classement des institutions financières selon leur contribution au risque du système comparativement à l'estimation du risque systémique basée sur une $\Delta CoVaR$ standard. De plus, nous pouvons constater qu'une régulation s'appuyant sur cette nouvelle mesure a un comportement contracyclique. En effet, relativement à une mesure $\Delta CoVaR$ usuelle, la demande de capitaux apparait plus importante en périodes calmes et moins importante lors des périodes de crises. Ces deux points sont attractifs puisqu'ils peuvent permettre une meilleure régulation et peuvent mener à une meilleure gestion du risque systémique de la part des institutions financières.

3.1 Introduction

Measuring systemic risk is at the very center of the current regulation since we note the impact of the Lehman Brothers collapse on the financial market and the worldwide economy. Actually, measuring systemic risk especially focuses on the identification of Systemically Important Financial Institutions (SIFIs hereafter). In that sense, it means the regulator wants to establish the ranking of financial institutions going from the one that contributes the less to the risk of the system to those that has the biggest externalities in case of default. However, since the last financial crisis of 2007-08, in addition to the market risk, the regulator has to focus on the liquidity risk. Indeed, it has been shown the liquidity may trigger systemic events. Moreover, considering the aim of the regulator to impose capital requirements for risky firms, they have to consider their ability to satisfy the regulatory constraints. As a consequence, the regulation should be countercyclical in order to prevent systemic risk events rather than tax financial institutions during crisis. We find a very large literature about the measurement of systemic risk as it has been surveyed in Bisias et al. (2012). Among all these measures, we have the cross-sectional measures as for example the $\Delta CoVaR$ of Adrian and Brunnermeier (2011) or the MES of Brownlees and Engle (2012). The former considers the behavior of the market return conditional to the bailout of a financial firm while the latter focuses on the impact of a large loss for the financial market on the return of the firm. More recently, in addition to the first two measures of systemic risk, Acharya et al. (2012) propose the $SRISK$, based on the MES which takes into account firm's liabilities and market capitalization. This latter proposes an alternative way to consider more information than that extracted from returns of a single equity market information.

In this paper, we propose to include the funding liquidity in a market-based systemic risk measure framework. According to my knowledge, this is the first time that a paper proposes a market-based measure of systemic risk including liquidity into its modeling.

Indeed, controlling for the impact of liquidity on the financial system appears to be a key challenge in order to improve the systemic risk measurement. The first idea should be to evaluate the amount of capital for the institution in order to pursue its activity despite a funding liquidity constraint. The *SRISK* measure, proposed by Acharya et al. (2012) perfectly fits this definition. Indeed, *SRISK* represents the need of capital for the firm in order to experience a financial crisis without impact on its activity. However, based on the *MES* and taking into account firm's liabilities and market capitalization, introducing funding liquidity into the estimation is not obvious and requires some strong assumptions. Effectively, the *SRISK* is based on the *MES* and focusing on this part, its estimation requires restrictive assumptions about the distribution of the liquidity indicators. As a consequence, we choose to study the $\Delta CoVaR$ that appears more flexible.

During the last financial crisis, the liquidity play a predominant role. We know the liquidity could either trigger a financial crisis and/or amplify the effects of the crisis leading to systemic risk events. However, Brunnermeier and Pedersen (2009) propose to distinguish the market liquidity from the funding liquidity. The first corresponds to the ability to trade an asset while the second corresponds to the ability for a trader to finance their trade. As a consequence, the link between the liquidity and the systemic risk is stronger in regards of the interconnectedness between all the financial firms due to their funding relationships. Thus, the regulation should introduce a liquidity dimension into the macroprudential framework that is developed since 2008 and especially focusing on the risk of funding liquidity. The contribution of this paper is to detail a way for including the liquidity into a market-based systemic risk framework and thus, propose a new measure. Indeed, the actual framework of systemic risk measurement is largely driven by the two well known measures: $\Delta CoVaR$ and *MES*. Both of them are only based on public data and are easily computable. In this paper, we focus on keeping these characteristics even adding a liquidity component. As a consequence, we have to choose a liquidity indicator publicly available representing the ability of financial firms to finance their trades. Moreover,

we propose a conditional estimation of the $\Delta CoVaR$. More precisely, we estimate the parameters of the systemic risk measure conditionally to the level of funding liquidity at the same date. The impact of the firm returns on the financial system returns is evaluated taking into account the level of funding liquidity using a threshold model [see Hansen (2000)]. Consequently, we distinguish different behaviors of the measure based on the ability of market participants to finance their trades leading to a better appreciation of the distress in which the financial system is.

All the results obtained in this paper are telling us that considering liquidity for measuring systemic risk implies a significant change in the identification of SIFIs. Indeed, the estimation of parameters largely differs and we observe a large difference between the two $\Delta CoVaR$ considering or not the funding liquidity risk. However, more than the simple evolution of the $\Delta CoVaR$ value, we show that the behavior of the new systemic risk measure is countercyclical. Effectively, the new systemic risk measure appears largely greater during calm periods leading to bigger capital requirements when the financial firms are easily able to satisfy them. But, in opposite, the capital requirements should be smaller during financial crisis when the institutions are funding constrained. Moreover, when we focus on the ranking of financial institutions as the regulator, we show that the new measure add information since the ranking largely differs from that obtained using the seminal $\Delta CoVaR$.

The remainder of this paper is organized as follows. The second section introduces the problematic of a systemic risk measure that takes into account liquidity, the different propositions already done and ours. Then, we present the data and summary statistics on the liquidity and systemic risk measures. Section IV exposes the results of the empirical case while section V concludes the paper.

3.2 Problematic

Every financial institutions are constrained to transform liquidity. In other words, assets and liabilities have not the same liquidity. As a consequence, they are exposed to funding liquidity risk and they may be constrained in the case of a funding liquidity drying. The systemic regulation, imposing to financial institutions capital requirements, has to take into account their ability to satisfy them. Indeed, considering the current regulation, the capital requirements are largely higher during crisis periods and relatively small during tranquil periods. The better solution should be opposite. In other words, we should impose high capital requirements during tranquil periods, when the financial institutions are able to satisfy these constraints. To sum up, a good systemic risk regulation has to be countercyclical and take into account the funding ability of the institutions.

However, measuring systemic risk only focusing on publicly available data implies some difficulties to consider liquidity risk. The standard systemic risk measurement framework proposes to model firm's contribution of systemic risk using conditional measures of extreme losses. Indeed, considering cross-sectional measures implies to model either market returns, conditionally to the firm returns or the opposite relation. As a consequence, we have to make some assumptions about the relation between the financial firms and market returns. According to these assumptions, modeling the dependence structure may lead to constraints about the distribution of variables. However, the methodology proposed by Adrian and Brunnermeier (2011) with the $\Delta CoVaR$ allows to consider the liquidity as a conditional variable since the estimation is made using a linear model allowing some changes. As a result, we propose to add to the standard model a dummy variable depending on the level of the funding liquidity perceived by the financial market.

3.2.1 Market Liquidity versus Funding Liquidity

In order to consider the liquidity risk, we have to make the distinction between market liquidity and funding liquidity. Indeed, each of these two risks induces particular economic implications. Based on the seminal paper of Brunnermeier and Pedersen (2009), we define the market liquidity of an asset as the ease to trade it while the funding liquidity for a manager is represented by the ease with which he can finance their trades. Since problems of liquidity may lead to vicious cycles of decreasing both funding and market liquidity [Gromb and Vayanos (2002), Gromb and Vayanos (2010) and Brunnermeier and Pedersen (2009)], we have to consider the ability for the financial firms to finance their activities. Moreover, in a systemic risk point of view, the links between financial firms appear as very relevant. Indeed, the interbank funding is a key point of the trading activities of financial firms. When the relations between institutions deteriorate, their ability to pursue their activities is largely impacted. As a consequence, this is more relevant to focus on the funding constraints that the financial firms may experience rather than the market liquidity problems that they may individually encounter and giving less information about their ability to satisfy capital requirements.

Considering the difference between these two kinds of liquidity, we have to focus on the measurement of funding liquidity. Indeed, even if we find in the literature a lot of market liquidity measures[1], there exist only few funding liquidity indicators only based on publicly available data. Then, we are able to define periods for which the institution is funding constrained. We want to know the impact on the systemic risk measure and the ranking of the institution taking into account its ability to finance their activities. In the case of a liquidity spiral, initial losses are amplified by the liquidity problem leading to greater losses. As a result considering funding liquidity appears to be useful in the systemic risk framework and funding liquidity add precious information about the likelihood for an individual problem to spread throughout the whole financial system.

[1] see for example Aitken and Winn (1997).

However, we see in the literature some papers considering the liquidity to measure the systemic risk. The main issue is the availability of data allowing to accurately treat this problem. During the last 5 years, the literature about funding liquidity largely grows up. As it was defined by Brunnermeier and Pedersen (2009), we may find different ways to measure the funding liquidity on the market. Boudt et al. (2011) propose two novel funding liquidity measures with a central bank measure defined as the excess broker loan rate and a market measure defined as the value-weighted average stock loan rate. They show the relation between the funding liquidity and the market liquidity taking care of endogeneity. Comerton-Forde et al. (2010) study the relation between market liquidity and funding constraints. They use as a proxy of funding liquidity, specialist equity trade summary data and especially trading revenues and inventories data. Concerning the measurement of the funding liquidity risk, Fontaine and Garcia (2012) propose a funding liquidity factor based on the difference of prices between two identical bonds only differentiated by their age. The literature is relatively extensive about measuring the funding liquidity. However, the TED spread[2] is widely used in applications needing a funding liquidity indicator. It indicates the spread between the funding rates of the financial firms and that of the US government.

3.2.2 Measuring systemic risk

The main part of the regulation still focuses on individual risk measures like for example, the well known Value-at-Risk. Using this kind of measure allows to define a minimum amount of capital requirement sufficient to cover the risk it imposes itself to. There exists a large literature about the measurement of individual market risks. However, since 2008, the regulator would like to internalize the externalities of financial firms. In other words, they are looking for a macroprudential regulation enabling to charge the financial institutions proportionally to the risk that they impose to the rest of the market. In

[2]The TED spread is the difference between the 3 months T-bill yield and the 3 months LIBOR

that sense, the firm has to prevent the risk that its bankruptcy might cause to the others financial institutions.

The most used systemic risk measures are the $\Delta CoVaR$ and the MES proposed by Adrian and Brunnermeier (2011) or Brownlees and Engle (2012), respectively. Both of these two measures focus on the conditional relation between the financial firm returns and the financial system returns focusing either on the losses of the firm conditional to an extreme event on the market or focusing on the losses of the market conditional to the bailout of the firm. As a consequence, in both cases, we have to make some assumptions about the dependence structure between the financial firm returns and the financial market returns.

The actual policy proposed by the regulator to control the systemic risk of individual institutions implies they measure it using publicly available data. Then, they rank the financial firms and finally, they group them into some buckets in order to impose regulatory constraints relative to their bucket. We already perceive two classifications leading the value of the systemic risk measure to have a very small impact. The first is to rank the financial firms, i.e. the value obtained from the measure does not have any importance since we focus on the rank of the firms. Thus, only the relative value matters. Secondly, the regulator groups the financial institutions and reduces again the impact of the systemic risk measure value. As a consequence, in addition to focus on the difference between values of distinct measures, we have to study the difference in terms of ranking in order to show whether considering funding liquidity may change the actual regulation.

Adrian and Brunnermeier (2011) propose to estimate the $\Delta CoVaR$ using quantile regression as a way to have a simple estimation but ensure an efficient use of the data. This methodology assumes a non linear dependence structure between the returns of the financial firms and those of the market. However, Benoit et al. (2013) show the assumption of linear dependence has a negligible impact on the ranking of SIFIs. Their results show that estimations that account for nonlinear dependence structure in return series are very sim-

ilar considering the identification of SIFIs whatever we model the nonlinear dependence or not. These findings are in line with Patro et al. (2013) suggesting that daily stock return correlation is a simple and a sufficiently informative indicator for assessing systemic importance of institutions and monitoring systemic risk. As a consequence, we then simplify the estimation by assuming that dependence is fully captured by the correlation and estimate the two measures using linear estimation methods.

Finally, assuming the dependence structure is linear considering the link between returns of financial firm and financial system, we are able to estimate the $\Delta CoVaR$ using Ordinary Least Square method rather than quantile regression. This assumption allows us to estimate our new model using the methodology of Hansen (2000)[3]. Thus, the $\Delta CoVaR$ is computed based on parameters from two regimes: the first one considering the funding liquidity as good enough and the second characterized by a lack of funding liquidity, expressed, in our case by a higher TED spread.

3.2.3 $\Delta CoVaR$ and a threshold model

The main contribution of this paper is to integrate the liquidity in the $\Delta CoVaR$ framework. Indeed, the model proposed in the seminal paper of Adrian and Brunnermeier (2011) describes the returns of the market conditionally to the financial firm returns whatever the liquidity of the market is. As a result, this methodology is not able to distinguish the contribution of the systemic risk from a financial firm when the funding liquidity of this firm is constrained or not. As we previously described, the natural way to model this kind of pattern is a threshold model. In our case, we assume there exist two distinct regimes and we distinguish them based on an exogenous variable representing the funding liquidity [Hansen (2000)].

Indeed, in addition to the standard methodology, we propose to distinguish two regimes

[3]The same model can be estimated using quantile regression using the methodology proposed by Kuan and Michalopoulos (2012).

in order to improve the measurement of systemic risk based on the impact of the funding liquidity on the financial firm returns. We assume that the systemic risk measure and either the capital requirements or the ranking induced by its value should differ based on the regime of funding liquidity in which the market is. That is especially the case since we could see that funding constraints lead to liquidity spirals and large losses for financial sector. However, as every firms are not constrained similarly by a funding liquidity problem, we need to estimate the threshold based on the financial firm individually.

Firstly, we have to define the Value-at-Risk of the firm i for a confidence level of α, $VaR^i(\alpha)$ as:

$$P\left[X^i \leq VaR^i(\alpha)\right] = \alpha \tag{3.1}$$

By convention, one switches the sign of the $VaR^i(\alpha)$ in order to treat a positive number.

As a consequence, in the general definition, the $CoVaR^{j|\mathbb{C}(X^i)}(\alpha)$ is expressed as:

$$P\left[X^j \leq CoVaR^{j|\mathbb{C}(X^i)}(\alpha)|\mathbb{C}(X^i)\right] = \alpha \tag{3.2}$$

where X^j is usually the market return and $\mathbb{C}(X^i)$ is defined as a distress event usually equal to the VaR of the institution i. As a result, the $\Delta CoVaR^{j|\mathbb{C}(X^i)}(\alpha)$ that denotes the contribution of firm i to firm j (or financial system usually) is defined as:

$$\Delta CoVaR^{j|\mathbb{C}(X^i)}(\alpha) = CoVaR^{j|\mathbb{C}(X^i)}(\alpha) - CoVaR^{j|X^i=median^i}(\alpha) \tag{3.3}$$

The estimation of this systemic risk measure is based on the modeling of the market return conditionally to the financial firm return i such that:

$$r_t^m = \delta^i + \beta^i r_t^i + \epsilon_t^i, \tag{3.4}$$

As a consequence, the $\Delta CoVaR$ is nothing but the product of the estimated beta and the difference of VaR as:

$$\Delta CoVaR_t^i(\alpha) = \hat{\beta}^i \left[VaR_t^{r^i}(\alpha) - VaR_t^{r^i}(0.5) \right]. \tag{3.5}$$

In order to consider the funding liquidity into the estimation of the $\Delta CoVaR$, we change the modeling of the market returns adding a trimmed variable taking the value of the firm's equity returns when the firm is constrained by the funding liquidity and 0 otherwise. We have:

$$r_t^m = \delta^i + \beta^i r_t^i + \Phi^i r_t^i \mathbb{1}_{\{y_{t-1} > \gamma^i\}} + \xi_t^i, \tag{3.6}$$

As a consequence and following the estimation of the standard $\Delta CoVaR$ presented in appendix, the $l{-}\Delta CoVaR$ or liquidity-adjusted $\Delta CoVaR$ is defined as:

$$l{-}\Delta CoVaR_t^i(\alpha) = \left(\hat{\beta}^i + \hat{\Phi}^i \mathbb{1}_{\{y_{t-1} > \hat{\gamma}^i\}} \right) \left[VaR_t^{r^i}(\alpha) - VaR_t^{r^i}(0.5) \right]. \tag{3.7}$$

where α is the risk threshold chosen. β^i and Φ^i are respectively the coefficient representing the impact of the financial firm returns on the financial market returns and the coefficient relative to the additional factor represented by the lagged level of liquidity (proxied here by y_{t-1}). Moreover, γ^i is the threshold that distinguishes two regimes in the relationship between financial firms and market returns.

We note in equation 3.7 that the second part does not differ from the standard $\Delta CoVaR$. However, the first one changes since we have an additional coefficient linked to the dummy

variable indicating the state of funding liquidity in which is the financial firm. We have already noted that the threshold γ^i depends on the financial firm i. Moreover, this threshold is endogenously defined by the model [Hansen (2000)]. As a result, at each date, one firm may be considered as funding constrained while the other is not.

3.3 Data and Summary Statistics

Our sample comprises 94 U.S. financial institutions with equity market capitalization greater than 5 bln USD as of June 30, 2007. Appendix D provides the list of the institutions categorized by industry groups. The sample period is from 01/03/2000 to 12/31/2011. Out of 94 financial firms 60 are continuously traded over the sample period. Data are obtained from CRSP. Finally, risk measures are computed at a risk threshold equal to 5%.

3.3.1 Summary statistics

Table 3.1 – Summary Statistics

	Mean	Min	Max	Std	Std Within	Std Between
Returns	0.0003	-0.9425	1.2838	0.0327	0.0005	0.0212
$\Delta CoVaR_{OLS}$	0.0102	0.0008	0.1389	0.0075	0.0028	0.0060
$\Delta CoVaR_{Quant}$	0.0103	0.0007	0.1876	0.0075	0.0028	0.0060
$l{-}\Delta CoVaR$	0.0112	-0.0114	0.1151	0.0072	0.0026	0.0056

This table presents summary statistics for the database as a whole. It consists of 94 U.S. financial institutions with equity market capitalization greater than 5 bln USD as of June 30, 2007. The sample is ranging from 01/03/2000 to 12/31/2011. In this table, we study the daily stock return (Returns), the daily $\Delta CoVaR$ estimated using Ordinary Least Square regression ($\Delta CoVaR_{OLS}$), the daily $\Delta CoVaR$ estimated using Quantile regression ($\Delta CoVaR_{Quant}$) and finally, the daily $\Delta CoVaR$ estimated using Ordinary Least Square regression based on a threshold model ($l{-}\Delta CoVaR$).

Table 3.1 presents some summary statistics about the returns of financial institutions previously presented. We see an average daily return equal to 3 bps, i.e. 7.79% annually. This table also provides summary statistics for the estimated systemic risk measures discussed in the previous section. The second and third lines report estimates of $\Delta CoVaR$

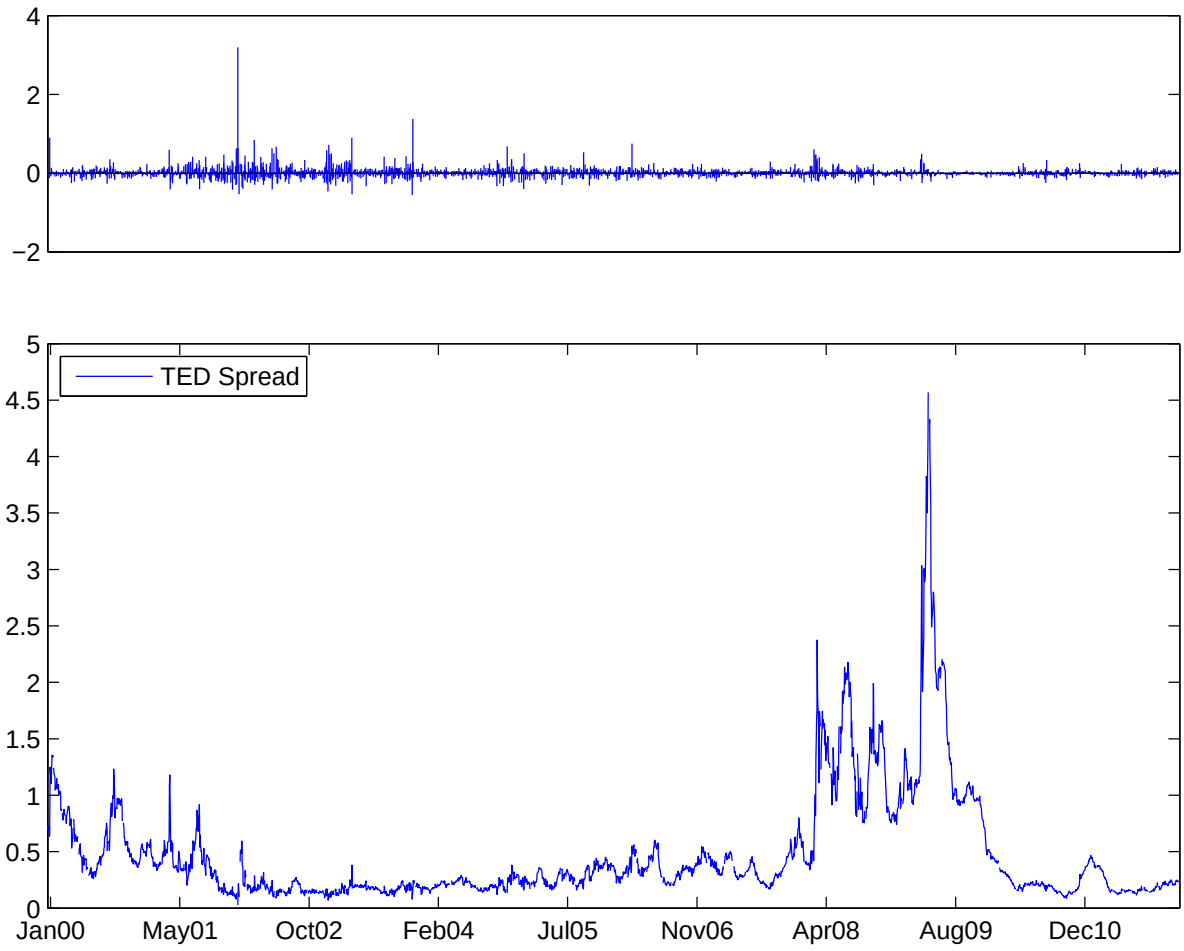
obtained via OLS (denoted $\Delta CoVaR_{OLS}$) and quantile regression (denoted $\Delta CoVaR_{Quant}$), respectively. Finally the last line presents estimates of the l–$\Delta CoVaR$ measure considering funding liquidity and using a threshold estimation model. We show the standard statistics of the estimated measures that account for nonlinear dependence are very close to those that do not capture nonlinear dependence feature in the data (respectively using quantile regression and OLS). However, looking at the maximum value of the systemic risk measures, nonlinear estimation method appears to accentuate extreme events. Nonetheless, as we discussed, the value does not really matter and even if the extreme events are better modeled, they may have no impact on the ranking of SIFIs. Table 3.1 also presents within standard deviation (across time) and between standard deviation (across firms). We see that dispersion is higher in time series than in cross-section in all cases.

3.3.2 The TED spread as a funding liquidity indicator

Goyenko (2012) explains the TED spread is often used as a proxy of funding liquidity and we already find several applications [Boyson et al. (2010), Teo (2011)]. Actually, it reflects the credit risk of the economy but, as the difference between interest rates of interbank loans and short-term U.S. government debt, it reflects the ability with which the bank are able to finance their trades and more generally their activities. Boudt et al. (2011) propose to use the TED spread in order to discriminate two regimes in the relation between market and funding liquidity. We use a similar framework to distinguish two states of the financial market assuming that the TED spread is relative to the funding liquidity on the U.S. financial market.

Figure 3.1 presents the dynamic of the TED spread from January 2000 to December 2011 at a daily frequency. We see a large increase at the end of 2008 and a high volatility of the spread during the crisis at the end of 2001. These two events experienced strong funding liquidity problems. The first part of our sample exhibits high changes of TED

Figure 3.1 – TED Spread



This figure presents the TED spread that is the difference between the interest rates on interbank loans and on short-term U.S. government debt (3 months T-Bills). They are computed based on daily data from January 2000 to December 2011.

spread even if the level remains relatively low. Then, between September 2002 and the summer 2007, the TED spread keep a low level before experiencing a strong increase. A surprising fact is the low volatility during the 2007-08 crisis. Even if the spread hits its highest level, the volatility is particularly low. The variations are gradual and very small.

3.4 Empirical Results and Discussion

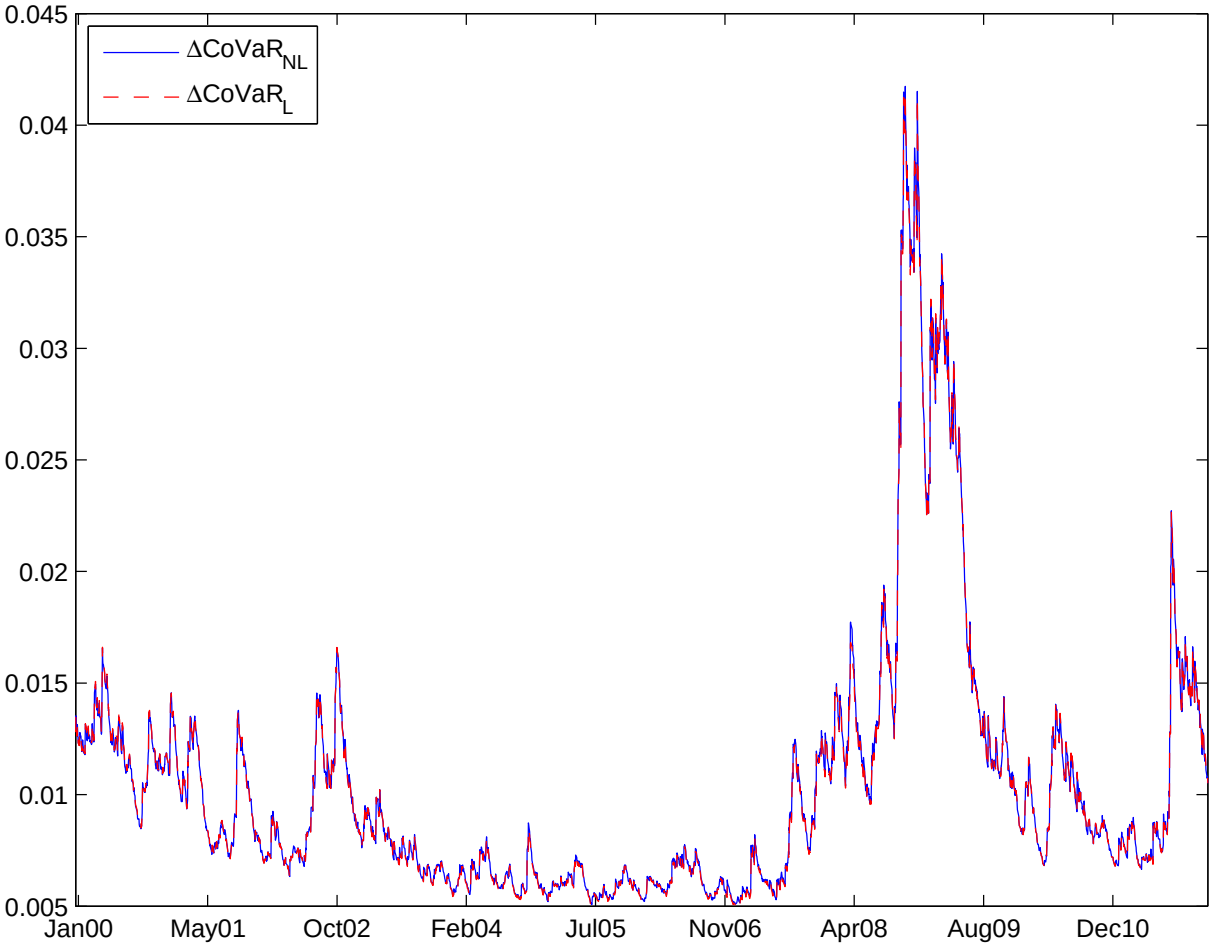
In this section, we discuss the results of the $l-\Delta CoVaR$. We explore the impact on the value and the ranking obtained using a model taking into account the funding liquidity. Thus, we study the mean value of all the $l-\Delta CoVaR$ and then, we focus on the ranking and the Top 10 of SIFIs for different dates of our sample.

3.4.1 Estimation parameters and dynamic

As we described in section 2, although the measure appears largely used, there exist many ways to estimate the so-called $\Delta CoVaR$ that may lead to different results due to their propensity to be manipulated. However, it has already been presented that taking into account the non linearity of the dependence structure between the financial firm and financial market returns does not impact the results [Benoit et al. (2013)]. Figure 3.2 confirms this results since we see a small difference between the $\Delta CoVaR$ estimated by OLS and those estimated using quantile regression. Indeed, as the β estimated either using OLS or quantile regression is constant over time, we know the ratio of the two $\Delta CoVaR$ does not depend on time (the ratio of the β). As we can guess, we see a large peak at the end of 2008 following the bankruptcy of Lehman Brothers but we see the systemic risk measure coming back to a normal state very quickly, at the middle of 2009.

However, considering a threshold model and the funding liquidity into the systemic risk framework leads to larger difference and mainly, a time varying difference between the $\Delta CoVaR$ proposed by Adrian and Brunnermeier (2011) and ours. Figure 3.4 presents the average $\Delta CoVaR$ across all the sample of financial institutions at a daily frequency. Thus, we are able to see that a strong difference appears depending on the model. Indeed, assuming that the market return is governed by a threshold model based on a funding liquidity indicator leads to large changes in terms of value. We will see whether these changes impact the ranking of financial institutions in the next subsection but we show

Figure 3.2 – $\Delta CoVaR$ estimated using OLS and Quantile regression



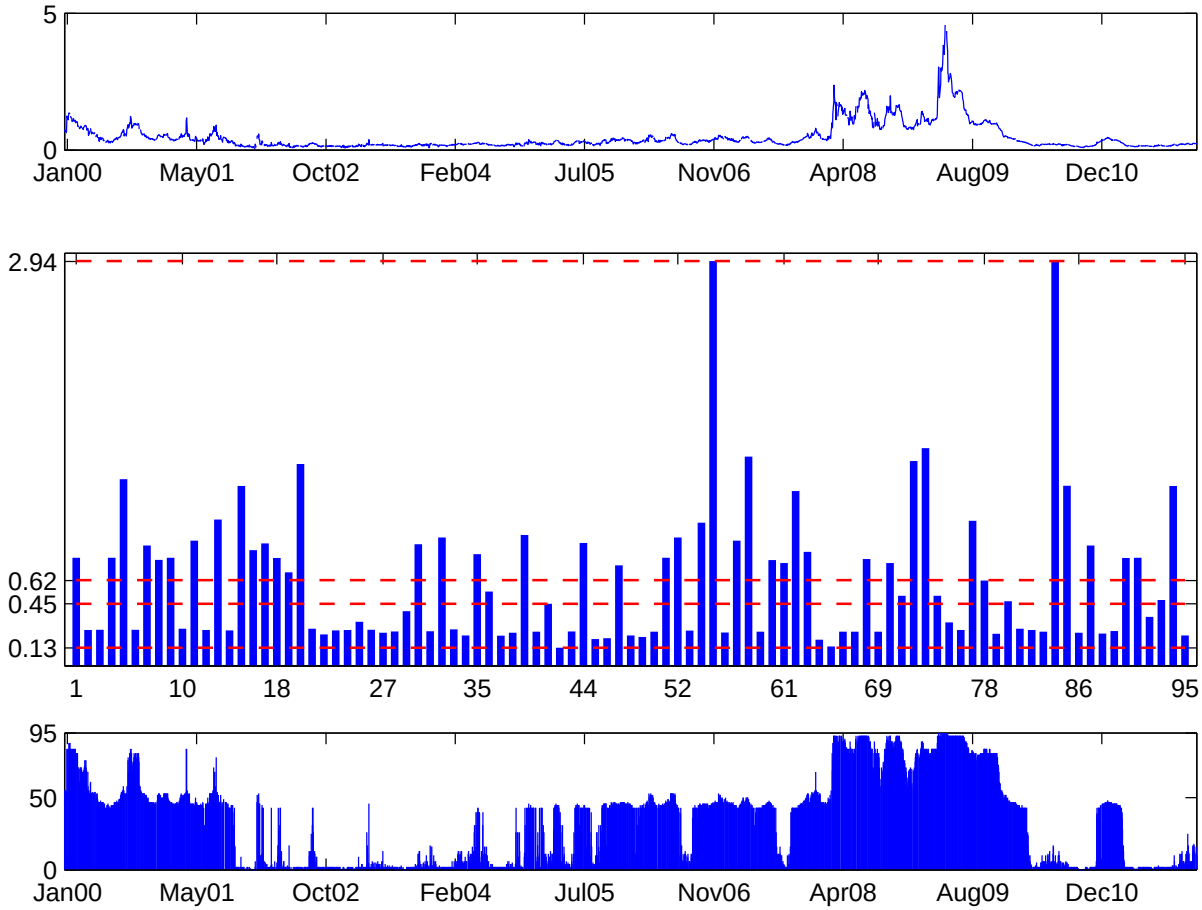
This figure presents *dcovar* estimated using OLS regression (L) and quantile regression (NL). The former assumes that the dependence structure is linear while the second take into account a possible nonlinear dependence between firms and market returns. They are computed based on daily data from January 2000 to December 2011.

an obvious countercyclical behavior of our new systemic risk measure. In other words, during calm periods the measure is higher and during crisis times, it is almost equal or even lower.

As we presented in the previous section, the $l-\Delta CoVaR$ implies the estimation of the γ^i parameter. As this latter is different for each financial firm, we present some summary statistics indicating the large difference of behavior according to the financial firm characteristics. Figure 3.3 graphically presents estimated values of γ^i from equation (14). We see the main part of financial firms has a funding liquidity threshold around 0.2

but some of them change their behavior when the TED spread is greater than 0.62. We also note that two firms are particularly insensitive to the funding liquidity since their γ threshold are very high, close to 2.9. Moreover, looking at the bottom graph of Figure 3.3, we see the number of firms that are constrained by funding liquidity at each date. We show this number is very high before the end of 2001 and after the end of 2008. There is a period of very low funding constrain between the end on 2001 with the end of internet bubble and the start of 2005 when we see a percentage of financial firms subject to funding constrains close to 50%.

Figure 3.3 – TED spread and γ^i thresholds



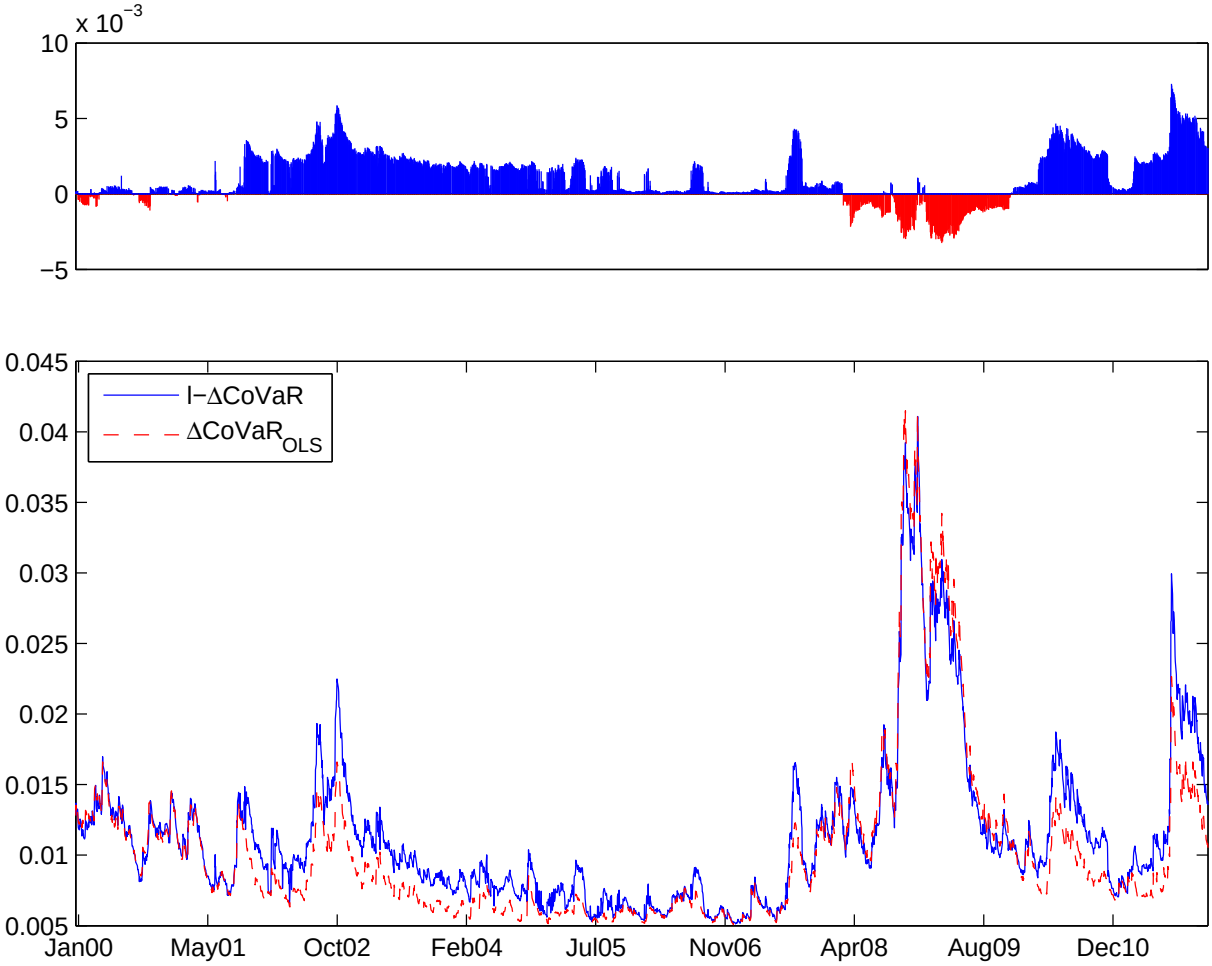
The top graph displays the TED spread from January 2000 to December 2011. The middle graph presents estimated threshold for each of the 95 financial institutions of our sample. The estimated value is $\hat{\gamma}^i$ obtained from equation (14). Minimum, median, mean and maximum values of the estimated threshold are displayed in dotted red line. Finally, the bottom graph presents the number of financial firms (among the 95) that are in a funding liquidity constraint state (TED spread greater than the threshold).

Consequently, Figure 3.4 also shows a large increase of the $l-\Delta CoVaR$ at the start of 2002 until July 2005. Then, the behaviors of $l-\Delta CoVaR$ and $\Delta CoVaR_{OLS}$ are really similar with a large difference in the summer 2007. At the end of 2008, the $l-\Delta CoVaR$ has a smaller value than the usual $\Delta CoVaR_{OLS}$. The first graph of Figure 3.4 allows to distinguish these periods and let appear that the systemic risk seems to be underestimated during tranquil periods. In blue, we have the positive difference while in red we have the negative one. We see that the red part is mainly focused in 2001 and 2008/09, corresponding to the internet bubble crisis and the subprime crisis, respectively. The impact of taking into account funding liquidity should lead to an additional premium. However, as we see on the previous figure, it appears during tranquil periods leading to a countercyclical behavior of the systemic risk measure.

Table 3.2 presents the average values of parameters obtained from both the standard model and the threshold model. As a result, we see the average value of β_1 is 0.28 considering the first model. However, the value of this parameter increases to 0.37. Thus, the parameter of the funding liquidity conditional variable is significantly negative and equal to -0.10. In other words, an increase of the return during a period for which the TED spread is high, leads to a significant decrease of the $\Delta CoVaR$ value. The countercyclical behavior is transcribed through this negative parameter.

These results let appear that the seminal measure is mainly driven by the univariate risk measures that largely increase during crisis periods. Indeed, the estimated β is based on a linear relationship between market return and financial firm return. As a result, it is overestimated in order to capture the large volatility during crisis times. Thus, when we add a component that distinguishes these periods, we obtain an estimated coefficient closer to the reality during calm periods.

Figure 3.4 – $\Delta CoVaR$ estimated using Threshold model and OLS regression



This figure presents $\Delta CoVaR$ estimated using a Threshold model based on funding liquidity and those using OLS regression as proposed in the seminal paper of Adrian and Brunnermeier (2011). The former assumes that the dependence structure is linear while the second take into account a possible nonlinear dependence between firms and market returns. They are computed based on daily data from January 2000 to December 2011.

3.4.2 Impact on the identification of SIFIs

In this subsection, we want to determine the impact of this modelling on the ranking of SIFIs. As we have seen, the systemic risk management is currently made using a bucketing approach. In other words, the ranking of the SIFIs is more relevant than the value of the systemic risk measure. Based on this conclusion, we compute for each dates of our sample, the ranking of the financial institutions based on the two systemic risk measures. In order to avoid the creation of a bias including some financial firms that are not present

Table 3.2 – Parameters of models

	Mean	t-statistic	Min	Median	Max	Percentage of reject
Panel A						
α	$8.27e^{-20}$**	3.903	$-5.18e^{-19}$	$6.83e^{-20}$	$7.41e^{-19}$	0.00%
β_1	0.280**	29.320	0.058	0.284	0.480	100.00%
Panel B						
α	$7.71e^{-06}$	0.784	$-3.69e^{-04}$	$1.57e^{-05}$	$3.55e^{-04}$	1.05%
β_1	0.367**	30.194	-0.029	0.371	0.636	96.84%
β_2	-0.103**	-6.095	-0.418	-0.151	0.432	98.95%
Number of obs.			95			

This table presents average value of the parameters, t-statistics, minimum, median and maximum values and the percentage of rejection for the hypothesis of the variable equal to 0. Panel A displays results for the standard $\Delta CoVaR$ and the panel B presents results for $l{-}\Delta CoVaR$. Results are based on the estimated parameters of the 95 firms available in our sample. t-statistics are robust (Newey and West (1987)) and computed for the mean value of estimated coefficients. ** corresponds to the value is significant at 5%.

over all the sample period, we focus only on the 60 firms being in our database from 2000 to 2011.

As the regulation is made using a bucketing approach, the Top 10 SIFIs appears to be the most important thing to assess. Indeed, only the most systemically important firms will be constrained to pay an additional amount of capital. As a consequence, better the identification of the Top 10 SIFIs is, greater is the chance to internalize the main part of externalities. Table 3.3 presents the Top10 SIFIs for 3 different dates and according to the $\Delta CoVaR_{OLS}$ and the $l{-}\Delta CoVaR$, respectively.

Table 3.3 shows some similarities across measures but we already find the counter-cyclical behavior of our measure through these results. Indeed, we see the number of concordant pairs[4] is larger at the 15/09/2008 than the two other dates that could be considered as tranquil periods. In other words, this results shows that the difference in average previously observed in figure 3.4 also appears in the Top 10 SIFIs for which we attach a greater importance.

Figure 3.5 presents the rankings obtained from $\Delta CoVaR_{OLS}$ and $l{-}\Delta CoVaR$ for Bank

[4]This is the number of firms that are simultaneously present in the two Top 10.

Table 3.3 – TOP 10 SIFIs

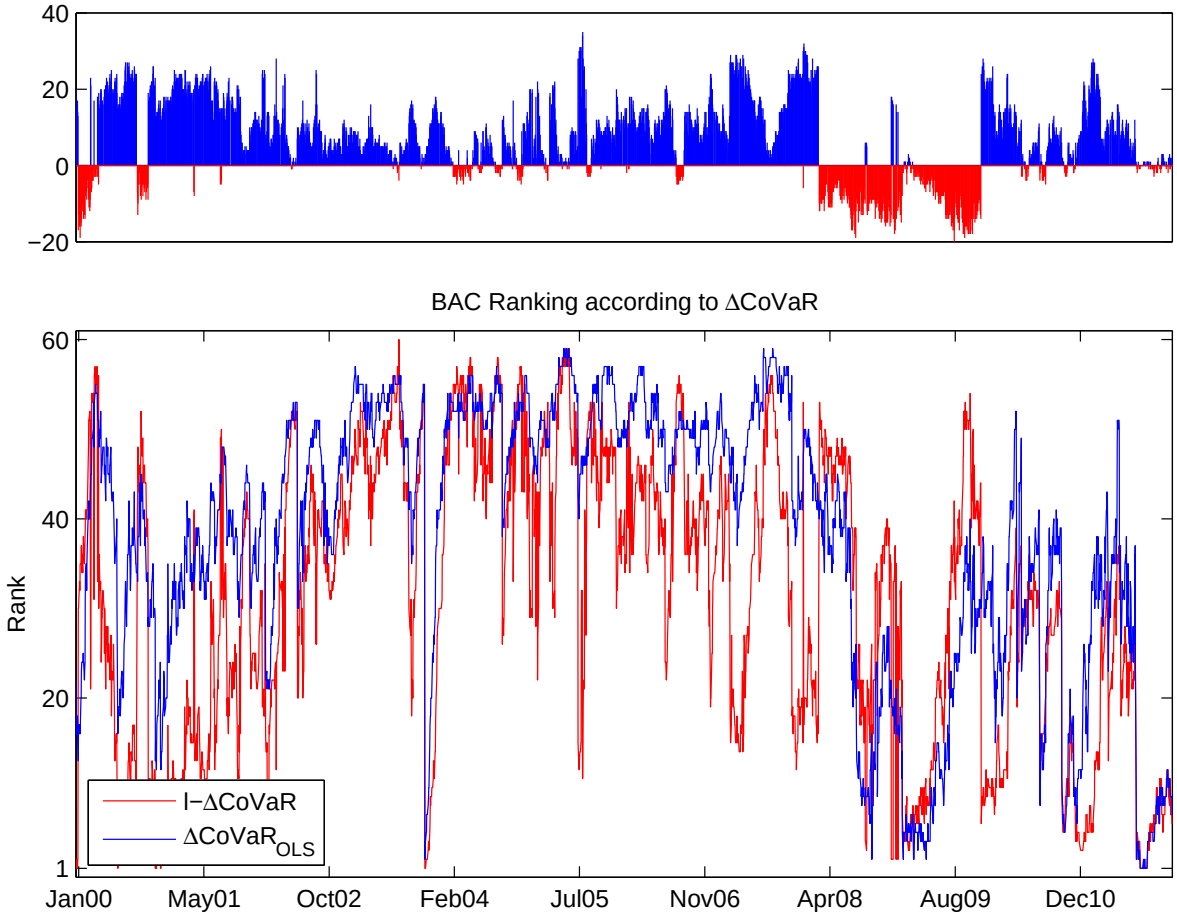
	15/09/2004		15/09/2008		15/09/2010	
Rank	$\Delta CoVaR_{OLS}$	$l-\Delta CoVaR$	$\Delta CoVaR_{OLS}$	$l-\Delta CoVaR$	$\Delta CoVaR_{OLS}$	$l-\Delta CoVaR$
1	PBCT	LM	LEH	MER	BBT	BBT
2	TRV	PGR	MER	FNM	EV	WFC
3	JNS	BEN	FNM	LEH	MTB	KEY
4	CME	ALL	AIG	NYX	BEN	ZION
5	BEN	JNS	FRE	FRE	AXP	LNC
6	SCHW	EV	BBT	NYB	TROW	RF
7	AMTD	STT	CMA	BLK	SNV	SNV
8	EV	TRV	NYX	JNS	HRB	STI
9	TROW	MTB	LM	CME	WU	STT
10	BLK	TROW	JNS	BBT	NYX	MTB
# of pairs	5		7		3	

Columns labeled $\Delta CoVaR_{OLS}$ display the Top 10 SIFIs according to 3 dates: 15/09/2004, 15/09/2008, 15/09/2010. The other columns, labeled $l-\Delta CoVaR$, display the same results based on the new $\Delta CoVaR$ approach with a threshold model. The last line shows the number of firms that are both in the Top 10 of $\Delta CoVaR_{OLS}$ and those of $l-\Delta CoVaR$.

of America. We see with the first part of the graph the difference between the two ranks at each date. Obviously, this difference is time-varying but, it is important to note that this difference is negative at approximately the same periods than the TED spread is over the threshold (corresponding to 2001 and 2008 financial crisis). In other words, Bank of America appears to be ranked too systemic during turmoils and that could lead to an overestimation of the capital requirement for this bank. Indeed, a rank equal to 60 means that the financial firms is considered as the less systemic firm of the system. As a consequence, we can see on the second graph of the figure that, based on the $\Delta CoVaR$, Bank of America is usually ranked less systemic than it should be, except during the last financial crisis.

Moreover, Figure 3.6 shows that the difference of rankings is very large. It presents the same results that the first part of figure 3.5 but we distinguish when this difference of rank is smaller than 3, 2 and 1. Indeed, we see in red, all the dates for which the difference between the rank obtained using one measure and the rank obtained using the other is greater than 3. We can consider that a difference smaller than 3 is not really relevant. But in this case, only few dates experience such a small difference of rankings. Adding

Figure 3.5 – $\Delta CoVaR$ estimated using Threshold model and OLS regression



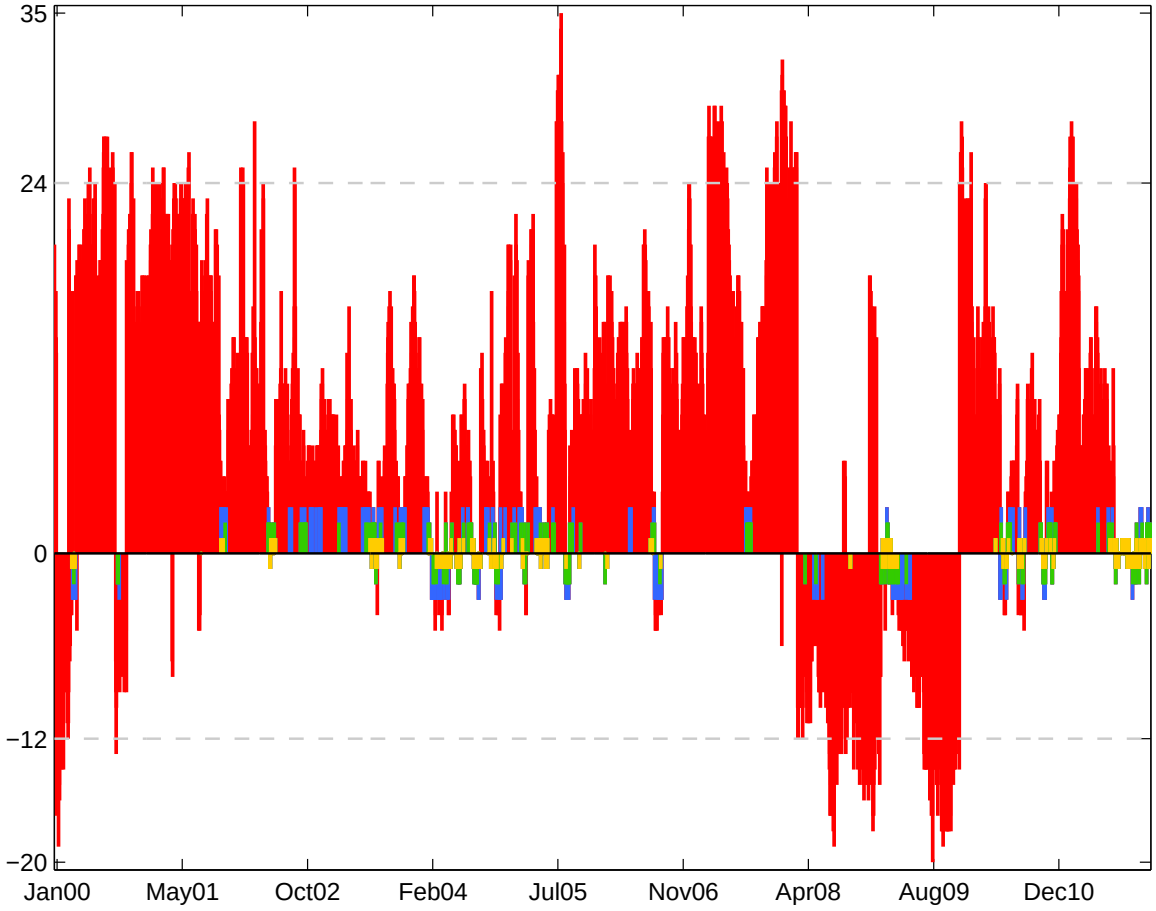
The top panel represents the difference between rankings from $\Delta CoVaR_{OLS}$ and $l-\Delta CoVaR$, respectively and the bottom panel time plots the daily ranking of Bank of America (BAC) according to $l-\Delta CoVaR$ (red line) and $\Delta CoVaR_{OLS}$ (blue line). The estimation period is from 01/03/2000 to 12/30/2011.

a liquidity component in the estimation of systemic risk appears to strongly change the results and economic conclusions about the financial firms.

3.4.3 Discussion

All these results are telling us that the modeling of financial market returns may lead to a strong variation in terms of ranking the SIFIs. Indeed, the changes of ranking show that the two measures largely differ, both in terms of value, but more importantly, in terms of ranking. The information that each measures propose is different proving the impact of

Figure 3.6 – $\Delta CoVaR$ estimated using Threshold model and OLS regression



This figure presents the difference of ranking according two $\Delta CoVaR$. The first is $\Delta CoVaR$ estimated using a Threshold model based on funding liquidity and the second is those using simple OLS regression and only taking into account for financial firm returns [as proposed in the seminal paper of Adrian and Brunnermeier (2011)]. The former assumes that the systemic risk also depends on the funding liquidity constraints of the financial firm. They are computed based on daily data from January 2000 to December 2011.

funding liquidity on the systemic risk measurement framework.

As we present in the previous section, the systemic risk of some financial firms appears to be drastically under evaluated during tranquil periods while it is over estimated during crisis times. But, considering the case of Bank of America, this result is particularly interesting in order to determine the amount of capital requirement. Indeed, if we follow this methodology, during tranquil times, the regulator should ask much more capital in order to satisfy future risk constraints. However, few times before the 2008 crisis,

we see that the rank of Bank of America according to the $l-\Delta CoVaR$ is higher than those obtained using the $\Delta CoVaR$. In other words, following our new measure, at this time, the capital requirements will be smaller than according to the $\Delta CoVaR$ proposed by Adrian and Brunnermeier (2011).

For regulators, this measure has for main interest to propose a countercyclical approach. Indeed, it behaves in the opposite direction to the individual risk firstly, but also to the systemic risk computed using standard $\Delta CoVaR$. Indeed, in the case of the standard $\Delta CoVaR$, the parameters appears to be mostly driven by the highly volatile returns during crisis periods. In a sense, the coefficient is overestimated and adding a dummy variable, strongly correlated with crisis periods, allows a better estimation of the β during calm periods. As a consequence, we show in this paper that the systemic risk also needs attention during tranquil periods since it could allow a better preparation for the next crisis event.

3.5 Conclusion

We propose in this paper a new modeling of the market returns depending on the financial firm returns and a funding liquidity component. This threshold estimation allows to consider the funding liquidity as a factor of systemic risk that is not usual in the standard framework of the $\Delta CoVaR$.

We choose to only focus on this measure in order to stay in a framework allowing the computation of the systemic risk measure only based on publicly available data. Moreover, the large literature about the $\Delta CoVaR$ enables to simplify the estimation method. As a consequence, we are able to add a funding liquidity component and we estimate a threshold model based on this factor.

In regards to the results, the new systemic risk measure adopts a countercyclical

behavior that is coherent with the liquidity needs of financial institutions, especially during turmoils.

C.1 $\Delta CoVaR$ estimation

The seminal paper of (Adrian and Brunnermeier, 2011) presents a standard approach to measure $\Delta CoVaR$. Thus, we have in this framework:

$$r_t^m = \delta^i + \beta^i r_t^i + \epsilon_t^i, \tag{8}$$

where r_t^m and r_t^i are respectively the time series of market returns and financial firm i returns. The set of parameters, $\{\delta^i, \beta^i\}$ is estimated using quantile regressions.

Thus, we have the conditional return of the financial market that is equal to:

$$\hat{r}_t^m(q) = \hat{\delta}^i(q) + \hat{\beta}^i(q) r_t^i \tag{9}$$

where q is the qth-quantile used to estimate parameters.

However, considering the definition of the value-at-risk, we know that:

$$VaR_t^m(q) | r_t^i = \hat{r}_t^m(q) \tag{10}$$

As a result, assuming that the conditioning event is $r_t^i = VaR_t^i$, we define the $CoVaR_t^i(\alpha)$ as:

$$CoVaR_t^i(\alpha) := VaR_t^m(q) | VaR_t^i = \hat{\delta}^i + \hat{\beta}^i VaR_t^{r^i}(\alpha), \tag{11}$$

where

$$VaR_t^{r_i}(\alpha) = \sigma_t^{r^i} F^{-1}(\alpha). \tag{12}$$

We assume that $r_t^i \sim F$, a location scale distribution and the estimation of σ_t^i is made using a TGARCH model [Zakoian (1994)] allowing to take into account asymmetric effects in the return's volatility.

Then, the $\Delta CoVaR_t^i(\alpha)$ can be expressed as:

$$\begin{aligned}
\Delta CoVaR_t^i(\alpha) &= CoVaR_t^i(\alpha) - CoVaR_t^i(0.5), \\
\Delta CoVaR_t^i(\alpha) &= \hat{\delta}^i + \hat{\beta}^i VaR_t^{r^i}(\alpha) - \hat{\delta}^i - \hat{\beta}^i VaR_t^{r^i}(0.5), \\
\Delta CoVaR_t^i(\alpha) &= \hat{\beta}^i \left(VaR_t^{r^i}(\alpha) - VaR_t^{r^i}(0.5) \right).
\end{aligned} \tag{13}$$

After having presented the general case already exposed in many papers, we introduce a regime component based on liquidity in the modeling of the returns that lead to express the return of the financial system. Thus, following the same way, we have:

$$r_t^m = \delta^i + \beta^i r_t^i + \Phi^i r_t^i \mathbb{1}_{\{y_{t-1} > \gamma^i\}} + \epsilon_t^i, \tag{14}$$

where y_{t-1} is the liquidity indicator chosen and γ^i is the threshold of the liquidity indicator over which we consider that the return of the market is modeled following a different dynamic.

As a result, we obtain the return of financial market conditional to the return of the financial firm expressed as :

$$r_t^m | r_t^i = \hat{\delta}^i + \left(\hat{\beta}^i + \hat{\Phi}^i \mathbb{1}_{\{y_{t-1} > \hat{\gamma}^i\}}\right) r_t^i. \tag{15}$$

As a consequence, we obtain from equation (11) that the liquidity-adjusted *CoVaR* or $l-CoVaR$ is equal to:

$$l-CoVaR_t^i(\alpha) = \hat{\delta}^i + \left(\hat{\beta}^i + \hat{\Phi}^i \mathbb{1}_{\{y_{t-1} > \hat{\gamma}^i\}}\right) VaR_t^i(\alpha). \tag{16}$$

As a result, we are able to define the liquidity-adjusted $\Delta CoVaR$ as :

$$l-\Delta CoVaR_t^i(\alpha) = \left(\hat{\beta}^i + \hat{\Phi}^i \mathbb{1}_{\{y_{t-1} > \hat{\gamma}^i\}}\right) \left(VaR_t^{r^i}(\alpha) - VaR_t^{r^i}(0.5)\right). \tag{17}$$

Concerning equation (17), the estimation of $\{\beta^i, \Phi^i, \gamma^i\}$ is made using Ordinary Least Square, i.e. assuming that the dependence structure between r_t^m and r_t^i is linear. Moreover, standard deviations are computed using a TGARCH model taking into account the asymmetric effect on financial returns.

C.2 Threshold Estimation

Following the estimation method proposed by Hansen (2000), we define a threshold model as:

$$\begin{cases} y_i = \theta_1 x_i + \epsilon_i & \text{if } q_i \leq \gamma \\ y_i = \theta_2 x_i + \epsilon_i & \text{if } q_i > \gamma \end{cases} \tag{18}$$

where q_i is the threshold variable used to split the sample into two parts. This variable may also be an element of x_i.

This model could be expressed as a single equation model. We have to define a dummy variable $d_i(\gamma)$ such that:

$$\begin{cases} d_i(\gamma) = 1 & \text{if } q_i \leq \gamma \\ d_i(\gamma) = 0 & \text{if } q_i > \gamma \end{cases} \tag{19}$$

Then, we set $x_i(\gamma) = x_i d_i(\gamma)$ and thus, rewrite the model as:

$$y_i = \theta_i x_i + \delta_i x_i(\gamma) + \epsilon_i \tag{20}$$

In a matrix notation we have:

$$Y = X\theta + X_\gamma \delta + \epsilon \tag{21}$$

Then, we define $S_n(\theta, \delta, \gamma)$ the squared errors function:

$$S_n(\theta, \delta, \gamma) = (Y - X\theta - X_\gamma \delta)'(Y - X\theta - X_\gamma \delta) \tag{22}$$

The definition of the OLS estimator implies that $\hat{\theta}$,$\hat{\delta}$ and $\hat{\gamma}$ jointly minimize (22). Define Γ as the set of possible values for γ as: $\Gamma = [\underline{\gamma}, \overline{\gamma}]$.

In order to estimate this model, the easiest way to compute it, is through concentration. So, conditional on γ, $\hat{\theta}(\gamma)$ and $\hat{\delta}(\gamma)$ are obtained by regression of Y on $X_\gamma^* = [X\ X_\gamma]$. Thus, the concentrated sum of squared errors function is defined as:

$$S_n(\gamma) = S_n(\hat{\theta}(\gamma), \hat{\delta}(\gamma), \gamma) = Y'Y - Y' X_\gamma^* (X_\gamma^{*\prime} X_\gamma^*)^{-1} X_\gamma^{*\prime} Y. \tag{23}$$

As a consequence, $\hat{\gamma}$ is the value minimizing $S_n(\gamma)$.

C.3 Dataset

Depositories (29)		Insurance (32)	
BAC	Bank of America Corp.	ABK	Ambac Financial Group
BBT	BB&T Corp.	AET	Aetna
BK	Bank of New York Mellon Corp.	AFL	AFLAC Inc.
C	Citigroup Inc.	AIG	American International Group Inc.
CBH	Commerce Bancorp	AIZ	Assurant
CMA	Comerica Inc.	ALL	Allstate Corp.
HBAN	Huntington Bancshares Inc.	AOC	Aon Corp.
HCBK	Hudson City Bankshares Inc.	BKLY	W.R. Berkley Corp.
JPM	JP Morgan Chase & Co.	BRK	Berkshire Hathaway Inc.
KEY	Keycorp New	CB	Chubb Corp.
MI	Marshall & Ilsley Corp.	CFC	Countrywide Financial
MTB	M&T Bank Corp.	CI	CIGNA Corp.
NCC	National City Corp.	CINF	Cincinnati financial Corp.
NTRS	Northern trust Corp.	CNA	CNA Financial Corp.
NYB	New York Community Bancorp Inc.	CVH	Coventry health Care Inc.
PBCT	Peoples United Financial Inc.	FNF	Fidelity National Financial
PNC	PNC Financial Services Grp Inc.	GNW	Genworth Financial
RF	Regions Financial Corp.	HIG	Hartford financial Svcs Grp Inc.
SNV	Synovus Financial Corp.	HNT	Health Net Inc.
SOV	Sovereign Bancorp	HUM	Humana Inc.
STI	Suntrust Banks Inc.	LNC	Lincoln National Corp.
STT	State Street Corp.	MBI	MBIA Inc.
UB	Unionbancal Corp.	MET	MetLife
USB	US Bancorp	MMC	Marsh & McLennan Cos Inc.
WB	Wachovia	PFG	Principal Financial Group
WFC	Wells Fargo & Co	PGR	Progressive Corp.
WM	Washington Mutual	PRU	Prudential Financial
WU	Western Union	SAF	Safeco
ZION	Zions Bancorp	TMK	Torchmark Corp.
		TRV	Travelers companies Inc.
		UNH	United Health Group Inc.
		UNM	Unum Group
Broker-Dealers (10)		Others (23)	
AGE	A.G. Edwards	ACAS	American Capital Ltd
BSC	Bear Stearns	AMP	Ameriprise Financial
ETFC	E*Trade Financial Corp.	AMTD	TD Ameritrade Holding Corp.
GS	Goldman Sachs group Inc.	AXP	American Express Co.
LEH	Lehman Brothers	BEN	Franklin Resources Inc.
MER	Merill Lynch	BLK	BlackRock Inc.
MS	Morgan Stanley Dean Witter & Co	BOT	CBOT Holdings
NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
SCHW	Schwab Charles Corp.	CBSS	Compass Bancshares
TROW	T. Rowe Price Group Inc.	CIT	CIT Group
		CME	CME Group
		COF	Capital One Financial Corp.
		EV	Eaton Vance Corp.
		FITB	Fifth Third Bancorp
		FNM	Fannie Mae
		FRE	Freddie Mac
		HRB	H&R Block
		ICE	Intercontinental Exchange
		JNS	Janus Capital
		LM	Legg Mason Inc.
		NYX	NYSE Euronext
		SEIC	SEI Investment Company
		SLM	SLM Corp.

Table 4 – Tickers and Company Names by Industry Groups

Chapter 4

Identifying SIFIs: Toward a Simpler Approach

Ce chapitre est issu d'un travail commun avec Sylvain Benoit et Manizha Sharifova.

Le risque systémique a pour but de déterminer l'impact d'un choc sur l'ensemble du système. Mais par définition, un évènement systémique n'est autre que l'arrêt du système suite à un choc. Néanmoins, nous n'avons jamais connu de tel évènement, ce qui le rend impossible à modéliser de la sorte. Dès lors, pour permettre de le mesurer, la littérature définit le risque systémique comme l'impact qu'une institution financière peut avoir sur l'ensemble du système lorsqu'elle même rencontre d'importantes difficultés.

Par conséquent, nous avons vu émerger de la littérature trois mesures du risque systémique ayant pour principal avantage de n'être basées que sur des données disponibles publiquement. La première est la $\Delta CoVaR$ qui mesure la contribution d'une firme sur le rendement du marché, comparant l'impact de la firme lorsqu'elle rencontre des problèmes à son impact lorsqu'elle est dans un état normal. La seconde mesure est le *Marginal Expected Shortfall* ou *MES*. Contrairement à la $\Delta CoVaR$, il estime l'impact d'un choc du marché sur le rendement de la firme. Ainsi, le *MES* n'est autre que la perte moyenne

enregistrée par la firme lorsque le marché est en crise. Pour finir, la troisième mesure est la *SRISK*. Elle permet de déterminer le montant de capital que la firme doit posséder pour traverser une nouvelle crise financière tout en conservant un fonctionnement normal. La *SRISK* a l'avantage de tenir compte des données de bilan de la firme, telles que le levier, les dettes et ne se concentre pas uniquement sur les rendements.

La relation entre les rendements de la firme et du marché est au coeur de l'estimation de ces mesures. Or, cette relation est non linéaire dès lors qu'une faible chute du rendement du marché n'a pas le même impact sur le rendement de la firme qu'une forte chute, et inversement. Sachant cela, la littérature s'efforce de capturer la non linéarité de cette structure de dépendance utilisant des méthodes de plus en plus complexes. Nous avons voulu, dans ce chapitre, voir l'impact sur la régulation du risque systémique de l'utilisation de la méthode d'estimation la plus simple possible. Pour cela, nous faisons l'hypothèse que la structure de dépendance est linéaire.

Bien que les chocs sur les rendements de la firme et du marché soient des évènements extrêmes, nos résultats montrent que l'impact de l'hypothèse de linéarité de la structure de dépendance est négligeable dans le cadre de la régulation du risque systémique. Cette dernière ne tient en effet pas compte de la valeur obtenue par la mesure mais se concentre sur le classement des firmes. Par la suite, le régulateur classe par groupes les institutions financières selon leur position. De ce fait, la valeur en elle même n'a que peu d'importance. Il faut s'intéresser à la valeur relative. Ce chapitre indique que le classement n'est que très peu modifié lorsqu'on fait l'hypothèse que la dépendance est linéaire. En effet, bien que les valeurs des mesures soient modifiées, le classement des firmes est très similaires et nous retrouvons un nombre de paires concordantes (classement identique pour les deux méthodes) particulièrement élevé.

Par conséquent, ce chapitre tend à montrer que la modélisation de la non linéarité de la structure de dépendance n'est pas aussi importante que la littérature le laisse paraître et ceci, particulièrement dans le cadre de la régulation actuelle.

4.1 Introduction

The 2007-2009 global financial crisis has made policymakers and regulators reconsider the institutional framework for overseeing the stability of financial systems. The crisis has clearly demonstrated that even though individual risks may be forecast and limited, financial shocks to a single firm can quickly spread across a large number of institutions and markets, threatening the system as a whole. Therefore, the focus of the reform agenda has now shifted to a macro-prudential approach in assuring the soundness of the financial system with a greater focus on individual institutions that are systemically important.

Consequently, a large body of literature has proposed various measures which would allow regulators to identify systemically important financial institutions (SIFIs) and allocate macro-prudential capital requirements in order to reduce their risk.[1] Among widely-cited measures of tail dependence in financial institutions' equity returns are the Delta Conditional Value-at-Risk ($\Delta CoVaR$) of Adrian and Brunnermeier (2011), the Marginal Expected Shortfall (MES) of Acharya et al. (2010) and the $SRISK$ of Brownlees and Engle (2012).[2] $\Delta CoVaR$ focuses on market losses conditional on particular institution being in distress whereas MES and $SRISK$ define the systemic risk contribution of an institution as the expected losses of this institution given a negative market shock. These three measures aim to evaluate the contribution of an institution to system-wide risk and have been widely discussed in terms of their ability to predict systemic risk ranking of financial institutions.

The key step in the estimation of the $\Delta CoVaR$ and MES is to model the joint distribution of individual firm's and market returns taking into account nonlinear dependence between returns. Indeed, markets may be more dependent during extreme downward movements then when they are moving upwards.[3] To account for this property of stock

[1] See Bisias et al. (2012) for a survey of systemic risk measures.

[2] In Adrian and Brunnermeier (2011) $\Delta CoVaR$ is estimated using asset returns.

[3] This is related to the notion of a financial contagion discussed, for example, in King and Wadhwani (1990), Rigobon (2001), Forbes and Rigobon (2002) and, Bekaert and Harvey (2003).

returns, papers that build on the $\Delta CoVaR$ and MES propose various estimation methods aimed to better capture possible nonlinear dependence structure of returns. In other words, they try to model the relationship between firm's and market returns during extreme events as accurately as possible in order to obtain a precise measure of the firm's systemic risk contribution. The approach may involve complicated estimation procedures.[4] Yet, the key questions is whether these attempts are justified given the objectives of the current macro-prudential regulation.

The banking regulation, so far, has focused on individual risk measure, like Value-at-Risk (VaR), as a way to determine the minimum capital a financial institution is required to put aside to cover the self-imposed risk. In this regard, it might be important that a financial firm estimates an accurate risk measure utilizing its internal risk model. In contrast, the recent improvements in the Basel III accord envision that capital surcharges be imposed on institutions that are identified as systemically risky according to their systemic relevance [Basel Committee (2011)].[5] More specifically, the percentage of additional capital that a firm is required to hold is determined by the systemic risk ranking of this institution and is not directly linked to the absolute value of its systemic risk contribution. As such the sufficient requirement for a systemic risk measure should be its ability to accurately identify and rank SIFIs.

This paper investigates the impact of nonlinear and linear methods of estimating the $\Delta CoVaR$, the MES and the $SRISK$ on the identification of SIFIs. First, we use the quantile regression and nonparametric tail estimator to capture nonlinear dependence

[4] Adrian and Brunnermeier (2011) model the tail dependence using the quantile regression, Brownlees and Engle (2012) apply the nonparametric tail estimator of Scaillet (2005) and Engle et al. (2012) use *Student t* copula. Jiang (2012) also suggest various copulas function to estimate the $\Delta CoVaR$, the MES and the $SRISK$ whereas Straetmans and Chaudhry (2013) as well as Balla et al. (2012) use extreme value theory for assessing systemic risk.

[5] Financial institutions are assessed based on the indicator-based measurement approach, which considers the individual factors such as the size of institutions, their interconnectedness, the lack of readily available substitutes or bank infrastructure, the global activity and the complexity. Using this methodology the total score for each institution is calculated as a simple average of its five category scores. Next, institutions whose overall score exceeds a cutoff level set by the Basel Committee are allocated into different equally-sized buckets according to their score rankings. The amount of additional capital requirement is then determined for each bucket [FSB (2011), FSB (2012)].

of returns in the calculation of these measures. Second, we model the dependence in a linear fashion by assuming that dependence is fully captured by the correlation coefficient which allows us to simplify the estimations. Our results show that estimations accounting for nonlinear dependence structure in return series do not improve in terms of identifying SIFIs compared to those that model the dependence structure linearly. The linear estimation methods of the market-based systemic risk measures are sufficient for the ranking of financial firms and identification of SIFIs. Their advantage is the ease of computation and lower estimation errors. Our results support a growing discussion about the simplicity in the systemic risk regulation and estimation. For example, Haldane (2011) highlights the three key principles of a good regulation: (i) simplicity, (ii) robustness and (iii) timeliness. Drehmann and Tarashev (2011), Drehmann (2013) and Rodríguez-Moreno and Pena (2013) argue that the regulation should focus on simple indicator(s) of monitoring systemic risk. Finally, our findings are also in line with Patro et al. (2013) suggesting that daily stock returns correlation is a simple and a sufficiently informative indicator for assessing systemic importance of institutions and monitoring systemic risk.

The remainder of the paper is structured as follows. Section 2 introduces the $\Delta CoVaR$, *MES* and *SRISK* measures and their nonlinear and linear estimation methods. Section 3 describes the data used in this paper and presents estimation results. Section 4 provides comparative analysis of the rankings of financial institutions obtained using the two estimation methods at different levels of risk. Section 5 concludes.

4.2 Estimation of Systemic Risk Measures

In this section we outline the framework and estimation methods of $\Delta CoVaR$ and *MES* that account for nonlinear dependence of returns introduced in the seminal papers of Adrian and Brunnermeier (2011) and Brownlees and Engle (2012). Based on this frame-

work we then discuss the linear estimation approaches to the computation of these two measures by assuming that dependence is fully captured by correlations.

4.2.1 Definitions

ΔCoVaR

The *CoVaR* is defined as the *VaR* of the financial system conditional on particular institution i being in financial distress. Given a distress event that the return of institution i is at its α percent *VaR* level, *CoVaR* is defined as:

$$Pr\Big(r_{mt} \leq CoVaR_{it}(q,\alpha)|r_{it} = VaR_{it}(\alpha)\Big) = q, \tag{4.1}$$

where r_{mt} denotes market return, r_{it} is the return of firm i and q is the conditional probability of market financial distress when firm i is under stress.

The contribution of firm i to system-wide risk, denoted by $\Delta CoVaR_{it}(q,\alpha)$, is then defined as the difference between *VaR* of the system given that institution i is in distress and *VaR* of the system given normal state of institution i:

$$\Delta CoVaR_{it}(q,\alpha) = CoVaR_{it}(q,\alpha) - CoVaR_{it}(q,0.5). \tag{4.2}$$

Hence, the $\Delta CoVaR$ measures additional risk that an individual institution imposes on the whole system. Adrian and Brunnermeier (2011) emphasize that a regulation based only on the risk of institutions in isolation can lead to an excessive risk-taking along systemic risk dimensions. We can think of two financial firms that have the same *VaR*s but different $\Delta CoVaR$s, and therefore, different level of contribution to the risk of the system. According to the Basel II regulation both firms would be subject to the same capital requirements based on their *VaR*s. However, capital surcharges should be higher

for firms that are systemically risky as measured by their $\Delta CoVaR$. Using this approach would force firms reduce activities that impose additional risk on the system.

MES

The *MES* is defined as the expected equity loss of an institution conditional on the market return falling below some threshold value. Setting the threshold at the *VaR* of the market at τ percent, we can express the *MES* of financial firm i at time t as:

$$MES_{it}(\tau) = E_{t-1}\Big(r_{it}|r_{mt} < VaR_{mt}(\tau)\Big). \tag{4.3}$$

In contrast to the *CoVaR*, which captures market losses when a particular financial firm experiences turmoil, the *MES* focuses on the institution's loss when market as a whole is in distress. *MES* can also be interpreted as a measure of the firm's sensitivity to a financial shock. More specifically, *MES* shows the sensitivity of a firm to the exceptionally bad returns of the financial system that it belongs to, which may not be necessarily attributed to a systemic event.

SRISK

The *SRISK* is defined as the expected capital shortfall of a given financial institution conditional on a shock to the financial system and can be expressed as:

$$SRISK_{it} = max[0; kD_{it} - (1-k)W_{it}(1 - LRMES_{it})] \tag{4.4}$$

where $0 < k < 1$ is the prudential capital ratio, D_{it} is the quarterly book value of the bank's total liabilities, and W_{it} is the bank's daily market capitalization or market value of its equity.

Financial institutions with the largest *SRISK* are considered the greatest contributors to the crisis and, hence, are the most systemically risky. Note that the *SRISK*, which is positive by convention, is an increasing function of the liabilities and a decreasing function of the market capitalization. So, *SRISK* can be viewed as an increasing function of the quasi-leverage (leverage thereafter) defined as the ratio of the book value of total liabilities to the market value of equity. The *SRISK* also considers a firm's interconnection with the rest of the system through the long-run marginal expected shortfall, denoted *LRMES*. *LRMES* corresponds to the expected drop in the equity value of a firm should the market fall by more than a given threshold within the next six months. Acharya et al. (2012) propose to approximate the *LRMES* as $1 - exp(18 \times MES)$ where *MES* is the daily loss as defined in Equation (4.3).

4.2.2 Nonlinear Estimation

Estimations of the systemic risk measures involve modeling the joint distribution of asset returns. The most common measure for dependency, correlation, can be efficiently used to model the dependence structure of returns when the distribution follows the strict assumptions of normality and constant dependency across quantiles. Existing empirical evidence suggests that asset prices exhibit skewed and heavy tail marginal distributions. Extreme co-movements also occur in multivariate distributions given by asymmetric dependence, which suggests that assets follow different levels of correlation during extreme downward market movements than during upward movements. Conclusions made by simply looking at linear correlation can be misleading for distributions that are not normally distributed due to outliers or strong nonlinear relationship. With these considerations, $\Delta CoVaR$ and *MES* are usually estimated accounting for possible nonlinear dependence between financial returns.

Adrian and Brunnermeier (2011) propose to estimate *CoVaR* via quantile regression (Koenker and Bassett Jr (1978)). The quantile regression models the nonlinear relationship between institution's and market returns for different quantiles of the return distribution. Using the quantile regression method $\Delta CoVaR$ is computed as follows:

$$\Delta CoVaR_{it}(q,\alpha) = \gamma_i^q \Big(VaR_{it}(\alpha) - VaR_{it}(0.5) \Big), \tag{4.5}$$

where γ_i^q is the estimated slope coefficient from the quantile regression of market return, r_{mt}, on firm's return, r_{it}, at the qth quantile and is constant over time. Equation (4.5) gives a dynamic $\Delta CoVaR$ because the estimated volatilities for each firm i, and therefore individual *VaR*s, are time-varying.[6] Appendix A describes the calculation of $\Delta CoVaR$ with the quantile regression.

The *MES* can be estimated using the linear market model of firm and market returns as in Brownlees and Engle (2012):

$$\begin{aligned} MES_{it}(\tau) &= \sigma_{it}\ \rho_{it}\ E_{t-1}\Big(\varepsilon_{mt} | \varepsilon_{mt} < VaR_{mt}(\tau)/\sigma_{mt}\Big) \\ &\quad + \sigma_{it}\ \sqrt{(1-\rho_{it}^2)}\ E_{t-1}\Big(\xi_{it} | \varepsilon_{mt} < VaR_{mt}(\tau)/\sigma_{mt}\Big), \end{aligned} \tag{4.6}$$

where ρ_{it} is the correlation between r_{mt} and r_{it}, σ_{mt} and σ_{it} are the volatilities of the market and the firm, respectively, and $(\varepsilon_{mt}, \xi_{it})$ are disturbances that follow an *i.i.d.* process with zero mean and identity covariance matrix and are not independent of each other at time t.[7]

The *MES* given by Equation (4.6) is a function of the tail expectation of the standardized market residual and the tail expectation of the standardized idiosyncratic firm residual and is dynamic given that estimated correlations, ρ_{it}, and return volatilities, σ_{it}

[6] The conditional volatilities are estimated by the GJR-GARCH model of Glosten et al. (1993) that takes into account asymmetric effects on the returns.

[7] This last assumption of dependence between the innovations, ε_{mt} and ξ_{it}, is valid given that extreme values of these distributions can happen simultaneously for systemically risky firms.

and σ_{mt} vary over time. Notice that any possible nonlinear dependence between market and firm returns is captured by the second term of Equation (4.6). Brownlees and Engle (2012) apply a nonparametric kernel estimator as in Scaillet (2003) and Scaillet (2005) to estimate the tail expectations. The step-by-step estimation procedure for *MES* is provided in Appendix B.

The *SRISK* is obtained according to Equation (4.4), details for *SRISK* are provided in Appendix C. As a result, any possible nonlinear dependence in returns is accounted for in the computation of nonlinear *MES* as given by Equation (4.6).

4.2.3 Linear Estimation

The linear version of $\Delta CoVaR$ can be obtained via the standard ordinary least squares (OLS) regression. Given its focus on mean response of the dependent variable OLS does not reflect the extreme quantile relationship between equity returns.

Using the OLS method we can express $\Delta CoVaR$ as follows:

$$\Delta CoVaR_{it}(\alpha) = \gamma_i \Big(VaR_{it}(\alpha) - VaR_{it}(0.5) \Big), \tag{4.7}$$

where γ_i is the estimated slope coefficient from the simple OLS regression of the market return, r_{mt} on firm i's return, r_{it}.

The linear *MES* is calculated using Equation (4.6), which when assuming that the dependence between firm and market returns is fully captured by the time-varying correlation, reduces to:

$$\begin{aligned} MES_{it}(\tau) &= \sigma_{it}\ \rho_{it}\ E_{t-1}\Big(\varepsilon_{mt} | \varepsilon_{mt} < VaR_{mt}(\tau)/\sigma_{mt} \Big) \\ &= \beta_{it}\ E_{t-1}\Big(r_{mt} | r_{mt} < VaR_{mt}(\tau) \Big), \end{aligned} \tag{4.8}$$

where $\beta_{it} = \rho_{it} \frac{\sigma_{it}}{\sigma_{mt}}$ is the conditional *beta* of firm i.[8] As seen from Equation (4.8), the linear *MES* is directly linked to the expected shortfall of the market.

As mentioned above, since the dependence structure of returns impacts the *SRISK* through the daily *MES* only, the linear counterpart of *SRISK* is calculated by using *MES*, as given by Equation (4.8), in the definition of *SRISK*.

4.3 Data and Estimation Results

Our sample comprises 94 U.S. financial institutions with equity market capitalization greater than 5 bln USD as of June 30, 2007. We extract daily data on equity return and market value of equity from CRSP and quarterly book value of liabilities from COMPUSTAT spanning over the period from 01/03/2000 to 12/31/2011. Out of all financial firms 60 had continuously traded over the sample period. Appendix D provides the list of institutions in the sample categorized by industry groups.

All risk measures are estimated at the $q = \alpha = \tau = 5\%$ confidence levels. We set the prudential capital ratio, k, to 8% in the calculation of the *SRISK* in accordance with current regulatory standards.

Existing evidence in financial modeling shows that asset prices are usually not normally distributed. Indeed, markets may be more dependent during extreme downward movements then when they are moving upwards. To check for the form of dependence in equity returns, we perform a series of quantile regressions of the market return on firm return Equation (A2) for different quantiles. If dependence between returns is linear the estimates of γ_i^q should be approximately the same across all quantiles. Figure 4.1 presents the estimated $\hat{\gamma}_i^q$ for Bank of America (BAC) for 99 quantiles ranging from 0 to 1. The figure shows that the estimated coefficients largely differ across various quantiles

[8]The unconditional beta can be computed using the Capital Asset Pricing Model.

and they are also statistically significant. We observe an inverse U-shape relationship between return series indicating that dependence in returns has a nonlinear form.

[Insert Figure 4.1]

To account for this evidence we, first, replicate the estimations of systemic risk measures used in the seminal papers that allow capturing the nonlinear dependence. To simplify estimations we, further, model the dependence in returns linearly. We do so in order to later compare the rankings of financial firms according to each of the three systemic risk measures, computed using both nonlinear and linear estimation methods.

Table 4.2 provides summary statistics for the estimated systemic risk measures discussed in Section 2.[9] The first two columns report estimates of $\Delta CoVaR$ obtained via quantile regression (denoted $\Delta CoVaR_{NL}$) and OLS regression (denoted $\Delta CoVaR_{L}$), respectively. Column 3 presents estimates of the *MES* measure that account for nonlinear dependence (denoted MES_{NL}) and column 4 contains the linear estimates of *MES* (denoted MES_{L}). The last two columns report estimates of nonlinear and linear *SRISK* (denoted $SRISK_{NL}$ and $SRISK_{L}$, respectively). As evident from Table 4.2 the standard statistics of the estimated measures that account for nonlinear dependence are very close to those that do not capture nonlinear dependence feature in the data. The only exception is the maximum values of the $\Delta CoVaR$s, which suggests that nonlinear dependence structure is better suited to capture extreme events in this case.

Table 4.2 also presents within standard deviations (across time) and between standard deviations (across firms). For the $\Delta CoVaR$s and *MES*s the volatility is larger in time series and for the *SRISK* the volatility is larger in cross section due to the strong dispersion across firms' liabilities. The estimated Pearson correlation coefficient between the two $\Delta CoVaR$s, $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$, is always equal to 1 and the average correlation coefficients are equal to 0.98 and 0.99 for *MES* and *SRISK*, respectively.

[9] Hereafter, we report *SRISK* estimates with both negative and positive values.

[Insert Table 4.2]

We next look at the dynamics of the estimated measures over time. Figure 4.2 time plots $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ averaged over all financial firms, Figure 4.3 displays the time plot of the mean MES_{NL} and MES_{L} measures and Figure 4.4 displays the time plot of the mean $SRISK_{NL}$ and $SRISK_{L}$ measures. In all graphs we observe a very close time-series dynamics of the each measure-pair over the sample period. There are only two periods for which the *SRISK* is positive: at the end of 2002 and from end of 2007 onward.

[Insert Figures 4.2, 4.3 and 4.4]

4.4 Comparison of Systemic Risk Rankings

In this section, we compare the daily rankings of financial institutions in our sample according to the three systemic risk measures that are computed using nonlinear and linear estimation techniques described in Section 2. The key objective is to determine whether the two contrasting methods of estimating systemic risk measures lead to the same conclusion.

4.4.1 Nonlinearity versus Linearity

Table 4.3 presents the rankings of financial institutions based on their contribution to systemic risk, as measured by $\Delta CoVaR$, *MES* and *SRISK*, for September 15, 2008. This date corresponds to the day of the collapse of Lehman Brothers. We report the results only for top 10 SIFIs for convenience.[10] The first two columns rank firms based on their $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$, respectively. Column 3 reports the ranking based on MES_{NL} and column 4 contains the ranking based on MES_{L}. The last two columns show the

[10]Results for all firms are available upon request.

ranking based on $SRISK_{NL}$ (column 5) and $SRISK_L$ (column 6). We observe that the ranking of SIFIs based on the nonlinear systemic risk measures are very close to their ranking based on the same measures estimated linearly. The percentage of concordant pairs between the $\Delta CoVaR_{NL}$ and $\Delta CoVaR_L$ is 8, which means that eight SIFIs out of ten are common to both measures. This number is even higher for the *MES*-pair and for the *SRISK*-pair. $\Delta CoVaR$ and *MES* rank Lehman Brothers as the most systemically risky firm on the date of its bankruptcy. AIG was ranked among top five riskiest financial firms the day before it was rescued by the Federal Reserve. Overall, financial institutions with large systemic risk contribution are identified by all systemic risk measures regardless of the methods we use to estimate them. The mean of the absolute difference in the rankings between nonlinear and linear versions of $\Delta CoVaR$ and *MES* is only 3 and less than 1 for the *SRISK*.

[Insert Table 4.3 and Figure 4.5]

To analyze the dynamics of systemic risk rankings over time we, first, examine the rankings obtained for Bank of America, the institution that has been continuously traded over the sample period. Figure 4.5 displays the time series evolution of the BAC rankings based on the six estimated systemic risk measures. We observe that the rankings based on nonlinear measures move closely with the rankings based on linear measures. Figure 4.6 further presents the time plot of the absolute daily differences between $\Delta CoVaR_{NL}$-based rankings and $\Delta CoVaR_L$-based rankings, MES_{NL} and MES_L-based rankings as well as $SRISK_{NL}$ and $SRISK_L$-based rankings for BAC. On most days the difference between the rankings of BAC obtained using nonlinear estimation methods and linear estimation methods equals 0 or 1. More specifically, the ranking of BAC based on $\Delta CoVaR_{NL}$ is the same as its ranking based on $\Delta CoVaR_L$ on 28% of days over the sample period. Similar results are obtained when we consider the MES-based rankings with the two rankings matching exactly on 26% of the days. $SRISK_{NL}$ and $SRISK_L$ produce the same rankings

of financial firms on 77% of the days. During some periods the difference between the rankings based on nonlinear measures and the rankings based on linear measures is large. These events are, however, rare. The difference in the ranking greater than 3 (shown by the red line) is observed on only 7%, 10% and 6% of the days for the $\Delta CoVaR$-pair, *MES*-pair and *SRISK*-pair rankings, respectively. The average difference in the rankings for BAC is 1.4 for the $\Delta CoVaR$-based rankings, 1.5 for the *MES*-based rankings and 1.1 for the *SRISK*-based rankings. Moreover, large differences in the *MES*-based rankings are usually observed in calm periods (from 10/2002 to 11/2006 and after 09/2009) when the nonlinear dependence in returns is less pronounced. This suggests that accounting for nonlinear dependence in calm periods may result in the overestimation of institution's systemic risk contribution and, consequently, to the inaccurate identification of SIFIs. The bottom panel of Figure 4.6 shows that there are large differences in the *SRISK*-based rankings before the end of 2002. Indeed, at this period and based on *SRISK*, financial institutions are closely ranked. As a consequence, a very small variation in the values of the SRISK may induce a large difference in terms of ranking. However, after October 2002 the *SRISK* had been mainly driven by the leverage and then by the total amount of liabilities resulting in relatively stable rankings, almost without difference between linear and nonlinear estimation methods as can be observe in Figure 4.5 because those quantities are clearly different from a firm to another.

[Insert Figure 4.6]

We further examine the rankings for all financial institutions in the sample. First, we estimate the percentage of concordant pairs between the rankings based on systemic risk measures estimated using nonlinear methods and the rankings based on systemic risk measures obtained using the linear methods. The percentage of concordant pairs equals 100 if the ranking of a financial institution according to the "nonlinear" systemic

risk measure exactly matches its ranking according "linear" systemic risk measure.[11] For example, concordance is 100% if both $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ produce the same ranking for a given institution. This would suggest that the nonlinear estimation of an institution's systemic risk measure has no value added over its linear estimation with respect to the ranking of this institution.

Figure 4.7 provides some insights to the $\Delta CoVaR$-based rankings analysis. The yellow line plots the percentage of concordant pairs between $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ for top 10 SIFIs, top 20 SIFIs and all 60 financial institutions that had continuously traded over the sample period. Table 4.4 further shows that the average percentage of concordance equals 18% when we consider all firms. In other words, the rankings based on $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ are exactly the same for 10 financial firms. Next we compare the rankings allowing for the deviations from full concordance in terms of one, two or three position changes in the ranking for each firm. Figure 4.7 shows the time plot of the percentage of concordance for these deviations. As given by Table 4.4 the percentage of concordant pairs more than doubles reaching 42% when we allow for one position change in the ranking. On average, the percentage increases by around 20 basis points for every additional difference in the position allowed for. Moreover, the percentage is much higher if we focus on top 10 riskiest firms, ranging from 37% when each institution's ranking is the same, to over 80% when we allow for two position changes in the ranking. These results indicate that there is no large difference in the identification and ranking of SIFIs between $\Delta CoVaR$ estimated using nonlinear method and $\Delta CoVaR$ computed linearly. The identification of SIFIs is not greatly affected by the methodology of estimating their systemic risk contribution.

[Insert Table 4.4 and Figure 4.7]

Figure 4.8 time plots the percentage of concordant pairs between MES_{NL} and MES_{L}-based rankings. As in the $\Delta CoVaR$ case we observe a large increase in the percentage when

[11] Quotation marks are used because only the estimation of the dependence is estimated with linear and nonlinear approaches, and not the measures themselves.

the deviation from concordance increases from 1 to 3 changes in institution's position in the overall ranking. Table 4.5 summarizes the results across the sample period and shows that on average the percentage of concordant pairs is equal to 19%. When we allow for 1, 2 or 3 differences in the ranking the percentage almost doubles growing from 43% to 71%. On average, the percentage growth is close to 20 basis points per additional difference in the position allowed. As shown above the percentage of concordant pairs is much higher for top 10 and top 20 SIFIs. On some dates, the percentage of concordant pairs reaches 100% for top 10, and 90% for top 20 SIFIs. Furthermore, when we allow for 3 position changes in the ranking, the percentage reaches 77% for top 20 SIFIs and increases further to 87% for top 10 SIFIs. This implies that despite of difference in the values of the MES_{NL} and MES_{L}, the rankings based on MES_{NL} are very close to those based on MES_{L}.

[Insert Table 4.5 and Figure 4.8]

Table 4.6 presents descriptive statistics for the percentage of concordant pairs between $SRISK_{NL}$ and $SRISK_{L}$-based rankings. The results show that the two *SRISK*s produce very similar rankings. On average, the percentage of concordant pairs equals 83%, which is twice as much as the percentage of full concordance obtained for the $\Delta CoVaR$ and *MES*-based rankings. This number increases to 99% for top 10 SIFIs when we allow for 3 position changes in each firm's ranking. The percentage of concordance remains high when we add more firms to the analysis. In particular, it equals 67% for top 20 risky firms and 59% for all 60 firms, and increases to 97% when we allow for 3 position changes in the rankings. Figure 4.9 further shows that the dynamics of concordance is pretty stable over time. The yellow line time plots the percentage of concordant pairs for top 10 SIFIs. On almost all days the concordance is greater than 75% and is close to 100%. It does not drop below a 50% mark on 84.03% and 80.39% of days when we consider top 20 SIFIs and all financial firms.

[Insert Table 4.6 and Figure 4.9]

We next report the time series evolution of the Kendall rank order correlation coefficient between the rankings based on nonlinear and linear systemic risk measures. Figure 4.10 shows that this coefficient is always greater than 74% for each measure-pair. On average, the Kendall correlation is 87.18% between $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ rankings, 87.55% between MES_{NL} and MES_{L} rankings, and 97.39% between $SRISK_{NL}$ and $SRISK_{L}$ rankings. These results confirm the irrelevance of computing systemic risk measures using nonlinear estimation methods for the identification of SIFIs. Although nonlinear techniques are better suited for the estimation of an accurate values of systemic risk measures, they do not improve in terms of the ranking of financial firms based on these measures.

[Insert Figure 4.10]

4.4.2 Impact of More Extreme Events

To check the significance of our previous results we re-estimate the nonlinear versions of all systemic risk measures at the 1% risk level. In other words, we chose the values of q, α, and τ to be equal to 0.01 in order to analyze 1% of worst days of the historical returns. By focusing on the farther left tale of the return distribution we expect that the difference between nonlinear and linear modeling of returns dependence might be larger and, consequently, have greater impact on the firms' rankings.

The top 10 SIFIs as identified by $\Delta CoVaR$s, MESs and $SRISK$s at the 1% risk level for September 15, 2008 are very similar to those presented in Table 4.3.[12] The number of concordant pairs between MES_{NL} and MES_{L} as well as $SRISK_{NL}$ and $SRISK_{L}$ remained 9 and 10, respectively, and is now only slightly lower between $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$, 7 instead of 8. This can be due to the larger difference between the estimates of γ^{OLS} and the $\gamma^{1\%}$ compared to the corresponding difference between the values of γ^{OLS} and the $\gamma^{5\%}$ as shown in Figure 4.1.

[12]Estimation results at the 1% level of risk are available on request.

Next we analyze the time series dynamics of the BAC rankings based on the systemic risk measures estimated at 1% risk threshold and the systemic risk computed linearly. There is larger difference between $\Delta CoVaR_{NL}$-based rankings and $\Delta CoVaR_{L}$-based rankings and MES_{NL} and MES_{L}-based rankings over the sample period compared to the corresponding differences in the rankings presented in Figure 4.6. In particular, the ranking of BAC based on $\Delta CoVaR_{NL}$ is the same as its ranking based on $\Delta CoVaR_{L}$ on 28.06% of days and *MES*-based rankings on 26.60% of the days. In contrast, the difference in the rankings between $SRISK_{NL}$ and $SRISK_{L}$ is still low corresponding to 77.08% of the days. Hence, the ranking of firms based on *SRISK* is not greatly altered by the risk threshold chosen to estimate nonlinear dependence between firm and market returns.

Finally, we examine the ranking of all firms by looking at the percentage of concordant pairs between systemic risk measures. On average, the percentages of concordant pairs are smaller when we consider 1% risk level. In particular, the average percentages of concordance for $\Delta CoVaR$, *MES* and *SRISK*-based rankings are equal to 9%, 12% and 38%, respectively and increase to 49%, 53% and 89% when we allow for a difference in the rankings being equal to 3. Hence, the nonlinear dependence in returns impacts the rankings when we focus on more extreme events. As expected the $\Delta CoVaR$ and *MES*-based rankings are more affected by the change in the risk threshold than the *SRISK*-based rankings. This is because the *SRISK* estimates are mainly driven by the total amount of firm's liabilities.

4.5 Conclusion

In this paper, we compare nonlinear and linear approaches to the estimation of the three market-based systemic risk measures, *MES*, $\Delta CoVaR$ and *SRISK*. Our results show that estimation methods that account for nonlinear dependence structure in return series do not greatly improve in terms of identifying SIFIs compared to those that model the

dependence structure linearly in a standard framework. However, the choice of the risk threshold has an impact on the results. We show that *SRISK*-based rankings do not change when we use the 1% threshold in the estimation of the systemic risk measures. Given the focus of the current regulation modeling the dependence structure of returns linearly appears to be sufficient to identify and rank SIFIs. These findings are similar to those of Patro et al. (2013) and suggest that the market-based systemic risk measures are mainly driven by stock return correlations.

D.1 $\triangle$CoVaR Framework and Estimation

The parameters of the quantile regression are estimated by minimizing the sum of residuals. These are weighted asymmetrically according to:

$$\min_{\beta} \sum_{i}^{N} \rho_q \Big(y_i - \eta(x_i, \beta) \Big) \tag{A1}$$

where y_i is dependent variable, $\eta(x_i, \beta)$ is a linear function of the parameters β associated with independent variables x_i, and $\rho_q(.)$ is a function that assigns weights to each observation depending on the given quantile.

Estimation procedure is as follows:

First, the quantile regression at the q percent quantile is performed on the following equation:

$$r_{mt} = \delta_i^q + \gamma_i^q r_{it} + \epsilon_{it} \tag{A2}$$

Second, the predicted values from Equation (A2) are used to compute *CoVaR* according to:

$$CoVaR_{it}(q, \alpha) = \hat{\delta}_i^q + \hat{\gamma}_i^q \, VaR_{it}(\alpha) \tag{A3}$$

where

$$VaR_{it}^i(\alpha) = \sigma_{it} F^{-1}(\alpha) \tag{A4}$$

assuming that $r_{it} \sim F$ a location-scale distribution and the estimation of σ_{it} is done using a GJR-GARCH model [(Glosten et al., 1993)].

Finally, to examine sensitivity of the system to a distressed institution i, $\Delta CoVaR$ is

computed as follows:

$$\Delta CoVaR_{it}(q,\alpha) = CoVaR_{it}(q,\alpha) - CoVaR_{it}(q,0.5) \quad \text{(A5)}$$

$$= \hat{\gamma}_i^q \Big(VaR_{it}(\alpha) - VaR_{it}(0.5) \Big) \quad \text{(A6)}$$

When we assume that dependence between r_{it} and r_{mt} is fully captured by the Pearson correlation coefficient, we can estimate the parameters of Equation (A2) using the Ordinary Least Square method.

D.2 MES Framework and Estimation

For the *MES*, our methodological framework is based on the linear market model defined by Brownlees and Engle (2012) in which, r_{mt} and r_{it} respectively the demeaned returns of the market and the financial firm i are defined as:

$$\begin{aligned} r_{mt} &= \sigma_{mt}\ \varepsilon_{mt} \\ r_{it} &= \sigma_{it}\ \rho_{it}\ \varepsilon_{mt} + \sigma_{it}\ \sqrt{1-\rho_{it}^2}\ \xi_{it} \\ (\varepsilon_{it}, \xi_{it}) &\sim F \end{aligned} \tag{B1}$$

where σ_{mt} and σ_{it} are the conditional volatilities of the market and the firm, ρ_{it} is the correlation between r_{mt} and r_{it}, and $(\varepsilon_{mt}, \xi_{it})$ are disturbances that follow an $i.i.d.$ process over time with zero mean vector and identity covariance matrix with $F(.)$ the bivariate distribution of the standardized innovations.

$$\begin{aligned} MES_{it}(\tau) &= \sigma_{it}\ \rho_{it}\ E_{t-1}\Big(\varepsilon_{mt} | \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\Big) + \sigma_{it}\ \sqrt{(1-\rho_{it}^2)}\ E_{t-1}\Big(\xi_{it} | \varepsilon_{mt} < \frac{C}{\sigma_{mt}}\Big) \\ &= \sigma_{it}\ \rho_{it}\ E_{t-1}\Big(\varepsilon_{mt} | r_{mt} < C\Big) + \sigma_{it}\ \sqrt{(1-\rho_{it}^2)}\ E_{t-1}\Big(\xi_{it} | r_{mt} < C\Big) \\ &= \beta_{it}\ E_{t-1}\Big(r_{mt} | r_{mt} < C\Big) + \sigma_{it}\ \sqrt{(1-\rho_{it}^2)}\ E_{t-1}\Big(\xi_{it} | r_{mt} < C\Big) \end{aligned} \tag{B2}$$

where $\beta_{it} = \rho_{it}\frac{\sigma_{it}}{\sigma_{mt}}$ is the beta of firm i.[13]

Concerning the estimation of the MES_{it}, σ_{it} and σ_{mt} come from a GJR-GARCH model [Glosten et al. (1993)], ρ_{it} is obtained with an asymmetric Dynamic Conditional Correlation (DCC) GARCH model [Engle (2002)] and both conditional expectations are computed with a nonparametric estimator [Scaillet (2003)]. When we assume the linearity of the dependence structure, the second part of Equation (B2) is null.

[13] An unconditional beta could be compute with the classic Capital Asset Pricing Model (CAPM).

D.3 SRISK Framework and Estimation

The *SRISK* is based on the same framework than described in Appendix B. According to Engle et al. (2012) the capital shortfall of a given financial firm i is defined as:

$$CS_{it} = k\ D_{it}\ -(1-k)\ (1-LRMES_{it})\ W_{it}\ , \tag{C1}$$

where D_{it} and W_{it} denote the value of the book value of total liabilities and equity of firm i and k is a prudential capital ratio of equity to assets. The *LRMES* is expressed by the following equation:

$$LRMES_{it} = LRMES_{i,t:t+T} = -\mathbb{E}_{t-1}\Big(\ R_{i,t:t+T} |\ R_{m,t:t+T} \leq -40\%\Big)\ , \tag{C2}$$

where $R_{i,t:t+T}$ and $R_{i,t:t+T}$ are cumulative returns defined as:

$$R_{i,t:t+T} = \exp\left(\sum_{j=1}^{T} r_{i,t+j}\right) - 1 \quad \text{and} \quad R_{m,t:t+T} = \exp\left(\sum_{j=1}^{T} r_{m,t+j}\right) - 1\ ,$$

$r_{i,t}$ and $r_{m,t}$ are the log-return of firm i and the market log-return, respectively. This *LRMES* is computed at a time horizon of six-month and T sets at 126 trading days. Then, the *LRMES* is approximated without simulation by:

$$LRMES_{it} = -\Big(\exp(18 \times MES_{it}(\tau)) - 1\Big) = 1 - \exp(18 \times MES_{it}(\tau)). \tag{C3}$$

Finally, the *SRISK* contribution of a given firm to the risk of the system is given by:

$$\begin{aligned} SRISK_{it} &= \max\Big(0\ ; CS_{it}\Big) \\ &= \max\Big(0\ ; k\ D_{i,t}\ -(1-k)\ \exp\Big(18 \times MES_{it}(\tau)\Big)\ W_{i,t}\Big)\ . \end{aligned} \tag{C4}$$

When we assume linearity, the second part of Equation B2 is null and we use this quantity to obtain the *SRISK* under the linearity assumption in the estimation methods.

D.4 Dataset

Depositories (29)		Insurance (32)	
BAC	Bank of America Corp.	ABK	Ambac Financial Group
BBT	BB&T Corp.	AET	Aetna
BK	Bank of New York Mellon Corp.	AFL	AFLAC Inc.
C	Citigroup Inc.	AIG	American International Group Inc.
CBH	Commerce Bancorp	AIZ	Assurant
CMA	Comerica Inc.	ALL	Allstate Corp.
HBAN	Huntington Bancshares Inc.	AOC	Aon Corp.
HCBK	Hudson City Bankshares Inc.	BKLY	W.R. Berkley Corp.
JPM	JP Morgan Chase & Co.	BRK	Berkshire Hathaway Inc.
KEY	Keycorp New	CB	Chubb Corp.
MI	Marshall & Ilsley Corp.	CFC	Countrywide Financial
MTB	M&T Bank Corp.	CI	CIGNA Corp.
NCC	National City Corp.	CINF	Cincinnati financial Corp.
NTRS	Northern trust Corp.	CNA	CNA Financial Corp.
NYB	New York Community Bancorp Inc.	CVH	Coventry health Care Inc.
PBCT	Peoples United Financial Inc.	FNF	Fidelity National Financial
PNC	PNC Financial Services Grp Inc.	GNW	Genworth Financial
RF	Regions Financial Corp.	HIG	Hartford financial Svcs Grp Inc.
SNV	Synovus Financial Corp.	HNT	Health Net Inc.
SOV	Sovereign Bancorp	HUM	Humana Inc.
STI	Suntrust Banks Inc.	LNC	Lincoln National Corp.
STT	State Street Corp.	MBI	MBIA Inc.
UB	Unionbancal Corp.	MET	MetLife
USB	US Bancorp	MMC	Marsh & McLennan Cos Inc.
WB	Wachovia	PFG	Principal Financial Group
WFC	Wells Fargo & Co	PGR	Progressive Corp.
WM	Washington Mutual	PRU	Prudential Financial
WU	Western Union	SAF	Safeco
ZION	Zions Bancorp	TMK	Torchmark Corp.
		TRV	Travelers companies Inc.
		UNH	United Health Group Inc.
		UNM	Unum Group
Broker-Dealers (10)		Others (23)	
AGE	A.G. Edwards	ACAS	American Capital Ltd
BSC	Bear Stearns	AMP	Ameriprise Financial
ETFC	E*Trade Financial Corp.	AMTD	TD Ameritrade Holding Corp.
GS	Goldman Sachs group Inc.	AXP	American Express Co.
LEH	Lehman Brothers	BEN	Franklin Resources Inc.
MER	Merill Lynch	BLK	BlackRock Inc.
MS	Morgan Stanley Dean Witter & Co	BOT	CBOT Holdings
NMX	Nymex Holdings	CBG	C.B. Richard Ellis Group
SCHW	Schwab Charles Corp.	CBSS	Compass Bancshares
TROW	T. Rowe Price Group Inc.	CIT	CIT Group
		CME	CME Group
		COF	Capital One Financial Corp.
		EV	Eaton Vance Corp.
		FITB	Fifth Third Bancorp
		FNM	Fannie Mae
		FRE	Freddie Mac
		HRB	H&R Block
		ICE	Intercontinental Exchange
		JNS	Janus Capital
		LM	Legg Mason Inc.
		NYX	NYSE Euronext
		SEIC	SEI Investment Company
		SLM	SLM Corp.

Table 1 – Tickers and Company Names by Industry Groups

Table 4.2 – Summary Statistics for the Estimated Systemic Risk Measures

	$\Delta CoVaR_{NL}$	$\Delta CoVaR_{L}$	MES_{NL}	MES_{L}	$SRISK_{NL}$	$SRISK_{L}$
Mean	0.0101	0.0103	0.0283	0.0262	-1.2768	-1.6548
Min	0.0007	0.0008	-0.0154	-0.0178	-159.22	-168.51
Max	0.1620	0.1052	0.5625	0.5522	164.73	164.28
Std.Dev	0.0074	0.0074	0.0252	0.0236	18.958	19.209
Between Std.Dev	0.0028	0.0028	0.0080	0.0075	9.652	9.837
Within Std.Dev	0.0060	0.0061	0.0203	0.0191	5.807	5.839

Notes: The table contains descriptive statistics for the estimated systemic risk measures for all firms in the sample. Within standard deviation is computed as the standard deviation of the time-series mean of individual Δ*CoVaR*s, *MES*s and *SRISK*s. Between standard deviation is the standard deviation of the cross-sectional average of Δ*CoVaR*, *MES* and *SRISK* over time. Δ*CoVaR*s and *MES*s are in percentages and *SRISK*s are in billion USD. Sample period is from 01/03/2000 to 12/31/2011.

Table 4.3 – Systemic Risk Rankings

Rank	$\Delta CoVaR_{NL}$	$\Delta CoVaR_{L}$	MES_{NL}	MES_{L}	$SRISK_{NL}$	$SRISK_{L}$
1	LEH	LEH	LEH	LEH	C	C
2	MER	MER	AIG	AIG	BAC	BAC
3	AIG	AIG	WM	WM	JPM	JPM
4	WM	BBT	ABK	MER	AIG	AIG
5	NYX	NYX	MER	ABK	MER	MER
6	EV	CMA	MBI	MBI	MS	MS
7	LM	LM	NYX	NYX	GS	GS
8	JNS	EV	CIT	LM	LEH	LEH
9	CMA	JNS	LM	BAC	PRU	MET
10	BEN	WM	SLM	JNS	MET	PRU

Pairs	$\Delta CoVaR_{NL}$	$\Delta CoVaR_{L}$	MES_{NL}	MES_{L}	$SRISK_{NL}$	$SRISK_{L}$
$\Delta CoVaR_{L}$	8	-				
MES_{NL}	7	6	-			
MES_{L}	7	6	9	-		
$SRISK_{NL}$	3	3	3	3	-	
$SRISK_{L}$	3	3	3	3	10	-

Notes: In the upper panel, the column labeled $\Delta CoVaR_{NL}$ displays the ranking of the top 10 financial institutions in terms of $\Delta CoVaR_{NL}$, listed from most to least risky. The following 5 columns display the top 10 financial institutions based on $\Delta CoVaR_{L}$, MES_{NL}, MES_{L}, $SRISK_{NL}$, and $SRISK_{L}$ respectively. In the lower panel, we report the number of concordant pairs between rankings based on systemic risk measures. Rankings are for September 15, 2008.

Table 4.4 – Percentage of Concordance for the $\Delta CoVaR$-based Rankings

	Rank Diff.	Mean	Std.Dev	Min	Max
Top 10 SIFIs	3	89	11	30	100
	2	82	14	30	100
	1	68	17	10	100
	0	37	19	0	100
Top 20 SIFIs	3	79	10	35	100
	2	69	12	25	100
	1	53	13	15	95
	0	26	11	0	70
All Firms	3	70	8	40	92
	2	58	8	30	82
	1	42	8	17	65
	0	18	6	3	40

Notes: This table presents descriptive statistics (mean, standard deviation, minimum and maximum values) of the percentage of concordant pairs for the rankings of financial institutions based on $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ for top 10 SIFIs, top 20 SIFIs and all financial institutions which had continuously traded over the sample period. The column labeled Rank Diff. shows the deviations from concordance in terms of 0, 1, 2 or 3 position changes in the ranking of each firm.

Table 4.5 – Percentage of Concordance for the *MES*-based Rankings

	Rank Diff	Mean	Std.Dev	Min	Max
Top 10 SIFIs	3	87	13	30	100
	2	81	17	20	100
	1	69	21	0	100
	0	39	21	0	100
Top 20 SIFIs	3	77	15	30	100
	2	68	17	15	100
	1	53	18	5	95
	0	26	14	0	90
All Firms	3	71	11	40	98
	2	59	12	27	95
	1	43	11	15	80
	0	19	7	3	52

Notes: This table presents descriptive statistics (mean, standard deviation, minimum and maximum values) in terms of the percentage of concordant pairs for the rankings of financial institutions based on nonlinear and linear *MES*s for top 10 SIFIs, top 20 SIFIs and all financial institutions which had continuously traded over the sample period. The column labeled Rank Diff. shows the deviations from concordance in terms of 0, 1, 2 or 3 position changes in the ranking of each firm.

Table 4.6 – Percentage of Concordance for the *SRISK*-based Rankings

	Rank Diff	Mean	Std.Dev	Min	Max
Top 10 SIFIs	3	99	4	60	100
	2	98	5	50	100
	1	96	8	40	100
	0	83	18	10	100
Top 20 SIFIs	3	97	5	50	100
	2	95	7	40	100
	1	89	12	25	100
	0	67	19	5	100
All Firms	3	97	3	77	100
	2	95	5	62	100
	1	86	8	47	100
	0	59	11	23	92

Notes: This table presents descriptive statistics (mean, standard deviation, minimum and maximum values) in terms of the percentage of concordant pairs for the rankings of financial institutions based on nonlinear and linear *SRISK*s for top 10 SIFIs, top 20 SIFIs and all financial institutions which had continuously traded over the sample period. The column labeled Rank Diff. shows the deviations from concordance in terms of 0, 1, 2 or 3 position changes in the ranking of each firm.

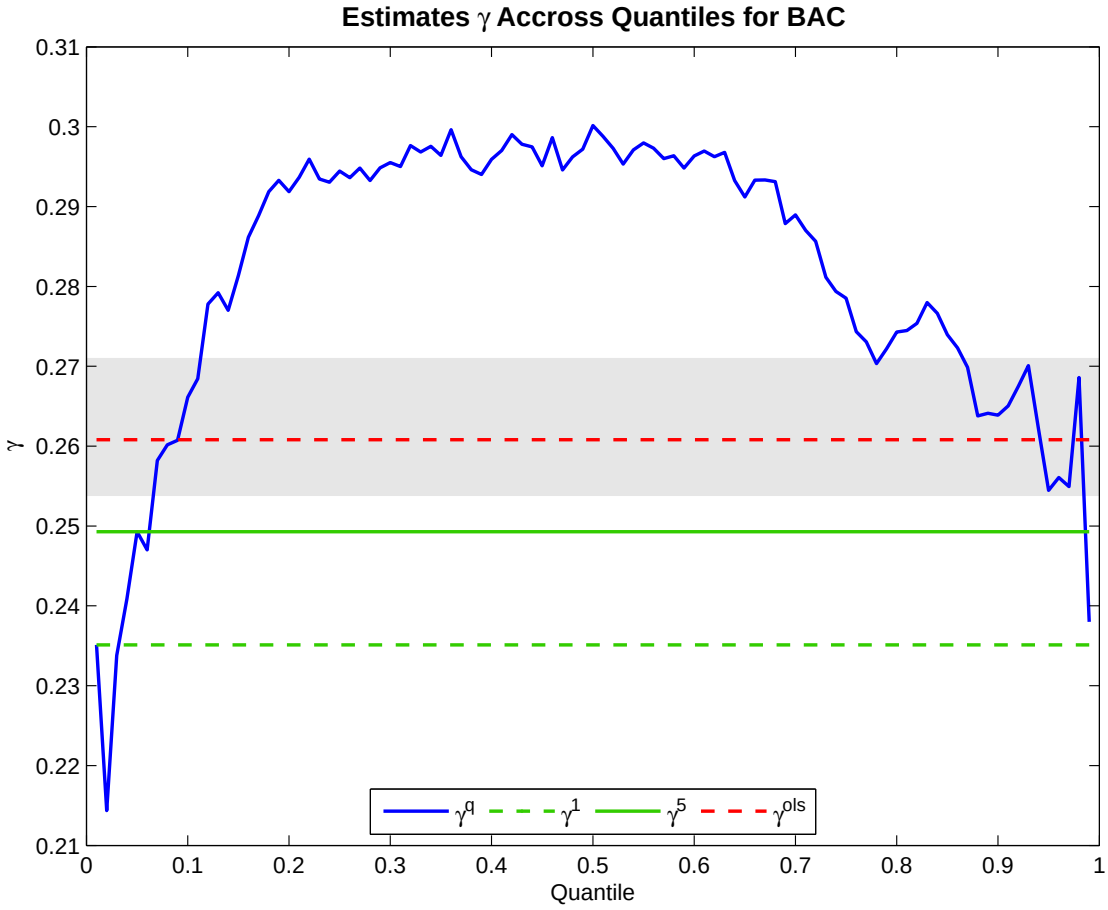


Figure 4.1 – This figure displays the gamma coefficient estimated with quantile regression, γ^q, (blue solid line) for quantiles ranging from 0.01 to 0.99 (x-axis), the gamma coefficient estimated at 1% quantile, γ^1, (green solid line), the gamma coefficient estimated at 5% quantile, γ^5, (green dashed line) and the gamma coefficient estimated by OLS, γ^{ols}, (red dashed line). The gray band is the area within which $\gamma^{0.05}$ and γ^{ols} are statistically not different from each other.

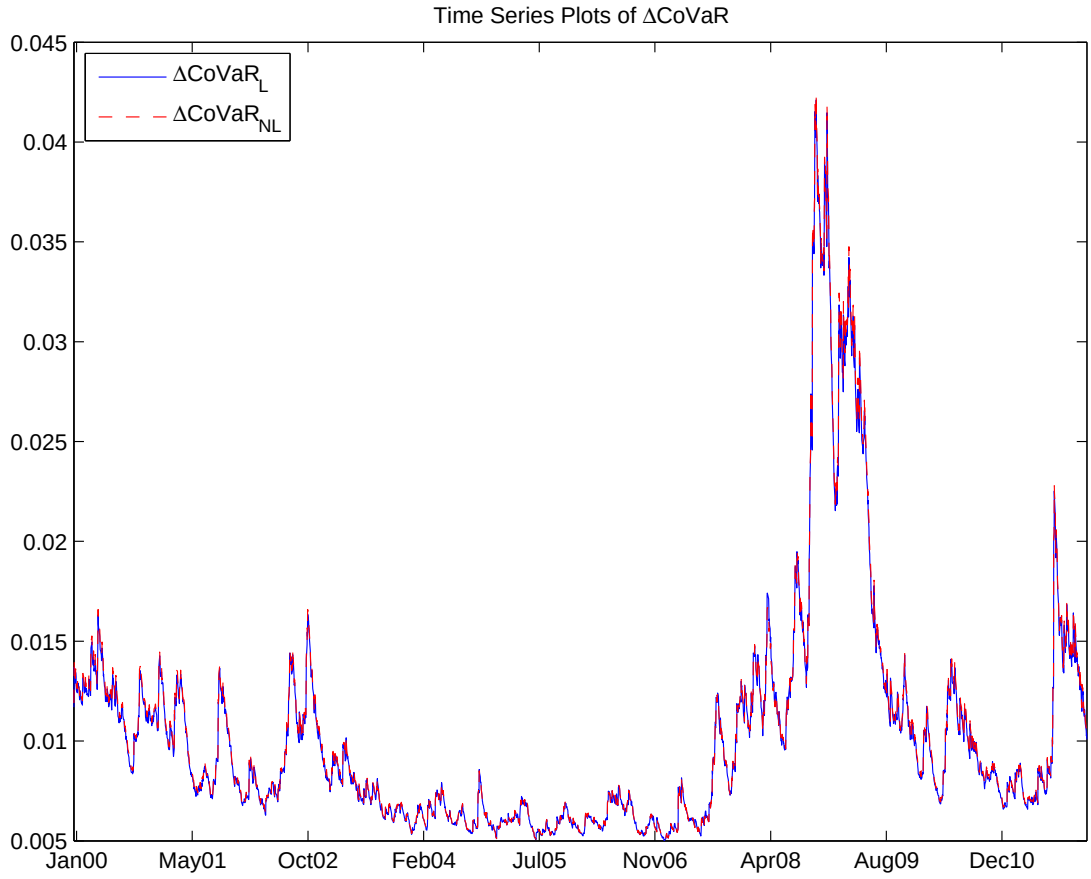


Figure 4.2 – This figure displays the mean values of $\Delta CoVaR_L$ (blue solid line) and the $\Delta CoVaR_{NL}$ (red dashed line). The estimation period is from 01/03/2000 to 12/30/2011.

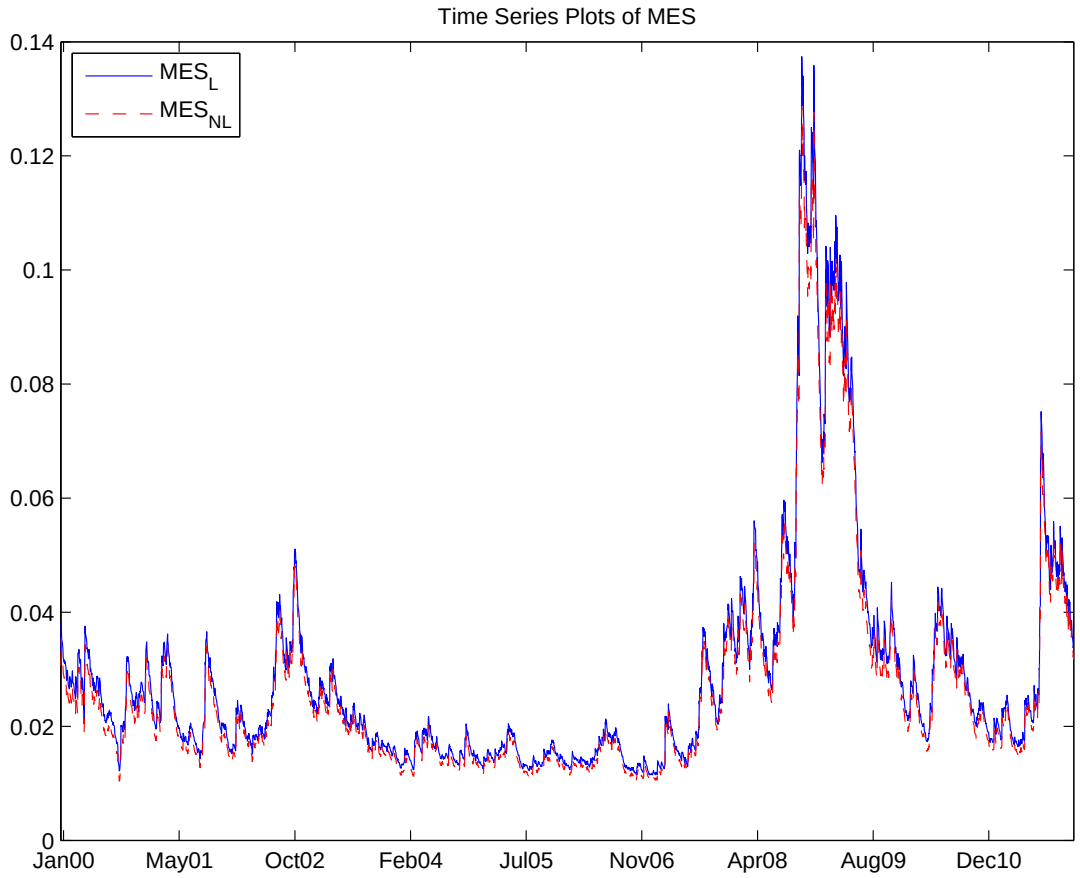


Figure 4.3 – This figure displays the mean values of MES_L (blue solid line) and the MES_{NL} (red dashed line). The estimation period is from 01/03/2000 to 12/30/2011.

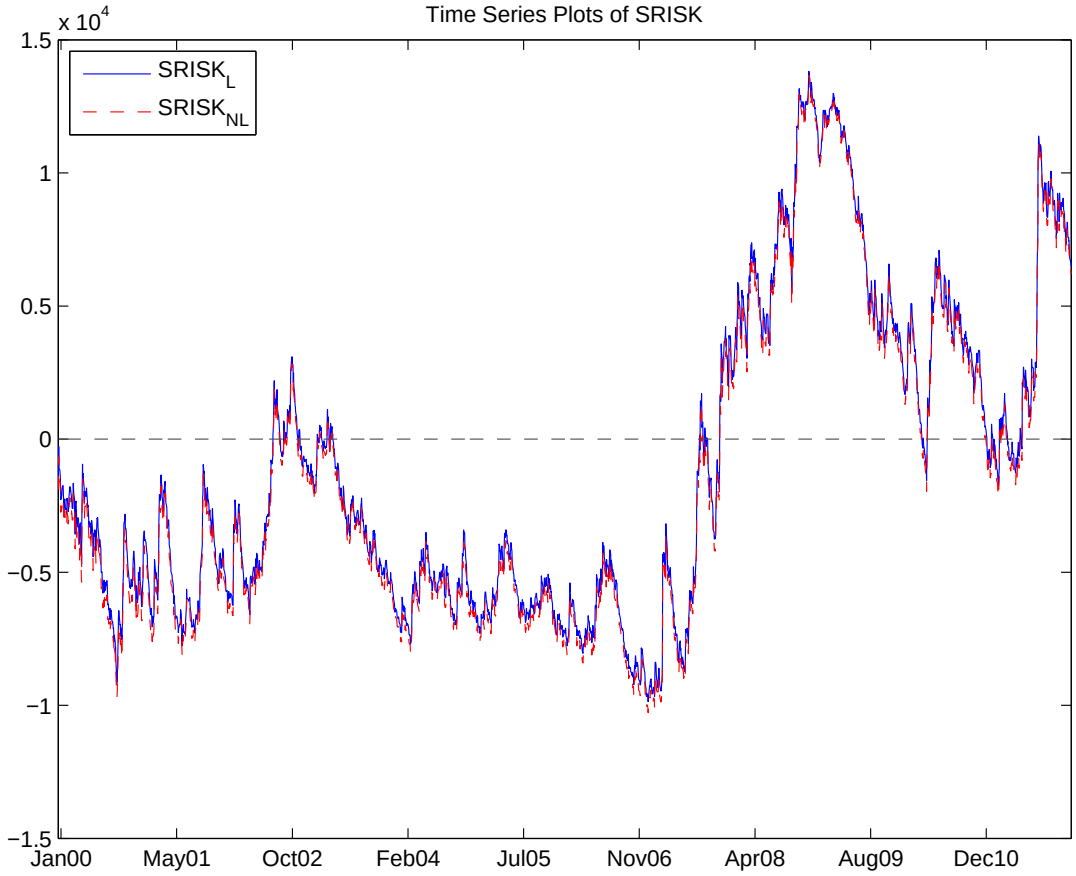


Figure 4.4 – This figure displays the mean values of $SRISK_L$ (blue solid line) and the $SRISK_{NL}$ (red dashed line). The estimation period is from 01/03/2000 to 12/30/2011.

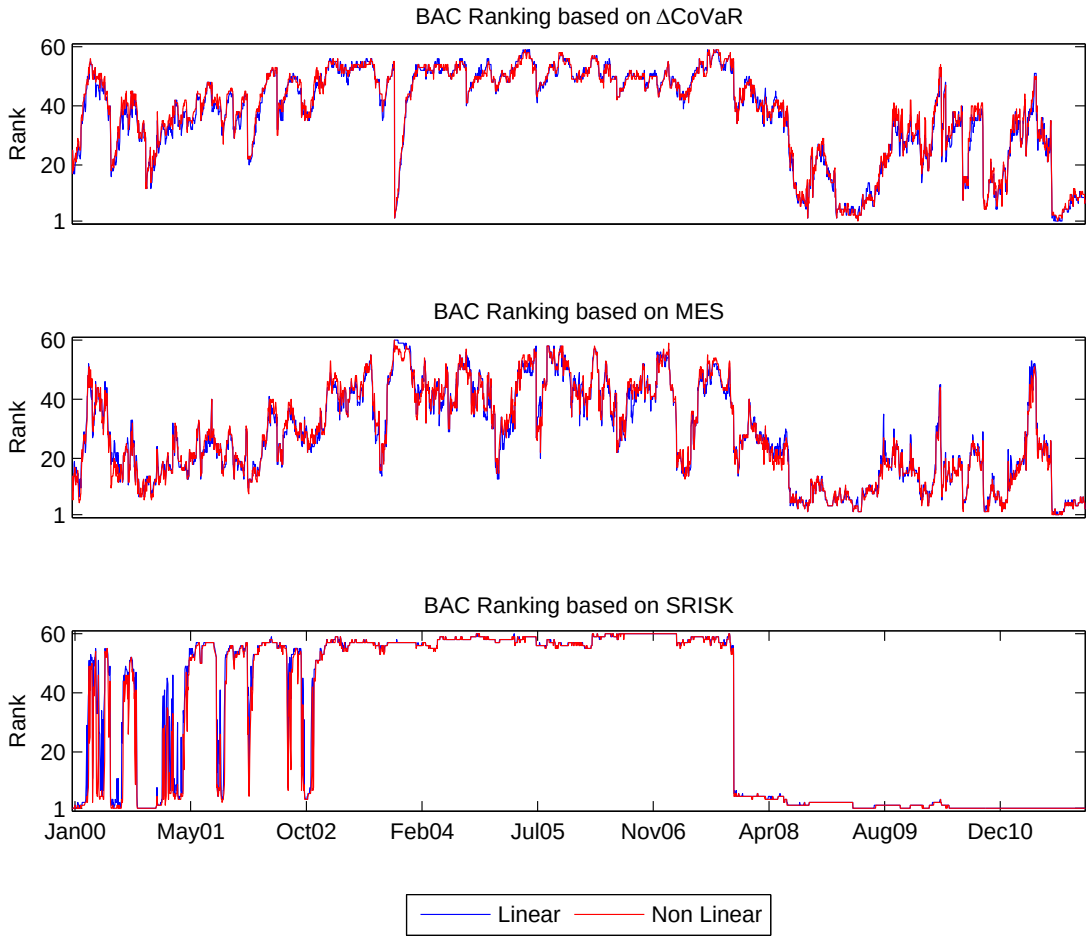


Figure 4.5 – The top panel time plots the daily ranking of Bank of America (BAC) based on $\Delta CoVaR_L$ (blue line) and $\Delta CoVaR_{NL}$ (red line), the middle panel displays the daily ranking of this institution based on its MES_L (blue line) and MES_{NL} (red line) and the bottom panel presents daily ranking based on its $SRISK_L$ (blue line) and $SRISK_{NL}$ (red line). The estimation period is from 01/03/2000 to 12/30/2011.

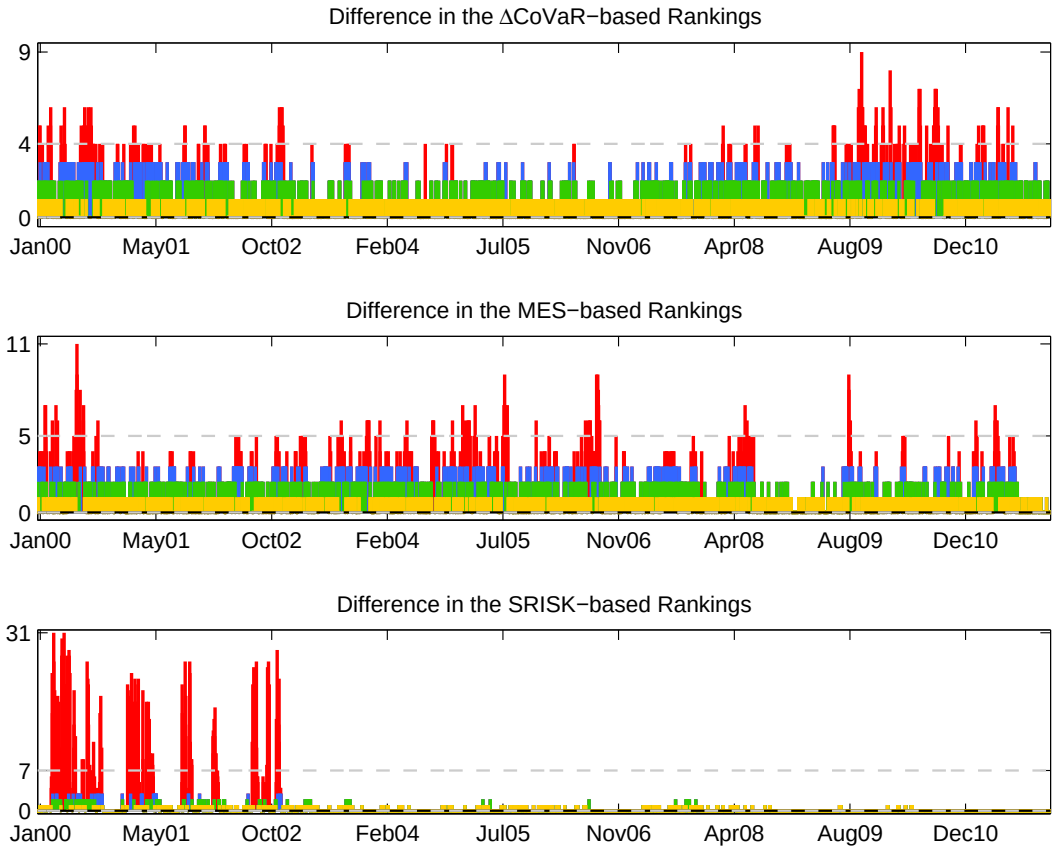


Figure 4.6 – The top figure shows the daily difference between the $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$-based rankings for Bank of America (BAC). The middle figure shows the daily difference in the MES_{NL} and MES_{L}-based rankings for BAC. The bottom figure plots the daily difference between $SRISK_{NL}$ and $SRISK_{L}$-based rankings for this institution. The 0, 1, 2, 3, and >3 differences in the rankings are plotted with black, yellow, green, blue and red lines, respectively. The estimation period is from 01/03/2000 to 12/30/2011.

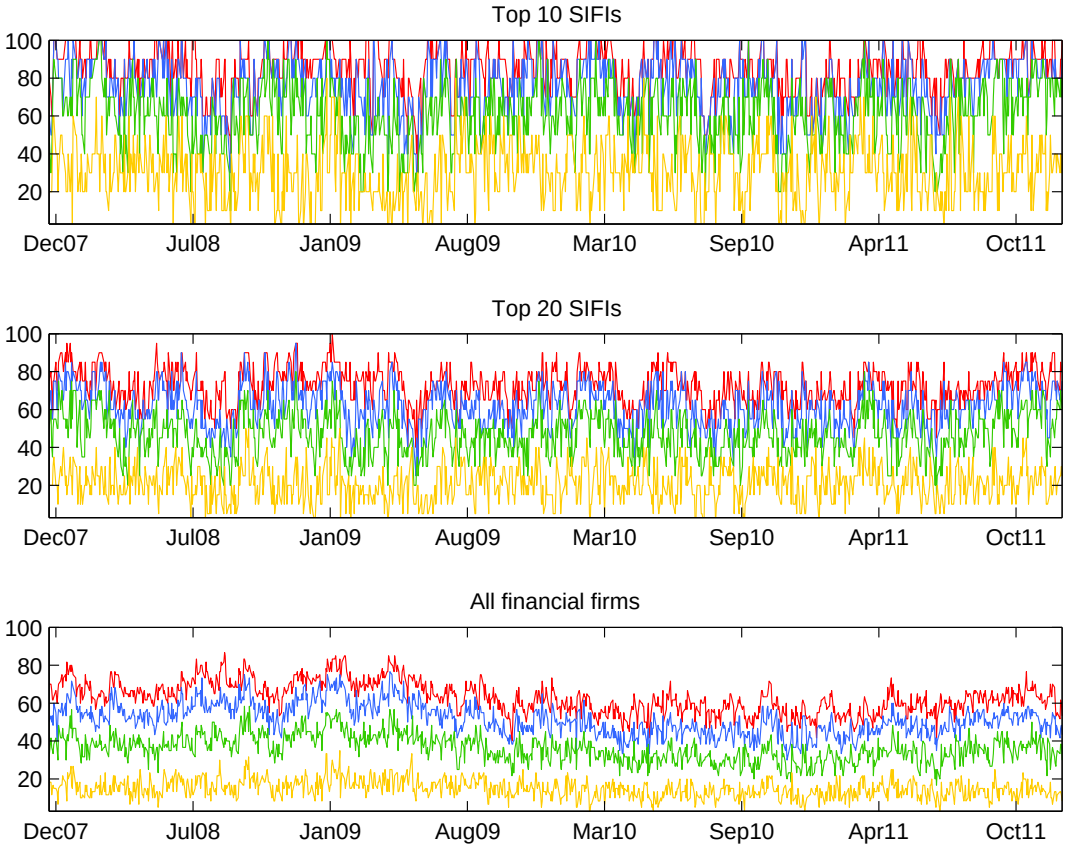


Figure 4.7 – The figure time plots the percentage of concordant pairs between $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ (yellow line). It also shows the percentage of concordance for deviations allowed in terms of 1 position change in the ranking (green line), 2 positions change in the ranking (blue line) and 3 positions change in the ranking (red line). The top panel considers the top 10 SIFIs, the middle panel focuses on the top 20 SIFIs and the bottom panel considers all 60 financial institutions that had continuously traded over the sample period. The estimation period is from 01/03/2000 to 12/30/2011.

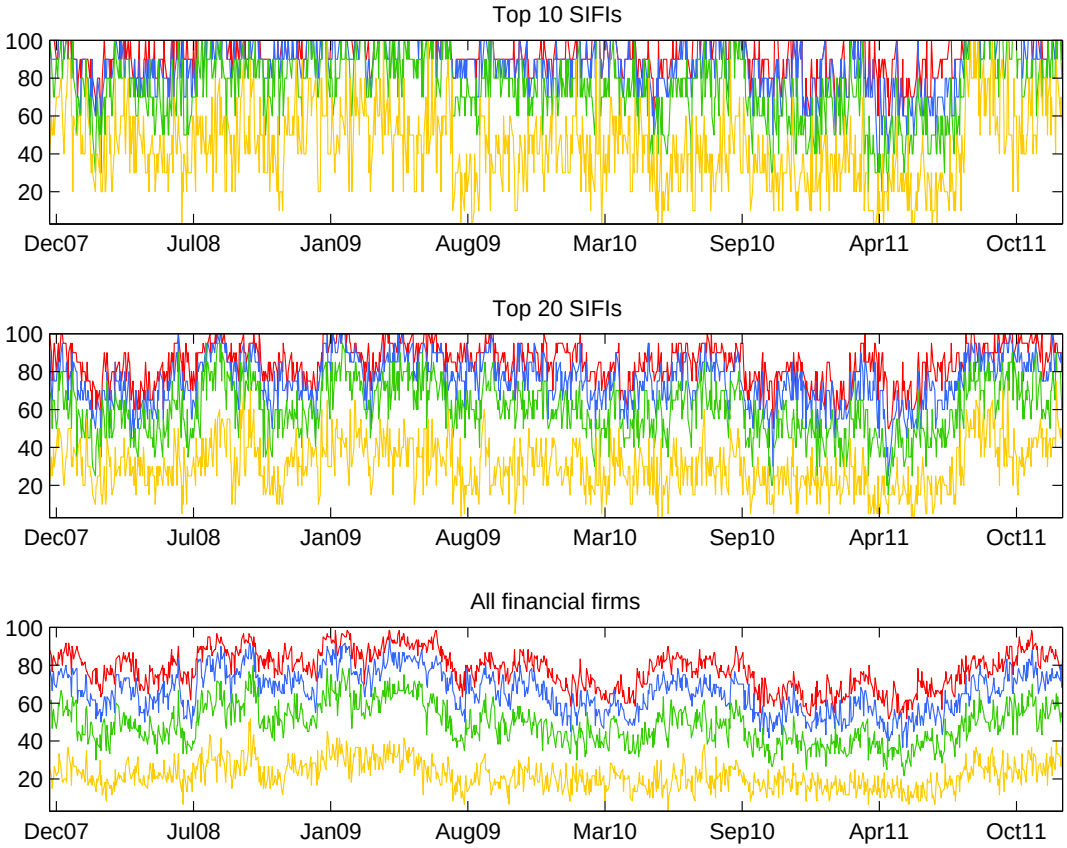


Figure 4.8 – The figure time plots the percentage of concordant pairs between MES_{NL} and MES_{L} (yellow line). It also shows the percentage of concordance for deviations allowed in terms of 1 position change in the ranking (green line), 2 positions change in the ranking (blue line) and 3 positions change in the ranking (red line). The top panel considers the top 10 SIFIs, the middle panel focuses on the top 20 SIFIs and the bottom panel considers all 60 financial institutions that had continuously traded over the sample period. The estimation period is from 01/03/2000 to 12/30/2011.

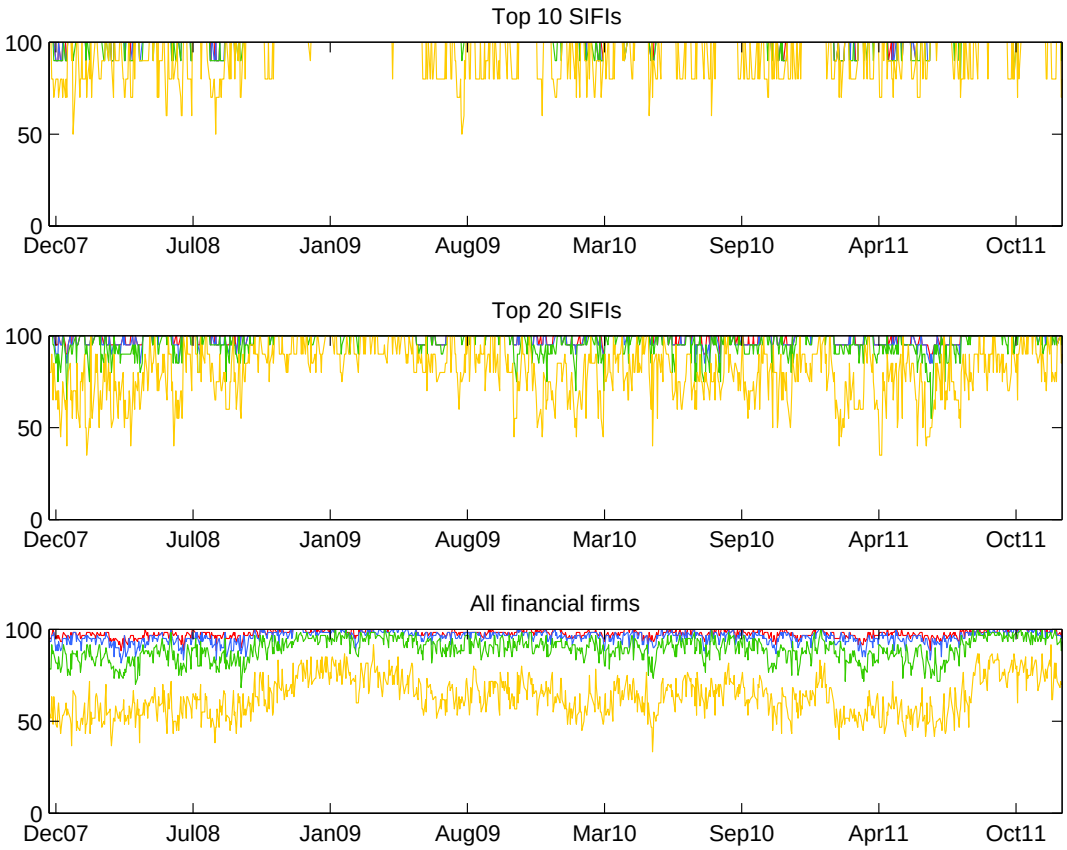


Figure 4.9 – The figure time plots the percentage of concordant pairs between $SRISK_{NL}$ and $SRISK_{L}$ (yellow line). It also shows the percentage of concordance for deviations allowed in terms of 1 position change in the ranking (green line), 2 positions change in the ranking (blue line) and 3 positions change in the ranking (red line). The top panel considers the top 10 SIFIs, the middle panel focuses on the top 20 SIFIs and the bottom panel considers all 60 financial institutions that had continuously traded over the sample period. The estimation period is from 01/03/2000 to 12/30/2011.

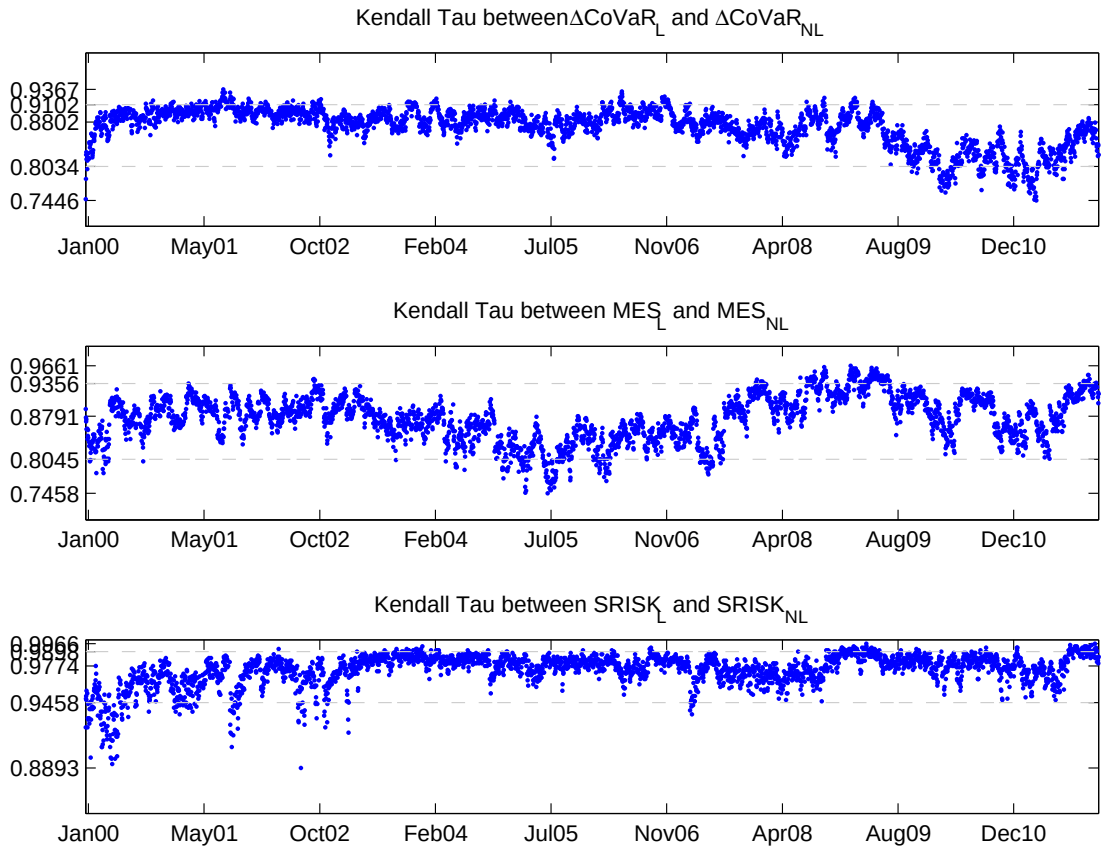


Figure 4.10 – The top figure shows the daily time-varying Kendall rank-order correlation coefficient difference between the $\Delta CoVaR_{NL}$ and $\Delta CoVaR_{L}$ whereas the middle figure shows the daily time-varying Kendall rank-order correlation coefficient difference between the MES_{NL} and MES_{L}, obtained for the 60 financial institutions which are continuously trading over the whole period. Finally, the bottom figure plots the results for the *SRISK*. The estimation period is from 01/03/2000 to 12/30/2011.

Conclusion

De tout temps, les marchés financiers ont rencontré de nombreuses perturbations. Cependant, les risques ayant un impact sur leur fonctionnement évoluent très largement au cours du temps. Les crises financières sont à l'origine des principaux changements et font évoluer la façon dont les intervenants perçoivent le marché. Ainsi, la crise des *subprimes* de 2007-08 a eu un impact retentissant sur la gestion des risques financiers. Ce travail est revenu sur un ensemble de risques qui, en 2008, focalisaient totalement l'attention des intervenants de marché au coeur même de la crise. Cependant, 5 ans plus tard, ces sujets sont restés d'actualité et comme nous pouvons le voir au travers de chacun des chapitres de cette thèse, ils le resteront encore. En effet, mesurer l'ensemble de ces risques demeurent d'une importance capitale. Ce travail a eu pour principal objectif et fil conducteur, la mise en perspective des différents risques financiers que représentent la contagion, la liquidité et le risque systémique. Ainsi, pour chacun d'entre eux, nous nous sommes efforcés de mettre en avant les définitions requises pour pouvoir les mesurer mais également le cadre de travail approprié à leur étude.

Dans le premier chapitre, nous avons construit un indicateur de liquidité de financement basé sur l'interprétation d'un effet de contagion en termes de risque de liquidité de marché. Ce travail propose une application aux fonds indiciels cherchant à répliquer la performance d'un portefeuille de dettes souveraines de pays émergents. Toutefois, ce cadre d'étude peut être étendu à de nombreuses situations. Nous utilisons la relation d'arbitrage qui existe entre les CDS (ou *Credit Default Swap*) et les taux obligataires. Ainsi, nous ob-

tenons un indicateur de la liquidité du marché de la dette émergente. De l'ensemble de ces indicateurs, nous filtrons un facteur de liquidité de financement par l'intermédiaire d'un modèle économétrique modélisant les phénomènes de contagion financière. Les résultats obtenus démontrent qu'il existe des phénomènes de pure contagion concernant les risques de liquidité des marchés de la dette souveraine des pays émergents. De plus, il est particulièrement intéressant de voir que les effets de contagion ne sont pas identiques à ceux présents en termes de prix démontrant que les canaux de transmission des chocs ont évolué et ne se cantonnent plus uniquement à une variation du prix des actifs. Une possible extension de ce travail pourrait être l'étude d'un autre univers d'investissement ou, méthodologiquement, rendre les probabilités de transition du modèle, dépendantes de facteurs tels que la volatilité.

La liquidité de financement qui était l'objet de notre étude dans le premier chapitre est à distinguer de la liquidité de marché. La première se définit comme la facilité à financer ses échanges alors que la seconde se rapporte à la facilité que l'actif a pour être échangé. Leurs modélisations respectives diffèrent très largement. Dans le second chapitre, nous proposons une méta-mesure de la liquidité de marché. En d'autres termes, nous modélisons une relation entre plusieurs mesures de liquidité afin de capturer un facteur de liquidité de marché tenant compte de l'ensemble des dimensions de sa définition. Ainsi, nous définissons un problème de liquidité par la probabilité d'une occurrence simultanée de problèmes de liquidité détectés par différentes mesures caractérisant parfaitement le comportement binaire de la liquidité : "liquide" ou "illiquide". Empiriquement, l'étude des rendements de titres entre 1964 et 2012 montre que notre méta-mesure capture une prime de liquidité significativement positive. De plus, la prime associée à cette méta-mesure n'est pas influencée ni par la taille de la firme, ni par le Bêta associé au titre. Pour finir, nous étudions la capacité que notre mesure a de détecter les titres ayant des problèmes de liquidité drastique. Ainsi, nous nous intéressons à la prime obtenue par une stratégie basée sur une mesure de liquidité standard et nous écartons les titres détectés

par notre mesure. Nous constatons alors que le ratio performance/risque est amélioré sur l'ensemble de la période et sur différents sous échantillons. Cette étude se concentre à démontrer la capacité de notre méta-mesure à estimer le risque de liquidité. Cependant, il serait intéressant dans un travail futur de replacer cette mesure dans une problématique de gestion des risques. Une autre extension serait d'introduire d'autres mesures de liquidité afin de pouvoir capturer d'autres informations concernant la liquidité de marché.

Abordés dans les deux premiers chapitres de cette thèse, la contagion financière et le risque de liquidité sont directement en lien avec la notion de risque systémique. Ce dernier, comme nous avons pu le définir, représente l'impact d'une firme sur l'ensemble du marché. L'utilisation de mesures conditionnelles est donc une solution pour mesurer le risque systémique. Les deux derniers chapitres de ce travail s'appliquent à démontrer l'importance de la prise en compte de la liquidité ou encore de la structure de dépendance entre les rendements de la firme et ceux du marché.

Le troisième chapitre de cette thèse propose une nouvelle modélisation des rendements du marché financier. Nous ajoutons un facteur de liquidité de financement dans le cadre de l'estimation de la $\Delta CoVaR$. Ainsi, les contraintes de capitaux propres des institutions financières sont déterminées tenant compte de la capacité des firmes à financer leurs activités. Les résultats de ce nouveau modèle laissent apparaître une mesure du risque systémique contracyclique. En effet, si nous nous basons sur les résultats de cette nouvelle méthodologie pour définir le niveau de capitaux requis pour chaque institution, ces derniers seraient, relativement aux capitaux demandés basés sur la $\Delta CoVaR$ standard, plus importants en période calme et moindres en période de crise. Ces résultats sont en accord avec les objectifs de la régulation actuelle. Ce travail s'inscrit dans le courant de la littérature sur les mesures du risque systémique. Cependant, selon mes connaissances, il est le premier à introduire directement la liquidité de financement dans l'estimation du risque systémique. Toutefois, une possible extension serait de parvenir à rendre conditionnelle à la liquidité, l'estimation de la $SRISK$. Bien que faisant face à quelques problèmes

méthodologiques, définir un montant de capitaux requis pour traverser une future crise sous contrainte de liquidité semble particulièrement intéressant aussi bien pour la firme que pour le régulateur.

Pour finir, un autre point important dans la mise en place d'une régulation macroprudentielle est la modélisation de la dépendance entre les rendements de marché et ceux de la firme. Pour cela, nous revenons dans le quatrième chapitre sur l'estimation de ces mesures de risque systémique. La structure de dépendance est supposée non linéaire et les méthodes d'estimation tenant compte de cette caractéristique peuvent devenir particulièrement compliquées. Nous avons voulu voir si l'hypothèse de linéarité de cette structure de dépendance pouvait avoir un impact dans l'identification des institutions systémiquement risquées. En étudiant un échantillon d'institutions financières, nos résultats démontrent que faire cette hypothèse n'a pas ou peu de répercutions dans le cadre de régulation standard. Peu importe la mesure utilisée parmi la $\Delta CoVaR$, le MES et la $SRISK$, faire l'hypothèse d'une structure de dépendance linéaire permet de simplifier les méthodes d'estimation et ainsi éviter les possibilités de manipulation des mesures. Ce travail s'étant seulement concentré sur le marché financier nord-américain, il serait intéressant de reproduire cette étude sur une base de données regroupant également des firmes européennes. Une autre extension pourrait être de cartographier les effets de cette hypothèse selon le secteur dans lequel se trouve la firme afin de différencier par exemple, le comportement des banques de celui des assurances.

Bibliographie

Acharya, V. V., Engle, R., and Richardson, M. (2012). Capital shortfall : A new approach to ranking and regulating systemic risks. *American Economic Review*, 102(3) :59–64.

Acharya, V. V., Pedersen, L. H., Philippon, T., and Richardson, M. (2010). Measuring systemic risk. *Federal Reserve Bank of Cleveland*.

Adrian, T. and Brunnermeier, M. K. (2011). Covar. Working paper.

Adrian, T. and Shin, H. S. (2008). Liquidity and financial contagion. *Banque de France Financial Stability Review : Special Issue on Liquidity*, 11 :1–7.

Aitken, M. and Comerton-Forde, C. (2003). How should liquidity be measured ? *Pacific-Basin Finance Journal*, 11(1) :45–59.

Aitken, M. and Winn, R. (1997). What is this thing called liquidity. *Securities Industry Research Center of Asia Pacific Sydney, Australia*.

Amihud, Y. (2002). Illiquidity and stock returns : cross-section and time-series effects. *Journal of Financial Markets*, 5(1) :31–56.

Ammer, J. and Cai, F. (2007). Sovereign CDS and bond pricing dynamics in emerging markets : does the cheapest-to-deliver option matter ? *International Finance Discussion Papers*.

Ang, A. and Bekaert, G. (2002). International asset allocation with regime shifts. *Review of Financial Studies*, 15(4) :1137.

Bai, J. and Collin-Dufresne, P. (2011). The determinants of the cds-bond basis during the financial crisis of 2007-2009. *SSRN eLibrary*.

Balla, E., Ergen, I., and Migueis, M. (2012). Tail dependence in the US banking sector and measures of systemic risk. *Working Paper*.

Basel Committee, o. B. S. (2011). Global systemically important banks : Assessment methodology and the additional loss absorbency requirement. *Bank for International Settlements*.

Batisdon, G., Brasseul, J., and Gilles, P. (2010). *Histoire de la globalisation financière*. A. Colin.

Bauwens, L. and Otranto, E. (2013). Modeling the dependence of conditional correlations on volatility. Technical report, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).

Bekaert, G. and Harvey, C. (2003). Emerging markets finance. *Journal of Empirical Finance*, 10(1-2) :3–56.

Benoit, S., Dudek, J., and Sharifovaz, M. (2013). Linear dependence estimation : a useful tool to identify sifis.

Billio, M. and Caporin, M. (2005). Multivariate Markov switching dynamic conditional correlation GARCH representations for contagion analysis. *Statistical methods and applications*, 14(2) :145–161.

Bisias, D., Flood, M., Lo, A., and Valavanis, S. (2012). A survey of systemic risk analytics. *Available at SSRN 1983602*.

Bollerslev, T. (1990). Modelling the coherence in short-run nominal exchange rates : a multivariate generalized arch model. *The Review of Economics and Statistics*, pages 498–505.

Boudt, K., Paulus, E. C., and Rosenthal, D. W. (2011). Funding liquidity, market liquidity and ted spread : A two-regime model. *Available at SSRN*.

Boyer, B., Gibson, M., and Loretan, M. (1999). Pitfalls in tests for changes in correlations. In *Federal Reserve Boars, IFS Discussion Paper No. 597R*. Citeseer.

Boyson, N. M., Stahel, C. W., and Stulz, R. M. (2010). Hedge fund contagion and liquidity shocks. *The Journal of Finance*, 65(5) :1789–1816.

Brandon, R. G. and Wang, S. (2012). Liquidity risk, return predictability and hedge funds'performance : an empirical study. *Journal of Financial and Quantitative Analysis, forthcoming*.

Brownlees, C. and Engle, R. (2012). Volatility, correlation and tails for systemic risk measurement. *SSRN Working Paper*.

Brunnermeier, M. K. and Pedersen, L. H. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22(6) :2201–2238.

Calvo, S. and Reinhart, C. (1996). Capital flows to Latin America : Is there evidence of contagion effects ? *Policy Research Working Paper Series*.

Chesnay, F. and Jondeau, E. (2001a). Does correlation between stock returns really increase during turbulent periods ? *Economic Notes*, 30(1) :53–80.

Chesnay, F. and Jondeau, E. (2001b). Does correlation between stock returns really increase during turbulent periods ? *Economic Notes*, 30(1) :53–80.

Chiang, T., Jeon, B., and Li, H. (2007). Dynamic correlation analysis of financial contagion : Evidence from Asian markets. *Journal of International Money and Finance*, 26(7) :1206–1228.

Chollete, L., Næs, R., and Skjeltorp, J. (2007). What captures liquidity risk ? a comparison of trade and order based liquidity factors. *Working Paper*.

Choudhry, M. (2006). *The credit default swap basis*. Bloomberg Pr.

Comerton-Forde, C., Hendershott, T., Jones, C. M., Moulton, P. C., and Seasholes, M. S. (2010). Time variation in liquidity : The role of market-maker inventories and revenues. *The Journal of Finance*, 65(1) :295–331.

Dempster, A., Laird, N., and Rubin, D. (1977). Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society. Series B (Methodological)*, pages 1–38.

Drehmann, M. (2013). Total credit as an early warning indicator for systemic banking crises. *BIS Quarterly Review*, page 41.

Drehmann, M. and Tarashev, N. A. (2011). Systemic importance : some simple indicators. *BIS Quarterly review*, pages 25–37.

Duffie, D. (1999). Credit swap valuation. *Financial Analysts Journal*, 55(1) :73–87.

Dungey, M., Fry, R., González-Hermosillo, B., and Martin, V. (2005). Empirical modelling of contagion : a review of methodologies. *Quantitative Finance*, 5(1) :9–24.

Dungey, M., Milunovich, G., Thorp, S., and Yang, M. (2012). Endogenous crisis dating and contagion using smooth transition structural garch. *FIRN Research Paper Forthcoming*.

Edwards, S. and Susmel, R. (2001). Volatility dependence and contagion in emerging equity markets. *Journal of Development Economics*, 66(2) :505–532.

Eichengreen, B., Rose, A. K., and Wyplosz, C. (1996). Contagious currency crises. Technical report, National Bureau of Economic Research.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica : Journal of the Econometric Society*, pages 987–1007.

Engle, R. (2002). Dynamic conditional correlation. *Journal of Business and Economic Statistics*, 20(3) :339–350.

Engle, R., Jondeau, E., and Rockinger, M. (2012). Systemic risk in europe. *Swiss Finance Institute, Research Paper Series NÂř12-45*.

Fama, E. F. and MacBeth, J. D. (1973). Risk, return, and equilibrium : Empirical tests. *The Journal of Political Economy*, pages 607–636.

Fazio, G., Bayoumi, T., Kumar, M. S., and MacDonald, R. (2003). *Fatal Attraction : A New Measure of Contagion*, volume 3. International Monetary Fund.

Fontaine, J.-S. and Garcia, R. (2012). Bond liquidity premia. *Review of Financial Studies*, 25(4) :1207–1254.

Fontana, A. and Scheicher, M. (2010). An analysis of euro area sovereign cds and their relation with government bonds. *SSRN eLibrary*.

Forbes, K. and Rigobon, R. (2001). Measuring contagion : Conceptual and empirical issues. *International financial contagion*, pages 43–66.

Forbes, K. J. and Chinn, M. D. (2004). A decomposition of global linkages in financial markets over time. *Review of Economics and Statistics*, 86(3) :705–722.

Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence : Measuring stock market comovements. *Journal of Finance*, 57(5) :2223–2261.

FSB (2011). Policy measures to address systemically important financial institutions. *Financial Stability Board, Basel*.

FSB (2012). Update of group of global systemically important banks (G-SIBs). *Financial Stability Board, Basel*.

Garcia, R. (1998). Asymptotic null distribution of the likelihood ratio test in markov switching models. *International Economic Review*, pages 763–788.

Garleanu, N. and Pedersen, L. (2009). Margin-based asset pricing and deviations from the law of one price. *Working Paper, UC Berkeley*.

Giamouridis, D. and Vrontos, I. (2007). Hedge fund portfolio construction : A comparison of static and dynamic approaches. *Journal of Banking & Finance*, 31(1) :199–217.

Glick, R. and Rose, A. K. (1999). Contagion and trade : Why are currency crises regional ? *Journal of International Money and Finance*, 18(4) :603–617.

Glosten, L., Jagannathan, R., and Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, pages 1779–1801.

Goyenko, R. (2012). Treasury liquidity and funding liquidity : Evidence from mutual fund returns. *Available at SSRN*.

Goyenko, R., Holden, C., and Trzcinka, C. (2009). Do liquidity measures measure liquidity ? *Journal of Financial Economics*, 92(2) :153–181.

Gromb, D. and Vayanos, D. (2002). Equilibrium and welfare in markets with financially constrained arbitrageurs. *Journal of Financial Economics*, 66(2) :361–407.

Gromb, D. and Vayanos, D. (2010). A model of financial market liquidity based on intermediary capital. *Journal of the European Economic Association*, 8(2-3) :456–466.

Haldane, A. G. (2011). Capital discipline. *Speech at the American Economic Association*, 9.

Hamao, Y., Masulis, R., and Ng, V. (1990). Correlations in price changes and volatility across international stock markets. *Review of Financial Studies*, 3(2) :281.

Hamilton, J. D. and Susmel, R. (1994). Autoregressive conditional heteroskedasticity and changes in regime. *Journal of Econometrics*, 64(1) :307–333.

Hansen, B. E. (2000). Sample splitting and threshold estimation. *Econometrica*, 68(3) :575–603.

Harris, L. (1990). Liquidity, trading rules and electronic trading systems. *Papers*.

Hasbrouck, J. (2004). Liquidity in the futures pits : Inferring market dynamics from incomplete data. *Journal of Financial and Quantitative Analysis*, 39(2).

Holl, T. and Winn, R. (1995). Comparability of different measures of liquidity on the australian stock exchange. Technical report, Working Paper, Security Inudstry Research Center of Asia-Pasific.

Holmes, R. (1991). On random correlation matrices. *SIAM journal on matrix analysis and applications*, 12 :239.

Hull, J. and White, A. (2000). Valuing credit default swaps I : No counterparty default risk. *Journal of Derivatives*, 8(1) :29–40.

Jiang, C. (2012). Does tail dependence make a difference in the estimation of systemic risk ? covar and mes.

Kenourgios, D., Samitas, A., and Paltalidis, N. (2010). Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. *Journal of International Financial Markets, Institutions and Money*.

King, M. and Wadhwani, S. (1990). Transmission of volatility between stock markets. *Review of Financial Studies*, 3(1) :5.

Knif, J., Kolari, J., and Pynnonen, S. (2005). What drives correlation between stock market returns ? international evidence. *NBER working paper*, (8109).

Koenker, R. and Bassett Jr, G. (1978). Regression quantiles. *Econometrica : journal of the Econometric Society*, pages 33–50.

Korajczyk, R. and Sadka, R. (2008). Pricing the commonality across alternative measures of liquidity. *Journal of Financial Economics*, 87(1) :45–72.

Kuan, C.-M. and Michalopoulos, C. (2012). Structural threshold quantile regression. *Working Paper*.

Lesmond, D., Ogden, J., and Trzcinka, C. (1999). A new estimate of transaction costs. *Review of Financial Studies*, 12(5) :1113.

Levy, A. (2009). The cds bond basis spread in emerging markets : Liquidity and counterparty risk effects. Working paper.

Mengle, D. (2007). Credit derivatives : An overview. *Economic Review*, (Q4) :1–24.

Menkveld, A. and Wang, T. (2011). Liquileaks.

Morris, S. and Shin, H. (2004). Coordination risk and the price of debt. *European Economic Review*, 48(1) :133–153.

Naoui, K., Liouane, N., and Brahim, S. (2010). A Dynamic Conditional Correlation Analysis of Financial Contagion : The Case of the Subprime Credit Crisis. *International Journal of Economics and Finance*, 2(3) :85–96.

Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica : Journal of the Econometric Society*, pages 703–708.

Patro, D., Qi, M., and Sun, X. (2013). A simple indicator of systemic risk. *Journal of Financial Stability*, 9(1) :1005–116.

Pelletier, D. (2006). Regime switching for dynamic correlations. *Journal of Econometrics*, 131(1-2) :445–473.

Ramchand, L. and Susmel, R. (1998). Volatility and cross correlation across major stock markets. *Journal of Empirical Finance*, 5(4) :397–416.

Rigobon, R. (2000). A simple test for stability of linear models under heteroskedasticity, omitted variable, and endogenous variable problems. *manuscript, MIT*.

Rigobon, R. (2001). Contagion : How to Measure It ? *NBER Working Papers*.

Rigobon, R. (2003a). Identification through heteroskedasticity. *Review of Economics and Statistics*, 85(4) :777–792.

Rigobon, R. (2003b). On the measurement of the international propagation of shocks : is the transmission stable ? *Journal of International Economics*, 61(2) :261–283.

Rodríguez-Moreno, M. and Pena, J. I. (2013). Systemic risk measures : The simpler the better ? *Journal of Banking & Finance*, 37(6) :1817 – 1831.

Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. *Journal of Finance*, 39(4) :1127–1139.

Scaillet, O. (2003). Nonparametric estimation and sensitivity analysis of expected shortfall. *Mathematical Finance*, 14(1) :115–129.

Scaillet, O. (2005). Nonparametric estimation of conditional expected shortfall. *Insurance and Risk Management Journal*, 74 :639–660.

Solnik, B., Boucrelle, C., and Le Fur, Y. (1996). International market correlation and volatility. *Financial analysts journal*, pages 17–34.

Straetmans, S. and Chaudhry, S. (2013). Tail risk and systemic risk of US and Eurozone financial institution in the wake of the global financial crisis. *Working Paper*.

Teo, M. (2011). The liquidity risk of liquid hedge funds. *Journal of Financial Economics*, 100(1) :24–44.

Van Rijckeghem, C. and Weder, B. (2001). Sources of contagion : is it finance or trade ? *Journal of International Economics*, 54(2) :293–308.

Wang, K. and Nguyen Thi, T. (2007). Testing for contagion under asymmetric dynamics : Evidence from the stock markets between US and Taiwan. *Physica A : Statistical Mechanics and its Applications*, 376 :422–432.

Zakoian, J. (1994). Threshold heteroskedastic models. *Journal of Economic Dynamics and control*, 18(5) :931–955.