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Essais sur la fraude à l'impôt sur le revenu

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L'évasion et la fraude fiscales mettent en péril les recettes des États du monde entier. [...] Le phénomène se traduit par une contraction des ressources disponibles pour financer les infrastructures et influe sur les conditions de vie de tous, tant dans les économies développées que dans les économies en développement. La mondialisation offre des perspectives d'accroissement de la richesse mondiale, mais multiplie aussi les risques.

Rapport de l'OCDE (2010)

Résumé

L'objectif central de cette thèse est d'étudier le comportement de fraude fiscale des contribuables quand ils ne déclarent qu'une partie de leur revenu.

Le premier chapitre complète la littérature existante en étudiant le niveau de déclaration du revenu et les effets de changements des taux de taxe, de pénalité et de probabilité de contrôle, en considérant des fonctions d'imposition et de pénalité non linéaire, dans le cadre de la théorie de l'espérance de l'utilité.

Le cadre fourni par la théorie des perspectives cumulatives est ensuite utilisé dans le second chapitre. L'accent est mis sur la dépendance des décisions du contribuable vis-à-vis du revenu de référence introduit par cette théorie.

Le troisième chapitre caractérise le barême optimal d'imposition du revenu et la stratégie de contrôle et de pénalité que doit mettre en place l'État quand le comportement de fraude des contribuables vérifie les propriétés de la théorie des perspectives.

Mots clés : fraude fiscale, non linéarité, théorie de l'espérance de l'utilité, théorie des perspectives, dépendance vis-à-vis du point de référence, stratégie de collecte de l'impôt sur le revenu.

Abstract

This dissertation analyzes the tax evasion behavior of taxpayers when they do not declare their entire income.

The first chapter studies the declaration of the taxpayer and the effects of changes in the tax rate, the penalty rate and the probability of audit. The tax and the penalty functions are assumed to be non linear. The setting is provided by expected utility theory.

The setting provided by cumulative prospect theory is used in the second chapter. Reference dependence, which is a central point in this theory, is particularly studied.

The third chapter characterizes the optimal income tax and audit schemes under tax evasion behavior, when of taxpayers behave as predicted by prospect theory.

Keywords : tax evasion, non linearity, expected utility theory, prospect theory, reference dependence, income tax enforcement.

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Introduction générale

Les enjeux de la fraude fiscale

L'impôt est un élément essentiel du lien qu'entretient l'État avec les membres de la société. Il dépend étroitement des institutions politiques, juridiques et économiques. Or, si le pouvoir fiscal est communément possédé par les représentants du peuple, ce qui assoit la légitimité du prélèvement, ce dernier se fait par voie d'autorité par l'administration, ce qui représente une forme de contrainte. L'absence de contrepartie directe et de visibilité sur l'utilisation des fonds récoltés, renforcent encore ce sentiment chez les contribuables. Cela explique pourquoi le phénomène d'évitement de l'impôt est sans doute aussi ancien que les premiers systèmes centralisés de prélèvement. Ainsi, certaines pratiques frauduleuses ont été recensées dans les sociétés grecque et romaine antiques.

Dans les sociétés d'aujourd'hui, les systèmes fiscaux sont complexes, avec une personnalisation et une volonté d'impôts synthétiques. La technique de la déclaration est parfois utilisée, comme par exemple en France depuis la réforme Caillaux de 1914-1917. Les déclarations ne pouvant être mises en doute qu'après vérification de la part de l'administration, cela accroît l'opportunité de fraude.

À des degrés divers, liés par exemple au degré de stabilité économique ou au niveau d'organisation de l'administration, tous les pays subissent le poids social et budgétaire créé par la fraude fiscale.

Un concept difficile à définir

Derrière l'idée de base que la fraude fiscale correspond au fait de se soustraire partiellement ou totalement au paiement de l'impôt, la notion reste difficile à cerner précisément à cause de l'ampleur, de la complexité et de la diversité des situations qu'elle regroupe. Le degré d'intention et de comportement actif peuvent par exemple varier de manière significative.

De plus, la frontière est mince avec le fait d'utiliser habilement des procédés juridiques permettant d'échapper à l'impôt sans contrevenir à la loi, correspondant à une situation dite d'évasion fiscale. L'évasion fiscale pouvant elle-même être admise ou refusée par les États, notamment lorsqu'elle concerne l'utilisation de divergences existantes entre les systèmes fiscaux de différents pays, la confusion entre ces notions peut facilement exister. Il faut d'ailleurs noter que la traduction du français à l'anglais peut tromper. Fraude fiscale se traduit par *tax evasion* et evasion fiscale par *tax avoidance*.

La difficulté à cerner le phénomène de fraude fiscale explique pourquoi sa définition peut différer d'une source à l'autre. On peut cependant mettre en avant trois éléments sur lesquels se fonde systématiquement sa qualification : la réalisation d'une opportunité de minimisation de l'impôt, l'intention et la violation de la loi.

Un phénomène difficile à quantifier

Même avec une définition précise, la fraude fiscale étant un acte illégal, elle est par principe emprunte de secret. Aucune mesure exacte de son ampleur ne peut donc être considérée comme totalement fiable. De plus, la multiplicité des pratiques rend le travail d'évaluation très complexe. Même en se concentrant par exemple sur le défaut de déclaration, qui représente la fraude la plus élémentaire, les actes frauduleux peuvent concerner des personnes physiques ou morales, des impôts directs ou indirects et peuvent correspondre à une sous-déclaration volontaire ou bien à une activité souterraine, chacune des situations pouvant nécessiter un traitement particulier.

Il existe deux types de méthode d'évaluation de l'ampleur de la fraude fiscale : les méthodes directes et les méthodes indirectes. Les premières utilisent la confrontation des informations comprises dans les déclarations des contribuables, avec un échantillon de déclarations intensivement contrôlé ou avec des données tirées d'enquêtes. Elles peuvent être mises en oeuvre directement par l'administration fiscale. Les autres évaluent le niveau d'économie occulte (travail et revenus dissimulés) en se servant des traces qu'elle laisse sur les marchés et agrégats économiques.

Voici quelques résultats permettant d'approcher le niveau réel de fraude fiscale dans différentes sociétés. L'INSEE en France pour l'année 2006, le HM Revenues and Customs au Royaume-Uni pour 2004 et l'Internal Revenue Service aux États-Unis pour 2001, ont estimé que la différence entre les taxes dûes et les taxes perçues s'était élevée respectivement à 8%, 9% et 16% du revenu total des taxes (INSEE [37], HMRC [36], IRS [38]). Il n'est pas rare de lire que les administrations fiscales des pays en cours de développement ne perçoivent pas la moitié des recettes fiscales initialement prévues, comme au Maroc dans les années 80 par exemple (Rahj [61]).

Les enjeux individuels de la fraude

Dans la plupart des pays, la fraude fiscale est une activité risquée. En France, elle peut par exemple être qualifiée de délit pénal à la suite d'une procédure judiciaire et peut être sanctionnée par une amende, l'obligation de publier le jugement dans des journaux nationaux, la perte temporaire des droits civiques ou même une peine de prison.

Parmi eux apparaît bien sûr en premier lieu l'intérêt financier. L'espérance du gain obtenu en cas d'absence de contrôle peut être mieux évalué par un contribuable, que le risque encouru. Mais ce n'est pas le seul motif pouvant être mis en évidence. Le goût du jeu peut expliquer un comportement frauduleux. Le refus de toute contrainte, la mentalité ou les opinions politiques (anti-étatique ou bien en désaccord avec les politiques publiques) sont également importantes dans la décision de fraude. Enfin, la conjoncture économique ou une pression fiscale trop forte, c'est-à-dire mal adaptée à la situation économique, peuvent expliquer le recours à la fraude.

Les enjeux collectifs de la lutte contre la fraude

Les choix du niveau et de l'assiette des impôts est une question sociale et politique autant qu'économique. La légitimité du système de prélèvement au sein d'une société, dépend en grande partie de celle des représentants des membres de celle-ci. Et les choix en matière d'imposition peuvent se révéler très différents d'un pays à l'autre. La mise en place d'un système fiscal adapté dépend de la situation économique et sociale.

De même que la fiscalité, l'existence de fraude fiscale est un problème social et politique autant qu'économique. En plus d'un problème évident de revenu public, la fraude fiscale pose une question de justice et d'équité, autant verticale, un contribuable aisé peut ainsi moins contribuer à l'impôt qu'un autre moins riche, qu'horizontale, deux contribuables de même niveau de richesse peuvent ainsi ne pas contribuer à l'impôt de la même manière.

La formalisation du comportement de fraude

Préciser quels schémas d'incitation peuvent favoriser l'honnêteté des contribuables, constitue donc l'un des principaux enjeux de la réflexion relative à la fraude fiscale. Dans un contexte où l'ampleur et l'impact réels des comportements fiscaux frauduleux ne peuvent être évalués que de manière approximative, une formalisation peut servir de point d'appui au raisonnement économique sur le comportement de fraude et sa prise en compte dans le choix d'un barême d'imposition. Plus précisément, la compréhension de la décision du contribuable à travers une modélisation mathématique, peut constituer un préliminaire aux actions concrètes permettant d'inclure le phénomène dans les décisions fiscales ou même de le contraindre. La présente thèse s'inscrit dans cette idée.

La tentation étant grande pour des agents de petite taille, isolés et dont le paiement de l'impôt dépend de leur propre déclaration, de se soustraire à celui-ci, l'imposition est nécessairement accompagnée d'une politique de contrôle, devant inciter à l'honnêteté. Cette dernière implique l'apparition d'un risque. L'analyse microéconomique de l'incertain fournit donc le cadre conceptuel nécessaire à l'obtention de réponses dépassant la simple observation des faits.

L'étude de la fraude n'a pas seulement été le domaine de l'économie. L'utilisation d'outils économiques modernes pour formaliser spécifiquement le comportement de fraude a commencé avec Allingham et Sandmo [2] et Srinivasan [72], qui ont adapté à la taxation le travail fondateur de Becker [7], en utilisant le modèle de l'espérance de l'utilité dû à Von Neumann et Morgenstern [81]. Ils ont ainsi montré qu'avec une aversion absolue au risque décroissante, l'hypothèse la plus communément admise, lors d'une augmentation du taux de taxe, les effets revenu et de substitution agissent en sens inverse sur la déclaration du revenu. Yitzhaki [83] est venu compléter ces modèles en considérant une pénalité appliquée non pas au revenu non déclaré, mais à l'impôt impayé, ce qui reflète mieux la réalité de beaucoup de pays. Ce travail montre que les effets revenu et de substitution agissent dans le même sens, avec une augmentation du revenu déclaré en cas d'uagmentation du taux de taxe. Depuis, la littérature sur la fraude fiscale s'est étoffée, avec notamment le travail théorique détaillé de Cowell [21], qui montre notamment que le montant du revenu non déclaré augmente avec le revenu si et seulement si l'aversion absolue au risque du contribuable est décroissante et que la part du revenu non déclaré augmente avec le revenu si et seulement si l'aversion relative au risque du contribuable est décroissante. Slemrod et Yitzhaki [71] étudie en particulier les différences entre fraude et évasion fiscale et montre que si ces notions sont bien distinctes théoriquement, elles sont parfois difficilement différenciables en pratique. Cowell [22] met en évidence les limites des modèles couramment employés et propose des hypothèses alternatives en se basant sur des études empiriques critiquant les quatre assertions suivantes : i) si la fraude est profitable alors tout le monde fraude, ii) les contribuables ayant une plus grande aversion au risque fraudent moins, iii) les contribuables à haut revenu fraudent plus et iv) augmenter l'un des paramètres du système fiscal (taux de taxe, probabilité de contrôle, pénalité) réduit la fraude. Marchese [49] quant à lui, s'intéresse particulièrement aux décisions publiques et Slemrod [70] fait le bilan de ce qui a été démontré et observé sur l'amplitude, la nature et les déterminants de la fraude fiscale, notamment aux États-Unis, et étudie différents modèles et leurs implications en terme de politiques publiques. Comme revue de la littérature on peut également citer Andreoni et al. [4] et Franzoni [31] qui exposent les avancées de la recherche théorique et empirique sur la fraude fiscale et la conformité à l'impôt, et Cohen et Tallon [20] qui analyse les avancées sur le champ plus général qu'est la théorie des décisions en environnement incertain.

L'étude de la sous-déclaration du revenu, la base d'une analyse générale

Comme évoqué plus haut, la question de la fraude dans les systèmes fiscaux modernes, est étroitement liée à la technique de la déclaration. Quand l'impôt est prélevé à la source, la fraude est plus difficile à organiser car elle implique quatre personnes ou entités : le contribuable, l'entreprise, la banque et l'administration. Elle nécessite donc qu'il y ait collusion avec les déclarants que sont l'entreprise et la banque. Dans un système déclaratif, comme en France par exemple, l'impôt auquel l'administration fiscale soumet un contribuable dépend en premier lieu de la déclaration qu'il fait concernant son revenu ou tout ce qui peut concerner son niveau de richesse ou ses contributions économiques à la société. Cela explique pourquoi la fraude la plus élémentaire et la plus courante, consiste en une sous-déclaration du revenu ou de la richesse. La multiplicité des opportunités et des stratégies de fraude en complique l'analyse, mais un préliminaire à une analyse plus large consiste à axer la réflexion sur la forme la plus élémentaire : la décision de fraude d'un contribuable par sous-déclaration volontaire des revenus lors de l'établissement de la déclaration d'impôt. C'est l'objet de la présente thèse.

L'analyse correspond à l'approche initiée par Allingham et Sandmo [2] et Srinivasan [72], et complétée par Yitzhaki [83]. Le contribuable est confronté à un problème de choix en présence de risque. Il connaît son revenu et les dispositions de la législation fiscale, donc les impôts qu'il devrait normalement acquitter et la pénalité qu'il encourt en minimisant son revenu en totalité, ou pour partie. Conscient de l'asymétrie d'information dont il bénéficie, il sait que c'est par sa déclaration que l'autorité fiscale prend connaissance de sa situation. Dès lors, il bénéficie d'une opportunité de fraude. S'il l'utilise, il est face à deux états de la nature. Soit l'administration ne détecte pas la fraude et son revenu final est plus élevé que ce qu'il aurait été sous honnêteté totale. Soit la fraude est détectée et son revenu est moins élevé, à cause de la pénalité à laquelle il est alors soumis.

Les facteurs de décision de la sous-déclaration du revenu

Dans ce contexte, la fraude fiscale correspond à une prise de décision dans l'incertain. Et plusieurs paramètres fondamentaux affectent la décision optimale du contribuable. Certains le concernent directement, comme son niveau réel de revenu, que l'administration fiscale ne connait pas, ce qui lui fournit l'opportunité de fraude, et son attitude face au risque, qui caractérise son comportement général face à une situation incertaine et qui peut être formalisé de différentes façons, comme il sera vu plus loin. Les autres paramètres affectant la décision du contribuable concernent le système fiscal mis en place par l'administration. Il s'agit du barême d'imposition, se présentant généralement sous la forme d'une fonction croissante du revenu, de la probabilité de détection, soit constante soit dépendante du revenu déclaré par le contribuable, et de la pénalité encourue en cas de détection, fonction de l'impôt impayé ou bien du revenu non-déclaré.

D'autres paramètres peuvent être introduits. Le plus couramment utilisé, comme chez Benjamini et Maital [8], Gordon [33] ou al-Nowaihi et Pyle [1], est la conséquence morale d'une détection, liée par exemple à la réputation au sein de la communauté. Le choix est fait ici d'étudier le cadre le plus élémentaire pour mettre en avant les interactions entre les paramètres qualifiés plus haut de fondamentaux et fournir un préliminaire aux études prenant en compte d'autres aspects.

La maximisation de l'utilité

Dans le modèle d'Allingham et Sandmo [2] et la plupart des analyses sur la question, le contribuable est supposé totalement rationnel. Sa décision est guidée par les même éléments que n'importe quelle autre décision de consommation, il compare gain et coût et cherche à maximiser l'espérance de l'utilité associée à la loterie fiscale. Plus précisément, sa fonction d'utilité ne dépend que de son revenu final et son comportement est conforme aux axiomes de von Neumann-Morgenstern. Son attitude face au risque est principalement caractérisée par les mesures d'aversion absolue et relative au risque, d'Arrow-Pratt. L'aversion absolue au risque est d'ailleurs généralement supposée décroissante avec le revenu, ce qui signifie que plus un contribuable est riche, plus il saisit l'opportunité de frauder. Ce cadre d'analyse est communément appelé théorie de l'espérance de l'utilité (*expected utility theory* en anglais).

De nombreuses études ont cependant mis en avant des failles importantes dans la prévision du comportement de fraude dans ce cadre (notamment le fait qu'une augmentation du taux de taxe aurait pour effet de diminuer la fraude fiscale). Des alternatives à cette théorie ont donc été développées sur le comportement en situation de risque en général et sur la fraude et l'évasion fiscale en particulier. On peut citer parmi d'autres Bernasconi [9], Eide [29] et Bernasconi et Zanardi [10]. Ces modèles prévoient une gamme plus large de réponses des contribuable à la question de la fraude en tenant compte par exemple de la surestimation de probabilités basses ou du traitement différent des gains et des pertes par rapport à un point de référence.

Plus exactement, une des principales alternatives proposées tire son origine des travaux de Kahneman et Tversky [40] et Tversky et Kahneman [79]. Il s'agit de ce qui peut être appelé la théorie des perspectives cumulatives (*cumulative prospect theory* en anglais). Le contribuable n'y est pas considéré comme raisonnant de la même manière dans la mesure où sa décision ne se fonde pas sur l'espérance de son utilité. Il ne réfléchi pas seulement en fonction de son revenu final, mais en terme de gains et de pertes par rapport à un revenu de référence, qu'il établit dès le début du processus de décision. Contrairement à la théorie de l'espérance de l'utilité, la théorie des perspectives cumulatives n'impose pas que l'utilité marginale soit décroissante. Celui-ci est supposé averse au risque dans le domaine des gains mais cherchant le risque dans le domaine des pertes, ce qui signifie que si son revenu disponible est inférieur à son revenu de référence, il préfère courir un risque important plutôt que de rester dans ce domaine. Autre différence significative avec la théorie de l'espérance de l'utilité, le cadre de la théorie des perspectives cumulatives considère que les individus ont tendance à minimiser la probabilité d'évènements très probables et à maximiser celle d'évènements peu probables. L'aversion au risque et l'utilité marginale décroissante ne sont plus autant liés et le revenu de référence et la manière d'appréhender les probabilités sont deux paramètres supplémentaires influant sur la décision de fraude du contribuable. La formalisation de son attitude face au risque est donc significativement différente et il sera montré plus loin que les prévisions de son comportement de fraude diffèrent également.

La recherche sur le comportement de fraude des individus

Bien que maniant des concepts très proches, la recherche sur la fraude venant des entreprises et celle sur la fraude venant des individus n'abordent pas exactement les mêmes questions et n'utilisent pas les mêmes outils, car les enjeux, les montants et les origines sont différents. Sur le comportement de fraude des individus, plusieurs axes de recherche importants peuvent actuellement être mis en évidence.

L'influence de différents facteurs sur la décision de frauder est toujours étudiée, comme dans Kim [43], qui montre que la fraude est influencée par la volonté du gouvernement de contrôler toute l'économie ou Tonin [75], qui montre que l'existence d'un salaire minimum diminue la fraude des contribuables à faible revenu.

Par ailleurs, comme dans d'autres domaines de la recherche en économie, le choix d'une approche dynamique plutôt que statique est régulièrement fait, avec un temps soit discret soit continu et un horizon de temps le plus souvent infini, sur le modèle de Merton [50]. Un des premiers travaux sur le sujet a été celui de Lin et Yang [48], qui dit résoudre le paradoxe soulevé par le résultat de Yitzhaki [83], résultat par ailleurs contesté par Dzhumashev et Gahramanov [28]. Caballé et Panadés [13] et Niepelt [52] peuvent être également cités.

Mais un des principaux axes de recherche se situe dans le champ d'interaction entre l'économie et la psychologie et plus exactement dans l'économie comportementale. Les travaux étudient comment la psychologie, les normes sociales et la morale peuvent influencer la décision de frauder. Concernant la psychologie, peuvent être cités Hammar et al. [34], qui utilise des données suédoises pour montrer que la confiance des contribuables envers les hommes politiques est une donnée indispensable de la collecte des impôts, Vihanto [80], qui fournit un cadre à l'étude de l'hypothèse selon laquelle le choix de frauder dépend de la perception par le contribuable de la justice du système fiscal, ou Cullis et al. [24], qui met en évidence l'existence d'un effet lié au ressentiment que peut avoir un contribuable envers un système fiscal qu'il trouve excessif. Traxler [77] étudie la fraude et les normes sociales en analysant l'interdépendance des comportements des contribuables et Richardson [63] mène des études empiriques dans différents pays sur le lien entre la culture et le niveau de fraude, ce qui étend l'étude internationale de Tsakumis et al. [78]. Concernant la morale, on peut notamment citer Eisenhauer [30], qui introduit un paramètre de moralité et analyse le degré de moralité pouvant expliquer les hauteurs et variations de fraude au cours des dernières décennies aux États-Unis. Dell'Anno [25] montre lui que tous ces aspects sont liés et influencent de manière significative la hauteur de la fraude.

Les enjeux et la structure de la thèse

Les principaux enjeux de la thèse

L'enjeu de cette thèse est de compléter l'analyse sur les déterminants de la décision individuelle de fraude. Plus exactement, il s'agit de savoir comment les modifications des paramètres fiscaux attachés au contrôle (barême de taxe, probabilité de contrôle, taux de pénalité), influent sur le comportement du contribuable, donc sur l'ampleur de la fraude, et de savoir comment ce comportement doit être pris en compte par l'État quand il met en place un barême de taxe optimal. L'analyse ne se situe pas dans l'axe psychologique de la recherche actuelle sur la fraude fiscale. Elle porte sur des questions déjà abordées et se fait directement dans le cadre fondateur d'Allingham et Sandmo [2] et Yitzhaki [83]. La volonté de rester dans un cadre et sur des problématiques ne correspondant pas aux derniers thèmes de recherche vient de l'observation que tous les aspects n'avaient pas encore été abordés et que mener une étude théorique approfondie de la théorie des perspectives dans ce cadre était un préliminaire à l'ajout de nouveaux paramètres. Cette analyse se fait selon trois axes privilégiés.

Le premier axe se situe dans la continuité des contributions initiales d'Allingham et Sandmo [2], Srinivasan [72] et Yitzhaki [83]. Dans le contexte de la première d'entre elles, l'administration impose une taxe proportionnelle au revenu du contribuable. Mais, ne connaissant pas son revenu réel, elle ne peut imposer la taxe qu'à son revenu déclaré. Cela donne la possibilité à ce dernier de minimiser le montant de la taxe qu'il doit payer, en choisissant de déclarer un revenu inférieur à son revenu réel. Pour inciter à l'honnêteté et recouvrer une partie des taxes non récoltées, l'administration contrôle le contribuable selon une probabilité constante et, en cas de fraude, impose une pénalité proportionnelle au revenu dissimulé. Le choix du revenu déclaré constitue donc une activité risquée pour le contribuable. Il l'effectue en maximisant son espérance de l'utilité, conforme aux axiomes de von Neumann-Morgenstern. Il est supposé averse au risque, au sens des mesures d'aversion au risque d'Arrow-Pratt. Les conditions d'existence d'une solution intérieure sont supposées réunies. Il est alors possible de déterminer les effets sur la déclaration, de variations du taux d'imposition, de la probabilité de détection et du taux de pénalité. Alors qu'une augmentation des variables de contrôle diminue la fraude, l'influence du taux de taxe n'est pas clairement établie. Sous l'hypothèse d'une aversion au risque croissante, une augmentation du taux de taxe diminue la fraude. Mais sous celle d'une aversion au risque décroissante, l'hypothèse la plus naturelle, une augmentation du taux de taxe a un effet ambigüe sur la déclaration du contribuable. L'effet revenu et l'effet de substitution agissent en fait de manière opposée. Le premier réduit la fraude car l'augmentation de taxe rend le contribuable moins riche et donc plus averse au risque. Le second l'augmente car elle est plus profitable à la marge. Srinivasan [72] poursuit l'analyse avec un contribuable neutre au risque, qui maximise donc l'espérance de son revenu.

Le choix de la forme de la pénalité est essentiel dans cette analyse. Si dans certains pays elle est appliquée au revenu non déclaré, comme au Canada par exemple, dans la plupart d'entre eux, l'administration l'applique à la taxe non payée. En l'appliquant à la taxe non payée plutôt qu'au revenu non déclaré, Yitzhaki [83] montre qu'une augmentation du taux de taxe a pour effet de diminuer la fraude d'un contribuable dont l'aversion au risque est décroissante, car dans ce cas, l'effet de substitution agit dans le même sens que l'effet revenu, la fraude étant moins profitable à la marge car la pénalité augmente en même temps que la taxe.

Le résultat de Yitzhaki [83] allant à l'encontre de l'intuition et de la plupart des études empiriques (voir Andreoni et al. [4] ou Slemrod et Yitzhaki [71] par exemple), de nombreuses analyses ont cherché à modifier l'une ou l'autre de ses hypothèses, la plupart en introduisant un nouveau paramètre. Parmi d'autres, Koskela [45] introduit un changement à la progressivité du barême de taxe, pour compenser l'effet de la fraude. Scotchmer et Slemrod [67] et Scotchmer [66] introduisent de l'incertitude sur le montant du revenu découvert par l'administration en cas de contrôle. Benjamini et Maital [8], Gordon [33] et al-Nowaihi et Pyle [1] ajoutent la conséquence morale et sociale pour un contribuable, qu'induit la détection d'une fraude. Jung et al. [39] considèrent une économie où co-existent deux secteurs d'emplois offrant différents degrés d'opportunité de fraude. Panadés [53] étudie l'équivalence Ricardienne. Lee [47] introduit la possibilité de s'assurer contre une éventuelle pénalité. Slemrod et Yitzhaki [71] et Bayer [6], enfin, supposent que la probabilité de contrôle dépend du revenu que déclare le contribuable à l'administration.

Depuis les contributions d'Allingham et Sandmo [2] et Yitzhaki [83], de nombreuses hypothèses alternatives ont été considérées, mais toujours avec des taux constants de taxe et de pénalité. L'enjeu de la première partie de cette thèse est d'étudier le problème de la déclaration du revenu des contribuables, en introduisant de la non-linéarité dans les fonctions de taxe et de pénalité. Cela correspond mieux aux fonctions réelles, la plupart des fonctions de taxe et un grand nombre de fonctions de pénalité étant convexes et cela permet de connaitre le role de la convexité de ces fonctions dans le choix de fraude du contribuable.

Le second enjeu de la thèse est d'étudier les déterminants du comportement de fraude dans un cadre différent du précédent. Il s'agit plus exactement, de considérer une manière plus souple de modéliser le comportement face au risque, en utilisant la théorie des perspectives cumulatives, plutôt que celle de l'espérance de l'utilité. Bien que cette dernière soit l'approche la plus communément retenue pour décrire les choix risqués, le contenu descriptif de son modèle de décision a été fréquemment critiqué, notamment son axiome d'indépendance à travers une expérience restée cèlèbre sous le nom de paradoxe d'Allais. La théorie de l'espérance de l'utilité dépendant du rang (*rank-dependent expected utility theory* en anglais) généralise la théorie de l'espérance de l'utilité en utilisant une transformation monotone de la distribution des probabilités cumulées, qui peut prendre en compte le fait que les évènements extrêmes et très peu probables sont en général sur-évalués par les individus. Ce cadre a principalement été développé par Quiggin [57] et [58]. Kahneman et Tversky [40] ont par ailleurs développé la théorie des perspectives (*prospect theory* en anglais), où un individu raisonne en terme de gains et de pertes par rapport à un point de référence, comme alternative à la théorie de l'espérance de l'utilité. Avec Tversky et Kahneman [79], l'idée centrale de la théorie de l'espérance de l'utilité dépendant du rang, a par la suite été intégrée dans la théorie des perspectives, pour donner la théorie des perspectives cumulées, une des principales alternatives à la théorie de l'espérance de l'utilité et qui donne le cadre d'analyse de la seconde partie de cette thèse.

L'étude des déterminants de la décision de fraude dans le cadre de la théorie des perspectives et de celle des perspectives cumulées a fait l'objet d'un petit nombre de contributions. Alm et al. [3] fournissent une étude expérimentale. Yaniv [82] analyse l'influence des prélèvements d'impôt à la source sur la décision des contribuables. Pour mettre en avant la dépendance du choix de fraude vis-à-vis du point de référence, Bernasconi et Zanardi [10] utilisent la théorie des perspectives cumulées avec un revenu de référence général, mais avec une fonction de pondération des probabilités et une fonction d'utilité particulières. Dhami et al-Nowaihi [26] fournissent une étude globale des déterminants de la décision de fraude avec le revenu après taxe légal comme revenu de référence, car c'est le seul pour lequel le contribuable est toujours dans le domaine des pertes en cas de contrôle et dans celui des gains sans contrôle.

L'enjeu de la seconde partie de la thèse est donc de mettre en avant les spécificités de la théorie des perspectives et de celle des perspectives cumulées, utilisées dans le cadre le plus général possible. L'accent est mis en particulier sur l'influence du point de référence sur la décision de fraude.

La troisième partie de cette thèse se distingue des deux premières par l'objet étudié. L'analyse ne porte plus sur la décision individuelle de fraude mais sur les choix que doit faire l'État concernant le barême d'impôt sur le revenu et la stratégie de contrôle, en intégrant la possibilité de fraude de la part des contribuables. Les comportements de fraude sont décrits par la théorie des perspectives car il apparaît dans la deuxième partie de la thèse que ses prévisions correspondent généralement aux résultats mis en avant par les études empiriques.

La caractérisation d'un barême de taxe optimal en présence d'un coût de mise en place dû à la fraude (à cause des contrôles qu'elle implique) a déjà été étudiée, notamment par Chander et Wilde [17]. En décrivant les décisions de fraude grâce à la théorie de l'espérance de l'utilité et en supposant que les contribuables sont neutres au risque, c'est-à-dire que seule l'espérance de leur revenu final compte, ils analysent la nature des relations qui existent entre les taux de taxes optimaux, les probabilités de contrôle et les pénalités imposées en cas de fraude avérée. Dans la continuité de cette contribution importante, Chander [15] introduit l'hypothèse généralement considérée comme plus satisfaisante, de contribuables averses au risque. Mais aucune contribution n'a jusqu'à maintenant étudié cette question en décrivant le comportement frauduleux grâce à la théorie des perspectives. Cela fait l'objet de la dernière partie de la présente thèse.

Le plan de la thèse

Chacun des trois axes présentés précédemment fait l'objet d'une contribution théorique présentée dans un chapitre de la thèse.

Le premier chapitre, intitulé **Tax evasion under expected utility theory**, est issu d'un travail réalisé en collaboration avec Alain Trannoy et complète la littérature en étudiant le niveau de défaut de déclaration du revenu et les effets de changements des taux de taxe, de pénalité et de probabilité, en considérant des fonctions d'imposition et de pénalité non linéaire, dans le cadre de la théorie de l'espérance de l'utilité. Tous les cas sont envisagés, avec une pénalité appliquée à l'impôt impayé ou au revenu non déclaré, avec une solution intérieure (quand le contribuable déclare une partie de son revenu) ou une solution de coin (quand il ne déclare rien ou déclare tout son revenu), avec une probabilité de contrôle exogène (quand elle ne dépend pas de la déclaration du contribuable) ou endogène (quand elle dépend de la déclaration). Ce travail permet de généraliser au maximum le modèle fondamental d'Allingham et Sandmo [2] et Yitzhaki [83] et nécessite l'utilisation du théorème des fonctions implicites dans un cadre non linéaire. Sont mises en évidence en particulier, des conditions sous lesquelles le sens de la relation entre le taux d'imposition et le niveau de fraude est clairement défini. Ces conditions portent sur les paramètres du schéma de taxation et de contrôle et sur la forme de l'utilité du contribuable, notamment quand celui-ci présente une aversion au risque décroissante, qui est le cas généralement considéré comme le plus réaliste et qui n'est pas systématiquement étudié quand la pénalité est appliquée au revenu non déclaré. Elles peuvent servir de support à un travail expérimental sur la modélisation du comportement de fraude par la théorie de l'espérance de l'utilité.

Le second chapitre, intitulé **Solving the Yitzhaki paradox : Tax evasion under cumulative prospect theory**, étudie les mêmes relations que le premier chapitre, mais dans le cadre de la théorie des perspectives cumulatives. Pour souligner les différences entre la théorie de l'espérance de l'utilité et celles des perspectives, la fonction de pondération des probabilités et le revenu de référence sont introduits l'un après l'autre. Par ailleurs, la dépendance des décisions de l'individu vis-à-vis du point de référence est un élément essentiel de la théorie des perspectives. Pour mettre en avant les déterminants de la fraude, différentes valeurs fondamentales du revenu de référence sont donc considérées : le revenu final minimum que le contribuable peut avoir, le revenu final légal et le revenu initial. Les résultats sont ensuite étendus aux autres valeurs possibles du revenu de référence. Il est ainsi mis en évidence que la théorie des perspectives offre un cadre adapté à l'analyse du comportement de sous-déclaration du revenu et que les relations obtenues dans le cadre de la théorie de l'espérance de l'utilité ne sont pas systématiquement vérifiées. En particulier, il est prédit que la fraude fiscale augmente avec le taux de taxe dès que l'aversion relative au risque (adaptée à la théorie des perspectives) est plus élevée en cas de contrôle que sans contrôle. L'intuition est qu'une augmentation du taux de taxe augmente les revenus calculés par rapport au revenu de référence et qu'avec une aversion relative au risque décroissante, le contribuable choisit donc d'augmenter la fraction de son revenu qu'il place dans l'alternative risquée. L'effet de substitution apparaissant dans Yitzhaki [83] s'annule, tandis que l'effet revenu agit dans le sens inverse grâce à la présence du revenu de référence. Cela correspond mieux aux résultats mis en évidence par la littérature empirique et montre ainsi que la théorie des perspectives est un cadre particulièrement adapté à la modélisation du comportement de fraude.

Enfin, le troisième chapitre, **Optimal tax enforcement under prospect theory**, est issu d'un travail réalisé en collaboration avec Amedeo Piolatto et caractérise le barême optimal d'imposition du revenu et la stratégie de contrôle et de pénalité que doit mettre en place l'État quand le comportement de fraude des contribuables vérifie les propriétés de la théorie des perspectives. Il y est démontré que les résultats mis en évidence par Chander et Wilde [17] et Chander [15] dans le cadre de la théorie de l'utilité espérée, sont toujours vrais sous de faibles conditions. Plus précisément, sous une condition généralement vérifiée concernant une mesure d'aversion au risque adaptée à la théorie des perspectives, la probabilité de contrôle doit être décroissante avec le revenu déclaré et la fonction d'imposition doit être croissante et concave. La motivation de ne pas déclarer tout son revenu est trop forte chez les contribuables riche, pour qu'il puisse être optimal de choisir une fonction d'imposition progressive.

Chapitre 1

Tax evasion under expected utility theory

1.1 Introduction

Tax evasion matters, as shown by its effect on government revenues. In the US, according to the Internal Revenue Service (IRS [38]), the tax gap for tax year 2001 can be estimated at U$D 345 billion, that is, almost 16% of total tax revenue. According to HM Revenue and Customs (HMRC [36]) in the UK, the tax gap for tax year 2004 is estimated to be at least 9% of total tax receipts. According to the official French statistical body (INSEE [37]) in France, the tax gap for tax year 2006 is estimated at more than 8% of total tax receipts. Crucial to the elaboration of optimal tax enforcement policies is an understanding of how the underreporting behavior of taxpayers is affected by changes in taxes or in fines. A basic model of the tax evasion decision is that developed by Allingham and Sandmo [2]. They studied the connection between tax rate and undeclared income when the penalty is a function of undeclared income. With a constant tax rate and a constant penalty rate applied to the undeclared amount, they showed that there is no simple link between the regular tax rate and tax evasion. Under increasing or constant absolute risk aversion, an increase in tax rate reduces tax evasion. Under decreasing absolute risk aversion – which is "the most attractive assumption" according to them –

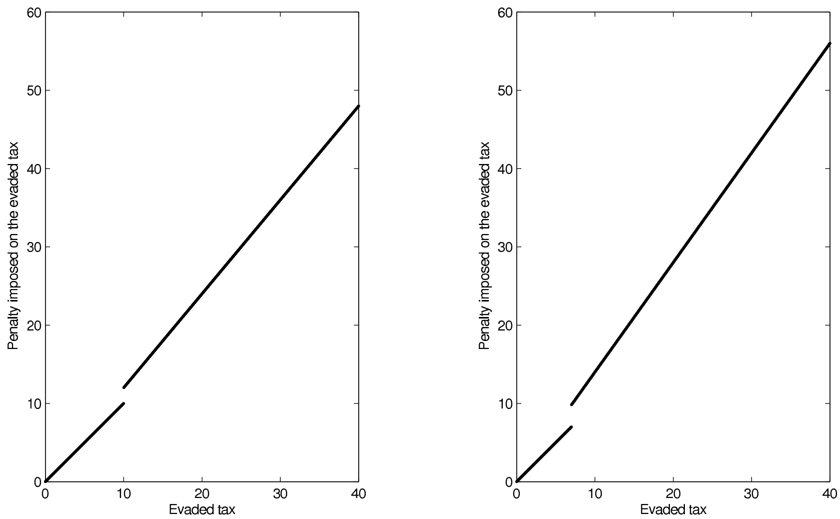
0. This chapter reviews a joint work with Alain Trannoy, registered as *IDEP working paper n° 1004* and submitted to publication under the title "Do high tax and tax evasion go hand in hand? The non-linear case".

an increased tax rate has an ambiguous effect on incentives to cheat due to competing substitution and income effects. The substitution effect increases tax evasion because it is more profitable on the margin. The income effect reduces evasion because the taxpayer is less wealthy and hence more risk averse.

When fraud is proven, the form of penalty chosen differs from country to country. It can be a function of the undeclared income, as in Poland and in Sweden, or of the evaded tax, as in Mexico, in Denmark, in the US and in France. Yitzhaki [83] revised the Allingham and Sandmo [2] model by imposing the fine on the evaded tax and concluded that as tax rate increases, evaded income decreases. [1] The income effect is unchanged with respect to the Allingham and Sandmo case and the substitution effect reduces tax evasion because the penalty increases proportionally with the tax rate.

Since the papers of Allingham and Sandmo [2] and Yitzhaki [83], many alternative assumptions have been considered but the question of linearity has not been adequately dealt with. The purpose of the present chapter is to introduce non-linearity into the analysis of the tax evasion decision. We apply the Allingham-Sandmo and Yitzhaki basic approaches to consider the widest possible range of tax and penalty functions providing testable conditions under which results are robust. Non-linearity has strong empirical support. Tax functions in most countries are convex which is also the case with a certain number of penalty functions. In Mexico, penalties range from 55% to 75% of the unpaid tax. In Denmark, for serious evasion, the penalty is between 100% and 200% of the evaded tax. In the US and in France, for non-criminal tax evasion, the penalty functions are represented by Figure 1.1. In addition, in both countries, when the violation is classified as a misdemeanor or felony, the penalty takes the form of a heavy fine and/or imprisonment plus the payment of the evaded taxes. Presumably, this occurs more frequently when the evaded tax is high. This second form of punishment can be modeled as a bend in the curvature of the function that increases its convexity. The class of convex functions which we consider are thus a good approximation of reality. We show that the extension of the results previously obtained in the literature depends on the slope and the convexity of tax and penalty functions, especially when the taxpayer optimum is a corner solution, that is when he totally declares or evades his income.

1. For a study on the comparison between these penalty structures, see for example Balassone and Jones [5] or Borck [11].



Penalty function in the US

Penalty function in France

FIGURE 1.1 – Penalty functions

The effect of a change in tax rate also depends on changes in risk aversion in relation to income. We assume as far as possible DARA (decreasing absolute risk aversion). According to Chiappori and Paiella [18], not only is absolute risk aversion decreasing with income empirically, but relative risk aversion is more or less constant. When the fine is imposed on undeclared income, that is, when income and substitution effects are competing, we reveal a situation where relative risk aversion is constant and a rise in tax rate increases tax evasion. This extends the result already obtained by Allingham and Sandmo [2] where absolute risk aversion is constant or increasing.

When the fine is imposed on the evaded tax, we examine the robustness of Yitzhaki's result of a positive relationship between a change in tax rate and undeclared income. We consider both interior and corner solutions in terms of tax evasion. The paper which comes closest to treating this subject is Pencavel [54], who used a specific non-linear income tax function with a penalty imposed on the evaded tax.

The proof of the results relies on the Implicit Function Theorem to obtain a local version of the results. Extensions to global results rely on the fact that the Implicit Function Theorem is valid on small intervals and not only at a point.

Our model considers the tax evasion decision in a relatively simple set-up under the expected utility assumption. This contrasts with the highly specialized extensions that have been proposed on, for example, endogenous income, dynamic effects, social norms, tax avoidance or uncertainty on the outcome of audit. [2] Among others, Srinivasan [72] used expected income as an objective function to maximize, taking into account tax evasion and fine. Kolm [44] argued that how tax revenues are used by the government will influence underreporting behavior of individuals. Koskela [45] considered a compensated change in the progressivity of the tax schedule, Bernasconi [9] explains the consistency of the Allinghman and Sandmo's results and the observed rate of tax compliance by the distinction between orders of risk aversion as defined by Segal and Spivak [68], Scotchmer and Slemrod [67] and Scotchmer [66] considered the effect of randomness in tax liability assessments. Benjamini and Maital [8], Gordon [33] and al-Nowaihi and Pyle [1] introduced the stigma associated with being caught evading taxes. Jung, Snow and Trandel [39] examined the impact of a change in tax rate in an economy with two employment sectors offering differing degrees of ease of tax evasion. Panadés [53] studied Ricardian equivalence. Lee [47] introduced an insurance against the possible fines. Slemrod and Yitzhaki [71] and Bayer [6] assumed that the probability of audit is endogenous.

Moreover, while this analysis are based on expected utility theory, recent works have investigated several alternative frameworks. Rank-dependent expected utility theory, as seen in Quiggin [58], uses a transformed cumulative probability distribution. More recently, Yaniv [82] or Dhami and al-Nowaihi [26] explored the cumulative prospect theory framework, where the utility goals are not final levels of wealth but gains and losses relative to some reference point. However, none of these theories has clearly emerged as the definitive alternative to expected utility.

The chapter is organized as follows. Section 1.2 sets out the model. Section 1.3 examines the effect of a change in tax rate on declared income in the case where the fine is imposed on the evaded tax. Section 1.4 addresses the same issue when the fine is imposed on the undeclared income. Section 1.5 is a partial extension of previous results to an endogenous probability of auditing, as in Yitzhaki [83].

2. For a survey, see Andreoni et al. [4], Slemrod and Yitzhaki [71] or Franzoni [31].

1.2 The model

The taxpayer has an exogenous income w, which is private information. He declares an income x, with $x \in [0, w]$. The income tax function T is twice continuously differentiable on $\mathbb{R}_+$, such that $T' < 1$ on $\mathbb{R}_+$ and $T' > 0$ on $\mathbb{R}_+^*$. It should be noted that no assumption about the sign of T'' and the values of $T(0)$ and $T'(0)$ is imposed. The tax authorities audit with the exogenous probability p, such that $0 < p < 1$. By auditing, they are certain to find out the exact amount of the actual income of the taxpayer, who is certain to pay a fine.

There are two types of penalty functions. F is imposed on the evaded tax and G on the undeclared income. They are twice continuously differentiable on $\mathbb{R}_+$, such that $F(0) = 0$, $G(0) = 0$, $F'' > 0$, $G'' > 0$ and $F' \geq 1$ and for all $s \in [x, w]$, $G'(s-x) \geq T'(s)$. The latter assumptions ensure that the penalty functions include payback. It should be noted that the latter one implies that $G'(w-x) \geq T'(x)$ because G is a convex function.

y is the net income without auditing : $y = w - T(x)$ and z the net income with auditing :

i. $z = w - T(x) - F(T(w) - T(x))$ when the fine is imposed on the evaded tax ;

ii. $z = w - T(x) - G(w - x)$ when it is imposed on the undeclared income.

The taxpayer's problem is assumed to maximize the von Neumann-Morgenstern expected utility function :

$$E[U] = (1 - p)\, U(y) + p\, U(z).$$

The utility U is assumed to be twice continuously differentiable on $\mathbb{R}_+$, such that $U' > 0$ and $U'' < 0$ which implies that the individual is risk averse. Denote $A(.) = -\frac{U''(.)}{U'(.)}$ the Arrow-Pratt absolute risk aversion measure. We will generally assume that this measure is decreasing with income (DARA) but other assumptions will be considered as well in the following, like IARA (increasing absolute aversion), CARA and CRRA.

1.3 When the fine is on the evaded tax

1.3.1 The taxpayer's behavior

The taxpayer selects x so as to maximize his expected utility $E[U] = (1-p)U(y) + pU(z)$ under the constraint (i).

We define the first and second derivatives of the objective :

$$\Phi(x) \equiv \frac{\partial E[U]}{\partial x} = (1-p)T'(x)\left[-U'(y) + C(x)U'(z)\right]$$

$$\text{and } D(x) \equiv \frac{\partial^2 E[U]}{\partial x^2} = (1-p)T''(x)\left[-U'(y) + C(x)U'(z)\right]$$

$$+(1-p)T'(x)\left[T'(x)U''(y) + C'(x)U'(z) + \left(\frac{1-p}{p}\right)C(x)^2T'(x)U''(z)\right],$$

where $C(x) = \frac{p}{1-p}\left(F'(T(w)-T(x)) - 1\right)$ is the marginal cost of tax evasion. It is the price of the risk of evasion. C is non-negative and decreasing with x.

Local maxima of $E[U]$ are not necessarily unique. The taxpayer chooses one of them or one of corner solutions 0 and w. x^* will denote the result of this maximization. We assume unicity of the solution.

The objective of the paper is to study the effect of a change in tax rates on tax evasion. Formally, it means that we want to study the effect on the declared income x^*, by applying some change to the average tax rate function $\frac{T(x)}{x}$ or to the marginal tax rate function T'. When T is a linear function, these tax rate functions are constant and equal and it is sufficient to use the derivative of x^* with respect to the tax rate. Here, T is non-linear, it is then necessary to introduce a change in the tax function, with a shift parameter and to differentiate x^* with respect to this parameter. In this way, we replace the initial tax function $T(x)$ with the following tax function $\tilde{T}(x,t) = T(x) + \Delta(x,t)$. The function Δ is assumed to be positive and infinitely continuously differentiable on $\mathbb{R}_+ \times \mathbb{R}_+$, such that for all $x \in \mathbb{R}_+$, $\Delta(x,0) = 0$ and $\frac{\partial\Delta}{\partial t} > 0$ and for all $t \in \mathbb{R}_+$, $T' + \frac{\partial\Delta}{\partial x} < 1$ and $\frac{\partial\Delta}{\partial x} > 0$. The variable $t \in \mathbb{R}_+$ is the shift parameter [3]. Assumptions about Δ ensure that tax rates are unchanged when t is equal to zero and increase with

3. The results detailed below would be unchanged if t was negative. The effects of a decrease in the tax rates would be highlighted instead.

t, and that for all t, $\tilde{T}(.,t)$ satisfies the same assumptions than T. It should be noted that Δ is *a priori* a non-linear function in x and in t.

We denote by $\tilde{\Phi}(x,t)$, $\tilde{D}(x,t)$ and $\tilde{C}(x,t)$ the functions obtained from $\Phi(x)$, $D(x)$ and $C(x)$ with the new tax function. They are such that $\tilde{\Phi}(x,0) = \Phi(x)$, $\tilde{D}(x,0) = D(x)$ and $\tilde{C}(x,0) = C(x)$. The new declared income x^* depends on the shift parameter t. To study the change in the tax evasion decision of the taxpayer, at the time when tax rates are modified, we compute the derivative of x^* with respect to t, with t close to 0.

The following proposition allows us to restrict to a particular form for Δ.

Proposition 1.1. *The derivative* $\frac{\partial x^*}{\partial t}\big|_{t=0}$ *is identical when replacing* $\Delta(t,x)$ *with* $t\delta(x)$, *where* $\delta(x) = \frac{\partial \Delta}{\partial t}(0,x)$.

Proof. See the Appendix. □

This proposition allows us to restrict to the use of the change function $\Delta(t,x) = t\delta(x)$, where δ is infinitely continuously differentiable, positive, increasing and such that $T^{'}(x) + t\delta^{'}(x) < 1$, without the loss of any generality.

1.3.2 Interior maxima

When the declared income x^* of the taxpayer is strictly included between 0 and w, it verifies the first-order condition :

$$\Phi(x^*) = 0, \tag{1.1}$$

which is equivalent to the following condition because $T^{'}(x^*) > 0$:

$$U^{'}(y) = C(x^*)U^{'}(z) \tag{1.1'}$$

and the second-order condition :

$$D(x^*) < 0. \tag{1.2}$$

It can be seen that if $F^{'}(T(w) - T(x)) = 1$, $\forall x \in [x_0, w]$ for some $x_0 > 0$ – as in the US and in France – then $\Phi(x) < 0$, $\forall x \in [x_0, w]$ and $x^* \leq x_0$. This condition $(1^{'})$ is verified only if $F^{'}(T(w) - T(x^*)) > 1$. To study the interior maxima we add the following assumption :

Assumption 1.1. There is $\overline{x}$ in $]0; w]$ such that for all x in $[0; \overline{x}]$, $F^{'}(T(w) - T(x)) > 1$.

This assumption means that the slope of the penalty function is higher than 1 on an interval including total evasion. This assumption combined with FOC (1.1) ensures that this slope is higher than 1 at the global solution of the taxpayer, that is $F^{'}(T(w) - T(x^*)) > 1$. Under this assumption, the marginal cost of tax evasion is positive.

The following conditions are sufficient for an interior solution :

$$\Phi(0) > 0 \Leftrightarrow T^{'}(0) > 0 \text{ and } \frac{U^{'}(w - T(0))}{U^{'}(w - T(0) - F(T(w) - T(0)))} < C(0) \tag{1.3}$$

$$\text{and } \ \Phi(w) < 0 \Leftrightarrow pF^{'}(0) < 1. \tag{1.4}$$

They generalize conditions obtained by Yitzhaki [83] when tax and penalty functions are linear. It is interesting to note that the second condition only concerns $F^{'}(0)$. This means that the benefit obtained from the first undeclared dollar outweighs the risk incurred and so it is in the best interests of the taxpayer to evade tax from the first dollar earned.

We now state a useful lemma which is a consequence of the implicit function theorem. It allows us to extend a local result to a global one.

Lemma 1.1. *Let f be a twice continuously differentiable function,*
$f : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$. Assume that :
i. for all t_0 in $\mathbb{R}$, there is a unique x_0^* in $\mathbb{R}_+$ such that $f(x_0^*, t_0) = 0$;
ii. for all (x, t) in $\mathbb{R}_+ \times \mathbb{R}$, $\frac{\partial f}{\partial x}(x, t) \neq 0$;
then there is a function twice continuously differentiable $x^* : \mathbb{R} \to \mathbb{R}_+$ such that for all t in $\mathbb{R}$, $f(x^*(t), t) = 0$.

Proof. See the Appendix. □

Because the maximization solution is unique, the Implicit Function Theorem can be successively implemented on small intervals, thus providing a complete picture of the behavior of the taxpayer when the tax rate is changing. The next proposition describes the behavior of the taxpayer when his declared income is interior.

Proposition 1.2. *Under Assumption 1.1, when U displays DARA or CARA, if the income declared by the taxpayer is interior, then it strictly increases with both average and marginal tax rates.*

Proof. Using first-order and second-order conditions (1) and (2) we can write :

$$\tilde{\Phi}(x^*, 0) = 0 \quad \text{and} \quad \frac{\partial \tilde{\Phi}}{\partial x}(x^*, 0) = \tilde{D}(x^*, 0) < 0.$$

The fact that x^* is an interior maximum guaranties $\tilde{\Phi}$ is continuously differentiable in a neighborhood of $(x^*, 0)$. Thus, by applying the Implicit Function Theorem we obtain :

$$\left.\frac{\partial x^*}{\partial t}\right|_{t=0} = -\frac{1}{\tilde{D}(x^*, 0)} \frac{\partial \tilde{\Phi}}{\partial t}(x^*, 0).$$

Then the sign of $\frac{\partial x^*}{\partial t}\big|_{t=0}$ is the same as that of $\frac{\partial \tilde{\Phi}}{\partial t}(x^*, 0)$.
After computing and using equation $(1^{'})$ we find :

$$\frac{\partial \tilde{\Phi}}{\partial t}(x^*, 0) = (1-p)U^{'}(y) \Bigg[-x^* T^{'}(x^*)A(y) - (w - x^*)\frac{C^{'}(x^*)}{C(x^*)} + \Big(x^* + (w - x^*)F^{'}(T(w) - T(x^*)) \Big) T^{'}(x^*)A(z) \Bigg].$$

Since $(1-p)U^{'}(y) > 0$, $\frac{\partial \tilde{\Phi}}{\partial t}(x^*, 0) > 0$ if and only if :

$$(w - x^*) \left[-\frac{C^{'}(x^*)}{C(x^*)} + F^{'}(T(w) - T(x^*))T^{'}(x^*)A(z) \right] + x^* T^{'}(x^*) \left[A(z) - A(y) \right] > 0 \quad (1.5)$$

which is always true because $F^{'}(T(w) - T(x^*)) > 1$, $F^{''} > 0$, $T^{'}(x^*) > 0$ and U displays CARA or DARA, with $z < y$.
From Lemma 1.1 and since the result of the maximization of the taxpayer is unique, x^* is a global function of t and the former local reasoning holds for all t. Then, finally we have : $\frac{\partial x^*}{\partial t} > 0$, for all t. □

The first of the two terms on the RHS of (1.5) represents the substitution effect. It is positive because for the same level of declared income, the fine on the evaded tax goes up with a rise in the average tax rate. The second term represents the income effect, which is positive when risk aversion is constant or decreasing with income because an increase in the tax rate makes the taxpayer less wealthy and more risk averse. He evades less when the tax rate increases because his expected penalty rises more than his marginal benefit from cheating.

The assumptions underlying proposition 1.2 are $T^{'} > 0$, $F^{''} > 0$; Assumption 1.1 and DARA or CARA are sufficient. In addition, they are almost necessary in the sense that if we relax one of them, another will have to be strengthened.

This result generalizes Yitzhaki [83] and Pencavel [54] to the case where tax and penalty functions are non-linear.

1.3.3 Corner maxima

Our aim here is to predict how a taxpayer will respond to an increase in tax rate in cases where he either totally evades or fully declares. Corner maxima have sometimes been considered, see for example Dhami and al-Nowaihi [26], but in the case where the tax and penalty rates are linear. Here, the use of the Implicit Function Theorem requires the function $\tilde{\Phi}(x,t)$ to be continuously differentiable in a neighborhood of $(x^*,0)$. It is verified under assumptions of the model as soon as x^* is strictly between 0 and w. To study corner solutions it is necessary to widen hypotheses on the differentiability of the penalty and tax functions at zero.

Assumption 1.2. The function F and its derivatives are extended on $]-\epsilon;0]$, with $\epsilon > 0$, such that F is twice continuously differentiable, $F^{''} > 0$ on $]-\epsilon;+\infty]$ and $F(0) = 0$.

A possible interpretation is that there is a small social benefit from being honest. [4]

The following result extends the previous one to the case where the taxpayer totally declares.

Proposition 1.3. *Under Assumption 1.2,*

$$\text{if } pF^{'}(0) = 1 \quad \text{or if} \quad \frac{T^{''}(w)}{T^{'}(w)^2} < \frac{F^{''}(0)}{F^{'}(0)}, \tag{1.6}$$

when the taxpayer declares all his income, then a rise in average and marginal tax rates has no effect on his declaration.

Proof. See the Appendix. □

It should be noted that no assumption about absolute risk aversion is required in Proposition 1.3. The result is verified when it is decreasing. It can also be seen that, once the tax function is concave, $T^{''}(w) \leq 0$, the condition (1.6) is verified. Roughly speaking, this condition means that the penalty function is more convex than the tax function. Despite the small social benefit from being honest, the tax payer has no incentive to declare more than his true income. In that sense, the reasoning of the proposition is

4. For instance, in France, economists who conceal part of their income cannot be members of certain advisory bodies on tax issues.

valid for the smallest possible social benefit. When the tax rate increases, the expected penalty rises as much as the marginal benefit from cheating.

To study the total evasion case we now extend the tax function on negative incomes.

Assumption 1.3. The function T and its derivatives are extended on $]-\epsilon;0]$, with $\epsilon > 0$, such that T is twice continuously differentiable on $]-\epsilon;0]$.

Up to now, we have not needed assumptions about the tax function for zero income. However, the shape of the tax function for no income matters to an individual who totally cheats. We thus look at two cases : when $T'(0) > 0$ and when $T'(0) = 0$.

Since the penalty function takes payback into account, this assumption is rather innocuous : it means that if a taxpayer has declared too much, he is going to be reimbursed by the fiscal authorities. The payback is negative in that case.

Proposition 1.4. *Under Assumption 1.3, when the taxpayer totally cheats,*
i. when $T'(0) > 0$, if $F'(T(w) - T(0)) > 1$ and

$$\frac{U'(w - T(0))}{U'(w - T(0) - F(T(w) - T(0)))} > C(0), \tag{1.7}$$

then the declared income of the taxpayer strictly increases with average and marginal tax rates,
ii. when $T'(0) = 0$, if condition (1.7) is verified, then a rise in average and marginal tax rates has no effect on declared income.

Proof. See the Appendix. □

Condition (1.7) is not difficult to satisfy through examples mimicking real situations, as for example when $U(\lambda) = a\ log(\lambda) + b$, with $a, b > 0$, $T(0) = 0$, $p \leq 10\%$, $F(\lambda) \leq 150\%\lambda$ and $T(\lambda) \leq 60\%\lambda$.

This result is the direct consequence of the fact that the fine depends on the evaded tax. Obviously, the marginal tax rate plays a major role in the optimum choice of the taxpayer. A point where it is zero is thus a point where neither the evaded tax nor the penalty changes. Hence, the previous optimum choice remains optimum.

Remark. Under the assumptions of Proposition 1.4 and the convexity of the tax function $T'' > 0$, the income declared by the taxpayer is necessarily zero.

Proof. Under these assumptions, $D < 0$ because the expression $\left[-U'(y) + C(x)U'(z)\right]$ is decreasing with x.
In addition, $\Phi(0) = 0$ because $T'(0) = 0$. So $\Phi(x) < 0$, for all $x \in]0; w]$.
0 is then the unique solution of the maximization problem of the taxpayer. □

Note that all results obtained for the corner solutions hold, whatever the risk aversion of the taxpayer.

1.4 When the fine is on the undeclared income

1.4.1 The taxpayer's behavior

The taxpayer selects x so as to maximize his expected utility $E[U] = (1-p)U(y) + pU(z)$ under the constraint ii.

We define the derivatives of this function :

$$\Phi(x) \equiv \frac{\partial E[U]}{\partial x} = (1-p)T'(x)\left[-U'(y) + \Gamma(x)U'(z)\right]$$

$$\text{and } D(x) \equiv \frac{\partial^2 E[U]}{\partial x^2} = (1-p)T''(x)\left[-U'(y) + \Gamma(x)U'(z)\right]$$
$$+(1-p)T'(x)\left[T'(x)U''(y) + \Gamma'(x)U'(z) + \left(\frac{1-p}{p}\right)\Gamma(x)^2 T'(x)U''(z)\right],$$

where $\Gamma(x) = \frac{p}{1-p}\left(\frac{G'(w-x)-T'(x)}{T'(x)}\right)$ is the marginal cost of tax evasion when the fine is imposed on the undeclared income. Γ is non-negative and decreasing with x once $G'(w-x)T''(x) + G''(w-x)T'(x) > 0$.

Local maxima of $E[U]$ are not necessarily unique. The taxpayer chooses one of them or one of corner solutions 0 and w. x^{**} will denote the result of this maximization. We assume unicity of the solution.

We study the effect of a change in tax rate on declared income. To this end, we replace the tax function $T(x)$ with $T(x) + tx$ and we denote $\tilde{\phi}(x,t)$ and $\tilde{\Delta}(x,t)$ the functions obtained in this way which verify $\tilde{\phi}(x,0) = \phi(x)$ and $\tilde{\Delta}(x,0) = \Delta(x)$. t is a shift parameter which represents a constant change in average tax rate.

1.4.2 Interior maxima

When the declared income x^{**} of the taxpayer is strictly included between 0 and w, it verifies the first-order condition :

$$\phi(x^{**}) = 0, \tag{1.8}$$

which is equivalent to the following condition because $T^{'}(x^{**}) > 0$:

$$U^{'}(y) = \Gamma(x^{**})U^{'}(z) \tag{1.8'}$$

and the second-order condition :

$$\Delta(x^{**}) < 0. \tag{1.9}$$

To study the interior maxima we add the following assumption :

Assumption 1.4. There is $\tilde{x}$ in $]0; w]$ such that for all x in $[0; \tilde{x}]$, $G'(w - x) > T^{'}(x)$.

This assumption combined with FOC (1.8) ensures that the slope of the penalty function is higher than that of the tax function in the global solution of the taxpayer. Under this assumption the marginal cost of evasion is positive.

The following conditions are sufficient for an interior solution :

$$\phi(0) > 0 \;\Leftrightarrow T^{'}(0) > 0 \text{ and } \frac{U^{'}(w - T(0))}{U'(w - T(0) - G(w))} < \Gamma(0) \tag{1.10}$$

$$\text{and} \quad \phi(w) < 0 \;\Leftrightarrow\; pG^{'}(0) < T^{'}(w). \tag{1.11}$$

They boil down to the conditions obtained by Allingham and Sandmo [2] when both tax and penalty functions are linear.

The following result extends Allingham and Sandmo to the case where either the tax or the penalty function is not linear.

Proposition 1.5. *Under Assumption 1.4, when U displays IARA or CARA, if the income declared by the taxpayer is interior, then it strictly decreases with average and marginal tax rates.*

Proof. Using first-order and second-order conditions and applying the Implicit Function Theorem we find that the sign of $\frac{\partial x^{**}}{\partial t}\big|_{t=0}$ is the same as that of $\frac{\partial \tilde{\phi}}{\partial t}(x^{**}, 0)$.
After computing we find :

$$\frac{\partial \tilde{\phi}}{\partial t}(x^{**}, 0) = (1-p)x^{**}T^{'}(x^{**})U^{'}(y)\left[A(z) - A(y)\right] - (1-p)U^{'}(y) - pU^{'}(z), \quad (1.12)$$

which is always negative because $T^{'} > 0$, $U^{'} > 0$ and U displays IARA or CARA, with $z < y$.
We conclude with Lemma 1.1. □

The first of the terms on the RHS of (1.12) represents the income effect. It is negative when risk aversion is constant or increasing with income because a rise in tax rate makes the taxpayer less wealthy and less risk averse. The last term, the substitution effect, is negative because for the same level of declared income, a rise in average tax rate increases the expected payment.

No clearcut result emerges when risk aversion is decreasing, although this is a priori the most realistic assumption. Besides, according to Chiappori and Paiella [18], it emerges from empirical data that not only is absolute risk aversion decreasing but relative risk aversion is constant and positive, with an average coefficient of around 2.

It is possible to provide a sufficient condition to extend the previous result to the situation where relative risk aversion is constant, with a strictly positive coefficient.

Let us denote $A_T(w) \equiv \frac{T(w)}{w}$ and $A_P(0) = \frac{G(w)}{w-T(0)}$, respectively the average tax in w and the average penalty in 0 (when the taxpayer totally evades).

The next proposition provides a completely new result when the penalty is levied on the undeclared income and risk preferences display decreasing absolute risk aversion.

Proposition 1.6. *Under Assumption 1.4, when U displays CRRA with the strictly positive coefficient τ,*

$$\textit{if } (1-p)\tau < (1 - A_T(w))\left(\frac{1}{A_P(0)} - 1\right),$$

if the declared income of the taxpayer is interior, then it strictly decreases with average and marginal tax rates.

Proof. Applying the Implicit Function Theorem, the sign of $\frac{\partial x^{**}}{\partial t}\big|_{t=0}$ is the same as that of $\frac{\partial \tilde{\phi}}{\partial t}(x^{**}, 0)$.

When U displays CRRA where τ is the coefficient,

$$\frac{\partial \tilde{\phi}}{\partial t}(x^{**},0) = U'(y)\Psi(x^{**}),$$

$$\text{where } \Psi(x) = (1-p)xT'(x)\tau\left[\frac{1}{z}-\frac{1}{y}\right]-(1-p)-p\left(\frac{y}{z}\right)^{\tau}.$$

For all $x \in]0;w[$, $y > z$, $y > w - T(w)$, $z > w - T(0) - G(w)$ and $T'(x) < 1$,

$$\text{then } \Psi(x) < (1-p)\tau w\frac{G(w)}{(w-T(w))(w-T(0)-G(w))}-1, \ \forall x \in]0;w[.$$

$$\text{Thus we have : if } (1-p)\tau < \frac{(w-T(w))(w-T(0)-G(w))}{wG(w)}, \text{ then } \frac{\partial \tilde{\phi}}{\partial t}(x^{**},0) < 0.$$

We conclude with Lemma 1.1. □

If the relative Arrow-Pratt coefficient is sufficiently low and remains positive, tax evasion goes hand in hand with a high tax rate because the increased risk aversion induced by the increased average tax rate is not sufficient to cancel out the marginal benefit from cheating.

1.4.3 Corner maxima

As in section 1.3.3, we study the effect on declared income of an increase in tax rate when the taxpayer either totally evades or fully declares. The use of the Implicit Function Theorem requires the function $\tilde{\phi}(x,t)$ to be continuously differentiable in a neighborhood of $(x^{**},0)$. It is necessary to use Assumptions 1.2 and 1.3 defined previously.

The following result extends the previous ones to the case where the taxpayer totally declares.

Proposition 1.7. *Under Assumption 1.2, if $T''(w) \geq 0$, when the taxpayer declares all his income, then it strictly decreases with average and marginal tax rates.*

Proof. See the Appendix. □

Convexity of the tax function for the highest income is sufficient for the decision not to declare to go hand in hand with a high tax rate.

Proposition 1.8. *Under Assumption 1.3, if $T''(0) \geq 0$, then a rise in average and marginal tax rates has no effect on the declaration of the taxpayer when he totally cheats.*

Proof. See the Appendix. □

Convexity of the tax function for a null income is sufficient to ensure that the behavior of the taxpayer does not change if he totally cheats initially.

Note that there is no assumption either about absolute risk aversion or about the value of the marginal tax function in zero. In addition, this form of penalty does not necessitate any assumption about the marginal tax function.

1.5 Extensions to an endogenous audit probability

The results obtained above supposed that the probability of auditing is given. The literature on optimal auditing (Reinganum and Wilde [62], Scothmer [65] and Sanchez and Sobel [64]) claims that it is optimal for the auditing decision to be based on the level of reported income. Here we study the robustness of the above results when the probability of auditing is decreasing with the level of reported income.

The relation between tax evasion and tax rate with a decreasing probability of detection has already been studied by Yitzhaki [84]. He showed that a compensated rise in marginal tax rate, other things being equal (including exogenous income), increases tax evasion.

To study the effect of a general change in tax rate on declared income, we consider the following assumptions. The tax function T and the probability function p are three times continuously differentiable on $\mathbb{R}_+$ and verify $T^{'} < 1$ on $\mathbb{R}_+$, $T^{'} > 0$ on $\mathbb{R}_+^*$ and $p^{'} < 0$ on $\mathbb{R}_+^*$. We assume, as Yitzhaki [83], that the probability of auditing is decreasing with the declared amount. For convenience, the penalty functions F and G are assumed to be linear with coefficients f and g such that $f > 1$ and $g > T^{'}$. These functions can thus obviously be extended below zero.

1.5.1 Fine on the evaded tax

A decreasing probability of auditing makes total evasion less likely. The following condition is sufficient to avoid a total evasion solution :

$$T^{'}(0)\left[CU^{'}(z(0)) - U^{'}(y(0))\right] - p^{'}(0)\left[U(y(0)) - U(z(0))\right] > 0, \qquad (1.13)$$

where $y(0) = w - T(0)$, $z(0) = w - T(0) - f(T(w) - T(0))$ and $C = \frac{p(f-1)}{1-p}$ is the marginal cost of tax evasion.

The following proposition shows that Proposition 1.2 does not hold in general with a decreasing audit probability.

Proposition 1.9. *When the income declared by the taxpayer is interior, suppose that the following conditions hold :*

i. U displays DARA or CARA,

ii. p is concave,

iii. $$A(w - T(w)) > -\frac{1}{f-1}\frac{p'(w)}{p(w)T'(w)}, \tag{1.14}$$

either iv. T' and p' are concave and $p''(0)T'(0) < p'(0)T''(0)$,

or v. T' and p' are convex and $p''(w)T'(w) < p'(w)T''(w)$,

then a rise in average and marginal tax rates strictly decreases the declared income.

Proof. See the Appendix. □

A decreasing audit probability affects the behavior of individuals who declare all their income.

Proposition 1.10. *When the taxpayer declares all his income, under condition (1.14) and if :*

$$p(0) < \frac{1}{f}, \tag{1.15}$$

then his declared income strictly decreases with average and marginal tax rates.

Proof. See the Appendix. □

1.5.2 Fine on the undeclared income

A decreasing audit probability makes total evasion less likely. The following condition is sufficient for a non-total evasion solution :

$$T'(0)\left[CU'(z(0)) - U'(y(0))\right] - p'(0)\left[U(y(0)) - U(z(0))\right] > 0, \tag{1.16}$$

where $y(0) = w - T(0)$, $z(0) = w - T(0) - gw$ and $\Gamma(0) = \frac{p(g-T'(0))}{(1-p)T'(0)}$ is the marginal cost of tax evasion in zero.

The following proposition shows that Propositions 1.5 and 1.6 hold with a decreasing probability.

Proposition 1.11. *When the income declared by the taxpayer is interior, when U displays DARA or CARA, if T and p are convex and if :*

$$A\left(w - T(0) - gw\right) < \frac{1}{wT'(w)}, \tag{1.17}$$

then a rise in average and marginal tax rates strictly decreases the declared income.

Proof. See the Appendix. □

A decreasing audit probability has no effect on the behavior of individuals who declare all their income.

Proposition 1.12. *When the taxpayer declares all his income, if T is convex and if :*

$$p(w) < \frac{T'(w)}{g}, \tag{1.18}$$

then his declared income strictly decreases with average and marginal tax rates.

Proof. See the Appendix. □

1.6 Conclusion

This chapter provides testable conditions on the relation between tax evasion and tax change that can be used in experimental econometrics studies under the EU hypothesis. Very weak conditions are set on the tax and penalty functions. The conditions are testable since they cover the parameters of the cheating game : risk aversion, marginal tax in zero, marginal fine when there is total evasion,...

Finally, these same methods of proof, using the Implicit Function Theorem in a non-linear framework, can be used to provide predictions about tax declaration under non expected utility theories, like rank-dependent expected utility theory or prospect theory. Such applications should be a promising avenue for future research.

1.7 Appendix

Proof of Proposition 1.1. Since Δ is analytic and $\Delta(0,x)=0$, it can be expanded in power series in a neighborhood of $(0,x^*)$ as :

$$\Delta(t,x)=\sum_{i,j=0}^{+\infty} a_{ij}(x-x^*)^i t^j=\delta_1(x-x^*)t+\delta_2(x-x^*)t^2+...,$$

where the δ_j's are power series.
x^* is the solution of the maximization problem of the taxpayer, then $\tilde{\Phi}(0,x^*)=0$. From the Implicit Function Theorem, $\frac{\partial x^*}{\partial t}\big|_{t=0}$ is obtained by differentiating this expression with respect to t. It thus depends on the partial derivatives of $\tilde{\Phi}$ and on $\Delta(0,x^*)=0$, $\frac{\partial \Delta}{\partial x}(0,x^*)=0$, $\frac{\partial^2 \Delta}{\partial x^2}(0,x^*)=0$, $\frac{\partial \Delta}{\partial t}(0,x^*)$ and $\frac{\partial^2 \Delta}{\partial x \partial t}(0,x^*)$.
That is, only δ_1 is involved in the expression of $\frac{\partial x^*}{\partial t}\big|_{t=0}$. Thus, to study the local behavior of x^* in a neighborhood of 0, Δ can be replaced with $t\delta$, where $\delta(x)=\frac{\partial \Delta}{\partial t}(0,x)$. □

Proof of Lemma 1.1. From the Implicit Function Theorem, for all t_0 in $\mathbb{R}$, there is a neighborhood V_{t_0} of t_0 and a function twice continuously differentiable $x^*_{t_0}: V_{t_0} \to \mathbb{R}$, such that for all t in V_{t_0}, $f(x^*_{t_0}(t),t)=0$.
Moreover, (i) asserts the uniqueness of $x^*_{t_0}$.
Therefore, $\mathbb{R}$ is covered by a family of open sets $(V_{t_0})_{t_0\in\mathbb{R}}$ and a function $x^*_{t_0}$ is defined on each of these open sets.
To construct the expected function x^*, we have to prove that for all t_0, $t_0^{'}$ in $\mathbb{R}$, the functions $x^*_{t_0}$ and $x^*_{t_0^{'}}$ coincide on $V_{t_0}\cap V_{t_0^{'}}$. This is a consequence of the uniqueness of the function $x^*_{t_0}$ on V_{t_0}. □

Proof of Proposition 1.3. When the taxpayer totally declares, $pF^{'}(0)\geq 1$.
When $pF^{'}(0)=1$ or when the condition (5) is verified, we find directly that $D(w)\neq 0$ and makes it possible to apply the Implicit Function Theorem to $\tilde{\Phi}$ [5]. Thus, there are three possibilities :

$$\Phi(w)=0 \text{ and } D(w)<0, \quad \text{or } \Phi(w)>0 \text{ and } D(w)<0,$$

$$\text{or } \Phi(w)>0 \text{ and } D(w)>0.$$

5. In the opposite case we would apply it to $\tilde{D}$ provided we assume that the tax and penalty functions are three times continuously differentiable and that $\frac{\partial \tilde{D}}{\partial x}(w,0)\neq 0$.

If $\Phi(w) = 0$ we can apply the Implicit Function Theorem to $\tilde{\Phi}(x,t)$.

If $\Phi(w) > 0$ we can do it to $\tilde{\Phi}(x,t) - \Phi(w)$.

Actually under Assumption 1.2, in both cases these functions are differentiable in a neighborhood of $(w,0)$ and vanish at this point. Moreover, by applying the theorem we obtain :

$$\left.\frac{\partial x^*}{\partial t}\right|_{t=0} = -\frac{1}{\tilde{D}(w,0)}\frac{\partial \tilde{\Phi}}{\partial t}(w,0) \text{ when } x^* = w.$$

Now $\dfrac{\partial \tilde{\Phi}}{\partial t}(w,0) = 0$,

therefore, from Lemma 1.1, $\frac{\partial x^*}{\partial t} = 0$ when $x^* = w$. □

Proof of Proposition 1.4. i. We find directly that $D(0) < 0$. As for the Proposition 2, by applying the Implicit Function Theorem to $\tilde{\Phi}(x,t)$ (modulo some constant if $\Phi(0) < 0$) in $(0,0)$, we obtain :

$$\left.\frac{\partial x^*}{\partial t}\right|_{t=0} = -\frac{1}{\tilde{D}(0,0)}\frac{\partial \tilde{\Phi}}{\partial t}(0,0) \text{ when } x^* = 0.$$

Now $$\frac{\partial \tilde{\Phi}}{\partial t}(0,0) = (1-p)T^{'}(0)U^{'}(y)w\left[\frac{F^{''}(T(w)-T(0))}{F^{'}(T(w)-T(0))-1} + F^{'}(T(w)-T(0))A(z)\right] > 0,$$

therefore, from Lemma 1.1, $\frac{\partial x^*}{\partial t} > 0$ when $x^* = 0$.

ii. $\Phi(0) = 0$ and $D(0) \neq 0$, we can then apply the Implicit Function Theorem to $\tilde{\Phi}(x,t)$ in $(0,0)$ and we obtain :

$$\left.\frac{\partial x^*}{\partial t}\right|_{t=0} = -\frac{1}{\tilde{D}(0,0)}\frac{\partial \tilde{\Phi}}{\partial t}(0,0) \text{ when } x^* = 0,$$

with $\dfrac{\partial \tilde{\Phi}}{\partial t}(0,0) = 0$ because $T^{'}(0) = 0$.

We conclude with Lemma 1.1. □

Proof of Proposition 1.7. If $T^{''}(w) \geq 0$, $\Delta(w) < 0$ and under Assumption 1.2, by applying the Implicit Function Theorem, we obtain :

$$\left.\frac{\partial x^{**}}{\partial t}\right|_{t=0} = -\frac{1}{\tilde{\Delta}(0,0)}\frac{\partial \tilde{\phi}}{\partial t}(0,0) \text{ when } x^{**} = 0.$$

$$\text{Now } \frac{\partial \tilde{\phi}}{\partial t}(w,0) = -U^{'}\left(w - T(w)\right) < 0, \text{ therefore } \left.\frac{\partial x^{**}}{\partial t}\right|_{t=0} < 0 \text{ when } x^{**} = w.$$

We conclude with Lemma 1.1. □

Proof of Proposition 1.8. When $T^{''}(0) \geq 0$, $\Delta(0) < 0$.
Applying the Implicit Function Theorem to $\tilde{\phi}(x,t)$ (modulo some constant if $\phi(0) \neq 0$) in $(0,0)$, the sign of $\frac{\partial x^{**}}{\partial t}\big|_{t=0}$ when $x^{**} = 0$, becomes the same as that of $\frac{\partial \tilde{\phi}}{\partial t}(0,0)$,

$$\text{with } \frac{\partial \tilde{\phi}}{\partial t}(0,0) = -(1-p)U^{'}\left(w - T(0)\right) - pU^{'}\left(w - T(0) - G(w)\right) < 0.$$

We conclude with Lemma 1.1. □

Proof of Proposition 1.9. Applying the IFT, the sign of $\frac{\partial x^{*}}{\partial t}\big|_{t=0}$ becomes the same as that of $\frac{\partial \tilde{\Phi}_p}{\partial t}(x^*,0)$, where Φ_p is the first derivative of the objective.
After computing and using the FOC :

$$p(x^*)T^{'}(x^*)U^{'}(z) = (1 - p(x^*))T^{'}(x^*)U^{'}(y) + p^{'}(x^*)\left[U(y) - U(z)\right], \text{ we find :}$$

$$\begin{aligned}\frac{\partial \tilde{\Phi}_p}{\partial t}(x^*,0) = \frac{p^{'}(x^*)}{T^{'}(x^*)}\left[U(y) - U(z)\right] &+ (1 - p(x^*))\, x^* T^{'}(x^*)U^{''}(y) + x^* p^{'}(x^*)U^{'}(y) \\ &- p^{'}(x^*)U^{'}(z)\left[x + (w-x)f\right]\left[1 + \frac{p(x^*)T^{'}(x^*)}{p^{'}(x^*)}(f-1)A(z)\right].\end{aligned}$$

With $p^{'} < 0$, $T^{'} > 0$, $U^{'} > 0$ and $U^{''} < 0$,

$$\frac{\partial \tilde{\Phi}_p}{\partial t}(x^*,0) < 0 \text{ as soon as } \left[1 + \frac{p(x^*)T^{'}(x^*)}{p^{'}(x^*)}(f-1)A(z)\right] < 0.$$

When U displays DARA or CARA, $A(z) \geq A(w - T(w))$ and under assumptions, $\frac{p^{'}}{pT^{'}}$ is decreasing,

$$\text{then } \left[1 + \frac{p(x^*)T^{'}(x^*)}{p^{'}(x^*)}(f-1)A(z)\right] < \left[1 + \frac{p(w)T^{'}(w)}{p^{'}(w)}(f-1)A(w - T(w))\right],$$

thus, under condition (1.14), $\frac{\partial x^{*}}{\partial t}\big|_{t=0} < 0$.
We conclude with Lemma 1.1. □

Proof of Proposition 1.10. Under condition (1.15), the second derivative of the objective in w is negative. Then, from IFT, the sign of $\frac{\partial x^{*}}{\partial t}\big|_{t=0}$ when $x^* = w$, is the same as that of $\frac{\partial \tilde{\Phi}_p}{\partial t}(w,0)$, which is negative under condition (1.14).
We conclude with Lemma 1.1. □

Proof of Proposition 1.11. Applying the IFT, the sign of $\frac{\partial x^{**}}{\partial t}\big|_{t=0}$ becomes the same as that of $\frac{\partial \tilde{\phi}_p}{\partial t}(x^{**}, 0)$.
After computing we find :

$$\frac{\partial \tilde{\phi}_p}{\partial t}(x^{**}, 0) = (1-p(x^{**}))T^{'}(x^{**})x^{**}U^{''}(y)+x^{**}A(z)p^{'}(x^{**})\left[U(y) - U(z)\right]$$

$$+\left[x^{**}A(z)(1-p(x^{**}))T^{'}(x^{**}) - (1-p(x^{**})) + p^{'}(x^{**})x^{**}\right]U^{'}(y) - \left[p + p^{'}(x^{**})x^{**}\right]U^{'}(z),$$

which is negative under condition (1.17) once p and T are convex.
We conclude with Lemma 1.1. □

Proof of Proposition 1.12. When T is convex, $\Delta_p(w) < 0$. Then, from the IFT, the sign of $\frac{\partial x^{**}}{\partial t}\big|_{t=0}$ when $x^{**} = w$ is the same as that of $\frac{\partial \tilde{\phi}_p}{\partial t}(w, 0)$, which is negative under condition (1.18).
We conclude with Lemma 1.1. □

Chapitre 2

Solving the Yitzhaki paradox : Tax evasion under cumulative prospect theory

2.1 Introduction

Concerns about tax enforcement policies have led most governments to set up a large tax evasion fighting system with audits and fines. The first step to optimize this system is to make best knowledge of evasion decisions of taxpayers. A substantial literature has already studied this issue, most often within an expected utility theory framework, the seminal applications of this theory to tax evasion problem being the ones by Allingham and Sandmo [2], where the fine is imposed on the undeclared income and Yitzhaki [83], where the fine is imposed on the evaded tax. The second case is the most frequently seen, like for example in the US and in France.

The model of Yitzhaki has been extended to include many alternative assumptions, see for example Franzoni [31]. However, the expected utility theory has been criticized a lot these last years. Many empirical studies have emphasized its disability to describe the observed behavior patterns in an adequate way, see for example Skinner and Slemrod [69], Alm et al. [3], Andreoni et al. [4] or Slemrod and Yitzhaki [71]. In particular, with a reasonable degree of risk aversion, it predicts a too large extent of tax evasion.

0. This chapter reviews a work whose title is "Solving the Yitzhaki paradox : Income tax evasion and reference dependence under cumulative prospect theory".

Additionally, under the assumption of decreasing absolute risk aversion, it predicts that an increase in the tax rate leads to a decrease in tax evasion. Therefore, a number of works have developped alternatives to expected utility theory to account for the behavior patterns observed in experiments. Among them, rank dependent expected utility theory allows for the overestimation of low probabilities. Prospect theory provides differential treatments of gains and losses with respect to a reference point. Cumulative prospect theory is a variant of the last one, weighting being applied to the cumulative probability distribution function, as in rank-dependent expected utility theory, rather than to the probabilities of individual outcomes.

Cumulative prospect theory has become one of the most prominent alternative to expected utility. Building upon prospect theory (Kahneman and Tversky [40]) and the works of Starmer and Sugden [74] and Tversky and Kahneman [79], it is widely used in empirical research. The carriers of utility are not final levels of income any more but differences between final levels of income and a determined reference income. It expresses the *framing effect* phenomenon. The utility function convex for gains and concave for losses expresses the *loss aversion* phenomenon : individuals care generally more about potential losses than potential gains. There is risk-averse behavior in case of gains and risk-seeking behavior in case of losses. Furthermore, individuals tend to overweight unlikely events but underweight average and likely ones. This last point differentiates cumulative prospect theory from prospect theory. The collection of papers in Kahneman and Tversky [41], for instance, provides empirical confirmation of these properties.

The tax evasion problem has already been dealt with the literature on prospect theory. Alm et al. [3] provides an experimental study. Among others, Yaniv [82] analyzes the influence of obligatory advance tax payments on the taxpayer's evasion decision. He applies prospect theory to a simple model of tax evasion, using the income after the payment of the tax advance and prior to the filing of a return for the reference income, and demonstrates that advance tax payments may substitute for costly detection efforts in enhancing compliance. Bernasconi and Zanardi [10] use cumulative prospect theory with a general reference point but with particular probability weighting and utility functions. Dhami and al-Nowaihi [26] also apply cumulative prospect theory to tax evasion. They consider the legal after-tax income to be the reference point because it is the only one with which the taxpayer is in the domain of gains if not caught and in the domain of losses if caught. Following Eide [29], they argue that it is the only case in which the paradoxical comparative static results of the Allingham and Sandmo - Yitzhaki model do not carry over to rank dependent expected utility theory. They use a probability of detection which depends on the amount of income evaded and introduce stigma costs of evasion. Using the power utility function of Tversky and Kahneman [79], they show that the predictions of prospect theory are consistent with the evidence. Using parameters estimated by the experimental literature and the weighting probability function of Prelec [55], they show that relative to expected utility theory, prospect theory provides a much better explanation of tax evasion.

The objective of the present paper is also to apply cumulative prospect theory to the tax evasion problem. More exactly, it provides comparisons between expected utility theory, prospect theory and cumulative prospect theory, when the fine is imposed on the evaded tax, as in Yitzhaki [83].[1] It differs from previous paper in several respects. The main are the use of a general reference income and a general utility function. Arguing that it can be possible for the taxpayer to be in the domain of gains or in the domain of losses with or without auditing, we show that the level and the expression of the reference income is an essential point in the use of prospect theory and that the paradoxical comparative static results of the Yitzhaki model do not carry all over the support of income distribution. The use of a general utility function let us to highlight general intuitions concerning tax evasion problem under prospect theory. In particular, introducing a suitable relative risk aversion measure, we show with several expressions for the reference income, that tax evasion is increasing in the tax rate as soon as it is larger with auditing, than without. It is because an increase in the tax rate causes the taxpayer to be richer in terms of outcomes (differences between final incomes and the reference income). With a decreasing relative risk aversion, he chooses to increase the fraction of his initial income in the risky alternative and tax evasion increases. Compared to Yitzhaki [83], the substitution effect vanishes, while the income effect goes to the opposite side. This is due to the reference income, one of the central items in prospect theory. With tax evasion problem, qualitative results under cumulative prospect

1. Results would be sensitively the same with a fine imposed on the undeclared income, see the conclusion.

theory does not differ from those provided by prospect theory. Under some conditions, qualitative experimental results are robust to the use of prospect theory.

With qualitative concerns, expected utility theory account of tax evasion contradicts the empirical evidence in both following main ways :

- With a positive expected return to tax evasion, expected utility theory predicts that all taxpayers should hide some income.
- Yitzhaki [83] showed that using expected utility theory under the reasonable assumption of decreasing absolute risk aversion, an increase in the tax rate leads to a decrease in tax evasion.

The present chapter shows how prospect theory provides a framework where results are in accordance with the qualitative empirical evidence. In particular, the well-known Yitzhaki paradox is solved.

The chapter is organized as follows. The next section sets up the basic model. Section 2.3 studies the effect of the use of a probability weighting function on the tax evasion decision of the taxpayer in an expected utility theory framework. Section 2.4 studies the effect of the use of a reference point. Section 2.5 studies tax evasion decision in a general setting of cumulative prospect theory. Section 2.6 makes the concluding remarks.

2.2 The model

2.2.1 Final incomes

A taxpayer has an exogenous taxable income $w > 0$ which is private information. He declares some amount $x \in [0, w]$. In particular, he can not declare a negative income or an income higher than the initial income.[2] The government levies a tax on declared income at the constant marginal rate t, $0 < t < 1$. The tax administration audits with the exogenous probability $p \in [0, 1]$. If he is caught cheating, the taxpayer must pay the evaded tax $t(w - x)$ and a fine $ft(w - x)$, where $f > 0$ is the fine rate on evaded taxes.[3] Denote by Y and Z, respectively, the net income of the taxpayer without and

2. This rules out the possibility to get some gain from being more than honest.
3. It is assumed that if an audit occurs, the actual income of the taxpayer is discovered without error.

with auditing :

$$Y = w - tx, \tag{2.1}$$

$$Z = w - tx - (1+f)t\,(w-x)\,. \tag{2.2}$$

Cumulative prospect theory differs from expected utility theory by the use of two main characteristics. A probability weighting function expresses that people tend to overreact to small probability events, but underreact to medium and large probabilities. A reference point expresses that individuals tend to think of possible outcomes relative to a certain reference point rather than to the final status. To highlight the differences between the application of the two theories, every characteristic is introduced alone before to study results in a general cumulative prospect theory setting. The next subsections precise the expressions of the probability weighting function and of the reference income.

2.2.2 Probability weighting function

The probability weighting function expresses that people tend to overreact to small probability events, but underreact to medium and large probabilities. A large literature backs up this fact, see for example Tversky and Kahneman [79] or Starmer [73].

The probability weighting function π is a continuous function on $[0,1]$, differentiable on $]0,1[$, strictly increasing from $[0,1]$ onto $[0,1]$, with $\pi(0) = 0$ and $\pi(1) = 1$. There exists p_0 and $p_1 \in]0,1[$, such that $p_0 \leq p_1$ [4] and such that for all $p \in]0, p_0[$, $\pi(p) \geq p$ and for all $p \in]p_1, 1[$, $\pi(p) \leq p$. Figure 2.1 represents a typical weighting function. It transforms objective cumulative probabilities into subjective cumulative probabilities.

It can be observed that to consider $\pi(p) = p$, for all $p \in [0,1]$, is equivalent to use objective cumulative probabilities, as in expected utility theory or in prospect theory.

The attitude towards risk depends on the curvature of the utility function and also on the shape of the probability weighting function. The random risk attitude of the taxpayer, determined by the shape of the probability weighting function, can be expressed by the following measure :

$$\Pi(p) = \frac{\pi(p)}{\pi(1-p)}, \text{ for } p \in [0,1[, \ \Pi(1) = +\infty. \tag{2.3}$$

4. It is commonly supposed that $p_0 = p_1 \simeq 0.3$. See for example Prelec [55].

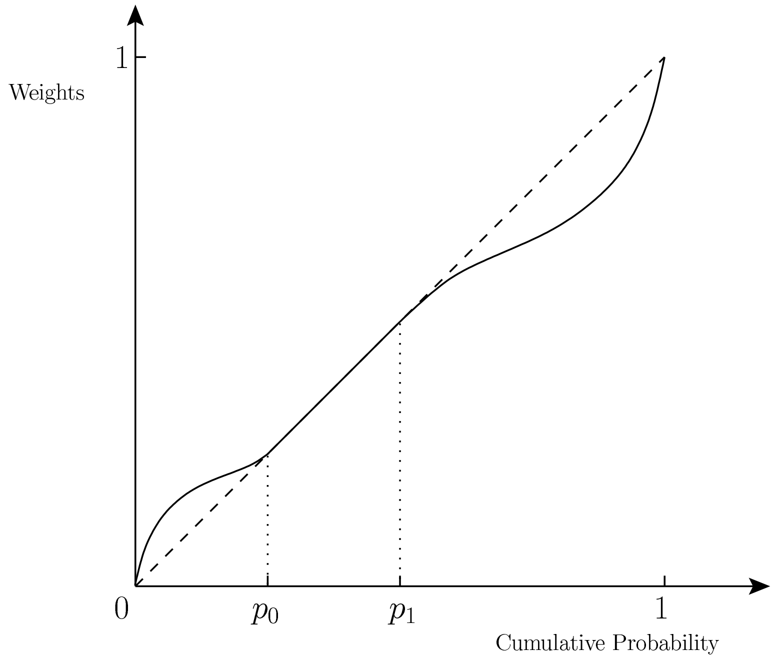


FIGURE 2.1 – Probability weighting function

This measure is positive. For a fixed value for the audit probability, the more the taxpayer overweights small probabilities and underweights large probabilities, the higher it is. It measures the subjectivity of the taxpayer considering probabilities of events.

The weighting function of Prelec [55] is consistent with much of the available empirical evidence. It will be useful to specify general results. It has the following form :

$$\pi(p) = \mathrm{e}^{-(-\ln p)^{\alpha}}, \text{ with } 0 < \alpha < 1, \text{ for } 0 < p \leq 1 \text{ and } \pi(0) = 0. \tag{2.4}$$

The lower is α, the higher is the degree of overweighting of small probabilities and of underweighting of large probabilities. As α is close to 0, the probability function approximates a function flat everywhere except at the endpoints of the probability interval. As α is close to 1, the probability function approximates the objective (linear) function. Figure 3 represents Prelec weighting function for different values for α. Figures 2.2 illustrates examples of Prelec weighting functions with different values for α.

With this Prelec form, the random risk attitude measure has the following expression :

$$\Pi(p) = \mathrm{e}^{(-\ln(1-p))^{\alpha}-(-\ln p)^{\alpha}}. \tag{2.5}$$

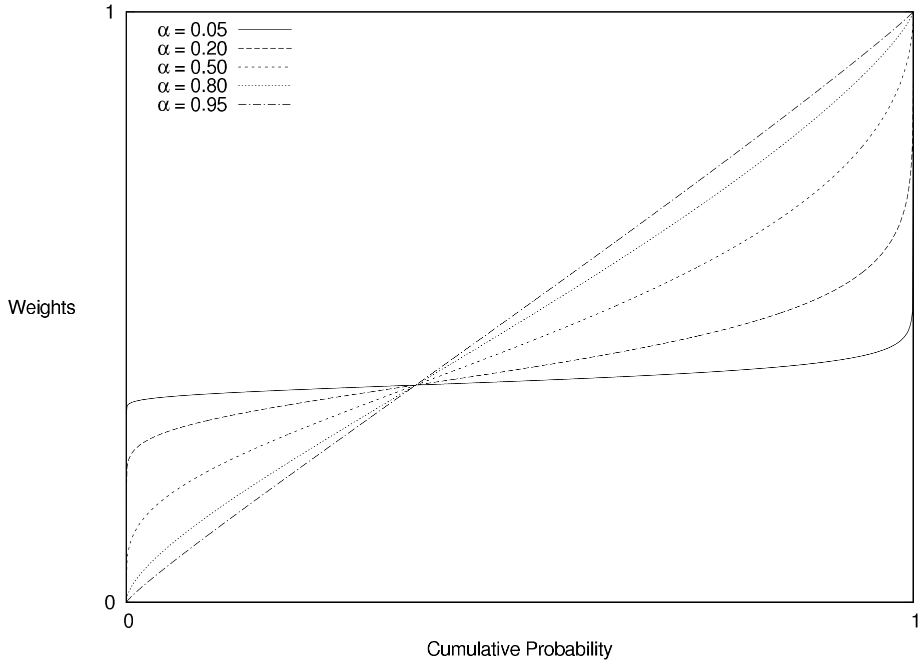


FIGURE 2.2 – Prelec probability weighting functions with $\alpha = 0.05,\ 0.2,\ 0.5,\ 0.8$ and 0.95.

2.2.3 Reference income and utility of an outcome

Many empirical studies have shown that individuals tend to think of possible outcomes usually relative to a certain reference point rather than to the final status, see for example Kahneman and Tversky [41]. Following prospect theory, the taxpayer evaluates potential losses and gains. He sets a reference income and consider larger outcomes as gains and lower as losses. The income with which the taxpayer represents his final income includes at once what he considers to deserve (or the price he is willing to pay for public goods) – his initial income and the tax rate – and the characteristics of the cheating game to which he subjects himself by not declaring his entire income – the penalty rate and the probability of auditing :

$$R = R(w, t, f, p)\,. \tag{2.6}$$

The incomes relative to the reference income without and with auditing are :

$$y = Y - R = w - tx - R, \tag{2.7}$$

$$z = Z - R = w - tx - (1+f)t(w-x) - R. \tag{2.8}$$

The final income of the taxpayer is always non-negative. [5] The initial income is the final income obtained by the taxpayer if he does not declare any income and he is not audited ($Y|_{x=0} = w$). It is then impossible, even if there is no auditing, that his final income is above his initial income. It is then quite natural to assume that the reference income chosen by the taxpayer is non-negative and below his initial income :

$$0 \leq R \leq w. \tag{2.9}$$

The legal after-tax income is the final income of the taxpayer if he is completely honest ($Y|_{x=w} = Z|_{x=w} = (1-t)w$). It is the only value of the reference income for which for all levels of declared income, the taxpayer is in the domain of gains if not caught and in the domain of losses if caught [6] and from now on, it is denoted by :

$$\bar{R} = (1-t)\,w. \tag{2.10}$$

The relative incomes can be rewritten. From (2.7), (2.8) and (2.13), the incomes relative to the reference income without and with auditing are :

$$y = Y - R = t\,(w-x) + \bar{R} - R, \tag{2.11}$$

$$z = Z - R = -ft\,(w-x) + \bar{R} - R. \tag{2.12}$$

If the reference income is below the legal after-tax income ($R \leq \bar{R}$), the outcome of the taxpayer without auditing is above the reference income, he is in the domain of gains ($y \geq 0$), while his outcome with auditing may be above or below the reference income, he may be in the domain of gains as well as of losses ($z \geq 0$ or $z \leq 0$). In the same manner, if the reference income is above the legal after-tax income ($R \geq \bar{R}$), with auditing the taxpayer is in the domain of losses ($z \leq 0$) while without auditing he may be in the domain of losses as well as of gains ($y \leq 0$ or $y \geq 0$).

An other particular value for the reference income and which can be compared with the general one is the final income of the taxpayer if he does not declare any income

5. The penalty rate is assumed to be not too high, to avoid that a taxpayer who declares an income equal to zero and is audited, pays more than his initial income. Formally, that is $f \leq \frac{1-t}{t}$. The tax administration can not use the strategy consisting of giving incentives to report honestly while the cost of auditing is minimized, by reducing the probability and imposing a huge fine.

6. See Dhami and al-Nowaihi [26].

and he is audited ($Z|_{x=0} = w - (1+f)tw$). From now on, it is denoted by :

$$\tilde{R} = w - (1+f)\,tw. \tag{2.13}$$

This values for the reference income can be ordered in this manner :

$$0 \leq \tilde{R} < \bar{R} \leq w. \tag{2.14}$$

It is empirically well-established that individuals have different risk attitudes towards gains (outcomes above the reference point) and losses (outcomes below the reference point) and care generally more about potential losses than potential gains, see for example Rabin [59] or Rabin and Thaler [60]. The taxpayer exhibits diminishing marginal sensitivity to increasing gains and losses : he is more sensitive to changes close to the reference point than to changes away from the reference point. He also exhibits loss aversion : he is more sensitive to losses than to gains, he prefers avoiding losses to making gains and he prefers risks that might possibly mitigate a loss.

The utility u associated with an outcome is thus assumed :

i. to be continuous on $\mathbb{R}$, twice continously differentiable on $\mathbb{R}^*$ and to vanish in zero : $u(0) = 0$,

ii. to be increasing, convex for losses and concave for gains : $u^{'} > 0$ on $\mathbb{R}^*$, $u^{''} > 0$ on $\mathbb{R}^*_-$ and $u^{''} < 0$ on $\mathbb{R}^*_+$ *(Diminishing marginal sensitivity)*,

iii. to be steeper for losses than for gains : $u^{'}(-k) > u^{'}(k)$ for $k \in \mathbb{R}^*_+$ *(Loss aversion)*.

The right-handed and left-handed limits of u' and u'' thus exist in $\mathbb{R} \cup \{-\infty, +\infty\}$ and are such that : $u^{'}(0_-) \geq u^{'}(0_+) \geq 0$, $u^{''}(0_-) \geq 0$ and $u^{''}(0_+) \leq 0$.

Figure 3.1 represents a typical utility function.

It can be observed that to consider $R = 0$, is equivalent to use the classical increasing and concave utility function of expected utility theory. In a prospect theory setting, this corresponds to a taxpayer who is in the domain of gains whatever is his final income. He has an extremely low propensity to evaluate a tax payment as a loss.

As in other cumulative theories, the attitude towards risk (risk aversion and risk seeking) depends on the curvature of the utility function and on the shape of the probability weighting function. The monetary risk attitude of the taxpayer, determined by the

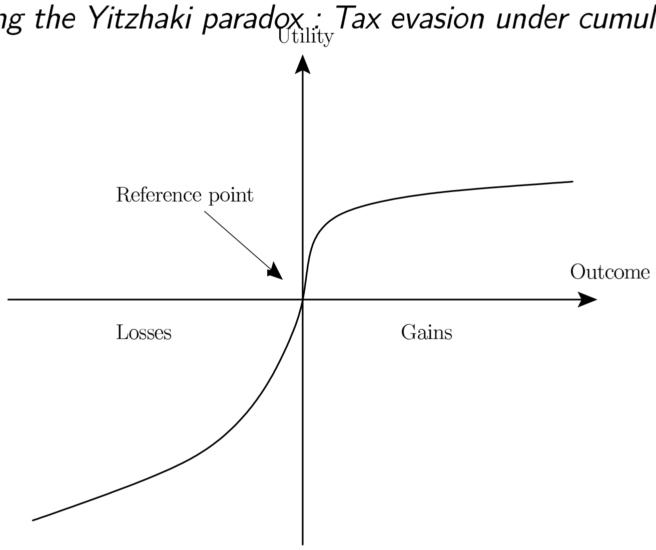


FIGURE 2.3 – Utility of an outcome

curvature of the utility function, is expressed by the following risk aversion measure :

$$\textit{Absolute risk aversion measure}: \ r_A(k) = -\frac{u''(k)}{u'(k)}, \ \text{for all } k \in \mathbb{R}. \tag{2.15}$$

$$\textit{Relative risk aversion measure}: \ r_R(k) = -k\frac{u''(k)}{u'(k)}, \ \text{for all } k \in \mathbb{R}. \tag{2.16}$$

Contrary to the classical Arrow-Pratt risk aversion measures in expected utility theory, this monetary measures do not determine entirely the attitude towards risk. They determine the attitude of the taxpayer concerning the values of the outcomes. The taxpayer is monetary risk averse for gains (the utility function is concave and the absolute risk aversion measure is positive) and monetary risk seeker for losses (the utility function is convex and the absolute risk aversion measure is negative). The relative risk aversion measure is positive in both domains of losses and gains.

From experimental motives, Tversky and Kahneman [79] states that the following power form is a satisfying form for the utility function to describe the behavior of individuals under risk :

$$u(k) = \begin{cases} k^{\gamma} & \text{if } \ k \geq 0, \\ -\mu(-k)^{\gamma} & \text{if } \ k < 0, \end{cases} \tag{2.17}$$

where $0 < \gamma < 1$, and $\mu > 1$ because of loss aversion.[7] It will be useful to specify general results.

7. More precisely, it suggests that empirically, $\gamma = 0.88$ and $\mu = 2.25$.

With this power form, the monetary risk aversion measures have very simple expressions :

$$\textit{Absolute risk aversion measure}: \ r_A(k) = \frac{1-\gamma}{k}, \ \text{for all } k \in \mathbb{R}. \tag{2.18}$$

$$\textit{Relative risk aversion measure}: \ r_R(k) = 1-\gamma, \ \text{for all } k \in \mathbb{R}. \tag{2.19}$$

The absolute risk aversion measure is negative in the domain of losses (the taxpayer is risk seeker), positive in the domain of gains (the taxpayer is risk averse) and decreasing in outcome. The relative one is constant. They depend only on the power parameter γ and not on the loss aversion parameter μ. $1-\gamma$ measures the strength of the monetary risk aversion in both domains of losses and gains.

2.2.4 Calibration assumptions

For sake of simplicity and because they are largely empirically verified, in all the following, this assumptions will be considered as true.

Assumption 2.1. The probability and penalty rates are such that :

$$f < \frac{1}{\Pi(p)}. \tag{2.20}$$

This is consistent with observed rates. [8] Actual values for the penalty rate range from 50% and 200% in most developped countries, while audit probabilities range from 1% to 3%. Assumption 2.1 is verified as soon as $\alpha \geq 0.2$, in the weighting function of Prelec [55], which seems to be a condition rather realistic about the degree of weighting of probabilities by a taxpayer.

Assumption 2.2. The probability, the tax and penalty rates, the probability weighting and utility functions are such that :

$$f, \ f^2 < -\frac{1}{\Pi(p)} \frac{u''(tw)}{u''(-ftw)}. \tag{2.21}$$

This expression relies the parameters of the cheating game and the behavior functions of the taxpayer. It is consistent with observed rates. Using the weighting function of

8. They depend a priori on signals sent by the taxpayer to the tax authorities about his wealth, but in the great majority of cases they stay in a fixed range of values.

Prelec [55] and the utility function of Tversky and Kahneman [79], it is verified with very reasonable values for the parameters α, γ and μ. [9]

Besides, the reference income of the taxpayer is modified by a change in the tax and penalty rates or the probability of auditing. Some assumptions about the direction of these modifications are natural :

Assumption 2.3. The modifications of the reference income are such that :

$$R_t \leq 0, \;\; R_f \leq 0 \text{ and } R_p \leq 0. \tag{2.22}$$

An increase of the taxable income (respectively, the tax and penalty rates or the probability of auditing) does not decrease (respectively, increase) the reference income which the taxpayer uses in assessing losses and gains. He assesses as being potentially richer (respectively, poorer).

Under expected utility theory, the Arrow-Pratt absolute risk aversion measure is decreasing with income. The same type of assumption can be made here :

Assumption 2.4. The monetary absolute risk aversion measure r_A is decreasing in outcome in both domains of losses and gains.

It means that the higher the losses are, the less the taxpayer is risk seeking. The lower the losses are, the more the taxpayer is risk seeking. In the same time, the higher the gains are, the less the taxpayer is risk averse. The lower the gains are, the more the taxpayer is risk averse. The power utility function of Tversky and Kahneman [79] confirms this assumption.

9. This two first assumptions are based in particular on the low values of probability of being audited. This values can be higher for certain taxpayers considered by the tax administration as evading more likely. However, the effective probability of an audit is possibly no more than 5%, according to Slemrod and Yitzhaki [71], while the assumptions are still verified with higher values, as 10% or 15%, taking reasonable values for others parameters (α around 0.5, γ around 0.88 and μ around 2.25).

2.3 Tax evasion decision with a probability weighting function

In this section, subjective cumulative probabilities represents the only difference with an expected utility theory setting. It is equivalent to apply cumulative prospect theory with zero for the reference income. The taxpayer maximizes the following utility of his declared income x :

$$U(x) = \pi(1-p)u(Y) + \pi(p)u(Z), \tag{2.23}$$

where Y and Z are the taxpayer's final incomes without and with auditing.

U is a continuous function on the interval $[0, w]$. In the general case, such a function may reach its maximum at several points in the interval. We assume here, without the lost of too much of generality, that this maximum is reached at only one point, denoted by x^*. Tax evasion is thus measured by $w - x^*$. The first and second derivatives of the function U are respectively described by :

$$U^{'}(x) = -\pi(1-p)tu^{'}(Y) + \pi(p)ftu^{'}(Z), \tag{2.24}$$

$$U^{''}(x) = \pi(1-p)t^2u^{''}(Y) + \pi(p)f^2t^2u^{''}(Z) < 0. \tag{2.25}$$

The following proposition describes the tax evasion decision of the taxpayer.

Proposition 2.1. *When $R = 0$, the taxpayer does not declare all his income.*

If $0 < f < \dfrac{1}{\Pi(p)}\dfrac{u^{'}(w)}{u^{'}(\tilde{R})}$, *the taxpayer totally cheats ($x^* = 0$).* [10]

If $\dfrac{1}{\Pi(p)}\dfrac{u^{'}(w)}{u^{'}(\tilde{R})} \leq f < \dfrac{1}{\Pi(p)}$, *the income declared by the taxpayer is interior*

($0 < x^* < w$).

Proof. See the Appendix. □

The taxpayer declares less than his actual income because of Assumption 2.1. The penalty rate is not high enough to motivate the taxpayer to be honest. It generalizes to

10. It is assumed in all the paper, that for equal values of utility, the taxpayer declares the higher level of income.

this setting, the condition obtained in an expected utility theory framework to ensure that the taxpayer evades. It means that the subjective expected payment on undeclared income underweights the subjective gain. The condition for an interior solution in the proposition generalizes also those obtained in expected utility theory. [11]

This proposition describes the change in the income declared by the taxpayer if the tax rate, the penalty rate or the probability of auditing increase.

Proposition 2.2. *When $R = 0$, tax evasion is decreasing in the tax rate, t, the penalty rate, f, and the probability of auditing, p.*

Proof. See the Appendix. □

The results obtained in an expected utility framework are robusts when a probability weighting function is introduced. The taxpayer staying in the domain of gains, the properties of the utility function are identical. Quantitative results about the level of tax evasion and the effects of changes in parameters would be slightly different, but they are qualitatively identical. Indeed, a change in the tax or penalty rate does not affect probabilities. In particular, the paradoxical result of Yitzhaki [83] remains. An increase in the tax rate makes tax evasion more risky but reduces the income of the taxpayer, which leads to a reduction of tax evasion activity, with a decreasing risk aversion in the domain of gains. In addition, the probability weighting function being increasing, a change in the probability of auditing has the same positive effect than with objective probabilities.

2.4 Tax evasion decision with a reference point

Subjective probabilities are not used here. The taxpayer is assumed to exactly weight the probability of events. The associated probability weighting function is the identical function ($\pi(p) = p$) and the tax evasion behavior is studied in a prospect theory framework, with a reference income and a utility function convex for losses and concave for gains. Then the risk aversion measures defined in (2.15) and (2.16) are complete risk aversion measures because they determine entirely the attitude towards risk. The

11. See for example Yitzhaki [83] for the linear case and Trannoy and Trotin [76] for the non-linear case.

taxpayer thinks of possible outcomes relative to a certain reference point, caring more about potential losses than potential gains. Compared to an expected utility theory framework, risk aversion is not computed from zero but on both sides of the reference income. The taxpayer now maximizes the following utility of his declared income x :

$$U(x) = (1-p)u(y) + pu(z). \tag{2.26}$$

U is a continuous function on $[0, w]$. It is assumed here that it reachs its maximum at only one point, denoted by x^*. $w - x^*$ measures tax evasion. The first and second derivatives of U are respectively :

$$U'(x) = -(1-p)tu'(y) + pftu'(z), \tag{2.27}$$

$$U''(x) = (1-p)t^2u''(y) + pf^2t^2u''(z). \tag{2.28}$$

Before to study the tax evasion problem in the present prospect theory setting, it is interesting to understand the differences with an expected utility theory, where the utility function u is increasing and concave and the absolute risk aversion measure of Arrow-Pratt is decreasing, the taxpayer maximizing the following utility :

$$E(x) = (1-p)u(Y) + pu(Z). \tag{2.29}$$

The following proposition explains the case where the results are similar.

Proposition 2.3. *When $\pi(p) = p$, $R \leq \tilde{R}$ and R does not include the tax and penalty rates and the probability of auditing (that is R_t, R_f and $R_p = 0$), in both frameworks, the taxpayer does not declare all his income and the necessary and sufficient conditions under which he declares an interior income are similar :*

$$f \geq \frac{1-p}{p}\frac{u'(w)}{u'(\tilde{R})}, \quad \textit{in expected utility theory,}$$

$$f \geq \frac{1-p}{p}\frac{u'(w-R)}{u'(\tilde{R}-R)}, \quad \textit{in prospect theory.}$$

In addition, in both frameworks, tax evasion decreases with the tax rate, the penalty rate and the probability of auditing.

Proof. See the Appendix. □

More generally, results brought by prospect theory differ from those under expected utility theory as soon as the reference income is above $\tilde{R}$, because of the possible convexity of u. [12] In particular, the utility function U is not concave everywhere and the second order conditions is not so easily verified. Comparative static results differ also as soon as the reference income includes the considered parameter (t, f or p) because its derivatives intercede. Concerning the present tax evasion problem, prospect theory differs from expected utility theory through two elements : the level of the reference income and its dependence in the parameters.

The income declared by the taxpayer thus depends on the expression of his reference income. As mentioned above, $\tilde{R}$, $\bar{R}$ and w are interesting because their values are equal to final incomes in extreme cases. They represent boundaries from which the behavior of the taxpayer changes. They are thus considered here. To interpret conditions more easily, it will be interesting to outline results using the probability weighting function of Prelec [55] and the utility function of Tversky and Kahneman [79].

First case : $R = \tilde{R}$

In this case, the taxpayer is in the domain of gains whatever is his final income. He has an extremely low propensity to evaluate a tax payment as a loss. It is linked for example to a high preference level for public goods. The outcomes without and with auditing are :

$$y = (1+f)tw - tx > 0_+, \tag{2.30}$$

$$z = ftx \geq 0_+. \tag{2.31}$$

The following proposition describes his tax evasion behavior.

Proposition 2.4. *When $\pi(p) = p$ and $R = \tilde{R}$, the taxpayer does not declare all his income.*

$$\text{If} \quad 0 < f < \frac{1-p}{p}\frac{u'(w-\tilde{R})}{u'(0_+)}, \text{ the taxpayer totally cheats } (x^* = 0).$$

12. As mentioned above, $\tilde{R}$ is the lower level of final income that the taxpayer can obtained because it corresponds to the case where he totally cheats and is audited.

If $\frac{1-p}{p}\frac{u'(w-\tilde{R})}{u'(0_+)} \leq f < \frac{1-p}{p}$, *the income declared by the taxpayer is interior* $(0 < x^* < w)$.

Proof. The proof, similar to the one for Proposition 2.1, is available from the author, upon request. □

As in the previous section, this condition are equivalent to those obtained in expected utility theory. With a so low reference income, properties about the utility functions are similar, the scheme of the tax evasion decision is then similar too.

Corollary 2.1. *When the utility function has the Tversky and Kahneman form and when* $\pi(p) = p$ *and* $R = \tilde{R}$, *the income declared by the taxpayer is interior* $(0 < x^* < w)$.

Proof. See the Appendix. □

The utility u rises infinitely from the income reached when there is total evasion and auditing, $\tilde{R}$.[13] This motivates the taxpayer to declare. Indeed, the benefit obtained from the first declared dollar, in case of auditing, infinitely outweighs the cost imposed by the first declared dollar without auditing.

Effects on tax evasion of changes in parameters, are described by the following proposition.

Proposition 2.5. *When* $\pi(p) = p$ *and* $R = \tilde{R}$,

i. *if* $r_R(z) > r_R(y)$, *tax evasion is increasing in the tax rate,* t,
   if $r_R(z) \leq r_R(y)$, *tax evasion is non-increasing in the tax rate,* t,
ii. *if* $f > \frac{r_R(z)-r_R(y)}{t} + (w-x^*)r_A(y)$, *tax evasion is decreasing in the penalty rate,* f,
   if $f \leq \frac{r_R(z)-r_R(y)}{t} + (w-x^*)r_A(y)$, *tax evasion is non-decreasing in the penalty rate,* f,
iii. *tax evasion decreases with the probability of auditing,* p.

Proof. See the Appendix. □

Many experimental and econometric studies have emphasized that a rise in the tax rate increases tax evasion.[14] Yitzhaki [83] showed that, using expected utility theory

13. It is because $\tilde{R}$ is the reference income and $u'(0_+) = +\infty$ with the utility function of Tversky and Kahneman [79].

14. See for example Friedland et al. [32], Clotfelter [19] and Pudney et al. [56].

under the assumption of decreasing absolute risk aversion of Arrow-Pratt, a rise in the tax rate leads to a decrease in tax evasion. With the present framework, with a so low reference income, a rise in the tax rate increases tax evasion if the relative risk aversion, as defined in (2.16), is larger with, than without auditing. It is verified, for instance, if the relative risk aversion is decreasing in outcome in the domain of gains. [15] The intuition is that an increase in the tax rate causes the taxpayer to be richer in terms of outcomes (y and z are increassing in t). With a decreasing relative risk aversion, he chooses then to increase the fraction of his initial income in the risky alternative and tax evasion increases. The use of the reference income implies that the outcomes can be increasing in the tax rate, contrary to incomes which are decreasing under the standard expected utility theory setting. The intuition behind the present relative risk aversion is similar to that behind the standard relative risk aversion of Arrow-Pratt. In addition, empirical studies show that a rise in the penalty rate decreases tax evasion. It is verified only if the penalty rate is sufficiently high. The empirically correct result is predicted when the probability of auditing increases, this leads to a decrease in tax evasion.

Corollary 2.2. *When the utility function has the Tversky and Kahneman form and when $\pi(p) = p$ and $R = \tilde{R}$,*

i. tax evasion is not modified by a change in the tax rate, t,

ii. if $f(1+f) > \frac{1-\gamma}{t}$, tax evasion is decreasing in the penalty rate, f.

Proof. See the appendix. □

The relative risk aversion, defined in (2.19), is constant. This implies that a change in the tax rate does not modify tax evasion. In addition, tax evasion decreases with the penalty rate if it is sufficiently high. It is verified with observed tax and penalty rates. [16]

All the results would be very similar for a taxpayer whose reference income is below $\tilde{R}$, ($0 \leq R \leq \tilde{R}$), up to minor changes caused by different values for R_t, R_f and R_p. Indeed, in the same manner, he would be always in the domain of gains.

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15. As with the Arrow-Pratt measures, the decrease of the relative risk aversion is a stronger assumption than the decrease of the absolute one.

16. For example, with $\gamma = 0.88$ and $f = 50\%$, it is verified as soon as $t > 16\%$. With $\gamma = 0.88$ and $f = 150\%$, it is verified as soon as $t > 3,2\%$.

Second case : $R = \bar{R}$

In this case, the taxpayer is in the domain of losses as soon as he is audited and in the domain of gains as soon as he is not audited, whatever is his declared income. With this reference income, if he is or not discovered cheating by the tax administration, is the central element for the taxpayer to evaluate his outcome. The outcomes without and with auditing are :

$$y = t(w - x) \geq 0_+, \tag{2.32}$$

$$z = -ft(w - x) \leq 0_-. \tag{2.33}$$

It is not possible to anticipate the tax evasion decision with this reference income as with others, because the incomes with and without auditing are always non-equal, even if the taxpayer is completely honest.[17] Signs of the maximized utility function U and its marginal functions are not easily computed. In particular, U is not concave everywhere. Conditions generalizing those obtained in expected utility theory, to ensure an interior solution, can not be highlighted here.

Effects on tax evasion of changes in parameters, are described by the following proposition.

Proposition 2.6. *When* $\pi(p) = p$ *and* $R = \bar{R}$,

i. when the declared income is interior,
   if $r_R(z) > r_R(y)$, *tax evasion is increasing in the tax rate,* t,
   if $r_R(z) \leq r_R(y)$, *tax evasion is non-increasing in the tax rate,* t,
   when there is total evasion, it is decreasing in the tax rate, t,
   when there is no evasion, it is not modified by a change in the tax rate, t,

ii. when the declared income is interior,
   if $r_R(z) < t$, *tax evasion is decreasing in the penalty rate,* f,
   if $r_R(z) \geq t$, *tax evasion is non-decreasing in the penalty rate,* f,

17. More precisely, the income with auditing z is always negative and the income without auditing y is always positive. Even if the taxpayer totally declares, $y(w) = 0_+$ and $z(w) = 0_-$, and a priori, the values of u, u' and u'' are be non-equal. $\bar{R}$ is the only reference income with which $U'(w)$ can be non-negative.

when there is total evasion,

if $r_A(-ftw) > -\frac{1}{w}$, *tax evasion is decreasing in the penalty rate,* f,

if $r_A(-ftw) \leq -\frac{1}{w}$, *tax evasion is not modified by a change in the penalty rate,* f,

when there is no evasion, it is not modified by a change in the penalty rate, f,

iii. when there is no evasion, it is not modified by a change in the probability of auditing, p,
otherwise, tax evasion is decreasing in the probability of auditing, p.

Proof. See the Appendix. □

As in the previous case, when the declared income is interior, tax evasion is increasing in the tax rate if the relative risk aversion is larger with auditing than without. In the present case, levels of the relative risk aversion measure in the two domains are compared. If the relative risk aversion is larger in the domain of losses than in that of gains, a rise in the tax rate increases tax evasion. Again, it is verified if the relative risk aversion is decreasing on both domains together. The intuition is similar because an increase in the tax rate causes the taxpayer to be richer in terms of outcomes. Indeed, the expected outcome, $(1-p)y+pz$, is increasing in the tax rate. In addition, tax evasion is decreasing in the penalty rate if the relative risk aversion with auditing is lower than the tax rate. Under the condition $r_R(y) < r_R(z) < t$, results are then consistent with empirical evidence, because the empirically correct result is predicted when the probability of auditing increases.

Corollary 2.3. *When the utility function has the Tversky and Kahneman form and when* $\pi(p) = p$ *and* $R = \bar{R}$,

i. when there is total evasion, it is decreasing in the tax rate, t,
otherwise, tax evasion is not modified by a change in the tax rate, t,

ii. when the declared income is interior,

if $t > 1-\gamma$, *tax evasion is decreasing in the penalty rate,* f,

if $t \leq 1-\gamma$, *tax evasion is non-decreasing in the penalty rate,* f,

when there is total evasion,

if $ft > 1-\gamma$, *tax evasion is decreasing in the penalty rate,* f,

if $ft \leq 1-\gamma$, *tax evasion is not modified by a change in the penalty rate,* f,

when there is no evasion, it is not modified by a change in the penalty rate, f.

Tax evasion is not modified by a change in the tax rate. It is due to the specific form of the risk aversion measure, which is due to the symetric form of the utility function in (3.12). In particular, the same power parameter, γ, is used. To find that a rise in the tax rate increases tax evasion, the power parameter in the domain of losses should be lower than in the domain of gains, because risk aversion should be stronger in the domain of losses than in the domain of gains,[18] as for instance :

$$u(k) = \begin{cases} k^{\gamma} & \text{if} \quad k \geq 0, \\ -\mu(-k)^{\rho} & \text{if} \quad k < 0, \end{cases} \tag{2.34}$$

where $0 < \rho < \gamma < 1$ and $\mu > 1$.

With a reference income just above $\tilde{R}$, the taxpayer considers the most of possible outcomes as gains. Remaining below the legal income $\bar{R}$, the outcome without auditing is always considered as a gain, while the higher the reference income is, the more probably the outcome with auditing is a loss. What we can observe is that, with a reference income such that $\tilde{R} < R < \bar{R}$, the result about the tax evasion decision would be very similar to that with $\tilde{R}$. Up to minor changes caused by different values for R_t, R_f and R_p, the schemes of results about changes in parameters would be very close to that with $\tilde{R}$ and $\bar{R}$.[19]

With a reference income higher than the legal income ($\bar{R} < R < w$), the taxpayer is always in the domain of losses when he is audited and also when he is not audited but declares a too high income. He has a high propensity to evaluate a tax payment as a loss. A priori, this is the most usual case in practice because it corresponds to a case where the taxpayer regards positively the situation where his final income exceeds the legal income while staying below his initial income. It is interesting to notice that with such a reference income, the taxpayer does not declare an income close to the legal one. At this level, the higher is the reference income, the lower is the maximum declaration of the taxpayer.[20] As mentioned before, $\bar{R}$ is the only one with which the taxpayer can be

18. Dhami and al-Nowaihi [26] show that when the declared income is interior, an increase in the tax rate increases tax evasion, because of the introduction of a stigma proportional to the evaded income.

19. Between $\tilde{R}$ and $\bar{R}$, results differs essentially at corner declarations. In fact, with $\tilde{R} < R < \bar{R}$, the taxpayer would not declare all his income, as with $\tilde{R}$. However, in case of total evasion, results would be similar to that with $\bar{R}$.

20. Formally, the income declared by the taxpayer stays below $\frac{w-R}{t}$.

honest because it is the only one with which the income with and without auditing are never equal, even if the taxpayer is honest. In addition, up to minor changes caused by different values for R_t, R_f and R_p, the schemes of results about changes in parameters would be very close to that with $\bar{R}$.

Third case : $R = w$

This extreme case corresponds to an extremely tax-averse taxpayer. Any payment to the tax administration is considered a loss. The outcomes without and with auditing are :

$$y = -tx \leq 0_-, \tag{2.35}$$

$$z = -tx - (1+f)t(w-x) < 0_-. \tag{2.36}$$

The following proposition describes the tax evasion behavior.

Proposition 2.7. *When $\pi(p) = p$ and $R = w$, the taxpayer evades all his income* $(x^* = 0)$.

Proof. See the Appendix. □

With a so high reference income, the taxpayer is completely dishonest. Formally, it is due to the convexity of the utility function U, linked to the convexity of u, the taxpayer staying in the domain of losses. He is a complete risk taker.

Effects on tax evasion of changes in parameters, are described by the following proposition.

Proposition 2.8. *When $\pi(p) = p$ and $R = w$,*

i. tax evasion is not modified by a change in the tax rate, t,

ii. if $\frac{(1+f)}{f} > \frac{r_R(z)}{t}$, tax evasion is decreasing in the penalty rate, f,
if $\frac{(1+f)}{f} \leq \frac{r_R(z)}{t}$, tax evasion is not modified by a change in the penalty rate, f,

iii. tax evasion is decreasing in the probability of auditing, p.

Proof. See the Appendix. □

The results differ from previous cases because of the complete evasion but we can observe that again, tax evasion is decreasing in the penalty rate if it sufficiently high.

Corollary 2.4. *When the utility function has the Tversky and Kahneman form and when $\pi(p) = p$ and $R = w$,*

if $f > \frac{-t}{t-(1-\gamma)}$, tax evasion is decreasing in the penalty rate, f,
if $f \leq \frac{-t}{t-(1-\gamma)}$, tax evasion is not modified by a change in the penalty rate, f.

Actually, as soon as $t > 1 - \gamma$, which seems to be an empirically correct condition, tax evasion is decreasing in the penalty rate.

In all this section, results differs significantly from those given in the previous one. This highlights that results given in an expected utility theory framework are not robust to the use of a reference income, whatever is the value of the reference income.

2.5 Tax evasion decision under cumulative prospect theory

The general setting of cumulative prospect theory is now used to study the tax evasion decision of the taxpayer. In fact, even when a change of the probability of auditing is studied, results do not really differ to those highlighted in the previous section. In particular, intuitions are exactly the same. This shows that the presence of a probability weighting function in this tax evasion problem does not transform the framework of expected utility theory as does the use of a reference income. It is because there are only two kind of outcomes, with or without auditing. In particular, the risk aversion measures defined in (2.15) and (2.16) determine entirely the attitude towards risk. Results differs only quantitatively from those without probability weighting function highlighted in the previous section.[21]

The taxpayer maximizes the following utility of his declared income x :

$$U(x) = \pi(1-p)u(y) + \pi(p)u(z). \tag{2.37}$$

U is a continuous function on $[0, w]$. It is assumed that it reachs its maximum at only one point, denoted by x^*. $w - x^*$ measures tax evasion. The first and second derivatives

21. Certain parts of proof, very similar than those in the previous section, are not given in this section.

of U are respectively :

$$U'(x) = -\pi(1-p)tu'(y) + \pi(p)ftu'(z), \tag{2.38}$$

$$U''(x) = \pi(1-p)t^2u''(y) + \pi(p)f^2t^2u''(z). \tag{2.39}$$

As in the previous section, the income declared by the taxpayer depends on the expression of his reference income.

First case : $R = \tilde{R}$

Proposition 2.9. *When $R = \tilde{R}$, the taxpayer does not declare all his income.*

If $0 < f < \frac{1}{\Pi(p)} \frac{u'(w-\tilde{R})}{u'(0_+)}$, *the taxpayer totally cheats ($x^* = 0$).*

If $\frac{1}{\Pi(p)} \frac{u'(w-\tilde{R})}{u'(0_+)} \leq f < \frac{1}{\Pi(p)}$, *the income declared by the taxpayer is interior ($0 < x^* < w$).*

This conditions are the equivalent in a cumulative prospect theory framework, of those obtained in expected utility theory. Under Assumption 2.1, the penalty is not high enough to motivate the taxpayer to be completely honest.

Proposition 2.10. *When $R = \tilde{R}$,*

i. if $r_R(z) \leq r_R(y)$, tax evasion is non-increasing in the tax rate, t,
if $r_R(z) > r_R(y)$, tax evasion is increasing in the tax rate, t,

ii. if $x^* r_A(z) < w\ r_A(y) + 1$, tax evasion is decreasing in the penalty rate, f,
if $x^* r_A(z) \geq w\ r_A(y) + 1$, tax evasion is non-decreasing in the penalty rate, f,

iii. tax evasion decreases with the probability of auditing, p.

Proof. See the Appendix. □

Second case : $R = \bar{R}$

Proposition 2.11. *When $R = \bar{R}$,*

i. when the declared income is interior,
if $r_R(z) \leq r_R(y)$, tax evasion is non-increasing in the tax rate, t,
if $r_R(z) > r_R(y)$, tax evasion is increasing in the tax rate, t,
when there is total evasion, it is decreasing in the tax rate, t,
when there is no evasion, it is not modified by a change in the tax rate, t,

ii. when the declared income is interior,

if $r_R(z) < t$, *tax evasion is decreasing in the penalty rate,* f,

if $r_R(z) \geq t$, *tax evasion is non-decreasing in the penalty rate,* f,

when there is total evasion,

if $r_A(-ftw) > -\frac{1}{w}$, *tax evasion is decreasing in the penalty rate,* f,

if $r_A(-ftw) \leq -\frac{1}{w}$, *tax evasion is not modified by a change in the penalty rate,* f,

when there is no evasion, it is not modified by a change in the penalty rate, f,

iii. when there is total evasion, tax evasion is decreasing in the probability of auditing, p,

otherwise, tax evasion is not modified by a change in the probability of auditing, p.

Proof. See the Appendix. □

Third case : $R = w$

Proposition 2.12. *When* $R = w$, *the taxpayer evades all his income.*

Proposition 2.13. *When* $R = w$,

i. tax evasion is not modified by a change in the tax rate, t,

ii. if $r_R(z) < \frac{t(1+f)}{f}$, *tax evasion is decreasing in the penalty rate,* f,

if $r_R(z) \geq \frac{t(1+f)}{f}$, *tax evasion is not modified by a change in the penalty rate,* f,

iii. tax evasion is decreasing in the probability of auditing, p.

Proof. See the Appendix. □

2.6 Conclusion

This paper characterizes tax evasion decision under a few or all properties of cumulative prospect theory, used in a very general way. It appears that prospect theory is a setting consistent with empirical results. In particular, considering a general reference income, under relatively weak conditions, it predicts that an increase in the tax rate rises tax evasion, contrary to predictions under expected utility theory. The point is the use of the reference income, which implies that the income effect goes to the opposite side : an increase in the tax rate causes the taxpayer to be richer in terms of outcome and with a decrease relative risk aversion, he chooses to increase the fraction of his initial income in the risky alternative.

The choice was made to use a fine imposed on the evaded tax because it is a more common case, but the main result would be sensitively unchanged with a fine imposed on the undeclared income. In fact, the values of the outcomes would be different but still increasing with the tax rate.

2.7 Appendix

Proof of Proposition 2.1. $U'' < 0$, and $U'(w) < 0$, then the taxpayer totally cheats if and only if $U'(0) < 0$ and his declared income is interior if and only if $U'(0) \geq 0$ and $U'(w) < 0$. □

Proof of Proposition 2.2. Denote $\Phi_p(x,t) = U'(x)$, when the declared income is interior, $\Phi_p(x^*,t) = 0$ and $\frac{\partial \Phi_p}{\partial x}(x^*,t) < 0$. Thus, by applying the Implicit Function Theorem (IFT), we obtain that the sign of $\frac{\partial x}{\partial t}\big|_{x=x^*}$ is the same as that of $\frac{\partial \Phi_p}{\partial t}(x^*,t)$. After computing we find :

$$\frac{\partial \Phi_p}{\partial t}(x^*,t) = \pi(1-p)tu'(Y)\left[x^*\left[r_A(Z) - r_A(Y)\right] + (1+f)(w-x^*)r_A(Z)\right] > 0.$$

When the taxpayer totally cheats, $U'(0) \leq 0$. By applying the IFT to $\Phi_p^0(x,t) = U'(x) - U'(0)$, we obtain that the sign of $\frac{\partial x}{\partial t}\big|_{x=0}$ is the same as that of $\frac{\partial \Phi_p^0}{\partial t}(0,t)$, with :

$$\frac{\partial \Phi_p^0}{\partial t}(0,t) = -\pi(1-p)t\Pi(p)u''(\tilde{R})(1+f)w > 0.$$

Denote $\Psi_p(x,f) = U'(x)$. By applying the IFT, we obtain that the sign of $\frac{\partial x}{\partial f}\Big|_{x=x^*}$ is the same as that of $\frac{\partial \Psi_p}{\partial f}(x^*,f)$, with :

$$\frac{\partial \Psi_p}{\partial f}(x^*,f) = \pi(1-p)t^2u'(Y)\left[(w-x^*)r_A(Z) + \frac{1}{f}\right] > 0.$$

When $x^* = 0$, by applying the IFT to $\Psi_p^0(x,f) = U'(x) - U'(0)$, we obtain that the sign of $\frac{\partial x}{\partial f}\Big|_{x=0}$ is the same as that of $\frac{\partial \Psi_p^0}{\partial f}(0,f)$, with :

$$\frac{\partial \Psi_p^0}{\partial f}(0,f) = \pi(1-p)t^2u'(w)\left[wr_A(\tilde{R}) + \frac{1}{f}\right] > 0.$$

Denote $\Gamma_p(x,p) = U^{'}(x)$. By applying the IFT, we obtain that the sign of $\left.\frac{\partial x}{\partial p}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Gamma_p}{\partial p}(x^*,p)$, with :

$$\frac{\partial \Gamma_p}{\partial p}(x^*,p) = \pi^{'}(1-p)tu^{'}(Y) + \pi^{'}(p)ftu^{'}(Z) > 0.$$

When $x^* = 0$, by applying the IFT to $\Gamma_p^0(x,p) = U^{'}(x) - U^{'}(0)$, we obtain that the sign of $\left.\frac{\partial x}{\partial p}\right|_{x=0}$ is the same as that of $\frac{\partial \Gamma_p^0}{\partial p}(0,p)$, with :

$$\frac{\partial \Gamma_p^0}{\partial p}(0,p) = \pi^{'}(1-p)tu^{'}(w) + \pi^{'}(p)ftu^{'}(\tilde{R}) > 0.$$

□

Proof of Proposition 2.3. U and E are concave, $U^{'}(w)$ and $E^{'}(w) < 0$, and $U^{'}(0) = -(1-p)tu^{'}(w-R) + pftu^{'}(\tilde{R}-R)$ and $E^{'}(0) = -(1-p)tu^{'}(w) + pftu^{'}(\tilde{R})$. In addition, under prospect theory, the sign of $\left.\frac{\partial x}{\partial t}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Phi}{\partial t}(x^*,t)$, with :

$$\frac{\partial \Phi}{\partial t}(x^*,t) = (1-p)tu^{'}(Y-R)\left[-xr_{_A}(Y-R) + (x+(1+f)(w-x))r_{_A}(Z-R)\right] > 0,$$

with $0 < r_{_A}(Y-R) < r_{_A}(Z-R)$, because $R < Z < Y$, and under expected utility theory, the sign of $\left.\frac{\partial x}{\partial t}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Delta}{\partial t}(x^*,t)$, with :

$$\frac{\partial \Delta}{\partial t}(x^*,t) = (1-p)tu^{'}(Y)\left[-xr_{_A}(Y) + (x+(1+f)(w-x))r_{_A}(Z)\right] > 0.$$

The proof is similar with f and p.

□

Proof of Corollary 2.1. When u has the power form as described in (3.12), $u^{'}(0_+) = +\infty$ and $u^{'}(w-\tilde{R}) = \frac{\gamma}{(w-\tilde{R})^{1-\gamma}} > 0$, then $\frac{1}{\Pi(p)}\frac{u^{'}(w-\tilde{R})}{u^{'}(0_+)} = 0$.

□

Proof of Proposition 2.5. The sign of $\left.\frac{\partial x}{\partial t}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*,t)$, with :

$$\frac{\partial \Phi_{\tilde{R}}}{\partial t}(x^*,t) = (1-p)u^{'}(y)\left[r_{_R}(y) - r_{_R}(z)\right].$$

When $x^* = 0$, the principle of proof is the same as that for Proposition 2.2.
The sign of $\left.\frac{\partial x}{\partial f}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Psi_{\tilde{R}}}{\partial f}(x^*,f)$, with :

$$\frac{\partial \Psi_{\tilde{R}}}{\partial f}(x^*,f) = (1-p)\frac{t^2}{f}u^{'}(y)\left[w\, r_{_A}(y) - x^* r_{_A}(z) + 1\right].$$

The sign of $\left.\frac{\partial x}{\partial p}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Gamma_{\tilde{R}}}{\partial p}(x^*,p)$, with :

$$\frac{\partial \Gamma_{\tilde{R}}}{\partial p}(x^*,p) = tu^{'}(y) + ftu^{'}(z) > 0.$$

□

Proof of Corollary 2.2. If $f > (w-x^*)\frac{1-\gamma}{y}$, tax evasion decreases with f. $(w-x^*)\frac{1-\gamma}{y}$ is decreasing in x^*, therefore, as soon as $f > w\frac{1-\gamma}{y(0)}$, tax evasion decreases with f.

□

Proof of Proposition 2.6. The sign of $\left.\frac{\partial x}{\partial t}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Phi_{\bar{R}}}{\partial t}(x^*,t)$, with :

$$\frac{\partial \Phi_{\bar{R}}}{\partial t}(x^*,t) = (1-p)yu^{'}(y)\left[fr_{A}(z) + r_{A}(y)\right].$$

When $x^* = 0$, the sign of $\left.\frac{\partial x}{\partial t}\right|_{x=0}$ is the same as that of $\frac{\partial \Phi^0_{\bar{R}}}{\partial t}(0,t)$, with :

$$\frac{\partial \Phi^0_{\bar{R}}}{\partial t}(0,t) = -tw\left[(1-p)u^{''}(tw) + fpu^{''}(-ftw)\right].$$

When $x^* = w$, the sign of $\left.\frac{\partial x}{\partial t}\right|_{x=w}$ is the same as that of $\frac{\partial \Phi^w_{\bar{R}}}{\partial t}(w,t) = 0$.
The sign of $\left.\frac{\partial x}{\partial f}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Psi_{\bar{R}}}{\partial f}(x^*,f)$, with :

$$\frac{\partial \Psi_{\bar{R}}}{\partial f}(x^*,f) = (1-p)\frac{t}{f}u^{'}(y)\left[-r_{R}(z) + t\right].$$

When $x^* = 0$, the sign of $\left.\frac{\partial x}{\partial f}\right|_{x=0}$ is the same as that of $\frac{\partial \Psi^0_{\bar{R}}}{\partial f}(0,f)$, with :

$$\frac{\partial \Psi^0_{\bar{R}}}{\partial f}(0,f) = (1-p)\frac{t^2}{f}u^{'}(tw)\left[w\, r_{A}(-ftw) + 1\right].$$

When $x^* = w$, the sign of $\left.\frac{\partial x}{\partial f}\right|_{x=w}$ is the same as that of $\frac{\partial \Psi^w_{\bar{R}}}{\partial f}(w,f)$, with :

$$\frac{\partial \Psi^w_{\bar{R}}}{\partial f}(w,f) = (1-p)u^{'}(0_+)\frac{t^2}{f}.$$

The sign of $\left.\frac{\partial x}{\partial p}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Gamma_{\bar{R}}}{\partial p}(x^*,p)$, with :

$$\frac{\partial \Gamma_{\bar{R}}}{\partial p}(x^*,p) = tu^{'}(y) + ftu^{'}(z) > 0.$$

When $x^* = 0$, the sign of $\left.\frac{\partial x}{\partial p}\right|_{x=0}$ is the same as that of $\frac{\partial \Gamma^0_{\bar{R}}}{\partial p}(0,p)$, with :

$$\frac{\partial \Gamma^0_{\bar{R}}}{\partial p}(0,p) = tu^{'}(tw) + ftu^{'}(-ftw) > 0.$$

When $x^* = w$, the sign of $\left.\frac{\partial x}{\partial p}\right|_{x=w}$ is the same as that of $\frac{\partial \Gamma^w_{\bar{R}}}{\partial p}(w,p)$, with :

$$\frac{\partial \Gamma^w_{\bar{R}}}{\partial p}(w,p) = t(1+f)u^{'}(0_+) > 0.$$

□

Proof of Proposition 2.7. With $R = w$, $z \leq y \leq 0$, u is thus increasing and convex and U is decreasing.

□

Proof of Proposition 2.8. $x^* = 0$, the sign of $\left.\frac{\partial x}{\partial t}\right|_{x=0}$ is then the same as that of $\frac{\partial \Phi^0_w}{\partial t}(0,t)$ with :

$$\frac{\partial \Phi^0_w}{\partial t}(0,t) = -(1+f)twpu^{''}(-(1+f)tw) < 0.$$

$x^* = 0$, the sign of $\left.\frac{\partial x}{\partial f}\right|_{x=0}$ is then the same as that of $\frac{\partial \Psi^0_w}{\partial f}(0,f)$ with :

$$\frac{\partial \Psi^0_w}{\partial f}(0,f) = t^2(1-p)u^{'}(0_+)\left[w\ r_{_A}(-(1+f)tw) + \frac{1}{f}\right].$$

$x^* = 0$, the sign of $\left.\frac{\partial x}{\partial p}\right|_{x=0}$ is then the same as that of $\frac{\partial \Gamma^0_w}{\partial p}(0,p)$ with :

$$\frac{\partial \Gamma^0_w}{\partial p}(0,p) = tu^{'}(0_+) + ftu^{'}(-(1+f)tw) > 0.$$

□

Proof of Proposition 2.10, 2.11 and 2.13. The sign of $\left.\frac{\partial x}{\partial p}\right|_{x=x^*}$ is the same as that of $\frac{\partial \Gamma_c}{\partial p}(x^*,p)$, with :

$$\frac{\partial \Gamma_c}{\partial p}(x^*,p) = \pi^{'}(1-p)tu^{'}(y) + \pi^{'}(p)ftu^{'}(z) > 0.$$

□

Chapitre 3

Optimal tax enforcement under prospect theory

3.1 Introduction

In most countries, income tax administrations rely on income reports from taxpayers. Thus, they have an incentive to misreport their income, in order to reduce their income tax liability. Losses to public budgets from tax evasion are indeed significant : the US Internal Revenue Service (IRS), for example, estimates the tax gap in 2001 at U$D 345 billion, i.e., almost 16% of the total tax revenue (IRS [38]). The main tools in the hand of the tax administration to limit possible misbehaviour are to audit taxpayers and to verify the information provided. Audits being costly,[1] the tax administration generally selects the reports to be audited. Assessed misreporting can result in penalties and fines ; the setting up of penalty schemes also satisfies an objective of both horizontal and vertical equity among taxpayers. On top of tax rates, an optimal tax policy includes therefore an audit strategy and a scheme of penalties.

This gives rise to interesting questions, that we aim at analysing, about the optimal audit strategies and penalty schemes, and the nature of interactions between tax rates and audit strategies. For that, we need to take into account taxpayers' attitude toward risk and uncertainty. Although expected utility theory has been considered for long

0. This chapter reviews a joint work with Amedeo Piolatto.

1. One main source of cost is the wage and the formation of the tax administration agents. Also, there are incentive problems related to the corruptibility of auditors (Hindriks et al. [35]).

time the most convenient framework, there is a growing consensus about the need of an alternative theory of the agents' behaviour under uncertainty. [2] Pioneered by Kahneman and Tversky [40], prospect theory has become one of the most prominent alternatives to expected utility theory, and it is widely used in empirical research. [3] According to prospect theory, the carrier of utility is the difference between the final level of income and a determined reference income, and not the final level of income (as suggested by expected utility theory). Agents think of gains and losses relative to this reference point : this phenomenon is known in cognitive sciences as the *framing effect.* [4] The utility function, convex for gains and concave for losses, expresses the *loss aversion* phenomenon : individuals care generally more about potential losses than potential gains. Prospect theory is nowadays commonly used in cognitive sciences and has become one of the standards in the Behavioural Economics literature.

The recent literature on taxation highlights problems in using the expected utility theory setting for tax evasion decision issues, because it contradicts the empirical evidence in several ways. In particular, with a reasonable degree of risk aversion, it overestimates the willingness of agents to misreport their income, therefore, it predicts more tax evasion than what really occurs. Furthermore, under the assumption of Decreasing Absolute Risk Aversion (DARA), it predicts that an increase in the tax rate leads to a decrease in tax evasion. [5] As a consequence, we observe a growing interest for prospect theory within the taxation literature. Kanbur et al. [42] study the optimal non-linear taxation under Prospect Theory, and show that the standard Mirrlees [51] results' are modified in several interesting ways. Dhami and al-Nowaihi [26] apply prospect theory to the taxpayers' decision to evade taxes, and show that predictions are both quantitatively and qualitatively more in line with the empirical evidence than under expected utility theory. In Dhami and al-Nowaihi [27] the tax rate is endogenous : one main finding is that the best description of the data is obtained by combining taxpayers behaving according to prospect theory and the government acting as predicted by expected utility theory.

To the best of our knowledge, ours is the first attempt to analyse the optimal audit

2. See, for instance, Mirrlees [51].
3. See, for example, Yaniv [82], Camerer [12] or Camerer and Loewenstein [14].
4. For more on that, see Tversky and Kahneman [79].
5. See Yitzhaki [83] for the linear case, and Trannoy and Trotin [76] for the non-linear case.

scheme under the more realistic assumption that agents behave according to prospect theory. For the case of expected utility theory, Cremer and Gahvari [23] focus on the moral hazard problem occurring when the labour supply choice is endogenous. The expected utility theory-works that are closer to ours are probably Chander and Wilde [17] and Chander [15], [16]. In the former, they characterise the optimal tax schedule in the presence of enforcement costs and clarify the nature of the interplay between optimal tax rates, audit probabilities and penalties for misreporting. In particular, under the (rather strong) assumption of risk neutral expected-utility-maximiser taxpayers, they show that the optimal tax function must generally be increasing and concave. This because a progressive tax function implies stronger incentives to misreport and thus it calls for larger audit probabilities. Chander [15], [16] studies the same issues for the case of risk averse taxpayers, when the incentive to misreport is weaker. By introducing a measure of aversion to large risks, he shows that the optimal tax function is increasing and concave if the taxpayer's aversion to such large risks is decreasing with income.

The chapter extends the optimal tax enforcement literature, considering agents that behave according to prospect theory. Reference dependence being a crucial element in prospect theory, we need to define a general reference income. The most natural choice is to restrict our attention, setting the legal income (i.e., the after-tax disposable income, under no tax evasion) as a lower-bound and the pre-tax income as an upper-bound. The chapter shows that Chander and Wilde [17]'s and Chander [15]'s results hold under a set of less restrictive assumptions when agents behave according to prospect theory, as opposed to expected utility theory. In particular, we show that the optimal audit probability function is always non-increasing. Concerning the optimal tax function, we show that it is always non-decreasing and concave when the pre-tax income is the used as a reference ; nevertheless, for the same result to hold when the reference income is the legal one, we need to impose a further restriction : we show that a sufficient condition is to have Decreasing Prospect Risk Aversion (DPRA).

The chapter is organised as follows. The next section describes a general model of income tax enforcement under prospect theory and introduces the definition of an optimal tax and audit scheme. Section 3 and 4 solves the model using as the reference income respectively the legal income and the pre-tax income. Section 5 concludes.

3.2 The model

Taxpayers income w is a random variable with distribution function g, defined over the interval $[0, \bar{w}]$, with $\bar{w} > 0$. The tax administration knows g but not w. Following prospect theory, when he sends a message $x \in [0, \bar{w}]$ to the tax administration about his income, the taxpayer tends to think of possible outcomes relative to a certain reference point rather than to the final status. [6] He sets a reference income R and consider larger outcomes as gains and lower as losses. He has a utility function $u(x - R)$ which is :

i. continuous on $\mathbb{R}$, twice continously differentiable on $\mathbb{R}^*$ and equal to zero in zero : $u(0) = 0$,

ii. increasing, convex for losses and concave for gains : $u^{'} > 0$ on $\mathbb{R}^*$, $u^{''} > 0$ on $\mathbb{R}^*_-$ and $u^{''} < 0$ on $\mathbb{R}^*_+$ *(Diminishing marginal sensitivity)*,

iii. steeper for losses than for gains : $u^{'}(-k) > u^{'}(k)$ for $k \in \mathbb{R}^*_+$ *(Loss aversion)*.

Figure 3.1 represents a typical utility function.

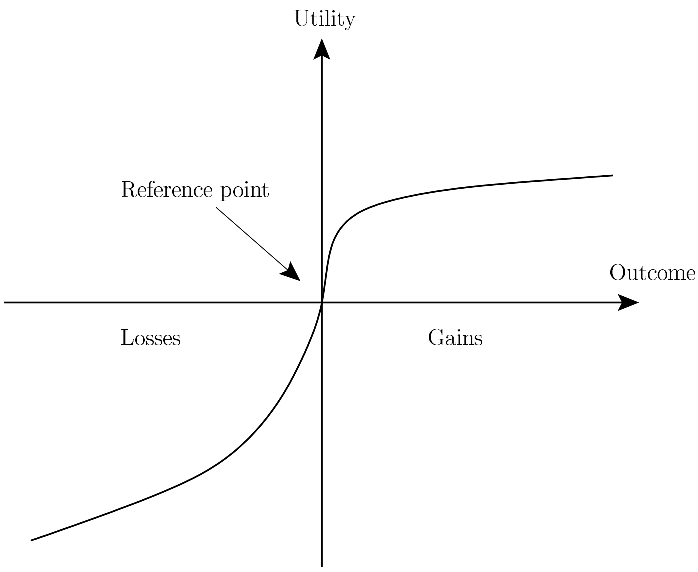

FIGURE 3.1 – Utility of an outcome

The tax administration may set up a mechanism consisting of a set $X \subset \mathbb{R}^*_+$ of messages, a tax function $t : X \to \mathbb{R}_+$, twice continuously differentiable, a probability function $p : X \to [0, 1]$ and a penalty function $f : X \times [0, \bar{w}] \to \mathbb{R}_+$. A taxpayer whose

6. See for example Kahneman and Tversky [41].

initial income is w and who sends the message $x \in X$, is audited with probability $p(x)$ and pays $t(x) \in [0, w]$ if no audit occurs and $f(w, x) \in [t(x), w]$ if an audit occurs.[7] The associated payment function for the taxpayers can be defined by :

$$r(w, x) = (1 - p(x))t(x) + p(x)f(w, x), \text{ for all } (w, x) \in [0, \bar{w}] \times X. \tag{3.1}$$

Audits are assumed to be costly, c being the cost for an audit. Others things being equal, the tax administration then prefers smaller audit probabilities to reduce audit costs.

The reference income with which the taxpayer represents his final income depends on his initial income :

$$R = R(w) \in [0, w]. \tag{3.2}$$

More exactly, it includes at once what he considers to deserve (or the price he is willing to pay for public goods) – the tax rate function – and also a priori the characteristics of the cheating game to which he subjects himself by not declaring his entire income – the probability and penalty functions :

$$R = R_{t,p,f}(w) \in [0, w]. \tag{3.3}$$

It is assumed that a taxpayer can never pay more than his true income.[8] In order to be feasible, a mechanism (X, t, p, f) must therefore satisfy that a taxpayer can send a message which does not require a payment larger than his initial income. The mechanisms which satisfy the following requirements are called *direct revelation mechanisms :*

i. *First feasibility requirement :* For all $w \in [0, \bar{w}]$, the set of feasible messages $X(w) = \{x \in X, t(x) \leq w\}$ contains at least one element and for all $x \in X(w)$, $f(w, x) \leq w$.

ii. *Second feasibility requirement :* The maximization problem of the taxpayer :

$$\max_{x \in X(w)} [(1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))]$$

has a solution for all $w \in [0, \bar{w}]$.

7. It is assumed that if an audit occurs, the actual income of the taxpayer is discovered without error.

8. This rules out a full-information optima with large penalties.

The associated first-order and second-order conditions are :

$$(1-p^{'}(x))u(w-t(x)-R(w))-t^{'}(x)(1-p(x))u^{'}(w-t(x)-R(w))$$
$$+p^{'}(x)u(w-f(w,x)-R(w))-\frac{\partial f}{\partial x}(w,x)p(x)u^{'}(w-f(w,x)-R(w))=0. \quad (3.4)$$
$$(1-p^{''}(x))u(w-t(x)-R(w))-2t^{'}(x)(1-p^{'}(x))u^{'}(w-t(x)-R(w))$$
$$-t^{''}(x)(1-p(x))u^{'}(w-t(x)-R(w))+t^{'}(x)^2(1-p(x))u^{''}(w-t(x)-R(w))$$
$$+p^{''}(x)u(w-f(w,x)-R(w))-2\frac{\partial f}{\partial x}(w,x)p^{'}(x)u^{'}(w-f(w,x)-R(w))$$
$$-\frac{\partial^2 f}{\partial x^2}(w,x)p(x)u^{'}(w-f(w,x)-R(w))+\frac{\partial f}{\partial x}(w,x)^2p(x)u^{''}(w-f(w,x)-R(w))<0. \quad (3.5)$$

A direct revelation mechanism is said to be *incentive compatible* if it is optimal for each taxpayer to report his income truthfully. As in most analyses of a principal-agent problem, the revelation principle can be applied to this setting.[9] It comes from the existence of a solution in the maximization problem defined in the second feasibility requirement and allows to confine the attention to the *incentive compatible direct revelation mechanisms.*

Proposition 3.1. *For each direct revelation mechanism* (X,t,p,f), *there exists a scheme* $(t^{'},p^{'},f^{'})$ *such that* $(X,t^{'},p^{'},f^{'})$ *is an incentive compatible direct revelation mechanism which is equivalent from the point of view of the tax administration and each taxpayer.*

Proof. See the Appendix. □

Without loss of generality, the attention can then be confined to mechanisms in which the taxpayers are asked to send their income and are provided incentives to report truthfully.[10] Taking into account the feasibility requirements mentioned above, the incentive compatible direct revelation mechanism are composed of schemes (t,p,f) such that for all $w \in [0,\bar{w}]$:

i. $t(w) \leq w$,

ii. $f(w,x) \leq w$, for all $x \in X(w)$,

9. For a detailed study, see for example Laffont and Martimort [46].
10. It is assumed that a taxpayer reports truthfully if reporting truthfully is optimal.

iii. $(1-p(w))u(w-t(w)-R(w))+p(w)u(w-f(w,w)-R(w)) \geq$

$(1-p(x))u(w-t(x)-R(w))+p(x)u(w-f(w,x)-R(w))$, for all $x \in X(w)$.

The third condition says that the utility of the taxpayer is maximized if he reports his income truthfully. From now on, it will be called *incentive constraint.*

With a incentive compatible direct revelation mechanisms, the payment function for the taxpayers is defined by :

$$r(w) = (1-p(w))t(w) + p(w)f(w,w), \text{ for all } w \in [0, \bar{w}]. \tag{3.6}$$

The objective of the tax administration is to maximize its revenue net of audit cost :

$$\max_{r,p} \left[\int_0^{\bar{w}} r(w)g(w)dw - c\int_0^{\bar{w}} p(w)g(w)dw \right]. \tag{3.7}$$

Denote by F the set of incentive compatible direct revelation mechanisms. Following Chander and Wilde [17], efficient schemes are defined in this manner : a scheme (t,p,f) is *efficient* in F if there is no other scheme $(t^{'},p^{'},f^{'}) \in F$ such that $p^{'} \leq p$, $r^{'} \geq r$ and $r^{'} \neq r$ or $p^{'} \neq p$, where r and $r^{'}$ are the payment functions corresponding to (t,p,f) and $(t^{'},p^{'},f^{'})$. It is not possible to not lower the payments and decrease the audit probability of a taxpayer without increasing any other audit probability and it is not possible to not increase the audit probabilities and raise the payment at some income level without lowering it at any other level.

Notice that an optimal scheme maximizes the tax administration's total revenue, net of audit cost, and by definition is efficient.

3.3 When the reference income is the legal income

Through this section, the reference income is the legal after-tax income : $R(w) = w - t(w)$. The legal after-tax income has the specificity that the taxpayer is in the domain of gains as soon as he pays less than the amount of tax initially planed for him, and in the domain of losses as soon as he pays more. The incentive constraints thus become :

$$p(w)u(t(w)-f(w,w)) \geq (1-p(x))u(t(w)-t(x)) + p(x)u(t(w)-f(w,x)),$$

$$\text{for all } x \in X(w). \tag{3.8}$$

This incentive constraint can be weakened. It is expressed by the following lemma.

Lemma 3.1. *The incentive constraint is equivalent to :*

$$(1-p(x))u(t(w)-t(x))+p(x)u(t(w)-w)\leq 0,\ \textit{for all } x\in X(w). \tag{3.9}$$

Proof. See the Appendix. □

F is the set of all schemes (t,p) that satisfy the conditions i and ii for a incentive compatible direct revelation mechanism and the new incentive constraint (3.9). An efficient scheme in F is now a scheme (t,p) for which there is no other scheme $(t',p')\in F$ such that $t'\geq t$, $p'\leq p$ and $t'\neq t$ or $p'\neq p$.

The following monotonicity and concavity results hold for any efficient (henceforth optimal) scheme.

Lemma 3.2. *A scheme $(t,p)\in F$ is efficient in F only if the incentive constraints for each income level $w\in[0,\bar{w}]$ is binding at some $x\in X(w)$.*

Proof. See the Appendix. □

Proposition 3.2. *A scheme $(t,p)\in F$ is efficient in F only if t is non-decreasing and p is non-increasing.*

Proof. See the Appendix. □

Lemma 3.3. *If for all $\hat{w}\in[0,\bar{w}]$, there exists an affine function $l_{\hat{w}}$ on $[0,\bar{w}]$ such that for all $w\in[0,\bar{w}]$, $l_{\hat{w}}(w)\geq t(w)$ and $l_{\hat{w}}(\hat{w})=t(\hat{w})$, then t is concave.*

Proof. See the Appendix. □

The incentive constraint (3.9), when binding, requires the utility of an agent to be the same when declaring all his income and when not. This can be seen as a lottery with an expected utility equal to zero, where gambling the legal income $w-t(w)$ against the gap between the two tax levels $t(w)-t(x)$.

Let us define an absolute risk aversion measure by :[11]

$$r_A(k)=-\frac{u''(k)}{u'(k)},\ \text{for all } k\in\mathbb{R}^*.^{12} \tag{3.10}$$

11. The measure is already defined in the second chapter.

12. This measure can also be defined on 0 in this manner : $r_A(0_-)=-\frac{u''(0_-)}{u'(0_-)}$ and $r_A(0_+)=-\frac{u''(0_+)}{u'(0_+)}$.

This measure slightly differs from the classical Arrow-Pratt absolute risk aversion measure in expected utility theory. In that it can take both be negative and positive values. The taxpayer is risk averse for gains (u is concave) and risk seeker for losses (u is convex).

The following condition concerns how the taxpayer takes his tax evasion decision. The utility function u satisfies *decreasing prospect risk aversion* (DPRA) in $p \in]0,1[$, on $\mathbb{R}_+^*$, if z is increasing in $y \in \mathbb{R}_+^*$, at a non-decreasing rate, where z is implicitely defined by equation (3.11) :

$$(1-p)u(y) + pu(-z) = 0. \tag{3.11}$$

In the setting of prospect theory, z is always increasing with y. In addition, the convexity condition generally holds. This is the case, for instance, of the power utility fonction in Tversky and Kahneman [79] used to describe the behavior of individuals under risk :

$$u(k) = \begin{cases} k^{\alpha} & \text{if} \quad k \geq 0, \\ -\mu(-k)^{\alpha} & \text{if} \quad k < 0, \end{cases} \tag{3.12}$$

where $0 < \alpha < 1$, and $\mu > 1$ because of loss aversion.[13]

More formally, the convexity condition is equivalent to :

$$(1-p)\frac{\mid r_A(-z) \mid}{u'(-z)} \geq p\frac{\mid r_A(y) \mid}{u'(y)}, \text{ where } y \text{ and } z \text{ are defined by (3.11).} \tag{3.13}$$

Weighted by probability coefficients, the risk loving behavior in case of loss must be larger than the risk aversion in case of gain. Actually, p is usually very close to zero, this condition is then very weak and easily holds.

The concavity result of Chander and Wilde [17] holds under this setting, at least when the utility function of taxpayers satisfies DPRA.

Proposition 3.3. *If u satisfies DPRA, then a scheme $(t,p) \in F$ is efficient in F only if t is concave.*

Proof. See the Appendix. □

13. More precisely, from experimental motives, it suggests that $\alpha = 0.88$ and $\mu = 2.25$.

3.4 When the reference income is the initial income

In this section we consider the initial income as the reference income : $R(w) = w$. This extreme case corresponds to an extremely tax-averse taxpayer. This implies that any payment to the tax administration always lies in the loss domain and that therefore the taxpayer is risk-lover. The incentive constraint becomes :

$$(1 - p(w))u(-t(w)) + p(w)u(-f(w, w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-f(w, x)),$$

$$\text{for all } x \in X(w). \quad (3.14)$$

Similarly to what we did for the previous case (see Section 3.3, Lemma 3.1), the incentive constraint can be weakened.

Lemma 3.4. *The incentive constraint is equivalent to :*

$$u(-t(w)) \geq (1 - p(x))u(-t(x)) + p(x)u(-w), \ \textit{for all } x \in X(w). \quad (3.15)$$

F is now the set of all schemes (t, p) that satisfy the conditions i and ii for a incentive compatible direct revelation mechanism and the incentive constraint (3.15). The notion of efficiency is the same as for when the reference income is the legal one.

Again, optimal schemes are characterized by the following monotonicity and concavity results.

Lemma 3.5. *A scheme $(t, p) \in F$ is efficient in F only if the incentive constraints for each income level $w \in [0, \bar{w}]$ are binding at some $x \in X(w)$.*

Proof. The proof, similar to the one for Lemma 3.2, is available from the authors, upon request. □

Proposition 3.4. *A scheme $(t, p) \in F$ is efficient in F only if t is non-decreasing and p is non-increasing.*

Proof. See the Appendix. □

Proposition 3.5. *A scheme $(t, p) \in F$ is efficient in F only if t is concave.*

Proof. See the Appendix. □

Results are very similar to those of Section 3.3. However, under the current framework, we do not need additional assumptions about the shape of the utility function to ensure that the tax function of a revenue maximizing scheme is concave. This comes directly from the convexity of the utility function, all its arguments being negative.

A priori, the most natural restriction for the reference income is to be not lower than the legal one and not higher than the initial one ($w - t(w) \leq R(w) \leq w$). Indeed, this corresponds to the case of a taxpayer whose final income will exceed the legal income while remaining below the initial one. The taxpayer derives an obvious disutility from paying the legal tax. Following the reasonings for the legal income and the initial income, it can be proved that, in a revenue maximizing framework, a. the probability function is non-increasing, b. the tax function is non-decreasing, and c. the interval on which the utility function is convexe is larger when the reference income increases. Therefore, the conditions for the tax function to be concave become less restrictive.

3.5 Conclusion

This chapter characterizes the optimal income tax and audit schemes when tax evasion decisions of taxpayers verify prospect theory. It advances the theory of risk aversion in prospect theory by introducing a useful risk aversion measure in this setting.

We conclude that the penalty for misreporting should take a high value at the optimum in tax enforcement. Although, this is not observed in practice, lowering this optimal penalty would only reinforce our results, because incentives for misreporting would be stronger and it would be harder to design a progressive tax function.

3.6 Appendix

Proof of Proposition 3.1. Denote by χ the function defined by the second feasibility requirement :

$$\chi(w) = \operatorname{argmax}_{x \in X(w)} \left[(1 - p(x))u(w - t(x) - R(w)) + p(x)u(w - f(w, x) - R(w))\right]. \tag{3.16}$$

Then, at each $w \in [0, \bar{w}]$, the scheme (t, p, f) associates $(t(\chi(w)), p(\chi(w)), f(w, \chi(w)))$.
Denote by (t', p', f') the scheme such that for all $w \in [0, \bar{w}]$,
$(t'(w), p'(w), f'(w, w)) = (t(\chi(w)), p(\chi(w)), f(w, \chi(w)))$.
Using the definition of χ, we have :

$$(1 - p'(w))u(w - t'(w) - R(w)) + p'(w)u(w - f'(w, w) - R(w)) \geq$$

$$(1 - p'(x))u(w - t'(x) - R(w)) + p'(x)u(w - f'(w, x) - R(w)), \text{ for all } x \in X(w).$$

(X, t', p', f') is an incentive compatible direct revelation mechanism and (t', p', f') is equivalent because the utility of each agent is maximized. □

Proof of Lemma 3.1. We can weaken the constraint by rising $f(w, x)$. This is possible as long as $f(w, x) < w$, and up to $f(w, x) = w$, for which equations (3.8) and (3.9) are equivalent.
In addition, the function $\phi(t) = p(w)u\left(\frac{t-r}{p(w)}\right) - (1 - p(x))u(t - t(x)) - p(x)u(t - w)$ is increasing with t when t is smaller but sufficiently close to r, for all $r > 0$. Then, $f(w, w)$ can decrease and $t(w)$ can rise, while keeping constant the payment $r(w)$, as long as $f(w, w) > t(w)$. The conditions in (3.8) and (3.9) are then equivalent when $f(w, w) = t(w) = r(w)$. □

Proof of Lemma 3.2. Suppose that $w \in [0, \bar{w}]$ exists such that for all $x \in X(w)$, the following inequality holds : $(1 - p(x))u(t(w) - t(x)) + p(x)u(t(w) - w) < 0$.
Because u is increasing, t' such that $t'(w) > t(w)$, $t'(x) = t(x)$ for all $x \in X(w) \setminus \{w\}$ and

$$(1 - p(x))u(t'(w) - t'(x)) + p(x)u(t'(w) - w) < 0,$$

can then be considered. This contradicts the efficiency of (t, p) in F. □

Proof of Proposition 3.2.

- Suppose that there exists $w, w' \in [0, \bar{w}]$ such that $w < w'$ and t is decreasing on $[w, w']$. According to Lemma 3.2, there exists $x' \in X(w')$ such that the incentive constraint (3.9) for w' is binding at x'. By the incentive constraint (3.9) for w,

$$(1 - p(x'))u(t(w) - t(x')) + p(x')u(t(w) - w) \leq 0,$$

and, because u is increasing,

$$(1-p(x^{'}))u(t(w^{'})-t(x^{'}))+p(x^{'})u(t(w^{'})-w^{'})<0.$$

This contradicts the fact that for $w^{'}$, (3.9) is binding at $x^{'}$.

– According to (3.9), for all $x \in X$, for all $w \in [0,\bar{w}]$ such that $x \in X(w)$,

$$p(x) \geq \frac{u(t(w)-t(x))}{u(t(w)-t(x))-u(t(w)-w)}.$$

Then, (t,p) being efficient,

$$p(x) = \sup_{w>t(x)} \frac{u(t(w)-t(x))}{u(t(w)-t(x))-u(t(w)-w)}.$$

t is non-decreasing, p is thus non-increasing. If there exists $x \in X$ which does not belong to any $X(w)$, $w \in [0,\bar{w}]$, then, according to (3.9), $p(z)=0$ for all $z \geq x$.

□

Proof of Lemma 3.3. Let there be some $\hat{w} \in [0,\bar{w}]$.

– The slope of $l_{\hat{w}}$ is $t^{'}(\hat{w})$. Indeed, for all $w \in [0,\bar{w}]$, the first order Taylor expansion of t near $\hat{w}$ is :

$$t(w)=t(\hat{w})+t^{'}(\hat{w})(w-\hat{w})+r_1(w), \text{ with } r_1(w) \ll w-\hat{w} \text{ (when } w \to \hat{w}).$$

Since $l_{\hat{w}}$ is an affine function which crosses t in $\hat{w}$, $l_{\hat{w}}(w)=t(\hat{w})+\lambda(w-\hat{w})$. For all $w \in [0,\bar{w}]$, $l_{\hat{w}}(w) \geq t(w)$, then :

$$\lambda(w-\hat{w}) \geq t^{'}(\hat{w})(w-\hat{w})+r_1(w).$$

For all $w>\hat{w}$, $\lambda \geq t^{'}(\hat{w})+r_0(w)$, with $r_0(w) \ll 1$, then $\lambda \geq t^{'}(\hat{w})$,
for all $w<\hat{w}$, $\lambda \leq t^{'}(\hat{w})+r_0(w)$, with $r_0(w) \ll 1$, then $\lambda \leq t^{'}(\hat{w})$, then $\lambda = t^{'}(\hat{w})$.

– For all $w \in [0,\bar{w}]$, $l_{\hat{w}}(w)=t(\hat{w})+t^{'}(\hat{w})(w-\hat{w})$. In addition, the second order Taylor expansion of t near $\hat{w}$ is :

$$t(w)=t(\hat{w})+t^{'}(\hat{w})(w-\hat{w})+t^{''}(\hat{w})\frac{(w-\hat{w})^2}{2}+r_2(w),$$
$$\text{with } r_2(w) \ll (w-\hat{w})^2.$$

$l_{\hat{w}}(w) \geq t(w)$, then $t^{''}(\hat{w})\frac{(w-\hat{w})^2}{2}+r_2(w) \leq 0$, then $t^{''}(\hat{w}) \leq 0$. This is verified for all $\hat{w} \in [0,\bar{w}]$, t is then concave on $[0,\bar{w}]$.

□

Proof of Proposition 3.3. Let there be some $\hat{w} \in [0, \bar{w}]$. Since (t, p) is efficient, according to Lemma 3.2, it exists some $\hat{x} \in [0, \bar{w}]$ such that $t(\hat{x}) \leq \hat{w}$ and $(1-p(\hat{x}))u(t(\hat{w})-t(\hat{x}))+p(\hat{x})u(t(\hat{w})-\hat{w}) = 0$. Three cases arise from the value of $p(\hat{x})$.

- *First case :* $p(\hat{x}) = 0$, then $u(t(\hat{w}) - t(\hat{x})) = 0$, then $t(\hat{w}) = t(\hat{x})$. In addition, according to the incentive constraints (3.9), for all $w \in [0, \bar{w}]$, $u(t(w) - t(\hat{x})) \leq 0$, then $t(w) \leq t(\hat{x})$. The (constant) affine function $l_{\hat{w}}(w) = t(\hat{x})$ satisfies the assumptions of Lemma 3.3.
- *Second case :* $p(\hat{x}) = 1$, then $u(t(\hat{w})-\hat{w}) = 0$, then $t(\hat{w}) = \hat{w}$. Then, since $t(w) \leq w$, for all $w \in [0, \bar{w}]$, the affine function $l_{\hat{w}}(w) = w$ satisfies the assumptions of Lemma 3.3.
- *Third case :* $0 < p(\hat{x}) < 1$, since u satisfies DPRA, the curve $C_{p(\hat{x})}$ defined by :

$$(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) = 0$$

is increasing and convex in the coordinate system $(0, y, z)$. Denote by $\hat{\Phi}$ the associated function and let there be some $\hat{z} \in [0, \bar{w}]$. Denote by $\hat{y}$ the real number such that $\hat{\Phi}(\hat{y}) = \hat{z}$. The tangent to $C_{p(\hat{x})}$ at $\hat{y}$ in $(0, y, z)$ is below itself. Denote by $\hat{k}$ the function associated to the tangent :

$$\hat{k}(y) = a(y - \tilde{y}), \text{ with } \hat{k}(\hat{y}) = \hat{\Phi}(\hat{y}) = \hat{z}, \ \tilde{y} \in [0, \bar{w}], \text{ and } a > 0.$$

For all $z \in [0, \bar{w}]$ such that $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) \leq 0$, $\hat{k}(y) \leq \hat{\Phi}(y) \leq z$, because u is increasing.

Consider $z = w - t(w)$, $\hat{z} = \hat{w} - t(\hat{w})$, $\hat{y} = t(\hat{w}) - t(\hat{x})$ and $y = t(w) - t(\hat{x})$, $(1 - p(\hat{x}))u(y) + p(\hat{x})u(-z) \leq 0$ according to (3.9) and
$(1 - p(\hat{x}))u(\hat{y}) + p(\hat{x})u(-\hat{z}) = 0$, then the affine function :

$$l_{\hat{w}}(w) = \frac{w + a(t(\hat{x}) + \tilde{y})}{a + 1}$$

satisfies the assumptions of Lemma 3.3.

This is verified for all $\hat{w} \in [0, \bar{w}]$, t is then concave on $[0, \bar{w}]$, according to Lemma 3.3. □

Proof of Proposition 3.4.

– Let us suppose that there exists $w, w^{'} \in [0, \bar{w}]$ such that $w < w^{'}$ and t is decreasing on $[w, w^{'}]$. According to Lemma 3.5, there exists $x^{'} \in X(w^{'})$ such that the incentive constraints (3.15) for $w^{'}$ are binding at $x^{'}$. But according to the incentive constraints (3.15) for w,

$$u(-t(w)) \geq (1 - p(x^{'}))u(-t(x^{'})) + p(x^{'})u(-w).$$

u being increasing and t being decreasing on $[w, w^{'}]$, the following function is increasing on $[w, w^{'}]$:

$$\psi(v) = u(-t(v)) - (1 - p(x^{'}))u(-t(x^{'})) - p(x^{'})u(-v).$$

Then, $\psi(w^{'}) > \psi(w) > 0$, which contradicts the fact that the constraints (3.15) for $w^{'}$ are binding at $x^{'}$.

– According to (3.15), for all $x \in X$, for all $w \in [0, \bar{w}]$ such that $x \in X(w)$,

$$p(x) \geq \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$

Then, (t, p) being efficient,

$$p(x) = \sup_{w > t(x)} \frac{u(-t(x)) - u(-t(w))}{u(-t(x)) - u(-w)}.$$

t is non-decreasing, p is thus non-increasing. If there exists $x \in X$ which does not belong to any $X(w)$, $w \in [0, \bar{w}]$, then, according to (3.15), $p(z) = 0$ for all $z \geq x$.

□

Proof of Proposition 3.5. Let there be some $\hat{w} \in [0, \bar{w}]$. Since (t, p) is efficient, according to Lemma 3.5, it exists some $\hat{x} \in [0, \bar{w}]$ such that $t(\hat{x}) \leq \hat{w}$ and $u(-t(\hat{w})) = (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-\hat{w})$. Three cases arise from the value of $p(\hat{x})$.

– *First case :* $p(\hat{x}) = 0$, then $u(-t(\hat{w})) = u(t(-\hat{x}))$, then $t(\hat{w}) = t(\hat{x})$. In addition, according to (3.15), for all $w \in [0, \bar{w}]$, $u(-t(w)) \geq u(-t(\hat{x}))$, then $t(w) \leq t(\hat{x})$. The (constant) affine function $l_{\hat{w}}(w) = t(\hat{x})$ satisfies the assumptions of Lemma 3.3.

– *Second case :* $p(\hat{x}) = 1$, then $u(-t(\hat{w})) = u(-\hat{w})$, then $t(\hat{w}) = \hat{w}$. Then, since $t(w) \leq w$, for all $w \in [0, \bar{w}]$, the affine function $l_{\hat{w}}(w) = w$ satisfies the assumptions of Lemma 3.3.

– *Third case :* $0 < p(\hat{x}) < 1$, then, u being convex on $\mathbb{R}_-^*$, for all $w \in [0, \bar{w}]$,

$$u(-t(w)) \geq (1 - p(\hat{x}))u(-t(\hat{x})) + p(\hat{x})u(-w) \geq u\left(-(1 - p(\hat{x}))t(\hat{x}) - p(\hat{x})w\right),$$

then, $t(w) \leq l_{\hat{w}}(w)$, where $l_{\hat{w}}$ is the affine function defined by $l_{\hat{w}}(w) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})w$.

In addition, following the incentive constraints (3.15), the expected utility for the initial income $\hat{w}$ is maximized by $\hat{x}$. The payment when declaring $\hat{x}$ is then lower than the one when declaring truthfully, that is :

$$r(\hat{w}, \hat{x}) = (1 - p(\hat{x}))t(\hat{x}) + p(\hat{x})\hat{w} \leq r(\hat{w}) = t(\hat{w}),$$

then $t(\hat{w}) = l_{\hat{w}}(\hat{w})$.

This is verified for all $\hat{w} \in [0, \bar{w}]$, t is then concave on $[0, \bar{w}]$, according to Lemma 3.3. $\square$

Conclusion générale

La fraude fiscale, bien que faisant régulièrement l'objet de débats sociaux et politiques, reste une donnée importante de la relation qu'entretiennent les États et leurs citoyens. Le défaut de déclaration du revenu en est la première manifestation.

Cela nous a amenés à aborder dans cette thèse, les questions du choix de déclaration des contribuables en présence du risque d'une pénalité et du choix de la stratégie d'imposition que peut faire l'État. A été en particulier étudiée, la question de savoir quel effet peut avoir une augmentation de la taxe sur la fraude par sous-déclaration du revenu. Le phénomène étant par nature très difficile à quantifier, toute information sur les effets d'un changement de politique d'imposition peut s'avérer très utile au décideur public. La théorie donnant le cadre de modélisation du comportement de fraude est un élément essentiel de la réponse apportée par les modèles économiques. Les trois contributions présentées ont ainsi pu apporter des éléments de réponse aux questions suivantes :

– Quelles prévisions apporte la théorie de l'espérance de l'utilité quand les fonctions de taxe et de pénalité ne sont pas linéaires ?
– Si la théorie de l'espérance de l'utilité ne fournit pas le cadre le plus adapté à la modélisation du comportement de sous-déclaration du revenu, les théories des perspectives sont-elles plus appropriées ?
– Quelle est la forme optimale de la politique d'imposition que doit mettre en place l'État si l'on suppose que le comportement de fraude des contribuables vérifie les propriétés de la théorie des perspectives ?

Le premier chapitre s'est attaché à compléter la littérature existante en introduisant de la non-linéarité dans le modèle classique de déclaration du revenu, impliquant l'utilisation de nouveaux outils de modélisation pouvant être utilisés dans de futures recherches. Il couvre l'ensemble des questions attenantes au problème de fraude par sous-déclaration dans le cadre de la théorie de l'espérance de l'utilité et fournit des conditions testables empiriquement (car portant sur les paramètres de la stratégie d'imposition), sous lesquelles le comportement du contribuable est prévisible.

L'utilisation de la théorie de l'espérance de l'utilité dans ce type de problème étant par ailleurs régulièrement critiquée dans la littérature, le second chapitre de cette thèse s'est posé la question de savoir si les théories des perspectives pouvaient s'avérer être plus indiquées, et si oui, sous quelles conditions. Après avoir mis en évidence les situations dans lesquelles les deux théories fournissent les mêmes résultats, il est apparu que la théorie des perspectives, avec des taux de taxe et de pénalité suffisamment élevés et une aversion relative au risque décroissante, permet d'offrir un cadre approprié au problème de la fraude fiscale par sous-déclaration.

Le troisième chapitre se place dans une optique légèrement différente des deux autres car il adopte un point de vue normatif et non descriptif. Comme mis en évidence par Chander et Wilde [17], dans le contexte de stratégie d'imposition en présence de fraude, la contrainte d'asymétrie d'information sur la valeur réelle du revenu des contribuables impose à l'État d'arbitrer entre des considérations d'équité et d'efficacité. Cela peut être comparé à la situation que crée la contrainte d'asymétrie d'information sur la valeur de la productivité réelle des contribuables, dont un exemple fondamental a été donné par Mirlees [51]. Avec des contribuables neutres au risque, comme dans Chander et Wilde [17], cela implique qu'à l'optimum, l'État doit mettre en place un barême de taxe non progressif, avec lequel les contribuables sont soumis à un taux d'imposition d'autant plus bas qu'ils sont riches, leur motivation à sous-déclarer leur revenu étant plus forte. Le troisième chapitre de la présente thèse suppose que la décision de fraude des contribuables vérifie les propriétés de la théorie des perspectives, il n'y a donc plus neutralité au risque mais aversion au risque ou recherche du risque selon s'ils se considèrent dans le domaine des gains ou celui des pertes. Avec une motivation à frauder différente, il est démontré que la fonction d'imposition doit toujours être concave.

Malgré une apparente indépendance entre les trois contribuations présentées dans cette thèse, les résultats mis en évidence complètent la littérature relative à la sous-déclaration du revenu, de manière cohérente. Le problème de la modélisation du choix des contribuables est en effet abordé de manière relativement complète et sous différents aspects dans les deux premiers chapitres. Ils servent ensuite d'appui à l'étude normative de la stratégie d'imposition de l'État en présence de fraude.

L'utilisation de la théorie des perspectives semble être un point intéressant pour de futures recherches sur la modélisation de la fraude fiscale. En particulier, la compréhension du lien existant entre l'aversion au risque dans ce cadre, le revenu de référence et le comportement de fraude, apparaît être une piste de recherche riche pour affiner la compréhension du choix que font les contribuables quand ils déclarent leur revenu. Par ailleurs, l'étude du schéma d'imposition optimal sous cette théorie n'a été que peu effectuée dans la littérature et le cadre mis en place dans cette thèse fournit de nouvelles ouvertures sur cette question.

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