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Si la plupart des taxes sont impopulaires, l'une d'entre elles l'est particulièrement : c'est la taxation de l'héritage. Or, l'impôt successoral s'adresse au caractère financier de l'héritage. Cependant, l'héritage est multiple, il peut être monétaire, patrimonial, culturel, éducatif ou encore, perçu comme un transfert de temps.

Ces transferts familiaux intergénérationnels, qui sont pour la plupart descendants suite à l'essor de la redistribution publique (ascendante) et à l'amélioration des niveaux de vie (voir Attias-Donfut et Lapierre (2000)), sont de plus en plus élevés. Tout d'abord, l'évolution des transferts familiaux est liée à la croissance des transferts financiers familiaux, à travers les donations ou les héritages. Mais, elle est aussi renforcée par la hausse des dépenses d'éducation privée des parents pour leurs enfants, dans un certain nombre de pays développés, tels que les États-Unis ou le Royaume-Unis. Enfin, la transmission familiale intergénérationnelle comprend aussi des transferts sous forme de temps et de services rendus, soutenus (voir Wolff et Attias-Donfut (2007)).

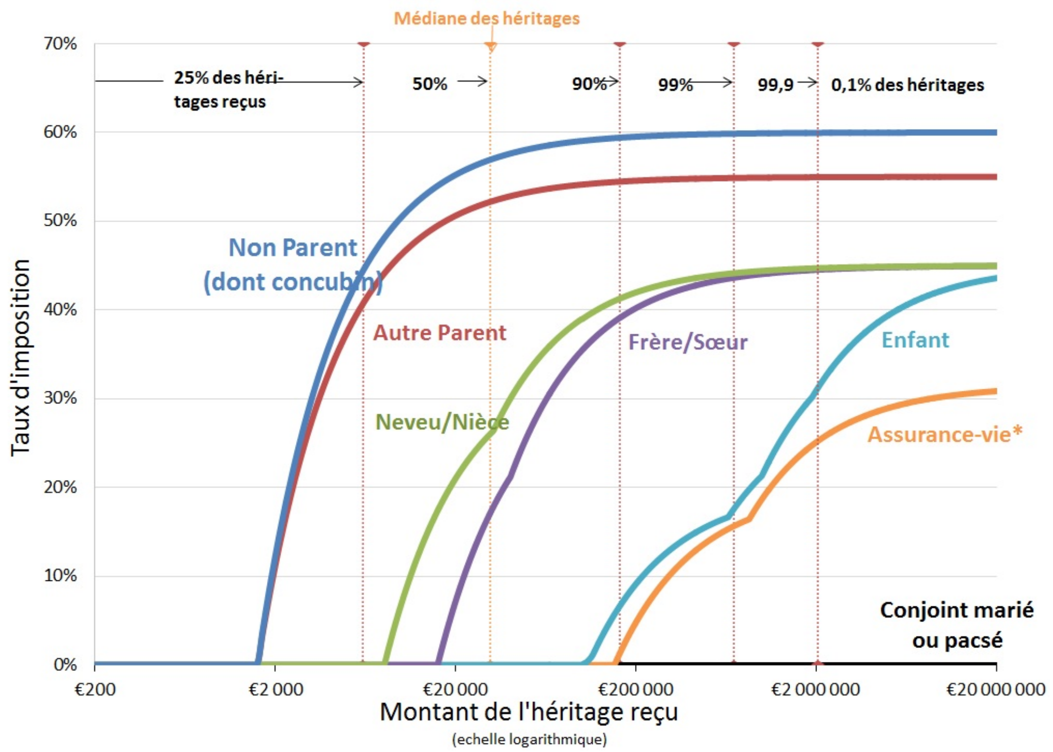
Toutes ces formes de solidarités familiales, génèrent des externalités qui impactent différemment la croissance économique et l'offre de travail des ménages. Les successions ou donations, par exemple, désincitent les héritiers à travailler suite à la hausse de leurs ressources disponibles tandis que la hausse des transferts en temps, en libérant du temps disponible, accroît cette incitation. En même temps, la hausse des héritages accroît l'accumulation de capital, tout comme l'investissement éducatif augmente le capital humain des descendants, ce dernier affecte directement leur productivité, et ainsi la croissance économique.

La thèse a pour objectif de prendre en compte la diversité des transferts familiaux pour analyser les politiques fiscales en matière d'héritage. En effet, l'impôt successoral, dont l'assiette fiscale est l'un des principaux transferts familiaux intergénérationnels, a un rôle différent dans la politique fiscale des gouvernements en fonction de la prise en compte ou non de la diversité de la solidarité familiale. De prime abord, l'introduction de ce type de taxe semble avoir un effet négatif sur la transmission intergénérationnelle puisqu'il réduit l'incitation des ménages à épargner et ainsi à transmettre des héritages. Mais si on considère l'arbitrage auquel font face les donateurs à travers la diversité des transferts, la taxation de l'héritage a un effet incitatif sur les transferts en temps, ou encore sur l'investissement éducatif des parents. Par conséquent, considérer le caractère multiple des transferts familiaux descendants peut modifier la façon dont on appréhende la taxation des héritages et ainsi son impact sur la croissance et l'offre de travail.

La taxation de l'héritage suscite la controverse à tous les niveaux : politique, économique, philosophique, mais aussi au sein de l'opinion publique. Plusieurs critiques importantes sont exposées par ses opposants. Ils considèrent en premier lieu, que l'impôt successoral est immoral car il amplifie la souffrance des familles endeuillées en accaparant une partie du patrimoine du défunt. Il pénalise aussi les parents qui souhaitent transmettre un héritage à leurs enfants ou à des personnes tiers, et réduit ainsi leur incitation à épargner. Cet effet négatif sur l'épargne diminue l'investissement domestique et par ricochet le stock de capital de l'économie, ce qui affecte négativement tous les individus quels qu'ils soient. L'objectif de réduction des inégalités, supposé avec ce genre de taxation, est donc remis en question. De plus, les fondements d'un système de taxation équitable reposent en partie sur l'idée que chaque revenu ne peut être taxé plus d'une fois. Or, la taxation de l'héritage viole ces fondements. L'impôt successoral est basé sur des revenus provenant de la rémunération du capital et du travail du défunt, eux-mêmes déjà initialement taxés. L'impôt sur les successions est également injuste dans le sens où le montant taxé est différent dans de nombreux pays, dont la France, selon la structure des actifs imposables des contribuables et selon la nature des liens affectifs ou de parenté avec les héritiers (voir graphique 1). Enfin, il peut entraîner la dissolution d'entreprise familiales et/ou y engendrer des instabilités financières et managériales qui peuvent être préjudiciables pour l'ensemble des salariés de l'entreprise. Tous les arguments, en défaveur de l'impôt successoral, sont amplifiés par la complexité de son recouvrement et par la faible part des droits de succession dans les recettes fiscales de nombreux pays.

D'un autre côté, les partisans de la taxation de l'héritage soutiennent qu'elle est, de loin, la plus efficace et la plus équitable. Ils affirment que la taxation de l'héritage

Figure 1 – Taux d'imposition des héritages reçus, par montant reçu et degré de parenté



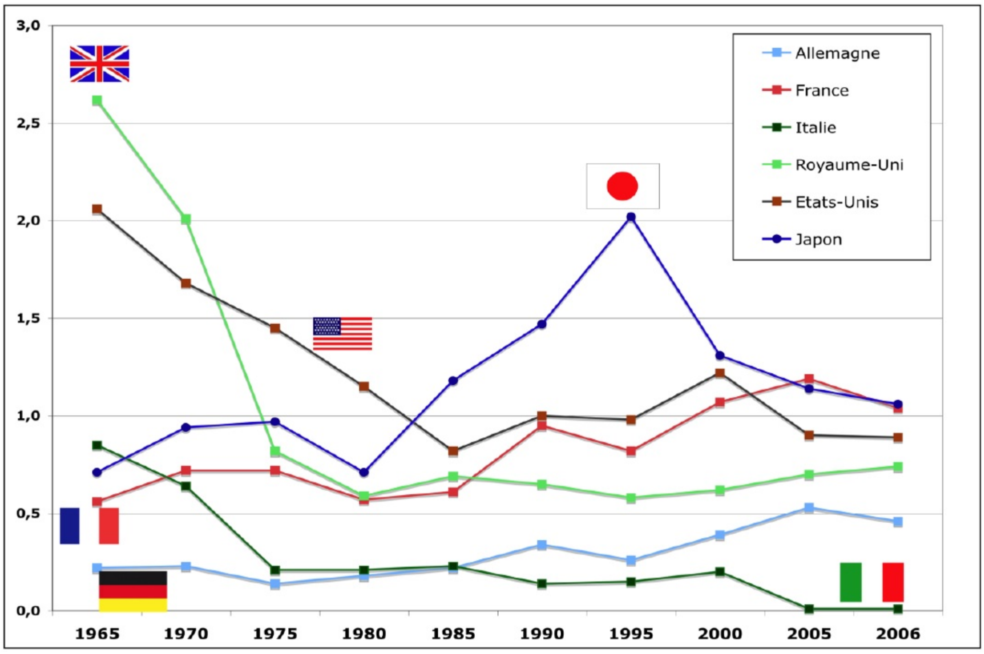
Note : les héritages ne sont pas taxés s'ils sont inférieurs à 1 594 euros pour les non-parents et autres parents, à 7 967 euros pour les neveux et nièces, à 15 932 euros pour les frères et sœurs, à 100 000 euros pour les enfants et à 152 500 euros pour les assurances-vie. Un héritage de 2 millions d'euros est taxé à 60 % pour un non-parent, à 30,8 % pour un enfant, à 0 % pour un conjoint marié.
Source : France Stratégie, d'après le Code des impôts, enquête Patrimoine 2010 pour la répartition des héritages.

doit jouer un rôle majeur dans la réduction des inégalités, étant donné son caractère hautement progressif. En effet, la distribution du flux successoral est fortement concentrée sur une faible part de la population dans la plupart des pays développés. D'après Piketty (2010), les 50% de la population française les moins bien lotis en terme d'héritage n'ont reçu que 5% du flux successoral global, alors que le décile le mieux loti a hérité de plus de 60% de ce total en 2010. De plus, ces patrimoines hérités représentent environ deux tiers du patrimoine total en France (voir Alvaredo *et al.* (2017)), qui lui-même, est concentré à une faible part de la population. La moitié de la population française par exemple ne détenait que 4% du patrimoine immobilier en 2010 (voir Piketty (2011)). Par conséquent, selon ces défenseurs, la taxation de l'héritage a un rôle décisif à jouer dans la réduction des inégalités de capital. Pour leur part, c'est aussi une taxe juste dans le sens où elle prélève des montants qui ne rémunèrent pas un travail ou un effort fournit par l'héritier (lorsqu'il est considéré comme le contribuable). Enfin, selon eux, elle est caractérisée par des effets désincitatifs faibles sur le comportement des ménages, étant donné que le prélèvement ne s'effectue qu'après la mort du défunt.

Ainsi, aucune autre forme d'imposition ne génère autant de polémiques que la taxation de l'héritage. Une partie des critiques, à son égard, est plus attachée à sa forme et à la procédure de son recouvrement plutôt qu'à sa nature. L'une d'entre elles est liée aux faibles recettes fiscales qu'elle génère. Dans une grande majorité des pays développés, sa part dans les recettes fiscales baisse depuis le milieu des années soixante et elle ne représente pratiquement plus que 1% des recettes fiscales totales dans une grande partie de ces pays. Le graphique 2 illustre ce phénomène pour un certain nombre de pays développés. Actuellement, un nombre croissant de pays ont supprimé l'impôt sur les successions (par exemple, le Canada en 1972, le Portugal en 2004 et la Suède en 2005) ou l'ont réduit significativement, comme les États-Unis ou le Royaume-Uni. Seulement une minorité de pays (dont la France, la Belgique, le Japon et l'Allemagne) n'ont pas suivi cette tendance au cours de ces dernières années. Toutefois, pour ces derniers, ce n'est pas la conséquence d'une volonté politique qui explique le maintien ou la hausse des recettes fiscales successorales, mais plutôt la croissance des patrimoines en terme réel, plus rapide que celle des revenus, augmentant ainsi l'assiette fiscale de l'impôt successoral.

La baisse pratiquement généralisée de l'utilisation de l'impôt successoral souligne son impopularité croissante. Le phénomène est d'autant plus important que l'assiette fiscale sur laquelle elle repose, a fortement augmenté dans la plupart des pays développés depuis la fin de la seconde guerre mondiale. En France, par exemple, alors qu'au début des années soixante, le flux annuel des transmissions à titre gratuit ne représentait que l'équivalent de quelque points de revenu national, en 2010 celui-ci est d'environ 12%, soit plus de 220 milliards d'euros (voir graphique 3). Cette même année 2010, les recettes fiscales rattachées à l'impôt successoral sont autour de 7,7

Figure 2 – Part des droits de succession et de donation dans les recettes fiscales totales

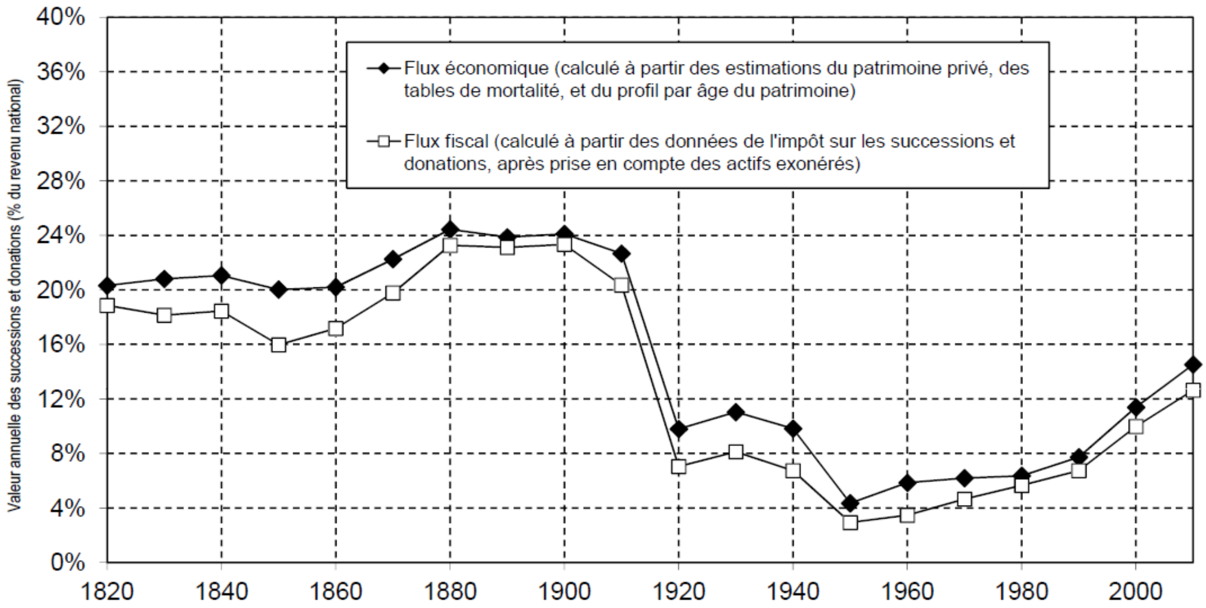


Note : Données de l'OCDE. Six pays de l'OCDE. Evolution de la part des droits de succession dans les recettes fiscales de ces pays entre 1965 et 2006. Graphique issu de Arrondel et Masson (2012).

milliards d'euros (source : Insee, comptes nationaux - base 2010), soit environ un taux d'imposition moyen de 3,5%. Par conséquent, les recettes fiscales provenant de l'impôt sur les successions sont faibles eu égard à des volontés politiques et non due à la faiblesse de leur assiette fiscale. Arrondel et Masson (2013) appuie ce constat, en montrant empiriquement que le flux d'héritage annuel est significatif par rapport au revenu national et augmente actuellement, alors que l'impôt sur les successions est très faible et suit la tendance inverse.

Ainsi, par la controverse qu'elle engendre et ses caractéristiques particulières, de nombreux économistes s'intéressent à la taxation de l'héritage en se reposant sur des critères d'efficacité et/ou d'équité. Les économistes perçoivent les flux successoraux comme des ressources supplémentaires pour les bénéficiaires pouvant être assimilées à une sorte d'épargne dont les motivations sont assurément familiales. L'impôt successoral, quant à lui, doit être efficace, équitable et juste, comparativement aux autres formes d'impôts (comme l'impôt sur le revenu, sur l'épargne et sur la

Figure 3 – Le flux successoral annuel exprimé en pourcentage du revenu national, France 1820-2010



Note : voir Piketty (2011) et aussi piketty.pse.ens.fr/capital21c.

consommation des ménages).

La littérature sur la taxation de l'héritage est souvent assimilée à celle sur la taxation du capital. Dans cette littérature, on retrouve deux modèles de référence, celui d'Atkinson et Stiglitz (1976) et celui de Judd (1985)-Chamley (1986), qui conduisent à un niveau optimal d'imposition sur le capital, hérité ou non, nul à long terme. Ce résultat standard est vérifié quelle que soit la distribution des richesses dans la population. Ainsi, une taxation du capital nulle est souhaitable même du point de vue des ménages ne possédant pas de capital et ne recevant pas d'héritage.

Atkinson et Stiglitz (1976) construisent ainsi un modèle de cycle de vie sans héritage dans lequel les individus ne se différencient qu'en terme de productivité. Dans ce modèle, le capital accumulé provient uniquement de la part du revenu du travail dévolu à l'épargne de cycle de vie du ménage, puisqu'il n'y a pas de transfert familial intergénérationnel. Ainsi, le revenu du travail est la seule source d'inégalité entre agents (en supposant des préférences homogènes pour l'épargne). Étant donné que l'impôt sur les revenus du capital est considéré comme une imposition d'un des facteurs de production, il est plus efficace de taxer le travail que le capital.

Chamley (1986) et Judd (1985) développent un modèle d'agents à durée de vie infinie où les ménages ont des préférences homogènes pour l'épargne. Ils montrent que tout niveau positif d'impôt sur les revenus du capital créé des distorsions, dont les effets sont exponentiels sur l'épargne, conduisant par la suite à un niveau de production nul à long terme. Ainsi, le niveau optimal de taxation du capital à long terme est nul, puisque l'élasticité-prix du capital par rapport au taux de l'impôt sur le capital est infinie.

Dans ces deux modèles, la taxation du capital distord le comportement d'épargne des ménages, ce qui induit une baisse du niveau du capital de l'économie à long terme, affectant l'ensemble des ménages. Ce résultat standard, suppose des marchés de capitaux parfaits, une information parfaite et des préférences homogènes des ménages. En considérant le modèle Chamley (1986) et Judd (1985), le résultat peut s'étendre à la taxation de l'héritage en interprétant l'hypothèse d'agents à durée de vie infinie à travers le prisme de la dynastie familiale. Ainsi, les ménages peuvent avoir des horizons de vie infinie via leur dynastie par leur comportement altruiste vis à vis de leur descendance. En effet, lorsque les individus se préoccupent du bien-être de leurs progénitures, leurs intérêts portent aussi sur celui des générations futures puisque la prochaine génération elle-même s'intéresse au bien-être de sa descendance. Cette forme d'altruisme est appelée « altruisme rationnel » et ses caractéristiques sont détaillées dans l'annexe. Comme le montre le modèle de Barro (1974), en présence de ce type d'altruisme, la taxation de l'héritage réduit l'incitation à épargner et a un effet négatif sur l'accumulation de capital de la même manière que la taxation du capital dans Chamley (1986) et Judd (1985). Dans ce contexte, l'impôt successoral semble être inefficace et devrait être nul à long terme.

De plus, le modèle Barro (1974) montre que les familles avec altruisme rationnel peuvent neutraliser toute tentative du gouvernement de redistribuer des ressources entre générations à travers les legs. En effet, dans ces modèles, les dynasties familiales lissent leurs consommations au fil du temps par le biais des transferts familiaux intergénérationnels menant à un effet compensatoire des transferts privés vis-à-vis des transferts publics. Cet effet aussi appelé «équivalence

Ricardienne» implique la neutralité de la dette publique.

Au cours des dernières années, une nouvelle littérature a remis en cause ce résultat standard, en relâchant certaines de ces hypothèses afin d'analyser le taux d'imposition optimal à long terme sur l'héritage, de manière différente et en introduisant un débat autour de «l'équivalence Ricardienne». Ils ont développé différents types de modèles en introduisant des imperfections sur le marché des capitaux, de l'hétérogénéité dans les préférences ou encore de l'incertitude liée à la mort, tout en s'efforçant de différencier les comportements d'accumulation de capital et de transmission des héritages en s'interrogeant sur les différents motifs de legs.

Certains auteurs se sont ainsi intéressés au niveau optimal de taxation de l'héritage à travers les différentes motivations de transmission familiale intergénérationnelles. Contrairement au fameux résultat de taxation optimale nulle de l'héritage à long terme, ces auteurs analysent le comportement du legs indépendamment de celui de l'épargne de cycle de vie. Cremer et Pestieau (2011) montrent que le niveau de taxation optimal de l'héritage dépend des motivations des ménages en termes de legs, ainsi que de l'ambiguïté liée à leur mort. En effet, les ménages, ne connaissant pas la date de leur décès, épargnent un montant plus important de leur revenu que ce qui leur est nécessaire au cours de leur vie. Il y a donc bien un legs accidentel sur lequel la taxation de l'héritage n'entraînera pas d'effet distordant sur le comportement d'épargne des ménages. Ainsi, si les legs sont accidentels, l'impôt successoral devient très efficace. Cependant, si les ménages sont motivés à travailler et à épargner afin de laisser à leur famille un héritage, alors la taxe sera source de distorsion. L'impact de la distorsion dépendra du motif du legs. En effet, si les parents souhaitent juste accroître les ressources de leurs enfants à travers l'héritage (altruisme familial), en dépit de leurs besoins, ou si c'est le geste de léguer qui leur importe (altruisme « joy of giving »), le résultat est différent de ce qu'il serait, si le montant légué était déterminé par le souci du bien-être de l'héritier (altruisme rationnel). Ainsi, le rôle de la taxation des héritages mais aussi les facteurs explicatifs de transmissions sont différents pour chaque forme d'altruisme, qui en conséquence de ses caractéristiques, implique un rejet ou non de «l'équivalence Ricardienne».

D'autres économistes ont obtenu une taxation successorale optimale positive en s'intéressant essentiellement à l'hétérogénéité des préférences en termes de legs. Par exemple, Piketty et Saez (2013) considèrent un modèle où les ménages vivent une seule période, mais peuvent épargner pour transmettre un héritage. Le comportement familial en terme de transmission intergénérationnelle est caractérisé par de l'altruisme rationnel, c'est à dire que le donneur s'intéresse au bien-être du bénéficiaire. Ils supposent aussi des préférences hétérogènes en termes de legs dans la population et une productivité différente des ménages concernant leur travail. Ils considèrent aussi une élasticité-prix du legs par rapport au taux d'impôt successoral finie, contrairement à Judd (1985) et Chamley (1986). Par conséquent, deux types d'inégalités découlent du travail fournit et de l'héritage reçu, ce qui rend une politique de redistribution basée exclusivement sur la taxation des revenus du travail non optimale. Ainsi, grâce à ces hypothèses, dont celle d'une élasticité-prix du legs finie, le taux d'imposition optimal du capital à long terme devient positif.

De plus, de nombreux auteurs soulignent l'importance de l'hypothèse de marchés des capitaux parfaits dans les modèles standards. Ils montrent que le niveau optimal de l'impôt sur le revenu du capital (hérité ou non) devient positif lorsqu'on tient compte d'un marché des capitaux imparfait. En effet, dans ce contexte, la taxation du capital est un moyen de mettre en place une politique de redistribution entre ceux qui ne sont pas contraints (les détenteurs de capitaux) et ceux qui sont contraints sur leur endettement (ne possédant pas de capital). Aiyagari (1995) et Chamley (2001) montrent que dans des modèles à horizon de vie infinie, où les ménages font face à une contrainte d'endettement et à de l'incertitude concernant leur niveau de revenu, le niveau optimal de taxation du capital est positif, plus la consommation des ménages est liée au niveau d'épargne.

Cependant, dans la plupart des modèles considérés jusqu'à présent, les transferts familiaux intergénérationnels sont seulement traités à travers les flux successoraux. Néanmoins, l'héritage en terme monétaire ou financier n'est pas le seul type de transfert familial intergénérationnel. En effet, cette transmission, prise dans sa globalité, concerne autant le legs en terme de patrimoine (mobilier ou immobilier) qu'une personne laisse à son décès, que l'héritage qu'il transmet au cours de sa vie. Ce dernier type de transfert, peut être étudié sous trois principaux aspects. Tout d'abord, le défunt peut décider de léguer une partie de son patrimoine au cours de sa vie, c'est-à-dire effectuer une donation. Mais il peut aussi transmettre ses connaissances ou investir dans l'éducation de sa descendance. On parle alors d'héritage culturel ou éducatif. Enfin, il peut donner un peu de son temps à ses futurs héritiers à travers la garde d'enfants ou en effectuant différentes tâches domestiques, ce qui peut se traduire comme un transfert de temps descendant, et ainsi libérer du temps aux héritiers pour, éventuellement, travailler plus.

Ainsi, la transmission familiale intergénérationnelle n'est pas seulement caractérisée par la part des revenus du travail épargnée le long d'une dynastie. Elle peut aussi être perçue comme un investissement éducatif ou un transfert de temps, qui affecte différemment l'offre de travail et les ressources de cycle de vie de la descendance. Par conséquent, la prise en compte par le futur défunt de plusieurs transferts familiaux intergénérationnels, implique que celui-ci doit faire face à un arbitrage entre ces transferts avec, pour objectif, d'obtenir la meilleure allocation des ressources octroyées à l'héritier. Dans ce contexte, la taxation de l'héritage n'est pas seulement un impôt sur la richesse accumulée par les dynasties, c'est aussi un instrument qui distord le choix de l'agent dans l'arbitrage entre les différents types de transferts. Dès lors, la taxation du capital et l'impôt successoral n'ont pas les mêmes effets sur le comportement des agents, ce qui implique que les gouvernements peuvent choisir des niveaux différents de ces deux types de taxes, contrairement aux modèles standards avec agents à horizon de vie infinie (Chamley, 1986, page 613).

De plus, la prise en compte de plusieurs types de transferts et de l'arbitrage des donneurs entre ces transferts, peut mener à des inégalités entre dynasties, c'est à dire des inégalités intra-générationnelles. Les inégalités sont alors bidimensionnelles, provenant d'une part, des différences de productivité des travailleurs et d'autre part, des différences des choix des donateurs en termes de legs. Dès lors, comme dans le modèle de Piketty et Saez (2013), avoir une taxation de l'héritage nulle peut être non optimal.

En outre, les relations entre les générations se manifestent aussi par d'importants transferts publics intergénérationnels. Ce type de transfert permet de garantir une certaine équité entre les générations en termes d'allocations de ressources, en prenant en considération la dimension de long terme de certaines politiques. Dans les faits, on constate que les transferts intergénérationnels publics sont plutôt ascendants dans la plupart des pays développés, à travers l'utilisation de la dette publique. En effet, les gouvernements peuvent se servir de la dette publique pour transférer des ressources des générations futures vers les générations présentes, celles-ci pouvant découler d'investissements contemporains, moteur de la croissance économique future. Cependant, la pression sur les finances publiques de ces dernières années, a contraint certains gouvernements à réduire le recours à la dette publique. En 2010, par exemple, survient la crise de la dette publique grecque, qui se propage rapidement à l'ensemble des pays de la zone Euro, suite à une perte de confiance des marchés financiers dans la capacité des gouvernements européens à honorer leurs engagements et à réduire dans un futur proche leur dette publique considérée comme excessive. Ainsi, les gouvernements doivent trouver d'autres solutions pour mettre en place leur politique de redistribution intergénérationnelle. Dans ces circonstances, ils peuvent décider de réduire les transferts financiers familiaux via la taxation de l'héritage. Par conséquent, les deux flux intergénérationnels (familial et étatique) peuvent être antinomiques.

Pour résumer, autant dans le débat public qu'entre économistes, la taxation de l'héritage fait polémique. Comme nous l'avons décrit précédemment, cette controverse est principalement liée, d'un part aux formes de la transmission familiale qui peuvent être diverses, tout comme les motifs de legs, et d'autre part au caractère intergénérationnel de l'héritage. Bien que la littérature économique sur ce sujet soit importante et couvre un certain nombre des questions que pose l'utilisation de l'impôt successoral, elle prend rarement en considération la diversité de la transmission familiale intergénérationnelle (par exemple, les transferts de temps descendants ou les dépenses en éducation). Pourtant, l'impôt successoral ne conduit pas aux mêmes effets sur ces différents types de transferts. De même, la relation entre les transferts familiaux descendants et les transferts publics intergénérationnels (souvent ascendants) à travers le prisme de la taxation des héritages, est peu analysée dans la littérature, malgré la crise de la dette grecque qui a affecté les transferts publics intergénérationnels de l'ensemble des économies de la zone Euro (en contraignant l'utilisation de leur dette publique).

Cette thèse a donc pour objectif d'apporter des éléments de réponse sur ces points en analysant l'effet de la mise en place de la taxation des héritages sur les transferts intergénérationnels ainsi que sur la croissance et l'offre de travail. Cette thèse est construite à travers des modèles théoriques à générations imbriquées (ou successives), ce qui permet d'appréhender au mieux la dimension inter-temporelle de la problématique de taxation des héritages.

Le premier chapitre permet d'étudier l'impact de la non disponibilité de la dette publique sur la politique de redistribution intergénérationnelle mise en place par le gouvernement en utilisant uniquement l'impôt sur les revenus du travail et l'impôt successoral. Il permet aussi d'analyser son effet sur la croissance économique et les transferts familiaux intergénérationnels, consistant en des legs et des dépenses d'éducation, en mettant en évidence le rôle central de la taxation de l'héritage. Le second chapitre (écrit en collaboration avec Pascal Belan) propose un modèle avec transferts de temps descendants et héritage monétaire, dont l'objectif est de montrer les différences entre la taxation de l'héritage et la taxation du capital de cycle de vie, sur le comportement des ménages. Dans certaines circonstances, l'utilisation de la taxation de l'héritage à la place de celle du capital peut être une reforme Pareto-améliorante. Enfin, le troisième chapitre (écrit également en collaboration avec Pascal Belan) s'intéresse comme le chapitre précédent à la comparaison entre taxation du capital et taxation de l'héritage, dans un modèle où les dynasties sont différentes en termes de productivité et de niveau d'altruisme vis à vis de la génération future. Ce chapitre démontre qu'appliquer l'impôt successoral à la place de celui du capital, peut améliorer à long terme le bien-être des moins altruistes et dans certains cas, peut être Pareto-améliorante. Enfin, nous proposons en annexe, une revue de la littérature sur les différentes formes d'altruisme, d'héritage, ainsi que d'impôts successoraux. Cette partie s'intéresse aussi à l'implication de ces différences vis-à-vis de la taxation des héritages.

Dans le premier article de cette thèse intitulé « **Intergenerational family transfers, tax policies and public debt** », nous proposons un modèle de croissance endogène du capital humain dans lequel les parents augmentent le revenu de leurs enfants en investissant dans leur éducation et en leur transférant des héritages (monétaires). Ce chapitre étudie l'impact de la dette publique sur les transferts familiaux intergénérationnels et sur la croissance économique optimale à long terme, dans un modèle à générations successives en économie fermée. La croissance du capital humain provient de l'amélioration du niveau d'éducation de l'agent représentatif, qui dépend des dépenses d'éducation des parents et des connaissances accumulées par la dynastie au cours des générations. Les motifs de legs des ménages correspondent à de l'altruisme familial. En effet, les parents, en ce qui concerne leurs enfants, se focalisent exclusivement sur leurs ressources disponibles (voir Lambrecht *et al.* (2006)). L'altruisme familial implique une externalité positive des dépenses d'éducation des parents vis-à-vis de toutes les générations futures. De plus, les deux taxes disponibles sont celles qui affectent directement les arbitrages des agents entre les deux transferts, c'est-à-dire la taxe sur le travail et celle sur les legs. Ces deux types d'impôts et la dette publique sont les seuls instruments fiscaux disponibles pour financer les dépenses publiques, corriger les externalités et mettre en œuvre une politique de redistribution intergénérationnelle. Lorsque la dette publique est un instrument disponible pour le gouvernement, la charge fiscale sur le travail et les héritages peut être ajustée, et le gouvernement peut promouvoir le développement du capital humain et améliorer la consommation des ménages des premières générations. Si la dette publique n'est pas disponible, le gouvernement internalise l'externalité positive sur le capital humain et poursuit une politique de redistribution en utilisant des taux de taxes moins élevés mais avec un écart plus important entre les deux types d'impôts (le taux optimal d'impôt successoral étant toujours supérieur à celui sur le revenu du travail afin d'internaliser l'externalité). A travers cet écart plus élevé entre les deux taxes, la croissance à long terme sera plus importante qu'au premier rang, avec un niveau de capital physique plus bas réduisant ainsi le ratio capital-travail de long terme. La baisse de ce ratio permet de mettre en place une redistribution intergénérationnelle bénéficiant aux premières générations, même si cette redistribution ascendante n'est pas équivalente à la solution de premier rang.

Dans le second article intitulé « **Inheritance taxation in a model with intergenerational time transfers** », nous considérons un modèle à générations imbriquées où les ménages vivent deux périodes avec de l'altruisme rationnel à la Barro (1974) et utilisent deux types de transferts intergénérationnels : l'héritage et le transfert en temps. Partant d'une situation initiale où les dépenses publiques et la charge de la dette sont financées par l'impôt sur le revenu du travail et celui sur le capital, nous analysons une réforme fiscale qui consiste en un transfert de la charge fiscale de l'impôt sur le revenu du capital vers la taxation des héritages. Comme dans le modèle standard de Barro (1974), l'impôt sur les successions réduit l'incitation à laisser des ressources à la génération suivante, diminuant ainsi l'accumulation de capital et le ratio capital-travail à long terme. Mais la baisse de l'impôt sur le revenu du capital peut compenser cet effet. En outre, la prise en compte des transferts de temps ajoute un effet de substitution puisque la taxe sur l'héritage affecte l'arbitrage entre les deux types de transmissions, rendant les transferts de temps plus attractifs. Ainsi, la réforme peut augmenter les ressources de long terme puisque l'impôt sur les successions a aussi un effet positif sur l'offre de travail. En supposant que la réforme fiscale est conçue pour maintenir constant le ratio capital-travail à l'état stationnaire, nous identifions des situations où l'utilité de cycle de vie des ménages augmente avec la réforme. Nous montrons que l'amélioration du bien-être des familles dépend principalement de l'ampleur de l'effet de la hausse des transferts de temps des parents sur l'offre de travail des jeunes. De plus, en maintenant constant le ratio capital-travail à l'état stationnaire, la charge de la dette publique initiale se déplace vers les premières générations. En utilisant un exemple numérique, nous montrons que l'effet de la réforme fiscale sur le bien-être des ménages de chaque génération le long de la dynamique de transition peut être positif.

Dans le troisième article intitulé « **Inheritance taxation with agents differing**

in altruism and productivity », nous analysons, comme dans la partie précédente, un déplacement de l'impôt sur le revenu du capital vers l'impôt sur les successions dans un modèle à générations imbriquées à deux périodes avec l'altruisme rationnel à la Barro (1974). Cependant, la population se compose de deux types de dynasties qui diffèrent en termes d'altruisme et de productivité. D'après Michel et Pestieau (2005), l'incidence fiscale de l'impôt sur les successions est susceptible d'empirer le bien-être de chaque ménage quel qu'il soit. En effet, l'effet distordant sur le comportement d'épargne des ménages, entraîne une baisse de l'accumulation de capital et du ratio capital-travail à l'état stationnaire, ce qui affecte négativement la consommation de toutes les dynasties, et se traduit par un impact négatif en termes de bien-être. Cependant, dans notre modèle, la réforme fiscale est mise en œuvre de telle sorte que le ratio capital-travail reste constant à l'état stationnaire. Nous montrons alors que la réforme (impliquant un ratio capital-travail inchangé) est nécessairement bénéfique pour toute dynastie, excepté la plus altruiste. En outre, la réforme ne peut améliorer le bien-être des agents les moins altruistes sans réduire l'utilité des ménages les plus altruistes. La principale raison étant que le maintien du ratio capital-travail constant avec une offre de main-d'œuvre inélastique, implique que les ressources disponibles restent constantes à l'état stationnaire. Nous proposons une extension du modèle permettant de modifier les ressources disponibles du ménage, tout en laissant le ratio capital-travail constant. Nous considérons pour cela un modèle avec transferts de temps des grands parents vers les parents et une offre de travail élastique. Comme le montre le chapitre 2, l'effet positif de la réforme sur les transferts en temps peut augmenter l'offre de travail des parents, ce qui atténue ou inverse l'effet potentiellement négatif de la taxation des héritages sur les ressources disponibles tout en gardant constant le ratio capital-travail à l'état stationnaire. Par conséquent, l'effet négatif de la réforme fiscale sur la dynastie la plus altruiste peut être atténué ou inversé par l'impact positif de l'augmentation des transferts de temps sur l'offre de travail des jeunes. Nous montrons qu'en fonction de la force de cet effet et de la répartition de la population, la réforme fiscale peut être Pareto-améliorante à l'état stationnaire.

Cette thèse est donc composée d'une revue de la littérature sur l'altruisme et la taxation de l'héritage, ainsi que de trois autres parties distinctes. La première d'entre-elles étudie l'impact de la dette publique sur les transferts intergénérationnels publics et privés ainsi que sur les instruments fiscaux (notamment la taxation des héritages). La seconde partie analyse l'effet du remplacement de la taxation du capital par la taxation des héritages en considérant des héritages monétaires et des transferts en temps descendant. Enfin, la troisième partie étudie la réforme fiscale de la seconde partie en considérant des agents différents en termes d'altruisme et de productivité. En dernier lieu, une conclusion générale reprend l'ensemble des résultats en lien avec la taxation des héritages obtenus dans les trois parties précédentes.

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Chapter 1

Intergenerational family transfers, tax policies and public debt

Abstract [1]

This paper studies the impact of public debt on intergenerational family transfers and on human capital growth, in a successive generation model of a closed economy, in which parents augment their children's income through education and bequests. We limit ourselves to simple tax structures with labor and bequest taxes. When public debt is an available instrument for the government, we show that the fiscal policy used to achieve the long run optimal endogenous growth improves the individuals' consumption of the first generations. In this case, the government reduces the tax burden on labor, encourages human capital development and implements a redistributive policy. If the public debt is not available, the government cannot completely satisfy these objectives such that the two taxes do not fully implement the intergenerational redistributive policy and the long run human capital growth is higher. In all cases, the optimal bequest tax rate is higher than the optimal tax rate on labor income.

Keywords: family transfers, debt, altruism, growth, optimal taxation.

JEL classifications: D64, H21, H23, H63.

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1.1 Introduction

Intergenerational family transfers in the form of bequests and education expenditures have an important influence on physical and human capital investments (Laitner and Juster, 1996). Both investments are engines of economic growth involving an increase in disposable resources for future generations. Endogenous growth models address the scenario in which human capital development induces positive externalities and spillover effects on economic growth. Further, public transfers in the form of public debt can be used to transfer these new resources from future generations to the present one, and distortive fiscal instruments can be implemented to internalize the external contribution of human capital investment to the production sector. Thus, public debt can be used by governments to implement an intergenerational redistributive policy. However, in many countries today, the pressure on public finance can make public debt unavailable. When public debt is not available, the intergenerational redistribution has to be conducted through other channels. When this occurs and the governments want to redistribute some resources from future generations to the first generation, they can only used tax instruments to correct externalities and arbitrate between generations. In this context, we address the issue of the best tax policy that allows simultaneously, on the one hand enhance physical capital and human capital investments and on the other hand redistribute.

The effectiveness of public debt in stimulating economic activity relies on the span of the forecasting horizons of households. Through intergenerational family transfers, families may be able to neutralize any attempt by the government to redistribute resources among generations as in the Barro (1974) model. This offsetting of public by private transfers involves the neutrality of public debt referred to as Ricardian equivalence. Intergenerational altruism extends the planning horizon of economic agents. In models of household's rational altruism *à la* Barro (1974), dynasties want to smooth their consumption over time through intergenerational family transfers, such as education and bequests. Dynastic households are the effective decision makers since they are as long lived as the government involving public debt neutrality. However, there exist a number of other bequest motives differently affecting the optimal fiscal policy (Cremer and Pestieau, 2011) in which the Ricardian equivalence may be rejected.

Therefore, the motives for intergenerational transfers are crucial to study the effect of public debt availability on economic activity. In Barro (1974), there is an altruistic feeling among generations. In our study, we consider family altruism, a less drastic approach than dynastic altruism *à la* Barro (1974), in which public debt is not neutral (see Becker (1991) or Mankiw (2000)). Lambrecht et al. (2006) implement this kind of bequest, wherein parents care only about their children's income and not about their utility levels. Halfway between a pure altruistic bequest *à la* Barro (1974) and a pure life cycle model *à la* Diamond (1965), the concept of family altruism involves the non-neutrality of the public debt. In addition, an empirical study from Arrondel et al. (1997) shows that family altruism is a significant fraction of bequest motives and that taking this perspective is a less drastic approach than rational altruism.

Various authors have highlighted the crucial importance of human capital investment in contributing to economic growth (see Romer (1986) and Lucas (1988)). In growth models, the latter is closely related to households' education levels. The human capital of individuals relies on their parents' knowledge and their parents' educational spending since these individuals are not able to self-finance their educations. Regardless of whether bequests take place, familial transfers of human capital are thus a significant altruistic behavior that impacts future generations and economic activity (Glomm and Ravikumar, 1992). Some authors, such as Lambrecht et al. (2005), consider transfers of education expenditures and bequests to analyze the effectiveness of fiscal policy in stimulating economic activity. Drazen (1978) shows that parents invest in education rather than bequest until the education return corresponds to the interest rate. The parents' trade-off between both transfers creates inequalities across agents and generations in accordance with their preferences and lifetime resources.

In this paper, we consider an endogenous growth model that includes both transfers, education and bequests. We investigate the impact of public debt availability on intergenerational family transfers and on optimal long-run economic growth, in a successive generation model of a closed economy, in which parents augment their children's income through education and bequest transfers. Human capital growth corresponds to the representative agent's improvement in education level, which depends on the parents' education expenditure and accumulated dynastic knowledge. Family altruism involves a positive externality of parents' education spending on all future generations. We assume that a fraction of production must be devoted to public spending necessary for the good development of the economy, such as justice or defence. The two available taxes are the ones affecting directly the agents' trade-offs between both transfers. Thus, the bequest tax, labor tax and public debt are the only available fiscal instruments to finance public spending, correct externalities and implement a redistributive policy. Public debt is required to achieve optimal human capital growth along the balanced growth path and to implement an intergenerational redistribution policy. Thanks to positive public debt, the tax burden on both labor and capital can be adjusted, and the government can promote human capital development and improve the household consumption of first generations. Otherwise, when public debt is not available, the social planner internalizes the positive human capital externality and pursues a redistributive policy by using lower bequest tax and labor tax but with a higher gap between both. This creates an over-investment in education and reduces the capital-labor ratio of each generation leading to increase the individual's consumption of first generation. However, this tax policy does not allow to achieve the first-best objective with respect to redistribution.

In Section 3.2, the framework and the dynamics of the model are developed. Section 3.3 analyzes the first-best optimum. Then, in Section 3.4, we present the second-best optimum. A numerical illustration is used to show that the transition dynamics jumps to the optimal solutions along the balanced growth path, in both cases. The final Section concludes.

1.2 The model

We consider one dynasty composed of successive generations of individuals in a closed economy. Each generation lives for one period and gives birth to a child. In addition, we concentrate on dynastic family altruism, meaning that parents care about their children's income.

1.2.1 Households

The representative household of generation t works, consumes and leaves intergenerational transfers to increase his offspring's disposable income. For this purpose, he invests in his child's education and leaves bequests to generation $t+1$, which are represented by e_{t+1} and x_{t+1}, respectively. In our model, bequests are the only motive for saving.

The labor supply is inelastic, and an individual's labor income depends on his human capital level. We focus on a private education regime in which parent's investment in their child's education e_{t+1}, as well as accumulated knowledge from the dynasty H_t, characterizes the child's human capital:

$$H_{t+1} = G\left(e_{t+1}, H_t\right), \; t \geq 0 \tag{1.2.1}$$

The fact that parent's knowledge influences child's human capital is consistent with a number of empirical studies, such as Hertz et al. (2007). Parents have to invest first in child's education in order to provide a positive human capital to their child. Thus, both human capital factors are imperfect substitutes. A Cobb-Douglas human capital function is used to represent the human capital technology:

$$G\left(e_{t+1}, H_t\right) = B(e_{t+1})^{\delta}(H_t)^{1-\delta}$$

where B is a strictly positive technological parameter and $\delta \in (0,1)$ represents the responsiveness of a child's human capital to a change in the parent's education spending.

The individual's resources come from two sources: work and bequest. The total after tax lifetime income is represented by Ω_t:

$$\Omega_t = (1 - \tau_t^B) R_t x_t + (1 - \tau_t^L) w_t H_t \tag{1.2.2}$$

where w_t is the real wage, R_t is the gross interest rate and x_t is the bequest received from his parent. τ_t^B and τ_t^L are the respective period-t tax rates on labor income and bequests. The individual born in t receives after tax labor income $(1 - \tau_t^L) w_t H_t$ and after tax bequest $(1 - \tau_t^B) R_t x_t$. These resources are allocated to consumption c_t, bequest x_{t+1} and the child's education expenditure e_{t+1}:

$$\Omega_t = c_t + e_{t+1} + x_{t+1} \tag{1.2.3}$$

In this framework, the parent derives utility from his offspring's resources. Unlike the joy-of-giving formulation, this form of altruism allows a trade-off between bequest and education spending driven by relative returns. Individual's preferences are represented by a logarithmic utility function that depends on consumption c_t and child's disposable income Ω_{t+1}:

$$u_t = \ln c_t + \gamma \ln \Omega_{t+1} \tag{1.2.4}$$

where $\gamma > 0$ is the intergenerational degree of altruism. The parent makes a trade-off between his own consumption and both family transfers. Since individual's level of human capital depends on his parent's human capital, every additional unit of education spending increases child's human capital and that of generations to come. However, the parent's transfer decision does not consider the impact of human capital on subsequent generations given the feature of family altruism.

Plugging (1.2.2)-(1.2.3) into (1.2.4) gives individual's utility as a function of education expenditure e_{t+1} and bequest x_{t+1}. The representative individual maximizes his utility with respect to these two variables. This leads to the following first-order conditions:

— with respect to e_{t+1}, for $t \geq 0$,

$$-\frac{1}{c_t} + (1 - \tau^L_{t+1}) w_{t+1} G'_e(e_{t+1}, H_t) \frac{\gamma}{\Omega_{t+1}} = 0 \tag{1.2.5a}$$

— with respect to x_{t+1}, for $t \geq 0$,

$$-\frac{1}{c_t} + (1 - \tau^B_{t+1}) R_{t+1} \frac{\gamma}{\Omega_{t+1}} = 0 \text{ if } x_{t+1} > 0, \ \leq 0 \ \text{ otherwise} \tag{1.2.5b}$$

Since the after tax marginal return of education expenditure is decreasing and close to infinite when the education level is equal to zero, education spending constraint for an interior solution is necessarily satisfied at equilibrium (involving equation (1.2.5a)). The individual first invests in human capital until the marginal return of education expenditure corresponds to that of bequest. Both rates of return are equal with operative bequest.

1.2.2 Production

The production sector consists in a representative firm that behaves competitively, and produces a homogeneous good with physical capital K_t and efficient labor L_t. The production function $F(K_t, L_t)$ is linear homogeneous and concave. Profit maximization implies that factor prices w_t and R_t are equal to their marginal products:

$$w_t = F'_L(K_t, L_t) \tag{1.2.6a}$$

$$R_t = F'_K(K_t, L_t) \tag{1.2.6b}$$

assuming total depreciation of the physical capital stock in one period. F'_L and F'_K stand for the partial derivatives of F with respect to efficient labor and physical capital, respectively. We use a Cobb-Douglas production function with the following form:

$$F(K_t, L_t) = A K_t^{\alpha} L_t^{1-\alpha} \tag{1.2.7}$$

where A is a strictly positive technological parameter and $0 < \alpha < 1$.

1.2.3 Government

In each period, the government incurs an amount of public spending that corresponds to a fraction Γ of total production. The available fiscal instruments are the bequest tax τ_t^B, the labor income tax τ_t^L and the public debt Δ_t with one period of maturity debt. The government's budget constraint at time t is:

$$\Gamma F\left(K_t, L_t\right) + R_t \Delta_{t-1} = \tau_t^B R_t x_t + \tau_t^L w_t L_t + \Delta_t \tag{1.2.8}$$

When public debt is not available, using the Cobb-Douglas production function (1.2.7), equation (1.2.8) can be rewritten as follows:

$$\Gamma = \tau_t^B \alpha + \tau_t^L (1 - \alpha) \tag{1.2.9}$$

1.2.4 Equilibrium

At equilibrium, household's bequest is divided into private capital and public debt:

$$\Delta_t + K_{t+1} = x_{t+1} \tag{1.2.10}$$

The assumption that government spending is proportional to production involves an externality created by the level of production. Considering the equilibrium on market for goods and taking account of the equilibrium on labor market which involves that $L_t = H_t$, we get in period t:

$$c_t + e_{t+1} + K_{t+1} = (1 - \Gamma) F\left(K_t, H_t\right) \tag{1.2.11}$$

The social marginal product of physical capital $(1 - \Gamma) F_K'\left(K_t, H_t\right)$, respectively human capital $(1 - \Gamma) F_H'\left(K_t, H_t\right)$, is lower than the private marginal product $F_K'\left(K_t, H_t\right)$ (resp. $F_H'\left(K_t, H_t\right)$) as stated by the first-order conditions of the representative firm (1.2.6a) and (1.2.6b). If the government sets the tax rates on bequest and labor income to $\tau_t^B = \tau_t^L = \Gamma$, these externalities are assimilated by the private sector.

1.2.5 Intertemporal equilibria with constant tax rates

We concentrate on intertemporal equilibria where bequests are positive. Tax rates on labor income τ_t^L and bequest τ_t^B are assumed to be constant for $t \geq 1$. At the initial period, τ_0^L and τ_0^B are lump-sum taxes since both taxes only affect the individual's resources. Both tax values implemented, are decisive for the initial public debt condition. Following this period, both taxes have an impact on the trade-off between both transfers. From the first-order conditions (1.2.5), the private rate of return on education is equal to the private rate of return on capital:

$$(1-\tau^B)R_{t+1} = (1-\tau^L)w_{t+1}G_e^{'}(e_{t+1}, H_t) \tag{1.2.12}$$

Let $\eta_{t+1} \equiv \frac{e_{t+1}}{H_t}$ denotes the individual's education spending per unit of human capital. At equilibrium, from equation (1.2.12), we get:

$$\eta_{t+1} = (\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} \tag{1.2.13}$$

where $\rho_{t+1} \equiv \phi_{t+1}\frac{w_{t+1}}{R_{t+1}} = \phi_{t+1}\frac{1-\alpha}{\alpha}k_{t+1}$ is the after-tax ratio of factor prices, $k_{t+1} \equiv \frac{K_{t+1}}{H_{t+1}}$ is the capital-labor ratio and ϕ_{t+1} corresponds to the ratio $\frac{1-\tau_{t+1}^L}{1-\tau_{t+1}^B}$. As we focus on constant tax rates along the dynamics, the ratio $\phi_{t+1} = \phi$. A higher ρ_{t+1} involves that the return on human capital $(1-\tau^L)w_{t+1}$ increases relative to the one on physical capital $(1-\tau^B)R_{t+1}$. Thus, after an increase of ρ_{t+1}, the individual is incited to invest in education rather than bequest increasing the human capital growth rate $g_{t+1}^H \equiv \frac{H_{t+1}}{H_t}$. From equation (1.2.1), the human capital growth rate in period t, corresponds to:

$$g_{t+1}^H = B\left(\eta_{t+1}\right)^{\delta} = B\left(\rho_{t+1}B\delta\right)^{\frac{\delta}{1-\delta}} \tag{1.2.14}$$

From equation (1.2.2), the child's disposable income per unit of human capital writes:

$$\frac{\Omega_{t+1}}{H_t} = (1-\tau^B)R_{t+1}\left(\rho_{t+1}g^H_{t+1} + \frac{x_{t+1}}{H_t}\right) \tag{1.2.15}$$

And using equation (1.2.3), we get the consumption per unit of human capital:

$$\frac{c_t}{H_t} = (1-\tau^B)R_t\left(\rho_t + \frac{x_t}{H_t}\right) - \eta_{t+1} - \frac{x_{t+1}}{H_t} \tag{1.2.16}$$

Then, equation (1.2.5b) gives simple expression for bequest per unit of human capital (using (1.2.15) and (1.2.16)):

$$\frac{x_{t+1}}{H_t} = \frac{1}{1+\gamma}\left[\gamma\left(1-\tau^B\right)R_t\left(\rho_t + \frac{x_t}{H_t}\right) - \left(\frac{1+\gamma\delta}{\delta}\right)\eta_{t+1}\right] \tag{1.2.17}$$

Dynamics

We combine the capital market equilibrium (1.2.10) and equation (1.2.17) (using the Cobb-Douglas production function (1.2.7)):

$$d_{t+1} + g^H_{t+1}k_{t+1} = \frac{1}{1+\gamma}\left[\gamma\left(1-\tau^B\right)A\alpha k_t^{\alpha-1}\left(\rho_t + \frac{d_t}{g^H_t} + k_t\right) - \left(\frac{1+\gamma\delta}{\delta}\right)\eta_{t+1}\right] \tag{1.2.18}$$

where $d_{t+1} = \frac{\Delta_t}{H_t}$ is the public debt per unit of human capital. In addition, using the capital market equilibrium (1.2.10), we can rewrite the government budget constraint (1.2.8) as follows:

$$d_{t+1} = \left(1-\tau^B\right)A\alpha k_t^{\alpha-1}\frac{d_t}{g^H_t} + Ak_t^\alpha\left[\Gamma - \tau^B\alpha - \tau^L(1-\alpha)\right] \tag{1.2.19}$$

Since the after-tax ratio of factor prices ρ_t is linear with respect to capital labor ratio k_t, it is equivalent to analyze its dynamics. From equations (1.2.18) and (1.2.19) and using equations (1.2.13) and (1.2.14), we get the two-dimensional dynamics of $(\rho_{t+1}, d_{t+1})_{t\geq 0}$:

$$(\rho_{t+1}B\delta)^{\frac{1}{1-\delta}} = \frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^\alpha}{D}\rho_t^\alpha\left[\gamma(1-\Gamma) - E - \frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}}\right] \tag{1.2.20a}$$

$$d_{t+1} = A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\rho_t\right)^{\alpha}\left[\frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}} + E\right] \tag{1.2.20b}$$

where $D = \frac{1}{\delta}\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta\right) > 0$, $J = \delta\left(1-\tau^L\right)(1-\alpha) > 0$ and $E = \Gamma - \tau^B\alpha - \tau^L(1-\alpha)$. The initial human capital $\overline{H}_{-1}$ and $\overline{H}_0$, physical capital $\overline{K}_0$ and public debt $\overline{\Delta}_{-1}$ are given and consequently the initial bequest $\overline{x}_0 = \overline{K}_0 + \overline{\Delta}_{-1}$. Then, $\overline{\rho}_0$ and $\overline{d}_0$ are given since $\overline{\rho}_0 = \phi\frac{1-\alpha}{\alpha}\frac{\overline{K}_0}{\overline{H}_0}$ and $\overline{d}_0 = \frac{\overline{\Delta}_{-1}}{\overline{H}_{-1}}$. Notice that the right-hand side (RHS) of equation (1.2.20a) needs to be positive ($i.e.$ $\gamma(1-\Gamma) - E - \frac{Jd_t}{(\rho_t B\delta)^{\frac{1}{1-\delta}}} > 0$) in order to get a positive level of education expenditures per unit of human capital η_{t+1} (see equation (1.2.13)) as well as positive human capital growth g^H_{t+1} (see equation (1.2.14)).

From the system of equations (1.2.20), we deduce the two-dimensional dynamics of $(X_{t+1}, \rho_{t+1})_{t\geq 0}$, where $X_{t+1} = d_{t+1}(\rho_{t+1}B\delta)^{\frac{-1}{1-\delta}}$. Thus, we get the following system of equations:

$$X_{t+1} = \psi(X_t) \equiv D\left(\frac{JX_t + E}{\gamma(1-\Gamma) - E - JX_t}\right) \tag{1.2.21a}$$

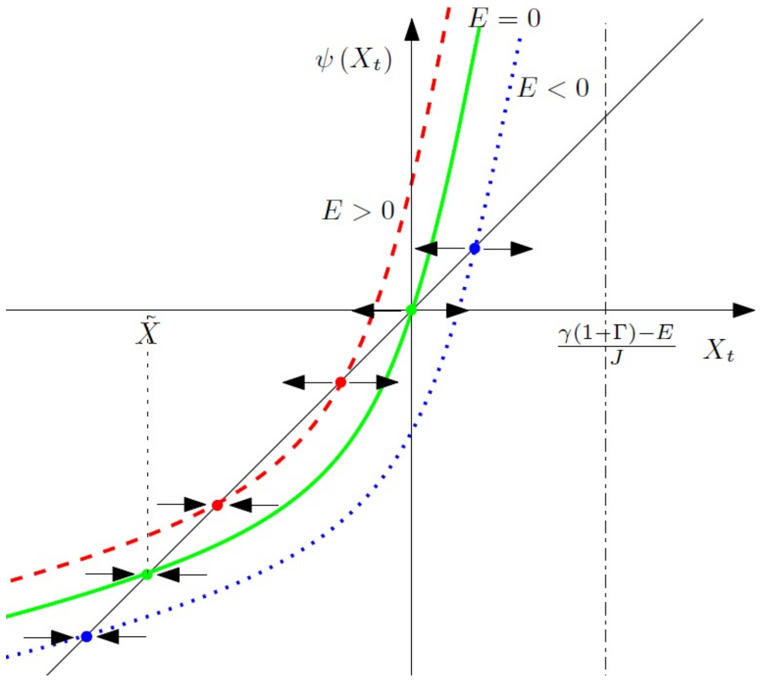
$$\rho_{t+1} = \frac{1}{B\delta}\left(\frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{\alpha}}{D}\rho_t^{\alpha}\left[\gamma(1-\Gamma) - E - JX_t\right]\right)^{1-\delta} \tag{1.2.21b}$$

where $\overline{X}_0 = \overline{d}_0(\overline{\rho}_0 B\delta)^{\frac{-1}{1-\delta}}$ is given. As shown in Appendix, the function $\psi(X_t)$ is strictly increasing and strictly convex if $JX + E < \gamma(1-\Gamma)$, which is a necessary condition to get positive human capital growth g^H_{t+1} and defines an upper bound on X above which production become zero in finite time. The autonomous backward dynamics of X_t, for $t \geq 0$, is monotonous.

When public debt is not used to finance public spending ($i.e.$ $E = 0$ or equivalently $\Gamma = \tau^B\alpha + \tau^L(1-\alpha)$), we show in Appendix that two steady states are achievable: $\tilde{X} \equiv \frac{\gamma(1-\Gamma)}{J} - D$ and $X = 0$. If $X_0 < 0$ ($i.e.$ with an initial public capital accumulation $d_0 < 0$), the dynamics of X_t converges towards a stable negative balanced growth path $\tilde{X}$. The other steady state $X = 0$ is unstable. Otherwise, if $X_0 > 0$ ($i.e.$ with an initial public debt $d_0 > 0$), a zero production is reached in finite time, since the dynamic of X_t goes beyond its upper bond. Figure 1.1 illustrates this dynamics.

Then, as described in Figure 1.1, the curve of function $\psi(X_t)$ shifts to left with $E > 0$. According to the magnitude of E, there are two, one or zero steady state and all of which negative. When there are two steady states, X_t reaches a locally stable negative balanced growth path as soon as X_0 is below the unstable one. Otherwise, the dynamics leads to zero production as seen previously.

Figure 1.1 – The dynamics of X depending on the sign of E



where $E = \Gamma - \tau^B\alpha - \tau^L(1-\alpha)$ and $X_{t+1} = \psi(X_t) \equiv D\left(\frac{JX_t+E}{\gamma(1-\Gamma)-E-JX_t}\right)$.

Lastly, the curve of function $\psi(X_t)$ shifts to right with $E < 0$ (see Figure 1.1). In this case, there are two steady states, one negative and one positive if it is below $\frac{\gamma(1-\Gamma)-E}{J}$. The dynamics of X_t converges towards a locally stable negative balanced growth path as soon as X_0 is below the unstable one. Above the unstable steady-state, a zero production is reached in finite time.

As a result, in each case, the dynamics of X_t may converge only towards a stable negative balanced growth path as soon as X_0 is below the unstable one. This implies a steady-state public capital accumulation (*i.e.* $d < 0$). The only way to have a positive or zero public debt along the balanced growth path is that government implements, in the first period, the fiscal policy which jumps directly to unstable steady-state. There are two different situations of unstable steady-state based on whether governments use a positive public debt or not:

(i) At the unstable steady-state, we have $X = 0$ ($i.e.$ $d_t = 0$ and $E = 0, \forall t \geq 1$). Given the government budget constraint (1.2.8), the initial tax instruments τ_0^B and τ_0^L are chosen such that $d_t = 0, \forall t \geq 1$, regardless of initial public debt d_0. Tax instruments satisfy $\Gamma = \tau^B \alpha + \tau^L (1-\alpha)$ for the following periods ($i.e.$ $E = 0$).

(ii) At the unstable steady-state, we have $X > 0$ ($i.e.$ $d_t > 0$ and $E < 0, \forall t \geq 1$). The initial tax instruments τ_0^B and τ_0^L is used by government to jump directly on the positive unstable balanced growth path ($i.e.$ $X > 0$), where $d_t > 0$ and $E < 0, \forall t \geq 1$.

Whether or not the dynamics of X_t jumps on unstable balanced growth path ($X > 0$ or $X = 0$) or converges towards a stable negative one, equation (1.2.21b) shows that ρ_t converges towards a strictly positive balanced growth path. Indeed, the RHS of equation (1.2.21b) is increasing and concave. The slope of the RHS of equation (1.2.21b) tends to infinity when ρ approaches zero and to zero when ρ approaches infinity. At steady state, we deduce from equation (1.2.21b), the following after-tax ratio of factor prices:

$$\rho = \left(\frac{1}{B\delta}\right)^{\frac{1}{1-\alpha(1-\delta)}} \left(\left(\frac{A\left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{\alpha}}{D}\right)\left[\gamma\left(1-\Gamma\right) - E - JX\right]\right)^{\frac{1-\delta}{1-\alpha(1-\delta)}}$$

As shown by the last equation, the after-tax ratio of factor prices is negatively affected by X. Thus, a negative X along the balanced growth path, leads to an higher ρ which implies a higher human capital growth g^H(see equation (1.2.14)). In contrast, a positive X involves a lower human capital growth g^H.

Since, in our paper, we analyze the effect of public debt constraint on steady state human capital growth and on the intergenerational family transfers, we focus on steady states with zero or positive public debt. Consider situation which jumps directly on positive X in period 0. In this case, $E < 0$ and the non-negativity constraint of bequests is satisfied since $d_t > 0, \forall t \geq 1$. Indeed, given the capital market equilibrium (1.2.10), a sufficient condition to satisfy the non-negativity constraint on bequests is to assumed a positive or zero public debt. The implementation of positive public debt has a negative effect on the after-tax ratio of factor prices ρ. Lower ρ decreases the capital-labor ratio k since $\rho = \phi \frac{1-\alpha}{\alpha} k$. As a result, implementing a positive public debt has a negative effect on education expenditure (*i.e.* $\eta = (\rho B \delta)^{\frac{1}{1-\delta}}$) and on human capital growth along the balanced growth path (*i.e.* $g^H = B\eta^\delta$). Thus, the human capital growth is higher without public debt (*i.e.* $X = 0$ and $E = 0$). We concentrates on this situation in the next Subsection.

Intertemporal equilibrium without public debt (*i.e.* $X = 0$ and $E = 0$)

In the following, we focus on intertemporal equilibria where bequests are positive, tax instruments are constant over time and public debt is not used to finance public spending (*i.e.* $E = 0$). The government implements τ_0^B and τ_0^L such that the dynamics of X_t jumps directly to the balanced growth path of $X = 0$ (*i.e.* $E = 0$ and $d_t = 0$ for $t \geq 1$). From equation (1.2.21b) with $X = 0$ and $E = 0$, the dynamics of the after-tax ratio of factor prices ρ reaches the following balanced growth path:

$$\rho = \left(\frac{1}{B\delta}\right)^{\frac{1}{1-\alpha(1-\delta)}} \left(\frac{\delta\gamma A \left(\frac{1}{\phi}\frac{\alpha}{1-\alpha}\right)^{\alpha}(1-\Gamma)}{\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta}\right)^{\frac{1-\delta}{1-\alpha(1-\delta)}}$$

Plugging the last equation into expression (1.2.14), we get the human capital growth rate along a balanced growth path:

$$g^H = \left(B^{1-\alpha}\delta^{\delta(1-\alpha)}\left[\gamma A \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}(1-\Gamma) f(\phi)\right]^{\delta}\right)^{\frac{1}{1-\alpha(1-\delta)}} \tag{1.2.22}$$

where $f(\phi) = \frac{\phi^{1-\alpha}}{(1+\gamma)\frac{\alpha}{1-\alpha} + \phi(1+\gamma\delta)}$. As stated by equation (1.2.22), the human capital growth is positively affected by the degree of altruism γ. Individuals leave more resources to their offsprings when they have a higher degree of altruism. Thus, they invest more in education which increases the human capital growth rate along a balanced growth path. However, individuals do not take into account the consequences of their transfer's decisions for all generations. By increasing the child's human capital through the education expenditures, the parent increases in the same way the human capital of generations to come by increasing the accumulated knowledge. Thus, the allocation of intergenerational family transfers plays an important role in the human capital growth level. The fiscal policy implemented affects this transfers' allocation (1.2.12) as well as the human capital growth g^H(see equation (1.2.22)). As a result, a fiscal policy should be set up in order to internalize the positive human capital externality on future generations.

When both taxes are identical and equivalent to the share of production devoted to the public spending, $\tau^B = \tau^L = \Gamma$, none intergenerational transfer is promoted. From equation (1.2.22), the effect of both tax instruments on the after-tax ratio of factor prices ρ along the balanced growth path (and also on the human capital growth) are given by function $f(\phi)$. The sign of $f'(\phi)$ corresponds to the one of $1 + \gamma - (1 + \gamma\delta)\phi$. When $\tau^B = \tau^L = \Gamma$, we have $f'(1) = \gamma(1 - \delta) > 0$ which implies that ρ is not maximized with this tax policy. Another fiscal policy must be implemented in order to get the highest human capital growth rate at equilibrium.

Since $f'(\phi) > 0$ when $\tau^B = \tau^L = \Gamma$, a marginal increase of the ratio ϕ has a positive impact on ρ until a threshold (where $f'(\phi) = 0$). The household is incited to invest more in education expenditures. Consequently, this new attractiveness for the education spending involves an increase of human capital and a decrease of physical capital used in production. This leads to fall down the marginal return on human capital relative to the one on physical capital. After achieving this threshold, this negative effect on the marginal return on labor overcompensates the positive effect of reducing tax on labor income, which decreases the after-tax ratio of factor prices ρ. Thus, the fiscal policy which maximizes ρ corresponds to the situation where $f'(\phi) = 0$, that is:

$$\phi = \widehat{\phi} \equiv \frac{1+\gamma}{1+\gamma\delta} > 1$$

We deduce from $\widehat{\phi} > 1$ that the bequest tax is higher than Γ and the labor income tax is lower than Γ. Thanks to the fiscal policy which satisfied $\phi = \widehat{\phi}$, we get the highest human capital growth along the balanced growth path g^H of $X = 0$.

However, the highest human capital growth rate at equilibrium is not necessarily the optimum. Furthermore, in this Subsection, we consider an equilibrium without public debt whereas public debt can be used into the fiscal policy to achieve optimal human capital growth rate. In the following, we focus on the optimal fiscal policy which maximizes the households' welfare.

1.3 First-best optimum

The social objective adopted depends on whether or not individual's altruism is taken into account. There are at least two types of social criteria. In models based on rational altruism *à la* Barro (1974), the social welfare function is usually assumed to exclude altruistic preferences in order to avoid undesirable double counting and thus to avoid increasing social weight with time (Hammond, 1988). For Harsanyi (1995), the same approach can be adopted excluding "all external preferences" (*i.e.* preferences for assignments of goods to others indiviuals). In other words, the social objective should include only the individual's life cycle utility for any form of altruism. Michel and Pestieau (2004) follow this approach in the context of paternalistic altruism. Hence, we concentrate our analysis on a social welfare function where the government eliminates the altruistic part of the individual's utility function. The government's social objective thus is:

$$SWF = \sum_{t=0}^{+\infty} \beta^t \ln c_t \tag{1.3.1}$$

where β is the social discount rate.

1.3.1 The first-best optimal solutions

The social planner has to maximize the social welfare function (1.3.1) with respect to $(c_t, e_{t+1}, H_{t+1}, K_{t+1})_{t\geq 0}$ subject to the human capital technology (1.2.1) and the resource constraint (1.2.11).

Proposition 1. *A sequence* $\left(\frac{c_t^*}{H_t^*}, k_{t+1}^*, g_{t+1}^{H*}\right)_{t\geq 0}$ *satisfies the first-best optimality conditions iff:*

$$\frac{c_t^*}{H_t^*} + \eta_{t+1}^* + g_{t+1}^{H*} k_{t+1}^* = (1-\Gamma) F\left(k_t^*, 1\right) \tag{1.3.2}$$

$$\frac{c_{t+1}^*}{H_{t+1}^*} / \frac{c_t^*}{H_t^*} g_{t+1}^{H*} = \beta (1-\Gamma) F_K^{'}\left(k_{t+1}^*, 1\right) \tag{1.3.3}$$

$$(1-\Gamma) F_K^{'}\left(k_{t+1}^*, 1\right) = G_e^{'}\left(\eta_{t+1}^*, 1\right) \left[(1-\Gamma) F_H^{'}\left(k_{t+1}^*, 1\right) + \frac{G_H^{'}\left(\eta_{t+2}^*, 1\right)}{G_e^{'}\left(\eta_{t+2}^*, 1\right)}\right] \tag{1.3.4}$$

where g_{t+1}^{H*} *and* η_{t+1}^* *are linked through the static relation:* $g_{t+1}^{H*} = G\left(\eta_{t+1}^*, 1\right)$.

Proof. Let us denote by μ_{t+1} and λ_{t+1} the respective Lagrange multipliers of the human capital technology (1.2.1) and the resource constraint (1.2.11). Then, the optimality conditions are:

— with respect to c_t, for $t \geq 0$,

$$\frac{\beta^t}{c_t} - \lambda_{t+1} = 0 \tag{1.3.5}$$

— with respect to e_{t+1}, for $t \geq 0$,

$$\mu_{t+1} G_e^{'}\left(\eta_{t+1}, 1\right) - \lambda_{t+1} = 0 \tag{1.3.6}$$

— with respect to H_{t+1}, for $t \geq 0$,

$$\lambda_{t+2} (1-\Gamma) F_H^{'}\left(k_{t+1}, 1\right) + \mu_{t+2} G_H^{'}\left(\eta_{t+2}, 1\right) - \mu_{t+1} = 0 \tag{1.3.7}$$

— with respect to K_{t+1}, for $t \geq 0$,

$$\lambda_{t+2} (1-\Gamma) F_K^{'}\left(k_{t+1}, 1\right) - \lambda_{t+1} = 0 \tag{1.3.8}$$

From the optimality conditions (1.3.5) and (1.3.8), we get the optimal consumption growth (1.3.3). Using (1.3.5), (1.3.6) and (1.3.7), we deduce:

$$\frac{\beta^t}{c_t} = \frac{\beta^{t+1}}{c_{t+1}} G_e^{'}\left(\eta_{t+1}, 1\right) \left[(1-\Gamma) F_H^{'}\left(k_{t+1}, 1\right) + \frac{G_H^{'}\left(\eta_{t+2}, 1\right)}{G_e^{'}\left(\eta_{t+2}, 1\right)}\right]$$

Using (1.3.3), we obtain the condition (1.3.4). Lastly, the resource constraint (1.2.11) can be rewritten as (1.3.2). □

Equation (1.3.4) shows that the optimal level of education expenditure is obtained when the social marginal returns of both transfers are equal. The individual's decision about investment in a child's education involves a positive human capital externality for future generations. As a reminder, the individual's transfer motive is to improve the lifetime resources of the recipient and not to increase his welfare, and given the human capital function (1.2.1), there is a positive human capital externality on all the next generations. Two different effects compose the social return of an extra unit of education: (i) the direct effect on a child's wage and (ii) the indirect effect on human capital level of future generations and its impacts on family transfer decisions.

As a result, the social planner's fiscal policy objectives are to finance public spending (1.3.2), to take positive human capital externality into account (1.3.4) and to pursue a redistributive policy that satisfies the optimal consumption growth (1.3.3). In the next Subsection, we analyze the fiscal policy used to decentralize the first-best optimal solutions such that individuals make the optimal choice regarding their transfer decisions.

1.3.2 Decentralization of the first-best optimal solutions

In this Section, we are seeking to achieve the fiscal policy that decentralizes the first best optimum. In addition to finance public spending Γ, the government has to implement incentives that allow to reach equilibrium paths for $\left(\frac{c_t}{H_t}, g^H_{t+1}, k_{t+1}\right)_{t\geq 0}$ that take positive externality into account (1.3.4) and ensure that optimal consumption growth is satisfied (1.3.3). This can be drawn by using the fiscal instruments $\left(\tau^B_{t+1}, \tau^L_{t+1}, d_{t+1}\right)_{t\geq 0}$.

Proposition 2. *The fiscal policy* $\left(\phi^*_{t+1}, d^*_{t+1}\right)_{t\geq 0}$ *that decentralizes the first-best* *optimal paths* $\left(\frac{c_t^*}{H_t^*}, g_{t+1}^{H*}, k_{t+1}^*\right)_{t\geq 0}$ *as an equilibrium, satisfies:*

$$\phi_{t+1}^* = 1 + \frac{1}{(1-\Gamma)\,F_H'\left(k_{t+1}^*, 1\right)}\left[\frac{G_H'\left(\eta_{t+2}^*, 1\right)}{G_e'\left(\eta_{t+2}^*, 1\right)}\right] \tag{1.3.9}$$

$$d_{t+1}^* = \gamma\,(1-\Gamma)\,F\left(k_t^*, 1\right) - (1+\gamma)\,g_{t+1}^{H*}k_{t+1}^* - \left(\frac{1+\gamma\delta}{\delta}\right)\eta_{t+1}^* \tag{1.3.10}$$

where $g_{t+1}^{H*} = G\left(\eta_{t+1}^*, 1\right)$.

Proof. Notice that the resource constraint (1.2.11) and the human capital technology (1.2.1) are satisfied at equilibrium and at the first-best optimum. Thus, the fiscal policy is only used to decentralize the first-best conditions (1.3.3) and (1.3.4).

At equilibrium with positive bequest, equation (1.2.12) is satisfied and can be rewritten as follows:

$$\phi_{t+1} = \frac{(1-\Gamma)\,F_K'\left(k_{t+1}, 1\right)}{(1-\Gamma)\,F_H'\left(k_{t+1}, 1\right)G_e'\left(\eta_{t+1}, 1\right)}$$

Then, the government uses the fiscal policy such that expression (1.2.12) coincides with the social condition (1.3.4). Plugging (1.3.4) into (1.2.12), we get the ratio ϕ_{t+1}^* (*i.e.* equation (1.3.9)) that internalizes the positive human capital externality at period t. In addition, using the capital market equilibrium (1.2.10) and the linear homogeneity of the production function F, the government's budget constraint (1.2.8) can be rewritten as follows:

$$d_{t+1} + (1-\Gamma)\,F\left(k_t, 1\right) = \left(1-\tau_t^B\right)F_K'\left(k_t, 1\right)\left(\rho_t + \frac{d_t}{g_t^H} + k_t\right)$$

Then, plugging the last equation into (1.2.18), we deduce (1.3.10). This concludes the proof. □

Equation (1.3.9) shows that the optimal bequest tax rate is higher than the labor income tax rate. The gap between the two tax rates is used to internalize the positive human capital externality in the individual's intergenerational transfer decision. Having a bequest tax rate higher than the labor income tax rate, provides an incentive to transfer more to education and less to bequest.

Furthermore, the degree of altruism γ has an impact on the fiscal policy implemented to decentralize the optimum solutions. Indeed, the optimal amount of public debt depends on this parameter. The government uses the public debt to support the tax instruments that internalize the positive human capital externality and pursue a redistributive policy. Therefore, the availability of public debt plays an important role in reaching the first-best optimal solution. In the following, we analyze first-best optimal fiscal policy at a steady state as to better grasp public debt impact.

1.3.3 The first-best optimal solutions and decentralization at steady state

We consider equilibria with operative bequests along the balanced growth path. Recall that at steady state equilibrium with positive public debt, the human capital growth is lower than the one without public debt (1.2.22). This human capital growth is, in general, different to the first-best human capital growth that internalizes the positive human capital externality.

Proposition 3. *At steady state, the first-best consumption growth corresponds to the first-best human capital growth.*

Proof. At steady state, the capital-labor ratio is strictly positive and constant: $k_{t+1} = k$. Then, the resource constraint (1.2.11) is equivalent to:

$$k = \frac{(1-\Gamma)\, F\,(k,1)}{g^H} - \frac{1}{g^H}\left(\frac{c}{H}\right) - \frac{1}{g^H}\eta$$

Since the human capital growth and the education expenditure per unit of human capital are constant along the balanced growth path, $\frac{c_t}{H_t}$ is also constant. Thus, the resource constraint involves that consumption growth and human capital growth are the same for every generation and are equivalent to each other. This concludes the proof. □

In order to achieve the first-best human capital growth as an equilibrium, the social planner implements the following fiscal policy, which decentralizes the optimal solutions $\left(\frac{c^*}{H^*}, g^{H*}, k^*\right)$ along the balanced growth path.

Proposition 4. *At steady state, a fiscal policy* $\left(\tau^{L*}, \tau^{B*}, d^*\right)$ *allows to decentralize the first-best optimal solutions. This optimal fiscal policy is fully described as follows:*

$$\tau^{L*} = 1 - \frac{(1-\Gamma)\,\beta}{\gamma\,(1-\beta)}\left[\frac{1}{1-\alpha\beta\,(1-\delta)} + \left(\frac{\gamma}{1+\gamma} - \beta\right)\frac{1+\gamma}{1-\beta\,(1-\delta)}\right] \tag{1.3.11}$$

$$\tau^{B*} = 1 - (1-\beta\,(1-\delta))\,\frac{(1-\Gamma)\,\beta}{\gamma\,(1-\beta)}\left[\frac{1}{1-\alpha\beta\,(1-\delta)} + \left(\frac{\gamma}{1+\gamma} - \beta\right)\frac{1+\gamma}{1-\beta\,(1-\delta)}\right] \tag{1.3.12}$$

$$d^* = (1+\gamma)\left(\frac{\gamma}{1+\gamma} - \beta\right)\left(\frac{1-\alpha\beta\,(1-\delta)}{1-\beta\,(1-\delta)}\right)(1-\Gamma)\,F\,(k^*, 1) \tag{1.3.13}$$

There are three possible fiscal policy cases:
(*) If $\frac{\gamma}{1+\gamma} = \beta$*, only the two taxes are implemented in order to decentralize the first-best human capital growth. (**) If* $\frac{\gamma}{1+\gamma} > \beta$ *(resp.* $\frac{\gamma}{1+\gamma} < \beta$*), the social planner uses also a positive (resp. negative) public debt into the first-best fiscal policy.*

Proof. As both consumption and human capital growths are equal along the balanced growth path, we deduce from the optimal consumption growth (1.3.3):

$$g^{H*} = \beta\,(1-\Gamma)\,F_K^{'}\,(k^*, 1) \tag{1.3.14}$$

Then, using equation (1.2.12) and given that $g^{H*} = B\,(\eta^*)^{\delta}$, we get:

$$\frac{\eta^*}{\beta\delta\phi^*} = (1-\Gamma)\,F_H^{'}\,(k^*, 1) \tag{1.3.15}$$

At steady state, equation (1.3.9) corresponds to:

$$\phi^* = 1 + \frac{1}{(1-\Gamma)\,F_H^{'}\,(k^*, 1)}\left[\frac{1-\delta}{\delta}\eta^*\right]$$

Plugging (1.3.15) into the last equation gives:

$$\phi^* = \frac{1}{1-\beta\,(1-\delta)} \tag{1.3.16}$$

Then, using equations (1.3.14)-(1.3.16) and the Cobb-Douglas production function (1.2.7), the optimal level of public debt per unit of human capital (1.3.10) can be rewritten as equation (1.3.13). Equation (1.3.13) shows that the sign of public debt depends on the gap between β and $\frac{\gamma}{1+\gamma}$. Indeed, $d^* > 0 \Leftrightarrow \frac{\gamma}{1+\gamma} > \beta$. From the optimal ratio ϕ^* (equation (1.3.16)), we get: $\tau^{B*} = \tau^{L*} + \left(1 - \tau^{L*}\right)\beta\left(1 - \delta\right)$. Then, we deduce the first-best optimal tax instruments $\left(\tau^{L*}, \tau^{B*}\right)$ from the government's budget constraint (1.2.19), taking into account equation (1.3.14) and the optimal level of public debt (1.3.13) along the balanced growth path. This concludes the proof. □

To interpret results in Proposition 4, recall that the first-best ratio ϕ^* internalizes the positive human capital externality. Equation (1.3.16) illustrates this point. A higher discount factor β increases the gap between taxes which increases the individual's incentive to invest in education relative to bequest. In addition, the gap between β and $\frac{\gamma}{1+\gamma}$ affects both taxes in the same way.

As a result, the social planner decentralizes the first-best human capital growth as an equilibrium such that the equilibrium human capital growth rate (1.2.14) corresponds to it is first-best optimal value (1.3.14), using the optimal fiscal policy describes in Proposition 4. Since the after-tax ratio of factor prices ρ is linear in relation to the capital-labor ratio k, we deduce from equation (1.3.14), the first-best capital-labor ratio along the balanced growth path (using expression (1.2.14), the first-best ratio ϕ^* (equation (1.3.16)) and the Cobb-Douglas production function (1.2.7)):

$$k^* = \left[\frac{\left(1-\beta\left(1-\delta\right)\right)^{\delta}\left(\beta\left(1-\Gamma\right)\alpha A\right)^{1-\delta}}{B\left(\frac{1-\alpha}{\alpha}\delta\right)^{\delta}}\right]^{\frac{1}{1-\alpha+\alpha\delta}} \tag{1.3.17}$$

Then, plugging the last equation into equation (1.3.14), we get the first-best human capital growth rate along the balanced growth path:

$$g^{H*} = \left[B^{1-\alpha}\left(\frac{\left(1-\alpha\right)\delta}{\alpha\left(1-\beta\left(1-\delta\right)\right)}\right)^{(1-\alpha)\delta}\left(\beta\left(1-\Gamma\right)\alpha A\right)^{\delta}\right]^{\frac{1}{1-\alpha+\alpha\delta}} \tag{1.3.18}$$

As shown in Proposition 3, the first-best human capital grows at the same rate than consumption along the balanced growth path and thus, both are positively affected by the discount factor β (see equation (1.3.18)). Indeed, the optimal growth is higher when the social planner gives more weight to future generations.

Notice that the first-best human capital growth rate is positively affected by the discount factor β whereas the human capital growth rate at equilibrium without public debt g^H (see equation (1.2.22)) positively depends on the degree of altruism γ. At steady state, the first-best human capital growth rate g^{H*} (see equation (1.3.18)) is lower than the human capital growth rate at equilibrium g^H (see equation (1.2.22)) iff:

$$\frac{\gamma}{1+\gamma}\left(\frac{\phi}{\phi^*}\right)^{1-\alpha} > \beta\left[\alpha+\phi\left(1-\alpha\right)\left(\frac{1+\gamma\delta}{1+\gamma}\right)\right] \tag{1.3.19}$$

When we consider the equilibrium situation without public debt in which both taxes implement the highest human capital growth ($i.e.$ $\phi=\widehat{\phi}\equiv\frac{1+\gamma}{1+\gamma\delta}$), the first-best ratio ϕ^* (equation (1.3.16)) is lower than $\widehat{\phi} \Leftrightarrow \frac{\gamma}{1+\gamma}>\beta$. Then, we deduce from the last inequality that $g^{H*}<\widehat{g}^H$ iff $\frac{\gamma}{1+\gamma}>\beta$. ($i.e.$ when the optimal public debt is positive). Thus, the key element is the weight given to future generations by the social planner and the representative individual. Therefore, only three cases of fiscal policy can arise.

(i) When $\frac{\gamma}{1+\gamma}>\beta$, for the social planner, the representative individual over-invests in the child's resources such that he penalizes his own consumption. As shown by inequality (1.3.19), the social planner promotes a lower human capital growth rate g^{H*} (see equation (1.3.18)) than the highest human capital growth at equilibrium without public debt $\widehat{g}^H$ (see equation (1.2.22)) in order to increase the individual's consumption in the current generation (using positive public debt d^* and both tax instruments $\left(\tau^{L*},\tau^{B*}\right)$). The first-best labor income tax rate τ^{L*} is still less than that on bequest τ^{B*} to take into account the positive externality, and both taxes are higher to reduce the individual's incentive to transfer resources to his child. Thus, this fiscal policy encourages individuals to consume rather than invest in future generations by reducing the incentive to pass on resources to the next generation using higher tax rates and positive public debt. Furthermore, when the gap between $\frac{\gamma}{1+\gamma}$ and β is reduced, the gap between both tax rates is higher and the public debt tends towards zero.

(ii) When $\frac{\gamma}{1+\gamma} = \beta$, the individual and government care about the next generation in the same way. Households transfer the optimum amount of resources but still under-invest in education. Individual make non-optimal trade-offs between intergenerational transfers. Thus, the government uses only both tax instruments $\left(\tau^{L*}, \tau^{B*}\right)$ to internalize the positive human capital externality. In this situation, the first-best fiscal policy ϕ^* that ensures the first-best human capital growth rate g^{H*} along the balanced growth path is that which promotes the highest human capital growth rate at equilibrium without public debt $\widehat{g}^H$. As $\phi^* = \widehat{\phi}$, we deduce from inequality (1.3.19) that g^{H*} corresponds to the highest human capital growth rate at equilibrium $\widehat{g}^H$. Given the government budget constraint (1.2.9) and the optimal ratio ϕ^* (equation (1.3.16)), we get:

$$\begin{aligned}
\tau^{B*} &= \Gamma + \frac{(1-\alpha)\,\beta\,(1-\delta)\,(1-\Gamma)}{1-\alpha\beta\,(1-\delta)} \\
\tau^{L*} &= \Gamma - \frac{\alpha\beta\,(1-\delta)\,(1-\Gamma)}{1-\alpha\beta\,(1-\delta)}
\end{aligned}$$

where the bequest tax is positive whereas the labor income tax can be either positive or negative depending on the values of the parameters.

(iii) When $\frac{\gamma}{1+\gamma} < \beta$, for the social planner, individuals are selfish in that they do not enough consider their child. Inequality (1.3.19) illustrates that both consumption and human capital growths at equilibrium without public debt, are low compared to their first-best optimal levels. Hence, the government wants to increase consumption of future generations. By improving the gap between both taxes (such that $\phi^* > \widehat{\phi}$), the government increases the incentives to invest in education expenditures, which negatively affects the capital-labor ratio. Thus, a negative public debt is necessary to achieving the first-best human capital growth rate g^{H*}. A negative public debt implies a public capital accumulation that increases the capital-labor ratio k and removes the negative impacts on the capital-lalor ratio to achieve the first-best one k^*.

Therefore, the first-best fiscal policy implemented depends on the discount factor

β and the degree of altruism γ. When both are not equal, public debt is required to decentralize the first-best optimum. The first-best human capital growth decentralized with positive public debt (*i.e.* when $\frac{\gamma}{1+\gamma} > \beta$), is lower than the first-best one in which public debt is zero. Thus, in the following, we analyze the impact of a positive public debt constraint on the human capital growth and on the tax policy used. Then, we compare results with the ones at equilibrium.

1.4 Second-best optimum

The government adopts as a social criteria the discounted sum of generational consumption's utility (1.3.1) as in the previous Section. As stated before, we focus on situations where the first-best fiscal policy uses a positive public debt in order to decentralize the first best optimal solutions (*i.e.* when $\frac{\gamma}{1+\gamma} > \beta$). However, in this Section, public debt is not available. Thus, the social planner only uses the two tax instruments to maximize the households' welfare. The household's bequest is used only as private capital.

From equation (1.2.5b), we get: $c_t = \left[\gamma\left(1-\tau^B_{t+1}\right)R_{t+1}\right]^{-1}\Omega_{t+1}$. Then, using equation (1.2.15) and the capital market equilibrium (1.2.10), the individual's consumption corresponds to:

$$c_t = \frac{1}{\gamma}\left(1+\frac{1-\alpha}{\alpha}\phi_{t+1}\right)K_{t+1} \tag{1.4.1}$$

Expression (1.2.12) gives the education expenditure:

$$e_{t+1} = \delta\frac{1-\alpha}{\alpha}\phi_{t+1}K_{t+1} \tag{1.4.2}$$

Using the consumption level (1.4.1) and education expenditure (1.4.2), the resource constraint (1.2.11) corresponds to:

$$K_{t+1}\left[1+\frac{1}{\gamma}\left(1+\frac{1-\alpha}{\alpha}\phi_{t+1}\right)+\delta\frac{1-\alpha}{\alpha}\phi_{t+1}\right] = (1-\Gamma)\,F\left(K_t,H_t\right) \tag{1.4.3}$$

As a result, the social planner maximizes the social welfare function (1.3.1) with respect to $(\phi_{t+1}, H_{t+1}, K_{t+1})_{t\geq 0}$ subject to the resource constraint (1.4.3) and the human capital technology (1.2.1) as well as taking into account the agent's consumption level (1.4.1). Let us denote by λ_{t+1} and μ_{t+1}, the respective Lagrange multipliers of both constraints. Taking into account the Cobb-Douglas production function (1.2.7) and derivatives, we get the following optimality conditions:

— with respect to H_{t+1}, for $t \geq 0$,

$$-\frac{\mu_{t+1}}{\mu_{t+2}} + \beta \frac{\lambda_{t+2}}{\mu_{t+2}} (1-\alpha)(1-\Gamma) F(k_{t+1}, 1) + \beta (1-\delta) g_{t+2}^{H} = 0 \qquad (1.4.4)$$

— with respect to K_{t+1}, for $t \geq 0$, (using the optimality condition (1.4.6)),

$$-1 + \beta \frac{\lambda_{t+2}}{\lambda_{t+1}} \alpha (1-\Gamma) \frac{F(k_{t+1}, 1)}{k_{t+1}} + \left[\frac{1}{c_t \lambda_{t+1}} - 1\right] \frac{1}{\gamma} = 0 \qquad (1.4.5)$$

— with respect to ϕ_{t+1}, for $t \geq 0$,

$$\left[\frac{1}{c_t \lambda_{t+1}} - 1\right] \frac{1}{\gamma} + \left[\frac{\mu_{t+1}}{\lambda_{t+1}} \delta \frac{H_{t+1}}{e_{t+1}} - 1\right] \delta = 0 \qquad (1.4.6)$$

1.4.1 Second-best optimum along the balanced growth path

In this Subsection, we analyze the second-best fiscal policy along the balanced growth path where bequests are positive. We concentrate on situations where public debt is positively used when it is not constrained (i.e when $\frac{\gamma}{1+\gamma} > \beta$).

Proposition 5. *Let us assume $\frac{\gamma}{1+\gamma} > \beta$. Along the balanced growth path of the second-best optimum, we get:*

(i) $\phi^{**} > \widehat{\phi} > \phi^{*}$;

(ii) $k^{**} < k^{*}$ and $k^{**} < \widehat{k}$;

(ii) $g^{H*} < g^{H**} < \widehat{g}^{H}$.

where ϕ^{**} is the second-best ratio ϕ, k^{**} is the second best capital-labor ratio and g^{H**} is the second-best human capital growth rate.

Proof. At steady state, we have $g^{H} = \frac{\lambda_{t+1}}{\lambda_{t+2}} = \frac{\mu_{t+1}}{\mu_{t+2}}$ and we define $\Lambda = \frac{\lambda_{t+1}}{\mu_{t+1}}$ and

$M = \lambda_{t+1} K_{t+1}$. Thus, under the optimality conditions (1.4.5) and (1.4.6), we get:

$$\Lambda = \frac{\delta^2 \frac{\left(g^{H**}\right)}{\eta^{**}}}{\delta - 1 + \frac{\beta}{g^{H**}} \alpha \left(1 - \Gamma\right) \frac{F(k^{**},1)}{k^{**}}} \tag{1.4.7}$$

Plugging equation (1.4.7) into the optimality condition (1.4.4), we obtain the following relationship between the second-best values of g^{H**}, ϕ^{**} and k^{**}:

$$\frac{\delta}{\phi^{**}\left[1 - \beta\left(1 - \delta\right)\right]} + \left(1 - \delta\right) \frac{g^{H**}}{\beta\alpha\left(1 - \Gamma\right) \frac{F(k^{**},1)}{k^{**}}} = 1 \tag{1.4.8}$$

At a steady state, the resource constraint (1.4.3) can be rewritten as follows:

$$g^H k \left[1 + \gamma + \left(1 + \gamma\delta\right) \frac{1 - \alpha}{\alpha} \phi\right] = \gamma \left(1 - \Gamma\right) F\left(k, 1\right)$$

Plugging the last expression into (1.4.8), we get:

$$\Psi\left(\phi^{**}\right) = 1 \tag{1.4.9}$$

where

$$\Psi\left(\phi\right) = \frac{\delta\phi^*}{\phi} + \left(1 - \delta\right) \frac{\frac{\gamma}{1+\gamma}}{\beta\left[\alpha + \left(1 - \alpha\right) \frac{\phi}{\widehat{\phi}}\right]} \tag{1.4.10}$$

which is decreasing and convex relative to the ratio ϕ. As a reminder, $\widehat{\phi} = \frac{1+\gamma}{1+\gamma\delta}$, $\phi^* = \frac{1}{1-\beta(1-\delta)}$ and $\widehat{\phi} > \phi^* \Leftrightarrow \frac{\gamma}{1+\gamma} > \beta$. Then, from (1.4.10):

$$\begin{aligned}
\Psi\left(\widehat{\phi}\right) > 1 \quad &\Leftrightarrow \quad \delta\frac{\phi^*}{\widehat{\phi}} + \left(1 - \delta\right) \frac{\gamma}{\beta\left(1 + \gamma\right)} > 1 \\
&\Leftrightarrow \quad \left(1 - \delta\right) \left[\frac{\gamma}{\beta\left(1 + \gamma\right)} - 1\right] > \delta \left(1 - \frac{\phi^*}{\widehat{\phi}}\right)
\end{aligned}$$

where $\frac{\phi^*}{\widehat{\phi}} = \frac{1 - \frac{\gamma}{1+\gamma}(1-\delta)}{1-\beta(1-\delta)}$. Thus, we get:

$$\Psi\left(\widehat{\phi}\right) > 1 \Leftrightarrow \left[\frac{\gamma}{\beta\left(1 + \gamma\right)} - 1\right] \left[1 - \frac{\delta\beta}{\delta\beta + 1 - \beta}\right] > 0$$

which is verified iff $\frac{\gamma}{1+\gamma} > \beta$. Since the second-best ratio ϕ^{**} satisfies equation (1.4.9) and given the function $\Psi(\phi)$ is decreasing and convex, a necessary and sufficient condition for $\phi^{**} > \widehat{\phi} > \phi^{*}$ is $\frac{\gamma}{1+\gamma} > \beta$.

At the steady state, the first-best optimal capital-labor ratio (1.3.17) can be rewritten as follows:

$$\frac{B^{\frac{1}{1-\delta}}\left(k^{*}\right)^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta\alpha\left(1-\Gamma\right)A} = \left(\frac{\alpha}{\delta\left(1-\alpha\right)\phi^{*}}\right)^{\frac{\delta}{1-\delta}}$$

For the second-best capital-labor ratio k^{**}, plugging human capital growth at equilibrium (1.2.14) into the second-best optimal condition (1.4.8), we get:

$$\frac{B^{\frac{1}{1-\delta}}\left(k^{**}\right)^{\frac{1-\alpha+\delta\alpha}{1-\delta}}}{\beta\alpha\left(1-\Gamma\right)A} = \frac{1-\delta\frac{\phi^{*}}{\phi^{**}}}{1-\delta}\left(\frac{\alpha}{\delta\left(1-\alpha\right)\phi^{**}}\right)^{\frac{\delta}{1-\delta}}$$

From the last two equations, we obtain:

$$k^{**} \leq k^{*} \Leftrightarrow \frac{1-\delta\frac{\phi^{*}}{\phi^{**}}}{1-\delta}\left(\frac{\phi^{*}}{\phi^{**}}\right)^{\frac{\delta}{1-\delta}} \leq 1$$

which is always satisfied since $\phi^{**} > \phi^{*}$.

Given condition (1.4.8), the previous result, regarding $\phi^{**} > \phi^{*}$ with $\frac{\gamma}{1+\gamma} > \beta$, leads to the following inequality:

$$g^{H**} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{**},1\right)}{k^{**}}$$

Thus, assuming $\frac{\gamma}{1+\gamma} > \beta$ and given that $k^{**} < k^{*}$, we deduce that the second-best economic growth rate g^{H**} (given condition (1.4.8)) is higher than the first-best one (1.3.14):

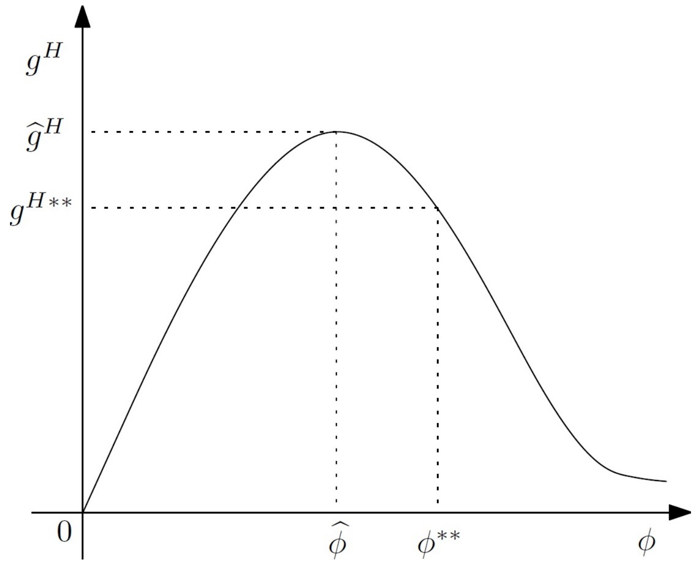
$$g^{H**} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{**},1\right)}{k^{**}} > \beta\alpha\left(1-\Gamma\right)\frac{F\left(k^{*},1\right)}{k^{*}} = g^{H*}$$

As stated before in Section 1.2.5, without public debt, the fiscal policy which satisfied $\phi = \widehat{\phi}$, leads to the highest human capital growth rate along the balanced growth path. Thus, $g^{H*} < g^{H**} < \widehat{g}^{H}$.

Lastly, given that $g^{H**} < \widehat{g}^{H}$ and $\phi^{**} > \widehat{\phi}$, we deduce from equation (1.2.14) that $k^{**} < \widehat{k}$ (since the human capital growth rate satisfies in both cases: $g^{H} =$

$B\left(\phi\frac{1-\alpha}{\alpha}kB\delta\right)^{\frac{\delta}{1-\delta}}$). This concludes the proof. □

Figure 1.2 – Human capital growth (1.2.22) along the balanced growth path of $X = 0$



Therefore, the social planner only uses both taxes to internalize the positive human capital externality, to finance public spending and to pursue a redistributive policy between generations. For these purposes, when $\frac{\gamma}{1+\gamma} > \beta$, the government adopts a second-best ratio ϕ^{**} which is higher than the one used to obtain the highest human capital growth rate without public debt $\widehat{\phi}$, as shown in Figure 1.2. Since, $\phi^{**} > \widehat{\phi}$, the agent's incentive to invest in education expenditure rather than bequest increases. This implies an over-investment in education and results in reducing the capital-labor ratio (*i.e.* $k^{**} < \widehat{k}$). Notice that the fall in capital-labor ratio has a positive effect on the first generation consumption. Since $k^{**} < k^*$, the government encourages agents to consume rather than invest in future generations by decreasing the capital-labor ratio using both tax instruments. However, the government cannot fully support its redistributive policy towards current generation (since $g^{H*} \neq g^{H**}$). Thus, the availability of public debt plays an essential role in optimizing the intergenerational family transfers between generations.

Therefore, the government implements a tax policy (*i.e.* $\phi = \phi^{**}$) which leads to a higher human capital growth g^{H**} than the first-best human capital growth g^{H*} but lower compared to the highest possible one $\widehat{g}^H$ along the balanced growth path.

1.4.2 Numerical illustration

We use a numerical example to show that optimal transition dynamics jumps to the balanced growth path and to analyze the optimal solutions given the availability of public debt. We focus on situation where achieving the first-best optimal solutions requires a positive public debt. As a result, we calibrate the optimal transition dynamics with $\frac{\gamma}{1+\gamma} > \beta$. For this purpose, the discount factor chosen should be quite low such that government cares more about the wellbeing of the present generations rather than that of the next one. This dynamics will result in a low β compared to the usual estimate given that γ cannot be too high. We assume $\gamma = 0.7$ such that households pay attention to their offspring. Table 1.1 presents value of all parameters.

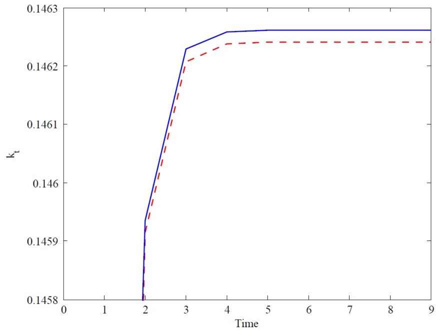
Table 1.1 – Base-case parameter value

Parameter		Value
Government		
Discount factor	β	0.4
Share of production devoted to public sector	Γ	0.1
Production function		
Technological parameter	A	3
Share parameter of physical capital	α	0.33
Representative individual		
Agent's human capital production function		
Technological parameter	B	3
Share parameter of education spending	δ	0.7
Taste		
Degree of altruism	γ	0.7

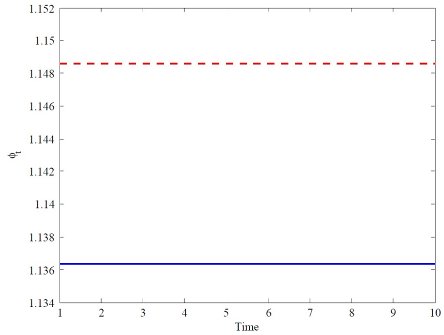
The results are reported in Figure 1.3. All the variables jump to the optimal balanced growth path in two periods. Because of optimal tax instruments, regardless of the availability of public debt, individuals have incentives to modify their family transfers toward the optimal choices.

Figure 1.3 – The optimal transition dynamics beginning in the first period

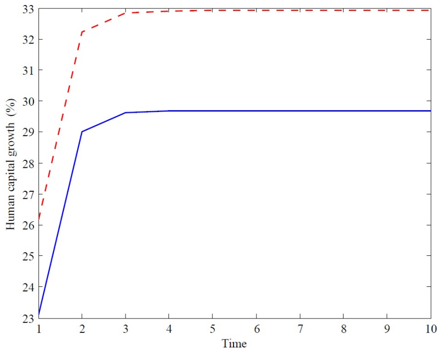
(a) Capital-labor ratio k_t



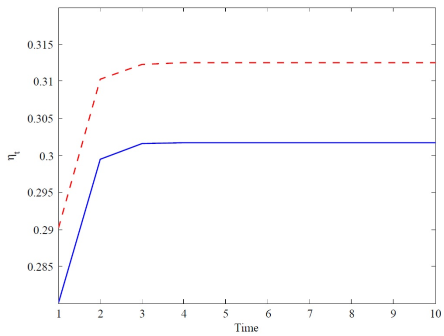
(b) Ratio ϕ_t



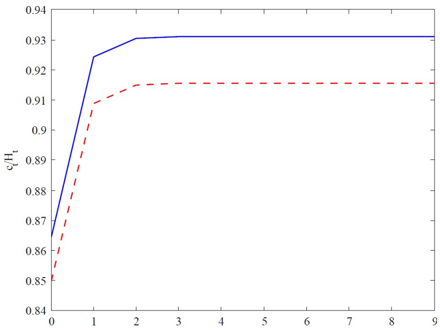
(c) Human capital growth rate g_t^H



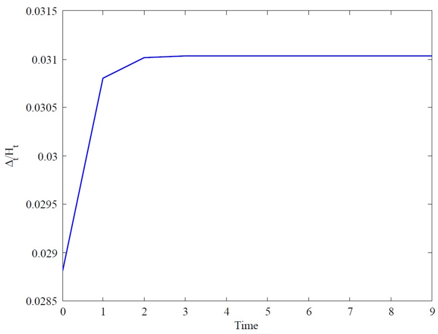
(d) Ratio η_t



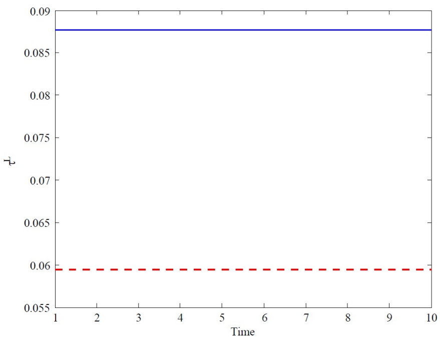
(e) Consumption per unit of human capital



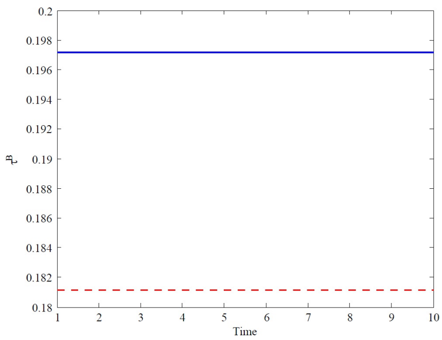
(f) Public debt per unit of human capital



(g) Labor income tax rate τ_t^L



(h) Bequest tax rate τ_t^B



Note: The first-best optimal solutions: bold line. The second best optimal solutions: dashed line.

Since the optimal capital-labor ratio is quite low in both cases (Figure 1.3(a)), this numerical illustration describes a knowledge economy that encourages agents to invest in child's education expenditure. In both situations, we get a human capital growth rate of approximately 30% per period along the balanced growth path (see Figure 1.3(c)). Assuming that a period lasts for approximately 30 years, the economic growth achieved corresponds to an annual growth rate of 1%. Then, Figures 1.3(g) and 1.3(h) illustrate the optimal tax instruments values. The labor income tax rate is really low in both cases whereas the bequest tax rate is approximately 20%. Concerning the effect of non-availability of public debt , this simulation illustrates the results obtained previously. When public debt is available, it is used to pursue the intergenerational redistributive policy and to internalize the positive externality (through supports on both taxes). Otherwise, the results given in Proposition 5 are satisfied. The absence of public debt involves higher growth levels of human capital (Figure 1.3(c)) and of consumption (Figure 1.3(e)). Indeed, the negative effect of increasing the ratio ϕ compared to the first-best one ϕ^* (Figure 1.3(b)) on the capital-labor ratio (Figure 1.3(a)) does not lead to the first best human capital growth (or equivalently the first-best consumption growth). As stated in Section 1.4.1, the second-best fiscal policy results in a over-investment in education expenditure (Figure 1.3(d)). Finally, the second-best tax rates are lower than the first-best ones since public debt can not be used to finance public spending.

Therefore, this numerical example describes the public debt availability issue correctly. It shows that the transitional dynamics jump to the balanced growth path and that both optimal tax rates are lower when public debt is not available.

1.5 Conclusion

The long-run optimal human capital growth that we are able to identify in this paper depends crucially on the availability of public debt. Public debt allows optimal distributions of intergenerational family transfers, through which the first-best optimal human capital growth is achieved. Because of a positive public debt, the government improves the consumption of the current generations without affecting the optimal human capital growth and uses taxes to internalize the positive human capital externality. When a positive public debt is not available, social planner cannot completely satisfy these objectives such that the two taxes do not fully implement the intergenerational redistribution policy. For this reason, the economic growth is higher than with public debt. Furthermore, the model reveals the necessity of public intervention to ensure that agents' decisions concerning their family transfers correspond to the optimal choices along the balanced growth path.

We focus on intergenerational redistribution policies. We do not analyze the effect of intragenerational inequality on the optimal human capital growth, which depends on the availability of public debt. These disparities across agents should modify the optimal fiscal policy used and the human capital growth along the balanced growth path. Additionally, we can analyze the optimal fiscal policy by relaxing the inelastic labor supply. Thus, studying the influence of the labor income tax rate on the household's labor supply and on transfer allocations is an interesting question to be explored in future investigations.

1.6 Appendix

1.6.1 Analysis of function ψ

As a reminder, $D = \frac{1}{\delta}\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha} + 1 + \gamma\delta\right) > 0$, $J = \delta\left(1-\tau^L\right)(1-\alpha) > 0$ and $E = \Gamma - \tau^B\alpha - \tau^L(1-\alpha)$. From equation (1.2.21a), we get the following first order derivative of $\psi(X)$:

$$\frac{DJ\gamma(1-\Gamma)}{(\gamma(1-\Gamma)-E-JX)^2} > 0 \tag{1.6.1}$$

and the following second order derivative:

$$\frac{D(3J)\gamma(1-\Gamma)}{(\gamma(1-\Gamma)-E-JX)^3} > 0 \text{ iff } JX+E < \gamma(1-\Gamma)$$

The condition $JX + E < \gamma(1-\Gamma)$ is always satisfied as soon as we focus on positive human capital growth g^H(see equation (1.2.14)). Under this condition, the function $\psi(X)$ is strictly increasing and strictly convex.

1.6.2 Steady states of X with $E = 0$

Using equation (1.2.21a), the steady states of X depend on the roots of the following polynomial of degree two:

$$P(X) = JX^2 - [E + JD - \gamma(1-\Gamma)]X + DE = 0$$

A sufficient condition to get two opposite sign roots for the polynomial P is that $E = 0$, (*i.e.* $\Gamma = \tau^B\alpha + \tau^L(1-\alpha)$). In this case, the polynomial P has two real roots, which are 0 and $\frac{\gamma(1-\Gamma)}{J} - D$. The sign of the second root is negative iff:

$$\begin{aligned}\frac{\gamma(1-\Gamma)}{J} - D < 0 \;\Leftrightarrow\; & \frac{\left(1-\tau^L\right)(1-\alpha)\left(\frac{1+\gamma}{\phi}\frac{\alpha}{1-\alpha}+1+\gamma\delta\right)}{\gamma(1-\Gamma)} > 1\\ \Leftrightarrow\; & \alpha\frac{1-\tau^B}{1-\Gamma} + \frac{1+\gamma\delta}{1+\gamma}(1-\alpha)\frac{1-\tau^L}{1-\Gamma} > \frac{\gamma}{1+\gamma}\end{aligned}$$

From the government budget constraint (1.2.9), we get:

$$\begin{aligned}
E = 0 \quad &\Leftrightarrow \quad 1 - \Gamma = \alpha\left(1 - \tau^B\right) + (1 - \alpha)\left(1 - \tau^L\right) \\
&\Leftrightarrow \quad \frac{1 - \tau^B}{1 - \Gamma} = \frac{1}{\alpha}\left[1 - (1 - \alpha)\left(\frac{1 - \tau^L}{1 - \Gamma}\right)\right] \\
&\Leftrightarrow \quad \phi = \frac{\alpha}{\frac{1-\Gamma}{1-\tau^L} - (1 - \alpha)} \\
&\Leftrightarrow \quad \frac{1 - \Gamma}{1 - \tau^L} = \frac{\alpha}{\phi} + 1 - \alpha = \frac{1}{\phi}\left(\frac{1 - \Gamma}{1 - \tau^B}\right)
\end{aligned}$$

Then, using the last equation, we deduce the sign of the second root:

$$\begin{aligned}
\frac{\gamma(1 - \Gamma)}{J} - D < 0 \quad &\Leftrightarrow \quad \alpha \frac{\frac{1}{\phi}}{\frac{\alpha}{\phi} + 1 - \alpha} + \left(\frac{1 + \gamma\delta}{1 + \gamma}\right) \frac{1 - \alpha}{\frac{\alpha}{\phi} + 1 - \alpha} > \frac{\gamma}{1 + \gamma} \\
&\Leftrightarrow \quad \frac{\alpha}{\phi} + (1 - \alpha)\frac{1 + \gamma\delta}{1 + \gamma} > \frac{\gamma}{1 + \gamma}\left[\frac{\alpha}{\phi} + 1 - \alpha\right] \\
&\Leftrightarrow \quad \frac{\alpha}{\phi} > (1 - \alpha)\left[\gamma(1 - \delta) - 1\right]
\end{aligned}$$

which is always satisfied. Thus, the sign of $\tilde{X} \equiv \frac{\gamma(1-\Gamma)}{J} - D$ is negative. Since $\psi(X)$ is strictly convex and one real root is strictly negative with $E = 0$, $\tilde{X}$ is a locally stable negative balanced growth path of the dynamics of X_t. The other steady state $X = 0$ is unstable. Then, the stable set of $\tilde{X}$ is $\mathbb{R}_-^*$. If $X_0 > 0$, X_t goes beyond the upper bond for positive human capital growth in finite time which leads the following period to zero production.

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Chapter 2

Inheritance taxation in a model with intergenerational time transfers

joint work with Pascal Belan.

Abstract [1]

We consider a two-period overlapping generation model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. Starting from a steady state where public spendings are financed through taxation on capital income and labor income, we analyze a tax reform that consists in a shift of the tax burden from capital income tax towards inheritance tax. In the standard Barro model with no time transfer and inelastic labor supply, such a policy decreases steady-state welfare. In our setting, inheritance tax modifies parent's trade-off between time transfers and bequests. We identify situations where the tax reform increases welfare for all generations. Welfare improvement mainly depends on the magnitude of the effect of higher time transfers on the labor supply of the young.

Keywords:family transfers, altruism, time transfers, inheritance tax.

JEL Classifications: D64, H22, H24, J22.

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2.1 Introduction

Inheritance taxation is a controversial subject in the public policy debate and among economists. Currently, an increasing number of countries have no inheritance tax or have significantly reduced it (United States and United Kingdom). For the opponents, the inheritance tax discourages capital accumulation, and it is an immoral tax which increases the pain suffered by mourning families. They claim that tax on bequest involves "double taxation" of income that has already been taxed. A second line of argument suggests that if people have a long enough horizon (through altruistic behavior), inheritance taxation is inefficient. This point relies on the well known result of Chamley (1986) and Judd (1985) who show that capital income taxation should be zero in the long run. In dynastic interpretation of the infinite-lived agent model, this implies that inheritance taxation should also be zero (see Chamley (1986, p. 613)).

Over the past few years, an extensive literature has shown that we can overturn the Chamley-Judd result of zero capital income (inheritance) taxation by relaxing some of their hypotheses.[2] Moreover, the previous theoretical literature about inheritance taxation has essentially focused on financial bequests as the single source of intergenerational transfers within family. Nevertheless, a number of empirical studies suggest that time transfers from parents to their children are substantial and on average almost as important as monetary transfers (Cardia and Ng (2003) and Schoeni et al. (1997)).[3] Some studies based on the Survey of Health, Ageing and Retirement in Europe (SHARE)[4], such as Attias-Donfut et al. (2005) or also Albertini et al. (2007), show that parent's time transfers to children consists mainly in childcare. According to Wolff and Attias-Donfut (2007), two-fifth of grandparents

2. A non-zero bequest tax result is potentially achieved by assuming other bequest motives (Cremer and Pestieau, 2011) or for example, focusing on a model with heterogeneous random tastes for bequest and for wealth per se (Piketty and Saez, 2013) or, lastly, considering imperfect competition on capital market (Farhi and Werning, 2010).

3. For example Cardia and Ng (2003, Table 1) uses the Health and Retirement Study of 1992 and report that the mean time transfer for total sample (7547 households) of 325 hours has a value of $1950 (using a time cost of $6 per hour), which is similar of the sample mean of $1868 for monetary transfers.

4. The SHARE Survey is conducted since 2004 in ten Western European countries.

keep their grandchildren every week. A common finding is that grandparents still support parents' home production with household tasks for instance.

Thanks to the intergenerational transfers of time in the form of grandparenting, parents free up more time for working and taking care of their children. Labor supply of the heirs as well as life cycle resources are affected differently by time transfers compared to inheritances as shown by Cardia and Michel (2004) or Belan et al. (2010). Taking into account time transfers allows to deal with the trade-off between both types of transfers. Time transfers have some macroeconomic implications through labor supply of the next generation, while bequests enhance its private wealth (Cardia and Ng, 2003).

Despite their importance and macroeconomic implications, the theoretical literature about fiscal incidence of inheritance tax has not devoted attention to the trade-off between giving time or giving money. Whether or not time transfers are introduced, inheritance taxation reduces the incentive to leave resources to the next generation. Taking account of time transfers adds a substitution effect since inheritance tax affects the trade-off between monetary and time transfers, making time transfers more attractive. From this point of view, taxing bequests may enhance the young's labor supply, giving room to an increase in resources disposable for market good consumption. Nevertheless, the positive effect on labor supply has to be balanced with the potential reduction in private wealth that may be detrimental for capital accumulation. At least, with inheritance tax, capital-labor ratio could be lower, moving the economy away from the Golden-rule of capital accumulation.

In this paper, considering time transfers in a second-best world, we analyze whether shifting from capital income tax towards inheritance tax may be a welfare-improving tax reform. The fall in capital income taxation may compensate the negative effects of inheritance tax on savings, capital accumulation and capital-labor ratio. But simultaneously, the reform may increase steady-state resources since the inheritance tax has a positive effect on labor supply. To analyze and disentangle the above effects, we consider a two-period overlapping generation model with rational altruism taking into account both types of family transfers (inheritance and time transfers) from grandparents to parents. Individuals work when young (*i.e.* parent) and then retire in their second period of life (*i.e.* when they are grandparents). In each period, every household consumes a composite good that aggregates market good and home production. Parent's labor supply decision depends on the trade-off between formal work and home production. Then, grandparents contribute to home production of the parents through both family transfers. Furthermore, the government finances public spending using taxation on inheritance, capital income and labor income.

As in the standard Barro model with rational altruism, inheritance tax decreases the accumulated capital stock and thus, reduces the capital-labor ratio at steady state. But the fall in capital income tax allows to neutralize this steady-state effect. Assuming that the tax reform is designed in order to leave the steady-state capital-labor ratio constant, we identify situations where life-cycle utility increases. First, steady-state utility is likely to increase when the substitution effect between consumption of market good and time devoted to home production is strong. Indeed, in this case, inheritance tax makes time transfers more attractive. Grandparents prefer to leave higher time transfers and lower bequests. The higher the substitution effect, the higher the increase in labor supply of the parents. Secondly, even if the substitution effect is not too strong, the tax reform may have a positive effect on steady-state utility through the size of the additional production of market goods generated by the increase in labor supply. We show that the strength of the latter effect depends crucially on the gaps between the marginal rates of transformation and the marginal rates of substitution between consumption in market goods and time devoted to home production. The reform is likely to increase utility if lower time devoted to home production allows the production sector to generate more market goods than necessary for leaving individual with the same level of utility. However, keeping the steady-state capital-labor ratio constant may shift the burden of the initial public debt towards the first generations and introduces some intergenerational redistributions. Using a numerical example, we illustrate that the effect of the tax reform on household's welfare of each generation can be positive along the transitional dynamics.

The paper is organized as follows. In Section 2.2, the model is presented. Section 2.3 analyzes the steady-state equilibrium with operative bequests and positive time transfers. Then, in Section 2.4, we present the tax reform and study its effects on households' utility in steady state. In Section 2.5, we conduct numerical illustrations in order to study the impact of the tax reform on the whole transitional dynamics. The final section concludes.

2.2 The model

We consider an overlapping generation model. Time is discrete. Population consists in one dynasty where the representative household of generation t lives two periods and has one child, born in $t+1$. We consider dynastic altruism *à la* Barro (1974) from parents to children.

2.2.1 Households

The representative household of generation t works during his first period of life (*i.e.* when young, or equivalently parent) and then retires (*i.e.* when old, or equivalently grandparent). Labor supply when young is elastic and depends on the allocation of a unit-time endowment between formal work and home production. In both periods, the household consumes a composite good that aggregates market good and home production. Life-cycle utility writes

$$u\left(f^y\left(c_t, T_t^y\right)\right) + v\left(f^o\left(d_{t+1}, T_{t+1}^o\right)\right)$$

where u and v are increasing and strictly concave. Function $f^y\left(c_t, T_t^y\right)$, respectively $f^o\left(d_{t+1}, T_{t+1}^o\right)$, is the quantity of composite good when young, resp. when old. The former is obtained with market good expenditures c_t and time devoted to home production T_t^y. In the latter, d_{t+1} represents market good expenditures when old, while T_{t+1}^o is time spent in home production. Home production functions f^y et f^o are assumed to be linear homogenous and concave. Marginal products are strictly positive and strictly decreasing.

Let ℓ_t denotes labor supply of the young in the formal sector. Household's decision for labor supply results from the trade-off between participation in the formal sector or to home production. Time devoted to home production when young, T_t^y, aggregates time spent by the household for its home production $1-\ell_t$ and time transfer from its parent (denoted by λ_t):

$$T_t^y = 1 - \ell_t + \mu\lambda_t \tag{2.2.1}$$

where $\mu > 0$ represents the relative efficiency of time transfer of the parent. Since the parent is retired, time spent in home production when old is the fraction of the unit-time endowment that is not transferred to his offspring

$$T_t^o = 1 - \lambda_t \tag{2.2.2}$$

In the following, τ_t^w, τ_t^x and τ_t^R are the respective period-t tax rates on wages, bequests and capital income. R_t and w_t denote the gross interest rate and the wage rate. When young, a household born in t receives after-tax wage income $(1-\tau_t^w)\, w_t\ell_t$ and after-tax bequest $(1-\tau_t^x)\, x_t$. These resources are allocated between consumption spendings c_t and saving s_t:

$$c_t + s_t = (1-\tau_t^w)\, w_t\ell_t + (1-\tau_t^x)\, x_t \tag{2.2.3}$$

When old, the household allocates after-tax capital income $\left(1-\tau_{t+1}^R\right) R_{t+1}s_t$ between consumption spendings d_{t+1} and bequest x_{t+1}:

$$d_{t+1} + x_{t+1} = \left(1-\tau_{t+1}^R\right) R_{t+1}s_t \tag{2.2.4}$$

Following Barro (1974), rational altruism means that households enjoy utility of their children. Utility of the household born in t, U_t, depends on consumptions in composite goods in both periods and utility of its offspring U_{t+1}:

$$U_t = u\left(f^y\left(c_t, T_t^y\right)\right) + v\left(f^o\left(d_{t+1}, T_{t+1}^o\right)\right) + \beta U_{t+1} \tag{2.2.5}$$

where β denotes the degree of altruism, $0 < \beta < 1$.

Using equations (2.2.1)-(2.2.4), both consumptions in market goods rewrite

$$c_t = \left(1 - \tau_t^w\right) w_t \left[1 - T_t^y + \mu\left(1 - T_t^o\right)\right] + \left(1 - \tau_t^x\right) x_t - s_t \tag{2.2.6}$$

$$d_{t+1} = \left(1 - \tau_{t+1}^R\right) R_{t+1} s_t - x_{t+1} \tag{2.2.7}$$

Plugging (2.2.6)-(2.2.7) into U_t gives household's utility as a function of s_t, x_{t+1}, T_t^y and T_{t+1}^o. The representative household maximizes U_t with respect to these four variables. For an interior solution, this leads to the following first-order conditions:

— with respect to s_t

$$-u_t' f_{c_t}^y + \left(1 - \tau_{t+1}^R\right) R_{t+1} v_{t+1}' f_{d_{t+1}}^o = 0 \tag{2.2.8}$$

where u_t', $f_{c_t}^y$, v_{t+1}' and $f_{d_{t+1}}^o$ respectively stand for the partial derivatives $\frac{\partial u_t}{\partial f_t^y}$, $\frac{\partial f_t^y}{\partial c_t}$, $\frac{\partial v_{t+1}}{\partial f_{t+1}^o}$ and $\frac{\partial f_{t+1}^o}{\partial d_{t+1}}$.

— with respect to T_t^y

$$-\left(1 - \tau_t^w\right) w_t f_{c_t}^y + f_{T_t^y}^y = 0, \text{ if } 0 < T_t^y < 1 + \mu\left(1 - T_t^o\right) \tag{2.2.9}$$

where $f_{T_t^y}^y$ stands for $\frac{\partial f_t^y}{\partial T_t^y}$.

— with respect to x_{t+1}

$$-v_{t+1}' f_{d_{t+1}}^o + \beta\left(1 - \tau_{t+1}^x\right) u_{t+1}' f_{c_{t+1}}^y = 0, \text{ if } x_{t+1} > 0 \tag{2.2.10}$$

— with respect to T_{t+1}^o

$$v_{t+1}' f_{T_{t+1}^o}^o - \beta\mu\left(1 - \tau_{t+1}^w\right) w_{t+1} u_{t+1}' f_{c_{t+1}}^y = 0, \text{ if } 0 < T_{t+1}^o < 1 \tag{2.2.11}$$

where $f^o_{T^o_{t+1}}$ stands for $\frac{\partial f^o_{t+1}}{\partial T^o_{t+1}}$.

Constraints for an interior solution are not necessarily satisfied at equilibrium. The less critical one is the constraint $T^y_t < 1 + \mu(1 - T^o_t)$, which is equivalent to positive labor supply ($\ell_t > 0$). Assuming that it is satisfied remains to consider equilibria where the production sector uses labor. Two other constraints should be satisfied with small additional assumptions: $T^y_t \geq 0$ and $T^o_{t+1} \geq 0$. Time spent in home production remains positive if substitutability with market goods is not too strong.

Finally, non-negativity constraints on bequests and time transfers deserve some discussion. It depends on the utility gains that parents may expect with both kinds of transfers. As shown by Weil (1987), in the standard Barro framework without time transfers, positive bequests are obtained at steady state, if the steady-state capital-labor ratio in the corresponding Diamond economy is below the modified Golden-rule. With time transfers, assuming logarithmic utility and Cobb-Douglas technology, Cardia and Michel (2004) have given conditions for the existence of intertemporal equilibria where both transfers are positive. They also state conditions for the case with zero bequest and positive time transfers. In the following, since our concern is the effect of inheritance tax on bequests and time transfers, we focus on a steady state where both transfers are positive.

2.2.2 Equilibrium

The production sector consists in a representative firm that behaves competitively, and produces output with labor and capital. The production function $F(k, \ell)$ is linear homogenous and concave, and includes capital depreciation. Marginal products are strictly positive and strictly decreasing. Profit maximization leads to the standard equality between factor prices and marginal products

$$w_t = F_L(k_t, \ell_t) \tag{2.2.12}$$

$$R_t = F_K(k_t, \ell_t) \tag{2.2.13}$$

where k_t is capital stock. F_L and F_K stand for the partial derivatives of F with respect to labor and capital.

At equilibrium, household savings s_t split into private capital that will be used in $t+1$ and public debt Δ_t

$$k_{t+1} + \Delta_t = s_t$$

In each period, government spendings amount to a fraction Γ of total production. Government resources come from taxation on labor income, capital income and bequests. Then, public debt accumulates according to the following law of motion:

$$\Delta_t = R_t \Delta_{t-1} + \Gamma F(k_t, \ell_t) - \tau_t^w \ell_t w_t - \tau_t^R R_t s_{t-1} - \tau_t^x x_t \tag{2.2.14}$$

where the initial public debt $\Delta_{-1} = \bar{\Delta}_{-1}$ is given.

The assumption that government spendings are proportional to production leads to an externality created by the level of production. Indeed, consider the resource constraint in period t

$$c_t + d_t + k_{t+1} = (1 - \Gamma) F(k_t, \ell_t) \tag{2.2.15}$$

By increasing capital (resp. labor) used in production, the social marginal product for consumption and investment is $(1 - \Gamma) F_K(k_t, \ell_t)$ (resp. $(1 - \Gamma) F_L(k_t, \ell_t)$), while the private marginal product are higher, equal to $F_K(k_t, \ell_t)$ (resp. $F_L(k_t, \ell_t)$) as stated by the first-order condition of the representative firm (2.2.12) and (2.2.13). Such externalities are internalized by the private sector if the government sets the tax rates on capital and labor incomes to $\tau_t^R = \tau_t^w = \Gamma$.

2.3 Steady state with positive transfers

We consider steady states with operative bequests and positive time transfers. Tax rates and public debt are assumed to be constant over time. From the marginal conditions (2.2.8) and (2.2.10), the gross interest rate satisfies the modified Golden-rule, and is equal to R_M defined as

$$\beta\left(1-\tau^x\right)\left(1-\tau^R\right)R_M=1 \tag{2.3.1}$$

which characterizes the capital-labor ratio $k/\ell = z_M$ and the wage rate $w_M = F_L\left(z_M,1\right)$.

From equations (2.2.8), (2.2.9) and (2.2.11), the other marginal conditions of the household problem can be rewritten as equalities between marginal rates of substitution and relative prices:

$$\frac{v' f_d^o}{u' f_c^y}=\beta\left(1-\tau^x\right)\equiv P^R \tag{2.3.2}$$

$$\frac{f_{T^y}^y}{f_c^y}=\left(1-\tau^w\right)w_M\equiv P^y \tag{2.3.3}$$

$$\frac{f_{T^o}^o}{f_d^o}=\mu\frac{\left(1-\tau^w\right)w_M}{1-\tau^x}\equiv P^o \tag{2.3.4}$$

where P^R is the relative price between market good consumption when old d and market good consumption when young c, and P^y (resp. P^o) is the relative price between time devoted to home production and market good consumption when young (resp. when old).

Time constraint when young (2.2.1) gives the household's labor supply

$$\ell=1-T^y+\mu\left(1-T^o\right). \tag{2.3.5}$$

Then, the resource constraint (2.2.15) becomes

$$c+d=C_M\left[1-T^y+\mu\left(1-T^o\right)\right] \tag{2.3.6}$$

where C_M denotes aggregate consumption per labor unit

$$C_M\equiv\left(1-\Gamma\right)F\left(z_M,1\right)-z_M$$

Consequently, for given tax rates $\left(\tau^w, \tau^R, \tau^x\right)$, marginal conditions (2.3.2)-(2.3.4) and the resource constraint (2.3.6) characterize household's choice at steady-state equilibrium for consumptions in market good, c and d, and times devoted to home production, T^y and T^o.

The household's intertemporal budget constraint (obtained by eliminating s_t from equations (2.2.6) and (2.2.7)) allows to compute steady-state bequest. Indeed, using the time constraint (2.3.5) and the relation between relative prices $P^R P^o = \beta\mu P^y$ (deduced from their definitions in (2.3.2)-(2.3.4)), the intertemporal budget constraint rewrites

$$c + P^y T^y + P^R \left(d + \beta^{-1} P^o T^o\right) = P^y \left(1+\mu\right) + \left(1-\tau^x\right)\left(1-\beta\right) x$$

Bequests are positive if the present value of market good spendings $c + P^R d$ is higher than net wage income $\left(1-\tau^w\right) w_M \ell$.

Finally, public debt is deduced from the budget constraint of the government

$$\Delta = \left(\left(1-\tau^R\right) R_M - 1\right)^{-1} \left(\tau^x x + \left[\tau^w w_M + \tau^R R_M z_M - \Gamma F\left(z_M, 1\right)\right] \ell\right) \tag{2.3.7}$$

For instance, if all tax rates are zero, steady-state public debt is negative, equal to $-\left(R_M - 1\right)^{-1} \Gamma F\left(z_M, 1\right) \ell$. This means that, at each period, the government uses interests on public capital to finance public spendings $\Gamma F\left(z_M, 1\right) \ell$. Of course, this is possible either if there is initial public capital that finances the whole sequence of public spendings, or if the government has taxed households in order to accumulate some public capital to this end.

As stated before, the case where capital and labor incomes are taxed at the same rate $\tau^R = \tau^w = \Gamma$ allows to eliminate the externality created by public spendings. Since F is linear homogenous, these taxes would then finance the whole current public spendings. Additionally, if the initial debt $\bar{\Delta}_{-1}$ is zero, a first-best optimum is obtained with a zero inheritance tax.

2.4 Fiscal reform

In the following, we assume that the government cannot set the tax rates, τ^w and τ^R, at Γ in order to eliminate the public spending externality. This can result, for instance, from the burden of a positive public debt that has to be distributed among generations through additional taxation. In this regard, before the tax reform, the government finances the debt burden using capital income tax at rate $\bar{\tau}^R > \Gamma$ constant over time, with $\bar{\tau}^w = \Gamma$ and $\bar{\tau}^x = 0$. Higher capital income tax rate distorts in household's saving decision, leading to a lower capital-labor ratio than the one obtained at a first-best optimum.

The issue we address is whether a tax shift from capital income tax towards inheritance tax would be welfare enhancing. The tax reform consists to set up a positive inheritance tax rate $\tau^x > 0$ and reduces the capital income tax τ^R in order to make it closer to Γ.

In overlapping-generation models with rational altruism, capital income is divided between second-period consumption and inheritance (see the second-period budget constraint (2.2.4) of the representative household). This implies that inheritance is lower than capital income. Therefore, if the government tries to keep the primary surplus (fiscal receipts minus public spendings) constant, it needs to increase inheritance tax rate by a larger amount than the fall in the capital income tax rate. This means that the product $(1 - \tau^x)\left(1 - \tau^R\right)$ decreases and becomes lower than $\left(1 - \bar{\tau}^R\right)$, leading to a fall in the capital-labor ratio (see equation (2.3.1)), moving the economy away from the Golden-rule.

In the following, we conduct the analysis by first assuming that the fiscal reform is designed in order to keep the capital-labor ratio constant. Therefore, the shift from capital income to inheritance tax is such that

$$(1 - \tau^x)\left(1 - \tau^R\right) = 1 - \bar{\tau}^R$$

This implies a proportional change in both tax rates. Consequently, the fiscal reform decreases the steady-state primary surplus and reduces the steady-state public debt (see equation (2.3.7)). The tax reform then shifts the burden of the initial debt toward the first generations. Otherwise stated, it introduces an intergenerational redistribution of resources from the first generations towards the ones far in the future.

At this stage, we will focus on the effect of the reform on steady-state life-cycle utility:

$$V = u\left(f^{y}\left(c, T^{y}\right)\right) + v\left(f^{o}\left(d, T^{o}\right)\right) \tag{2.4.1}$$

and postpone the issue of intergenerational redistribution to the next section, through numerical illustrations.

The rest of the section decomposes the marginal effect of the tax reform on steady-state household's life cycle utility in different settings. We start from the Barro model, assuming inelastic labor supply and no time transfer. We then extend the discussion to elastic labor supply, still without time transfer. Finally, we consider the complete framework with elastic labor supply, assuming that both private intergenerational transfers are positive.

2.4.1 Tax reform without time transfer at steady state

To decompose the different effects of a tax reform, we first analyze a shift from capital income taxation toward inheritance taxation (leaving constant the capital-labor ratio) in an economy where time transfers are inoperative. We thus leave aside the fact that inheritance taxation modifies the trade-off between both parental transfers. At a steady-state equilibrium with positive bequests and zero time transfer, *i.e.* $x > 0$ and $T^{o} = 1$, the capital-labor ratio, the gross interest rate and the wage rate are at their modified Golden-rule levels. Market good consumptions (c and d) and time spent to home production when young T^{y} are characterized by marginal conditions (2.3.2), (2.3.3) and the resource constraint (2.3.6) and may change with the tax reform implemented. Thus, the marginal changes in τ^{x} reduces the relative price P^{R} and then modifies the household's intertemporal allocation of resources between consumptions in market good when young and old. The magnitude of the effect crucially depends on the elasticity of substitution between the composite goods f^y and f^o. Let us denote by σ^u, the absolute value of this intertemporal elasticity of substitution. Then

$$\frac{df^y}{f^y} - \frac{df^o}{f^o} = \sigma^u \frac{d\left(\frac{f_c^y P^R}{f_d^o}\right)}{\frac{f_c^y P^R}{f_d^o}} = \sigma^u \left(\frac{df_c^y}{f_c^y} - \frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R}\right) \tag{2.4.2}$$

Tax reform in the standard Barro model

We first show that the tax reform in the standard Barro (1974) model with inelastic labor supply ($T^y = 1$) has a negative effect on household's welfare.

Proposition 6. *At a steady-state equilibrium with no time transfer and inelastic labor supply, consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first-period consumption in the market good c decreases, while the second-period consumption d increases. Moreover, steady-state life-cycle utility (2.4.1) decreases.*

Proof. Differentiating steady-state life-cycle utility $V = u\left(f^y\left(c,1\right)\right) + v\left(f^o\left(d,1\right)\right)$, and using marginal condition (2.3.2), dV has the same sign as

$$dc + P^R dd$$

Moreover, differentiating the resource constraint (2.3.6), one gets

$$c\frac{dc}{c} + d\frac{dd}{d} = 0 \tag{2.4.3}$$

Thus dV has the same sign as

$$\left(P^R - 1\right)\frac{dd}{d}$$

We now need to state the sign of dd. Let us define the shares of market good cost in the total cost of production of the composite good for the young $\alpha^y \equiv f_c^y c/f^y$ and the old $\alpha^o \equiv f_d^o d/f^o$. Equation (2.4.2) then rewrites as

$$\alpha^y \frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(\frac{f^y_{cc}(c,1)\, c}{f^y_c(c,1)} \frac{dc}{c} - \frac{f^o_{dd}(d,1)\, d}{f^o_d(d,1)} \frac{dd}{d} + \frac{dP^R}{P^R} \right)$$

using the following relations:

$$\frac{df^y_c}{f^y_c} = \frac{f^y_{cc}(c,1)\, c}{f^y_c(c,1)} \frac{dc}{c} \text{ and } \frac{df^o_d}{f^o_d} = \frac{f^o_{dd}(d,1)\, d}{f^o_d(d,1)} \frac{dd}{d}$$

$$\frac{df^y}{f^y} = \alpha^y \frac{dc}{c} \text{ and } \frac{df^o}{f^o} = \alpha^o \frac{dd}{d}$$

Then, from equation (2.4.3), one easily checks that dd has an opposite sign to dP^R. Since the tax reform considered implies a fall in $P^R = \beta(1-\tau^x)$, one gets $dV < 0$, which concludes the proof. □

The fall in the relative price between both intertemporal market good consumptions P^R increases the market good consumption when old d and pushes down the market good consumption when young c. Both effects are stronger when the substitutability between composite goods is important (*i.e.* high σ^u). In addition, from equation (2.4.3), the marginal rate of transformation between d and c ($MRT_{d/c}$) is equal to one. As the marginal rate of substitution between d and c ($MRS_{d/c} = P^R$) is lower than the $MRT_{d/c}$ and declines with the tax reform, household's welfare is negatively affected by the reform.

Tax reform with elastic labor supply

Extending the model to elastic labor supply when young ($T^y \leq 1$) modifies the effect of the tax reform, introducing labor supply effects. From equation (2.3.3), since home production functions are linear homogeneous, one deduces that the ratio c/T^y can be written as a function of P^y: $c/T^y = \phi^y(P^y)$. Since the tax reform does not modify the relative price P^y, market good consumption c varies in the same proportion as time devoted to home production T^y. Then any reallocation of resources from c to d is associated with a reduction in T^y by the same percentage as the reduction in c. One gets the following result.

Proposition 7. *At a steady-state equilibrium with no time transfer, let us consider a switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, first period consumption in the market good* (c) *and time spent in home production* (T^y) *decrease, while the second period consumption* (d) *increases. Moreover, steady-state utility increases iff*

$$C_M - \frac{P^y}{P^R} > \phi^y\left(P^y\right)\left(\frac{1}{P^R} - 1\right). \tag{2.4.4}$$

Proof. Since the home production function when young f^y is linear homogenous and $dP^y = 0$, we deduce from (2.3.3) that

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

Then, equation (2.4.2) rewrites as

$$\frac{dc}{c} - \alpha^o \frac{dd}{d} = \sigma^u \left(-\frac{f_{dd}^o\left(d,1\right)d}{f_d^o\left(d,1\right)}\frac{dd}{d} + \frac{dP^R}{P^R}\right)$$

Differentiating the resource constraint (2.3.6), one gets

$$\left(c + C_M T^y\right)\frac{dc}{c} = -d\frac{dd}{d} \tag{2.4.5}$$

Thus, straightforward computations lead to

$$\left[\frac{-f_{dd}^o\left(d,1\right)d}{f_d^o\left(d,1\right)}\sigma^u + \frac{d}{c + C_M T^y} + \alpha^o\right]\frac{dd}{d} = -\sigma^u \frac{dP^R}{P^R}$$

which shows that the sign of dd is opposite to dP^R, while dc and dT^y have the same sign as dP^R. Moreover, the sign of dV is the same as

$$dc + P^y dT^y + P^R dd = \left(c + P^y T^y\right)\frac{dc}{c} + P^R d\frac{dd}{d}$$

Using equation (2.4.5), $dV > 0$ is equivalent to condition (2.4.4), since the tax reform considered implies a fall in $P^R = \beta\left(1 - \tau^x\right)$. □

To interpret results in Proposition 7, recall that the tax reform consists in a fall in second-period consumption price P^R that increases d and reduces c and T^y. The fall in T^y improves total resources for market good consumption $C_M(1 - T^y)$ through the increase in labor supply. The positive effect of the tax reform on labor supply attenuates or reverses the Barro-model effect on utility highlighted in Proposition 6. Notice that the increase in labor supply should be stronger when the substitutability between both periods is important (*i.e.* high σ^u).

Since the capital-labor ratio is kept constant, the increase in labor supply is associated with an increase in the capital stock, and thus in savings. The young work more, consume less and then save more for their second period of life.

The consumption per additional labor unit C_M corresponds to the marginal rate of transformation between T^y and d, while P^y/P^R corresponds to the marginal rate of substitution between both variables. Then, $C_M > P^y/P^R$ means that the fall in T^y allows to produce more market goods for second-period consumption than the amount necessary to preserve the same welfare.

The condition $C_M > P^y/P^R$ is sufficient to guarantee welfare improvement if $P^R > 1$. But, with the initial values of the instruments that we consider ($\tau^w = \Gamma$, $\tau^x = 0$ and $\bar{\tau}^R > \Gamma$), the relative price P^R is equal to β, and is lower than 1. In this case, the condition $C_M > P^y/P^R$ is no longer sufficient: welfare increases if the ratio ϕ^y is small enough. Indeed, a low ϕ^y corresponds to a situation where the first-period market good consumption c is relatively small to T^y. Thus, the proportional reduction of c and T^y leads to a small reduction in c (small negative effect on welfare) and a sharp increase in labor supply.

In a country where people consume a large (resp. small) amount of market goods, the ratio ϕ^y would be high (resp. low) and then the tax reform would be detrimental for welfare (resp. welfare enhancing). The situation where consumption relies essentially on market goods can be associated with a developped country. By contrast, in a developing country, time devoted to home production becomes more important and consumption in market goods lower, leading to a small ratio ϕ^y. Following this interpretation, under the condition $C_M > P^y/P^R$, the tax reform is likely to be welfare enhancing in developing rather than developped countries.

2.4.2 Tax reform when both transfers are positive

Let us now introduce time transfers by considering the tax reform at steady state where both private transfers are positive: $x > 0$ and $T^o < 1$. Compared with the preceding Section without time transfers, the marginal shift from capital income tax towards inheritance tax also modifies the parent's trade-off between bequests and time transfers. As we shall see, this adds new positive or negative effects on the young's labor supply.

The steady state is characterized by marginal conditions (2.3.2)-(2.3.4) and the resource constraint (2.3.6). In these equations, the tax reform not only decreases the relative price P^R between both market good consumptions, but also increases P^o, the relative price between market good and time used in home production when old. In the following, consequences of the fall in P^R will be named *interperiod* effects, while those resulting from higher P^o will be named *intraperiod* effects.

We first detail the interperiod effects. The fall in P^R has similar consequences on labor supply than those stressed in the preceding Subsection 2.4.1, but also introduces an additional effect through changes in the time transfer. Indeed, lower P^R involves a negative effect on c and T^y and a positive effect on d and T^o. The elasticity of substitution σ^u between both composite goods may amplify these effects. The resulting impact on the young's labor supply is ambiguous: the negative effect on T^y affects positively the labor supply whereas the positive effect on T^o leads to a negative impact on time transfers, hence on the young's labor supply.

We now turn to the intraperiod effects, that come from the increase in P^o. The equality between marginal rate of substitution and relative price, $MRS_{T^o/d} = P^o$, implies that the marginal rate of substitution between d and T^o increases with the tax reform. This has a positive impact on d and a negative effect on T^o. The negative effect on T^o affects positively the labor supply. The magnitude of the intraperiod effect on T^o depends crucially on the elasticity of substitution between T^o and d.

Let us denote by σ^o, the absolute value of this elasticity of substitution associated with home production technology f^o. By definition:

$$\frac{dd}{d} - \frac{dT^o}{T^o} = \sigma^o \frac{dP^o}{P^o} = -\sigma^o \frac{dP^R}{P^R} \tag{2.4.6}$$

The following Lemma signs the marginal effect on the second-period consumption d.

Lemma 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Then, marginal effect on second-period consumption d is positive and given by*

$$\frac{dd}{d} = -\left[\left(1 - \frac{d}{(1+\mu)\, C_M}\right)\sigma^o + \frac{c + C_M T^y}{(1+\mu)\, C_M}\alpha^o\left(\sigma^u - \sigma^o\right)\right]\frac{dP^R}{P^R} > 0 \tag{2.4.7}$$

where $\alpha^o \equiv f_d^o d / f^o$.

Proof. As the home production function when old f^o is linear homogenous,

$$\frac{df^o}{f^o} = \alpha^o \frac{dd}{d} + (1-\alpha^o)\frac{dT^o}{T^o} = \frac{dd}{d} + (1-\alpha^o)\,\sigma^o \frac{dP^R}{P^R}$$

where the second equality is obtained with equation (2.4.6). Since $dP^y = 0$ and the home production function when young f^y is linear homogenous, one deduces

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \frac{df^y}{f^y} \text{ and } df_c^y = 0$$

Then, equation (2.4.2) rewrites as

$$\frac{dc}{c} - \frac{dd}{d} - (1-\alpha^o)\,\sigma^o \frac{dP^R}{P^R} = \sigma^u\left(-\frac{df_d^o}{f_d^o} + \frac{dP^R}{P^R}\right)$$

Linear homogeneity of f^o implies $T^o f^o_{dT^o}(d, T^o) = -d f^o_{dd}(d, T^o)$ and $\frac{-f^o_{dd} d}{f^o_d}\sigma^o = 1 - \alpha^o$.

Then, one gets

$$\frac{df_d^o}{f_d^o} = \frac{f_{dd}^o dd + f_{dT^o}^o dT^o}{f_d^o} = \frac{-f_{dd}^o d}{f_d^o} \sigma^o \frac{dP^R}{P^R} = (1 - \alpha^o) \frac{dP^R}{P^R}$$

Consequently, the preceding relation between $\frac{dc}{c}$ and $\frac{dd}{d}$ becomes

$$\frac{dc}{c} - \frac{dd}{d} = [\sigma^u \alpha^o + (1 - \alpha^o) \sigma^o] \frac{dP^R}{P^R} \tag{2.4.8}$$

Differentiation of the resource constraint (2.3.6) yields

$$c\frac{dc}{c} + d\frac{dd}{d} + C_M \left(T^y \frac{dT^y}{T^y} + \mu T^o \frac{dT^o}{T^o} \right) = 0$$

and, combining with equation (2.4.8), allows to compute dd/d:

$$\frac{dd}{d} = -\frac{(c + C_M T^y) [\sigma^u \alpha^o + (1 - \alpha^o) \sigma^o] + \mu C_M T^o \sigma^o}{c + d + C_M (T^y + \mu T^o)} \frac{dP^R}{P^R} > 0$$

which is equivalent to equation (2.4.7). □

Lemma 1 shows that tax reform results in an increase in d whatever the initial values of the instruments, as soon as they allow for positive bequests and positive time transfers. We now turn to the variations of c, T^y and T^o that depend crucially on both elasticities of substitution σ^u and σ^o, that respectively drive up the size of the interperiod and intraperiod effects.

Lemma 2. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady state satisfies $\mu C_M > P^o$. Then, one gets the following sufficient conditions:*

(i) If $\sigma^o \geq \sigma^u$, the marginal effect on time devoted to home production when old T^o is negative.

(ii) If $\sigma^u \geq \sigma^o$, the marginal effects on first-period consumption in market good c and time devoted to home production T^y are negative.

(iii) If σ^o/σ^u is close to zero, c and T^y decrease, while T^o increases.

(iv) If σ^o/σ^u is close to unity, then c, T^y and T^o decrease.

(v) If σ^o/σ^u tends to infinity, c and T^y increase, while T^o decreases.

Proof. Marginal effects on c, T^y and T^o can be computed from expressions (2.4.6), (2.4.7) and (2.4.8):

$$\frac{dc}{c} = \frac{dT^y}{T^y} = \sigma^o \frac{d + \mu C_M T^o}{(1+\mu)\, C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) + \frac{d}{d + \mu C_M T^o}\right] \frac{dP^R}{P^R}$$

$$\frac{dT^o}{T^o} = -\ \sigma^o \frac{c + C_M T^y}{(1+\mu)\, C_M} \left[\alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) - \frac{d}{c + C_M T^y}\right] \frac{dP^R}{P^R}$$

This proves results (i)-(iv). Let us show result (v). Assuming that σ^o/σ^u tends to infinity, one gets that dc and dT^y are positive iff

$$\alpha^o > \frac{d}{d + \mu C_M T^o}$$

which is equivalent to $\mu C_M > P^o$, since $\alpha^o = d/\left(d + P^o T^o\right)$. The proof is complete. □

Notice that the assumption $\mu C_M > P^o$ is satisfied at the initial steady state, that is, with $\tau^x = 0$, $\tau^w = \Gamma$ and $\bar{\tau}^R > \Gamma$. Indeed, since $P^o = \beta\mu P^y/P^R$, straightforward calculations using linear homogeneity of the technology F show that the inequality $\mu C_M > P^o$ is always true.[5] At equilibrium, the relative price P^o is equal to the marginal rate of substitution between T^o and d ($MRS_{T^o/d}$). Moreover, from the resource contraint, the marginal rate of transformation between T^o and d is: $MRT_{T^o/d} = \mu C_M$. Thus, the assumption $\mu C_M > P^o$ means that the MRT between T^o and d is higher than the MRS, that is, for given (c, T^y), any fall in T^o increases labor supply, and then leaves enough additional resources for second-period consumption, to increase utility.

5. With $\tau^x = 0$, inequality $\mu C_M > P^o$ is equivalent to $C_M > P^y$. Using the linear homogeneity of F, one gets

$$C_M = (1-\Gamma)\, F_L + \left[(1-\Gamma)\, F_K - 1\right] z_M > P^y$$

where the last inequality is obtained using $\tau^w = \Gamma$ and $(1-\Gamma)\, F_K > \left(1 - \bar{\tau}^R\right) F_K = \frac{1}{\beta} > 1$.

From the proof of the preceding Lemma, one may notice that increases in all variables c, d, T^y and T^o cannot arise simultaneously, since $dc > 0$ requires $\sigma^u < \sigma^o$, which implies $dT^o < 0$. Therefore, only three cases can arise:

— $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o > 0$. This case arises when σ^o/σ^u is close to zero. Intergenerational time transfers have been reduced by the increase in the inheritance tax.

— $dc < 0$, $dT^y < 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u is close to one, as with logarithmic utility.[6] It induces a rise in intergenerational time transfers.

— $dc > 0$, $dT^y > 0$, $dd > 0$ and $dT^o < 0$. This case arises when σ^o/σ^u tends to infinity. Intergenerational time transfers increase with the inheritance tax.

We now analyze the marginal effect of the tax reform on the household life-cycle utility in each of these three cases. In the following Proposition, we establish the condition for the tax reform to be welfare improving.

Proposition 8. *At a steady state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant, the marginal effect on utility dV has the same sign as*

$$\left[P^R - \Theta\right] d - \alpha^o \left(\frac{\sigma^u}{\sigma^o} - 1\right) \left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] \tag{2.4.9}$$

where

$$\Theta \equiv \frac{c + P^y T^y + P^R d + \beta\mu P^y T^o}{(1+\mu)\, C_M} \tag{2.4.10}$$

Proof. Using the marginal conditions of the household problem (2.3.2)-(2.3.4), dV has the same sign as

$$dc + P^y dT^y + P^R dd + \beta\mu P^y dT^o$$

6. This is the case, for instance, if the life-cycle utility function is:

$$\alpha^y \ln c + (1-\alpha^y)\ln(1-T^y) + \gamma\left[\alpha^o \ln d + (1-\alpha^o)\ln(1-T^o)\right]$$

where α^y, α^o and γ are positive parameters, $\alpha^y < 1$ and $\alpha^o < 1$.

Since $dP^y = 0$, relative changes dc/c and dT^y/T^y are equal. Consequently, replacing equations (2.4.6) and (2.4.8) in the latter equation and using expression (2.4.7) in Lemma 1, one obtains that dV has the same sign as

$$-\sigma^o \left[P^R - \Theta\right] d\frac{dP^R}{P^R} + \alpha^o \left(\sigma^u - \sigma^o\right) \left[\left(c + P^y T^y\right) - \Theta \left(c + C_M T^y\right)\right] \frac{dP^R}{P^R}$$

which concludes the proof. □

To interpret condition (2.4.9), we distinguish the above three cases according to the value of the elasticity ratio σ^o/σ^u.

Tax reform with $\sigma^u = \sigma^o$

In this situation, that encompasses the case of a logarithmic utility function, the second-period consumption d increases thanks to lower c, T^y and T^o. From expression (2.4.9), welfare increases if and only if $P^R > \Theta$, which can be rewritten as:

$$dV > 0 \Leftrightarrow C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y + \left(\mu C_M - P^o\right)\frac{T^o}{T^y} > 0$$

In the latter inequality, we observe the same term as in (2.4.4): $C_M - \frac{P^y}{P^R} - \left(\frac{1}{P^R} - 1\right)\phi^y$. The tax reform increases welfare in the model with elastic labor supply and no time transfer iff this term is positive. This leads to the same kind of interpretation: the fall in the second-period consumption price P^R reduces c and T^y and increases d. Then, the reduction in T^y increases the young's labor supply involving a positive effect on resources in market good.

Moreover, the positive effect on labor supply is reinforced by the increase in time transfers since T^o decreases with the reform. This positive effect on welfare appears in the second-term of the latter inequality. As stated before, the substitution from T^o to d is welfare enhancing since the initial equilibrium satisfies $\mu C_M > P^o$, that is, $MRT_{T^o/d} > MRS_{T^o/d}$.

Therefore, taking the Barro model with elastic labor supply as a benchmark, the introduction of intergenerational time transfers creates an additional positive effect on steady-state welfare. As soon as condition (2.4.4) is satisfied, the tax reform improves steady-state welfare. The falls in T^y and T^o involve a rise in labor supply. Simultaneously, reducing c and increasing d imply higher savings, and lead to higher capital stock. All these additional inputs allow to produce more market goods, that will be consumed in second-period of life.

Tax reform with $\sigma^u >> \sigma^o$

In this case, interperiod effects (from the decrease in P^R) dominate intraperiod effects (from the increase in P^o). This arises with a high elasticity of substitution between both composite goods σ^u, or with a low elasticity of substitution σ^o.

A high σ^u involves a significant shift in resources from the first to the second period of life. Thus, the market good consumption d and the time devoted to home production when old T^o strongly increase thanks to lower c and T^y.

For a low elasticity of substitution σ^o, the tax reform has a negative effect on time transfers. Indeed, the increase in d associated with strong complementarity between d and T^o results in an increase in T^o as $\phi^o = d/T^o$ remains constant.

In both cases, the effect on labor supply is ambiguous as the labor supply is positively affected by the reduction in T^y and negatively by the increase in T^o.

The marginal effect on household life-cycle utility may be worse off than with a logarithmic utility as the effect on labor supply is attenuated or reversed. From expression (2.4.9), the welfare is improved iff:

$$dV > 0 \Leftrightarrow -\left[(c + P^y T^y) - \Theta\left(c + C_M T^y\right)\right] > 0$$

Using expression (2.4.10), one gets

$$C_M - \frac{P^y}{P^R} > \left(\frac{\phi^o + \mu C_M}{\phi^o + P^o} \frac{1}{P^R} - 1 \right) \phi^y \tag{2.4.11}$$

With the initial values of the instruments: $\mu C_M > P^o$ and $P^R < 1$. Therefore, the difference $C_M - \frac{P^y}{P^R}$ has to be positive for the tax reform to improve welfare. Comparing inequalities (2.4.4) and (2.4.11), the right-hand side in inequality (2.4.11) is higher. Consequently, situations where the tax reform has a positive effect on welfare are less likely to happen with operative time transfers than in the Barro model with elastic labor supply. Increase in T^o reduces time transfer to the young and then affects negatively their labor supply.

The ratio ϕ^y has still to be low in order to get a positive effect of the tax reform. As in the preceding Sections, low ϕ^y means a sharp decrease in T^y, and thus an important increase in labor supply. With time transfers, the ratio ϕ^o has also an impact. Indeed, since the tax reform increases P^o, the ratio ϕ^o also increases. Thus, if ϕ^o is initially high, the rise in T^o will be small and has also a small negative effect on labor supply.

Tax reform with $\sigma^u << \sigma^o$

Here, intraperiod effects (through higher P^o) dominate interperiod effects (through lower P^R). This case arises if σ^o is high, or if σ^u is small.

On the one hand, for a high elasticity of substitution σ^o, increasing relative price P^o involves higher second-period consumption of market good d, lower time devoted to home production T^o, and so, higher time transfer to the young. The young enjoy more resources, and then consume more composite good, increasing both c and T^y.

On the other hand, a low elasticity of substitution between both periods σ^u means that both composite goods are complements. This involves a small effect of P^R and a small shift of resources from the first to the second period. But, higher P^o increases the ratio d/T^o, leading to a fall in T^o. The latter involves a positive effect on labor supply of the young.

Corollary 1. *At a steady-state equilibrium with positive bequests and positive time transfers, consider a marginal switch from capital income tax towards inheritance tax leaving the capital-labor ratio constant. Let us assume that the initial steady* *state satisfies $\mu C_M > P^o$. If the ratio σ^o/σ^u tends to infinity, the marginal effect of the tax reform on utility is positive.*

Proof. Putting σ^u/σ^o at zero in condition (2.4.9), one gets that dV is positive iff

$$\left[P^R - \Theta\right] d + \alpha^o \left[(c + P^y T^y) - \Theta \left(c + C_M T^y\right)\right] > 0$$

Then, plugging Θ, from expression (2.4.10) into the preceding inequality yields

$$(1+\mu)\, C_M > c + C_M T^y + d + P^o T^o$$

which is true if $\mu C_M > P^o$. □

In the case with σ^o/σ^u closed to unity, both T^y and T^o were reduced by the tax reform. With higher σ^o/σ^u, the negative effect of the tax reform on time devoted to home production by the old, T^o, is strengthened. This increases first-period resources, and allows a rise in time devoted to home production by the young T^y. This shows that the effect on welfare is likely to be positive if the increase in labor supply only comes from a rise in time transfers from the grandparents.

2.5 Numerical illustrations when $\sigma^u << \sigma^o$

As stated before, our aim is to identify situations where a tax shift from capital income tax towards inheritance tax would be Pareto-improving. We use numerical examples to analyze the impact of the tax reform on welfare along the transitional dynamics. Welfare of any generation t corresponds to the infinite sum

$$W_t = \sum_{i=t}^{+\infty} \beta^{i-t} V_i$$

where V_i is life-cycle utility of generation $i \geq t$. Then a Pareto-improvement is achieved if the tax reform does not reduce W_t, for any generation $t \geq -1$, and increases W_t for at least one generation.

We start from the same values of the instruments as those considered in the steady-state analysis, for any $t \geq 0$: $\tau_t^R = \bar{\tau}^R > \Gamma$, $\tau_t^w = \Gamma$, $\tau_t^x = 0$. We consider an initial public debt $\bar{\Delta}_{-1} = 0.1$ whose debt burdens is equally shared between generations through an higher capital income tax such that we get $\tau_t^R = \bar{\tau}^R > \Gamma$.

In a first illustration, we focus on the same kind of fiscal reform as in Section 2.4, that is a fiscal reform that keeps the capital-labor ratio constant in the long-run. However, this tax reform reduces fiscal receipts at steady state, thus shifting the burden of the initial public debt to the first generations. It involves some intergenerational redistribution towards generations living in the long run.

Then, we turn to a second illustration where the tax reform can reduce the capital-labor ratio in the long-run. This attenuates the intergenerational redistribution due to the reallocation of public debt burden.

Furthermore, we concentrate on situations where the tax reform increases c, T^y and the labor supply through the positive effect on time transfer (*i.e.* $\sigma^u << \sigma^o$). In this case, the fiscal reform implemented in Section 2.4, involves an increase of the steady state households' life-cycle utility. This is likely the most favorable situation to improve the household welfare. For this purpose, we assume that d and T^o are substitutes and that the elasticity of substitution between both periods σ^u is low. Table 2.1 presents values of all parameters.

2.5.1 Tax reform with constant steady-state capital-labor ratio

A shift from capital income tax towards inheritance tax is implemented by introducing $\tau_t^x = 0.03$ for any period $t \geq 0$ and leaving the steady-state capital-labor ratio constant. The new capital income tax rate is $\tilde{\tau}_t^R = \frac{\bar{\tau}^R - \tau^x}{1-\tau^x}$ for any generation $t \geq 1$, with adjustment of τ_0^R in order to satisfy the intertemporal budget constraint

Table 2.1 – Base-case parameter value

Parameter		Value
Government		
Initial public debt	$\bar{\Delta}_{-1}$	0.1
Fraction of production devoted to public sector	Γ	0.1
Production function[a]		
Technological parameter	A	20
Share parameter of physical capital	a	0.4
Share parameter of labor supply	b	1
Elasticity of substitution between production factors	σ^F	0.5
Representative household		
Home production function when young[b]		
Share parameter of market good c	a^y	0.1
Elasticity of substitution between c and T^y	σ^y	0.5
Home production function when old[c]		
Share parameter of market good d	a^o	0.1
Elasticity of substitution between d and T^o	σ^o	10
Taste		
Degree of altruism	β	0.7
Efficiency of time transfer	μ	0.7
Elasticity of substitution[d] between f^y and f^o	σ^u	0.2
Time preference	γ	0.5

Note: We consider CES production and utility functions:

[a] $F(K,L) = A\left(aK^{\rho^F} + bL^{\rho^F}\right)^{\frac{1}{\rho^F}}$, with $\rho^F = 1 - \frac{1}{\sigma^F}$.

[b] $f^y(c,T^y) = \left(a^y c^{\rho^y} + (T^y)^{\rho^y}\right)^{\frac{1}{\rho^y}}$, with $\rho^y = 1 - \frac{1}{\sigma^y}$.

[c] $f^o(d,T^o) = \left(a^o d^{\rho^o} + (T^o)^{\rho^o}\right)^{\frac{1}{\rho^o}}$, with $\rho^o = 1 - \frac{1}{\sigma^o}$.

[d] $u(x) = \left(1 - \frac{1}{\sigma^u}\right)^{-1} x^{1-\frac{1}{\sigma^u}}$ and $v(x) = \gamma u(x)$.

of the government.[7]

Figure 2.1 describes the effect of the tax reform on households' welfare. The steady-state household's life-cycle utility increases. The tax reform involves an increase of the first generation's capital income tax τ_0^R since the burden of the public debt is

7. From equation (2.2.14) and given tax instruments values considered in Subsection 2.5.1, τ_0^R is solution of the following intertemporal budget constraint:

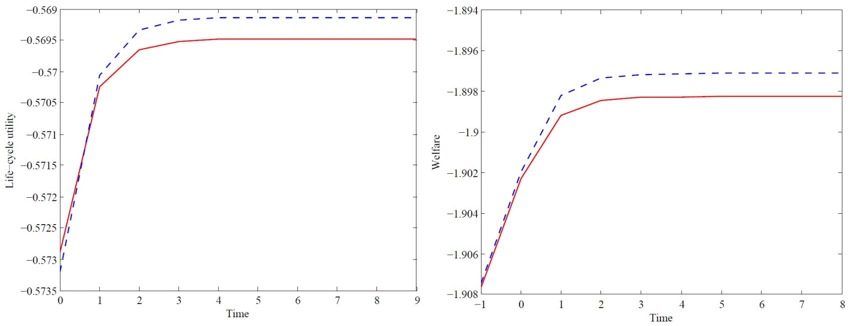
$$\left(1-\tau_0^R\right)R_0\bar{\Delta}_{-1}+\Gamma F\left(\bar{k}_0,\ell_0\right)-\Gamma\ell_0 w_0-\tau_0^R R_0\bar{k}_0-\tau^x x_0+\sum_{t=1}^{+\infty}P_t\left[\Gamma F\left(k_t,\ell_t\right)-\Gamma\ell_t w_t-\tilde{\tau}^R R_t k_t-\tau^x x_t\right]=0$$

with $P_0 = 1$ and $P_t\left(1-\tilde{\tau}^R\right)R_t = P_{t-1}$, for any period $t \geq 1$.

shifted towards them. This results in a negative impact on the life-cycle utility of the first generation (decreasing from -1.7103 to -1.7109) while the effect on life-cycle utilities along the transitional dynamics is positive. Taking account of the altruistic component of utility, the tax reform increases welfare of each generation (see Figure 2.1(b)) and then involves efficiency gains.

Figure 2.1 – Tax reform with constant steady-state capital-labor ratio

(a) Life-cycle utility V_t (b) Welfare W_t



Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. The introducion of a positive inheritance tax $\tau_t^x = 0.03$ for any $t \geq 0$, implies that $\tau_0^R \simeq 0.2895$ and $\tilde{\tau}_t^R \simeq 0.2603$ for any $t \geq 1$ after the reform.

However, the adjustment of the first generation's capital income tax τ_0^R allows to shift part of the burden of the initial debt towards a lump-sum tax. One may wonder whether the result comes from the fact that the tax reform shifts the public debt burden on a lump-sum tax. For this reason, we thereafter focus on another fiscal reform that keeps the lump-sum tax τ_0^R constant.

2.5.2 Tax reform with constant tax rate on capital income from period 1

The tax reform now consists to set up a positive inheritance tax rate $\tau_t^x = 0.03$ for any period $t \geq 0$, keeping the initial tax rate on capital income constant $\tau_0^R = \bar{\tau}^R$. This implies a decrease of the capital income tax $\tau_t^R = \widehat{\tau}^R$ for any period $t \geq 1$ such as $\widehat{\tau}^R$ balances the intertemporal budget constraint of the government.[8]

Since the tax reform is not conducted to leave the capital-labor ratio constant in steady-state, a shift from capital income tax towards inheritance reduces the capital-labor ratio, and thus the resource available for market good consumption. In order to attenuate this negative effect on the whole dynamics and in the long run, we consider that production factors are complements.

The results are reported in Figure 2.2. We get similar effects on utility and welfare to the preceding illustration. The tax reform implemented illustrates the trade-off for the government between keeping the steady-state primary surplus constant (constant steady-state public debt) and leaving the capital-labor ratio constant (lower steady-state public debt). The tax reform considered involves a decrease of the steady-state capital-labor ratio: the increase of the capital stock (thanks to the fall in the relative price between both intertemporal market goods consumptions P^R) is lower than the rise in labor supply. This positive effect on labor supply relies on the rise of time transfers (through the increase of P^o). In addition, implementing positive inheritance tax rate reduces bequests (since households have more incentive to transfer time).

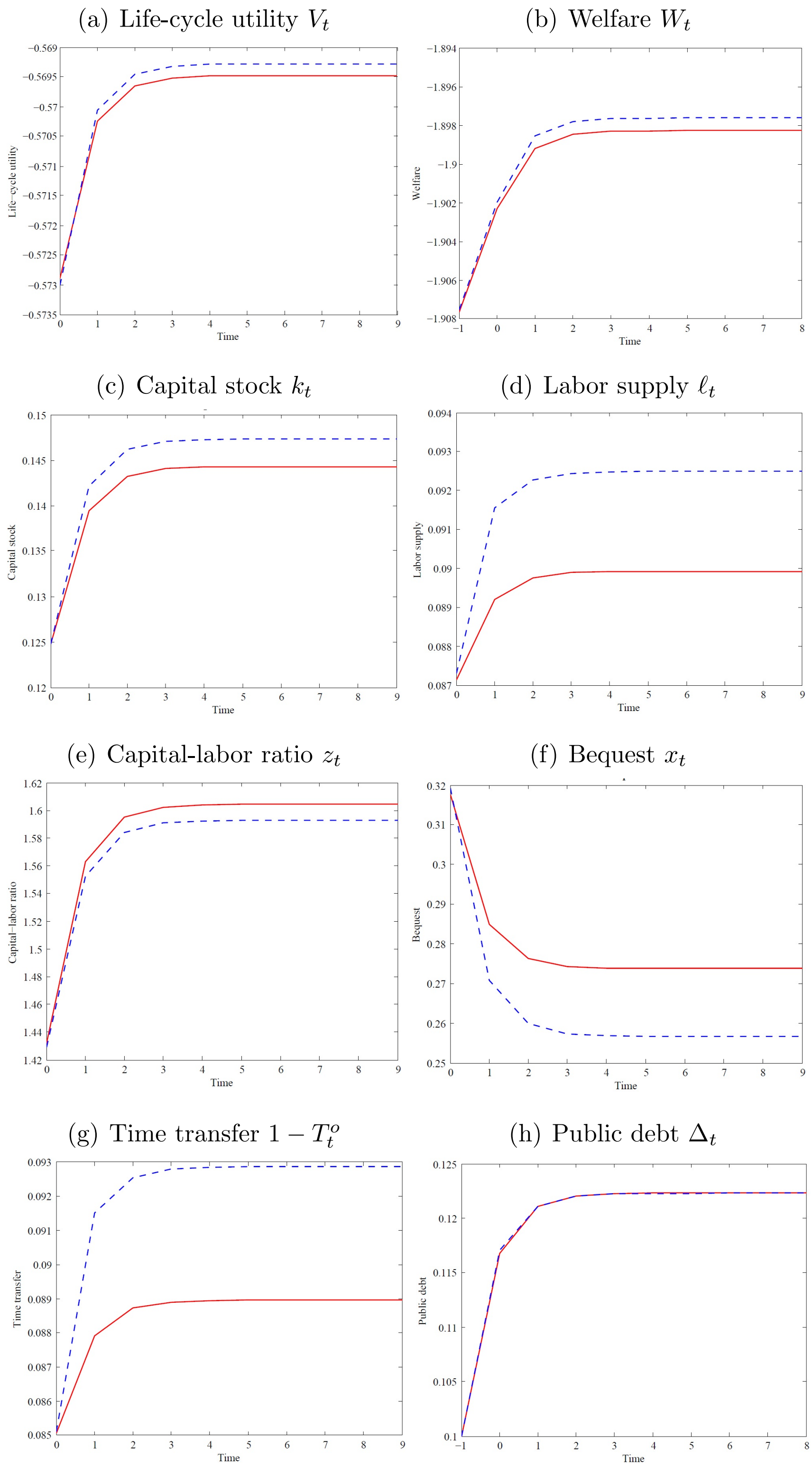
As in the previous Subsection, life cycle utility is improved for each generation except for the first old (their utility decreases from -1.7103 to -1.7113) and welfare of all generations increases with the tax reform.

8. From equation (2.2.14) and given tax instruments values considered in Subsection 2.5.2, $\widehat{\tau}^R$ is solution of the following intertemporal budget constraint:

$$\left(1-\bar{\tau}^R\right) R_0 \bar{\Delta}_{-1} + \Gamma F\left(\bar{k}_0, \ell_0\right) - \Gamma \ell_0 w_0 - \bar{\tau}^R R_0 \bar{k}_0 - \tau^x x_0 + \sum_{t=1}^{+\infty} P_t \left[\Gamma F\left(k_t, \ell_t\right) - \Gamma \ell_t w_t - \widehat{\tau}^R R_t k_t - \tau^x x_t\right] = 0$$

with $P_0 = 1$ and $P_t\left(1-\widehat{\tau}^R\right) R_t = P_{t-1}$, for any period $t \geq 1$.

Figure 2.2 – The tax reform effect with constant tax rate on capital income



Note: The transitional path before the reform: bold line. After the reform: dashed line. Before the reform, we get $\bar{\tau}^R \simeq 0.2825$. The introdution of a positive inheritance tax $\tau_t^x = 0.03$ for any $t \geq 0$, implies that $\tau_0^R = \bar{\tau}^R \simeq 0.2825$ and $\hat{\tau}^R \simeq 0.2689$ for any $t \geq 1$, after the reform.

2.6 Conclusion

To summarize our results, we consider a tax reform starting from an intertemporal equilibrium where the capital income tax is above its efficient level in order to finance the burden of an initial public debt. We have then addressed the following issue: should the government increase inheritance tax in order to reduce the capital income tax?

In the Barro model, the tax reform reduces steady-state welfare. The driving force is the change in the marginal rate of substitution between young and old consumptions, leading to a fall in the first-period consumption and a rise in the second-period one.

With elastic labor supply, the tax reform may be Pareto-improving. The most favorable cases are those where the fall in first-period consumption is associated with a fall in time devoted to domestic production (*i.e.* leisure in the usual terminology), allowing for an increase in the young labor supply.

Introducing time transfers enhances the positive effect on the young labor supply. Indeed, in this framework, inheritance tax also modifies the trade-off between time transfers and bequests. Grandparents are incited to transfer more time and less money to the next generation, that will benefited from higher time resources and will be able to work more.

With familial time transfers, we have shown that a shift from capital income tax towards inheritance tax can be Pareto-improving. The Pareto improvement strongly depends on the strength of the positive effect of time transfers on the young's labor supply and on the strength of the effect of higher labor supply on the production of market goods.

For further researches, a closer look to the intragenerational heterogeneity would allow to address redistribution issues. Heterogeneity could be introduced at least in the two following dimensions. First, empirical studies show differences in the distributions of time transfers and distributions of bequests. They suggest that bequests are more concentrated than time transfers. Secondly, capital income tax may affect a larger part of the population than inheritance tax.

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Chapter 3

Inheritance taxation with agents differing in altruism and productivity

joint work with Pascal Belan.

Abstract [1]

We analyze a shift from capital income tax towards inheritance tax in a two-period overlapping generation model with rational altruism *à la* Barro, where the population consists of two types of dynasties that differ in altruism and productivity. The tax reform is implemented in a way that leaves the capital-labor ratio constant at steady state. It increases welfare of less altruistic dynasties, but decreases welfare of the most altruistic one. We then extend the model by considering time transfers from the old to the young generation and assuming that the young have elastic labor supply. We discuss the condition for the tax reform to be Pareto-improving in steady state.

Keywords: altruism, bequests, heterogeneity, inheritance tax, redistribution.

JEL Classifications: D64, H22, H24, J22.

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3.1 Introduction

Standard models with infinite horizon agents do not make much difference between capital income tax and inheritance tax leading to optimal values equal to zero in the long run (see Chamley (1986, p. 613)). Most of these models consider homogeneous households. However, the distribution of inheritances is highly concentrated in most developed countries and only concerns a part of the population while life cycle savings concern everyone. According to Piketty (2010), in France in 2010, the bottom 50% poorest with respect to inherited wealth received about 5% of aggregate bequests whereas the top 10% richest received about 60%. In addition, one quarter of total bequests is transmitted to the top 1% while a third of deceased people leave no bequests. Hence, these strong disparities in terms of bequests lead to intragenerational inequalities. From this point of view, inheritance tax could play a crucial role in reducing inequality, in a society where dynasties do not have the same accumulation behavior. Another positive argument for stepping inheritance tax against capital income tax, is that it promotes fairness since it concerns unearned resources which does not compensate any effort or work.

With these elements in mind, this paper analyzes whether a shift from capital income tax towards inheritance tax may be welfare enhancing for infinite horizon households (through rational altruism *à la* Barro (1974)), in an economy where bequests are concentrated on a part of population. We consider for this purpose that dynasties have different degrees of altruism, meaning that households within generation care differently about their descendants (see Michel and Pestieau (1998) and Vidal (1996)).

Theoretical literature on rational altruism *à la* Barro (1974) with intragenerational heterogeneity suggests that redistributive incidence of inheritance taxation is likely to worsen welfare of every household even those who behave like life-cyclers. As shown by Michel and Pestieau (2005), if households have homothetic preferences, a uniform lump-sum transfer financed through inheritance tax reduces the steady-state welfare of all dynasties which differ in altruism degree and in productivity. Although inheritance taxation allows to redistribute wealth, the distortive effect of inheritance tax concerning households choice on bequests pushes down the steady-state capital-labor ratio. The fall in capital-labor ratio affects negatively the consumption of all dynasties which results in a negative impact of the tax reform on steady-state households welfare.

Considering a shift from capital income taxation towards inheritance taxation allows to attenuate the fall in the capital-labor ratio. Indeed, this tax reform leads to opposite effects of both taxes on its steady state value: a fall in capital income tax increases the capital-labor ratio, while raising inheritance tax decreases it. In addition, the policy is redistributive since all households pay the capital income tax (in a two-period life cycle model) while inheritance is paid only by dynasties that leave bequests.

We first show that at steady state, a switch from capital income taxation to inheritance taxation leaving constant the capital-labor ratio, is necessarily welfare enhancing for every altruistic dynasty, except for the most altruistic one. We consider the same kind of framework as Michel and Pestieau (2005): a two-period overlapping generation model with rational altruism *à la* Barro (1974) characterized by two types of dynasties which differ in terms of altruism degree and of human capital level. Both dynasties consume in both periods, work during the first one and then retire in the second period of life. The government finances public spending and a uniform lump-sum transfer using taxes on inheritance, capital income and labor income. We show that the tax reform cannot improve the welfare of agents who do not leave bequests without reducing utility of agents who leave bequests. The main reason is that keeping the capital-labor ratio constant with inelastic labor supply involves constant disposable resources in steady state.

We then propose an extension of the model that allows to modify the household's disposable resources at steady state, even if the capital-labor ratio is constant. We consider a model with elastic labor supply of the young, taking account of family time transfers from old to young adults that may allow the inheritance tax to increase labor supply of the young. Indeed, considering family time transfers, which are substantial,[2] introduces a trade-off between bequests and time transfers since both transfers differently affect the labor supply of the recipient and his life-cycle resources. The model that we are going to consider has many similarities with Cardia and Ng (2003) and Cardia and Michel (2004). In this context, taxing bequests makes time transfers more attractive since inheritance tax affects the trade-off between both family transfers. As shown by Belan and Moussault (2018) in a model with homogeneous agents, the positive effect on time transfers may increase the young's labor supply which attenuates or reverses the potential negative effect of inheritance tax on disposable resources. They show that a shift from capital income tax towards inheritance tax can increase steady-state welfare, depending on the magnitude of the effect of higher time transfers on the labor supply of the young. Consequently, with time transfers, constant capital-labor ratio does not imply constant disposable resources. In the present paper, considering heterogeneous dynasties, we analyze the impact of the tax reform on different dynasties taking into account both types of transfers (in time and money). We identify situations where tax reform can be Pareto-improving. Indeed, the negative effect of the tax reform on the most altruistic dynasty may be attenuated or reversed by the positive impact of the increase of time transfers on the young's labor supply. Thus, depending on the strength of this effect as well as the population distribution, we show that the tax reform can be Pareto improving at steady state.

The paper is organized as follows. Section 3.2 presents the model. In Section 3.3, we analyze the steady-state effect of tax reform on the welfare of the two types of dynasties. Then in Section 3.4, the model is extended to time transfers and we study tax reform impacts on both types of dynasties. Final Section concludes.

2. Numbers of empirical studies indicate that time transfers from parents to their children are on average almost as important as bequests in monetary equivalent in European countries and the United States, such as Schoeni et al. (1997), Cardia and Ng (2003), Attias-Donfut et al. (2005) and Wolff and Attias-Donfut (2007).

3.2 Equilibrium

3.2.1 Dynasties and generations

We consider a two-period overlapping generation model. Time is discrete. The population size is constant and normalized to unity. Each parent has only one child. We consider dynastic altruism *à la* Barro (1974) from parents to children. The economy consists of two types of dynasties (types 1 and 2). All agents that belong to the same type of dynasty i ($i \in \{1, 2\}$), whatever the generation, have the same degree of altruism β_i, and the same level of human capital h_i. We assume $0 \leq \beta_1 < \beta_2 < 1$. We define p_i as the proportion of type $i's$ agents in each generation: $0 < p_i < 1$ and $p_1 + p_2 = 1$.

3.2.2 Household behavior

An individual born in t that belongs to a type-i dynasty works in period t and retires in period $t + 1$. During its working life, he/she allocates income between market good consumption c_{it} and savings s_{it}

$$c_{it} + s_{it} = (1 - \tau^w)\, h_i w_t + (1 - \tau^x)\, x_{it} + a_t$$

where w_t is the real wage, x_{it} is the bequest received from his parent and a_t is a lump-sum transfer. Tax rates on labor income τ^w and bequest τ^x are assumed to be constant.

When retired, the individual divides return on savings between market good consumption d_{it+1} and bequest to his child x_{it+1},

$$d_{it+1} + x_{it+1} = \left(1 - \tau^R\right) R_{t+1} s_{it}$$

where R_{t+1} is the gross interest rate. The tax rate on capital income τ^R is also assumed to be constant.

Utility U_{it} of a type-i individual born in t is

$$U_{it} = u(c_{it}, d_{it+1}) + \beta_i U_{it+1}$$

where the lifetime utility function u is increasing in both arguments, strictly quasi-concave and satisfies Inada conditions. We also assume that both consumptions are normal goods.

It is assumed that parents cannot leave negative bequest to their children:

$$x_{it+1} \geq 0$$

If $\beta_i > 0$, optimality conditions with respect to s_{it} and x_{it+1} write

$$-u_c^{'}(c_{it}, d_{it+1}) + \left(1 - \tau^R\right) R_{t+1} u_d^{'}(c_{it}, d_{it+1}) = 0 \quad (3.2.1)$$

$$-u_d^{'}(c_{it}, d_{it+1}) + \beta_i (1 - \tau^x) u_c^{'}(c_{it+1}, d_{it+2}) \leq 0 \; (= 0 \text{ if } x_{it+1} > 0) \quad (3.2.2)$$

If $\beta_1 = 0$, then type-1 dynasties consist of life-cyclers. Their bequest is zero and their saving satisfies (3.2.1).

3.2.3 Firms and production

The production sector consists in a representative firm that behaves competitively and combines capital K_t and efficient labor L_t to produce output. Technology $F(K_t, L_t)$ is linear homogenous. Profit maximization of the representative firm leads to equality between marginal products and real input prices

$$R_t = F_K(K_t, L_t) \text{ and } w_t = F_L(K_t, L_t) \quad (3.2.3)$$

assuming total depreciation of the capital stock in one period. F_K and F_L stand for the partial derivative of F with respect to capital and efficient labor.

3.2.4 Government

The government has to finance public spendings defined as a fraction of the production of the private sector, $\Gamma F(K_t, L_t)$, with $0 \leq \Gamma < 1$. Let Δ_t denote the public debt at the beginning of period t. The government budget constraint writes

$$\Delta_{t+1} + \tau^R \sum_i p_i R_t s_{it-1} + \tau^w L_t w_t + \tau^x \sum_i p_i x_{it} = R_t \Delta_t + a_t + \Gamma F(K_t, L_t) \quad (3.2.4)$$

Notice that production of the private sector creates an externality on public spendings. Capital and efficient labor, by increasing production, also increase public spendings and reduce product disposable for consumption and investment. Therefore, capital and labor demands have a social marginal product lower than the real input prices R_t and w_t.

3.2.5 Market equilibrium

In period t, the labor market equilibrium is given by

$$L_t = \sum_i p_i h_i = \bar{h}$$

where $\bar{h}$ is the average productivity. The resource constraint in period t writes

$$(1-\Gamma) F\left(K_t, \bar{h}\right) = \sum_i p_i c_{it} + \sum_i p_i d_{it} + K_{t+1} \quad (3.2.5)$$

The Walras' law implies equilibrium on the capital market is also satisfied:

$$K_{t+1} + \Delta_{t+1} = \sum_i p_i s_{it} \quad (3.2.6)$$

Indeed, it is obtained from the household budget constraints, the government budget constraint and linear homogeneity of the production function F, which implies $F\left(K_t, \bar{h}\right) = R_t K_t + w_t \bar{h}$.

The instruments considered allow the social planner to reach a Pareto optimum. Indeed, tax rates on capital income and labor income can be set in order to equalize social and private marginal products of capital and labor. It will be done with $\tau^R = \tau^w = \Gamma$.

Then, with a zero tax rate on bequest ($\tau^x = 0$), the government budget constraint for all $t \geq 0$ reduces to $\Delta_{t+1} = (1-\Gamma) R_t \Delta_t + a_t$. This means that an initial public debt would be shared among generations and dynasties through uniform lump-sum tax a_t ($t \geq 0$). Inheritance tax would create inefficiency by distorting the household choice on bequests. Nevertheless, inheritance tax may help to reduce wealth inequalities in an economy where dynasties do not have the same accumulation behavior.

3.2.6 Steady state

As stressed by Becker (1980), Altig and Davis (1992), Vidal (1996), Michel and Pestieau (1998, 2005) and Nourry and Venditti (2001), the dynasties that leave positive bequests, at steady-state equilibrium, can only be those with the highest degree of altruism. Other dynasties behave as life-cyclers and accumulate no wealth. The same result applies in our model. At steady state, optimality conditions (3.2.1) and (3.2.2) imply

$$\beta_i (1-\tau^x) \left(1-\tau^R\right) R \leq 1 \; (= 1 \text{ if } x_i > 0)$$

Since $\beta_1 < \beta_2$, the preceding condition implies that type-1 dynasties leave no bequest at steady state: $x_1 = 0$. As shown by Nourry and Venditti (2001), bequests of type-2 dynasties are positive iff the capital stock K_M consistent with the modified Golden-rule – *i.e.* the capital stock that satisfies $\beta_2 (1-\tau^x) \left(1-\tau^R\right) F_K \left(K_M, \bar{h}\right) = 1$ – is higher than savings that would be obtained if all agents were life-cyclers, with $R = F_K \left(K_M, \bar{h}\right)$ and $w = F_L \left(K_M, \bar{h}\right)$. We need to extend this result to take account of fiscal instruments.

In the following, we assume that the government chooses the tax instruments $\left(\tau^R, \tau^w, \tau^x, a\right)$.

Then, considering situations with positive bequests of type-2 dynasties ($x_2 > 0$), a steady-state equilibrium is a vector $(c_1, d_1, c_2, d_2, K_M, x_2, R_M, w_M)$ such that

$$\beta_2 (1-\tau^x) \left(1-\tau^R\right) R_M = 1 \tag{3.2.7}$$

$$MRS_i^{d/c} = \beta_2(1-\tau^x) \tag{3.2.8}$$

$$c_1 + \beta_2 (1-\tau^x) d_1 = (1-\tau^w) h_1 w_M + a \tag{3.2.9}$$

$$c_2 + \beta_2 (1-\tau^x) d_2 = (1-\tau^w) h_2 w_M + a + (1-\tau^x) x_2 \tag{3.2.10}$$

$$w_M = F_L(K_M, \bar{h}), \quad \text{and} \quad R_M = F_K(K_M, \bar{h}) \tag{3.2.11}$$

$$\sum_i p_i(c_i + d_i) = (1-\Gamma) F(K_M, \bar{h}) - K_M \tag{3.2.12}$$

where $MRS_i^{d/c} = u'_d(c_i, d_i)/u'_c(c_i, d_i)$ is the marginal rate of substitution of type-i between d and c, for $i = 1, 2$. The public debt Δ then results from the budget constraint of the government (3.2.4) at steady state:

$$\left[1 - \left(1-\tau^R\right) R_M\right] \Delta = \Gamma F\left(K_M, \bar{h}\right) + a - \tau^R R_M K_M - \tau^w w_M \bar{h} - \tau^x p_2 x_2 \tag{3.2.13}$$

We get the following Lemma.

Lemma 3. *Assume there exists a capital stock K_M that corresponds to the modified Golden-rule, i.e. that satisfies equality (3.2.7). Consider an inheritance tax rate τ^x close to zero. The steady-state bequest of type-2 agents x_2 is positive iff*

$$K_M + \Delta > \sum_{i=1}^{2} p_i s\left((1-\tau^w) h_i w_M + a, 0, \left(1-\tau^R\right) R_M\right) \tag{3.2.14}$$

where the public debt Δ satisfies equation (3.2.13).

Proof. Let us first define the following saving function

$$s(\omega_1, \omega_2, R) = \arg\max_z u(\omega_1 - z, \omega_2 + Rz)$$

Under the normal good assumption, s increases with ω_1 and decreases with ω_2. The capital market equilibrium (3.2.6) can be rewritten as

$$\begin{aligned} K_M + \Delta &= p_1 s\left(\left(1-\tau^w\right) h_1 w_M + a, 0, \left(1-\tau^R\right) R_M\right) \\ &\quad + p_2 s\left(\left(1-\tau^w\right) h_2 w_M + a + \left(1-\tau^x\right) x_2, -x_2, \left(1-\tau^R\right) R_M\right) \end{aligned} \tag{3.2.15}$$

where Δ depends on x_2. Differentiating equation (3.2.13) with respect to Δ and x_2 leads to:

$$d\Delta = \frac{\tau^x p_2}{\left(\beta_2(1-\tau^x)\right)^{-1} - 1} dx_2$$

Then, assuming τ^x close to zero leads to a small effect of x_2 on Δ.

Moreover, the saving function s increases with its first argument and decreases with the second one. Consequently, the right-hand side of (3.2.15) increases with x_2. Then, under condition (3.2.14), equality (3.2.15) is satisfied iff $x_2 > 0$. □

For high inheritance tax, the condition in Lemma 3 may be not sufficient. Indeed, as τ^x increases, the public debt Δ may become an increasing function of x_2. This positive effect on the left-hand side of (3.2.14) may then dominate the positive effect of x_2 on the right-hand side. Consequently, condition (3.2.14) may become irrelevant for guaranteeing positive bequests x_2.

The effect of τ^w and a on condition (3.2.14) can also be analyzed. Indeed, the public debt increases with τ^w and decreases with a, while the saving function on the right-hand side of (3.2.14) varies in the opposite direction. Then condition (3.2.14) is likely to be satisfied with low value of a and high value of τ^w. As the young receive low after-tax labor income or low public transfer, inheritance is likely to be positive.

3.3 Fiscal reform at steady state

With coexistence of dynasties that leaves bequest or behaves like life-cyclers, the inception of an inheritance tax allows to redistribute wealth. Nevertheless, it also reduces the capital-labor ratio. Indeed, equation (3.2.7) leads to a negative relation between the capital-labor ratio and the inheritance tax rate. Considering homothetic preferences, Michel and Pestieau (2005) have shown that a uniform lump-sum transfer financed through inheritance tax reduces the steady-state lifetime utility of all dynasties. One may explain the result in the following way. Two forces affect welfare of the life-cyclers. First, they receive a lump-sum public transfer. Second, the fall in the capital-labor ratio increases the real interest rate and pushes down the real wage rate. The latter effect on the wage rate overcompensates the other forces leading to a fall in the well-being of the life-cyclers. The main driving force here is the fact that, at a steady state with underaccumulation, any fall in the capital-labor ratio reduces the product disposable for consumption $((1-\Gamma) F\left(K_M, \bar{h}\right) - K_M)$. Dynasties that leave bequests also experience a fall in their welfare, for two additional reasons: (i) the inheritance tax creates a distortion in their bequest decision and (ii) the lump-sum transfer they receive is lower than their contribution.

The point we want to stress in this paper is that there exist fiscal reforms that include an increase in the inheritance tax combined with changes in the other tax rates that attenuate or eliminate the fall in the capital-labor ratio. In the following, we explore the consequence of a tax reform that consists in a switch from capital income taxation towards inheritance taxation. These changes have opposite effects on the capital-labor ratio. A fall in the capital income tax rate τ^R increases the capital-labor ratio while raising the inheritance tax rate τ^x decreases it. Moreover, such a policy still allows to redistribute wealth since the capital income tax is paid by all agents while the inheritance tax is paid only by the dynasties that leave bequests.

It is possible for instance to set both tax rates in order to leave the capital-labor ratio constant. If, additionally, we assume constant labor income tax rate and constant lump-sum transfer, the reduction of τ^R financed through an increase in τ^x is necessarily welfare enhancing for life-cyclers. They do not pay inheritance tax and pay less capital income taxes, while wage rate and interest rate remain unchanged. We can then state the following Proposition, only using the intertemporal budget constraint (3.2.9) of type-1 agents.

Proposition 9. *At steady state, any increase in the inheritance tax τ^x that leaves the first-period income of type-1 agents (i.e. $(1-\tau^w) h_1 w_M + a$) constant increases* *steady-state life-cycle utility of type-1 agents.*

First-period income of type-1 agents is constant if, for instance, the capital-labor ratio is left unchanged (constant w_M) as well as the instruments τ^w and a. Such a situation can be obtained by setting τ^R in order to keep the product $(1-\tau^x)\left(1-\tau^R\right)$ constant. In this case, the capital stock K_M, characterized by equation (3.2.7), is unchanged, as well as the wage rate. Nevertheless, keeping $(1-\tau^x)\left(1-\tau^R\right)$ constant also modifies fiscal receipts. Indeed, the fiscal base of the inheritance tax is p_2x_2 while the fiscal base of the capital income tax is

$$R_M \sum_i p_i s_i = \frac{p_1 d_1 + p_2\left(d_2 + x_2\right)}{1-\tau^R}$$

The latter is higher than p_2x_2 at least if the capital income tax rate is positive. Type-2 individuals save not only in order to leave bequests to their offsprings, but also to consume during old-age. To keep $(1-\tau^x)\left(1-\tau^R\right)$ constant, the fall in fiscal receipts from capital income tax will be larger than the increase in fiscal receipts from the inheritance tax. This results in a decrease in the steady-state public debt [3] that involves intergenerational redistribution from the first generations towards the ones living at a time where the economy is closed to the steady state.

We now complete the preceding Proposition by analyzing how life-cycle utility of type-2 agents is affected.

Proposition 10. *At steady state, for given τ^w and a, a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant increases the steady-state life-cycle utility of type-1 agents and reduces the one of type-2 agents.*

3. Differentiating equation (3.2.13) and assuming $d\tau^w = 0$, $da = 0$ and initially $\tau^x = 0$, one gets

$$\left(1-\frac{1}{\beta_2}\right) d\Delta = -R_M\left(K_M+\Delta\right) d\tau^R - p_2x_2 d\tau^x$$

where, to keep the capital-labor ratio constant: $d\left[\left(1-\tau^R\right)\left(1-\tau^x\right)\right]=0$. This implies

$$\left(1-\frac{1}{\beta_2}\right) d\Delta = \left[\left(1-\tau^R\right) R_M\left(K_M+\Delta\right) - p_2x_2\right] d\tau^x$$

Then, $d\Delta$ and $d\tau^x$ have opposite signs.

Proof. Since τ^w and a are not modified, the first-period income of type-1 agents is unchanged. Then applying Proposition 9 allows to state the result for type-1 agents.

For type-2 agents, differentiation of lifetime utility $u_2 = u(c_2, d_2)$ leads to

$$du_2 = u'_{c_2}\left[dc_2 + \beta_2\left(1-\tau^x\right)dd_2\right]$$

From the resource constraint (3.2.5) ,

$$p_1\left(c_1+d_1\right)+p_2\left(c_2+d_2\right) = \left(1-\Gamma\right)F\left(K_M,\bar{h}\right)-K_M$$

one gets

$$dc_2 + dd_2 = -\frac{p_1}{p_2}\left(dc_1+dd_1\right)$$

Moreover, equality (3.2.8) implies

$$\frac{\partial MRS_2^{d/c}}{\partial c_2}dc_2 + \frac{\partial MRS_2^{d/c}}{\partial d_2}dd_2 = -\beta_2 d\tau^x$$

The last two equalities lead to

$$\begin{aligned}&\left(\frac{\partial MRS_2^{d/c}}{\partial c_2}-\frac{\partial MRS_2^{d/c}}{\partial d_2}\right)\left[dc_2+\beta_2\left(1-\tau^x\right)dd_2\right]\\ = &-\left(1-\beta_2\left(1-\tau^x\right)\right)\beta_2 d\tau^x - \left[\beta_2\left(1-\tau^x\right)\frac{\partial MRS_2^{d/c}}{\partial c_2}-\frac{\partial MRS_2^{d/c}}{\partial d_2}\right]\frac{p_1}{p_2}\left(dc_1+dd_1\right)\end{aligned}$$

As shown in Appendix 3.6.1, strict concavity of u implies $\beta_2\left(1-\tau^x\right)\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2} > 0$ and normal good assumption implies $\frac{\partial MRS_2^{d/c}}{\partial c_2} > 0 > \frac{\partial MRS_2^{d/c}}{\partial d_2}$. To conclude, one needs to sign $dc_1 + dd_1$. Differentiation of the intertemporal budget constraint of type-1 agents (3.2.9) and of the marginal condition $MRS_1^{d/c} = \beta_2\left(1-\tau^x\right)$ leads to

$$dc_1 + dd_1 = \frac{1-\beta_2\left(1-\tau^x\right)+\left(\frac{\partial MRS_1^{d/c}}{\partial c_1}-\frac{\partial MRS_1^{d/c}}{\partial d_1}\right)d_1}{\frac{\partial MRS_1^{d/c}}{\partial c_1}\beta_2\left(1-\tau^x\right)-\frac{\partial MRS_1^{d/c}}{\partial d_1}}\beta_2 d\tau^x > 0$$

since the strict concavity of u implies that the denominator is positive and the normal good assumption implies that $\frac{\partial MRS_1^{d/c}}{\partial c_1} > 0 > \frac{\partial MRS_1^{d/c}}{\partial d_1}$. Consequently $dc_2 + \beta_2 (1 - \tau^x) dd_2 < 0$. This concludes the proof. □

For type-2 agents, the introduction of the inheritance tax reduces the relative price of old-age consumption. This positive effect on their utility is overcompensated by a fall in after-tax bequest $(1 - \tau^x) x_2$, leading to a reduction in utility.

Leaving the capital-labor ratio constant with the tax reform involves that aggregate resources available for consumption are constant. Thus, any consumption gain for one type of agent is offset by a loss of consumption for the other. For both types of dynasties, the fall in the relative price of old-age consumption leads the agents to shift part of their resources from the youth period to old-age. In addition, the marginal rate of transformation between d and c $(MRT_i^{d/c})$ is equal to one whereas the marginal rate of substitution $(MRS_i^{d/c} = \beta_2 (1 - \tau^x))$ is lower than one. As a result, any shift of consumption from c to d creates an inefficiency in the resource allocation for consumption.

As the utility of type-1 agents increases with the tax reform considered in Proposition 10, the utility of type-2 agents decreases both because of the transfer of resources to type 1-agents and also because of a greater inefficiency in the resource allocation between consumption when young and consumption when old.

The following Proposition states that a tax reform leaving the steady-state capital-labor ratio constant cannot increase lifetime utility of type-2 agents. The tax reform considered allow for changes in the labor income tax rate τ^w or the lump-sum transfer a, that we have kept constant until now.

Proposition 11. *Consider an initial steady-state equilibrium where $\beta_2 (1 - \tau^x) < 1$. Assume that government implements a tax reform that consists in a marginal increase in inheritance tax rate ($d\tau^x > 0$) and marginal changes in other tax instruments ($d\tau^R$, $d\tau^w$, da) such that the capital-labor ratio remains constant. If the reform does not reduce the lifetime utility of type-1 agents at steady state, then lifetime utility of type-2 agents necessarily decreases.*

Proof. The fiscal reform $\left(d\tau^x, d\tau^R, d\tau^w, da\right)$ is such that

— $d\tau^x > 0$.

— The capital-labor ratio remains unchanged, that is $d\left[(1-\tau^x)\left(1-\tau^R\right)\right] = 0$, or equivalently (since initially $\tau^x = 0$):

$$\frac{d\tau^R}{1-\tau^R} = -\frac{d\tau^x}{1-\tau^x}.$$

We consider the extreme case where the reform does not change lifetime utility of type-1 agents ($du_1 = 0$). We then check whether lifetime utility of type-2 agents can increase ($du_2 > 0$). Recall that du_i has the same sign as

$$dc_i + \beta_2 (1-\tau^x)\, dd_i$$

Then, type-1 utility does not change iff $dc_1 + \beta_2 (1-\tau^x)\, dd_1 = 0$. To compute dc_2 and dd_2, we combine the last equation with

— differentiation of the resource constraint (3.2.12) under the assumption of constant capital-labor ratio:

$$p_1 (dc_1 + dd_1) + p_2 (dc_2 + dd_2) = 0$$

— and differentiation of the marginal conditions $MRS_i^{d/c} = \beta_2 (1-\tau^x)$, for $i = 1, 2$, that is

$$\frac{\partial MRS_i^{d/c}}{\partial c_i} dc_i + \frac{\partial MRS_i^{d/c}}{\partial d_i} dd_i = -\beta_2 d\tau^x, \text{ for } i = 1, 2$$

Straightforward calculations lead to

$$dc_1 + dd_1 = \frac{\left[1-\beta_2 (1-\tau^x)\right] \beta_2 d\tau^x}{\beta_2 (1-\tau^x) \frac{\partial MRS_1^{d/c}}{\partial c_1} - \frac{\partial MRS_1^{d/c}}{\partial d_1}} > 0$$

and

$$\begin{aligned} dc_2 &+ \beta_2 (1-\tau^x)\, dd_2 \\ &= \frac{-\left[1-\beta_2 (1-\tau^x)\right] \beta_2 d\tau^x}{\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}} - \frac{\beta_2 (1-\tau^x) \frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}}{\frac{\partial MRS_2^{d/c}}{\partial c_2} - \frac{\partial MRS_2^{d/c}}{\partial d_2}} \frac{p_1}{p_2} (dc_1 + dd_1) \end{aligned}$$

which implies $du_2 < 0$. This concludes the proof. □

Therefore, the tax reform cannot increase the utility of type-2 agents whereas, as we have seen in Propositions 9 and 10, it is possible to design that reform in a way that increases lifetime utility of type-1 agents.

The crucial point in the preceding result is that disposable resources for consumption cannot vary since we have assumed a constant capital-labor ratio.

3.4 Time transfers and elastic labor supply

We now consider a case where keeping the capital-labor ratio constant does not imply that resources for consumption are fixed. The tax reform that consists in introducing an inheritance tax is reconsidered in a framework that combines elastic labor supply of the young and intergenerational time transfers from the old to the young as an alternative to leaving money.

Indeed, considering time transfers introduces a substitution effect of the inheritance tax on the trade-off between both types of intergenerational transfers. Inheritance tax makes time transfers more attractive and may increase time transfers and thus the young's labor supply.

3.4.1 A framework with time transfers

Households of generation t that belongs to type-i dynasties ($i \in \{1, 2\}$) consume a composite good that aggregates market good c_{it} when young (resp. when old d_{it+1}) and time spent in home production when young T_{it}^{y} (resp. T_{it+1}^{o} when old). Labor supply is elastic and the agent's labor supply decision depends on the trade-off between formal work and home production. The lifetime utility function becomes:

$$u\left(f^{y}\left(c_{it}, T_{it}^{y}\right), f^{o}\left(d_{it+1}, T_{it+1}^{o}\right)\right) \tag{3.4.1}$$

where u is increasing and strictly quasi-concave and f^y and f^o are linear homogeneous functions, with positive and decreasing first-order derivatives. f^y (resp. f^o) is the quantity of composite good when young (resp. when old).

The household's budget constraint during his working life is rewritten as follows:

$$c_{it} + s_{it} = (1 - \tau^w)\, h_i w_t \ell_{it} + (1 - \tau^x)\, x_{it} + a_t \tag{3.4.2}$$

where ℓ_{it} denotes type-i agent's labor supply in the formal sector, and satisfies:

$$\ell_{it} = 1 - T^y_{it} + \mu_i\,(1 - T^o_{it}) \tag{3.4.3}$$

where μ_i represents the productivity parameter of time transfer in home production of the young. We assume μ_i is the same for all dynasties of the same type.

When retired, type-i agent's budget constraint corresponds to the one of the preceding Section:

$$d_{it+1} + x_{it+1} = \left(1 - \tau^R\right) R_{t+1} s_{it} \tag{3.4.4}$$

The first-order conditions of type-i agent are given in the Appendix section 3.6.2.

On the production side, the factor prices w_t and R_t of the representative firm are equal to their marginal products (see equation (3.2.3)). The budget constraint of the government is the same as equation (3.2.4). The labor market equilibrium becomes:

$$L_t = \sum_i p_i h_i \ell_{it}$$

and the resource constraint is the same as equation (3.2.5) where $\bar{h}$ has been replaced with $\sum_i p_i h_i \ell_{it}$. The capital market equilibrium (3.2.6) is then satisfied as a consequence of the Walras' law.

3.4.2 Steady state

As shown in Appendix 3.6.2, only the more altruistic dynasties can leave bequests in the long-run. In the following, we assume that type-2 agents make positive bequests. Thus, $x_1 = 0$ and $x_2 > 0$, and the gross interest rate satisfies the modified Golden rule such that:

$$\beta_2 \left(1 - \tau^x\right) \left(1 - \tau^R\right) R_M = 1 \tag{3.4.5}$$

The steady-state capital-labor ratio z_M is then characterized by the equality between marginal product of capital $F_K\left(z_M, 1\right)$ and the gross interest rate R_M. The resource constaint at steady state then rewrites as

$$\sum_{i=1}^{2} p_i \left(c_i + d_i\right) = C_M \sum_{i=1}^{2} p_i h_i \left(1 + \mu_i\right) \ell_i \tag{3.4.6}$$

where

$$C_M \equiv \left(1 - \Gamma\right) F\left(z_M, 1\right) - z_M \tag{3.4.7}$$

We also assume that the productivity parameters of time transfers μ_i are high enough for all dynasties to leave time transfers: $T_i^o < 1$, for $i = 1, 2$.

Under non-negativity of time transfers and bequests at steady state, the first-order conditions of type-i agents can be rewritten as follows, for $i = 1, 2$,

$$MRS_i^{d/c} = \beta_2 \left(1 - \tau^x\right) \equiv P^R \tag{3.4.8}$$

$$MRS_i^{T^y/c} = \left(1 - \tau^w\right) h_i w \equiv P_i^y \tag{3.4.9}$$

$$MRS_i^{T^o/d} = \frac{\beta_i \mu_i \left(1 - \tau^w\right) h_i w}{\beta_2 \left(1 - \tau^x\right)} \equiv P_i^o \tag{3.4.10}$$

where P^R, P_i^y and P_i^o denote the relative prices respectively between d and c, between T^y and c, and between T^o and d.

3.4.3 Tax reform with time transfers

The tax reform is the same as in Section 3.3. To present its consequences, it is useful to distinguish interperiod and intraperiod effects:

1. *Interperiod effects.* The introduction of an inheritance tax decreases the relative price of the second-period market-good consumption P^R for both types agents. The fall in P^R is an *interperiod effect* which involves a negative effect on the consumption in composite good when young (negative effect on c_i and T_i^y) whereas the effect is positive for the composite good consumed when old (positive effect on d_i and T_i^o). The effect on young's labor supply is ambiguous since the fall of T_i^y increases the labor supply while the increase of T_i^o leads to the opposite effect. The magnitude of these effects depends on the intertemporal elasticity of substitution σ_i^u between the composite goods when young f^y and old f^o:
$$\sigma_i^u = \frac{d\ln\left(f^y/f^o\right)}{d\ln\left(u'_{f^o}/u'_{f^y}\right)} \tag{3.4.11}$$
where u'_{f^o} and u'_{f^y} stands for the marginal utilities of both composite goods.

2. *Intraperiod effects.* The tax reform leads also to increase the relative prices P_i^o $(i = 1, 2)$ between market good and time used in home production when old. This *intraperiod effect* has a positive impact on d_i and a negative effect on T_i^o. The negative effect on T_i^o affects positively time transfers and, therefore, the young's labor supply. Both dynasties have incentives to increase the young's labor supply through higher time transfers. The magnitude of this effect depends on the elasticity of substitution σ_i^o between d_i and T_i^o:
$$\sigma_i^o = \frac{d\ln\left(d_i/T_i^o\right)}{d\ln P_i^o} \tag{3.4.12}$$

The interperiod effect (through the fall in P^R) has some similarity with the one obtained in the model of the preceding Section with inelatic labor supply and no time transfers, but adds in the present framework new consequences on labor supply that depend on the relative changes in T_i^y and T_i^o, since $\ell_i = 1 - T_i^y + \mu_i\left(1 - T_i^o\right)$.

Indeed, interperiod effects involves lower T^y and higher T^o. Moreover, labor supply of the young is also modified by intraperiod effects through the fall in P_i^o, that also affect both times devoted to home production: higher T_i^y and lower T_i^o.

In the standard Barro (1974) model with inelastic labor supply, implemented a tax reform leaving the capital-labor ratio constant involves that the resources available for consumption are fixed. Indeed, as stated in Section 3.3, any consumption gain for one type of agent involves a loss of consumption for the other.

With time transfers and elastic labor supply, the tax reform affects the aggregate labor supply and thus may increase the aggregate resources for consumption of market goods. We then ask whether a Pareto-improving tax reform is possible. The most favorable situations for higher steady-state resources are those where the aggregate labor supply increases significantly.

In the following Proposition, we first consider the consequence of the tax reform on lifetime utility of type-1 agents.

Proposition 12. *For given τ^w and a, a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant increases the steady-state welfare of type-1 agents when the intraperiod effects dominate the interperiods effects, i.e. $\sigma_1^u \leq \sigma_1^o$.*

Proof. Differentiating steady-state life-cycle utility V_i and using marginal conditions (3.4.8)-(3.4.10), dV_i, for $i = 1, 2$, has the same sign as:

$$dV_i = dc_i + P_i^y dT_i^y + P^R dd_i + \beta_i \mu_i P_i^y dT_i^o \tag{3.4.13}$$

Since τ^w and the capital-labor ratio are kept constant with the tax reform, the relative prices P_i^y remains unchanged: $dP_i^y = 0$. Then, linear homogeneity of f^y implies that the ratio c_i/T_i^y is also unchanged, that is,

$$\frac{dc_i}{c_i} = \frac{dT_i^y}{T_i^y}, \text{ for } i = 1, 2. \tag{3.4.14}$$

Moreover, since $d \ln P_i^o = d \ln P_i^y - d \ln P^R$ and $dP_i^y = 0$, the definition of σ_i^o (see equation (3.4.12)) implies

$$\frac{dd_i}{d_i} - \frac{dT_i^o}{T_i^o} = -\sigma_i^o \frac{dP^R}{P^R}. \tag{3.4.15}$$

Finally, from the marginal condition (3.4.8), we get

$$\frac{u_{f_i^o}}{u_{f_i^y}} = \frac{f_{c_i}^y P^R}{f_{d_i}^o}.$$

which implies, using the definition of σ_i^u (see equation (3.4.11)),

$$\frac{df_i^o}{f_i^o} - \frac{df_i^y}{f_i^y} = -\sigma_i^u \left(\frac{df_{c_i}^y}{f_{c_i}^y} - \frac{df_{d_i}^o}{f_{d_i}^o} + \frac{dP^R}{P^R} \right).$$

Then, we obtain

$$\frac{dT_i^y}{T_i^y} - \frac{dT_i^o}{T_i^o} = -\alpha_i^o \left(\sigma_i^o - \sigma_i^u\right) \frac{dP^R}{P^R} \tag{3.4.16}$$

using the following relations deduced from linear homogeneity of f^y and f^o:

$$\frac{df_{c_i}^y}{f_{c_i}^y} = 0, \; \frac{df_{d_i}^o}{f_{d_i}^o} = \frac{f_{d_i d_i}^o d_i}{f_{d_i}^o} \left(\frac{dd_i}{d_i} - \frac{dT_i^o}{T_i^o} \right) \text{ and } \frac{-d_i f_{d_i d_i}^o}{f_{d_i}^o} = \frac{1-\alpha_i^o}{\sigma_i^o}$$

where $\alpha_i^o \equiv d_i f_{d_i}^o \,/\, f_i^o$.

Replacing equations (3.4.14)-(3.4.16) in (3.4.13) yields

$$dV_i = \Omega_i \left[\frac{dT_i^o}{T_i^o} + S_i \alpha_i^o \left(\frac{-dP^R}{P^R} \right) \right] \tag{3.4.17}$$

where

$$S_i \equiv \gamma_i \left(\sigma_i^o - \sigma_i^u\right) + \left(1-\gamma_i\right) \sigma_i^o, \quad \gamma_i \equiv \frac{c_i + P_i^y T_i^y}{\Omega_i} \text{ and } \Omega_i \equiv c_i + P_i^y T_i^y + P^R \left(d_i + P_i^o T_i^o\right)$$

Furthermore, differentiating the intertemporal budget constraint of type-1 agents

$$c_1 + P_1^y T_1^y + P^R d_1 + \mu_1 P_1^y T_1^o = P_1^y \left(1 + \mu_1\right) + a$$

leads to (using equations (3.4.14)-(3.4.16)):

$$\frac{dT_1^o}{T_1^o} = \frac{\alpha_1^o (1 - \gamma_1 - S_1)}{1 + (1 - \gamma_1) \frac{(1-\beta_1)(1-\alpha_1^o)}{\beta_1}} \left(\frac{-dP^R}{P^R} \right) \tag{3.4.18}$$

Thus, replacing in dV_1 (see equation (3.4.17) for $i = 1$), and given that the tax reform implies $dP^R < 0$, life-cycle utility of type-1 agents increases iff

$$1 + S_1 \frac{(1 - \beta_1)(1 - \alpha_1^o)}{\beta_1} > 0$$

which is true if $\sigma_1^u \leq \sigma_1^o$ since this implies $S_1 > 0$. This concludes the proof. □

From equation (3.4.18), notice that an increase in time transfer of type-1 dynasties arises iff $S_1 > 1 - \gamma_1$, or equivalently

$$\gamma_1 (\sigma_1^o - \sigma_1^u) + (1 - \gamma_1)(\sigma_1^o - 1) > 0$$

Therefore if the elasticity σ_1^o is larger than σ_1^u and 1, time resources of the young type-1 agents increase.

This suggests that, for high σ_1^o, intraperiod effect leads type-1 agents to increase the ratio d_1/T_1^o through the rise of time transfers. This may involve higher resources for the young type-1 agents. Additionnaly, if σ_1^u is high (for instance $\sigma_1^u = \sigma_1^o > 1$), the interperiod effect (fall in c_1 and T_1^y due to the fall in P^R) may dominate the intraperiod effect (increase in c_1 and T_1^y due to higher time transfers). In fact, the higher σ_1^u, the lower T_1^y, reinforcing the positive effect on labor supply of type-1 young agents ($\ell_1 = 1 - T_1^y + \mu_1 (1 - T_1^o)$).

Compared to the previous Section, the resources available for consumption of private goods may increase with the tax reform. Thus, the consumption gain for type-1 agents is not necessarily offset by a loss of consumption for type-2 agents.

Proposition 13. *For given τ^w and a, a shift from capital income tax towards inheritance tax that leaves the capital-labor ratio constant is Pareto-improving iff the two following conditions are satisfied (with a strict inequality for at least one of* *the two conditions)*

— $dV_1 \geq 0$, *or equivalently*

$$\theta_1 \equiv 1 + \left(\frac{1}{\beta_1} - 1\right)(1 - \alpha_1^o)\, S_1 \geq 0 \tag{3.4.19}$$

— $dV_2 \geq 0$, *or equivalently*

$$\sum_{i=1}^{2} p_i \Omega_i^M \alpha_i^o \left(S_i - S_i^M\right) \geq \frac{p_1 \Omega_1^M \alpha_1^o (1 - \gamma_1)}{1 + \left(\frac{1}{\beta_1} - 1\right)(1 - \alpha_1^o)(1 - \gamma_1)} \theta_1 \tag{3.4.20}$$

where $S_i^M \equiv \gamma_i^M (\sigma_i^o - \sigma_i^u) + (1 - \gamma_i^M)\, \sigma_i^o \left[1 - \left(1 - \alpha_i^{oM}\right)\left(1 - \frac{P_i^o}{C_M h_i \mu_i}\right)\right]$, $\gamma_i^M \equiv \frac{c_i + C_M h_i T_i^y}{\Omega_i^M}$, $\alpha_i^{oM} \equiv \frac{d_i}{d_i + C_M h_i \mu_i T_i^o}$, $\Omega_i^M \equiv c_i + d_i + C_M h_i (T_i^y + \mu_i T_i^o)$, *and* C_M *defined by (3.4.7).*

Proof. From equation (3.4.17), the marginal effect of the tax reform on the steady-state life-cycle utility of type-2 agents is positive iff:

$$\frac{dT_2^o}{T_2^o} \geq -S_2 \alpha_2^o \left(\frac{-dP^R}{P^R}\right)$$

To compute dT_2^o / T_2^o, we use the resource constraint (3.4.6):

$$\sum_{i=1}^{2} p_i \Omega_i^M = C_M \sum_{i=1}^{2} p_i h_i (1 + \mu_i)$$

where Ω_i^M is defined in the statement of Proposition 13 and C_M corresponds to production of market goods disposable for consumption per efficient labor unit (see equation (3.4.7)).

Differentiation of the resource constraint yields

$$\sum_{i=1}^{2} p_i \Omega_i^M \frac{d\Omega_i^M}{\Omega_i^M} = 0$$

where

$$\frac{d\Omega_i^M}{\Omega_i^M} = \frac{dT_i^o}{T_i^o} + S_i^M \alpha_i^o \left(\frac{-dP^R}{P^R}\right)$$

This implies

$$\frac{dT_2^o}{T_2^o} = -\frac{p_1\Omega_1^M}{p_2\Omega_2^M}\frac{dT_1^o}{T_1^o} - \left(\frac{p_1\Omega_1^M}{p_2\Omega_2^M}S_1^M\alpha_1^o + S_2^M\alpha_2^o\right)\left(\frac{-dP^R}{P^R}\right)$$

Then $dV_2 \geq 0$ iff

$$\frac{p_1\Omega_1^M}{p_2\Omega_2^M}\frac{dT_1^o}{T_1^o} + \left(\frac{p_1\Omega_1^M}{p_2\Omega_2^M}S_1^M\alpha_1^o + S_2^M\alpha_2^o\right)\left(\frac{-dP^R}{P^R}\right) \leq S_2\alpha_2^o\left(\frac{-dP^R}{P^R}\right)$$

where $\frac{dT_1^o}{T_1^o}$ is given by equation (3.4.18). One gets that $dV_2 \geq 0$ iff

$$p_1\Omega_1^M\alpha_1^o\left(\frac{1-\gamma_1-S_1}{1+(1-\gamma_1)\frac{(1-\beta_1)\left(1-\alpha_1^o\right)}{\beta_1}} + S_1^M\right) + p_2\Omega_2^M\alpha_2^o\left(S_2^M - S_2\right) \leq 0$$

which is equivalent to condition (3.4.20). This concludes the proof. □

Let us consider, as a benchmark, the case where there is no type-1 dynasties, i.e. $p_1 = 0$. Condition (3.4.20) rewrites as $S_2 > S_2^M$. We show in Appendix 3.6.3 that, under the condition $\mu_2 C_M h_2 > P_2^o$,[4] this inequality is satisfied in at least one of the two following cases: (i) σ_2^o relatively high with respect to σ_2^u; (ii) $\gamma_2 < \gamma_2^M$. These results are the same as the ones obtained in Belan and Moussault (2018) in an economy with homogeneous agents.

Notice that the assumption $\mu_2 C_M h_2 > P_2^o$ means that the marginal rate of transformation $(MRT_2^{T^o/d} = \mu_2 C_M h_2)$ is higher than the marginal rate of substitution $(MRS_2^{T^o/d} = P_2^o)$, that is, for a given (c_2, T_2^y) any reduction in T_2^o leads

4. From the definitions of C_M and P_2^o (equations (3.4.7) and (3.4.10)), the inequality $\mu_2 h_2 C_M > P_2^o$ is equivalent to

$$(1-\Gamma)F(z_M,1) - z_M > \frac{(1-\tau^w)F_L(z_M,1)}{1-\tau^x}.$$

It is satisfied for instance if $\tau^w = \Gamma$, $\tau^x = 0$ and $\tau^R \geq 0$ at the initial steady state. Indeed, in this case, it rewrites as

$$[(1-\Gamma)F_K(z_M,1) - 1]\,z_M > 0$$

or equivalently (from equation (3.4.5))

$$\beta\left(1-\tau^R\right) < 1.$$

to higher time transfers which increase the labor supply and then leaves enough additional resources for second-period consumption to maintain the type-2 agents with the same utility.

Let us give some interpretations of conditions (i) and (ii):

— $\sigma_2^o >> \sigma_2^u$ implies that the intraperiod effects (through higher P_2^o) dominate the interperiod effects (through the fall in P^R). In this context, the tax reform introduces a strong substitution effect between consumption of market good d_2 and time devoted to home production T_2^o, that increases time transfers and labor supply of the young, enhancing type-2 agents' welfare.

— The condition $\gamma_2 < \gamma_2^M$ is equivalent to

$$\frac{\frac{c_2}{T_2^y} + P_2^y}{\frac{c_2}{T_2^y} + C_M h_2} < P^R \frac{\frac{d_2}{T_2^o} + P_2^o}{\frac{d_2}{T_2^o} + \mu_2 C_M h_2}$$

A necessary condition for the last inequality is $C_M h_2 > \frac{P_2^y}{P^R}$ which implies that $C_M h_2 > P_2^y$ and hence $\mu_2 C_M h_2 > P_2^o$. Assuming $\tau^x = 0$ at the initial steady state, the inequality $\mu_2 C_M h_2 > P_2^o$ is equivalent to $C_M h_2 > P_2^y$. Then the condition $\gamma_2 < \gamma_2^M$ is likely to be satisfied if $\frac{c_2}{T_2^y}$ is low and $\frac{d_2}{T_2^o}$ is high, that is, domestic production is more intensive in time when young than when old.

By continuity, with small values of p_1, the tax reform creates a Pareto improvement, since it increases type-2 life-cycle utility without reducing type-1 utility (assuming $\sigma_1^o \geq \sigma_1^u$). However, for higher values of p_1, the increase in type-2 life-cycle utility is no longer guaranteed.

Nevertheless, differences between Ω_i^M can also make the tax reform Pareto improving. Indeed

$$\Omega_i^M = c_i + d_i + C_M h_i \left(T_i^y + \mu_i T_i^o\right)$$

Consumptions in time and market goods depend on the distribution of resources among both types of dynasties.

Using the intertemporal budget constraint of type-1 agents

$$c_1 + P_1^y T_1^y + P^R \left(d_1 + P_1^o T_1^o\right) = \Omega_1 = \left(1-\tau^w\right) h_1 w \left(1 - \left(1-\beta_1\right) \mu_1 T_1^o\right) + a$$

and marginal conditions (3.4.8)-(3.4.10), consumptions of the bundle (c_1, d_1, T_1^y, T_1^o) increase with Ω_1 under the normal good assumption. Then the higher Ω_1, the higher Ω_1^M.

Similarly, type-2 agents consumptions (c_2, d_2, T_2^y, T_2^o) that satisfy the intertemporal budget constraint

$$c_2+P_2^y T_2^y+P^R \left(d_2 + P_2^o T_2^o\right) = \Omega_2 = \left(1-\tau^w\right) h_2 w \left(1 - \left(1-\beta_2\right) \mu_2 T_2^o\right)+a+\left(1-\beta_2\right)\left(1-\tau^x\right) x_2$$

and marginal conditions (3.4.8)-(3.4.10), increase with Ω_2. Consequently differences between Ω_i's involve differences between Ω_i^M's. Then high bequests for type-2 agents can make the reform Pareto improving (as well as higher productivity for type-2 agents).

Therefore, there exist situations where inequality (3.4.20) is satisfied which involves that the tax reform may be Pareto-improving.

3.5 Conclusion

By considering the inheritance taxation in a fiscal reform that keeps the capital-labor ratio constant, we have shown that inheritance taxation is welfare enhancing for every altruistic dynasty except for the more altruistic one. However, keeping the capital-labor ratio constant with inelastic labor supply involves that disposable resources are constant. Thus, the tax reform consists in a transfer of resources across dynasties (through inheritance taxation) which can not be Pareto-improving. When we extend the model by introducing time transfers in a economy with elastic labor supply, the aggregate resources can increase with the tax reform (keeping the capital-labor ratio constant). Indeed, the implementation of inheritance tax makes time transfers more attractive which may increase the young's labor supply through the positive effect on time transfers. Finally, we have shown that the Pareto improvement of the tax reform strongly depends on the population distribution, as well as wealth distribution, between both types of agents and on the strength of the positive effect on the labor supply of every agent.

3.6 Appendix

3.6.1 Properties of the utility function

Let us consider a strictly concave utility function that satisfies normal goods assumption. Let us define the marginal rate of substitution as

$$MRS^{d/c} = \frac{u'_d(c,d)}{u'_c(c,d)}$$

Then, a bundle (c,d) that satisfies

$$MRS^{d/c} = P$$

is such that

$$\frac{\partial MRS^{d/c}}{\partial c}P - \frac{\partial MRS^{d/c}}{\partial d} > 0$$

and

$$\frac{\partial MRS^{d/c}}{\partial c} > 0 \text{ and } \frac{\partial MRS^{d/c}}{\partial d} < 0$$

The former inequality results from concavity, while the latter comes from the normal good assumption.

Indeed,

$$\begin{aligned} \frac{\partial MRS^{d/c}}{\partial c}P - \frac{\partial MRS^{d/c}}{\partial d} &= \frac{u'_d}{u'_c}\left(\frac{u''_{cd}}{u'_c} - \frac{u'_d}{u'_c}\frac{u''_{cc}}{u'_c}\right) - \left(\frac{u''_{dd}}{u'_c} - \frac{u'_d}{u'_c}\frac{u''_{cd}}{u'_c}\right) \\ &= \frac{1}{u'_c}\left[-\left(\frac{u'_d}{u'_c}\right)^2 u''_{cc} + 2\frac{u'_d}{u'_c}u''_{cd} - u''_{dd}\right] > 0 \end{aligned}$$

where the last inequality results from strict concavity of u. To establish the condition on u that implies c is a normal good, let us add a budget constraint $c + Pd = I$, where I would be the life-cycle income. Then

$$\frac{dc}{dI} = 1 - P\frac{dd}{dI}$$

where, from $MRS^{d/c} = P$,

$$\frac{\partial MRS^{d/c}}{\partial c}\frac{dc}{dI} + \frac{\partial MRS^{d/c}}{\partial d}\frac{dd}{dI} = 0$$

Consequently

$$\begin{aligned} \frac{dc}{dI} &= \left(\frac{\partial MRS^{d/c}}{\partial c}P - \frac{\partial MRS^{d/c}}{\partial d}\right)^{-1}\left(-\frac{\partial MRS^{d/c}}{\partial d}\right) \\ \frac{dd}{dI} &= \left(\frac{\partial MRS^{d/c}}{\partial c}P - \frac{\partial MRS^{d/c}}{\partial d}\right)^{-1}\frac{\partial MRS^{d/c}}{\partial c} \end{aligned}$$

Thus, both derivatives are positive iff

$$\frac{\partial MRS^{d/c}}{\partial c} > 0 \text{ and } \frac{\partial MRS^{d/c}}{\partial d} < 0$$

3.6.2 First-order conditions of the dynastic problem with time transfers

Plugging the consumptions c_{it}, d_{it+1} and c_{it+1} from the budget constraints (3.4.2)-(3.4.4) into the utility function (3.4.1), we get the following marginal conditions, for $i = 1, 2$:

— with respect to s_{it}

$$-u_{f_{it}^y} f_{c_{it}}^y + \left(1 - \tau^R\right) R_{t+1} u_{f_{it+1}^o} f_{d_{it+1}}^o = 0$$

— with respect to T_{it}^y (assuming interior solution $\mu_i\left(1 - T_{it}^o\right) < T_{it}^y < 1 + \mu_i\left(1 - T_{it}^o\right)$)

$$-\left(1 - \tau^w\right) h_i w_t f_{c_{it}}^y + f_{T_{it}^y}^y = 0$$

— with respect to x_{it+1}

$$-u_{f_{it+1}^o} f_{d_{it+1}}^o + \beta_i\left(1 - \tau^x\right) u_{f_{it+1}^y} f_{c_{it+1}}^y \leq 0, \; = 0 \text{ if } x_{it+1} > 0$$

— with respect to T^o_{it+1} (assuming $T^o_{it+1} > 0$)

$$u_{f^o_{it+1}} f^o_{T^o_{it+1}} - \beta_i \mu_i (1-\tau^w) h_i w_{t+1} u_{f^y_{it+1}} f^y_{c_{it+1}} \geq 0, \ = 0 \text{ if } T^o_{it+1} < 1$$

At steady state, marginal conditions with respect to x_{it+1} imply

$$\frac{u_{f^o_i} f^o_{d_i}}{u_{f^y_i} f^y_{c_i}} \geq \beta_2 (1-\tau^x) > \beta_1 (1-\tau^x)$$

Therefore, type-1 agents cannot leave positive bequests. In the text, we will assume that

— type-2 agents leave positive bequests;

— all time transfers are positive.

3.6.3 Tax reform with homogeneous agents

Consider the case with $p_1 = 0$. Steady-state lifetime utility of type-2 agents increases if and only if

$$S_2 > S^M_2$$

where

$$\begin{aligned} S_2 &= \gamma_2 (\sigma^o_2 - \sigma^u_2) + (1-\gamma_2) \sigma^o_2 \\ S^M_2 &= \gamma^M_2 (\sigma^o_2 - \sigma^u_2) + \left(1-\gamma^M_2\right) \sigma^o_2 \left[1 - \left(1-\alpha^{oM}_2\right)\left(1 - \frac{P^o_2}{C_M h_2 \mu_2}\right)\right] \end{aligned}$$

The inequality then rewrites

$$\frac{\sigma^o_2}{\sigma^u_2}\left(1-\gamma^M_2\right)\left(1-\alpha^{oM}_2\right)\left(1-\frac{P^o_2}{C_M h_2 \mu_2}\right) > \gamma_2 - \gamma^M_2$$

Under the condition $\mu_2 C_M h_2 > P^o_2$, the inequality is true if $\gamma_2 < \gamma^M_2$. By contrary, if $\gamma_2 > \gamma^M_2$, then the inequality is satisfied if σ^o_2 is high with respect to σ^u_2, that is, if intraperiod effects dominate interperiod effects.

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Conclusion générale

Cette thèse avait pour ambition principale d'analyser l'impact de l'introduction de l'impôt successoral sur la croissance et l'offre de travail, en prenant en compte la diversité des transferts familiaux intergénérationnels.

En particulier, l'objectif était d'étudier la taxation de l'héritage en comparant les effets désincitatifs qu'elle entraîne sur l'accumulation de capital aux effets liés à l'augmentation de l'attractivité des autres types de transferts, et ainsi d'analyser la répercussion de l'impôt successoral sur la croissance et l'offre de travail. L'impact de la taxation de l'héritage sur les autres types de transferts peut contrebalancer l'effet négatif sur l'accumulation de capital, en termes de ressources disponibles et de bien-être des ménages. A travers la diversité des transferts familiaux intergénérationnels, nous voulions également montrer le caractère différent de la taxation de l'héritage comparativement à celle du capital. Enfin, nous souhaitions aussi étudier la relation entre les transferts intergénérationnels publics et privés via l'impôt successoral.

Au fil des chapitres de ce travail doctoral, nous avons pu souligner plusieurs facettes de l'impôt sur les successions en prenant en compte la diversité des héritages. Nous avons montré que le passage de l'impôt sur les revenus du capital vers l'impôt successoral peut être Pareto-améliorant à long terme, en considérant des transferts de temps et des legs. Nous avons aussi mis en évidence le rôle central de la taxation des héritages comme outil de redistribution intergénérationnel ascendant ou comme outil de redistribution intragénérationnel. Enfin, nous avons souligné le caractère incitatif de la taxation des héritages au regard des autres types de transferts familiaux intergénérationnels et relevé que cet impact sur les autres transferts familiaux, tels que les dépenses d'éducation ou les transferts de temps, peut affecter la croissance économique de long terme, mais aussi l'offre de travail des ménages et/ou leur productivité.

Le premier chapitre nous a permis de mettre en évidence la nécessité d'une intervention publique pour s'assurer que les décisions des agents concernant leurs transferts familiaux correspondent aux choix optimaux le long de la trajectoire de croissance équilibrée. Dans le cadre de notre modèle, nous avons pu voir le rôle majeur de la taxation des héritages dans cette politique afin d'internaliser l'externalité positive sur le capital humain. Lorsque la dette publique permet une répartition optimale des transferts intergénérationnels, nous avons montré que la non-disponibilité de celle-ci pouvait entraîner un taux de croissance du capital humain plus élevé à long terme, résultant d'un accroissement de l'écart entre taxation du travail et du legs. Cette croissance plus élevée qu'au premier rang permet de réduire le ratio capital-travail et ainsi d'encourager les premières générations à consommer plus. En conséquence, cette politique de second rang utilise l'impôt successoral pour mettre en place une politique de redistribution ascendante. Ce chapitre a ainsi souligné le rôle de la taxation de l'héritage dans la répartition optimale des transferts familiaux intergénérationnels dans une économie basée sur la croissance du capital humain.

Dans les deux autres chapitres nous avons considéré des modèles à deux périodes avec altruisme rationnel, en s'interrogeant sur les différences entre la taxation du capital et celle sur les successions. Pour cela, nous avons étudié le même type de politique fiscale, dans laquelle l'introduction de l'impôt successoral conduit à une réduction de celui sur le revenu du capital, le tout en maintenant constant le ratio capital-travail à l'état stationnaire (réduisant ainsi l'impact négatif de la réforme fiscale sur le niveau de capital à long terme).

Dans le second chapitre, dans lequel les parents lèguent un héritage financier et transfèrent du temps à leurs enfants, nous avons considéré une réforme fiscale partant d'un équilibre intertemporel où l'impôt sur le revenu du capital est supérieur à son niveau efficace afin de financer une dette publique initiale. Nous avons ensuite regardé si la réforme fiscale pouvait être Pareto-améliorante. Dans ce cadre, nous avons montré que l'introduction de l'impôt sur les successions modifiait l'arbitrage entre les transferts de temps et les legs, ainsi que celui entre les consommations des deux périodes de vie. Dans le modèle de Barro, sans transfert de temps et offre de travail inélastique, l'effet sur l'arbitrage entre consommations implique une baisse du bien-être à l'état stationnaire. Cependant, avec des transferts de temps et une offre de travail élastique, nous avons montré que les grands-parents pouvaient être incités à transférer plus de temps et moins d'argent à la génération suivante, laquelle bénéficirait de plus de ressources en temps et serait capable de travailler plus. Ainsi, la réforme est Pareto-améliorante en fonction de la force de l'effet positif des transferts de temps sur l'offre de travail des jeunes, et de la force de l'effet de l'augmentation de l'offre de travail sur la production des biens marchands. En conséquence, à travers la réforme fiscale considérée, nous avons souligné l'importance de bien prendre en compte la diversité des transferts familiaux intergénérationnels lorsqu'on veut comparer les effets distordants de la taxation du capital et celle de l'impôt des successions, et leurs impacts sur la croissance et l'offre de travail.

Dans le dernier chapitre, dans lequel nous avons considéré des dynasties qui diffèraient en termes d'altruisme et de productivité, nous avons montré que l'impôt sur les successions améliorait le bien-être de toutes les dynasties, excepté celui des plus altruistes. Ce résultat a souligné l'importance de la taxation de l'héritage en tant qu'outil de redistribution intragénérationel lorsque la distribution des héritages est fortement concentrée comparativement à celle des revenus du capital. Cependant, maintenir le ratio capital-travail constant avec une offre de travail inélastique implique que les ressources disponibles soient constantes. Ainsi, la réforme fiscale consiste en un transfert de ressources entre les dynasties, ce qui ne conduit pas à une réforme Pareto-améliorante. L'introduction de transferts de temps avec une offre de travail élastique rend possible l'augmentation des ressources avec la réforme. Dans ce contexte, nous avons montré que la réforme pouvait être Pareto-améliorante et dépendait fortement de la force de l'effet de la hausse des transferts de temps sur l'offre de travail de chaque agent, ainsi que de la distribution de la population et de la répartition de la richesse. Par conséquent, la prise en compte des dons de temps peut renforcer l'impact positif de l'impôt successoral en tant qu'outil de redistribution intragénérationel.

De plus, dans le premier chapitre, nous avions mis l'accent sur les politiques de redistribution intergénérationnelles. Nous n'avions pas analysé l'effet de l'hétérogénéité intragénérationelle sur la croissance optimale de long terme et la politique fiscale à mettre en œuvre pour l'atteindre, bien que l'hypothèse d'agents homogènes en termes de degré d'altruisme et de productivité semblait réfutable, n'étant pas vérifié empiriquement, de même que peu réaliste. Nous pourrions désormais, par exemple, envisager le modèle du premier chapitre en considérant des dynasties à horizons de vie infinie dans lesquelles les individus de chaque génération peuvent être altruistes ou non. Ces disparités entre les agents et les dynasties devraient modifier la politique fiscale optimale et ainsi la croissance du capital humain le long de la trajectoire de croissance équilibrée. De plus, il serait également intéressant d'introduire une offre de travail élastique dans le modèle. Ainsi, l'impôt sur le revenu du travail modifierait aussi l'offre de travail des ménages, ce qui devrait impacter l'allocation des transferts familiaux et modifier la politique fiscale optimale ainsi que la croissance. Enfin, ce chapitre s'intéresse aux dépenses d'éducation privée des ménages. Cependant, dans la plupart des pays européens, une part non négligeable de l'investissement éducatif consacré aux générations futures est réalisée dans le domaine publique. L'étude de la prise en charge d'une part des investissements éducatifs privés par les dépenses publiques semblerait être un domaine d'approfondissement intéressant pour de futures recherches, et notamment pour comparer la politique fiscale et le niveau de capital humain de second rang à long terme, par rapport à ceux recueillis dans ce chapitre.

Concernant les deux derniers chapitres, dans lesquels on a considéré des transferts de temps et de legs, la question de la politique fiscale optimale reste en suspens. Il serait donc intéressant de l'étudier afin d'analyser le niveau de taxation optimal des héritages. En outre, il serait intéressant d'analyser empiriquement l'effet d'une variation de l'impôt successoral sur les transferts de temps pour vérifier si l'augmentation de l'attractivité des transferts de temps suite à l'introduction de l'impôt successoral est significative. Par exemple, nous pourrions analyser le cas de l'Italie qui, en plus d'être un pays caractérisé par des transferts en temps familiaux descendants importants, a aussi abrogé l'impôt successoral en 2001 pour le réintroduire en 2006.

Ainsi, ces divers thèmes à approfondir devraient faire l'objet de futures recherches et constituent une suite logique à ces travaux de thèse.

Annexe: altruismes, héritages et impôts successoraux

Dans cette partie, nous présentons les différentes formes de taxation de l'héritage ainsi que les principaux types d'altruisme que nous retrouvons dans la littérature afin de mieux pouvoir appréhender l'impôt successoral et ses effets dans toutes ces situations. En premier lieu, nous commençons par présenter les différentes natures et formes de la taxation de l'héritage puis les principaux motifs de legs et leurs implications en termes d'impôt successoral et pour enfin nous concentrer sur les situations qui intègrent plusieurs motifs de legs.

Nature et forme de l'impôt successoral

La richesse héritée est sans aucun doute affectée par la nature et la forme de l'impôt successoral, celles-ci variant fortement d'un pays à l'autre. La structure de la taxation de l'héritage distingue deux caractéristiques principales : la liberté de léguer (la nature) et le type de taxes (la forme).

Dans certains pays, le cadre légal contraint la liberté de léguer, en obligeant le légataire à transmettre une partie ou la totalité de l'héritage aux descendants directs ou ayant un lien de parenté avec le défunt. En France par exemple, les enfants sont héritiers réservataires, c'est-à-dire qu'ils ne peuvent être déshérités et qu'une partie non négligeable de l'héritage leur est réservée. De plus, le cadre légal peut imposer un partage équitable entre enfants (ou héritiers), comme c'est le cas en France. Inversement, il existe des pays où la liberté de léguer à quiconque est absolue, comme les Etats-Unis ou le Royaume Uni.

En outre, il y a deux façons de prélever l'impôt successoral. La première est de prélever directement la masse successorale, peu importe les caractéristiques et le nombre de bénéficiaires, on parle alors d'« estate taxation ». Cette forme de taxation est utilisée par les Etats-Unis, le Royaume-Uni ou le Danemark. La seconde impose le montant reçu par chaque héritier. Cette forme de taxation permet de mettre en place un taux différencié en fonction des liens de parenté des héritiers avec le défunt. On parle alors d' « inheritance taxation ». On la retrouve dans la plupart des pays Européens, comme la France et l'Allemagne.

De manière générale, nature et forme de l'impôt sur les successions vont de pair. D'une part, les pays qui imposent la masse successorale à travers l'« estate taxation » se distinguent par une liberté de léguer pleine et entière, sans distinction entre bénéficiaires, induisant ainsi une non-différenciation des taux d'imposition en fonction des liens de parenté (comme les Etats-Unis et le Royaume-Uni). D'autre part, les pays qui imposent directement les héritiers à travers l'« inheritance taxation » encadrent plus ou moins fortement cette liberté et distinguent différents taux d'imposition selon le degré de parenté entre le défunt et les bénéficiaires. Globalement, plus la proximité avec le défunt est grande, plus l'imposition est faible.

Ces deux types de taxation des héritages soulignent deux visions distinctes des gouvernements concernant la transmission familiale intergénérationnelle. Certains pays considèrent que laisser les parents décider du choix des bénéficiaires est juste et permet de réduire les inégalités intrafamiliales à travers les différents montants légués aux héritiers. D'autres choisissent en revanche de mettre en place un cadre légal afin de rendre la transmission équitable entre les descendants, en imposant par exemple des parts équivalentes entre les héritiers, ne pensant pas que le défunt fasse les bons choix au crépuscule de sa vie.

Une des questions intéressantes en économie est alors de savoir laquelle de ces formes de taxation est optimale. Il semble qu'en réalité le niveau optimal de taxation des héritages est bien différent de ces deux manières de voir l'impôt successoral. Cependant, lorsque le planificateur social et les individus accordent la même importance aux générations futures et qu'aucun d'entre eux ne souhaite mettre en œuvre une différenciation intragénérationnelle, imposer la masse successorale à travers l'« estate taxation » peut être alors Pareto Optimal. Lorsque des poids différents sont accordés aux générations futures, alors la taxation optimale se rapprochera plus d'une forme d'« inheritance taxation ». Pourtant, dans chacun de ces cas, l'inverse est aussi possible.

Les motifs de legs et l'impôt successoral

Les motifs de legs sont multiples et peuvent affecter différemment la taxation des héritages. On peut différencier deux types de motivations : les legs non prévus et ceux motivés par un sentiment considéré comme altruiste vis-à-vis de la descendance. Ce dernier, l'altruisme, est défini comme un acte désintéressé qui ne procure pas de bénéfice apparent à l'exécutant. Cependant, en économie, l'altruisme se caractérise comme un acte bénéfique à autrui qui accroît en retour le bien-être de la personne à l'origine de cet acte. On considère alors plusieurs types d'altruisme regroupant les principales motivations d'un individu altruiste. Les différentes formes sont : l'altruisme rationnel, paternaliste, familial et stratégique. Dans la suite, nous allons donc nous intéresser à toutes ces formes de motivations et leurs implications en termes de taxation de l'héritage. Nous commencerons par étudier les différentes formes d'altruisme puis nous regarderons ensuite les legs non prévus dont l'objectif initial n'est pas la transmission, tels que le legs accidentel ou le goût de l'individu pour la richesse.

L'altruisme rationnel

C'est la forme d'altruisme qui se rapproche le plus de la façon dont est perçu l'altruisme de manière générale. L'altruisme rationnel est lorsque le légataire est directement motivé par la hausse de l'utilité de sa descendance et ainsi par le bien-être des générations futures. La façon standard d'analyser cette forme d'altruisme est de considérer un modèle dynastique à horizon infinie, caractérisé par une infinité de générations liée par les legs.

La présence d'altruisme rationnel implique le plus souvent une égalité entre les facteurs d'actualisation sociaux/individuels et une contrainte de non négativité des legs. Cette forme de motivation a des effets positifs sur les inégalités intrafamiliales puisque les donateurs auront tendance à laisser différents montants de legs aux héritiers afin d'égaliser leurs revenus. De plus, l'altruisme rationnel conduit à l'équivalence Ricardienne puisque les parents peuvent compenser n'importe quelle politique de redistribution intergénérationnelle en utilisant les legs. Ce concept d'altruisme rationnel implique donc la neutralité de la dette publique sur les comportements d'accumulation du capital (voir Barro (1974)). La modélisation de cette forme d'altruisme étant similaire à un modèle d'agent à horizon de vie infinie, le résultat est le même que celui de Chamley (1986), c'est-à-dire une taxation nulle de l'héritage. En effet, l'impôt successoral crée des distorsions sur le comportement d'épargne des ménages et affecte ainsi négativement l'accumulation de capital. Cependant, ce résultat est basé sur des hypothèses fortes qui ont été remises en cause par une partie de la littérature.

L'altruisme paternaliste

Une autre façon de voir le comportement altruiste est le legs paternaliste, qui se définit comme la joie de donner. Avec cette forme d'altruisme, les parents sont motivés par l'utilité directe que procure l'acte de donner. Les bénéfices pour les héritiers ne sont pas pris en compte. Formellement, le legs paternaliste apparaît comme une consommation au cours de la dernière période de vie du défunt. Il y a cependant deux façons d'appréhender ce type d'altruisme. D'une part, lorsque le légataire s'intéresse à la valeur nette du don (c'est-à-dire à la valeur hors taxation des héritages), on parle d'une vision altruiste de ce motif. D'autre part, quand c'est la masse successorale totale transmise qui compte pour le donateur, on suggère une vision plus égoïste du legs paternaliste qui se rapproche du goût pour la richesse ou du legs accidentel (que nous allons étudier par la suite).

Quelle que soit la vision de ce motif, elle ne permet pas de réduire les inégalités intrafamiliales puisque la décision de l'agent en termes d'héritage ne prend pas en considération les bénéficiaires. En conséquence, cette forme d'altruisme n'affecte pas les politiques de redistribution intergénérationnelle. Cependant, les différences en termes de vision, modifient complètement l'effet de la taxation des héritages sur ce motif. Dans le cas d'un agent « égoïste », l'impôt successoral ne modifie pas son comportement en termes de legs. Ainsi, comme pour les legs accidentels, l'impôt successoral n'entraîne pas d'effet distordant et une taxation positive des héritages est possible. Inversement, l'impôt successoral affecte la décision d'un légataire qui s'intéresse à la valeur nette de sa succession et implique des effets distordants.

Néanmoins, ces deux façons d'appréhender l'altruisme paternaliste génèrent des externalités vis-à-vis des bénéficiaires en augmentant leurs ressources disponibles. La prise en compte ou non de ces externalités par le planificateur social peut modifier le niveau optimal de taxation. Harsanyi (1995) et Hammond (1988) ont préconisé « d'exclure toutes les préférences externes » de la fonction d'utilité sociale du planificateur, ce qui inclut toutes les formes d'altruisme afin d'éviter de donner trop de poids aux générations futures. Ainsi, en excluant l'altruisme de la fonction de bien-être social, les legs paternalistes perdent leur utilité sociale directe (voir Michel et Pestieau (2004)). Dans ce contexte, un niveau positif et élevé de taxation des héritages est obtenu pour un individu « égoïste » avec legs paternaliste. Cependant le résultat est plus incertain lorsque cela concerne un agent « altruiste » qui utilise ce motif d'héritage. En outre, lorsque la fonction de bien-être social prend en compte l'externalité, il n'est pas impossible d'avoir une taxation successorale négative (voir Farhi et Werning (2010)).

L'altruisme familial

L'altruisme familial peut être perçu comme à mi-chemin entre l'altruisme rationnel et l'altruisme paternaliste. Avec cette forme d'altruisme, le donateur souhaite améliorer les ressources de sa descendance et non son utilité. Formellement, les auteurs qui s'intéressent à l'altruisme familial, tels que Glomm et Ravikumar (1992), Lambrecht *et al.* (2005) et Lambrecht *et al.* (2006), prennent en compte les revenus de l'héritier dans la fonction d'utilité du légataire.

Les individus avec altruisme familial souhaitent augmenter les ressources disponibles de tous leurs enfants et sont ainsi incités à léguer plus aux enfants les moins bien dotés en revenus initialement. En conséquence, l'altruisme familial permet une réduction des inégalités intrafamiliales puisqu'il incite les donateurs à égaliser les ressources disponibles des héritiers. Comme avec l'altruisme rationnel, toute augmentation de l'impôt successoral sur les héritiers est compensée par une augmentation du legs des légataires. Cependant, à la différence de l'altruisme rationnel, le donateur s'intéresse uniquement aux ressources disponibles de la génération suivante et non à celles de toute la dynastie. Cette caractéristique de l'altruisme familial ne conduit pas à l'équivalence Ricardienne et implique ainsi la non-neutralité de la dette publique, comme avec de l'altruisme paternaliste.

Le legs stratégique

C'est une forme alternative de legs qui s'éloigne de la notion d'altruisme. Le motif de legs stratégique, développé par Bernheim *et al.* (1985), est lié à un concept d'échange réciproque entre parents et enfants. En effet, les enfants choisissent un niveau d'attention à fournir à leurs parents et en échange, les parents les rémunèrent à travers un legs prospectif. Le legs stratégique dépend de la richesse et des besoins du donateur. Les échanges peuvent impliquer différents types de services et ce type de legs donne lieu à des jeux stratégiques entre parents et enfants. Si les parents ont plus d'un enfant, ils peuvent extraire tout le surplus de leurs enfants en les menant à jouer l'un contre l'autre. En conséquence, l'héritage devient plus élevé pour les enfants qui apportent la forme d'attention particulière désirée par les parents. Avec des legs stratégiques, l'héritage n'est pas forcément égal entre les héritiers et n'est pas compensatoire entre les différentes générations (contrairement à l'altruisme rationnel). Ainsi, il ne s'intéresse pas aux inégalités intrafamiliales (qui peuvent soit augmenter ou diminuer en fonction du résultat des jeux stratégiques entre parents et enfants).

Comme pour les legs paternaliste ou accidentel, un légataire avec ce type de motif ne s'intéresse pas au bien-être de l'héritier. Ainsi, la politique de redistribution intergénérationnelle via la dette publique n'est pas affectée par les comportements individuels au regard de la transmission familiale intergénérationnelle. En ce qui concerne la taxation des héritages, elle peut être positive ou négative en fonction des élasticités de chacun (parents et enfants) vis-à-vis de celle-ci et aussi de la taxation sur la consommation future (puisque cette dernière forme de taxation affecte le gain de l'enfant).

Le legs accidentel

Ce type de legs non planifié provient de l'épargne de précaution ou de celle consacrée à la retraite qui n'a pas été consommée suite à la mort de l'individu. On représente ce motif en utilisant des modèles de cycles de vie dans lesquels les ménages ont de l'incertitude vis-à-vis de la date de leur décès. Les individus épargnent pour financer leur consommation future en étant confrontés à une probabilité de mourir prématurément. Ce type de legs se produit donc uniquement parce qu'il y a des imperfections sur le marché des annuités.

C'est, de par sa nature, un héritage non souhaité qui n'a pas pour objectif de réduire les inégalités intrafamiliales puisqu'il ne s'intéresse pas au bien-être des bénéficiaires. L'effet en termes d'inégalités de ce type de transfert reste ainsi incertain. Au regard de ses caractéristiques, les gouvernements peuvent imposer ce type de legs sans avoir des effets distordants sur le comportement d'épargne du donateur. En présence d'héritages accidentels, l'impôt sur les successions peut ainsi taxer dans leur totalité ces héritages sans avoir d'effet dissuasif sur le comportement des ménages. De plus, ce type d'altruisme ne perturbe pas une politique de redistribution intergénérationnelle ascendante via la dette publique puisque cette forme de transmission n'est pas motivée par le bien-être des générations futures.

Le goût pour la richesse

Cette forme de legs indirect, aussi appelée « esprit du capitalisme », survient lorsque la richesse elle-même procure un gain pour l'individu. Dans ce contexte, l'agent souhaite accumuler de la richesse tout au long de sa vie, comme une fin en soi ou afin de pouvoir quantifier son succès. Comme les legs accidentel ou paternaliste, ce motif de transmission ne s'intéresse pas au bénéficiaire du legs et n'a ainsi pas comme objectif la réduction des inégalités intrafamiliales. Le goût pour la richesse se rapproche du cas paternaliste avec agents « égoïstes » puisque c'est la richesse accumulée qui intéresse le défunt et non sa valeur hors-taxe. De la même façon, la richesse peut être considérée comme un bien de consommation de dernière période de vie. Cependant, l'une des différences majeures avec les autres motifs de legs est, qu'en présence du goût pour la richesse, le motif d'épargne initiale importe peu. En effet, quel que soit le motif (retraite ou legs) le résultat en termes de richesse accumulée est le même. Comme pour l'altruisme paternaliste ou accidentel, il y a une externalité positive envers la descendance en termes de ressources et la présence de ce motif de legs n'affecte pas la politique de redistribution intergénérationnelle (puisque le revenu ou le bien-être de la descendance ne sont pas pris en compte par ce type de motif). De plus, le goût pour la richesse se rapprochant du legs paternaliste avec agents « égoïste », il induit un possible niveau positif de taxation des héritages lorsque l'altruisme est exclu de la fonction de bien-être social (voir Michel et Pestieau (2004)).

Combinaison de plusieurs motifs de legs

Dans la littérature, il n'existe pas de consensus sur le motif de legs à adopter ou sur le fait de savoir si empiriquement l'un d'entre-eux l'emporte sur les autres. Ce constat est vrai aussi bien au niveau de l'individu qu'au niveau de la société. En effet, les ménages peuvent avoir différents motifs de legs mais aussi ils peuvent avoir tous individuellement plusieurs motivations de transmettre. Ainsi, certains auteurs développent des modèles avec des individus qui diffèrent en termes de motivations de léguer et d'autres considèrent un même type d'individus avec plusieurs motifs de legs dans sa fonction d'utilité.

On peut en effet supposer que les héritages actuels sont un mix en termes de préférence individuelle entre une motivation altruiste et des motifs de legs non voulus (comme l'accidentel ou le goût pour la richesse) puisque la forme d'altruisme caractérisant l'individu est exclusive vis-à-vis des autres motifs d'altruisme. La forme habituellement utilisée est un mix entre un motif de legs accidentel et un autre paternaliste « altruiste » (voir Pestieau et Sato (2008)). Etant données les caractéristiques de chaque motif, la transmission familiale intergénérationnelle ne permet pas la réduction des inégalités intrafamiliales mais cependant une politique de redistribution intergénérationnelle est possible. En ce qui concerne le niveau optimal de l'impôt successoral, il est issu d'un compromis entre l'objectif d'équité rendu possible à travers le motif accidentel et le souhait de ne pas décourager l'accumulation de capital via l'altruisme paternaliste. Ainsi, la taxation de l'héritage n'est pas forcément souhaitable.

Il est aussi important d'analyser les situations dans lesquelles les individus ont des préférences différentes en termes d'altruisme. On définit le plus souvent deux types d'individus : un agent avec de l'altruisme rationnel et un autre qui ne se préoccupe que de son propre bien-être (c'est-à-dire un individu sans altruisme). En outre, il y a aussi des modèles avec plusieurs types d'individus qui diffèrent en termes d'altruisme. Cependant, il n'y a que le type d'individu le plus altruiste qui souhaite transmettre des legs à l'état stationnaire, comme le montre Vidal (1996), Michel et Pestieau (1998) ou encore Nourry et Venditti (2001). Une politique de redistribution intergénérationnelle via la dette publique quoique neutre en terme agrégé, augmente l'inégalité intragénérationelle à l'état stationnaire. En effet, une dette publique plus élevée entraîne une hausse de la consommation des agents altruistes (à travers la hausse des paiements d'intérêts) et réduit celle de l'autre type d'agents (via la hausse des taxes pour financer la dette publique). En ce qui concerne la taxation de l'héritage à l'état stationnaire, payée uniquement par les dynasties les plus altruistes, cette forme de taxation réduit l'utilité non seulement des plus altruistes mais aussi celle des autres, même si le revenu de l'impôt successoral est redistribué uniformément (voir Michel et Pestieau (2005)). En effet, la taxation de l'héritage réduit le capital à l'état stationnaire, ce qui diminue le revenu du travail de tous les individus et ainsi affecte négativement leur consommation. Cet effet rend indésirable tout impôt successoral, même du point de vue des personnes qui ne transfèrent/reçoivent pas d'héritage et bénéficient d'un transfert redistributif.

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Résumé

Ce travail de thèse a pour objectif d'étudier l'impact de l'introduction de la taxation des héritages sur la croissance et l'offre de travail, en considérant la diversité des transferts familiaux intergénérationnels. En effet, la transmission familiale peut être éducative, culturelle, patrimoniale, ou encore perçue comme un transfert en temps. Toutes ces formes de solidarités familiales génèrent des externalités, qui impactent différemment la croissance et l'offre de travail, ce qui peut affecter l'efficacité des politiques fiscales. Ainsi, l'impôt successoral réduit l'incitation à épargner mais peut accroître l'investissement éducatif ou les transferts en temps, ce qui peut affecter positivement la productivité des ménages et l'offre de travail. Nous développons ici des modèles théoriques à générations imbriquées avec altruisme envers les descendants. La thèse est composée de trois chapitres. Le premier chapitre permet d'étudier l'impact de la non-disponibilité de la dette publique sur la politique de redistribution intergénérationnelle mise en place par le gouvernement, en utilisant uniquement l'impôt sur les revenus du travail et l'impôt successoral. Il permet aussi d'analyser son effet sur la croissance économique et les transferts familiaux intergénérationnels, consistant en des legs et des dépenses d'éducation, en mettant en évidence le rôle central de la taxation de l'héritage. Le second chapitre propose un modèle avec legs et transferts de temps descendants, dont l'objectif est de montrer les différences entre la taxation de l'héritage et la taxation du capital de cycle de vie, sur le comportement des ménages. Nous montrons que l'utilisation de la taxation de l'héritage à la place de celle du capital peut être une reforme Pareto-améliorante, en fonction de l'effet de la réforme sur l'offre de travail. Enfin, le troisième chapitre s'intéresse aussi à la comparaison entre taxation du capital et taxation de l'héritage, dans un modèle où les dynasties sont différentes en termes de productivité et de niveau d'altruisme. Ce chapitre démontre qu'appliquer l'impôt successoral à la place de celui du capital, peut améliorer à long terme, le bien-être des moins altruistes et, dans certains cas, peut être Pareto-améliorante, si les ressources disponibles pour les plus altruistes augmentent avec la réforme.

Mots clés : altruisme, transferts familiaux, taxation des héritages.

Abstract

This thesis analyzes the impact of inheritance taxation on growth and labor supply, considering the diversity of intergenerational family transfers, such that bequests, parent's education spendings or time transfers. These forms of family solidarity generate externalities, which impact growth and labor supply, and affect the effectiveness of tax policies. Concerning inheritance tax which reduces the incentive to save, it can also increase educational investment or time transfers, which can positively affect household productivity and labor supply. For this purpose, we use overlapping generations models with altruism towards offspring. The thesis is divided into three chapters. The first chapter studies the impact of public debt on intergenerational transfers and on human capital growth, using a simple tax structure with labor and bequest taxes. In this model, parents augment their children's income through education and bequest. When public debt is not available, we show that the long run growth is higher thanks to an increase of the gap between the two taxes, which underlines the role of inheritance taxation. The second chapter proposes a model with rational altruism *à la* Barro, where time transfers and bequests are available to parents. We analyze a shift from capital income tax towards inheritance tax, leaving constant the capital labor ratio. We show that this reform may increase welfare of all generations. Welfare improvement mainly depends on the effect of the reform on the labor supply. This tax reform is also implemented in the third chapter where we consider that dynasties differ in productivity and altruism. We show that the tax reform increases the welfare of less altruistic dynasties but decreases welfare of the most altruistic one. Extending the model with time transfers and elastic labor supply, we identify situations where the tax reform is Pareto improving.

Keywords: altruism, family transfers, inheritance tax.