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# MODÉLISATION MULTIVARIÉE HÉTÉROSCÉDASTIQUE ET TRANSMISSION FINANCIÈRE

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# Résumé

Cette thèse de doctorat composée de trois chapitres contribue au développement de tests statistiques et à analyser la transmission financière dans un cadre multivarié hétéroscédastique.
Le premier chapitre propose deux tests du multiplicateur de Lagrange de constance des corrélations conditionnelles dans les modèles GARCH multivariés. Si l'hypothèse nulle repose sur des corrélations conditionnelles constantes, l'hypothèse alternative propose une première spécification basée sur des réseaux de neurones artificiels et une seconde représentée par une forme fonctionnelle inconnue qui est linéarisée à l'aide d'un développement de Taylor.
Dans le deuxième chapitre, un nouveau modèle est introduit dans le but de tester la non-linéarité des (co)variances conditionnelles. Si l'hypothèse nulle repose sur une fonction linéaire des innovations retardées au carré et des (co)variances conditionnelles, l'hypothèse alternative se caractérise quant à elle par une fonction de transition non-linéaire : exponentielle ou logistique ; une configuration avec effets de levier est également proposée. Dans les deux premiers chapitres, les expériences de simulations et les illustrations empiriques montrent les bonnes performances de nos tests de mauvaise spécification.
Le dernier chapitre étudie la transmission d'information en séance et hors séance de cotation en termes de rendements et de volatilités entre la Chine, l'Amérique et l'Europe. Le problème d'asynchronicité est considéré avec soin dans la modélisation bivariée avec la Chine comme référence.

**Mots clés** : Scalar BEKK, transition lissée, effets de levier, GARCH multivarié, corrélation conditionnelle, covariance conditionnelle, test statistique, multiplicateur de Lagrange, Chine, rendements cotés en séance et hors séance, transmission de rendements et de volatilités.

# Abstract

This Ph.D. thesis composed by three chapters contributes to the development of test statistics and to analyse financial transmission in a multivariate heteroskedastic framework.
The first chapter proposes two Lagrange multiplier tests of constancy of conditional correlations in multivariate GARCH models. Whether the null hypothesis is based on constant conditional correlations, the alternative hypothesis proposes a first specification based on artificial neural networks, and a second specification based on an unknown functional form linearised by a Taylor expansion.
In the second chapter, a new model is introduced in order to test for nonlinearity in conditional (co)variances. Whether the null hypothesis is based on a linear function of the lagged squared innovations and the conditional (co)variances, the alternative hypothesis is characterised by a nonlinear exponential or logistic transition function; a configuration with leverage effects is also proposed.
In the two first chapters, simulation experiments and empirical illustrations show the good performances of our misspecification tests.
The last chapter studies daytime and overnight information transmission in terms of returns and volatilities between China, America and Europe. The asynchronicity issue is carefully considered in the bivariate modelling with China as benchmark.

**Keywords** : Scalar BEKK, smooth transition, leverage effects, multivariate GARCH, conditional correlation, conditional covariance, test statistic, Lagrange multiplier, China, daytime and overnight returns, volatility and return spillovers.

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# Introduction générale

La globalisation financière est un phénomène marquant de l'économie contemporaine. Les capitaux qui circulent à une vitesse et dans un volume de plus en plus élevés sont une des illustrations récentes des nouvelles problématiques que les investisseurs, les gestionnaires de portefeuilles et d'actifs, les analystes économiques et les chercheurs en économie et en finance doivent appréhender. L'apparition de modèles économétriques qui permettent d'analyser les volatilités a permis une nouvelle approche du risque. La tendance des marchés financiers peut dès lors être prédite grâce aux modèles à hétéroscédasticité conditionnelle (Engle (1982) et Bollerslev (1986), par exemple) qui fournissent de manière très élégante une façon de paramétriser la volatilité dynamique dans un cadre univarié (cf. Teräsvirta, 2007, pour une revue de la littérature). Cependant, ces modèles dans leur forme initiale ne prennent pas en compte les relations pouvant exister entre les séries ; c'est alors que l'approche multivariée prend tout son sens.

L'analyse économétrique de la volatilité dans un cadre multivarié, apparue à la fin des années 1980, est d'une grande importance pour la gestion, l'allocation, la diversification, le calcul de la value-at-risk et le choix de portefeuille optimal, ainsi que pour la prédiction des risques financiers (volatilités et co-volatilités). Les modèles AutoRégressifs Conditionnellement Hétéroscédastiques Généralisés (GARCH) multivariés paramétrisent les variances, covariances et corrélations conditionnelles. La littérature autour de ces modèles qui est notamment recensée dans les articles de Bauwens, Laurent and Rombouts (2006) et Silvennoinen and Teräsvirta (2009a) continue de se développer de nos jours.

Si les modèles GARCH endogénéisent la volatilité, il existe d'autres façons de l'appréhender. Les modèles à volatilité stochastique font de la volatilité une variable latente possédant une dynamique propre. Il existe une spécification, couramment utilisée, suffisamment riche pour reproduire les principales caractéristiques des séries financières et susceptible d'extensions. Cette spécification, appelée forme canonique, est

introduite par Taylor (1982) dans la littérature économétrique (voir également Taylor, 1986), tandis que les méthodes statistiques permettant de l'utiliser n'ont été développées qu'au cours des années 1990. Elle consiste à supposer que le logarithme de la volatilité, ou de son carré, suit un modèle autorégressif d'ordre 1. Lorsqu'on observe la volatilité d'un actif, il n'est pas toujours évident de dire que l'actif et le processus de volatilité soient totalement corrélés. Il y aurait donc un sens à modéliser la volatilité comme un processus stochastique à part entière, sans se limiter à des fonctions déterministes. Ainsi, Heston (1993) modélise la volatilité d'un actif comme n'étant ni constante, ni déterministe, mais suivant un processus aléatoire. Si les propriétés probabilistes de ces processus sont simples à établir, du moins par comparaison aux modèles GARCH, l'inférence statistique pose quant à elle de nombreux problèmes pouvant justifier l'utilisation de méthodes sophistiquées. De plus, comme la volatilité n'est pas directement observable, l'estimation des paramètres du modèle reste un problème difficile à résoudre. Comme pour les modèles GARCH, les modèles de volatilité stochastique peuvent être améliorés en incluant des sauts, capturer de l'asymétrie ou encore intégrer de la mémoire longue. Il existe également de nombreux modèles de volatilité stochastique multivariés recensés dans Asai, McAleer and Yu (2006).

Les modèles à hétéroscédasticité conditionnelle et les modèles de volatilité stochastique sont essentiellement paramétriques et sont utilisés pour estimer la volatilité avec des données de même fréquence. Depuis que la disponibilité des bases de données s'est répandue, il est possible d'utiliser les prix intra-journaliers d'actifs financiers à une très haute fréquence pour calculer des mesures de la volatilité à une fréquence plus faible (Bauwens, Hafner and Laurent, 2012, pour une revue des modèles de volatilité). Cette mesure *ex post* introduite par Andersen *et al.* (2003) est connue sous le nom de variance réalisée et permet ainsi de faire face à des problématiques contemporaines liées aux échanges à très haute fréquence.

Tous ces modèles, très brièvement exposés, présentent différentes manières (non exhaustives) de représenter la volatilité. Cependant, cette thèse se concentre essentiellement sur les modèles multivariés de type GARCH. Ces modèles filtrent l'auto-corrélation de second-ordre pour supprimer toutes dépendances résiduelles. De plus, leurs avantages (entre autres) sont une bonne prise en compte des propriétés des séries financières telles que la leptokurticité ou l'asymétrie, une étude complète de la structure du modèle, une estimation et des propriétés asymptotiques des estimateurs assez bien maitrisées. C'est donc les avantages d'un point de

vue statistique et également l'interprétation des résultats d'estimation de ces modèles qui nous motivent à contribuer à la littérature des modèles GARCH multivariés.

Contrairement aux modèles GARCH univariés, il n'existe que peu de tests spécifiques aux modèles GARCH multivariés. L'un des problèmes récurrents est le nombre important de paramètres nécessaires à la construction de ce type de modèles. Comme rappelé dans Bauwens, Laurent and Rombouts (2006), pour réduire le nombre de paramètres dans l'estimation des modèles GARCH multivariés, il est d'usage d'introduire des restrictions. En effet, un modèle non-contraint est généralement plus difficile à estimer qu'un modèle contraint, étant donné le nombre conséquent de paramètres.

Si l'estimation des modèles GARCH multivariés semble ne pas poser de problèmes particuliers (sinon les problèmes habituels que peut poser l'estimation de modèles non-linéaires complexes notamment en termes de convergence), il est préférable de tester des hypothèses pouvant conduire à réduire le nombre de paramètres. En effet, ne pas utiliser *ex ante* de tests de mauvaise spécification sur les données peut conduire à avoir des paramètres non identifiés. Il est à noter que le calcul, la programmation et l'estimation de ces modèles est assez chronophage pour ne pas mener préalablement quelques tests, et ainsi déterminer le bon modèle à utiliser. Par exemple, il existe quelques tests de constance de corrélations conditionnelles tels que ceux de Tse (2000), Bera and Kim (2002), Berben and Jansen (2005) ou Silvennoinen and Teräsvirta (2014, 2009b).

Si dans cette thèse la procédure du multiplicateur de Lagrange (LM) est utilisée, il est tout à fait possible d'utiliser les tests statistiques de Wald et du rapport de vraisemblance (LR). Comme discuté dans Francq and Zakoïan (2009), dès lors que sous l'hypothèse nulle certains paramètres ne sont pas identifiés ou à la limite de l'espace des paramètres, la distribution des tests n'est pas standard et requiert une correction des valeurs critiques habituelles. L'utilisation de la procédure *bootstrap* permet de déterminer la distribution non-standard de ces tests.

Cette thèse est constituée de trois chapitres. Dans les deux premiers chapitres l'accent est mis sur le développement de tests statistiques dans un cadre GARCH multivarié. Le dernier chapitre, qui utilise les tests développés dans le premier chapitre, porte sur l'analyse empirique de la transmission financière entre la Chine et le reste du monde, sur la base des modèles multivariés de type GARCH. Nous en venons maintenant à la présentation succincte du modèle GARCH multivarié général qui est utilisé dans l'ensemble de cette thèse. Les différents chapitres seront

ensuite également décrits.

La formulation générale du modèle GARCH multivarié peut être présentée comme suit. Considérons $\boldsymbol{\varepsilon}_t$ un processus stochastique vectoriel de dimension $N \times\ 1$ avec une moyenne conditionnelle $\mathsf{E}(\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}) = 0$. Notons $\mathcal{F}_t$ la tribu générée par toute l'information jusqu'à $t$ et incluant également la période $t$. Supposons que $\boldsymbol{\varepsilon}_t$ soit conditionnellement hétéroscédastique :

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t \tag{1}$$

compte tenu de l'ensemble d'information $\mathcal{F}_{t-1}$, où $\mathbf{H}_t$ est la matrice de covariances conditionnelles de dimension $N \times N$, $\boldsymbol{\eta}_t \sim iid(0, \mathbf{I}_N)$, et $\mathbf{I}_N$ est la matrice identité de dimension $N \times N$.

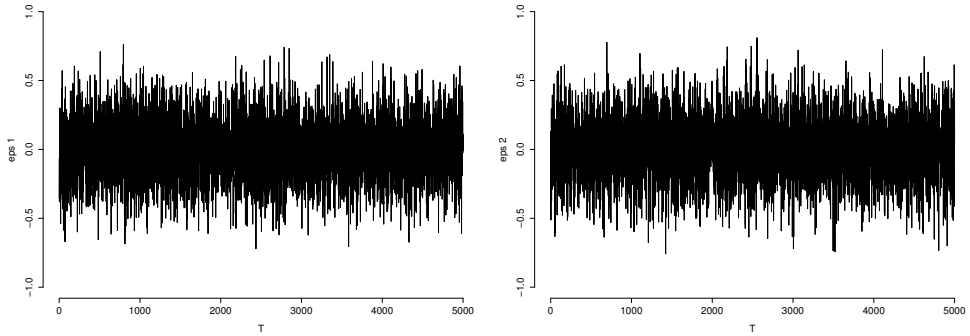
Les spécifications de la matrice $\mathbf{H}_t$ utilisées dans cette thèse appartiennent aux modèles de corrélations et de covariances conditionnelles.

Les modèles de corrélations conditionnelles utilisent l'écriture de la matrice de covariances conditionnelles sous la forme $\mathbf{H}_t = \mathbf{S}_t\mathbf{P}_t\mathbf{S}_t$, où $\mathbf{S}_t$ est la matrice diagonale des écarts-types dynamiques conditionnels, et $\mathbf{P}_t$ est la matrice de corrélations conditionnelles dynamiques définie positive de dimension $N \times N$. Cette modélisation permet d'expliciter les variances et les corrélations conditionnelles séparément. De nombreuses spécifications de la matrice de corrélations conditionnelles sont possibles. Elle peut être constante (Bollerslev, 1990, $\mathbf{P}_t \equiv \mathbf{P}$), dynamique (Aielli, 2013; Engle, 2002; Tse and Tsui, 2002) ou encore non-linéaire (Silvennoinen and Teräsvirta, 2014, 2009b), pour n'en citer que quelques unes.

Directement en lien avec cette famille de modèles, dans le Chapitre 1 intitulé "*Testing the constancy of conditional correlations in multivariate GARCH-type models*"[1] nous développons deux tests statistiques de constance des corrélations conditionnelles qui ne requièrent que peu d'information sur la fonction spécifiant ces corrélations. Les deux tests sont construits selon la procédure du multiplicateur de Lagrange où seule l'estimation du modèle sous l'hypothèse nulle de constance des corrélations est nécessaire. La matrice de corrélations conditionnelles sous l'hypothèse alternative est basée sur des réseaux de neurones artificiels pour un test et sur une approximation de Taylor d'une fonction inconnue pour l'autre test. La première configuration fait référence à une technique statistique proposée par Lee, White and Granger (1993). La seconde configuration est une expansion de Taylor de chaque corrélation conditionnelle inconnue autour d'un point donné dans un espace défini.

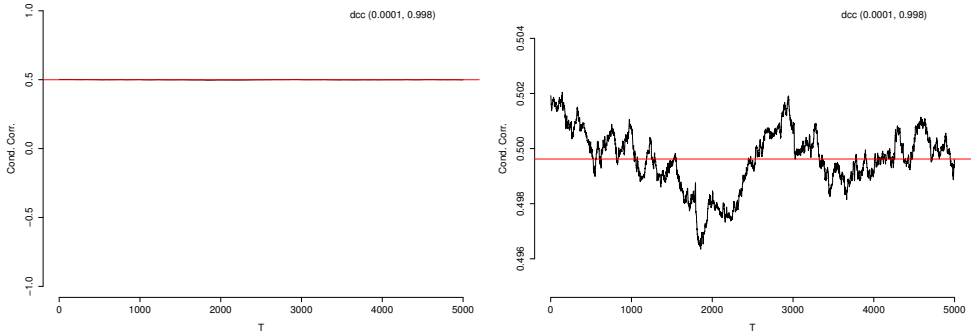
[1] Ce chapitre est tiré d'un papier co-écrit avec Anne Péguin-Feissolle (GREQAM-AMSE)

Si nous simulons deux séries de 5000 observations (Figures 1a et 1b), pouvant correspondre à des rendements par exemple, il paraît difficile de déterminer si la corrélation conditionnelle est constante ou dynamique sans utiliser de tests adéquats. En effet, lorsqu'on trace la corrélation conditionnelle selon différentes échelles (Figure 2), la Figure 2a qui correspond à l'échelle verticale de -1 à 1 nous ferait accepter la constance de cette corrélation conditionnelle, alors qu'une échelle plus petite, comme celle de la Figure 2b, montre bien que la corrélation varie dans le temps.




(a) $\varepsilon_{1t}$ (b) $\varepsilon_{2t}$

Figure 1: Génération d'un DCC-GARCH ($dcc:\ \alpha = 0.0001,\ \beta = 0.998$)




(a) $y\ axis = [-1, 1]$ (b) $y\ axis = [0.496, 0.504]$

Figure 2: Corrélation conditionnelle dynamique (DCC-GARCH) selon deux echelles verticales différentes (la ligne droite (rouge) représente la corrélation constante)

L'un des résultats de ce chapitre est que le test basé sur les réseaux de neurones artificiels sera le seul comme le montre la Table 1 à détecter

Table 1: Résultats des tests de constance des corrélations conditionnelles

| | Stat. | $p$value |
|---|---|---|
| NEURAL | 9.6122 | 0.0221 |
| TAYLOR | 3.6381 | 0.6026 |
| TSE | 0.1955 | 0.6583 |
| STCC | 0.1334 | 0.7148 |

*Note : $NEURAL$ est le test basé sur les réseaux de neurones artificiels, $TAYLOR$ est celui basé sur une expansion de Taylor, $TSE$ est le test développé par Tse (2000) et $STCC$ est celui développé par Silvennoinen and Teräsvirta (2014).*

la dynamique de la corrélation conditionnelle lorsque sa variabilité est faible (comme l'illustrent les deux graphes de la Figure 2), contrairement aux tests de Tse (2000) et de Silvennoinen and Teräsvirta (2014). Il est donc très fortement encouragé d'utiliser le test basé sur les réseaux de neurones artificiels conjointement au test basé sur une expansion de Taylor ; nos expériences de simulation montreront que ce dernier rejettera mieux l'hypothèse de constance des corrélations conditionnelles lorsque la variabilité est élevée.

Les propriétés en échantillon fini sont étudiées par des expériences de type Monte Carlo. La taille et la puissance, sous différents modèles et différentes distributions du processus générateur de données, montrent que nos deux nouveaux tests ont des résultats intéressants comparés aux tests de Tse (2000) et Silvennoinen and Teräsvirta (2014). Nos deux nouveaux tests présentent des caractéristiques fondamentales : ils ne nécessitent que peu de connaissance sur la relation fonctionnelle déterminant les corrélations, ils sont faciles à implémenter et fonctionnent assez bien dans nos simulations en échantillon fini. Ces deux tests peuvent être considérés comme des tests de mauvaise spécification générale pour un grand nombre de modèles très différents de type GARCH multivariés. En effet, un rejet de l'hypothèse nulle de constance des corrélations conditionnelles n'implique pas que les données aient été générées à partir d'un modèle où les corrélations conditionnelles sont spécifiées comme des fonctions neurales ou des fonctions spécifiques. Il est ainsi tout à fait possible de déterminer si les variables économiques ou financières auxquelles on s'intéresse suivent un modèle de corrélations conditionnelles constantes ou dynamiques. Il est également intéressant de remarquer que ces tests

pourraient être facilement utilisés pour tester l'hypothèse de corrélations partielles constantes (par exemple, lorsqu'on a trois rendements, pour tester la constance de la corrélation conditionnelle uniquement entre les deux premiers rendements). Dans l'analyse empirique de ce chapitre, trois séries sont testées de manière bivariée. Les résultats montrent que ces tests peuvent être très utiles pour rejeter l'hypothèse nulle de constance des corrélations dans une modélisation multivariée conditionnellement hétéroscédastique.

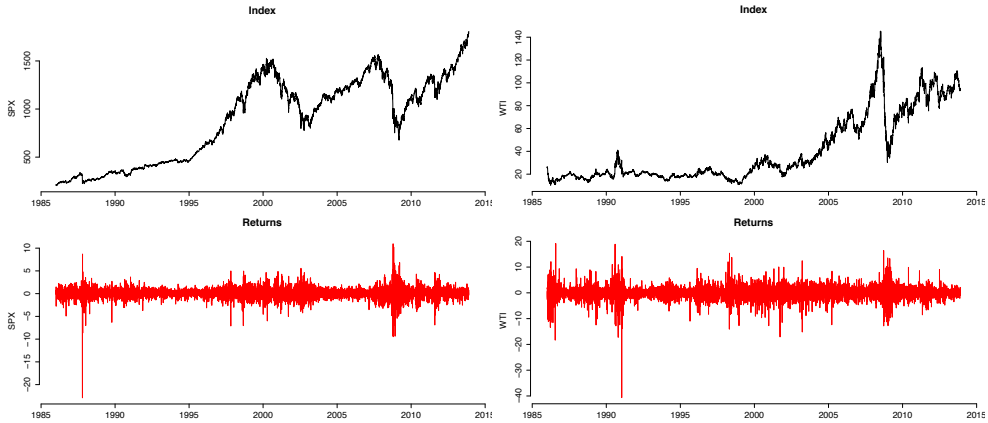
Les modèles de covariances conditionnelles, quant à eux, modélisent directement les covariances conditionnelles tout en assurant que la matrice des variances et covariances conditionnelles est définie positive. L'un des modèles les plus connus est le modèle BEKK de Engle and Kroner (1995), qui peut inclure, par exemple, de l'asymétrie (Kroner and Ng, 1998). Un nouveau modèle faisant partie de la famille des modèles de covariances conditionnelles est développé dans le Chapitre 2 intitulé "*Testing for nonlinearity in conditional (co)variances*"[2], dans le but de tester la non-linearité des variances et covariances conditionnelles. Comme précédemment, les tests développés dans ce chapitre suivent la procédure du multiplicateur de Lagrange. Deux tests sont ainsi construits sur la base de deux fonctions de transition qui servent à spécifier les covariances conditionnelles sous l'hypothèse alternative : la fonction logistique et la fonction exponentielle. Le modèle sous l'hypothèse nulle est le scalar BEKK de Ding and Engle (2001), une spécification particulière du modèle BEKK, dans lequel les co-volatilités des séries temporelles sont déterminées par une fonction linéaire de leurs propres retards et des innovations retardées au carré. Par ailleurs, les propriétés asymptotiques ainsi que les moments marginaux d'ordre quatre sont étudiés. Cependant, il se peut que le modèle sous l'hypothèse alternative ne soit pas le vrai modèle générateur de données. En effet, une covariance conditionnelle impliquant une relation non-linéaire qui ne serait ni exponentielle ni logistique serait tout à fait envisageable. Suivant la même procédure, deux autres tests robustes aux effets de levier sont également développés. Ainsi, grâce à une procédure simplifiée en deux étapes qui seront explicitées dans ce second chapitre, il est possible de discriminer entre les effets de levier, la non-linéarité et la linéarité au sein des covariances conditionnelles.

Les propriétés en échantillon fini des tests sont étudiées par des simulations de Monte Carlo et montrent de bons résultats pour leurs tailles et leurs puissances dès lors que la taille de l'échantillon atteint 1000 observations. Dans l'analyse empirique de ce chapitre nous considérons

[2]Ce chapitre est issu d'un article soumis à publication.

13 séries financières composées d'actions, d'indices et de devises. L'étude porte sur tous les couples d'actions possibles, ainsi que sur un nombre de variables endogènes égal à 3, 4, 8 et 9. Les tests rejettent assez souvent l'hypothèse nulle de linéarité motivant l'utilisation de covariances conditionnelles non-linéaires et montrant ainsi l'utilité des test introduits dans ce chapitre.

Considérons à titre d'exemple deux séries financières : le S&P500 et le WTI de 1986 à 2014 (valeurs d'indices et rendements sont exposés dans la Figure 3). Nous estimons un modèle bivarié de type scalar BEKK ; les résultats de l'estimation sont illustrés dans la Figure 4 où nous voyons l'estimation des variances conditionnelles et de la covariance conditionnelle.




(a) S&P500 (b) WTI

Figure 3: S&P500 et WTI (valeurs d'indices et rendements)

Il est très difficile de déterminer visuellement si nous sommes en présence de linéarité, de non-linéarité ou d'effets de levier dans les variances et covariances conditionnelles – bien que les chocs de rendements laissent présager que les variances et covariances conditionnelles n'obéissent pas à des spécifications linéaires. Les tests développés dans ce chapitre quant à eux permettent de discriminer clairement entre ces notions en rejetant dans ce cas l'hypothèse nulle de linéarité, et en indiquant la présence de non-linéarité et d'effets de levier.

Le dernier chapitre intitulé "*Volatility spillovers across daytime and overnight information between China and world equity markets*"[3] traite de la question de la transmission des rendements et des volatilités en prenant

[3]Ce chapitre est extrait d'un papier co-écrit avec Jian Hua (GREQAM-AMSE)

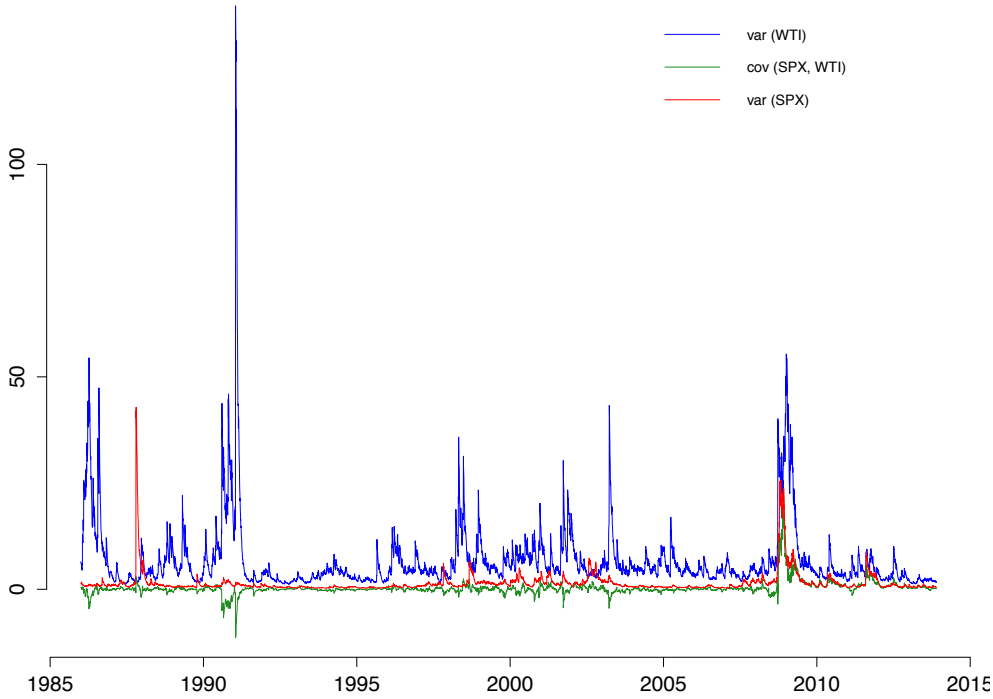



Figure 4: Résultats de l'estimation par le modèle scalar BEKK

la Chine comme référence dans un cadre hétéroscédastique multivarié. En effet, dans le contexte de mondialisation financière, l'intégration des marchés financiers a été une question très controversée en finance internationale compte tenu de son importance pour la gestion des risques et la diversification de portefeuille que les investisseurs internationaux et les gestionnaires de portefeuilles et d'actifs doivent considérer. Ceci est d'autant plus pertinent que les marchés boursiers à travers le monde sont de plus en plus intégrés dans le temps, en particulier dans les pays émergents. La récurrence et la fréquence élevées des crises financières, caractérisées par des *clusters* de volatilité, sont des sujets très actuels. Si c'est vrai pour un marché financier particulier, les périodes de crises sont aussi caractérisées par la transmission de chocs de volatilité d'un marché à l'autre, en raison de l'intégration financière mondiale.

Les modèles GARCH multivariés sont un outil puissant pour l'étude de la transmission financière. Dès lors que la matrice des paramètres des modèles de corrélations ou de covariances conditionnelles est pleine, il est possible de mesurer la direction et l'amplitude de la transmission des chocs et des volatilités.

Le dernier chapitre s'intéresse tout particulièrement à la Chine. La

Chine est un pays émergent qui a vu accroître considérablement son poids dans l'économie mondiale et il nous semble intéressant de mesurer les interactions existantes avec neuf autres marchés financiers internationaux situés en Amérique, en Asie et en Europe, en tenant compte du problème récurrent d'asynchronicité, qui définit le décalage entre l'ouverture et la fermeture des marchés financiers, dans ce type d'analyse. Pour réaliser cette étude nous séparons préalablement les rendements d'indices en fonction de l'information fournie en séance et hors séance de cotation, dans le but d'identifier le type d'information transmis. Pour obtenir une vision parcimonieuse, nous étudions les relations que la Chine a avec le reste du monde, avant, durant et après la crise financière globale de la fin des années 2000. Il est à noter qu'après 2005 la libéralisation du marché financier de la Chine avait été mise en place, ce qui nous permet de mesurer sa réelle intégration au sein du marché global. Après avoir spécifié les équations de moyennes conditionnelles sous certaines hypothèses, nous modélisons les seconds moments conditionnels avec un modèle de corrélations conditionnelles constantes (Bollerslev, 1990) ou dynamiques (Engle, 2002) dans leur forme étendue c'est-à-dire, avec des matrices de paramètres pleines. Le test développé dans le Chapitre 1 est utilisé et permet donc de discriminer entre une matrice de corrélations conditionnelles constantes ou dynamiques. Ainsi, pour chaque relation bivariée les paramètres adéquats sont estimés en fonction du modèle approprié. Il est important de prendre en compte le problème potentiel d'asynchronicité dans notre échantillon. En effet, l'interprétation des résultats économétriques dépend du fait que certains marchés ont différentes heures d'échange : par exemple, un marché encore ouvert ne peut pas influencer un marché déjà fermé. Les résultats empiriques montrent qu'à travers les transmissions d'information en séance de cotation la Chine a des relations plus proches avec les marchés asiatiques qu'avec les marchés non-asiatiques, tandis qu'à travers les transmissions d'information hors séance de cotation ces relations semblent être inverses.

# Chapter 1

# Testing the constancy of conditional correlations in multivariate GARCH-type models[1]

## 1.1 Introduction

During the last decade, there has been a rapid increase in the amount of literature on theoretical and empirical derivatives of multivariate GARCH-type modelling (see, for example, extensive surveys of Bauwens, Laurent and Rombouts (2006) and Silvennoinen and Teräsvirta (2009a). This econometric modelling is an important issue, particularly in finance. Indeed, it is largely accepted that empirical models are more relevant in a multivariate than an univariate framework. By taking into account the possible dependence between individual equations, one can determine for example option pricing, asset pricing, hedging and risk management or portfolio selection, as noted in Bauwens, Laurent and Rombouts (2006). For instance, a crucial problem in financial studies is the correlation structure among different national stock returns because it determines the gains from international portfolio diversification. Although few tests are specific to multivariate models, it would be desirable to check *ex ante* the properties of the data especially since estimating multivariate GARCH (MGARCH) models is time-consuming. Moreover, it is worth noting that, if we don't test first the hypothesis of the constancy of conditional correlations, the estimation of some multivariate type-GARCH models

[1]This chapter is part of joint work with Anne Péguin-Feissolle.

could lead to have unidentified parameter estimates.

This chapter is thus concerned with developing two tests for constancy of the conditional correlations.

The first test is based on artificial neural networks (ANN) or, more precisely, on single hidden layer perceptrons (later on, we use the generic term of artificial neural networks, following the terminology employed in the econometric literature). Their main interest is that they are universal approximators. This ANN-based test relies on a statistical technique proposed by Lee, White and Granger (1993). The ANN framework has been already used for some tests: see, for example, Lee, White and Granger (1993) and Teräsvirta, Lin and Granger (1993) for linearity tests, Kamstra (1993), Caulet and Péguin-Feissolle (2000) and Péguin-Feissolle (1999) for conditional heteroscedasticity tests, Lebreton and Péguin-Feissolle (2007) for heteroscedasticity test and Péguin-Feissolle and Teräsvirta (1999) for causality tests.

The second test we introduce is based on a Taylor expansion of each unknown conditional correlation around a given point in a sample space; it is a way of linearisation the testing problem by approximating the true relationships by a Taylor series expansion. Thus, because of the linearisation of the unknown relationship determining each conditional correlation, this test is not computationally more difficult to carry out than traditional tests. This kind of test has already been introduced in order to test the causality (Péguin-Feissolle and Teräsvirta, 1999; Péguin-Feissolle, Strikholm and Teräsvirta, 2013), the heteroskedasticity (Lebreton and Péguin-Feissolle, 2007) and the conditional heteroskedasticity (Caulet and Péguin-Feissolle, 2000; Péguin-Feissolle, 1999).

Therefore, the two tests present four fundamental characteristics. First, they require little knowledge of the functional relationship determining the correlations. Secondly, they are easy to implement and perform well in our small-sample simulations; indeed, we show in the simulations that they are relevant in small samples and can be useful in investigating potential conditional correlation. Thirdly, they generalize well to high dimensions, i.e. with a high number of endogenous variables. Fourthly, given the properties of the ANN to be universal approximators and given that the Taylor expansion permits the linearisation of an unknown relationship, a rejection of the null hypothesis of constancy of conditional correlations does not imply that the data have been generated from a model where the conditional correlations are specified as neural functions or specific functions; these tests can thus be seen as general misspecification tests of very different GARCH-type multivariate models. It is worthwhile to

remark too that these tests could be used easily to test also the hypothesis of partially constant correlations.

Finite-sample properties of the two new tests are examined using Monte Carlo methods by comparing them to two alternative conditional correlation tests: the tests of Tse (2000) and Silvennoinen and Teräsvirta (2014). Using a variety of different specifications for the MGARCH model, we show that they perform well in our small-sample simulations; they approximate quite well unknown specifications of the conditional correlations, even in the case where the normality hypothesis is not verified. The ANN-based test has good properties when the variability of the conditional correlation coefficients is moderate and low, while the Taylor expansion-based test is better when the variability is high. The main explanation of their good performance is probably because their original form is very general; the first test is based on artificial neural networks that are universal approximators, and the second one comes from the linearisation of an unknown relationship determining each conditional correlation. Empirical illustrations using real data point out that these tests can be useful to reject the constant correlation hypothesis in multivariate modelling of conditional heteroskedasticity.

The chapter is organized as follows. Section 1.2 makes a review of different conditional correlation tests based on the Lagrange Multiplier (LM) procedure. In Section 1.3, we introduce the ANN-based constant correlation test and the Taylor-expansion based test. Section 1.4 reports the results of a simulation study; we investigate the size and the power by Monte Carlo experiments in small samples. In Section 1.5, we study the robustness of size and power of all the tests to nonnormality by simulating some models such as the $GARCH - t$ model of Bollerslev (1987), or the $Beta - t - EGARCH$ model of Harvey and Sucarrat (2014) and Harvey and Chakravarty (2008). Section 1.6 describes some illustrative empirical examples based on financial data. Section 1.7 contains some conclusions. Technical derivations of the test statistics presented in the chapter can be found in the Appendix 1.A.

## 1.2 Literature review of the different LM conditional correlation tests

Most of the tests for the constancy of conditional correlations are based on the LM procedure. Some other tests are based on Likelihood Ratio procedure (see, among others, Engle and Sheppard, 2001; Longin and

Solnik, 1995). But, as noted by Tse (2000), computing the LM test statistics necessitates only the estimation of the constant-correlation model and is thus generally computationally convenient.

Tse (2000) introduces a Lagrange Multiplier test for the constant-correlation hypothesis in a multivariate GARCH model; he extends the constant-correlation model to one in which the correlations are allowed to be time varying and he tests for the zero restrictions on the key parameters. Bera and Kim (2002) apply the White's (1982) and Orme (1990) information matrix (IM) test to the constant correlation bivariate GARCH model and establish the linkage between the IM and score tests. Silvennoinen and Teräsvirta (2014) derive an LM test for the constancy of correlations against the Smooth Transition Conditional Correlation (STCC) GARCH model in the multivariate case with an endogenous or exogenous transition variable. Berben and Jansen (2005) introduce a bivariate GARCH model with smoothed time-varying correlations based on logistic function, called the Smooth-Transition Correlation GARCH (STC-GARCH) model, and derive a new test for constant correlation, building on the Lagrange Multiplier test developed by Tse (2000). Their model is bivariate and the variable controlling the transition between the extreme regimes is simply the time. Silvennoinen and Teräsvirta (2009b) extend the test of Silvennoinen and Teräsvirta (2014) for the constancy of correlations against the DSTCC-GARCH (double smooth transition conditional correlation GARCH) model, i.e. by allowing the conditional correlations to vary according to two transition variables that can be stochastic or deterministic (for example, exogenous variables or lagged elements of endogenous variables). To circumvent the nuisance parameter problem, the last three papers replace the logistic transition function by a first-order Taylor approximation to the logistic function around one of the transition parameters.

## 1.3 The ANN-based test and the Taylor expansion-based test

In order to fix the model that we consider and the notations, we present a general MGARCH model. Let $\{\mathbf{y}_t\}$ be a multivariate time series, where $\mathbf{y}_t$ is a $N \times 1$ vector. Consider the following model defined by, for $t = 1, ..., T$:

$$\begin{aligned} \mathbf{y}_t &= E[\mathbf{y}_t|\Omega_{t-1}] + \boldsymbol{\varepsilon}_t \\ Var\left[\mathbf{y}_t\,|\Omega_{t-1}\right] &= \mathbf{H}_t \end{aligned} \tag{1.1}$$

where $E[\mathbf{y}_t|\Omega_{t-1}]$ and $Var\left[\mathbf{y}_t\,|\Omega_{t-1}\right]$ are respectively the conditional expectation and the conditional variance of $\mathbf{y}_t$ with respect to $\Omega_{t-1}$, the sigma-field generated by all the information until time $t-1$. To simplify the notations, we will use below $E_{t-1}[\mathbf{y}_t]$ and $Var_{t-1}\left[\mathbf{y}_t\right]$ instead of $E[\mathbf{y}_t|\Omega_{t-1}]$ and $Var\left[\mathbf{y}_t\,|\Omega_{t-1}\right]$ respectively. The process $\mathbf{y}_t$ is strictly stationary and ergodic. To simplify the discussion (but the test exposed in this chapter could be generalized easily to a large variety of time-varying structures of the conditional expectation of $\mathbf{y}_t$), we assume that the observations are of zero means: for $t = 1, ..., T$, $E_{t-1}[\mathbf{y}_t] = 0$ or

$$\mathbf{y}_t = \boldsymbol{\varepsilon}_t; \tag{1.2}$$

$\boldsymbol{\varepsilon}_t$ is defined by

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t \tag{1.3}$$

with $\boldsymbol{\eta}_t \sim iid\,(0, \mathbf{I}_N)$. We assume for simplicity that the conditional variances follow an GARCH(1,1) process:

$$h_{iit} = \alpha_{i0} + \alpha_i\varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1} \qquad i = 1, ...N \tag{1.4}$$

where the standard positivity and covariance stationary constraints are verified, i.e. $\alpha_{i0} > 0$, $\alpha_i \geq 0$, $\beta_i \geq 0$, and $\alpha_i + \beta_i < 1$ for $i = 1, ..., N$. Moreover, the conditional covariances are:

$$h_{ijt} = \rho_{ijt}\sqrt{h_{iit}h_{jjt}} \qquad 1 \leq i < j \leq N \tag{1.5}$$

where $\rho_{ijt}$ is the conditional correlation defined by

$$\rho_{ijt} = Cor[\varepsilon_{it}, \varepsilon_{jt}|\Omega_{t-1}] = \frac{h_{ijt}}{\sqrt{h_{iit}h_{jjt}}}. \tag{1.6}$$

Let $\mathbf{P}_t = (\rho_{ijt})_{i,j=1,...,N}$ be the $N \times N$ conditional correlation matrix for the $\boldsymbol{\varepsilon}_t$; we can write

$$\mathbf{H}_t = \mathbf{S}_t\mathbf{P}_t\mathbf{S}_t \tag{1.7}$$

where $\mathbf{S}_t$ is the $N \times N$ matrix given by

$$\mathbf{S}_t = diag(\sqrt{h_{11t}}, ..., \sqrt{h_{NNt}}). \tag{1.8}$$

We assume that the conditional correlation matrix $\mathbf{P}_t$ is positive definite at each $t$; it assures the positive definiteness of $\mathbf{H}_t$. Moreover, we will assume the conditional normality of the $\boldsymbol{\varepsilon}_t$:

$$\boldsymbol{\varepsilon}_t|\Omega_{t-1} \sim N\,(0, \mathbf{H}_t) \tag{1.9}$$

which implies

$$\mathbf{z}_t|\Omega_{t-1} \sim N\left(0, \mathbf{P}_t\right), \tag{1.10}$$

with $\mathbf{z}_t = \mathbf{S}_t^{-1}\boldsymbol{\varepsilon}_t$.

In order to test the constancy of conditional correlations, we specify the null hypothesis:

$$H_0 : \rho_{ijt} = \rho_{ij}.$$

In the next two subsections we propose two original specifications for $\rho_{ijt}$.

### 1.3.1 The ANN-based test

The first test we present is an ANN based LM-type test. In order to build the test, we extend here the ideas of the test for the constant correlation hypothesis of Tse (2000), Silvennoinen and Teräsvirta (2014, 2009b) and Berben and Jansen (2005) to a model where the specification of the conditional correlations $\rho_{ijt}$ given by (1.6) is defined by a neural function. Artificial neural network models provide another way of dealing with situations where the functional form of a potential relationship between two variables is not assumed known in advance. As noted in Péguin-Feissolle (1999), the main theoretical support for the use of ANN models in the present context is the universal mapping theorem: it states that under mild regularity conditions, ANN models provide arbitrarily accurate approximations to nonlinear mappings (see Carroll and Dickinson, 1989; Cybenko, 1989; Hornik, 1991; Hornik, Stinchcombe and White, 1989, 1990; Stinchcombe and White, 1989, among others). Indeed, as noted in Hornik, Stinchcombe and White (1989), the ANN can be used as universal approximators: standard multilayer feedforward networks are capable of approximating any measurable function to any desired degree of accuracy and are not restricted to the approximation of particular forms of functions. Therefore, the test can be seen as a general misspecification test of a large set of GARCH-type multivariate models. Because the ANN are universal approximators, a rejection of the null hypothesis of constancy of conditional correlations does not imply that the data have been generated from a model where the conditional correlations are specified as neural functions.

We are going to specify the time-varying structure of the conditional correlations. We assume that the conditional correlations $\rho_{ijt}$ are changing smoothly over time depending on a neural function as follows

$$\rho_{ijt} = \rho_{ij} + \sum_{k=1}^{p} \delta_{ijk}(1 + \exp\{-w'_{ijt}\gamma_{ijk}\})^{-1} \tag{1.11}$$

where $1 \leq i < j \leq N$, $p < \infty$, $w_{ijt}$ and $\gamma_{ijk}$ are $(2q+1) \times 1$ vectors, $\delta_{ijk}$ is a scalar, and $w_{ijt}$ is given by

$$w_{ijt} = (1, \widetilde{w}'_{ijt})' = (1, \varepsilon_{i,t-1}, ..., \varepsilon_{i,t-q}, \varepsilon_{j,t-1}, ..., \varepsilon_{j,t-q})'. \tag{1.12}$$

The formula (1.11) represents the output of a single hidden layer feedforward network: input units of the network send signals, amplified or attenuated by weighting factors $\gamma_{ijk}$ to $p$ hidden units (or hidden nodes) that sum up the signals and generate a squashing function, or activation function; this function is assumed here to be a logistic function but other kinds of activation functions can be found in the literature (hyperbolic tangent function, radial basis functions, threshold function, Gaussian function, ...).

The conditional correlations $\rho_{ijt}$ should respect the restrictions $|\rho_{ijt}| \leq 1$, $1 \leq i < j \leq N$ and $t = 1, ...T$, but these constraints are very difficult to quantify in (1.11). Because we test the null hypothesis of constant conditional correlation, i.e., $\rho_{ijt} = \rho_{ij}$, we will not try to search for these restrictions but we will assume that they are verified in a neighbourhood of the null hypothesis (as noted by Tse, 2000, p. 111).

As was previously mentioned, it is important to remark that neural functions of the form (1.11) may approximate arbitrary functions quite well, given a sufficiently large number $p$ of hidden units and a suitable choice of the parameters $\delta_{ijk}$ and $\gamma_{ijk}$.

Following the definition (1.11) of the conditional correlations, the null hypothesis of constant conditional correlation, i.e., $\rho_{ijt} = \rho_{ij}$, can be formulated as:

$$H_{01} : \delta_{ijk} = 0,\ 1 \leq i < j \leq N,\ 1 \leq k \leq p. \tag{1.13}$$

Under $H_{01}$, the $\gamma_{ijk}$, for $i = 1, ..., N-1$, $j = i+1, ..., N$ and $k = 1, ..., p$, are not identified. For this reason, the conventional maximum likelihood theory for deriving the test procedure is not applicable, as noted in Lebreton and Péguin-Feissolle (2007). To solve this problem, we use the method given by Lee, White and Granger (1993). Following these authors, we draw the unidentified parameters, i.e. the hidden unit weights $\gamma_{ijk}$, for $i = 1, ..., N-1$, $j = i+1, ..., N$ and $k = 1, ..., p$, randomly from the uniform $[-\mu, \mu]$ distribution. Notice that in Monte Carlo simulations (Sections 1.4 and 1.5) and the empirical illustration (Section 1.6) we choose $\mu = 2$. According to Davies (1977) and Lundbergh and Teräsvirta (2002), if we do not circumvent this identification issue, the test will not be valid as its asymptotic null distribution is unknown.

Therefore, the vector $\theta$ of all the parameters of the model is composed by the $3 \times 1$ vectors determining the conditional variances, $\omega_i = (\alpha_{i0}, \alpha_i, \beta_i)'$ for $i = 1, ..., N$, the conditional correlation $\rho_{ij}$ with $1 \leq i < j \leq N$ and the parameters in the neural function, $\delta_{ijk}$ for $1 \leq i < j \leq N$ and $p < \infty$.

The null hypothesis can be tested using the LM procedure. Under standard regularity conditions, the test statistic is given by:

$$NEURAL = \frac{1}{T}\left(\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \theta}\right)' \Im(\theta)^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \theta}\right) \tag{1.14}$$

where $\Im(\theta)$ is replaced by the consistent estimator:

$$\Im(\theta) = \frac{1}{T}\sum_{t=1}^{T} E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \theta}\frac{\partial l_t(\theta)}{\partial \theta'}\right] \tag{1.15}$$

and the log-likelihood for the observation $t$, $l_t(\theta)$, is given by:

$$l_t(\theta) = -\frac{N}{2}\ln(2\pi) - \frac{1}{2}\ln(|\mathbf{P}_t|) - \frac{1}{2}\sum_{i=1}^{N}\ln(h_{iit}) - \frac{1}{2}\mathbf{z}_t'\mathbf{P}_t^{-1}\mathbf{z}_t. \tag{1.16}$$

Notice that whether to build our statistic we use the consistent estimator of the score's outer product (1.15), in practice (see Tse, 2000), $\Im(\theta)$ may be replaced using the consistent estimator of the negative of the Hessian matrix:

$$\Im(\theta) = -\frac{1}{T}\sum_{t=1}^{T} E_{t-1}\left[\frac{\partial^2 l_t(\theta)}{\partial\theta\partial\theta'}\right]. \tag{1.17}$$

We also could use a third configuration by using the consistent estimator of the product of the score's outer product (1.15) and the negative Hessian matrix (1.17), the well-known sandwich matrix, in order to capture the specification error.

The statistic $NEURAL$ has an asymptotic $\chi^2$ distribution under the null with $p\frac{N(N-1)}{2}$ degrees of freedom. Appendix 1.A.1 presents the details of the technical derivations of the test statistic.

As Lee, White and Granger (1993) and Péguin-Feissolle and Teräsvirta (1999) pointed out, the elements of some matrices used to build the ANN-based test statistic can lead to collinearity problems when the number of hidden units $p$ is large. Therefore, in our case, the $p \times 1$ vectors $g_{ijt}$

defined as, for $1 \leq i < j \leq N$, $p < \infty$:

$$g_{ijt} = \begin{pmatrix} g_{ijt,1} \\ \vdots \\ g_{ijt,p} \end{pmatrix} = \begin{pmatrix} (1+\exp\{-w'_{ijt}\gamma_{ij1}\})^{-1} \\ \vdots \\ (1+\exp\{-w'_{ijt}\gamma_{ijp}\})^{-1} \end{pmatrix} \tag{1.18}$$

tend to be collinear among themselves especially when $p$ is large. The conditional correlations $\rho_{ijt}$ given by (1.11) can be written as follows

$$\rho_{ijt} = \rho_{ij} + \sum_{m=1}^{p} \delta_{ijm} g_{ijt,m}.$$

Let us define $G_{ij}$ the $p \times T$ matrix given by:

$$G_{ij} = (g_{ij1}, ..., g_{ijT}).$$

Thus we conduct the test using the main principal components of each $G_{ij}$ matrix. Only the largest principal components that together explain at least 90% of the variation in this matrix are used; the number of principal components are determined automatically for each $i$ and $j$ (see Péguin-Feissolle and Teräsvirta (1999), Lebreton and Péguin-Feissolle (2007), Péguin-Feissolle, Strikholm and Teräsvirta (2013); see also Castle and Hendry (2010) for discussions on using principal components to solve the collinearity problem). More precisely, instead of $G_{ij}$, we build the $p^*_{ij} \times T$ matrix $G^*_{ij}$ of the $p^*_{ij}$ principal components chosen according to the preceding rule, i.e.

$$G^*_{ij} = \left(g^*_{ij1}, ..., g^*_{ijT}\right).$$

The null hypothesis is now that the parameters associated to the main principal components are equal to zero in the following specification of the conditional correlations:

$$\rho_{ijt} = \rho_{ij} + \sum_{m=1}^{p^*_{ij}} \delta_{ijm} g^*_{ijt,m}.$$

The statistic $NEURAL$ will have in this case an asymptotic $\chi^2$ distribution under the null with $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} p^*_{ij}$ degrees of freedom.

### 1.3.2 The Taylor expansion-based test

We assume that the functional form $f_{ij}$ determining $\rho_{ijt}$, for $i, j = 1, ..., N$ and $t = 1, ..., T$, is unknown and is adequately represented by the following equation:

$$\rho_{ijt} = f_{ij}\left(\widetilde{w}_{ijt}, \theta^*_{ij}\right) \tag{1.19}$$

where, as given in (1.12), for each couple of indices $(i, j)$, $\widetilde{w}_{ijt}$ is a $2q \times 1$ vector given by

$$\widetilde{w}_{ijt} = (\varepsilon_{i,t-1}, ..., \varepsilon_{i,t-q}, \varepsilon_{j,t-1}, ..., \varepsilon_{j,t-q})' \tag{1.20}$$

and $\theta^*_{ij}$ is a $r_{\theta^*_{ij}} \times 1$ unknown parameter vector. The conditional correlations $\rho_{ijt}$ should respect the restrictions $|\rho_{ijt}| \leq 1$, $1 \leq i < j \leq N$ and $t = 1, ...T$; but, like in the ANN-based test, we will simply assume that these restrictions are verified in a neighbourhood of the null hypothesis.

The test is based on a finite-order Taylor expansion. Following Péguin-Feissolle and Teräsvirta (1999) and Péguin-Feissolle, Strikholm and Teräsvirta (2013), we assume that all the functions $f_{ij}$ have a convergent Taylor expansion at any arbitrary point of the sample space for every $\theta^*_{ij} \in \Theta_{ij}$ (the parameter spaces) in order to ensure that, when the order of the Taylor expansion increases, the remainder of the Taylor expansion converges to zero, for each couple $(i, j)$.

In order to linearise now $f_{ij}$ in (1.19), we expand the function into a $k$th-order Taylor series around an arbitrary fixed point in the sample space. After approximating $f_{ij}$, merging terms and reparametrizing, we obtain:

$$\begin{aligned}
\rho_{ijt} = \; & \rho_{ij} + \sum_{m=1}^{q} \lambda_m \varepsilon_{i,t-m} + \sum_{m=1}^{q} \phi_m \varepsilon_{j,t-m} + \sum_{m_1=1}^{q} \sum_{m_2=m_1}^{q} \lambda_{m_1 m_2} \varepsilon_{i,t-m_1} \varepsilon_{i,t-m_2} \\
& + \sum_{m_1=1}^{q} \sum_{m_2=1}^{q} \psi_{m_1 m_2} \varepsilon_{i,t-m_1} \varepsilon_{j,t-m_2} + \sum_{m_1=1}^{q} \sum_{m_2=m_1}^{q} \phi_{m_1 m_2} \varepsilon_{j,t-m_1} \varepsilon_{j,t-m_2} \\
& + ... + \sum_{m_1=1}^{q} \sum_{m_2=m_1}^{q} ... \sum_{m_k=m_{k-1}}^{q} \lambda_{m_1 ... m_k} \varepsilon_{i,t-m_1} ... \varepsilon_{i,t-m_k} \\
& + ... + \sum_{m_1=1}^{q} \sum_{m_2=m_1}^{q} ... \sum_{m_k=m_{k-1}}^{q} \phi_{m_1 ... m_k} \varepsilon_{j,t-m_1} ... \varepsilon_{j,t-m_k} + R_t^{(k)}(\widetilde{w})
\end{aligned} \tag{1.21}$$

where $R_t^{(k)}$ is the remainder, and $q \leq k$ for notational convenience; in the expansion (1.21) we find all possible combinations of lagged values of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$.

The assumption that the conditional correlation $\rho_{ijt}$ is constant means that all terms involving functions of elements of lagged values of $\varepsilon_{i,t}$ and

$\varepsilon_{j,t}$ in (1.21) must have zero coefficients, i.e. all the parameters except $\rho_{ij}$ are equal to zero. Therefore, the null hypothesis of interest is:

$$H_{02}: \begin{cases} \lambda_m = 0,\ m = 1, ..., q \\ \phi_m = 0,\ m = 1, ..., q \\ \lambda_{m_1 m_2} = 0,\ m_1 = 1, ..., q,\ m_2 = m_1, ..., q \\ \psi_{m_1 m_2} = 0,\ m_1 = 1, ..., q,\ m_2 = 1, ..., q \\ \phi_{m_1 m_2} = 0,\ m_1 = 1, ..., q,\ m_2 = m_1, ..., q \\ \vdots \\ \phi_{m_1 ... m_k} = 0,\ m_1 = 1, ..., q,\ m_2 = m_1, ..., q, ...,\ m_k = m_{k-1}, ..., q. \end{cases}$$

The number of parameters to be tested under the null hypothesis is:

$$N^* = 2\left[\sum_{r=1}^{k} \begin{pmatrix} 2q + r - 1 \\ r \end{pmatrix} - \sum_{r=1}^{k} \begin{pmatrix} q + r - 1 \\ r \end{pmatrix}\right] \tag{1.22}$$

with $\begin{pmatrix} m \\ r \end{pmatrix} = \frac{m!}{r!(m-r)!}$.

The null hypothesis can be tested based on the LM procedure as before in the case of the $NEURAL$ statistic. Under standard regularity conditions, the test statistic is given by:

$$TAYLOR = \frac{1}{T}\left(\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\right)' \Im(\theta)^{-1} \left(\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\right) \tag{1.23}$$

where $\Im(\theta)$ is replaced by the consistent estimator (1.15), the log-likelihood $l_t(\theta)$ is given by (1.16), and $\theta$ is the vector of all the parameters of the model, i.e. the parameters for the conditional variances $\omega_i$ for $i = 1$, ..., $N$, the conditional correlations $\rho_{ij}$ with $1 \leq i < j \leq N$ and the $N^* \times 1$ parameters that are equal to zero under the null hypothesis.

When the remainder $R_t^{(k)}(\widetilde{w}) \equiv 0$, the statistic $TAYLOR$ has an asymptotic $\chi^2$ distribution with $N^*$ degrees of freedom under the null hypothesis. Appendix 1.A.2 presents the details of the technical derivations of the test statistic.

Following Péguin-Feissolle, Strikholm and Teräsvirta (2013), there are two practical difficulties when the order of the Taylor expansion $k$ is increasing, first the regressors tend to be highly collinear, and second the dimension of the null hypothesis may become rather large because the number of these regressors increases rapidly with $k$. More precisely, the conditional correlations $\rho_{ijt}$ given in (1.21) can be written as

$$\rho_{ijt} = \rho_{ij} + \sum_{m=1}^{N^*} \delta_{ijm} g_{ijt,m} + R_t^{(k)}(\widetilde{w})$$

where the $N^* \times 1$ vectors $g_{ijt}$ correspond to all terms involving functions of elements of lagged values of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$. We replace the matrix composed by the $N^* \times 1$ vectors $g_{ijt}$ by its largest principal components: like in the case of the ANN-based test, only the largest principal components that together explain at least 90% of the variation in the matrix are used, the number of principal components being determined automatically for each $i$ and $j$. Instead of the $N^* \times T$ matrix $G_{ij}$ composed by the $g_{ijt}$ vectors:

$$G_{ij} = (g_{ij1}, ..., g_{ijT})$$

for $1 \leq i < j \leq N$, $p < \infty$, we build the $N^*_{ij} \times T$ matrix $G^*_{ij}$ of the $N^*_{ij}$ chosen principal components, i.e.

$$G^*_{ij} = \left(g^*_{ij1}, ..., g^*_{ijT}\right).$$

The null hypothesis will be that the parameters associated to the main principal components are equal to zero in the following relationship determining $\rho_{ijt}$:

$$\rho_{ijt} = \rho_{ij} + \sum_{m=1}^{N^*_{ij}} \delta_{ijm} g^*_{ijt,m}$$

The statistic $TAYLOR$ will follow an asymptotic $\chi^2$ distribution under the null with $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} N^*_{ij}$ degrees of freedom.

## 1.4 Monte Carlo experiments

In this section, we investigate the small-sample performances of the ANN-based test and the Taylor expansion-based test for the constancy of conditional correlations in multivariate type-GARCH models. By using Monte-Carlo experiments, we compare both the sizes and the powers of these tests to two alternative conditional correlation tests: the tests of Tse (2000) and Silvennoinen and Teräsvirta (2014).

### 1.4.1 Simulation design

The data generating process (DGP) is based on the general multivariate GARCH model introduced before where the $N \times 1$ vector of residuals is given by

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2} \boldsymbol{\eta}_t$$

with $\boldsymbol{\eta}_t \sim nid\,(0, \mathbf{I}_N)$, $\forall t = 1, \ldots, T$. In other words, we generate several MGARCH-type models generally used in financial time series analysis, based on the normality assumption; we will relax the normality assumption in Section 1.5. For each simulation, $\boldsymbol{\eta}_t$ thus follows a multivariate Gaussian distribution with zero mean vector and covariance matrix $\mathbf{I}_N$ $(N(0, \mathbf{I}_N))$. For each model, the DGP is used to compute the empirical sizes and powers in order to compare the performances of both of the tests we propose (ANN-based test and Taylor expansion-based test) to two well-known tests (Tse (2000)'s test and Silvennoinen and Teräsvirta (2014)'s test).

For all the Monte Carlo simulations, we use the analytical expressions of the score and the score's outer product for all the tests (for instance, see Appendices 1.A.1 and 1.A.2 for the derivatives of the ANN-based test and the Taylor expansion-based test, respectively); Hafner and Herwartz (2008) show in a simulation study that analytical derivatives clearly outperform numerical methods. The sample sizes are $T = 1000, 1500$ and 2500 and the number of endogenous variables is $N = 2$. Moreover, we remove the first observations in order to eliminate initialization effects. The number of replications is $S = 2000$ except in Table 1 below; in this table, we generate $S = 1000$ replications in order to determine the optimal order of the Taylor expansion ($k$) and the optimal number of lags of residuals ($q$) in both the new tests. Concerning the artificial neural network based tests, following Lee, White and Granger (1993), the number of hidden units is $p = 20$; moreover, we generate the hidden unit weights, i.e. the different $\gamma_{ijk}$ in (1.11), for $1 \leq i < j \leq N$ and $1 \leq k \leq p$, randomly from the uniform $[-\mu, \mu]$ distribution. Following the same authors, we choose $\mu = 2$. For a fixed number of lags of residuals in the case of the artificial neural network based test and for a fixed number of lags of residuals and a fixed order of the Taylor expansion in the case of the Taylor expansion-based test, only the largest principal components that together explain at least 90% of the variation in the corresponding matrices are used, the number of principal components being determined automatically.

The performances of the two new tests are compared to two well-known tests:

- **Tse (2000) test**: The author defines an alternative conditional correlation:
$$\rho_{ijt} = \rho_{ij} + \delta_{ij} z_{it-1} z_{jt-1},$$
where the null hypothesis is obtained by setting $\delta_{ij} = 0$. $TSE$ denotes the LM statistic developed by Tse (2000) given by (we

report the paper's notations, see Tse (2000) for further details):

$$TSE = \mathbf{s}'(\mathbf{S}'\mathbf{S})^{-1}\mathbf{s} \tag{1.24}$$

where $\mathbf{s} = \mathbf{S}'\mathbf{l}$, $\mathbf{l}$ is the $T \times 1$ column vector of ones and $\mathbf{S}$ is the $T \times N$ matrix of which the rows are the partial derivatives $\frac{\partial l_t(\theta)}{\partial \theta'}$ for $t = 1, .., T$; $l_t(\theta)$ is given by (1.16). $\theta$ is evaluated at $\hat{\theta}$ which is the maximum likelihood estimator of $\theta$ under the null hypothesis.

- **Silvennoinen and Teräsvirta (2014) test**: these authors assume that the conditional correlation matrix $\mathbf{P}_t$ is given by

$$\mathbf{P}_t = (1 - \mathbf{G}_t)\mathbf{P}_1 + \mathbf{G}_t\mathbf{P}_2$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are positive definite correlation matrices; $\mathbf{G}_t$ is a logistic transition function:

$$\mathbf{G}_t = \left(1 + e^{-\gamma(s_t - c)}\right)^{-1}$$

where $\gamma$ is the transition parameter ($\gamma > 0$), $s_t$ is the transition variable and $c$ is the location parameter. Using a first-order Taylor approximation, $\mathbf{P}_t$ can be approximated as $\mathbf{P}_t^* = \mathbf{P}_1^* - s_t\mathbf{P}_2^*$ where $\mathbf{P}_1^*$ and $\mathbf{P}_2^*$ are depending on $\gamma$, $\mathbf{P}_1$ and $\mathbf{P}_2$. The null hypothesis can be written $vecl(\mathbf{P}_2^*) = 0$, where the $vecl$ operator stacks the columns of the strict lower diagonal of the square argument matrix (i.e. by excluding the diagonal elements). The LM statistic, $STCC$, is given by:

$$STCC = \frac{1}{T}\left(\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \theta}\right)' \Im_T(\theta)^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \theta}\right); \tag{1.25}$$

where $l_t(\theta)$ is given by (1.16). $\theta$ is evaluated at $\hat{\theta}$ which is the maximum likelihood estimator of $\theta$ under the null hypothesis (see Silvennoinen and Teräsvirta (2014) for more details).

### 1.4.2 Size study

We present here the results concerning the size of the different tests, i.e. in the case where the data are generated under the null hypothesis using a constant conditional correlation model. Following Silvennoinen and Teräsvirta (2014), the transition variable of the $STCC$ test will be generated in the simulations from an exogenous GARCH(1,1) process such that $s_t = h_t^{1/2}z_t$ where $z_t \sim N(0,1)$ and $h_t = 0.02 + 0.03s_{t-1}^2 + 0.94h_{t-1}$. We thus consider two models.

- The first one is an extended CCC-GARCH model (ECCC-GARCH), which is a generalization of the multivariate GARCH model with constant conditional correlations (CCC-GARCH) proposed by Bollerslev (1990) and Baillie and Bollerslev (1990); the individual conditional variance equations depend on the past squared returns and conditional variances of all series. Therefore, the elements of the conditional variance matrix are characterized for $N = 2$ by:

$$\begin{pmatrix} h_{11t} \\ h_{22t} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon^2_{2,t-1} \end{pmatrix} + \begin{pmatrix} g_{11} & g_{12} \\ g_{12} & g_{22} \end{pmatrix} \begin{pmatrix} h_{11,t-1} \\ h_{22,t-1} \end{pmatrix} \tag{1.26}$$

  and

$$h_{12t} = \rho_{12}\sqrt{h_{11t}h_{22t}} \tag{1.27}$$

  where $\rho_{12}$ is the constant conditional correlation. This model will be noted $ECCC$.

- The second model is the CCC-GARCH model. We will denote by $CCC$ the case where the off-diagonal parameters in (1.26) are equal to zero i.e. $a_{12} = g_{12} = 0$. We will denote by $GARCH_1$ the case with low persistence and by $GARCH_2$ the case with high persistence, the models that are composed of two univariate independent GARCH(1,1) models, i.e. $a_{12} = g_{12} = \rho_{12} = 0$ in (1.26) and (1.27).

To determine the optimal order of the Taylor expansion ($k$) and the optimal number of lags of residuals ($q$) used in both the new tests, we generate $S = 1000$ replications of the $ECCC$ model.

Table 1.1 summarizes the 1%, 5% and the 10% rejection frequencies for various $q$ lags of residuals and $k$ orders of Taylor expansion. We base the selection criterion on the size distortions of the both tests from the nominal sizes for different sample sizes. Regarding the simulation results, the choice of $q = 3$ for $NEURAL$ seems to be the best choice. The test statistic $NEURAL$ is therefore simulated for a number of lags $q$ in (1.12) equal to 3. Among other possibilities, the order of the Taylor expansion $k = 3$ accompanied by a number of lags in (1.20) $q = 2$, is likely to be a good choice, leading to $N^* = 50$ before the main component analysis. Therefore, we summarize the different parameters for the two tests that will be used in the rest of the chapter as follows:

- For the test statistic $NEURAL$: the number of hidden units is $p = 20$ and the number of lags of residuals is $q = 3$,

Table 1.1: Determination of the optimal Taylor expansion order ($k$) and the optimal number of lags of residuals ($q$) used in the ANN-based and Taylor expansion-based tests

| **S = 1000** | **T = 1000** | | | **T = 1500** | | | **T = 2500** | | |
|---|---|---|---|---|---|---|---|---|---|
| | **1%** | **5%** | **10%** | **1%** | **5%** | **10%** | **1%** | **5%** | **10%** |
| **TAYLOR k = 1** | | | | | | | | | |
| **q = 1** | 0.009 | 0.050 | 0.113 | 0.012 | 0.043 | 0.089 | 0.008 | 0.039 | 0.094 |
| **q = 2** | 0.016 | 0.063 | 0.123 | 0.013 | 0.055 | 0.097 | 0.006 | 0.059 | 0.124 |
| **q = 3** | 0.023 | 0.086 | 0.144 | 0.012 | 0.042 | 0.087 | 0.010 | 0.046 | 0.102 |
| **q = 4** | 0.017 | 0.070 | 0.131 | 0.010 | 0.059 | 0.110 | 0.006 | 0.044 | 0.088 |
| **TAYLOR k = 2** | | | | | | | | | |
| **q = 1** | 0.083 | 0.159 | 0.225 | 0.081 | 0.166 | 0.231 | 0.007 | 0.052 | 0.102 |
| **q = 2** | 0.005 | 0.028 | 0.061 | 0.008 | 0.031 | 0.073 | 0.008 | 0.051 | 0.105 |
| **q = 3** | 0.022 | 0.058 | 0.122 | 0.006 | 0.035 | 0.079 | 0.012 | 0.060 | 0.114 |
| **q = 4** | 0.020 | 0.068 | 0.114 | 0.008 | 0.049 | 0.103 | 0.011 | 0.063 | 0.131 |
| **TAYLOR k = 3** | | | | | | | | | |
| **q = 1** | 0.032 | 0.089 | 0.148 | 0.037 | 0.098 | 0.150 | 0.005 | 0.062 | 0.109 |
| **q = 2** | 0.012 | 0.042 | 0.083 | 0.014 | 0.053 | 0.097 | 0.007 | 0.049 | 0.106 |
| **q = 3** | 0.032 | 0.096 | 0.144 | 0.011 | 0.065 | 0.131 | 0.018 | 0.056 | 0.119 |
| **q = 4** | 0.024 | 0.086 | 0.151 | 0.022 | 0.093 | 0.155 | 0.021 | 0.067 | 0.126 |
| **TAYLOR k = 4** | | | | | | | | | |
| **q = 1** | 0.049 | 0.095 | 0.133 | 0.036 | 0.073 | 0.111 | 0.009 | 0.056 | 0.104 |
| **q = 2** | 0.014 | 0.045 | 0.090 | 0.009 | 0.050 | 0.096 | 0.011 | 0.058 | 0.108 |
| **q = 3** | 0.026 | 0.083 | 0.146 | 0.008 | 0.056 | 0.117 | 0.016 | 0.061 | 0.122 |
| **q = 4** | 0.024 | 0.080 | 0.160 | 0.024 | 0.094 | 0.146 | 0.018 | 0.069 | 0.122 |
| **NEURAL** | | | | | | | | | |
| **q = 1** | 0.004 | 0.018 | 0.035 | 0.003 | 0.018 | 0.039 | 0.006 | 0.058 | 0.109 |
| **q = 2** | 0.009 | 0.027 | 0.055 | 0.007 | 0.026 | 0.057 | 0.013 | 0.058 | 0.107 |
| **q = 3** | 0.007 | 0.052 | 0.086 | 0.008 | 0.048 | 0.083 | 0.009 | 0.052 | 0.103 |
| **q = 4** | 0.009 | 0.036 | 0.080 | 0.007 | 0.035 | 0.072 | 0.009 | 0.052 | 0.108 |

NOTE: The number of replications is $S = 1000$. The generation parameters of the $ECCC$ model are:

| $a_{10}$ | $a_{20}$ | $a_{11}$ | $a_{12}$ | $a_{22}$ | $g_{11}$ | $g_{12}$ | $g_{22}$ | $\rho_{12}$ |
|---|---|---|---|---|---|---|---|---|
| 0.02 | 0.02 | 0.01 | 0.02 | 0.01 | 0.85 | 0.02 | 0.85 | 0.60 |

- For the test statistic $TAYLOR$: the order of the Taylor expansion is $k = 3$ and the number of lags of residuals is $q = 2$.

Table 1.2 shows the empirical sizes of the different tests, i.e. the rejection probabilities under the null hypothesis of constant conditional correlations, assuming nominal sizes of 1%, 5% and 10% and different sample sizes ($T = 1000$, 1500 and 2500). Moreover, we consider models with different constant correlations: high with $\rho_{12} = 0.60$ for the ECCC-GARCH, medium with $\rho_{12} = 0.30$ for CCC-GARCH, and of course $\rho_{12} = 0$ for the bivariate independent GARCH models. The DGP models correspond to different persistency for example $a_{11}+g_{11} = a_{22}+g_{22} = 0.99$ for the CCC-GARCH model, and $a_{11} + g_{11} = 0.95$ and $a_{22} + g_{22} = 0.90$, and $a_{11} + g_{11} = a_{22} + g_{22} = 0.99$ in the bivariate GARCH models.

For all the tests, we can observe that the the empirical sizes in small samples are close to the nominal sizes, and the small sample sizes converge towards the nominal sizes when the number of observations $T$ is increasing, for most of the tests. More precise conclusions can be given:

- In the ECCC case, for the different sample sizes, **the $TSE$ test statistics over-rejects** the null hypothesis while a multivariate constant conditional correlation model is generated. The $NEURAL$, the $TAYLOR$ **and the** $STCC$ test statistics show similar performances and clearly outperform **the $TSE$ test**.

- In the CCC case, **the small sample sizes seem close to the nominal sizes for all the test statistics when the sample sizes reach $T = 2500$.**

- The bivariate GARCH case permits to study the possibility of no correlation at all, i.e. the nullity of the conditional covariances. The rejection frequencies show clear signs of over-rejections for all the sample sizes for **the $TSE$ test statistic; the $NEURAL$, $TAYLOR$ and $STCC$ test statistics clearly outperform the $TSE$ test statistics**, which shed light on their good size properties.

- The bivariate $GARCH_2$ has the same parameters than the $CCC$ model with a null conditional correlation. One can conclude that when the persistence is high, the $TAYLOR$ and $TSE$ tests have their empirical sizes closer to the nominal sizes compared to $GARCH_1$ for $T = 2500$. When the conditional correlation is null the performance are very close to those of $CCC$ ($\rho_{12} = 0.30$), except for the $TSE$ test for $T = 2500$.

Table 1.2: Small sample sizes of the different constant conditional correlation tests

| $T$ | 1000 | | | 1500 | | | 2500 | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| **ECCC** | | | | | | | | | |
| $NEURAL$ | 0.008 | 0.052 | 0.087 | 0.009 | 0.049 | 0.085 | 0.011 | 0.054 | 0.105 |
| $TAYLOR$ | 0.012 | 0.042 | 0.083 | 0.014 | 0.053 | 0.097 | 0.009 | 0.053 | 0.104 |
| $TSE$ | 0.027 | 0.084 | 0.134 | 0.023 | 0.080 | 0.139 | 0.086 | 0.144 | 0.201 |
| $STCC$ | 0.011 | 0.054 | 0.107 | 0.012 | 0.042 | 0.094 | 0.011 | 0.049 | 0.104 |
| **CCC** | | | | | | | | | |
| $NEURAL$ | 0.018 | 0.077 | 0.134 | 0.017 | 0.061 | 0.118 | 0.008 | 0.056 | 0.104 |
| $TAYLOR$ | 0.021 | 0.083 | 0.141 | 0.017 | 0.070 | 0.133 | 0.010 | 0.046 | 0.105 |
| $TSE$ | 0.018 | 0.060 | 0.117 | 0.015 | 0.062 | 0.107 | 0.017 | 0.052 | 0.105 |
| $STCC$ | 0.015 | 0.059 | 0.112 | 0.013 | 0.059 | 0.106 | 0.008 | 0.047 | 0.098 |
| **GARCH$_1$** | | | | | | | | | |
| $NEURAL$ | 0.014 | 0.053 | 0.107 | 0.015 | 0.057 | 0.113 | 0.011 | 0.055 | 0.106 |
| $TAYLOR$ | 0.013 | 0.053 | 0.105 | 0.015 | 0.056 | 0.103 | 0.013 | 0.058 | 0.111 |
| $TSE$ | 0.093 | 0.193 | 0.264 | 0.089 | 0.189 | 0.265 | 0.089 | 0.192 | 0.264 |
| $STCC$ | 0.006 | 0.045 | 0.101 | 0.011 | 0.046 | 0.095 | 0.008 | 0.054 | 0.105 |
| **GARCH$_2$** | | | | | | | | | |
| $NEURAL$ | 0.013 | 0.059 | 0.115 | 0.019 | 0.069 | 0.127 | 0.015 | 0.057 | 0.115 |
| $TAYLOR$ | 0.015 | 0.055 | 0.104 | 0.024 | 0.066 | 0.113 | 0.015 | 0.048 | 0.103 |
| $TSE$ | 0.013 | 0.053 | 0.104 | 0.023 | 0.060 | 0.109 | 0.019 | 0.066 | 0.119 |
| $STCC$ | 0.014 | 0.061 | 0.115 | 0.016 | 0.063 | 0.113 | 0.011 | 0.048 | 0.097 |

NOTE: The number of replications is $S = 2000$. For $NEURAL$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q = 3$, and for $TAYLOR$ the order of the Taylor expansion is $k = 3$ and the number of lags of residuals is $q = 2$. The generation parameters are:

| | $a_{10}$ | $a_{20}$ | $a_{11}$ | $a_{12}$ | $a_{22}$ | $g_{11}$ | $g_{12}$ | $g_{22}$ | $\rho_{12}$ |
|---|---|---|---|---|---|---|---|---|---|
| $ECCC$ | 0.02 | 0.02 | 0.01 | 0.02 | 0.01 | 0.85 | 0.02 | 0.85 | 0.60 |
| $CCC$ | 0.02 | 0.02 | 0.01 | – | 0.01 | 0.98 | – | 0.98 | 0.30 |
| $GARCH_1$ | 0.40 | 0.20 | 0.15 | – | 0.20 | 0.80 | – | 0.70 | 0.00 |
| $GARCH_2$ | 0.02 | 0.02 | 0.01 | – | 0.01 | 0.98 | – | 0.98 | 0.00 |

### 1.4.3 Power study

To illustrate the behaviour of all tests under the alternative hypothesis, we generate different time-varying conditional correlations multivariate type-GARCH models, chosen to represent a variety of situations. Following Silvennoinen and Teräsvirta (2014), the transition variable of the $STCC$ test is defined as a linear combination of lags of squared returns: $s_t = (0.2, 0.2, 0.2, 0.2, 0.2) \times \left(\overline{\boldsymbol{\varepsilon}}^2_{t-1}, \overline{\boldsymbol{\varepsilon}}^2_{t-2}, \overline{\boldsymbol{\varepsilon}}^2_{t-3}, \overline{\boldsymbol{\varepsilon}}^2_{t-4}, \overline{\boldsymbol{\varepsilon}}^2_{t-5}\right)'$ where $\overline{\varepsilon}^2_t$ is the mean of $\boldsymbol{\varepsilon}$ over $N$ of its squared elementwise. We consider the following models:

- the $BEKK(1,1,1)$ model (Engle and Kroner, 1995) defined by:

$$\mathbf{H}_t = \mathbf{C}'\mathbf{C} + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}\mathbf{A}' + \mathbf{G}\mathbf{H}_{t-1}\mathbf{G}' \tag{1.28}$$

  where $\mathbf{C}$, $\mathbf{A}$ and $\mathbf{G}$ are $N \times N$ matrices and $\mathbf{C}$ is upper triangular. With $N = 2$, we have:

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}, \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \text{ and } \mathbf{G} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \tag{1.29}$$

  Note that the conditional covariance matrices $\mathbf{H}_t$ are positive definite by construction. This model will be noted $BEKK$.

- the DCC-GARCH model of Engle (2002) (denoted by $DCC$): $\mathbf{H}_t$ is defined as in (1.7), i.e. $\mathbf{H}_t = \mathbf{S}_t\mathbf{P}_t\mathbf{S}_t$ with $\mathbf{P}_t$ the $N \times N$ conditional correlation matrix and $\mathbf{S}_t$ is the $N \times N$ matrix given by $\mathbf{S}_t = diag(\sqrt{h_{11t}}, ..., \sqrt{h_{NNt}})$, with

$$\mathbf{P}_t = diag\left(q_{11,t}^{-1/2}, ..., q_{NN,t}^{-1/2}\right)\mathbf{Q}_t diag\left(q_{11,t}^{-1/2}, ..., q_{NN,t}^{-1/2}\right) \tag{1.30}$$

  where $\mathbf{Q}_t = (q_{ij,t})$ is a symmetric positive definite $N \times N$ matrix given by

$$\mathbf{Q}_t = (1 - \alpha - \beta)\,\bar{\mathbf{Q}} + \alpha\mathbf{z}_{t-1}\mathbf{z}'_{t-1} + \beta\mathbf{Q}_{t-1}; \tag{1.31}$$

  $\bar{\mathbf{Q}}$ is the $N \times N$ constant matrix (see Aielli (2013) for more details on $\bar{\mathbf{Q}}$) and $\alpha$ and $\beta$ are non-negative scalar parameters satisfying $\alpha + \beta < 1$. Moreover, $h_{iit}$ is a simple GARCH(1,1) model for $i = 1, \ldots, N$.

- the EDCC-GARCH (denoted by $EDCC$) is the extended specification of DCC-GARCH model, allowing for volatility spillovers, i.e. the individual conditional variance equations depend on the past

squared returns and variances of all series (see formula (1.26)). In the case $N = 2$, we have for $i, j = 1, 2$:

$$h_{iit} = c_{ii} + a_{ii}\varepsilon^2_{i,t-1} + a_{ij}\varepsilon^2_{j,t-1} + g_{ii}h_{ii,t-1} + g_{ij}h_{jj,t-1}. \qquad (1.32)$$

We take two different data generation processes for each of these three models (called $BEKK_1$ and $BEKK_2$, $DCC_1$ and $DCC_2$, $EDCC_1$ and $EDCC_2$), characterized by different generation parameters and different variabilities of conditional correlation coefficients. Table 1.3 gives the parameter values used to generate the different models; following Tse (2000), we give in the second part of the Table 1.3 the variability of the conditional correlation coefficients by calculating the range (maximum and minimum) of these coefficients in each simulated sample. The variability is high with $DCC_2$ and $EDCC_1$, moderate with $BEKK_2$ and $DCC_1$ and low with $BEKK_1$ and $EDCC_2$. Table 1.4 summarizes the empirical powers of the ANN-based test, the Taylor expansion-based test, the $TSE$ test and the $STCC$ test. In other words, it shows the rejection probabilities of the null hypothesis of constant conditional correlations assuming nominal sizes of 1%, 5% and 10% and different sample sizes ($T = 1000$, 1500 and 2500). For all the tests, we can observe in Table 1.4 that the rejection probabilities of the null hypothesis increase with the number of observations $T$.

- For the two BEKK-GARCH specifications, $BEKK_1$ and $BEKK_2$, all the tests reject the null hypothesis of constancy of conditional correlation; nevertheless, even if they all present good properties with a rejection probability of 1 when the sample size is equal to 1500, the $TSE$ test seems to have less good results, especially for $T = 1000$.

- The DCC models, $DCC_1$ and $DCC_2$, are characterized by parameters implying very different ranges of variability of the conditional correlation coefficients: the intervals are very large in the case of the $DCC_2$ model. Nevertheless, the $TAYLOR$ test presents the best performance in both DGP models followed by the $NEURAL$ test in the first one and the $TSE$ test in the second one. On the other hand, the tests presenting very poor performances are the $TSE$ and $STCC$ tests in the $DCC_1$ model, and the $NEURAL$ and $STCC$ tests in the $DCC_2$ model.

- Concerning the extended DCC models, $EDCC_1$ and $EDCC_2$, the tests showing the best performances are the $TAYLOR$ and $TSE$

Table 1.3: MGARCH parameters for the generation of the DGP for the small sample powers

| | **BEKK$_1$** | **BEKK$_2$** | **DCC$_1$** | **DCC$_2$** | **EDCC$_1$** | **EDCC$_2$** |
|---|---|---|---|---|---|---|
| $c_{11}$ | 0.20 | 0.20 | 0.01 | 0.01 | 0.01 | 0.01 |
| $c_{12}$ | 0.01 | 0.04 | – | – | – | – |
| $c_{22}$ | 0.20 | 0.20 | 0.02 | 0.02 | 0.02 | 0.02 |
| $a_{11}$ | 0.10 | 0.30 | 0.001 | 0.001 | 0.01 | 0.02 |
| $a_{12}$ | 0.10 | 0.10 | – | – | 0.01 | 0.05 |
| $a_{21}$ | 0.10 | 0.10 | – | – | 0.01 | 0.05 |
| $a_{22}$ | 0.10 | 0.30 | 0.002 | 0.002 | 0.02 | 0.02 |
| $g_{11}$ | 0.70 | 0.30 | 0.98 | 0.98 | 0.80 | 0.75 |
| $g_{12}$ | 0.10 | 0.20 | – | – | 0.15 | 0.10 |
| $g_{21}$ | 0.10 | 0.20 | – | – | 0.15 | 0.10 |
| $g_{22}$ | 0.70 | 0.30 | 0.90 | 0.90 | 0.75 | 0.55 |
| $\alpha$ | – | – | 0.05 | 0.15 | 0.15 | 0.01 |
| $\beta$ | – | – | 0.94 | 0.80 | 0.84 | 0.94 |
| $\rho_{12}$ | – | – | 0.80 | 0.30 | 0.80 | 0.30 |
| $corr_{min_{1000}}$ | 0.33 | 0.18 | 0.26 | −0.66 | −0.65 | 0.21 |
| $corr_{max_{1000}}$ | 0.53 | 0.77 | 0.94 | 0.86 | 0.99 | 0.38 |
| $corr_{min_{1500}}$ | 0.33 | 0.17 | 0.19 | −0.69 | −0.75 | 0.21 |
| $corr_{max_{1500}}$ | 0.54 | 0.78 | 0.94 | 0.87 | 0.99 | 0.39 |
| $corr_{min_{2500}}$ | 0.33 | 0.15 | 0.11 | −0.71 | −0.81 | 0.20 |
| $corr_{max_{2500}}$ | 0.56 | 0.80 | 0.95 | 0.88 | 0.99 | 0.39 |

Table 1.4: Small sample powers of the different constant conditional correlation tests

| $T$ | 1000 | | | 1500 | | | 2500 | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| **BEKK$_1$** | | | | | | | | | |
| $NEURAL$ | 0.998 | 0.998 | 0.998 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $TAYLOR$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $TSE$ | 0.832 | 0.986 | 0.995 | 0.989 | 0.998 | 0.999 | 1.000 | 1.000 | 1.000 |
| $STCC$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| **BEKK$_2$** | | | | | | | | | |
| $NEURAL$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $TAYLOR$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $TSE$ | 0.983 | 0.987 | 0.990 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $STCC$ | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| **DCC$_1$** | | | | | | | | | |
| $NEURAL$ | 0.392 | 0.536 | 0.612 | 0.410 | 0.546 | 0.619 | 0.433 | 0.585 | 0.659 |
| $TAYLOR$ | 0.394 | 0.534 | 0.616 | 0.446 | 0.602 | 0.694 | 0.525 | 0.648 | 0.722 |
| $TSE$ | 0.060 | 0.149 | 0.241 | 0.100 | 0.234 | 0.344 | 0.191 | 0.375 | 0.487 |
| $STCC$ | 0.197 | 0.329 | 0.416 | 0.213 | 0.350 | 0.442 | 0.241 | 0.383 | 0.466 |
| **DCC$_2$** | | | | | | | | | |
| $NEURAL$ | 0.144 | 0.265 | 0.359 | 0.163 | 0.303 | 0.397 | 0.243 | 0.409 | 0.501 |
| $TAYLOR$ | 0.970 | 0.988 | 0.992 | 0.999 | 0.999 | 0.999 | 1.000 | 1.000 | 1.000 |
| $TSE$ | 0.829 | 0.943 | 0.966 | 0.967 | 0.991 | 0.997 | 0.997 | 0.999 | 1.000 |
| $STCC$ | 0.237 | 0.369 | 0.454 | 0.232 | 0.388 | 0.463 | 0.249 | 0.397 | 0.456 |
| **EDCC$_1$** | | | | | | | | | |
| $NEURAL$ | 0.693 | 0.784 | 0.830 | 0.711 | 0.797 | 0.835 | 0.756 | 0.832 | 0.870 |
| $TAYLOR$ | 0.923 | 0.956 | 0.969 | 0.969 | 0.981 | 0.986 | 0.994 | 0.996 | 0.996 |
| $TSE$ | 0.799 | 0.865 | 0.895 | 0.874 | 0.909 | 0.920 | 0.911 | 0.917 | 0.929 |
| $STCC$ | 0.454 | 0.566 | 0.631 | 0.509 | 0.604 | 0.654 | 0.615 | 0.708 | 0.747 |
| **EDCC$_2$** | | | | | | | | | |
| $NEURAL$ | 0.410 | 0.512 | 0.573 | 0.418 | 0.527 | 0.593 | 0.503 | 0.605 | 0.676 |
| $TAYLOR$ | 0.023 | 0.088 | 0.163 | 0.031 | 0.104 | 0.178 | 0.127 | 0.188 | 0.247 |
| $TSE$ | 0.022 | 0.081 | 0.138 | 0.025 | 0.094 | 0.158 | 0.131 | 0.199 | 0.262 |
| $STCC$ | 0.064 | 0.156 | 0.238 | 0.069 | 0.165 | 0.243 | 0.154 | 0.250 | 0.335 |

NOTE: The number of replications is $S = 2000$. For $NEURAL$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q = 3$, and for $TAYLOR$ the order of the Taylor expansion is $k = 3$ and the number of lags of residuals is $q = 2$.

tests in the $EDCC_1$ case and the $NEURAL$ tests in the $EDCC_2$ case. In the first case, the $NEURAL$, $TAYLOR$ and $TSE$ outperform the $STCC$ test. In the second DGP model, the performances are altogether poor for each test.

Concerning the advantages of the $NEURAL$ test with respect to the $TAYLOR$ test, the $NEURAL$ test seems to have better properties when the variability of the conditional correlation coefficients is moderate ($DCC_1$) or low ($EDCC_2$); it is important to note that in this last case, the $NEURAL$ test is the only one that can detect the non constancy of conditional correlations. At the opposite, the $TAYLOR$ test seems better in the case where the variability of the conditional correlation coefficients is high ($DCC_2$ and $EDCC_1$). Therefore, we can conclude that even if one or the other of the new tests is not the best each time, it is important to apply jointly both of them because at least one of them can detect the time-varying conditional correlations. The idea is that, if at least one of the two tests rejects the null hypothesis of constant conditional correlations, we can try a more complex model with time-varying conditional correlations.

All the multivariate GARCH-type models have thus been tested satisfactorily using the ANN-based test and the Taylor expansion-based test compared to $TSE$ and $STCC$ tests. This finding suggests that whatever the GARCH-type model, both tests developed in this chapter show relevant properties to identify time-varying conditional correlations. To summarize our results, the ANN-based and Taylor expansion-based tests permit to generalize the acceptance or rejection of the hypothesis of constant correlations in a general multivariate GARCH framework.

## 1.5 Robustness to nonnormality

The tests developed in this chapter have been tested in the above section with a DGP following a Gaussian distribution. In order to investigate the robustness of the different tests to nonnormality, we simulate in this section some models with a nonnormal conditional distribution: the multivariate versions of $GARCH - t$ model of Bollerslev (1987) and $Beta - t - EGARCH$ model of Harvey and Sucarrat (2014) and Harvey and Chakravarty (2008). In all the Monte Carlo simulations, the number of replications is $S = 2000$, the sample sizes are $T = 1000, 1500$ and 2500 and the number of endogenous variables is $N = 2$; we consider the nominal sizes of 1%, 5% and 10%.

### 1.5.1 Size results

We first study the empirical sizes of the different tests when the following models are generated with null correlations:

- Two independent univariate Student $t_\nu$–GARCH models; they are characterized by $a_{12} = g_{12} = \rho_{12} = 0$ in (1.26) and thus $h_{12t} = 0$.

- Two independent univariate $Beta-t-EGARCH$ with and without a leverage effect. We use the same notations as Harvey and Sucarrat (2014) who developed a more general framework than Harvey and Chakravarty (2008). Each $Beta-t-EGARCH$ model without a leverage effect is defined as follows, for $i = 1, 2$:

$$y_{it} = \mu_i + \varepsilon_{it} \exp\left(\lambda_{i,t|t-1}\right),\ t = 1, \ldots, T, \tag{1.33}$$

  where $\varepsilon_{it}$ has a $t_{\nu_i}$–distribution and is serially independent and $\nu_i$, the number of degrees of freedom, is positive. Let us define the conditional score

$$u_{it} = \frac{(\nu_i + 1)(y_{it} - \mu_i)^2}{\nu_i \exp(2\lambda_{i,t|t-1}) + (y_{it} - \mu_i)^2} - 1,\ -1 \leq u_{it} \leq \nu_i,\ \nu_i > 0. \tag{1.34}$$

  We consider the first order model given by

$$\lambda_{i,t|t-1} = \delta_i + \phi_i \lambda_{i,t-1|t-2} + \kappa_i u_{i,t-1}; \tag{1.35}$$

  with the stationarity condition $|\phi_i| < 1$. The first order $Beta-t-EGARCH$ with a leverage effect is defined in Harvey and Sucarrat (2014) as follows

$$\lambda_{i,t|t-1} = \delta_i + \phi_i \lambda_{i,t-1|t-2} + \kappa_i u_{i,t-1} + \kappa_i^* sign(-(y_{i,t-1} - \mu_i))(u_{i,t-1} + 1). \tag{1.36}$$

  As noted in Harvey and Sucarrat (2014), if an increase in the absolute values of a standardized observation leads to an increase in volatility the restriction $\kappa_i \geq \kappa_i^* \geq 0$ may be imposed (see Harvey and Chakravarty, 2008; Harvey and Sucarrat, 2014, for a complete definition of these models and their properties).

The generation parameters are given in Table 1.5. Table 1.6 shows the small sample sizes of $t-GARCH$ and $Beta-t-EGARCH$ for 2000 replications and $T = 1000$, 1500 and 2500:

- When the DGP follows a Student distribution, the results of $t-GARCH$ model highlight the relative robustness to nonnormality of the $TAYLOR$, $NEURAL$ and $STCC$ tests compared to the $TSE$ test, even for small numbers of observations.
- When the observations are simulated with a first order $Beta-t-EGARCH$ model, the $TSE$ test over-reject the null hypothesis of constant conditional correlations. The $NEURAL$, $TAYLOR$ and $STCC$ tests are very close to the nominal sizes.
- When the leverage effects are added in the first order $Beta-t-EGARCH$, i.e. $Beta-t-EGARCH^*$, the $TSE$ test statistic over-reject again the null hypothesis for all the sample sizes. The $NEURAL$, $TAYLOR$ and $STCC$ test statistics show again better results.

Generally, the $TAYLOR$, $NEURAL$ and $STCC$ tests outperform the $TSE$ tests, because their small sample sizes are closer to the nominal sizes.

### 1.5.2 Power results

We study the empirical powers in small samples with the same models studied for the sizes except that the conditional correlations are no longer constant. The conditional expectation model can be written, for $t = 1, \ldots, T$, and $i = 1$, ..., $N$,

$$y_{it} = \sqrt{h_{iit}}\varepsilon_{it} \tag{1.37}$$

where, for the $GARCH(1,1)$ models,

$$h_{iit} = \alpha_{i0} + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1} \tag{1.38}$$

and for the $Beta-t-EGARCH$ models,

$$h_{iit} = \left[\exp\left(\lambda_{i,t|t-1}\right)\right]^2. \tag{1.39}$$

following the formula (1.33); $\lambda_{i,t|t-1}$ is determined as a first order model defined by (1.35) or (1.36) respectively without or with a leverage effect. Moreover, the $\varepsilon_t$ have a conditional multivariate $t_\nu$–distribution with positive degrees of freedom $\nu$:

$$\boldsymbol{\varepsilon}_t|\Omega_{t-1} \sim t_\nu\left(0, \mathbf{H}_t\right). \tag{1.40}$$

Table 1.5: MGARCH parameters for the generation of the DGP for the small sample sizes and powers

| | $t_\nu$-*GARCH* | *Beta-t-EGARCH* | *DCC-t* | *DCC-Beta-t-EGARCH* | *DSC-Beta-t-EGARCH*$^*$ |
|---|---|---|---|---|---|
| $\alpha$ | – | – | 0.05 | 0.15 | 0.15 |
| $\beta$ | – | – | 0.94 | 0.80 | 0.80 |
| $\rho_{12}$ | – | – | 0.70 | 0.60 | – |
| $\nu$ | 10 | 10 | 10 | 10 | 10 |
| $a_{10}$ | 0.40 | – | 0.30 | – | – |
| $a_{20}$ | 0.20 | – | 0.20 | – | – |
| $a_{11}$ | 0.15 | – | 0.10 | – | – |
| $a_{12}$ | – | – | – | – | – |
| $a_{22}$ | 0.20 | – | 0.20 | – | – |
| $g_{11}$ | 0.80 | – | 0.70 | – | – |
| $g_{12}$ | – | – | – | – | – |
| $g_{22}$ | 0.70 | – | 0.60 | – | – |
| $\mu_1$ | – | 0.00 | – | 0.00 | 0.00 |
| $\mu_2$ | – | 0.00 | – | 0.00 | 0.00 |
| $\delta_1$ | – | 0.007 | – | 0.02 | 0.02 |
| $\delta_2$ | – | 0.05 | – | 0.04 | 0.04 |
| $\phi_1$ | – | 0.99 | – | 0.95 | 0.95 |
| $\phi_2$ | – | 0.90 | – | 0.90 | 0.90 |
| $\kappa_1$ | – | 0.05 | – | 0.05 | 0.05 |
| $\kappa_2$ | – | 0.07 | – | 0.07 | 0.07 |
| $\kappa_1^*$ | – | 0.02 or 0.00$^*$ | – | 0.04 or 0.00$^*$ | 0.04 |
| $\kappa_2^*$ | – | 0.04 or 0.00$^*$ | – | 0.06 or 0.00$^*$ | 0.06 |
| $\psi$ | – | – | – | – | 0.90 |
| $q_{ij,\,0}$ | – | – | – | – | 0.60 |

NOTE: * without leverage effect in the *Beta-t-EGARCH* and *DCC-Beta-t-EGARCH*

Table 1.6: Small sample sizes of constant conditional correlation tests under nonnormality ($t-GARCH$, $Beta-t-EGARCH$, and $Beta-t-EGARCH^*$)

| **T** | 1000 | | | 1500 | | | 2500 | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| **t − GARCH** | | | | | | | | | |
| $NEURAL$ | 0.014 | 0.058 | 0.116 | 0.016 | 0.059 | 0.102 | 0.016 | 0.056 | 0.102 |
| $TAYLOR$ | 0.014 | 0.064 | 0.115 | 0.013 | 0.061 | 0.118 | 0.022 | 0.072 | 0.118 |
| $TSE$ | 0.056 | 0.112 | 0.168 | 0.056 | 0.125 | 0.191 | 0.075 | 0.153 | 0.219 |
| $STCC$ | 0.012 | 0.055 | 0.097 | 0.011 | 0.043 | 0.091 | 0.012 | 0.051 | 0.101 |
| **Beta − t − EGARCH** | | | | | | | | | |
| $NEURAL$ | 0.024 | 0.057 | 0.107 | 0.017 | 0.064 | 0.117 | 0.014 | 0.053 | 0.100 |
| $TAYLOR$ | 0.026 | 0.070 | 0.116 | 0.018 | 0.071 | 0.125 | 0.014 | 0.056 | 0.100 |
| $TSE$ | 0.088 | 0.154 | 0.215 | 0.072 | 0.156 | 0.220 | 0.077 | 0.152 | 0.220 |
| $STCC$ | 0.015 | 0.053 | 0.101 | 0.017 | 0.061 | 0.110 | 0.011 | 0.049 | 0.107 |
| **Beta − t − EGARCH*** | | | | | | | | | |
| $NEURAL$ | 0.018 | 0.052 | 0.105 | 0.017 | 0.057 | 0.114 | 0.015 | 0.052 | 0.111 |
| $TAYLOR$ | 0.024 | 0.063 | 0.119 | 0.015 | 0.063 | 0.117 | 0.015 | 0.055 | 0.107 |
| $TSE$ | 0.064 | 0.133 | 0.189 | 0.061 | 0.138 | 0.211 | 0.065 | 0.141 | 0.195 |
| $STCC$ | 0.012 | 0.052 | 0.103 | 0.012 | 0.058 | 0.102 | 0.011 | 0.048 | 0.099 |

NOTE: The number of replications is $S = 2000$. For $NEURAL$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q = 3$, and for $TAYLOR$ the order of the Taylor expansion is $k = 3$ and the number of lags of residuals is $q = 2$.

We use the same notations as in the DCC-GARCH model: $\mathbf{H}_t$ is given by (1.7) (i.e. $\mathbf{H}_t = \mathbf{S}_t\mathbf{P}_t\mathbf{S}_t$), the $N \times N$ conditional correlation matrix $\mathbf{P}_t$ is written as in (1.30), $\mathbf{S}_t$ has the following form:

$$\mathbf{S}_t = diag\left(\hat{\sigma}_{11,t}, \ldots, \hat{\sigma}_{NN,t}\right); \tag{1.41}$$

Harvey and Chakravarty (2008) defined $\sigma^*_{ii,t} = \sqrt{h_{iit}}$ as the time-varying scale parameter, which is not necessarily equal to the standard deviation. We resample the variance of residuals $\varepsilon^*_{i,t} \sim IID(0, \sigma^2_{\varepsilon^*})$ which does not necessarily have unit variance by the sample variance of residuals $\hat{\sigma}^2_{\varepsilon^*} = \sum\left(\hat{\varepsilon}^*_{i,t} - \bar{\hat{\varepsilon}}^*_i\right)^2 \times (T-1)^{-1}$, and we obtain straightforwardly the conditions which ensure the existence of a unit variance with $\hat{\sigma}_{ii,t} = \hat{\sigma}_{\varepsilon^*}\sigma^*_{ii,t}$.

$\mathbf{Q}_t = (q_{ij,t})$ is a symmetric positive definite $N \times N$ matrix given by (1.31) with $\bar{\mathbf{Q}}$ a $N \times N$ constant matrix. Thus, we have:

$$q_{ij,\,t} = \overline{q}_{ij} + \alpha(z_{i,t-1}z_{j,t-1} - \overline{q}_{ij}) + \beta(q_{ij,\,t-1} - \overline{q}_{ij}). \tag{1.42}$$

We also design a model with a dynamic stochastic correlation, and $Beta - t - EGARCH$ with leverage effects for variances (for a review, see Asai, McAleer and Yu, 2006). Instead of (1.42) we have:

$$q_{ij,\,t} = q_{ij0} + \psi q_{ij,\,t-1} + \Xi_{t-1}\Xi'_{t-1} \tag{1.43}$$

where $0 < \psi < 1$ is a scalar, $\Xi_t \sim N(0, 0.1 \odot \mathbf{I}_N)$ and $\odot$ denotes the (element-wise) Hadamard product. Note that by definition, the correlations follow a random walk. This model is denoted $DSC - Beta - t - EGARCH^*$.

The generation parameters are given in Table 1.5. Table 1.7 shows the empirical powers of the different tests, i.e. the rejection probabilities of the null hypothesis of constant correlations:

- For the $DCC - t - GARCH$ model, all the tests reject the null hypothesis of constancy of conditional correlation, but the $STCC$ and especially the $TSE$ tests present less good results and the $TAYLOR$ test the best one for each sample size.

- For the $DCC - Beta - t - EGARCH$ model with and without leverage effects, the performances of the tests present the same characteristics: the two new tests show the best empirical powers, even for $T = 1000$ or 1500, and the $TSE$ test again seems the least powerful.

- For the $DSC - Beta - t - EGARCH^*$ model with leverage effects, the two tests developed in this chapter clearly outperform the $STCC$ test, while the $TSE$ test shows good performances when the sample size reaches $T = 2500$.

Table 1.7: Small sample powers of constant conditional correlation tests under nonnormality ($DCC - t - GARCH$, $DCC - Beta - t - EGARCH$, $DCC - Beta - t - EGARCH^*$, and $DSC - Beta - t - EGARCH^*$)

| **T** | 1000 | | | 1500 | | | 2500 | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| **DCC − t − GARCH** | | | | | | | | | |
| $NEURAL$ | 0.283 | 0.450 | 0.536 | 0.374 | 0.526 | 0.619 | 0.490 | 0.643 | 0.727 |
| $TAYLOR$ | 0.511 | 0.668 | 0.739 | 0.636 | 0.772 | 0.834 | 0.788 | 0.885 | 0.922 |
| $TSE$ | 0.113 | 0.223 | 0.329 | 0.183 | 0.338 | 0.436 | 0.245 | 0.432 | 0.530 |
| $STCC$ | 0.246 | 0.380 | 0.461 | 0.301 | 0.427 | 0.493 | 0.339 | 0.455 | 0.529 |
| **DCC − Beta − t − EGARCH** | | | | | | | | | |
| $NEURAL$ | 0.601 | 0.736 | 0.806 | 0.68 | 0.807 | 0.857 | 0.779 | 0.862 | 0.901 |
| $TAYLOR$ | 0.792 | 0.886 | 0.928 | 0.878 | 0.942 | 0.961 | 0.965 | 0.986 | 0.994 |
| $TSE$ | 0.107 | 0.256 | 0.376 | 0.188 | 0.349 | 0.471 | 0.284 | 0.463 | 0.567 |
| $STCC$ | 0.459 | 0.548 | 0.595 | 0.594 | 0.663 | 0.704 | 0.591 | 0.669 | 0.720 |
| **DCC − Beta − t − EGARCH*** | | | | | | | | | |
| $NEURAL$ | 0.676 | 0.801 | 0.851 | 0.762 | 0.855 | 0.897 | 0.854 | 0.917 | 0.943 |
| $TAYLOR$ | 0.856 | 0.931 | 0.959 | 0.917 | 0.967 | 0.978 | 0.982 | 0.994 | 0.997 |
| $TSE$ | 0.156 | 0.251 | 0.342 | 0.193 | 0.292 | 0.386 | 0.224 | 0.338 | 0.430 |
| $STCC$ | 0.569 | 0.643 | 0.690 | 0.658 | 0.729 | 0.759 | 0.753 | 0.813 | 0.842 |
| **DSC − Beta − t − EGARCH*** | | | | | | | | | |
| $NEURAL$ | 0.994 | 0.997 | 0.997 | 0.995 | 0.997 | 0.998 | 0.999 | 0.999 | 0.999 |
| $TAYLOR$ | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| $TSE$ | 0.431 | 0.550 | 0.624 | 0.545 | 0.662 | 0.727 | 0.681 | 0.797 | 0.849 |
| $STCC$ | 0.071 | 0.178 | 0.262 | 0.084 | 0.180 | 0.265 | 0.09 | 0.194 | 0.274 |

NOTE: The number of replications is $S = 2000$. For $NEURAL$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q = 3$, and for $TAYLOR$ the order of the Taylor expansion is $k = 3$ and the number of lags of residuals is $q = 2$.

Everything considered, it is worthwhile noting here that the rejection probabilities of the null hypothesis increase again with the number of observations $T$. To conclude about the small sample results in the case of

time-varying conditional correlation and nonnormality assumptions, the tests showing the best performances are the $TAYLOR$ and $NEURAL$ tests. On the other hand, the tests presenting very poor performances are the $STCC$ and especially the $TSE$ tests.

## 1.6 Empirical illustration

We illustrate the applicability of the two constant correlation tests that we propose, that is the ANN-based test and the Taylor-expansion based test, by comparing them to Tse (2000)'s test and Silvennoinen and Teräsvirta (2014)'s test in an empirical example. We consider the asset returns of three of "blue-chip" US daily stocks used to compute the Dow Jones Industrial Average index: JPMorgan Chase & Co. (JPM), The Coca-Cola Company (KO) and Exxon Mobil Corporation (XOM) from January 03, 2000 to October 12, 2012 ($T = 3217$ daily observations). The series correspond to closing prices adjusted for dividends and splits and are obtained from Yahoo Finance.

We compute the returns of these components of the Dow Jones index by the usual formula expressed for one day lag: $100 \times log(P_t/P_{t-1})$, where $P_t$ represents the daily closing price at time $t$, $t = 1, \ldots, T$. To avoid a singularity issue of the Hessian matrix for the multivariate optimization and in order to have a comparable metrics between the four tests, we removed the few peaks, caused by the market structure, by cutting them off at $-10$. Table 1.8 presents the summary statistics of the asset returns: mean, median (Med.), standard deviation (S.D.), skewness (Skew.) and kurtosis (Kurt.) take into account the cut off. The returns exhibit a positive excess of kurtosis.

Table 1.8: Summary statistics of the daily asset returns

| | Mean | Med. | Max. | Min. | S.D. | Skew. | Kurt. |
|---|---|---|---|---|---|---|---|
| JPM | 0.01 | -0.02 | 22.39 | -41.13 | 2.75 | 0.91 | 8.08 |
| KO | 0.00 | 0.02 | 13.00 | -69.56 | 1.46 | 0.01 | 8.12 |
| XOM | 0.02 | 0.06 | 15.86 | -67.07 | 1.67 | 0.18 | 8.25 |

We compute all the tests on the asset returns as bivariate and trivariate combinations in order to determine whether these series are conditionally correlated over time or not.

Because we don't want to use tests suffering from size distortions, we choose for $TAYLOR$ $k$ expansion orders and $q$ lags which have their empirical sizes close to the nominal size at the 5% level for $T = 2500$

in Table 1.1 because this sample size is close to the sample size of this empirical illustration ($T = 3217$). For $NEURAL$, the number of hidden units is $p = 20$ and we consider all the lags for illustration purposes. Following Silvennoinen and Teräsvirta (2014), the $STCC$ test uses two different transition variables: the lagged absolute daily returns over seven days ($STCC_1$) and the contemporaneous squared daily returns ($STCC_2$).

Table 1.9 shows the results of the four constant conditional correlation tests.

Table 1.9: Constant conditional correlation tests of the daily asset returns

| | KO-JPM | | JPM-XOM | | XOM-KO | | JPM-KO-XOM | |
|---|---|---|---|---|---|---|---|---|
| | Stat | p-v | Stat | p-v | Stat | p-v | Stat | p-v |
| $NEURAL(1)$ | 6.39 | 0.093 | 4.05 | 0.255 | 9.41 | 0.024 | 32.9 | 1e-04 |
| $NEURAL(2)$ | 17.8 | 0.003 | 21.3 | 6e-04 | 7.77 | 0.169 | 45.6 | 3e-05 |
| $NEURAL(3)$ | 22.6 | 0.001 | 14.7 | 0.022 | 15.6 | 0.015 | 53.8 | 6e-05 |
| $NEURAL(4)$ | 5.63 | 0.582 | 12.8 | 0.076 | 15.4 | 0.030 | 74.4 | 1e-06 |
| $TAYLOR(1,3)$ | 4.94 | 0.423 | 10.3 | 0.067 | 7.57 | 0.181 | 24.8 | 0.053 |
| $TAYLOR(2,1)$ | 21.9 | 2e-04 | 21.8 | 2e-04 | 27.9 | 1e-05 | 47.9 | 3e-06 |
| $TAYLOR(2,2)$ | 23.0 | 0.010 | 37.3 | 5e-05 | 52.7 | 8e-08 | 68.7 | 7e-05 |
| $TAYLOR(3,2)$ | 39.6 | 0.005 | 73.7 | 8e-08 | 63.8 | 3e-06 | 126 | 2e-06 |
| $TSE$ | 3.48 | 0.062 | 4.36 | 0.037 | 0.00 | 0.985 | 16.8 | 7e-04 |
| $STCC_1$ | 0.91 | 0.339 | 0.53 | 0.468 | 2.44 | 0.118 | 24.4 | 2e-05 |
| $STCC_2$ | 0.21 | 0.642 | 2.46 | 0.117 | 6.53 | 0.011 | 6.59 | 0.086 |

NOTE: Stat and p-v denote the value of the test statistic and the associated $p$-value, respectively. For $NEURAL(q)$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q$, and for $TAYLOR(k, q)$ the order of the Taylor expansion is $k$ and the number of lags of residuals is $q$. The transition variables used in $STCC$ are the lagged absolute daily returns of the first variable over seven days for $STCC_1$, and the contemporaneous squared daily returns of the first variable for $STCC_2$.

In the case of the relationship between JPMorgan Chase (JPM) and Coca-Cola returns (KO), the null hypothesis of constancy of conditional correlations is rejected at the 1% level for $NEURAL(q = 2, 3)$, $TAYLOR(k = 2, q = 1, 2)$ and $TAYLOR(k = 3, q = 2)$ ($p$-values are respectively 0.003, 0.001, 0.0002, 0.010 and 0.005), and at the 10% level for $NEURAL(q = 1)$ and $TSE$. It is important to remark that $NEURAL(q = 4)$, $TAYLOR(k = 1, q = 3)$, $STCC_1$ and $STCC_2$ do not

reject the null hypothesis considering that conditional correlations are not time-varying. If one of these tests rejects the null hypothesis of constant conditional correlations, it is a good reason to try to consider a nonlinear correlations modeling.

In the case of the JPMorgan Chase (JPM) and Exxon Mobil (XOM) returns, the $NEURAL(q = 2)$, $TAYLOR(k = 2, q = 1, 2)$ and $TAYLOR(k = 3, q = 2)$ tests reject the constant correlation hypothesis at the nominal size of 1% like the $TSE$ test at the 5% level ($p$-value = 0.037), and the $NEURAL(q = 3, 4)$ and the $TAYLOR(k = 1, q = 3)$ tests at the 10% level. However, the $NEURAL(q = 2)$, the $STCC_1$ and the $STCC_2$ tests do not reject the constancy of conditional correlations.

When we consider Exxon Mobil (XOM) and The Coca-Cola Company (KO) returns, the test based on artificial neural network for $q = 1, 3, 4$ and the $STCC$ test with contemporaneous squared XOM returns ($STCC_2$) reject the hypothesis of constancy of conditional correlation at the nominal size of 5% ($p$-value = 0.024, 0.015, 0.030 and 0.011, respectively). The test based on a Taylor expansion for $k = 2, q = 1, 2$ and $k = 3, q = 2$ rejects the null at the 1% level ($p$-value = 1e-05, 8e-08 and 3e-06, respectively) whereas the ANN-based test for $q = 2$, the Taylor expansion-based test for $k = 1, q = 3$ and the TSE test do not reject the constancy.

For the trivariate analysis, JPMorgan Chase, Coca-Cola Company and Exxon Mobil are tested. All the tests reject the null hypothesis at the 1% level except $TAYLOR(k = 1, q = 3)$ and $STCC_2$ which reject the constancy at the 10% level.

Although we have not reported all the combinations of $k$ expansion orders and $q$ lags, one can remark that, as expected, for the Taylor expansion-based test the power increase with the number of expansion order (see Luukkonen, Saikkonen and Teräsvirta, 1988, for further details). However, it is important to take into account the eventual size distortions that are reported in Table 1.1.

Therefore, if we had considered only the $STCC$ test in the two first cases and the $TSE$ test in the third one, we would have accepted the constancy of conditional correlations while main of the other tests conclude to the time-varying conditional correlations. These four short illustrations can thus show that the tests developed have nice properties. This implies that whether we want to determine that the correlation is constant or not, all the tests introduced in this chapter can be useful; moreover, they can be considered as general tests against misspecification.

In a second configuration, we filter each return $R$, following a very

simple intertemporal capital asset pricing model:

$$R_t = \alpha + \beta cov_{t-1}(R_{t-1}, SPX_{t-1}) + u_t,$$

where $cov_t(R_t, SPX_t) = \rho_{ijt}\sqrt{h_{i,t}h_{j,t}}$ is a dynamic measure of the market with the S&P500 index return ($SPX$) and is given by a DCC-GARCH process; it allows us to test whether the residuals of the returns ($u_t$) are conditionally correlated over-time or not (a possible extension would be to allow $\beta$ to vary over time, although it is unlikely that this parameter depends on $t$).

Table 1.10 indicates that for most of the cases, the $NEURAL$ and $TAYLOR$ tests reject the null hypothesis of constancy of conditional correlations of the filtered residuals ($u_t$), while the $TSE$ test fails to reject the null for two cases. Even though we had filtered the daily asset returns, this specification confirms the likelihood of having dynamic in the conditional correlations of the return residuals.

Table 1.10: Constant conditional correlation tests of the filtered daily asset returns

| | KO-JPM | | JPM-XOM | | XOM-KO | | JPM-KO-XOM | |
|---|---|---|---|---|---|---|---|---|
| | Stat | p-v | Stat | p-v | Stat | p-v | Stat | p-v |
| $NEURAL(1)$ | 8.25 | 0.041 | 8.03 | 0.045 | 25.1 | 1e-05 | 38.6 | 1e-05 |
| $NEURAL(2)$ | 23.4 | 2e-04 | 31.7 | 2-e06 | 20.6 | 3e-04 | 79.3 | 9e-11 |
| $NEURAL(3)$ | 12.6 | 0.049 | 25.0 | 3e-04 | 36.4 | 2e-06 | 78.3 | 1e-08 |
| $NEURAL(4)$ | 31.5 | 5e-05 | 40.0 | 1e-06 | 23.8 | 5e-04 | 83.1 | 6e-08 |
| $TAYLOR(1,3)$ | 30.1 | 1e-05 | 28.6 | 2e-05 | 21.1 | 7e-04 | 78.9 | 1e-10 |
| $TAYLOR(2,1)$ | 25.3 | 1e-04 | 17.9 | 0.001 | 31.1 | 2e-08 | 96.1 | 3e-15 |
| $TAYLOR(2,2)$ | 100 | 4e-17 | 58.9 | 5e-09 | 107 | 2e-18 | 155 | 7e-19 |
| $TAYLOR(3,2)$ | 179 | 1e-27 | 140 | 1e-19 | 187 | 1e-28 | 314 | 2e-35 |
| $TSE$ | 1.27 | 0.258 | 3.91 | 0.047 | 0.09 | 0.754 | 11.8 | 0.007 |
| $STCC_1$ | 17.8 | 2e-05 | 3.19 | 0.074 | 5.74 | 0.016 | 22.6 | 4e-05 |
| $STCC_2$ | 0.13 | 0.709 | 3.89 | 0.048 | 3.48 | 0.062 | 4.62 | 0.201 |

NOTE: Stat and p-v denote the value of the test statistic and the associated $p$-value, respectively. For $NEURAL(q)$ the number of hidden units is $p = 20$ and the number of lags of residuals is $q$, and for $TAYLOR(k, q)$ the order of the Taylor expansion is $k$ and the number of lags of residuals is $q$. The transition variables used in $STCC$ are the lagged absolute daily returns of the first variable over seven days for $STCC_1$, and the contemporaneous squared daily returns of the first variable for $STCC_2$.

## 1.7 Concluding remarks

In this chapter, we propose two tests for the constancy of conditional correlation in the MGARCH models: the first one is based on artificial neural networks and the second on a Taylor expansion of each unknown conditional correlation around given point in a sample space. The main practical findings in this paper are that these two new tests perform well in our small-sample simulations and that they keep very good performances in case of nonnormality: the simulations show their robustness to nonnormality when the DGP models are multivariate versions of $GARCH - t$ and $Beta - t - EGARCH$ models. They show better performances with respect to the $STCC$ and $TSE$ tests in most of cases probably because their original form is very general. Indeed, the first test is based on artificial neural networks that are universal approximators, and the second one comes from the linearisation of an unknown relationship determining each conditional correlation.

In the empirical applications, both tests should be applied jointly because they have different finite-sample properties, as shown by the small sample simulations and the financial application. When we consider the powers, the $NEURAL$ test seems to have better properties when the variability of the conditional correlation coefficients is moderate or low; it is the opposite for the $TAYLOR$ test. Therefore, it is important to apply both of them jointly. The idea is that we can try a more complex model than a constant conditional correlations model if at least one of the two tests reject the null hypothesis of constant conditional correlations.

We can thus conclude that they approximate quite well unknown specifications of the conditional correlations and therefore are useful in investigating potential time-varying conditional correlations between economic or financial time series.

# 1.A Mathematical derivations

## 1.A.1 Derivation of the statistic of the artificial neural network test

In order to compute the $NEURAL$ statistic (1.14), we use broadly Abadir and Magnus (2005), Anderson (2003) and Lütkepohl (1996) for the derivations based on the rules of matrix algebra used to compute the different derivatives $t = 1, ..., T$.

The test statistic can be written (we simplify by $T$):

$$NEURAL = \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} \right)' \Im(\theta)^{-1} \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} \right); \tag{1.A.1.1}$$

therefore, we have

$$NEURAL = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix}' \begin{pmatrix} M_{\omega\omega} & M'_{\rho\omega} & M'_{\delta\omega} \\ M_{\rho\omega} & M_{\rho\rho} & M'_{\delta\rho} \\ M_{\delta\omega} & M_{\delta\rho} & M_{\delta\delta} \end{pmatrix}^{-1} \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix} \tag{1.A.1.2}$$

where $\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}$ is a $(3N + (1+p)\frac{N(N-1)}{2}) \times 1$ vector as follows

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix} \tag{1.A.1.3}$$

and $\Im(\theta)$ is a $(3N + (1+p)\frac{N(N-1)}{2}) \times (3N + (1+p)\frac{N(N-1)}{2})$ matrix given by

$$\Im(\theta) = \begin{pmatrix} M_{\omega\omega} & M'_{\rho\omega} & M'_{\delta\omega} \\ M_{\rho\omega} & M_{\rho\rho} & M'_{\delta\rho} \\ M_{\delta\omega} & M_{\delta\rho} & M_{\delta\delta} \end{pmatrix}. \tag{1.A.1.4}$$

1. The different sub-vectors of the $\left[3N + (1+p)\frac{N(N-1)}{2}\right] \times 1$ vector $\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}$ are:

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix}$$

with

$$V_\omega = \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \omega_1'} \quad \cdots \quad \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \omega_N'} \right)', \; V_\rho = \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})} \right),$$

and

$$V_\delta = vecl^* \left[ \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \delta_{ij}} \right)_{i,j} \right],$$

where $V_\omega$, $V_\rho$ and $V_\delta$ are respectively $3N \times 1$, $\frac{N(N-1)}{2} \times 1$ and $p\frac{N(N-1)}{2} \times 1$ vectors. $vecl$ is an operator stacking the columns of the strict lower diagonal, i.e. excluding the diagonal elements of the matrix. The notation $vecl^*$ (instead of $vecl$) is used to denote the vec-operator applied to the strictly lower triangular part (obtained by excluding the diagonal elements) of the partitioned matrix $\left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \delta_{ij}} \right)_{i,j}$ and is a $\frac{N(N-1)}{2} \times 1$ vector. Moreover, we have, for $i = 1, ..., N$ and $1 \leq i < j \leq N$:

$$\begin{cases} \frac{\partial l_t(\theta)}{\partial \omega_i} = \frac{1}{2} \frac{1}{h_{iit}} \frac{\partial h_{iit}}{\partial \omega_i} \left[ -1 + z_{it} 1_i' \mathbf{P}_t^{-1} \mathbf{z}_t \right] \\ \frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})} = -\frac{1}{2} \mathbf{U}' \left[ vec\left(\mathbf{P}_t^{-1}\right) - \left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) (\mathbf{z}_t \otimes \mathbf{z}_t) \right] \\ \frac{\partial l_t(\theta)}{\partial \delta_{ij}} = \frac{\partial l_t(\theta)}{\partial \delta_{ji}} = -\frac{1}{2} W_{ijt} \left[ vec\left(\mathbf{P}_t^{-1}\right) - \left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) (\mathbf{z}_t \otimes \mathbf{z}_t) \right] \end{cases}$$

where $\frac{\partial l_t(\theta)}{\partial \omega_i}$, $\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})}$ and $\frac{\partial l_t(\theta)}{\partial \delta_{ij}}$ are respectively $3 \times 1$, $\frac{N(N-1)}{2} \times 1$ and $p \times 1$ vectors. Moreover, $\mathbf{z}_t = \mathbf{S}_t^{-1} \boldsymbol{\varepsilon}_t$ and the matrix $\mathbf{U}$ is an $N^2 \times \frac{N(N-1)}{2}$ matrix of zeros and ones whose columns are defined as

$$\left( vec\left(1_i 1_j' + 1_j 1_i'\right) \right)_{\substack{i=1,\ldots,N-1 \\ j=i+1,\ldots,N}}$$

and the columns appear in the same order from left to right as the indices in $vecl(\mathbf{P})$. The matrix $W_{ijt}$ is an $p \times N^2$ matrix given by

$$W_{ijt} = diag\left(g_{ijt}\right) \left\{ \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \otimes \left( vec\left(1_i 1_j' + 1_j 1_i'\right) \right)' \right\}$$

where $g_{ijt}$ is a $p \times 1$ vector defined as

$$g_{ijt} = \begin{pmatrix} g_{ijt,1} \\ \vdots \\ g_{ijt,p} \end{pmatrix} = \begin{pmatrix} (1 + \exp\{-w_{ijt}' \gamma_{ij1}\})^{-1} \\ \vdots \\ (1 + \exp\{-w_{ijt}' \gamma_{ijp}\})^{-1} \end{pmatrix}$$

2. The different elements of the matrix $\Im(\theta)$ are:

- $M_{\omega\omega}$ is the following $3N \times 3N$ matrix:

$$M_{\omega\omega} = \sum_{t=1}^{T} \begin{pmatrix} M_{\omega\omega,11} & M'_{\omega\omega,12} & \cdots & M'_{\omega\omega,1N} \\ M_{\omega\omega,12} & M_{\omega\omega,22} & \cdots & M'_{\omega\omega,2N} \\ \vdots & \vdots & & \vdots \\ M_{\omega\omega,1N} & M_{\omega\omega,2N} & \cdots & M_{\omega\omega,NN} \end{pmatrix}$$

where, for $i = 1, ..., N$ and $1 \leq i < j \leq N$, $M_{\omega\omega,ii}$ and $M_{\omega\omega,ij}$ are $3 \times 3$ matrices defined as

$$\begin{cases} M_{\omega\omega,ii} = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial \omega'_i}\right] = \frac{1}{4}\frac{1}{h_{iit}^2}\frac{\partial h_{iit}}{\partial \omega_i}\left[1'_i \mathbf{P}_t^{-1} 1_i + 1\right]\frac{\partial h_{iit}}{\partial \omega_i}' \\ M_{\omega\omega,ij} = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial \omega'_j}\right] = \frac{1}{4}\frac{1}{h_{iit}h_{jjt}}\frac{\partial h_{iit}}{\partial \omega_i}\left[\rho_{ijt} 1'_i \mathbf{P}_t^{-1} 1_j + 1\right]\frac{\partial h_{jjt}}{\partial \omega_j}' \end{cases}$$

- $M_{\rho\rho}$ is the following $\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ matrix:

$$M_{\rho\rho} = \sum_{t=1}^{T} \frac{1}{4}\mathbf{U}'\left[\left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right)\mathbf{K}(\mathbf{P}_t^{-1} \otimes \mathbf{I})\right]\mathbf{U}$$

- $M_{\rho\omega}$ is a $\frac{N(N-1)}{2} \times 3N$ matrix defined as

$$M_{\rho\omega} = \sum_{t=1}^{T} \begin{pmatrix} M'_{\rho\omega,1} & M'_{\rho\omega,2} & ... & M'_{\rho\omega,N} \end{pmatrix}$$

where each $3 \times \frac{N(N-1)}{2}$ matrix $M_{\rho\omega,i}$, for $i = 1, ..., N$, is:

$$\begin{aligned} M_{\rho\omega,i} &= E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})'}\right] \\ &= \frac{1}{4}\frac{1}{h_{iit}}\frac{\partial h_{iit}}{\partial \omega_i}\left[1'_i\left(1'_i \otimes \mathbf{P}_t^{-1}\right) + 1'_i\left(1'_i \otimes \mathbf{I}\right)\mathbf{K}(\mathbf{P}_t^{-1} \otimes \mathbf{I})\right]\mathbf{U} \end{aligned}$$

- $M_{\delta\rho}$ is a $p\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ matrix given by

$$M_{\delta\rho} = \sum_{t=1}^{T} vecl^*\left[\left(M_{\delta\rho,ij}\ \right)_{i,j}\right]$$

where each $p \times \frac{N(N-1)}{2}$ matrix $M_{\delta\rho,ij}$ is

$$\begin{aligned} M_{\delta\rho,ij} = M_{\delta\rho,ji} &= E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \delta_{ij}}\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})'}\right] \\ &= \frac{1}{4}W_{ijt}\left[\left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right)\mathbf{K}\left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right)\right]\mathbf{U} \end{aligned}$$

- $M_{\delta\delta}$ is the following $p\frac{N(N-1)}{2} \times p\frac{N(N-1)}{2}$ matrix:

$$M_{\delta\delta} = \sum_{t=1}^{T} \left[M_{\delta\delta}(i,j,k,m)\right]_{i,j,k,m}$$

with $M_{\delta\delta}(i,j,k,m)$ the following $p \times p$ matrix:

$$\begin{aligned} &M_{\delta\delta}(i,j,k,m) = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \delta_{ij}} \frac{\partial l_t(\theta)}{\partial \delta'_{km}}\right] \\ &= \tfrac{1}{4} W_{ijt}\left[\left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right) \mathbf{K}(\mathbf{P}_t^{-1} \otimes \mathbf{I})\right] W'_{kmt} \end{aligned}$$

and, for each couple $(i,j)$ taken in the same order as for $V_\delta$, take the couple $(k,m)$ in the same order.

- $M_{\delta\omega}$ is a $p\frac{N(N-1)}{2} \times 3N$ matrix given by:

$$M_{\delta\omega} = \sum_{t=1}^{T} \left[M_{\delta\omega}(i,j,k)\right]_{i,j,k}$$

with $M_{\delta\omega}(i,j,k)$ a $p \times 3$ matrix defined by:

$$\begin{aligned} &M_{\delta\omega}(i,j,k) = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \delta_{ij}} \frac{\partial l_t(\theta)}{\partial \omega'_k}\right] \\ &= -\tfrac{1}{4h_{kkt}} W_{ijt}\left[vec(\mathbf{P}_t^{-1}) - \left\{(\mathbf{I} \otimes 1_k) + \left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right) \mathbf{K}(\mathbf{I} \otimes \mathbf{P}_t 1_k)\right.\right. \\ &\left.\left. + vec(\mathbf{P}_t^{-1}) vec(1'_k \mathbf{P}_t)'\right\} \mathbf{P}_t^{-1} 1_k\right] \tfrac{\partial h_{kkt}}{\partial \omega'_k} \end{aligned}$$

and, for each couple $(i,j)$ taken in the same order as for $V_\delta$, take all the $k = 1, 2, ..., N$.

For the computation of all these vectors and matrices, we have:

$$\frac{\partial h_{iit}}{\partial \omega_i} = \nu_{i,t-1} + \beta_i \frac{\partial h_{ii,t-1}}{\partial \omega_i}.$$

with $\nu_{it} = (1, \varepsilon_{it}^2, h_{iit})'$ for $i = 1, ..., N$ (we compute recursively $\frac{\partial h_{iit}}{\partial \omega_i}$).

Following Silvennoinen and Teräsvirta (2014), we have for a model with general correlation matrix $\mathbf{P}_t$

$$\begin{cases} E_{t-1}\left[\mathbf{z}_t\mathbf{z}'_t \otimes \mathbf{z}_t\mathbf{z}'_t\right] = (\mathbf{P}_t \otimes \mathbf{P}_t) + (\mathbf{I} \otimes \mathbf{P}_t)\mathbf{K}(\mathbf{I} \otimes \mathbf{P}_t) + vec\mathbf{P}_t(vec\mathbf{P}_t)' \\ E_{t-1}(z_{it}z'_{jt} \otimes \mathbf{z}_t\mathbf{z}'_t) = (1'_i \otimes \mathbf{I})\, E_{t-1}(\mathbf{z}_t\mathbf{z}'_t \otimes \mathbf{z}_t\mathbf{z}'_t)\, (1_j \otimes \mathbf{I}) \\ E_{t-1}\left[z_{it}\mathbf{z}'_t \otimes \mathbf{z}_t\mathbf{z}'_t\right] = (1'_i \otimes \mathbf{I})\, E_{t-1}\left[\mathbf{z}_t\mathbf{z}'_t \otimes \mathbf{z}_t\mathbf{z}'_t\right] \end{cases}$$

with $\mathbf{K}$ a $N^2 \times N^2$ matrix defined as

$$\mathbf{K} = \begin{bmatrix} 1_1 1_1' & \cdots & 1_i 1_1' & \cdots & 1_N 1_1' \\ \vdots & & \vdots & & \vdots \\ 1_1 1_j' & \cdots & 1_i 1_j' & \cdots & 1_N 1_j' \\ \vdots & & \vdots & & \vdots \\ 1_1 1_N' & \cdots & 1_i 1_N' & \cdots & 1_N 1_N' \end{bmatrix}$$

Following (1.A.1.2), and using the rules of the partitioned matrices and the null hypothesis, the test statistic can be written

$$NEURAL = V_\delta' \left[ M_{\delta\delta} - \begin{pmatrix} M_{\delta\omega} & M_{\delta_\rho} \end{pmatrix} \begin{pmatrix} M_{\omega\omega} & M_{\rho\omega}' \\ M_{\rho\omega} & M_{\rho\rho} \end{pmatrix}^{-1} \times \begin{pmatrix} M_{\delta\omega}' \\ M_{\delta\rho}' \end{pmatrix} \right]^{-1} V_\delta. \tag{1.A.1.5}$$

### 1.A.2 Derivation of the statistic of the Taylor expansion-based test

In order to determine the $TAYLOR$ statistic (1.23), we use the same computation as before for the $NEURAL$ statistic.

The test statistic can again be written (we simplify by $T$):

$$TAYLOR = \left(\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\right)' \Im(\theta)^{-1} \left(\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\right); \tag{1.A.2.1}$$

therefore, we have

$$TAYLOR = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix}' \begin{pmatrix} M_{\omega\omega} & M'_{\rho\omega} & M'_{\delta\omega} \\ M_{\rho\omega} & M_{\rho\rho} & M'_{\delta\rho} \\ M_{\delta\omega} & M_{\delta\rho} & M_{\delta\delta} \end{pmatrix}^{-1} \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix} \tag{1.A.2.2}$$

where $\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}$ is a $\left[3N + (1+N^*)\frac{N(N-1)}{2}\right] \times 1$ vector as follows

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix} \tag{1.A.2.3}$$

and $\Im(\theta)$ is a $(3N + (1+N^*)\frac{N(N-1)}{2}) \times (3N + (1+N^*)\frac{N(N-1)}{2})$ matrix given by

$$\Im(\theta) = \begin{pmatrix} M_{\omega\omega} & M'_{\rho\omega} & M'_{\delta\omega} \\ M_{\rho\omega} & M_{\rho\rho} & M'_{\delta\rho} \\ M_{\delta\omega} & M_{\delta\rho} & M_{\delta\delta} \end{pmatrix}. \tag{1.A.2.4}$$

1. The different sub-vectors of $\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}$ are:

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} = \begin{pmatrix} V_\omega \\ V_\rho \\ V_\delta \end{pmatrix}$$

with

$$V_\omega = \begin{pmatrix} \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \omega'_1} & \cdots & \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \omega'_N} \end{pmatrix}', \; V_\rho = \left(\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})}\right),$$

and

$$V_\delta = vecl^* \left[ \left( \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \delta_{ij}} \right)_{i,j} \right],$$

where $V_\omega$, $V_\rho$ and $V_\delta$ are respectively $3N \times 1$, $\frac{N(N-1)}{2} \times 1$ and $N^* \frac{N(N-1)}{2} \times 1$ vectors. Moreover, we have, for $i = 1, ..., N$ and $1 \leq i < j \leq N$:

$$\begin{cases} \frac{\partial l_t(\theta)}{\partial \omega_i} = \frac{1}{2} \frac{1}{h_{iit}} \frac{\partial h_{iit}}{\partial \omega_i} \left[ -1 + z_{it} 1_i' \mathbf{P}_t^{-1} \mathbf{z}_t \right] \\ \frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})} = -\frac{1}{2} \mathbf{U}' \left[ vec\left(\mathbf{P}_t^{-1}\right) - \left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) (\mathbf{z}_t \otimes \mathbf{z}_t) \right] \\ \frac{\partial l_t(\theta)}{\partial \delta_{ij}} = \frac{\partial l_t(\theta)}{\partial \delta_{ji}} = -\frac{1}{2} W_{ijt} \left[ vec\left(\mathbf{P}_t^{-1}\right) - \left(\mathbf{P}_t^{-1} \otimes \mathbf{P}_t^{-1}\right) (\mathbf{z}_t \otimes \mathbf{z}_t) \right] \end{cases}$$

where $\frac{\partial l_t(\theta)}{\partial \omega_i}$, $\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})}$ and $\frac{\partial l_t(\theta)}{\partial \delta_{ij}}$ are respectively $3 \times 1$, $\frac{N(N-1)}{2} \times 1$ and $N^* \times 1$ vectors. Moreover, $\mathbf{z}_t = \mathbf{S}_t^{-1} \boldsymbol{\varepsilon}_t$. The matrix $\mathbf{U}$ is an $N^2 \times \frac{N(N-1)}{2}$ matrix of zeros and ones whose columns are defined as

$$\left( vec\left(1_i 1_j' + 1_j 1_i'\right) \right)_{\substack{i=1,...,N-1 \\ j=i+1,...,N}}$$

and the columns appear in the same order from left to right as the indices in $vecl(\mathbf{P})$. The matrix $W_{ijt}$ is an $N^* \times N^2$ matrix given by

$$W_{ijt} = diag\left(g_{ijt}\right) \left\{ \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \otimes \left( vec\left(1_i 1_j' + 1_j 1_i'\right) \right)' \right\}$$

where the $N^* \times 1$ vector $g_{ijt}$ is composed by all terms involving functions of elements of lagged values of $\varepsilon_{i,t}$ and $\varepsilon_{j,t}$ with

$$N^* = 2 \left[ \sum_{r=1}^{k} \begin{pmatrix} 2q + r - 1 \\ r \end{pmatrix} - \sum_{r=1}^{k} \begin{pmatrix} q + r - 1 \\ r \end{pmatrix} \right]$$

2. The different elements of the matrix $\Im(\theta)$ are:

- $M_{\omega\omega}$ is the following $3N \times 3N$ matrix:

$$M_{\omega\omega} = \sum_{t=1}^{T} \begin{pmatrix} M_{\omega\omega,11} & M'_{\omega\omega,12} & \cdots & M'_{\omega\omega,1N} \\ M_{\omega\omega,12} & M_{\omega\omega,22} & \cdots & M'_{\omega\omega,2N} \\ \vdots & \vdots & & \vdots \\ M_{\omega\omega,1N} & M_{\omega\omega,2N} & \cdots & M_{\omega\omega,NN} \end{pmatrix}$$

where, for $i = 1, ..., N$ and $1 \leq i < j \leq N$, $M_{\omega\omega,ii}$ and $M_{\omega\omega,ij}$ are $3 \times 3$ matrices defined as

$$\begin{cases} M_{\omega\omega,ii} = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial \omega_i'}\right] = \frac{1}{4}\frac{1}{h_{iit}^2}\frac{\partial h_{iit}}{\partial \omega_i}\left[1_i'\mathbf{P}_t^{-1}1_i + 1\right]\frac{\partial h_{iit}}{\partial \omega_i}' \\ M_{\omega\omega,ij} = E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial \omega_j'}\right] = \frac{1}{4}\frac{1}{h_{iit}h_{jjt}}\frac{\partial h_{iit}}{\partial \omega_i}\left[\rho_{ijt}1_i'\mathbf{P}_t^{-1}1_j + 1\right]\frac{\partial h_{jjt}}{\partial \omega_j}' \end{cases}$$

- $M_{\rho\rho}$ is the following $\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ matrix:

$$M_{\rho\rho} = \sum_{t=1}^{T}\frac{1}{4}\mathbf{U}'\left[\left(\mathbf{P}_t^{-1}\otimes\mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1}\otimes\mathbf{I}\right)\mathbf{K}(\mathbf{P}_t^{-1}\otimes\mathbf{I})\right]\mathbf{U}$$

- $M_{\rho\omega}$ is a $\frac{N(N-1)}{2} \times 3N$ matrix defined as

$$M_{\rho\omega} = \sum_{t=1}^{T}\begin{pmatrix} M_{\rho\omega,1}' & M_{\rho\omega,2}' & ... & M_{\rho\omega,N}' \end{pmatrix}$$

where each $3 \times \frac{N(N-1)}{2}$ matrix $M_{\rho\omega,i}$, for $i = 1, ..., N$, is:

$$\begin{aligned} M_{\rho\omega,i} &= E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \omega_i}\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})'}\right] \\ &= \frac{1}{4}\frac{1}{h_{iit}}\frac{\partial h_{iit}}{\partial \omega_i}\left[1_i'\left(1_i'\otimes\mathbf{P}_t^{-1}\right) + 1_i'\left(1_i'\otimes\mathbf{I}\right)\mathbf{K}(\mathbf{P}_t^{-1}\otimes\mathbf{I})\right]\mathbf{U} \end{aligned}$$

- $M_{\delta\rho}$ is a $N^*\frac{N(N-1)}{2} \times \frac{N(N-1)}{2}$ matrix given by

$$M_{\delta\rho} = \sum_{t=1}^{T} vecl^*\left[\left(M_{\delta\rho,ij}\ \right)_{i,j}\right]$$

where each $N^* \times \frac{N(N-1)}{2}$ matrix $M_{\delta\rho,ij}$ is

$$\begin{aligned} M_{\delta\rho,ij} = M_{\delta\rho,ji} &= E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \delta_{ij}}\frac{\partial l_t(\theta)}{\partial vecl(\mathbf{P})'}\right] \\ &= \frac{1}{4}W_{ijt}\left[\left(\mathbf{P}_t^{-1}\otimes\mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1}\otimes\mathbf{I}\right)\mathbf{K}\left(\mathbf{P}_t^{-1}\otimes\mathbf{I}\right)\right]\mathbf{U} \end{aligned}$$

- $M_{\delta\delta}$ is the following $N^*\frac{N(N-1)}{2} \times N^*\frac{N(N-1)}{2}$ matrix:

$$M_{\delta\delta} = \sum_{t=1}^{T}\left[M_{\delta\delta}(i,j,k,m)\right]_{i,j,k,m}$$

with $M_{\delta\delta}(i,j,k,m)$ the following $N^* \times N^*$ matrix:

$$\begin{aligned} M_{\delta\delta}(i,j,k,m) &= E_{t-1}\left[\frac{\partial l_t(\theta)}{\partial \delta_{ij}}\frac{\partial l_t(\theta)}{\partial \delta_{km}'}\right] \\ &= \frac{1}{4}W_{ijt}\left[\left(\mathbf{P}_t^{-1}\otimes\mathbf{P}_t^{-1}\right) + \left(\mathbf{P}_t^{-1}\otimes\mathbf{I}\right)\mathbf{K}(\mathbf{P}_t^{-1}\otimes\mathbf{I})\right]W_{kmt}' \end{aligned}$$

and, for each couple $(i,j)$ taken in the same order as for $V_\delta$, take the couple $(k,m)$ in the same order.

- $M_{\delta\omega}$ is a $N^* \frac{N(N-1)}{2} \times 3N$ matrix given by:

$$M_{\delta\omega} = \sum_{t=1}^{T} \left[ M_{\delta\omega}\left(i,j,k\right) \right]_{i,j,k}$$

with $M_{\delta\omega}\left(i,j,k\right)$ a $N^* \times 3$ matrix defined by:

$$\begin{aligned} M_{\delta\omega}\left(i,j,k\right) &= E_{t-1}\left[ \frac{\partial l_t(\theta)}{\partial \delta_{ij}} \frac{\partial l_t(\theta)}{\partial \omega_k'} \right] \\ &= -\frac{1}{4h_{kkt}} W_{ijt} \left[ vec(\mathbf{P}_t^{-1}) - \left\{ (\mathbf{I} \otimes 1_k) + \left(\mathbf{P}_t^{-1} \otimes \mathbf{I}\right) \mathbf{K}(\mathbf{I} \otimes \mathbf{P}_t 1_k) \right. \right. \\ &\quad \left. \left. + vec(\mathbf{P}_t^{-1}) vec(1_k' \mathbf{P}_t)' \right\} \mathbf{P}_t^{-1} 1_k \right] \frac{\partial h_{kkt}}{\partial \omega_k'} \end{aligned}$$

and, for each couple $(i,j)$ taken in the same order as for $V_\delta$, take all the $k = 1, 2, ..., N$.

For the computation of all these vectors and matrices, we have:

$$\frac{\partial h_{iit}}{\partial \omega_i} = \nu_{i,t-1} + \beta_i \frac{\partial h_{ii,t-1}}{\partial \omega_i}.$$

with $\nu_{it} = (1, \varepsilon_{it}^2, h_{iit})'$ for $i = 1, ..., N$ (we compute recursively $\frac{\partial h_{iit}}{\partial \omega_i}$).

Following Silvennoinen and Teräsvirta (2014), we have for a model with general correlation matrix $\mathbf{P}_t$

$$\begin{cases} E_{t-1}\left[\mathbf{z}_t\mathbf{z}_t' \otimes \mathbf{z}_t\mathbf{z}_t'\right] = (\mathbf{P}_t \otimes \mathbf{P}_t) + (\mathbf{I} \otimes \mathbf{P}_t)\mathbf{K}(\mathbf{I} \otimes \mathbf{P}_t) + vec\mathbf{P}_t(vec\mathbf{P}_t)' \\ E_{t-1}(z_{it}z_{jt}' \otimes \mathbf{z}_t\mathbf{z}_t') = (1_i' \otimes \mathbf{I})\, E_{t-1}(\mathbf{z}_t\mathbf{z}_t' \otimes \mathbf{z}_t\mathbf{z}_t')\, (1_j \otimes \mathbf{I}) \\ E_{t-1}\left[z_{it}\mathbf{z}_t' \otimes \mathbf{z}_t\mathbf{z}_t'\right] = (1_i' \otimes \mathbf{I})\, E_{t-1}\left[\mathbf{z}_t\mathbf{z}_t' \otimes \mathbf{z}_t\mathbf{z}_t'\right] \end{cases}$$

with $\mathbf{K}$ a $N^2 \times N^2$ matrix defined as

$$\mathbf{K} = \begin{bmatrix} 1_1 1_1' & \cdots & 1_i 1_1' & \cdots & 1_N 1_1' \\ \vdots & & \vdots & & \vdots \\ 1_1 1_j' & \cdots & 1_i 1_j' & \cdots & 1_N 1_j' \\ \vdots & & \vdots & & \vdots \\ 1_1 1_N' & \cdots & 1_i 1_N' & \cdots & 1_N 1_N' \end{bmatrix}$$

Following (1.A.2.2), and using the rules of the partitioned matrices and the null hypothesis, the test statistic can be written

$$\begin{aligned} TAYLOR = V_\delta' \Bigg[ M_{\delta\delta} - \begin{pmatrix} M_{\delta\omega} & M_{\delta\rho} \end{pmatrix} \begin{pmatrix} M_{\omega\omega} & M_{\rho\omega}' \\ M_{\rho\omega} & M_{\rho\rho} \end{pmatrix}^{-1} \\ \times \begin{pmatrix} M_{\delta\omega}' \\ M_{\delta\rho}' \end{pmatrix} \Bigg]^{-1} V_\delta. \end{aligned} \tag{1.A.2.5}$$

# Chapter 2

# Testing for nonlinearity in conditional (co)variances

## 2.1 Introduction

Since the last couple of decades an important amount of multivariate models handling the conditional second moments of (financial) time series have been developed (Bauwens, Laurent and Rombouts, 2006; Silvennoinen and Teräsvirta, 2009a, for large surveys).

Caporin and McAleer (2008, 2012) ask about the pertinence to discriminate among the two most used multivariate GARCH models: the conditional covariance BEKK (Engle and Kroner, 1995) and the conditional correlation DCC (Engle, 2002). The authors exhibit the tight relationship exiting between the scalar BEKK (Ding and Engle, 2001) and the DCC models, and shed light on the existence of asymptotic theory for the BEKK model, which is a key determinant for any econometricians who want to explore the asymptotic limits. It can be demonstrated straightforwardly that the scalar BEKK, a large scale conditional covariance model dealing with the curse of dimensionality, is a special case of the BEKK model for which Comte and Lieberman (2003) have proved strong consistency and asymptotic normality of the quasi-maximum likelihood estimator. Moreover, Hafner and Preminger (2009) have recently derived the general multivariate GARCH model nesting, among many special cases, the BEKK model.

While most of the conditional variance-covariance matrices in multivariate GARCH frameworks involve a linear function of their own lags and lagged squared innovations, a specification in which conditional variances and covariances are driven by a nonlinear function of the lagged squared

innovations could be appropriate. We are inspired in this chapter by the idea that to capture asymmetric effects of negative and positive shocks of time series, the smooth transition univariate GARCH model allows intermediate regimes passing smoothly from one to the other using a nonlinear function (see van Dijk, Teräsvirta and Franses, 2002, for a survey on smooth transition models).

Although only few misspecification tests specific to multivariate models are available, it would be interesting to check the properties of the data. The literature covering the nonlinearity tests in the multivariate framework remains relatively scarce and based on threshold models (Kwan, Li and Ng, 2010; Tsay, 1998); there are also few papers for testing the constancy of conditional correlations (for instance, Chapter 1 and Silvennoinen and Teräsvirta, 2014).

To address this issue, we develop two Lagrange multiplier (LM) tests based on an exponential or a logistic smooth transition function in order to test the nonlinearity in conditional variances and covariances. We propose a new model under the alternative hypothesis, the smooth transition scalar BEKK in which conditional covariances of time series are driven by their own lags and by a nonlinear function of the lagged squared innovations. Under the null hypothesis, the model is the scalar BEKK in which conditional covariances are driven by a linear function of their own lags and lagged squared innovations, for which we define the asymptotic properties and determine the unconditional second and fourth-order moments. Then, we develop two other LM tests to check the robustness to leverage effects. Through a simple two-step procedure we are able to discriminate whether conditional covariances are characterized by linearity, nonlinearity or leverage effects. Finite-sample properties of all the new tests are examined by using Monte Carlo methods. We show that the tests perform very well in our small-sample simulations. Empirical illustrations using a set of 13 series in bivariate and multivariate analyses point out that these tests are useful to reject the linearity, and encourage the use of nonlinear conditional covariance models.

The chapter is organized as follows. We introduce the model in the next section. In Section 2.3 the asymptotic properties of the QMLE under the null hypothesis are presented. Section 2.4 gives the LM test statistics and provides Monte Carlo experiments. Two other LM tests robust to leverage effects are developed in Section 2.5. Section 2.6 contains empirical illustration. Section 2.7 concludes. Details of calculus can be found in the Appendices.

## 2.2 The model

Let the observations $\{\mathbf{y}_t\}$ be multivariate time series of $\mathsf{R}^N$ defined by

$$\mathbf{y}_t = \mu_t + \boldsymbol{\varepsilon}_t, \qquad \text{for} \quad t = 1, \ldots, T \tag{2.1}$$

where the conditional mean $\mu_t = \mathsf{E}(\mathbf{y}_t|\mathcal{F}_{t-1}) = 0$ to simplify the discussion, and $\mathcal{F}_t$ is the $\sigma$-field generated by all the information until time $t$.

Consider $\boldsymbol{\varepsilon}_t$ an $N \times 1$ disturbance vector with conditional mean

$$\mathsf{E}(\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}) = 0$$

and

$$\mathsf{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\mathcal{F}_{t-1}) = \mathbf{H}_t;$$

$\mathbf{H}_t$ is the conditional covariance matrix defined by $\mathbf{H}_t = \mathbf{H}_t(\theta)$ where $\theta$ is a parameter vector.

We define a multivariate GARCH process such that

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t, \tag{2.2}$$

where $\boldsymbol{\eta}_t \sim iid(0, \mathbf{I}_N)$, and $\mathbf{I}_N$ is the $N \times N$ identity matrix. The conditional covariance is defined, for the first-order model (i.e. for one lag), by

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + (\alpha + \varphi G(s_t; \gamma, c))\, \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta\mathbf{H}_t, \tag{2.3}$$

where $\theta = (\text{vech}(\mathbf{C})', \alpha, \varphi, \beta, \gamma, c)'$. $\mathbf{C}\mathbf{C}'$ is a positive-definite matrix of parameters composed by two $N \times N$ nonsingular lower triangular matrices $\mathbf{C}$, $\alpha$, $\varphi$, $\beta$, $\gamma$ and $c$ are scalars.

The transition function $G(s_t; \gamma, c)$ depends on a transition variable $s_t$, a slope parameter $\gamma$ which controls the transition speed, and a location parameter $c$. It can adopt a first-order exponential form:

$$G\left(s_t; \gamma, c\right) = 1 - \exp\left\{-\gamma\left(s_t - c\right)^2\right\}, \quad \gamma > 0. \tag{2.4}$$

When $s_t \to -\infty$ or $\infty$ : $G(s_t; \gamma, c) \to 1$, and when $s_t \to c$ : $G(s_t; \gamma, c) \to 0$. It also can adopt a first-order logistic form:

$$G\left(s_t; \gamma, c\right) = \left[1 + \exp\left\{-\gamma\left(s_t - c\right)\right\}\right]^{-1}, \quad \gamma > 0. \tag{2.5}$$

When $s_t \to -\infty$ : $G(s_t; \gamma, c) \to 0$, when $s_t \to c$ : $G(s_t; \gamma, c) \to 1/2$, and when $s_t \to \infty$ : $G(s_t; \gamma, c) \to 1$. When $\gamma \to \infty$ : $G(s_t; \gamma, c)$ becomes a step function from one extreme state to another. Both transition functions are bounded by $[0, 1]$. According to van Dijk, Teräsvirta and Franses (2002),

the transition variable $s_t$ can be an exogenous variable, or a function of lagged endogenous variables, or a function of time $t$ corresponding to a frequency $(t/T)$. The transition variable $s_t$ is assumed to be stationary and to have a continuous distribution.

The sufficient conditions to ensure the positivity of the model are $\beta \geq 0$, $\alpha > 0$ and $\alpha + \varphi \geq 0$ (or $\alpha \geq 0$ and $\alpha + \varphi > 0$). Theorem 1 establishes the sufficient condition for the process to be weakly stationary.

**Theorem 1.** *Let $s_t$ and $\boldsymbol{\varepsilon}_t$ be independent, and assume $s_t$ weakly exogenous. The sufficient condition for the process $\boldsymbol{\varepsilon}_t$ defined by (2.2) and (2.3) to be second-order stationary is given by*

$$\alpha + \varphi + \beta < 1, \tag{2.6}$$

*where $\varphi$ can be negative, and $0 \leq \mathsf{E}\left[G(s_t; \gamma, c)\right] \leq 1$.*

The proof is straightforwardly obtained by applying the unconditional expectation operator on both sides of (2.3). Hence, the covariance matrix $\mathbf{H}_t$ has an unconditional mean:

$$\overline{\mathbf{H}} = \{1 - (\alpha + \varphi \mathsf{E}\left[G(s_t; \gamma, c)\right] + \beta)\}^{-1}\mathbf{C}\mathbf{C}', \tag{2.7}$$

where $\mathsf{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \mathsf{E}[\mathbf{H}_t] = \mathsf{E}[\mathbf{H}_{t+1}] = \overline{\mathbf{H}}$.

**Remark 1.** *The representation given by (2.7) allows to the unconditional covariance matrix to change smoothly over-time.*

To avoid redundancy we can consider only one side of the off-diagonal elements of matrices $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$ and $\mathbf{H}_t$. As seen in (2.3), the matrices involved are symmetric and can be defined following a vech representation without any modification in the estimates such that

$$\text{vech}(\mathbf{H}_{t+1}) = \text{vech}(\mathbf{C}\mathbf{C}') + (\alpha + \varphi G(s_t; \gamma, c))\,\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') + \beta\text{vech}(\mathbf{H}_t), \tag{2.8}$$

where vech, the vector-half operator, stacks the lower triangular half into a single vector of length $N(N+1)/2$. The conditional covariance designed by (2.3) or (2.8) is driven by the same parameters. This model is actually the 'Scalar-Diagonal or Two-parameters Model' commonly named the 'Scalar BEKK' of Ding and Engle (2001), itself derived from the general BEKK model, and augmented by a smooth transition function.

In order to test the nonlinearity in conditional variances and covariances, we specify the null hypothesis by setting

$$H_0 : \varphi = 0 \tag{2.9}$$

in (2.3). We also could specify $H_0 : \gamma = 0$ but in this case (2.5) has to be defined as $G(s_t; \gamma, c) = [1 + \exp\{-\gamma(s_t - c)\}]^{-1} - 1/2, \quad \gamma > 0$. Thus, we obtain the restricted conditional covariance of the scalar BEKK model defined by

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \alpha \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \beta \mathbf{H}_t. \tag{2.10}$$

Notice that under $H_0$ we are face to unidentified parameters: $\gamma$ and $c$. Following Luukkonen, Saikkonen and Teräsvirta (1988), to circumvent this issue we develop in Section 2.4 auxiliary tests.

This can be seen as a misspecification test needed to well specify the conditional covariance modelling, since time-consumption, singularity of the Hessian matrix and non-convergence of the objective function are well-known issues in the multivariate GARCH literature, leading to estimation issue.

## 2.3 Moment conditions under the null

Since the aim of this chapter is to build a Lagrange multiplier test, only the restricted model have to be estimated. We give the sufficient conditions for consistency and asymptotic normality of the quasi-maximum likelihood estimator (QMLE) of the model defined by (2.2) and (2.10).

Strong consistency and asymptotic normality of the QMLE in multivariate GARCH models have been studied by Ling and McAleer (2003) for the CCC-GARCH, Comte and Lieberman (2003) for the BEKK, Hafner and Preminger (2009) for the VEC, Avarucci, Beutner and Zaffaroni (2013) for the BEKK-ARCH, and Pedersen and Rahbek (2014) for the variance targeting BEKK (see also Francq and Zakoïan (2010), Chapter 11). In this section, we give the moment conditions under the null hypothesis, when there is no smooth transition in the conditional (co)variances. The model given by (2.1) to (2.3) defined in Section 2.2 is, under the null, the so-called scalar BEKK of Ding and Engle (2001) defined by $\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t$, $\boldsymbol{\eta}_t \sim iid(0, \mathbf{I}_N)$, where the conditional covariance matrix is given by (2.10). The quasi-loglikelihood function is given by

$$L_T(\theta) = -\frac{1}{2T}\sum_{t=1}^{T}\left(\log|\mathbf{H}_t(\theta)| + \boldsymbol{\varepsilon}_t'\mathbf{H}_t(\theta)^{-1}\boldsymbol{\varepsilon}_t\right), \tag{2.11}$$

where $\theta = (\mathrm{vech}(\mathbf{C})', \alpha, \beta)'$ is the parameter vector, $\theta \in \Theta \subset \mathsf{R}^p$, $p = \frac{N(N+1)}{2} + 2$, and $\theta_0$ denotes the true unknown parameter vector. The estimator of the QML is defined as $\widetilde{\theta}_T = \arg\max_{\theta\in\Theta} L_T(\theta)$.

In the case where all orders are set to one, for easiness of notation and without any loss of generality, we can state that the scalar BEKK is a particular case of the BEKK model, for which Comte and Lieberman (2003) have developed asymptotic theory, since (2.10) is equivalent to

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \mathbf{A}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{A}' + \mathbf{B}\mathbf{H}_t\mathbf{B}', \tag{2.12}$$

where $\mathbf{A} = \sqrt{\alpha}\mathbf{I}_N$ and $\mathbf{B} = \sqrt{\beta}\mathbf{I}_N$.

### 2.3.1 Strong consistency

Theorem 2.1 of Jeantheau (1998) proves strong consistency of the QML estimator when the process is described by a multivariate GARCH model. We make the following classical assumptions for the scalar BEKK process defined by (2.2) and (2.10):

**Assumption 2.3.1.** *The parameter space $\Theta$ is compact, and $\alpha + \beta < 1$.*

**Assumption 2.3.2.** *The rescaled errors $\boldsymbol{\eta}_t$ admit a density absolutely continuous with respect to the Lebesgue measure and positive in a neighbourhood of the origin.*

**Assumption 2.3.3.** *$var(\boldsymbol{\eta}_t) = \mathbf{I}_N$.*

**Assumption 2.3.4.** *The model is identifiable: $\alpha > 0$.*

Under Assumptions 2.3.1 – 2.3.4, the consistency is obtained:

$$\widetilde{\theta}_T \underset{T\to\infty}{\longrightarrow}_{a.s.} \theta_0.$$

**Remark 2.** *Assumptions 2.3.1 and 2.3.2 are in line with the literature on QML estimation of multivariate GARCH (see Comte and Lieberman, 2003; Hafner and Preminger, 2009; Pedersen and Rahbek, 2014) implying a unique stationary and ergodic solution $\{\boldsymbol{\varepsilon}_t\}$ to the process defined by (2.2) and (2.10) where the stationary solution has finite second-order moments, $\mathsf{E}||\boldsymbol{\varepsilon}_t||^2 < \infty$. Assumption 2.3.3 is made to ensure identifiability and is not restrictive, and Assumption 2.3.4 implies the identification of the process (see Comte and Lieberman, 2003; Hafner and Preminger, 2009, for details and proofs).*

### 2.3.2 Asymptotic normality

To establish asymptotic normality two additional assumptions are made:

**Assumption 2.3.5.** *$\theta_0$ is an interior point of $\Theta$.*

**Assumption 2.3.6.** $\mathsf{E}||\varepsilon_t||^6 < \infty$.

Under Assumptions 2.3.1 – 2.3.6, the asymptotic distribution of the QML estimator is given by

$$\sqrt{T}(\widetilde{\theta}_T - \theta_0) \xrightarrow[T\to\infty]{D} N(0, \mathcal{I}(\theta_0)^{-1}\mathcal{J}(\theta_0)\mathcal{I}(\theta_0)^{-1}) \tag{2.13}$$

with

$$\mathcal{J}(\theta_0) = \mathsf{E}\left[\left.\frac{\partial l_t(\theta)}{\partial\theta}\right|_{\theta=\theta_0}\left.\frac{\partial l_t(\theta)}{\partial\theta'}\right|_{\theta=\theta_0}\right] \tag{2.14}$$

and

$$\mathcal{I}(\theta_0) = -\mathsf{E}\left[\left.\frac{\partial^2 l_t(\theta)}{\partial\theta\partial\theta'}\right|_{\theta=\theta_0}\right], \tag{2.15}$$

consistently estimated by

$$\widetilde{\mathcal{J}}(\widetilde{\theta}_T) = \frac{1}{T}\sum_{t=1}^{T}\left.\frac{\partial l_t(\theta)}{\partial\theta}\right|_{\theta=\widetilde{\theta}_T}\left.\frac{\partial l_t(\theta)}{\partial\theta'}\right|_{\theta=\widetilde{\theta}_T} \tag{2.16}$$

and

$$\widetilde{\mathcal{I}}(\widetilde{\theta}_T) = -\frac{1}{T}\sum_{t=1}^{T}\left.\frac{\partial^2 l_t(\theta)}{\partial\theta\partial\theta'}\right|_{\theta=\widetilde{\theta}_T}. \tag{2.17}$$

**Remark 3.** *In line with Pedersen and Rahbek (2014), the moment conditions in Assumptions 2.3.5 and 2.3.6 are those found in existing literature on asymptotic normality of the QMLE. Hafner and Preminger (2009) provide regularity conditions under the sixth-order moment of the innovations for the QMLE of the VEC model which nests, among many others, the BEKK model; Stelzer (2008) presents some results on the relationship between the VEC and BEKK models, and Engle and Kroner (1995) have analyzed in detail these models. Avarucci, Beutner and Zaffaroni (2013) claim that these conditions could be stronger than effectively needed. However they show that to establish asymptotic normality below the fourth-order moment is hard to achieve in multivariate models. For details and proofs see Comte and Lieberman (2003).*

**Remark 4.** *If the innovations $\varepsilon_t$ are conditionally Gaussian, the ML estimation provides the most efficient estimators and the sandwich matrix becomes $\mathcal{J}(\theta_0)^{-1}$, the Cramer-Rao lower bound.*

### 2.3.3 Fourth-order moment

Following He and Teräsvirta (1999a,b) and Ling and McAleer (2002a,b) in univariate GARCH, and Hafner (2003) and He and Teräsvirta (2004) in multivariate GARCH, we don't consider moments of higher order since the fourth moments are probably more interesting in practice than any higher-order ones, in particular because they permit to compute the kurtosis and the autocorrelation functions of residuals and squared residuals.

We consider the matrix reduced-form of the scalar BEKK model defined by (2.2) and (2.10), corresponding to the null hypothesis of linear conditional covariances:

$$\mathbf{H}_t = \mathbf{C}\mathbf{C}' + \Gamma_{t-1}\mathbf{H}_{t-1}, \tag{2.18}$$

where $\Gamma_t = \alpha\boldsymbol{\eta}_t\boldsymbol{\eta}_t' + \beta\mathbf{I}_N$.

We introduce into the next theorem the conditions and the expressions of the unconditional second and fourth-order moments.

**Theorem 2.** *Let* $\mathsf{E}[\boldsymbol{\eta}_t\boldsymbol{\eta}_t'] = \mathbf{I}_N$ *and* $\mathsf{E}[\boldsymbol{\eta}_t\boldsymbol{\eta}_t' \otimes \boldsymbol{\eta}_t\boldsymbol{\eta}_t'] = \mathbf{I}_{N^2}$ *by assumption (2.2). Then, the second and fourth-order moments of* $\boldsymbol{\varepsilon}_t$ *exist if and only if*

$$\alpha + \beta < 1. \tag{2.19}$$

*Under condition (2.19) the unconditional second-order moment is*

$$\mathbf{H} = (\mathbf{I}_N - \Gamma)^{-1}\,\mathbf{C}\mathbf{C}', \tag{2.20}$$

*where* $\Gamma = \mathsf{E}[\Gamma_t] = (\alpha + \beta)\mathbf{I}_N$.

*The unconditional fourth-order moment is given by*

$$\mathsf{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' \otimes \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \mathbf{I}_N[(\mathbf{C}\mathbf{C}' \otimes \mathbf{C}\mathbf{C}') + 2(\mathbf{C}\mathbf{C}' \otimes \Gamma\mathbf{H})]\,[\mathbf{I}_{N^2} - \Gamma^\star]^{-1} \tag{2.21}$$

*where* $\Gamma^\star = (\alpha + \beta)^2\mathbf{I}_{N^2}$.

The proof for Theorem 2 is obtained by trivial matrix calculus not reported here.

**Remark 5.** *Similar results can be found in Theorem 1 of He and Teräsvirta (2004) (also used in Nakatani and Teräsvirta, 2009) which defines the second-order moment for the ECCC-GARCH (see also Francq and Zakoïan, 2010, p. 430 (11.7))*

**Remark 6.** *Theorem 2 is in line with Theorem 2 of Hafner (2003) which gives the sufficient conditions and the expression of the unconditional fourth-order moment for the general mutivariate GARCH(p,q) process in vector specification nesting the BEKK and the factor GARCH models, while Theorem 2 of He and Teräsvirta (2004) establishes them for the ECCC-GARCH.*

## 2.4 Nonlinearity test

### 2.4.1 Approximation of the null hypothesis

Under the null hypothesis of linear conditional covariances, $H_0 : \varphi = 0$, some parameters in the functions (2.4) and (2.5) are not identified: $\gamma$ and $c$. Following Luukkonen, Saikkonen and Teräsvirta (1988), we approximate the transition functions by Taylor expansion around $\gamma = 0$. Appendices 2.C.1 and 2.C.2 present the details of the technical derivations of the two test statistics developed below.

#### 2.4.1.1 Exponential-based linear conditional covariance test

The exponential function defined by (2.4) is approximated by a first-order Taylor expansion:

$$G(s_t; \gamma, c) \cong \gamma(s_t - c)^2. \tag{2.22}$$

Reparametrizing (2.3) with (2.22) yields to

$$\mathbf{H}_{t+1} = \mathbf{CC}' + \left(\varphi_1 + \varphi_2 s_t + \varphi_3 s_t^2\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \beta \mathbf{H}_t, \tag{2.23}$$

where $\varphi_1 = \alpha + \gamma c^2$, $\varphi_2 = -2\gamma c$ and $\varphi_3 = \gamma$.

The auxiliary null hypothesis of the exponential-based test, say $LCC(E)$, is as follows:

$$H_{01} : \varphi_2 = \varphi_3 = 0. \tag{2.24}$$

The LM statistic, say $LM_{LCC(E)}$, for testing $H_{01}$ is asymptotically distributed as a $\chi^2$ with 2 degrees of freedom ($\chi^2_2$) when the unrestricted model contains an exponential transition function.

#### 2.4.1.2 Logistic-based linear conditional covariance test

The logistic function defined by (2.5) is approximated by a second-order Taylor expansion:

$$G(s_t; \gamma, c) \cong 1/2 + 1/4(s_t - c)\gamma. \tag{2.25}$$

Reparametrizing (2.3) with (2.25) yields to

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + (\phi_1 + \phi_2 s_t)\,\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta\mathbf{H}_t, \tag{2.26}$$

where $\phi_1 = \alpha + 1/2 - 1/4\gamma c$ and $\phi_2 = 1/4\gamma$.

The auxiliary null hypothesis of the logistic-based test, say $LCC(L)$, is as follows:

$$H_{02} : \phi_2 = 0. \tag{2.27}$$

The LM statistic, say $LM_{LCC(L)}$, for testing $H_{02}$ is asymptotically distributed as a $\chi^2$ with 1 degree of freedom ($\chi_1^2$) when the unrestricted model contains a logistic transition function.

### 2.4.2 LM test statistic

Assuming that there is only one lag in the innovations and the conditional covariances, we construct a Lagrange multiplier (LM) test for smooth transition in the conditional covariances. Under the null hypothesis the model is the scalar BEKK defined by (2.2) and (2.10), while under the alternative it is the scalar BEKK with smooth transition in the conditional covariances defined by (2.2) and (2.3).

We consider the Gaussian loglikelihood function $L(\theta) = \sum_{t=1}^{T} l_t(\theta)$ with

$$l_t(\theta) = -\frac{1}{2}[N\ \log(2\pi) + \log|\mathbf{H}_t(\theta)| + \boldsymbol{\varepsilon}_t'\mathbf{H}_t(\theta)^{-1}\boldsymbol{\varepsilon}_t].$$

Under mild regularity conditions, the LM test statistic is given by

$$LM = T^{-1}\mathcal{S}(\widehat{\theta}_T)'\widehat{\mathcal{I}}(\widehat{\theta}_T)^{-1}\mathcal{S}(\widehat{\theta}_T). \tag{2.28}$$

$\widehat{\theta}_T$ is the Gaussian maximum likelihood estimator of $\theta_0$, the true unknown parameter vector under the null hypothesis. The score evaluated at $\widehat{\theta}_T$ equals

$$\mathcal{S}(\widehat{\theta}_T) = \sum_{t=1}^{T} \left.\frac{\partial l_t(\theta)}{\partial\theta}\right|_{\theta=\widehat{\theta}_T}.$$

The population information matrix $\widehat{\mathcal{I}}(\widehat{\theta}_T)$ is the consistent estimator of the score's outer product (2.14), or the consistent estimator of the negative of the Hessian matrix (2.15), or the sandwich matrix which is the product of these matrices. For all the tests developed in this chapter, we apply the rules of matrix partition to the inverse of the consistent estimator of the negative of the Hessian matrix (2.15) and extract the relevant part.

Once the LM statistic is computed analytically, we estimate the restricted model under the null hypothesis. Then, we compute the LM

statistic (2.28) by replacing $\theta_0$, the true unknown parameter vector, by the consistent estimator $\widehat{\theta}$ under the null hypothesis, i.e. in the scalar BEKK model given by (2.2) and (2.10). Appendix 2.B presents the details of derivations of the score and the Hessian matrix.

### 2.4.3 Monte Carlo experiments

In this section, we investigate the small-sample performances of the exponential-based ($LCC(E)$) and logistic-based ($LCC(L)$) linear conditional covariance test statistics.

#### 2.4.3.1 Simulation design

The true data generating process is based on the general multivariate GARCH model given by (2.1) and (2.2) with different specifications and Gaussian distribution. For all the Monte Carlo simulations, the sample sizes are $T = 100$, 250, 500 and 1000. Before generating the actual observations, we remove the first 1000 observations from the series in order to eliminate initialization effects. The number of endogenous variables is $N = 2$, and the number of replications is $S = 5000$, except in Table 2.3 below where $N = 5$ and $S = 1000$. The generation parameters reported in Table 2.1 are intended to be close to estimated parameters. Note that other sets of parameters have been tried with similar performance to those in Table 2.2. Moreover, for all the Monte Carlo simulations, we use the analytical expressions of the score and the negative Hessian; Hafner and Herwartz (2008) show that analytical derivatives clearly outperform numerical methods.

#### 2.4.3.2 Size results

The empirical size is investigated with a bivariate first-order scalar BEKK model (Ding and Engle, 2001). Following Silvennoinen and Teräsvirta (2014), the transition variable for the size analysis is generated from an exogenous GARCH$(1,1)$ process, such that $s_t = h_t^{1/2} z_t$, where $z_t \sim N(0,1)$ and $h_t = 0.02 + 0.03 s_{t-1}^2 + 0.94 h_{t-1}$. The results of Monte Carlo simulations are reported in Table 2.2. We observe in this table that the size of the logistic-based ($LCC(L)$) and exponential-based ($LCC(E)$) linear conditional covariance tests is already very close to the nominal size for the sample size of 500. The empirical sizes in small samples are close to the nominal sizes, and the small sample sizes converge towards the nominal sizes when the number of observations $T$ is increasing, showing

Table 2.1: Generation parameters

| | *Size* | *Power* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | *Scalar* | *Log* | *Exp* | *Lev* | *Exp* ($N = 5$) | | | | |
| $\mathbf{C}$ | $\begin{bmatrix} 0.07 & \\ 0.03 & 0.09 \end{bmatrix}$ | $\begin{bmatrix} 0.07 & \\ 0.03 & 0.09 \end{bmatrix}$ | | | $\begin{bmatrix} 0.5 & & & & \\ 0.9 & 0.5 & & & \\ 0.1 & 0.2 & 0.7 & & \\ 0.2 & 0.8 & 0.3 & 0.1 & \\ 0.6 & 0.9 & 0.9 & 0.3 & 0.4 \end{bmatrix}$ | | | | |
| $\alpha$ | 0.06 | 0.02 | 0.02 | 0.02 | 0.10 | | | | |
| $\varphi$ | – | 0.05 | 0.05 | – | 0.20 | | | | |
| $\delta$ | – | – | – | 0.05 | – | | | | |
| $\beta$ | 0.84 | 0.91 | 0.91 | 0.91 | 0.65 | | | | |
| $\gamma$ | – | 2.00 | 2.00 | – | 5.00 | | | | |
| $c$ | – | 0.00 | 0.00 | – | 0.00 | | | | |

*Scalar*: $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \alpha\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta\mathbf{H}_t$.
*Log*: $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\alpha + \varphi\left[1 + \exp\left\{-\gamma\left(s_t - c\right)\right\}\right]^{-1}\right)\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta\mathbf{H}_t$.
*Exp*: $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\alpha + \varphi\left[1 - \exp\left\{-\gamma\left(s_t - c\right)^2\right\}\right]\right)\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \beta\mathbf{H}_t$.
*Lev*: $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \alpha\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t' + \delta\boldsymbol{\nu}_t\boldsymbol{\nu}_t' + \beta\mathbf{H}_t$.

similar performance for both tests. However, until the sample size reaches $T = 500$, $LCC(L)$ shows better properties. A simple and intuitive explanation is related to the construction of the auxiliary test based on Taylor approximation. Under the null, $LCC(L)$ is preferred to $LCC(E)$ because the latest contains the variable $s_t^2$ in (2.23) adding nonlinearity for the size analysis. Note that this explanation remains valid for the next tests robust to leverage effects developed in Section 2.5.

Table 2.2: Small sample sizes of the logistic-based test ($LCC(L)$) and the exponential-based test ($LCC(E)$) when the simulated model is a scalar BEKK.

| | $LCC(L)$ | | | $LCC(E)$ | | |
|---|---|---|---|---|---|---|
| T | 1% | 5% | 10% | 1% | 5% | 10% |
| 100 | 0.0170 | 0.0570 | 0.0960 | 0.0468 | 0.0980 | 0.1492 |
| 250 | 0.0108 | 0.0440 | 0.0848 | 0.0188 | 0.0602 | 0.1116 |
| 500 | 0.0116 | 0.0510 | 0.0972 | 0.0110 | 0.0546 | 0.1058 |
| 1000 | 0.0108 | 0.0462 | 0.0902 | 0.0118 | 0.0550 | 0.1042 |

#### 2.4.3.3 Power results

The empirical power is investigated with a bivariate first-order logistic and exponential smooth transition scalar BEKK, a bivariate first-order scalar BEKK with leverage effects, and a five-variate first-order exponential smooth transition scalar BEKK, denoted by *Log*, *Exp*, *Lev* and *Exp* ($N = 5$) in Table 2.1, respectively, where Table 2.1 reports the generation parameters. In line with Silvennoinen and Teräsvirta (2014), a natural choice for the transition variable is to use a linear combination of lags of squared returns containing relative information of covariances. The transition variable is defined such that $s_t = (0.2, 0.2, 0.2, 0.2, 0.2) \times (\bar{\varepsilon}_{t-1}^{(2)}, \bar{\varepsilon}_{t-2}^{(2)}, \bar{\varepsilon}_{t-3}^{(2)}, \bar{\varepsilon}_{t-4}^{(2)}, \bar{\varepsilon}_{t-5}^{(2)})'$, where $\bar{\varepsilon}_t^{(2)}$ is the mean of $\boldsymbol{\varepsilon}_t$ over $N$ of its squared elementwise.

Figures 2.1 and 2.2 summarize the empirical powers of the exponential-based and logistic-based tests, respectively, when we simulate an exponential and a logistic smooth transition scalar BEKK, while Figure 2.3 shows the powers of both tests when a scalar BEKK with leverage effects is simulated. Table 2.3 reports the results of both tests, for $N = 5$, when an exponential smooth transition scalar BEKK is simulated. When we consider Figures 2.1 to 2.3 and Table 2.3, we remark that the power

of the two tests decreases when dimension raises; actually, the scalar BEKK is known to be ill-conditioned when the dimension reaches the observations. The performances increase with the sample size, and the rejection frequencies are close to one for $T = 1000$. The test based on an exponential function ($LCC(E)$) is likely more powerful, whatever we simulate a logistic or an exponential smooth transition scalar BEKK model, or a scalar BEKK model with leverage effects. One can explain this difference by the fact that $LCC(E)$ has a useful extra variable $s_t^2$ which adds nonlinearity in the expression of the conditional covariance matrix given by (2.23). This remains valid for the tests robust to leverage effects developed in the next section.

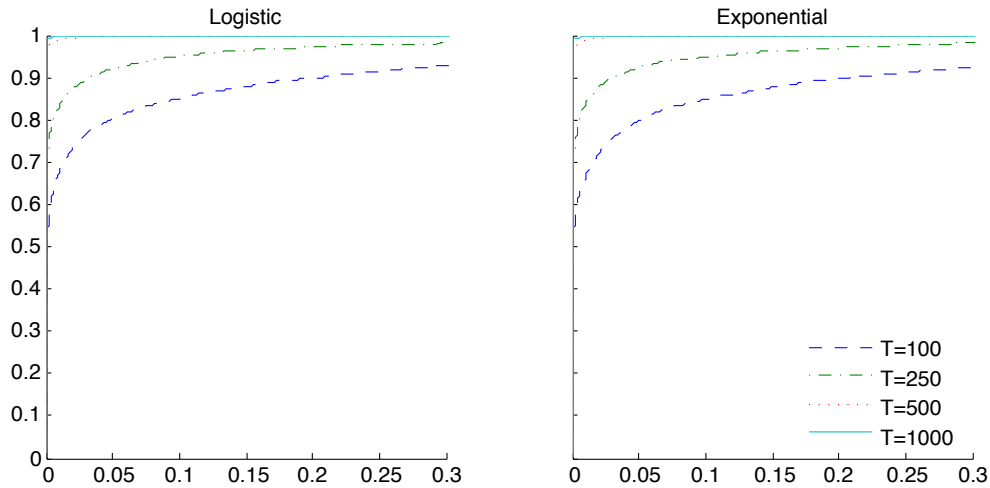



Figure 2.1: Powers curves of the exponential-based test ($LCC(E)$) when the simulated models are a logistic (left) and an exponential (right) smooth transition scalar BEKK.

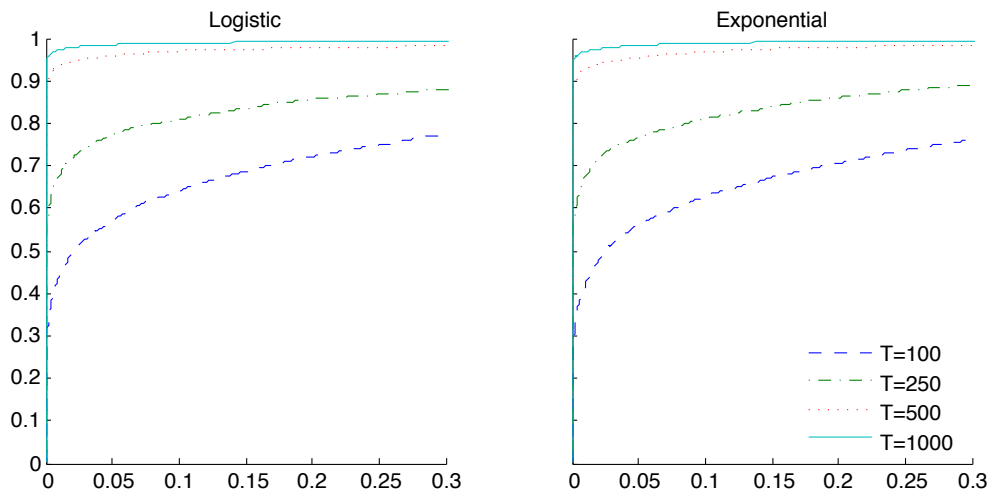



Figure 2.2: Powers curves of the logistic-based test ($LCC(L)$) when the simulated models are a logistic (left) and an exponential (right) smooth transition scalar BEKK.

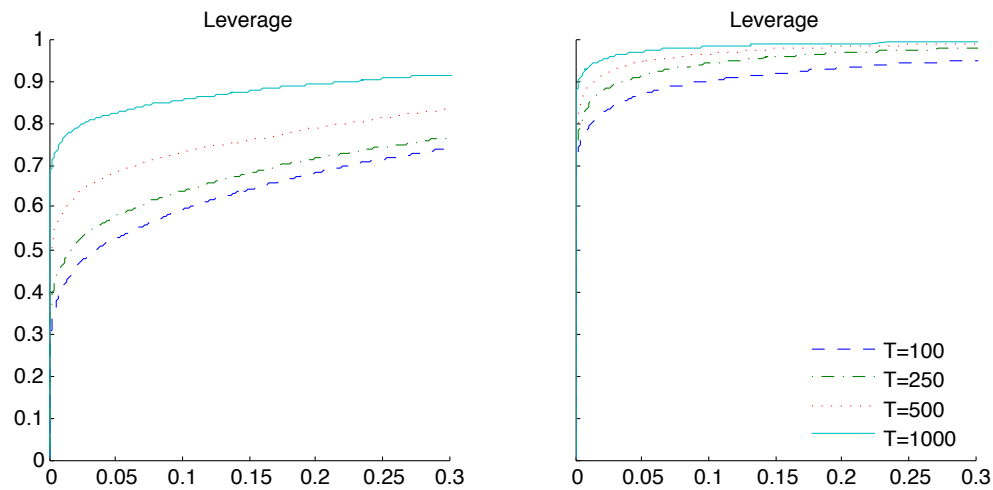



Figure 2.3: Powers curves of the logistic-based test ($LCC(L)$) (left) and the exponential-based test ($LCC(E)$) (right) when the simulated model is a scalar BEKK with leverage effects.

Table 2.3: Small sample powers of the logistic-based test ($LCC(L)$) and the exponential-based test ($LCC(E)$) when the simulated model is an exponential smooth transition scalar BEKK, for $N = 5$.

| | $LCC(L)$ | | | $LCC(E)$ | | |
|---|---|---|---|---|---|---|
| T | 1% | 5% | 10% | 1% | 5% | 10% |
| 100 | 0.322 | 0.472 | 0.566 | 0.461 | 0.588 | 0.682 |
| 250 | 0.662 | 0.817 | 0.871 | 0.620 | 0.772 | 0.832 |
| 500 | 0.918 | 0.971 | 0.986 | 0.889 | 0.955 | 0.973 |
| 1000 | 0.998 | 0.998 | 0.999 | 0.998 | 1.000 | 1.000 |

## 2.5 Robustness to leverage effects

In order to deal with asymmetry, an extension of the univariate GJR-GARCH model of Glosten, Jagannathan and Runkle (1992) to the multivariate model is proposed by Kroner and Ng (1998). In this section we introduce the smooth transition scalar BEKK with leverage effects.

Without loss of generality, we develop two new LM tests robust to leverage effects by adding a leverage term under the null and the alternative hypotheses. Notice that under the null hypothesis, the asymptotic theory remains valid by adding a leverage parameter as shown by McAleer, Hoti and Chan (2009) for the constant conditional correlation vector ARMA-asymmetric GARCH model, and by Francq and Zakoïan (2012).

The conditional covariance matrix defined by (2.3) is augmented as follows:

$$\mathbf{H}_{t+1} = \mathbf{CC}' + (\alpha + \varphi G(s_t; \gamma, c))\, \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t, \tag{2.29}$$

where $\delta$ is a scalar and $\boldsymbol{\nu}_t = I(\boldsymbol{\varepsilon}_t < 0) \odot \boldsymbol{\varepsilon}_t$. $I()$ is a $N \times 1$ indicator function which takes value one if the argument is true and zero otherwise, while $\odot$ indicates the Hadamard product. Under the null hypothesis, $H_{0^\star} : \varphi = 0$, the model is defined such that

$$\mathbf{H}_{t+1} = \mathbf{CC}' + \alpha \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t \tag{2.30}$$

is the conditional covariance matrix of the scalar BEKK model with leverage effects. This model is a direct extension of the GJR-GARCH to the multivariate case. When $\delta = 0$, the model given by (2.30) collapses to the scalar BEKK defined by (2.10). Appendices 2.C.3 and 2.C.4 introduce the details of the technical derivations of the two following test statistics.

### 2.5.1 Exponential-based linear conditional covariance test robust to leverage

After Taylor approximation and reparametrizing (2.29) with (2.22) we obtain

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\varphi_1 + \varphi_2 s_t + \varphi_3 s_t^2\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t. \tag{2.31}$$

The auxiliary null hypothesis of the exponential-based test robust to leverage effects, say $LCC(E)^\star$, is as follows:

$$H_{03} : \varphi_2 = \varphi_3 = 0.$$

The LM statistic, say $LM_{LCC(E)^\star}$, for testing $H_{03}$ is asymptotically distributed as a $\chi^2$ with 2 degrees of freedom ($\chi^2_2$) when the unrestricted model contains an exponential transition function and leverage effects.

### 2.5.2 Logistic-based linear conditional covariance test robust to leverage

After Taylor approximation and reparametrizing (2.29) with (2.25) we obtain

$$\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\phi_1 + \phi_2 s_t\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t. \tag{2.32}$$

The auxiliary null hypothesis of the logistic-based test robust to leverage effects, say $LCC(L)^\star$, is as follows:

$$H_{04} : \phi_2 = 0.$$

The LM statistic, say $LM_{LCC(L)^\star}$, for testing $H_{04}$ is asymptotically distributed as a $\chi^2$ with 1 degree of freedom ($\chi^2_1$) when the unrestricted model contains a logistic transition function and leverage effects.

### 2.5.3 A two-step procedure

In order to identify what kind of nonlinearity we deal with, we propose a two-step procedure (see Figure 2.4):

**Step 1.** We perform $LCC(E)$ and $LCC(L)$, where the null hypothesis is the linearity of conditional covariances. If the null is rejected for at least one of the two tests then the conditional covariances may follow a nonlinear process.

**Step 2.** If the null of linearity in conditional covariances is rejected in Step 1, we apply $LCC(E)^\star$ or $LCC(L)^\star$ to check if the series have leverage effects in their conditional covariances. When the null hypothesis is not rejected for both tests, we conclude that the conditional covariances are characterized by leverage effects.

By using this simple two-step procedure, we are able to characterize the second conditional moment of time series, and to propose a better conditional covariance modelling which considers linearity, nonlinearity or leverage effects.

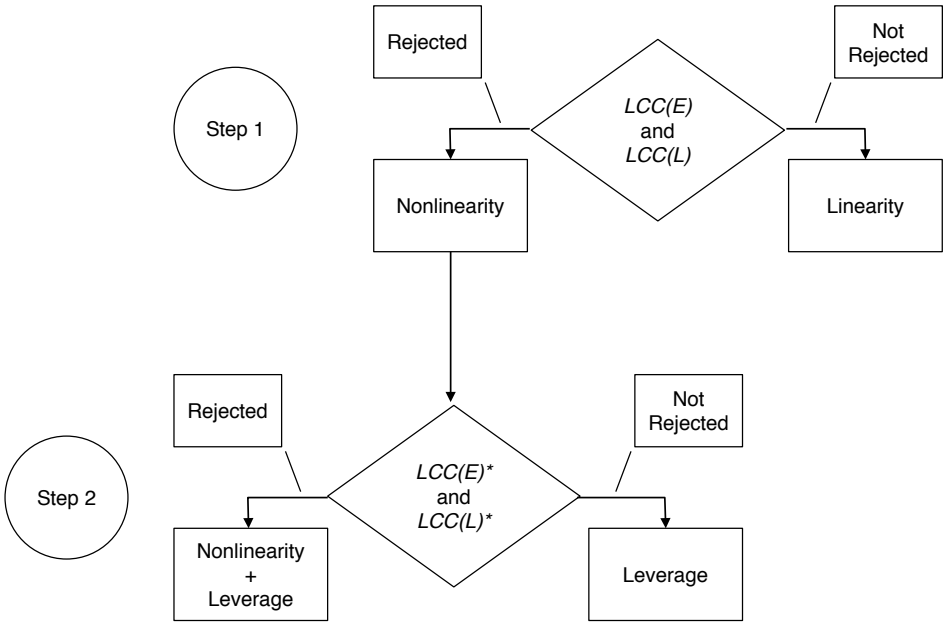



Figure 2.4: Diagrammatical repesentation of the two-step procedure using $LCC(E)$ and $LCC(L)$, and $LCC(E)^\star$ and $LCC(L)^\star$.

### 2.5.4 Monte Carlo simulations

We explore in this section the small-sample performances of the exponential-based ($LCC(E)^\star$) and logistic-based ($LCC(L)^\star$) linear conditional covariance test statistics robust to leverage effects. The simulation design is the same as described in Section 2.4.3.1, but we only consider two endogenous variables: $N = 2$. Table 2.4 details the generation parameters.

#### 2.5.4.1 Size study

We study the empirical size with a bivariate first-order scalar BEKK model with leverage effects and normal errors. As for the previous tests, the transition variable is generated from an exogenous GARCH(1, 1) process such that $s_t = h_t^{1/2} z_t$, where $z_t \sim N(0,1)$, and $h_t = 0.02 + 0.03 s_{t-1}^2 + 0.94 h_{t-1}$.

Table 2.4: Generation parameters (leverage)

| | *Size* | *Power* | | | |
|---|---|---|---|---|---|
| | *Lev* | *Log*$^\star$ | *Exp*$^\star$ | *MS*$^\star$ | |
| | | | | *Regime* 1 | *Regime* 2 |
| $\mathbf{C}$ | $\begin{bmatrix} 0.05 & \\ 0.02 & 0.07 \end{bmatrix}$ | $\begin{bmatrix} 0.06 & \\ 0.01 & 0.08 \end{bmatrix}$ | $\begin{bmatrix} 0.06 & \\ 0.04 & 0.08 \end{bmatrix}$ | $\begin{bmatrix} 0.07 & \\ 0.03 & 0.09 \end{bmatrix}$ | |
| $\alpha$ | 0.06 | 0.01 | 0.05 | 0.30 | 0.05 |
| $\varphi$ | – | 0.02 | 0.04 | 0.03 | 0.02 |
| $\delta$ | 0.05 | 0.02 | 0.03 | 0.02 | 0.01 |
| $\beta$ | 0.79 | 0.93 | 0.84 | 0.60 | 0.90 |
| $\gamma$ | – | 2.00 | 1.00 | 2.00 | 5.00 |
| $c$ | – | 0.00 | 0.00 | 0.00 | 0.00 |
| $\mathbf{P}$ | – – | – | – | $\begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}$ | |

*Lev* : $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \alpha \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t$.
*Log*$^\star$ : $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\alpha + \varphi \left[1 + \exp\left\{-\gamma\left(s_t - c\right)\right\}\right]^{-1}\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t$.
*Exp*$^\star$ : $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\alpha + \varphi \left[1 - \exp\left\{-\gamma\left(s_t - c\right)^2\right\}\right]\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t$.
*MS*$^\star$ : $\mathbf{H}_{t+1} = \mathbf{C}\mathbf{C}' + \left(\alpha + \varphi \left[1 - \exp\left\{-\gamma\left(s_t - c\right)^2\right\}\right]\right) \boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t' + \delta \boldsymbol{\nu}_t \boldsymbol{\nu}_t' + \beta \mathbf{H}_t$.
Note: *MS*$^\star$ denotes the Markov-switching smooth transition scalar BEKK model with leverage effects passing from *Regime* 1 to *Regime* 2 with the transition matrix ($\mathbf{P}$).

The results reported in Table 2.5 indicate that the size of the logistic-based ($LCC(L)^\star$) and exponential-based ($LCC(E)^\star$) linear conditional covariance tests robust to leverage effects is very close to the nominal size for the sample size of 1000. We remark that the number of observations needs to be increased compared to the previous tests since the leverage effects include only the negative innovations. Note that, as mentioned in Section 2.4.3.2, $LCC(L)^\star$ shows better performances until the sample

size reaches $T = 1000$.

Table 2.5: Small sample sizes of the logistic-based ($LCC(L)^\star$) and the exponential-based ($LCC(E)^\star$) tests robust to leverage effects.

| | $LCC(L)^\star$ | | | $LCC(E)^\star$ | | |
|---|---|---|---|---|---|---|
| T | 1% | 5% | 10% | 1% | 5% | 10% |
| 100 | 0.0322 | 0.0888 | 0.1490 | 0.0732 | 0.1726 | 0.2518 |
| 250 | 0.0142 | 0.0630 | 0.1176 | 0.0234 | 0.0842 | 0.1488 |
| 500 | 0.0128 | 0.0512 | 0.1048 | 0.0124 | 0.0616 | 0.1136 |
| 1000 | 0.0120 | 0.0548 | 0.1168 | 0.0106 | 0.0576 | 0.1130 |

#### 2.5.4.2 Power study

For the both new tests we study the power by simulating a bivariate first-order logistic and exponential smooth transition scalar BEKK with leverage effects. The generation parameters are reported in Table 2.4. The previous framework using a linear combination of lags of squared returns as transition variable described in Section 2.4.3.3 is applied here.

Figures 2.5 and 2.6 show the results of our small sample power simulations for the exponential and the logistic-based tests robust to leverage effects, respectively. As previously noticed in Section 2.4.3.3, $LCC(E)^\star$ shows better performances than $LCC(L)^\star$. However, when the sample size reaches $T = 1000$, the rejection frequencies are close to 1 for both tests.

We also simulate a Markov switching smooth transition scalar BEKK model with leverage effects based on the specification of Klaassen (2002). The parameters and the model (denoted $MS^\star$) are reported in Table 2.4; the matrix of probability $\mathbf{P}$ represents the case where regime switches occur often. Table 2.6 gives the empirical power of the tests robust to leverage effects when this model is generated indicating that rejection frequencies are already close to one when the sample size is equal to $T = 1000$ for both tests.

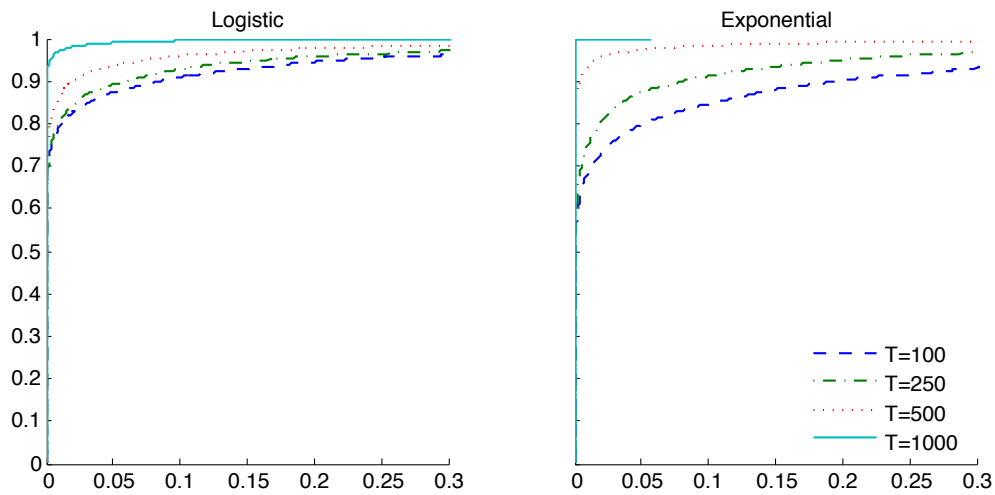



Figure 2.5: Powers curves of the exponential-based test robust to leverage effects ($LCC(E)^\star$) when the simulated models are a logistic (left) and an exponential (right) smooth transition scalar BEKK with leverage effects.

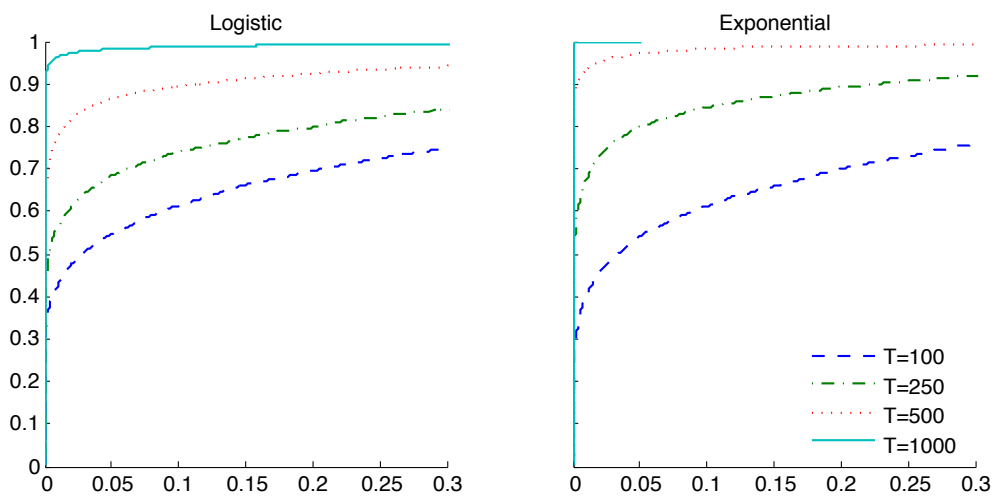



Figure 2.6: Powers curves of the logistic-based test robust to leverage effects ($LCC(L)^\star$) when the simulated models are a logistic (left) and an exponential (right) smooth transition scalar BEKK with leverage effects.

Table 2.6: Small sample powers of the logistic-based ($LCC(L)^\star$) and the exponential-based ($LCC(E)^\star$) tests robust to leverage effects when the simulated model is a Markov switching smooth transition scalar BEKK model with leverage effects.

| | $LCC(L)^\star$ | | | $LCC(E)^\star$ | | |
|---|---|---|---|---|---|---|
| T | 1% | 5% | 10% | 1% | 5% | 10% |
| 100 | 0.8002 | 0.8204 | 0.8434 | 0.8448 | 0.8568 | 0.8662 |
| 250 | 0.8488 | 0.8562 | 0.8620 | 0.8690 | 0.8842 | 0.9020 |
| 500 | 0.8652 | 0.9310 | 0.9554 | 0.9190 | 0.9608 | 0.9760 |
| 1000 | 0.9706 | 0.9900 | 0.9936 | 0.9234 | 0.9656 | 0.9888 |

## 2.6 Empirical illustration

We illustrate the performance of our tests in an empirical study. We consider 13 series:

- The S&P500 (SPX) index,
- Eight assets: Apple Inc. (AAPL), Bank of America Corp. (BAC), The Coca-Cola Company (KO), Ford Motor (F), International Business Machines Corp. (IBM), JPMorgan and Chase & Co. (JPM), McDonald's Corp. (MCD), Exxon Mobil Corp. (XOM),
- Three exchange rates: the Australian Dollar (AUD), the British Pound (GBP) and the New-Zealand Dollar (NZD),
- The West Texas Intermediate (WTI) oil price,

over the period from May 29, 1986 to January 3, 2014. Daily transactions are turned into returns: $100 \times \log(P_t/P_{t-1})$, where $P_t$ represents the daily closing price at time $t$; the number of observations is 6960.

Table 2.7 presents the summary statistics of the series. In order to identify nonlinearity without outliers, the extreme values are removed. Outliers are likely to come from market structure, and considering them increase the likelihood of nonlinearity. The few peaks are winsorized such that each return is in the range $[-20, +20]$ in order to also avoid any singularity issue of the Hessian matrix for the multivariate optimization (see Silvennoinen and Teräsvirta, 2014, and Chapter 1). The summary statistics with cutoff are mentioned by the superscript $^{(co)}$. The number of truncations is mentioned (Trunc.) in the last column of Table 2.7.

Table 2.7: Summary statistics

| Ret. | Mean | Mean$^{(co)}$ | Max. | Min. | Std. | Std.$^{(co)}$ | Skew. | Skew$^{(co)}$ | Kurt. | Kurt.$^{(co)}$ | Trunc. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SPX | 0.0287 | 0.0291 | 10.957 | -22.899 | 1.1882 | 1.1806 | -1.2961 | -0.9712 | 30.595 | 22.847 | 1 |
| AAPL | 0.0385 | 0.0661 | 28.679 | -73.124 | 3.3217 | 2.9195 | -4.6872 | -0.1954 | 101.10 | 9.1049 | 10 |
| BAC | -0.0173 | 0.0072 | 30.209 | -71.376 | 3.0664 | 2.6219 | -5.2771 | -0.1964 | 132.70 | 18.865 | 22 |
| KO | -0.0152 | 0.0265 | 17.958 | -107.69 | 2.6193 | 1.6530 | -20.328 | -1.2850 | 681.35 | 26.785 | 6 |
| F | -0.0239 | 0.0004 | 25.865 | -70.086 | 2.8753 | 2.4927 | -5.4628 | -0.3332 | 129.05 | 14.635 | 12 |
| IBM | 0.0028 | 0.0182 | 12.363 | -71.130 | 2.1530 | 1.8160 | -10.119 | -0.5618 | 328.94 | 15.585 | 3 |
| JPM | 0.0013 | 0.0132 | 22.391 | -71.131 | 2.6802 | 2.4869 | -3.2894 | -0.0412 | 91.935 | 13.535 | 7 |
| MCD | -0.0012 | 0.0265 | 10.305 | -73.438 | 2.2873 | 1.7074 | -14.089 | -1.3733 | 409.40 | 21.325 | 5 |
| XOM | 0.0075 | 0.0292 | 16.475 | -69.848 | 2.1012 | 1.5959 | -14.839 | -0.9522 | 481.80 | 24.868 | 4 |
| AUD | 0.0031 | – | 7.7035 | -8.2120 | 0.7482 | – | -0.6327 | – | 15.727 | – | – |
| GBP | 0.0012 | – | 4.4349 | -4.9662 | 0.6079 | – | -0.2988 | – | 6.8727 | – | – |
| NZD | 0.0057 | – | 5.9322 | -6.1782 | 0.7670 | – | -0.4768 | – | 9.1762 | – | – |
| WTI | 0.0266 | 0.0296 | 19.150 | -40.639 | 2.4719 | 2.4352 | -0.7602 | -0.2095 | 19.318 | 9.9912 | 1 |

The returns exhibit a positive excess of kurtosis and a negative excess of skewness. When the cutoff is applied, the positive excess of kurtosis and the negative excess of skewness hugely decrease.

The test is designed to be easy to implement. The relevant transition variable has been chosen among different candidates as discussed in Silvennoinen and Teräsvirta (2014). For our tests, we set it as the lagged two-day absolute S&P500 returns for all cases, except for $N = 3$; we set the squared GBP returns as transition variable. To save space, the results for the absolute S&P500 returns and for the lagged seven-day absolute S&P500 returns as transition variable are not reported here.

It is interesting to see what the conditional variances and covariances between two or more returns are. Table 2.8 shows the two tests ($LCC(E)$ and $LCC(L)$) in bivariate cases. The both tests applied on returns are in favour of nonlinearity in conditional covariances in most of bivariate relationships. For some of them, one or both tests fail to reject the null hypothesis of linearity. This is the case for AAPL–F for the both tests. In general, and as expected, $LCC(E)$ rejects the null of linearity more often than $LCC(L)$, as shown in Section 2.4.3.3. Actually, for 10 of the 29 pairs, the logistic-based test fails to reject the null hypothesis of linearity, while the exponential-based test fails to reject the null at the 10% level of significance for only 2 of them.

Table 2.9 shows the results for the both tests in multivariate analyses for $N > 2$. The tests fail to reject the hypothesis of linearity on exchange rate returns. For $N = 4, 8$ and 9, $LCC(E)$ rejects the null at the nominal size of 1%, and $LCC(L)$ fails to reject the null for AAPL-BAC-KO-F.

In order to identify the nonlinearity provided by leverage effects we apply the both tests robust to leverage effects ($LCC(E)^\star$ and $LCC(L)^\star$).

Globally, Tables 2.10 and 2.11 show that the results given by the new tests robust to leverage effects are likely near to those given by Tables 2.8 and 2.9, respectively, confirming the presence or not of nonlinearity in conditional covariances.

When we apply the two-step procedure described in Section 2.5.3 on KO-XOM and JPM-XOM, $LCC(L)$ rejects the null hypothesis of linearity at the 10% level of significance in Table 2.8, while $LCC(L)^\star$ fails to reject the null hypothesis of linearity robust to leverage effects in Table 2.10. So, we can conclude on these two examples that conditional covariances are driven by leverage effects.

Surprisingly, when $LCC(L)$ in Tables 2.8 and 2.9 fails to reject the null hypothesis for F-IBM, F-XOM and AAPL-BAC-KO-F, $LCC(L)^\star$ in Tables 2.10 and 2.11, instead of confirming the linear relationship between

returns and covolatilities, rejects the null at 5%, 1% and 10%, respectively. This means that the nonlinearity is more complex than simple leverage effects.

Note that our tests are more generally considered as misspecification tests. In numerous cases, these tests show that a modelling of the conditional covariances with an exponential or a logistic smooth transition BEKK (or more generally with nonlinearity) should be preferred, which is uneasy to determine without applying our tests. Thus, the tests developed in this chapter help to improve the specification of the conditional covariances.

## 2.7 Conclusion

We propose two Lagrange multiplier tests for nonlinearity of conditional variances and covariances in multivariate GARCH models, and two other tests robust to leverage effects. These tests are based on an exponential or a logistic transition function. Under the null hypothesis, we define the asymptotic properties and give the analytical expressions of the test statistics. The main practical findings in this chapter are that these new tests perform very well in our small sample Monte Carlo experiments. An empirical illustration shows that our tests can discriminate among linearity, nonlinearity, and leverage effects in conditional covariances through a simple two-step procedure. By applying these tests, we can conclude for many cases that conditional covariances should adopt a nonlinear modelling.

Table 2.8: Linear conditional covariance tests for $N = 2$, the transition variable is the absolute value of the two days lagged S&P500 returns.

| | $LCC(L)$ | | $LCC(E)$ | |
|---|---|---|---|---|
| | $LM_{LCC(L)}$ | $p$-value | $LM_{LCC(E)}$ | $p$-value |
| SPX-WTI | 0.0424 | 0.8369 | 8.8682 | 0.0119 |
| AAPL-BAC | 4.1798 | 0.0409 | 39.714 | 0.0000 |
| AAPL-KO | 14.892 | 0.0001 | 17.874 | 0.0001 |
| AAPL-F | 0.0023 | 0.9619 | 1.2840 | 0.5262 |
| AAPL-IBM | 17.570 | 0.0000 | 19.551 | 0.0001 |
| AAPL-JPM | 3.3776 | 0.0661 | 3.3787 | 0.1846 |
| AAPL-MCD | 16.125 | 0.0001 | 17.992 | 0.0001 |
| AAPL-XOM | 28.237 | 0.0000 | 30.580 | 0.0000 |
| BAC-KO | 10.037 | 0.0015 | 72.443 | 0.0000 |
| BAC-F | 44.067 | 0.0000 | 136.80 | 0.0000 |
| BAC-IBM | 11.556 | 0.0007 | 85.012 | 0.0000 |
| BAC-JPM | 7.0902 | 0.0078 | 30.577 | 0.0000 |
| BAC-MCD | 5.1405 | 0.0234 | 54.303 | 0.0000 |
| BAC-XOM | 22.046 | 0.0000 | 108.79 | 0.0000 |
| KO-F | 5.9015 | 0.0151 | 24.189 | 0.0000 |
| KO-IBM | 0.2634 | 0.6078 | 13.795 | 0.0010 |
| KO-JPM | 0.1272 | 0.7214 | 8.1833 | 0.0167 |
| KO-MCD | 2.1329 | 0.1442 | 12.265 | 0.0022 |
| KO-XOM | 2.8932 | 0.0890 | 22.710 | 0.0000 |
| F-IBM | 1.5854 | 0.2080 | 28.757 | 0.0000 |
| F-JPM | 7.9544 | 0.0048 | 14.194 | 0.0008 |
| F-MCD | 3.3461 | 0.0674 | 25.898 | 0.0000 |
| F-XOM | 1.5972 | 0.2063 | 18.001 | 0.0001 |
| IBM-JPM | 0.3578 | 0.5497 | 19.275 | 0.0001 |
| IBM-MCD | 4.8987 | 0.0269 | 12.463 | 0.0020 |
| IBM-XOM | 1.9868 | 0.1587 | 18.377 | 0.0001 |
| JPM-MCD | 1.5093 | 0.2192 | 4.7999 | 0.0907 |
| JPM-XOM | 3.0398 | 0.0812 | 8.4500 | 0.0146 |
| XOM-MCD | 8.9608 | 0.0028 | 15.697 | 0.0004 |

Table 2.9: Linear conditional covariance tests for $N > 2$, the transition variable is the absolute value of the two days lagged S&P500 returns (GBP squared returns for the trivariate case).

| | $LCC(L)$ | | $LCC(E)$ | |
|---|---|---|---|---|
| | $LM_{LCC(L)}$ | $p$-value | $LM_{LCC(E)}$ | $p$-value |
| AUD-NZD-GBP | 0.3246 | 0.5689 | 0.7443 | 0.6892 |
| AAPL-BAC-KO-F | 1.3356 | 0.2478 | 57.157 | 0.0000 |
| IBM-JPM-MCD-XOM | 11.284 | 0.0008 | 18.119 | 0.0001 |
| AAPL-JPM-KO-XOM | 27.102 | 0.0000 | 30.718 | 0.0000 |
| IBM-BAC-MCD-F | 17.640 | 0.0000 | 35.585 | 0.0000 |
| AAPL-JPM-KO-XOM-IBM-BAC-MCD-F | 20.502 | 0.0000 | 39.700 | 0.0000 |
| SPX-AAPL-JPM-KO-XOM-IBM-BAC-MCD-F | 28.742 | 0.0000 | 59.367 | 0.0000 |

Table 2.10: Linear conditional covariance tests robust to leverage effects for $N = 2$, the transition variable is the absolute value of the two days lagged S&P500 returns.

| | $LCC(L)^\star$ | | $LCC(E)^\star$ | |
|---|---|---|---|---|
| | $LM_{LCC(L)^\star}$ | $p$-value | $LM_{LCC(E)^\star}$ | $p$-value |
| SPX-WTI | 0.4046 | 0.5247 | 6.9076 | 0.0316 |
| AAPL-BAC | 2.7533 | 0.0971 | 37.073 | 0.0000 |
| AAPL-KO | 13.847 | 0.0002 | 18.959 | 0.0001 |
| AAPL-F | 0.5692 | 0.4506 | 0.7993 | 0.6705 |
| AAPL-IBM | 14.841 | 0.0001 | 20.899 | 0.0000 |
| AAPL-JPM | 5.4795 | 0.0192 | 6.3836 | 0.0411 |
| AAPL-MCD | 13.387 | 0.0003 | 17.014 | 0.0002 |
| AAPL-XOM | 20.426 | 0.0000 | 25.523 | 0.0000 |
| BAC-KO | 4.7261 | 0.0297 | 52.406 | 0.0000 |
| BAC-F | 43.497 | 0.0000 | 132.33 | 0.0000 |
| BAC-IBM | 8.1928 | 0.0042 | 75.267 | 0.0000 |
| BAC-JPM | 3.2652 | 0.0708 | 24.329 | 0.0000 |
| BAC-MCD | 4.1769 | 0.0410 | 55.375 | 0.0000 |
| BAC-XOM | 17.844 | 0.0000 | 95.204 | 0.0000 |
| KO-F | 6.6065 | 0.0102 | 28.860 | 0.0000 |
| KO-IBM | 1.7533 | 0.1855 | 13.108 | 0.0014 |
| KO-JPM | 0.6099 | 0.4348 | 8.1641 | 0.0169 |
| KO-MCD | 1.5581 | 0.2119 | 15.676 | 0.0004 |
| KO-XOM | 0.0028 | 0.9579 | 13.557 | 0.0011 |
| F-IBM | 4.3002 | 0.0381 | 40.329 | 0.0000 |
| F-JPM | 8.1674 | 0.0043 | 16.283 | 0.0003 |
| F-MCD | 7.0672 | 0.0079 | 37.898 | 0.0000 |
| F-XOM | 7.1985 | 0.0073 | 32.588 | 0.0000 |
| IBM-JPM | 0.0525 | 0.8187 | 18.067 | 0.0001 |
| IBM-MCD | 4.4857 | 0.0342 | 14.859 | 0.0006 |
| IBM-XOM | 1.8616 | 0.1724 | 35.580 | 0.0000 |
| JPM-MCD | 0.9324 | 0.3342 | 7.8510 | 0.0197 |
| JPM-XOM | 1.2004 | 0.2733 | 8.8210 | 0.0121 |
| XOM-MCD | 7.3284 | 0.0068 | 16.394 | 0.0003 |

Table 2.11: Linear conditional covariance tests robust to leverage effects for $N > 2$, the transition variable is the absolute value of the two days lagged S&P500 returns (GBP squared returns for the trivariate case).

| | $LCC(L)^\star$ | | $LCC(E)^\star$ | |
|---|---|---|---|---|
| | $LM_{LCC(L)^\star}$ | $p$-value | $LM_{LCC(E)^\star}$ | $p$-value |
| AUD-NZD-GBP | 0.5700 | 0.4503 | 0.8208 | 0.6634 |
| AAPL-BAC-KO-F | 3.7626 | 0.0524 | 75.419 | 0.0000 |
| IBM-JPM-MCD-XOM | 9.4167 | 0.0022 | 19.853 | 0.0000 |
| AAPL-JPM-KO-XOM | 24.680 | 0.0000 | 30.616 | 0.0000 |
| IBM-BAC-MCD-F | 14.560 | 0.0001 | 90.786 | 0.0000 |
| AAPL-JPM-KO-XOM-IBM-BAC-MCD-F | 99.719 | 0.0000 | 531.31 | 0.0000 |
| SPX-AAPL-JPM-KO-XOM-IBM-BAC-MCD-F | 162.64 | 0.0000 | 855.07 | 0.0000 |

## 2.A Matrix appendix

The matrix calculus rules applied and summarized here can be found in Petersen and Pedersen (2008) and Lütkepohl (1996), among others.

For matrices $\mathbf{A}, \mathbf{B}, \mathbf{E}, \mathbf{D}$ of appropriate dimensions

$$\mathrm{Tr}(\mathbf{AB}) = \mathrm{vec}(\mathbf{A}')'\mathrm{vec}(\mathbf{B})$$

$$\mathrm{Tr}(\mathbf{A} + \mathbf{B}) = \mathrm{Tr}(\mathbf{A}) + \mathrm{Tr}(\mathbf{B})$$

$$\mathrm{Tr}(\mathbf{ABED}) = \mathrm{vec}(\mathbf{A}')'(\mathbf{D}' \otimes \mathbf{B})\mathrm{vec}(\mathbf{E}).$$

Let matrix $\mathbf{X}$ nonsingular, squared and symmetric, and $\mathbf{U} = f(\mathbf{X})$, the chain rule can be written as:

$$\frac{\partial g(\mathbf{U})}{\partial X_{ij}} = \mathrm{Tr}\left[\left(\frac{\partial g(\mathbf{U})}{\partial \mathbf{U}}\right)' \frac{\partial \mathbf{U}}{\partial X_{ij}}\right]$$

(Petersen and Pedersen, 2008, p. 14, 2.8.1 (126)).

$$\frac{\partial \log|\mathbf{X}|}{\partial \mathbf{X}} = (\mathbf{X}^{-1})'$$

$$\frac{\partial a'\mathbf{X}^{-1}b}{\partial \mathbf{X}} = -(\mathbf{X}^{-1})'ab'(\mathbf{X}^{-1})$$

$$\frac{\partial \mathbf{X}^{-1}}{\partial x} = -\mathbf{X}^{-1}\frac{\partial \mathbf{X}}{\partial x}\mathbf{X}^{-1}$$

(Petersen and Pedersen, 2008, p. 8, 2.2 (53)).

$\mathbf{L}_N$ is the $(\frac{N(N+1)}{2} \times N^2)$ elimination matrix such that

$$\mathrm{vech}(\mathbf{X}) = \mathbf{L}_N\mathrm{vec}(\mathbf{X}).$$

$\mathbf{D}_N^+$ is the $(\frac{N(N+1)}{2} \times N^2)$ Moore-Penrose inverse such that

$$\mathbf{D}_N^+ = (\mathbf{D}_N'\mathbf{D}_N)^{-1}\mathbf{D}_N',$$

and $\mathbf{D}_N$ is the $(N^2 \times \frac{N(N+1)}{2})$ duplication matrix such that

$$\mathrm{vec}(\mathbf{X}) = \mathbf{D}_N\mathrm{vech}(\mathbf{X}).$$

If $\mathbf{C}$ is a lower triangular matrix

$$\frac{\partial \mathrm{vech}(\mathbf{CC}')}{\partial \mathrm{vech}(\mathbf{C})'} = 2\mathbf{D}_N^+(\mathbf{C} \otimes \mathbf{I}_N)\mathbf{L}_N'$$

(Lütkepohl, 1996, p. 196, 10.5.4).

## 2.B Derivatives from the dynamic specification of the model

We compute the derivatives of the loglikelihood of $\mathbf{H}_t$ with respect to $\theta$, the vector of parameters. All the rules of matrix calculus can be found in the *Handbook of Matrices* of Lütkepohl (1996) and in *The Matrix Cookbook* of Petersen and Pedersen (2008), and are summarized in Appendix 2.A. Moreover, Comte and Lieberman (2003) found similar results.

We calculate the score and the Hessian from the dynamic specification of the model. The Gaussian loglikelihood is

$$l_t(\theta) = -\frac{1}{2}[N \ \log(2\pi) + \log|\mathbf{H}_t(\theta)| + \boldsymbol{\varepsilon}_t'\mathbf{H}_t(\theta)^{-1}\boldsymbol{\varepsilon}_t], \text{ for } t = 1, \ldots, T.$$

For easiness of notation, $\mathbf{H}_t$ denotes $\mathbf{H}_t(\theta)$. We compute the first derivatives of the loglikelihood with respect to $\theta_i$, $i = 1, \ldots, m$, the elements of the parameter vector $\theta$. Thus, we obtain the $i$-th element of the score

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta_i} = -\frac{1}{2}\sum_{t=1}^{T} \mathrm{Tr}\left[\frac{\partial \mathbf{H}_t}{\partial \theta_i}\mathbf{H}_t^{-1} - \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1}\frac{\partial \mathbf{H}_t}{\partial \theta_i}\mathbf{H}_t^{-1}\right], \tag{2.B.1}$$

or

$$\begin{aligned}\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta} = -\frac{1}{2}\sum_{t=1}^{T} \Bigg[&\left(\mathbf{D}_N\frac{\partial \mathrm{vech}\mathbf{H}_t}{\partial \theta}\right)'\mathbf{D}_N\mathrm{vech}\mathbf{H}_t^{-1} \\ &- \left(\mathbf{D}_N\frac{\partial \mathrm{vech}\mathbf{H}_t}{\partial \theta}\right)'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\mathrm{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\Bigg],\end{aligned} \tag{2.B.2}$$

for $\theta = (\theta_1, \ldots, \theta_m)'$.

**Remark 7.** *In line with Comte and Lieberman (2003), when the conditional expectation operator is applied on the $i$-th element of the score when evaluated at $\theta_0$,*

$$\mathsf{E}_{t-1}\left[-\frac{1}{2}Tr\left(\frac{\partial \mathbf{H}_t}{\partial \theta_i}\mathbf{H}_t^{-1} - \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1}\frac{\partial \mathbf{H}_t}{\partial \theta_i}\mathbf{H}_t^{-1}\right)\right]_{\theta=\theta_0} = 0 \qquad a.s.$$

*which is a martingale difference sequence where $\mathbf{H}_t$ denotes $\mathbf{H}_t(\theta_0)$.*

The $ij$-th element of the Hessian is given by the first derivative of (2.B.1) with respect to $\theta_j$, $j = 1, \ldots, n$, the elements of the parameter

vector $\theta$.

$$\frac{\partial^2 l_t(\theta)}{\partial\theta_i\partial\theta_j} = -\frac{1}{2}\mathrm{Tr}\left[\frac{\partial^2\mathbf{H}_t}{\partial\theta_i\partial\theta_j}\mathbf{H}_t^{-1} - \frac{\partial\mathbf{H}_t}{\partial\theta_i}\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_j}\mathbf{H}_t^{-1} + \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_j}\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_i}\mathbf{H}_t^{-1} - \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1}\frac{\partial^2\mathbf{H}_t}{\partial\theta_i\partial\theta_j}\mathbf{H}_t^{-1} + \boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_i}\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_j}\mathbf{H}_t^{-1}\right]. \tag{2.B.3}$$

Since $\mathsf{E}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'|\mathcal{F}_{t-1}] = \mathsf{E}_{t-1}[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \mathbf{H}_t(\theta_0)$, under the expectation operator, conditioned on the information set $\mathcal{F}_{t-1}$, the expression above simplifies to

$$\mathsf{E}_{t-1}\left.\frac{\partial^2 l_t(\theta)}{\partial\theta_i\partial\theta_j}\right|_{\theta=\theta_0} = -\frac{1}{2}\mathrm{Tr}\left[\frac{\partial\mathbf{H}_t}{\partial\theta_i}\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_j}\mathbf{H}_t^{-1}\right]$$

when evaluated at the true unknown value of $\theta = \theta_0$. Then, the consistent estimator is given by

$$\widehat{\mathcal{I}}(\widehat{\theta}_T) = \frac{1}{2T}\sum_{t=1}^{T}\mathrm{Tr}\left[\frac{\partial\mathbf{H}_t}{\partial\theta_i}\mathbf{H}_t^{-1}\frac{\partial\mathbf{H}_t}{\partial\theta_j}\mathbf{H}_t^{-1}\right]_{\theta=\widehat{\theta}_T} \tag{2.B.4}$$

$$= \frac{1}{2T}\sum_{t=1}^{T}\left[\left(\frac{\partial\mathrm{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\frac{\partial\mathrm{vech}\mathbf{H}_t}{\partial\theta'}\right]_{\theta=\widehat{\theta}_T} \tag{2.B.5}$$

for $\theta = (\theta_1, \ldots, \theta_m; \theta_1, \ldots, \theta_n)'$.

## 2.C Mathematical derivations of the LM test statistics

### 2.C.1 Exponential-based test

**Derivatives of $\mathbf{H}_t$**

As $\mathbf{H}_t$ and $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$ are symmetric, we use the vech representation of (2.23)

$$\mathrm{vech}\mathbf{H}_t = \mathrm{vech}(\mathbf{C}\mathbf{C}') + \left(\varphi_1 + \varphi_2 s_{t-1} + \varphi_3 s_{t-1}^2\right)\mathrm{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \beta\mathrm{vech}\mathbf{H}_{t-1}.$$

We derive $\mathrm{vech}\mathbf{H}_t$ with respect to the parameters in $\theta = (\mathrm{vech}(\mathbf{C})', \varphi_1, \varphi_2, \varphi_3, \beta)'$, the vector of all the parameters of the model such that

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \text{vech}(\mathbf{C})'} = 2\mathbf{D}_N^+(\mathbf{C}\otimes\mathbf{I}_N)\mathbf{L}_N' + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \text{vech}(\mathbf{C})'} \tag{2.C.1}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_1} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_1} \tag{2.C.2}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_2} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')s_{t-1} + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_2} \tag{2.C.3}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_3} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')s_{t-1}^2 + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_3} \tag{2.C.4}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \beta} = \text{vech}(\mathbf{H}_{t-1}) + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \beta}. \tag{2.C.5}$$

These derivatives can be easily implemented in the score (2.B.2) and the information matrix (2.B.5). Thereafter, we present the LM test statistic.

**First, consider the score**

$$\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta} = -\frac{1}{2}\sum_{t=1}^{T}\left[\left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N\text{vech}\mathbf{H}_t^{-1} - \left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\right],$$

with

$$\underset{\left(\frac{N(N+1)}{2}+4\right)\times 1}{\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}} = \left[\begin{array}{c|c} & \textit{Dimension} \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \text{vech}(\mathbf{C})'} & \frac{N(N+1)}{2}\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_1} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\beta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_2} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_3} & 1\times 1 \end{array}\right] = \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}. \tag{2.C.6}$$

**Second, consider the information matrix**

$$\mathcal{I}(\theta) = \frac{1}{2T}\sum_{t=1}^{T}\left(\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta'}.$$

**Third, consider the analytical computation of the LM test**

The LM test statistic of linear conditional covariances based on an exponential transition function, say $LM_{LCC(E)}$, can be expressed as

$$\underset{1\times 1}{LM_{LCC(E)}} = \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}' \begin{bmatrix} M_{CC} & M'_{\varphi_1 C} & M'_{\beta C} & M'_{\varphi_2 C} & M'_{\varphi_3 C} \\ M_{\varphi_1 C} & M_{\varphi_1\varphi_1} & M'_{\beta\varphi_1} & M'_{\varphi_2\varphi_1} & M'_{\varphi_3\varphi_1} \\ M_{\beta C} & M_{\beta\varphi_1} & M_{\beta\beta} & M'_{\varphi_2\beta} & M'_{\varphi_3\beta} \\ M_{\varphi_2 C} & M_{\varphi_2\varphi_1} & M_{\varphi_2\beta} & M_{\varphi_2\varphi_2} & M'_{\varphi_3\varphi_2} \\ M_{\varphi_3 C} & M_{\varphi_3\varphi_1} & M_{\varphi_3\beta} & M_{\varphi_3\varphi_2} & M_{\varphi_3\varphi_3} \end{bmatrix}^{-1} \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix} \tag{2.C.7}$$

where $M_{..}$ is the information matrix with respect to its parameters.

**Last, consider the LM test**

Under the null hypothesis, we maximize the likelihood function with respect to vech$(\mathbf{C})'$, $\varphi_1$ and $\beta$. Then, the first derivatives of the likelihood function with respect to these three parameters are set to zero:

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\bigg|_{\theta=\widehat{\theta}_T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \varphi_2} \\ \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \varphi_3} \end{bmatrix}_{\theta=\widehat{\theta}_T} \tag{2.C.8}$$

and

$$\begin{aligned} \underset{1\times 1}{LM_{LCC(E)}} = \begin{pmatrix} V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}' \Bigg[ \begin{pmatrix} M_{\varphi_2\varphi_2} & M'_{\varphi_3\varphi_2} \\ M_{\varphi_3\varphi_2} & M_{\varphi_3\varphi_3} \end{pmatrix} - \begin{pmatrix} M_{\varphi_2 C} & M_{\varphi_2\varphi_1} & M_{\varphi_2\beta} \\ M_{\varphi_3 C} & M_{\varphi_3\varphi_1} & M_{\varphi_3\beta} \end{pmatrix} \\ \times \begin{pmatrix} M_{CC} & M'_{\varphi_1 C} & M'_{\beta C} \\ M_{\varphi_1 C} & M_{\varphi_1\varphi_1} & M'_{\beta\varphi_1} \\ M_{\beta C} & M_{\beta\varphi_1} & M_{\beta\beta} \end{pmatrix}^{-1} \begin{pmatrix} M'_{\varphi_2 C} & M'_{\varphi_3 C} \\ M'_{\varphi_2\varphi_1} & M'_{\varphi_3\varphi_1} \\ M'_{\varphi_2\beta} & M'_{\varphi_3\beta} \end{pmatrix} \Bigg]^{-1} \begin{pmatrix} V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix} \end{aligned} \tag{2.C.9}$$

using the matrix partition (Lütkepohl, 1996, p.30) and extracting the relevant part of the inverse of the information matrix corresponding to the bottom right block, for which $\varphi_2$ and $\varphi_3$ are set to zero under the null hypothesis.

Thus, the test statistic is given by

$$LM_{LCC(E)} = T^{-1} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial(\varphi_2, \varphi_3)'} \right) \left[ \widehat{\mathcal{I}}(\widehat{\theta}_T) \right]^{-1}_{(\varphi_2\varphi_3, \varphi_2\varphi_3)} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial \varphi_2, \varphi_3} \right), \tag{2.C.10}$$

where $LM_{LCC(E)} \xrightarrow{D} \chi^2_2$ under $H_{01} : \varphi_2 = \varphi_3 = 0$.

### 2.C.2 Logistic-based test

**Derivatives of $\mathbf{H}_t$**

As $\mathbf{H}_t$ and $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$ are symmetric, we use the vech representation of (2.26)

$$\text{vech}\mathbf{H}_t = \text{vech}(\mathbf{C}\mathbf{C}') + (\phi_1 + \phi_2 s_{t-1})\,\text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \beta\text{vech}\mathbf{H}_{t-1}.$$

We derive $\text{vech}\mathbf{H}_t$ with respect to the parameters in $\theta = (\text{vech}(\mathbf{C})', \phi_1, \phi_2, \beta)'$, the vector of all the parameters of the model such that

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \text{vech}(\mathbf{C})'} = 2\mathbf{D}_N^+(\mathbf{C}\otimes\mathbf{I}_N)\mathbf{L}_N' + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \text{vech}(\mathbf{C})'} \tag{2.C.11}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \phi_1} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \phi_1} \tag{2.C.12}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \phi_2} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')s_{t-1} + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \phi_2} \tag{2.C.13}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \beta} = \text{vech}(\mathbf{H}_{t-1}) + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \beta}. \tag{2.C.14}$$

These derivatives can be easily implemented in the score (2.B.2) and the information matrix (2.B.5). Thereafter, we present the LM test statistic.

**First, consider the score**

$$\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta} = -\frac{1}{2}\sum_{t=1}^{T}\left[\left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N\text{vech}\mathbf{H}_t^{-1} - \left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\right],$$

with

$$\underset{\left(\frac{N(N+1)}{2}+3\right)\times 1}{\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}} = \left[\begin{array}{c|c} & Dimension \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \text{vech}(\mathbf{C})'} & \frac{N(N+1)}{2}\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \phi_1} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \beta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \phi_2} & 1\times 1 \end{array}\right] = \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\beta \\ V_{\phi_2} \end{pmatrix}. \tag{2.C.15}$$

**Second, consider the information matrix**

$$\mathcal{I}(\theta) = \frac{1}{2T}\sum_{t=1}^{T}\left(\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)' \mathbf{D}_N'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta'}.$$

**Third, consider the inversion of the matrix $\mathcal{I}(\theta)$ and the analytical computation of the LM test**

The LM test statistic of linear conditional covariances based on a logistic transition function, say $LM_{LCC(L)}$, can be expressed as

$$\underset{1\times1}{LM_{LCC(L)}} = \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\beta \\ V_{\phi_2} \end{pmatrix}' \begin{bmatrix} M_{CC} & M_{\phi_1 C}' & M_{\beta C}' & M_{\phi_2 C}' \\ M_{\phi_1 C} & M_{\phi_1\phi_1} & M_{\beta\phi_1}' & M_{\phi_2\phi_1}' \\ M_{\beta C} & M_{\beta\phi_1} & M_{\beta\beta} & M_{\phi_2\beta}' \\ M_{\phi_2 C} & M_{\phi_2\phi_1} & M_{\phi_2\beta} & M_{\phi_2\phi_2} \end{bmatrix}^{-1} \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\beta \\ V_{\phi_2} \end{pmatrix} \tag{2.C.16}$$

where $M_{..}$ is the information matrix with respect to its parameters.

**Last, consider the LM test**

Under the null hypothesis, we maximize the likelihood function with respect to vech$(\mathbf{C})'$, $\varphi_1$ and $\beta$. Then, the first derivatives of the likelihood function with respect to these three parameters are set to zero:

$$\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}\bigg|_{\theta=\widehat{\theta}_T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\phi_2} \end{bmatrix}_{\theta=\widehat{\theta}_T} \tag{2.C.17}$$

and

$$\begin{aligned} \underset{1\times1}{LM_{LCC(L)}} = \; & V_{\phi_2}' \Bigg[ M_{\phi_2\phi_2} - \begin{pmatrix} M_{\phi_2 C} & M_{\phi_2\phi_1} & M_{\phi_2\beta} \end{pmatrix} \\ & \times \begin{pmatrix} M_{CC} & M_{\phi_1 C}' & M_{\beta C}' \\ M_{\phi_1 C} & M_{\phi_1\phi_1} & M_{\beta\phi_1}' \\ M_{\beta C} & M_{\beta\phi_1} & M_{\beta\beta} \end{pmatrix}^{-1} \begin{pmatrix} M_{\phi_2 C}' \\ M_{\phi_2\phi_1}' \\ M_{\phi_2\beta}' \end{pmatrix} \Bigg]^{-1} V_{\phi_2} \end{aligned} \tag{2.C.18}$$

using the matrix partition (Lütkepohl, 1996, p.30) and extracting the relevant part of the inverse of the information matrix corresponding to the bottom right block, for which $\phi_2$ is set to zero under the null hypothesis.

Thus, the test statistic is given by

$$LM_{LCC(L)} = T^{-1}\left(\sum_{t=1}^{T}\frac{\partial l_t(\widehat{\theta}_T)}{\partial \phi_2'}\right)\left[\widehat{\mathcal{I}}(\widehat{\theta}_T)\right]^{-1}_{(\phi_2,\phi_2)}\left(\sum_{t=1}^{T}\frac{\partial l_t(\widehat{\theta}_T)}{\partial \phi_2}\right), \tag{2.C.19}$$

where $LM_{LCC(L)} \xrightarrow{D} \chi^2_1$ under $H_{02}: \phi_2 = 0$.

### 2.C.3 Exponential-based test robust to leverage effects

**Derivatives of $\mathbf{H}_t$**

As $\mathbf{H}_t$, $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'$ and $\boldsymbol{\nu}_t\boldsymbol{\nu}_t'$ are symmetric, we use the vech representation of (2.31)

$$\begin{aligned}\text{vech}\mathbf{H}_t = \text{vech}(\mathbf{C}\mathbf{C}') + \left(\varphi_1 + \varphi_2 s_{t-1} + \varphi_3 s_{t-1}^2\right)\text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') \\ + \delta\text{vech}(\boldsymbol{\nu}_{t-1}\boldsymbol{\nu}_{t-1}') + \beta\text{vech}\mathbf{H}_{t-1}.\end{aligned}$$

We derive vech$\mathbf{H}_t$ with respect to the parameters in $\theta = (\text{vech}(\mathbf{C})', \varphi_1, \varphi_2, \varphi_3, \delta, \beta)'$, the vector of all the parameters of the model such that

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \text{vech}(\mathbf{C})'} = 2\mathbf{D}_N^+(\mathbf{C}\otimes\mathbf{I}_N)\mathbf{L}_N' + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \text{vech}(\mathbf{C})'} \tag{2.C.20}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_1} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}') + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_1} \tag{2.C.21}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_2} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')s_{t-1} + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_2} \tag{2.C.22}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \varphi_3} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}')s_{t-1}^2 + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \varphi_3} \tag{2.C.23}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \delta} = \text{vech}(\boldsymbol{\nu}_{t-1}\boldsymbol{\nu}_{t-1}') + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \delta} \tag{2.C.24}$$

$$\frac{\partial \text{vech}\mathbf{H}_t}{\partial \beta} = \text{vech}(\mathbf{H}_{t-1}) + \beta\frac{\partial \text{vech}\mathbf{H}_{t-1}}{\partial \beta}. \tag{2.C.25}$$

These derivatives can be easily implemented in the score (2.B.2) and the information matrix (2.B.5).Thereafter, we present the LM test statistic.

**First, consider the score**

$$\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta} = -\frac{1}{2}\sum_{t=1}^{T}\left[\left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N\text{vech}\mathbf{H}_t^{-1} - \left(\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')\right],$$

with

$$\underset{\left(\frac{N(N+1)}{2}+5\right)\times 1}{\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}} = \left[\begin{array}{c|c} & Dimension \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial \text{vech}(\mathbf{C})'} & \frac{N(N+1)}{2}\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_1} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\delta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\beta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_2} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\varphi_3} & 1\times 1 \end{array}\right] = \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\delta \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}. \quad (2.C.26)$$

**Second, consider the information matrix**

$$\mathcal{I}(\theta) = \frac{1}{2T}\sum_{t=1}^{T}\left(\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta'}.$$

**Third, consider the analytical computation of the LM test**

The LM test statistic of linear conditional covariances robust to leverage effects based on an exponential transition function, say $LM_{LCC(E)^\star}$, can be expressed as

$$\underset{1\times 1}{LM_{LCC(E)^\star}} = \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\delta \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}' \begin{bmatrix} M_{CC} & M_{\varphi_1 C}' & M_{\delta C}' & M_{\beta C}' & M_{\varphi_2 C}' & M_{\varphi_3 C}' \\ M_{\varphi_1 C} & M_{\varphi_1\varphi_1} & M_{\delta\varphi_1}' & M_{\beta\varphi_1}' & M_{\varphi_2\varphi_1}' & M_{\varphi_3\varphi_1}' \\ M_{\delta C} & M_{\delta\varphi_1} & M_{\delta\delta} & M_{\beta\delta}' & M_{\varphi_2\delta}' & M_{\varphi_3\delta}' \\ M_{\beta C} & M_{\beta\varphi_1} & M_{\beta\delta} & M_{\beta\beta} & M_{\varphi_2\beta}' & M_{\varphi_3\beta}' \\ M_{\varphi_2 C} & M_{\varphi_2\varphi_1} & M_{\varphi_2\delta} & M_{\varphi_2\beta} & M_{\varphi_2\varphi_2} & M_{\varphi_3\varphi_2}' \\ M_{\varphi_3 C} & M_{\varphi_3\varphi_1} & M_{\varphi_3\delta} & M_{\varphi_3\beta} & M_{\varphi_3\varphi_2} & M_{\varphi_3\varphi_3} \end{bmatrix}^{-1} \begin{pmatrix} V_C \\ V_{\varphi_1} \\ V_\delta \\ V_\beta \\ V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix} \quad (2.C.27)$$

where $M_{..}$ is the information matrix with respect to its parameters.

**Last, consider the LM test**

Under the null hypothesis, we maximize the likelihood function with respect to vech$(\mathbf{C})'$, $\varphi_1$, $\delta$ and $\beta$. Then, the first derivatives of the likelihood function with respect to these four parameters are set to zero:

$$\sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \theta}\bigg|_{\theta=\widehat{\theta}_T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \varphi_2} \\ \sum_{t=1}^{T} \frac{\partial l_t(\theta)}{\partial \varphi_3} \end{bmatrix}_{\theta=\widehat{\theta}_T} \tag{2.C.28}$$

and

$$\underset{1\times 1}{LM_{LCC(E)^\star}} = \begin{pmatrix} V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix}' \left[ \begin{pmatrix} M_{\varphi_2\varphi_2} & M'_{\varphi_3\varphi_2} \\ M_{\varphi_3\varphi_2} & M_{\varphi_3\varphi_3} \end{pmatrix} - \begin{pmatrix} M_{\varphi_2 C} & M_{\varphi_2\varphi_1} & M_{\varphi_2\delta} & M_{\varphi_2\beta} \\ M_{\varphi_3 C} & M_{\varphi_3\varphi_1} & M_{\varphi_3\delta} & M_{\varphi_3\beta} \end{pmatrix} \right.$$
$$\left. \times \begin{pmatrix} M_{CC} & M'_{\varphi_1 C} & M'_{\delta C} & M'_{\beta C} \\ M_{\varphi_1 C} & M_{\varphi_1\varphi_1} & M'_{\delta\varphi_1} & M'_{\beta\varphi_1} \\ M_{\delta C} & M_{\delta\varphi_1} & M_{\delta\delta} & M'_{\beta\delta} \\ M_{\beta C} & M_{\beta\varphi_1} & M_{\beta\delta} & M_{\beta\beta} \end{pmatrix}^{-1} \begin{pmatrix} M'_{\varphi_2 C} & M'_{\varphi_3 C} \\ M'_{\varphi_2\varphi_1} & M'_{\varphi_3\varphi_1} \\ M'_{\varphi_2\delta} & M'_{\varphi_3\delta} \\ M'_{\varphi_2\beta} & M'_{\varphi_3\beta} \end{pmatrix} \right]^{-1} \begin{pmatrix} V_{\varphi_2} \\ V_{\varphi_3} \end{pmatrix} \tag{2.C.29}$$

using the matrix partition (Lütkepohl, 1996, p.30) and extracting the relevant part of the inverse of the information matrix corresponding to the bottom right block, for which $\varphi_2$ and $\varphi_3$ are set to zero under the null hypothesis.

Thus, the test statistic is given by

$$LM_{LCC(E)^\star} = T^{-1} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial(\varphi_2, \varphi_3)'} \right) \left[ \widehat{\mathcal{I}}(\widehat{\theta}_T) \right]^{-1}_{(\varphi_2\varphi_3, \varphi_2\varphi_3)} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial \varphi_2, \varphi_3} \right), \tag{2.C.30}$$

where $LM_{LCC(E)^\star} \xrightarrow{D} \chi^2_2$ under $H_{01} : \varphi_2 = \varphi_3 = 0$.

## 2.C.4 Logistic-based test robust to leverage effects

**Derivatives of $\mathbf{H}_t$**

As $\mathbf{H}_t$, $\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t$ and $\boldsymbol{\nu}_t\boldsymbol{\nu}'_t$ are symmetric, we use the vech representation of

(2.32)

$$\begin{aligned}\text{vech}\mathbf{H}_t = \text{vech}(\mathbf{C}\mathbf{C}') + (\phi_1 + \phi_2 s_{t-1})\,\text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) \\ + \delta\text{vech}(\boldsymbol{\nu}_{t-1}\boldsymbol{\nu}'_{t-1}) + \beta\text{vech}\mathbf{H}_{t-1}.\end{aligned}$$

We derive $\text{vech}\mathbf{H}_t$ with respect to the parameters in $\theta = (\text{vech}(\mathbf{C})', \phi_1, \phi_2, \delta, \beta)'$, the vector of all the parameters of the model such that

$$\frac{\partial\text{vech}\mathbf{H}_t}{\partial\text{vech}(\mathbf{C})'} = 2\mathbf{D}_N^+(\mathbf{C}\otimes\mathbf{I}_N)\mathbf{L}'_N + \beta\frac{\partial\text{vech}\mathbf{H}_{t-1}}{\partial\text{vech}(\mathbf{C})'} \tag{2.C.31}$$

$$\frac{\partial\text{vech}\mathbf{H}_t}{\partial\phi_1} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1}) + \beta\frac{\partial\text{vech}\mathbf{H}_{t-1}}{\partial\phi_1} \tag{2.C.32}$$

$$\frac{\partial\text{vech}\mathbf{H}_t}{\partial\phi_2} = \text{vech}(\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}'_{t-1})s_{t-1} + \beta\frac{\partial\text{vech}\mathbf{H}_{t-1}}{\partial\phi_2} \tag{2.C.33}$$

$$\frac{\partial\text{vech}\mathbf{H}_t}{\partial\delta} = \text{vech}(\boldsymbol{\nu}_{t-1}\boldsymbol{\nu}'_{t-1}) + \beta\frac{\partial\text{vech}\mathbf{H}_{t-1}}{\partial\delta} \tag{2.C.34}$$

$$\frac{\partial\text{vech}\mathbf{H}_t}{\partial\beta} = \text{vech}(\mathbf{H}_{t-1}) + \beta\frac{\partial\text{vech}\mathbf{H}_{t-1}}{\partial\beta}. \tag{2.C.35}$$

These derivatives can be easily implemented in the score (2.B.2) and the information matrix (2.B.5). Thereafter, we present the LM test statistic.

**First, consider the score**

$$\begin{aligned}\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta} = -\frac{1}{2}\sum_{t=1}^{T}\Bigg[\left(\mathbf{D}_N\frac{\partial\text{vech}\mathbf{H}_t}{\partial\theta}\right)'\mathbf{D}_N\text{vech}\mathbf{H}_t^{-1} \\ - \left(\mathbf{D}_N\frac{\partial\text{vech}\mathbf{H}_t}{\partial\theta}\right)'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\text{vech}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}'_t)\Bigg],\end{aligned}$$

with

$$\underset{\left(\frac{N(N+1)}{2}+4\right)\times 1}{\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}} = \left[\begin{array}{c|c} & \textit{Dimension} \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\text{vech}(\mathbf{C})'} & \frac{N(N+1)}{2}\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\phi_1} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\delta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\beta} & 1\times 1 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\phi_2} & 1\times 1 \end{array}\right] = \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\delta \\ V_\beta \\ V_{\phi_2}\end{pmatrix}. \tag{2.C.36}$$

**Second, consider the information matrix**

$$\mathcal{I}(\theta) = \frac{1}{2T}\sum_{t=1}^{T}\left(\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta}\right)' \mathbf{D}_N'(\mathbf{H}_t^{-1}\otimes\mathbf{H}_t^{-1})\mathbf{D}_N\frac{\partial \text{vech}\mathbf{H}_t}{\partial\theta'}.$$

**Third, consider the inversion of the matrix $\mathcal{I}(\theta)$ and the analytical computation of the LM test**

The LM test statistic of linear conditional covariances robust to leverage effects based on a logistic transition function, say $LM_{LCC(L)^\star}$, can be expressed as

$$\underset{1\times 1}{LM_{LCC(L)^\star}} = \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\delta \\ V_\beta \\ V_{\phi_2} \end{pmatrix}' \begin{bmatrix} M_{CC} & M_{\phi_1 C}' & M_{\delta C}' & M_{\beta C}' & M_{\phi_2 C}' \\ M_{\phi_1 C} & M_{\phi_1\phi_1} & M_{\delta\phi_1}' & M_{\beta\phi_1}' & M_{\phi_2\phi_1}' \\ M_{\delta C} & M_{\delta\phi_1} & M_{\delta\delta} & M_{\beta\delta}' & M_{\phi_2\delta}' \\ M_{\beta C} & M_{\beta\phi_1} & M_{\beta\delta} & M_{\beta\beta} & M_{\phi_2\beta}' \\ M_{\phi_2 C} & M_{\phi_2\phi_1} & M_{\phi_2\delta} & M_{\phi_2\beta} & M_{\phi_2\phi_2} \end{bmatrix}^{-1} \begin{pmatrix} V_C \\ V_{\phi_1} \\ V_\delta \\ V_\beta \\ V_{\phi_2} \end{pmatrix} \tag{2.C.37}$$

where $M_{..}$ is the information matrix with respect to its parameters.

**Last, consider the LM test**

Under the null hypothesis, we maximize the likelihood function with respect to $\text{vech}(\mathbf{C})'$, $\varphi_1$, $\delta$ and $\beta$. Then, the first derivatives of the likelihood function with respect to these four parameters are set to zero:

$$\sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\theta}\bigg|_{\theta=\widehat{\theta}_T} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sum_{t=1}^{T}\frac{\partial l_t(\theta)}{\partial\phi_2} \end{bmatrix}_{\theta=\widehat{\theta}_T} \tag{2.C.38}$$

and

$$\begin{aligned} \underset{1\times 1}{LM_{LCC(L)^\star}} = \; & V_{\phi_2}' \Bigg[ M_{\phi_2\phi_2} - \begin{pmatrix} M_{\phi_2 C} & M_{\phi_2\phi_1} & M_{\phi_2\delta} & M_{\phi_2\beta} \end{pmatrix} \\ & \times \begin{pmatrix} M_{CC} & M_{\phi_1 C}' & M_{\delta C}' & M_{\beta C}' \\ M_{\phi_1 C} & M_{\phi_1\phi_1} & M_{\delta\phi_1}' & M_{\beta\phi_1}' \\ M_{\delta C} & M_{\delta\phi_1} & M_{\delta\delta} & M_{\beta\delta}' \\ M_{\beta C} & M_{\beta\phi_1} & M_{\beta\delta} & M_{\beta\beta} \end{pmatrix}^{-1} \begin{pmatrix} M_{\phi_2 C}' \\ M_{\phi_2\phi_1}' \\ M_{\phi_2\delta}' \\ M_{\phi_2\beta}' \end{pmatrix} \Bigg]^{-1} V_{\phi_2} \end{aligned} \tag{2.C.39}$$

using the matrix partition (Lütkepohl, 1996, p.30) and extracting the relevant part of the inverse of the information matrix corresponding to the bottom right block, for which $\phi_2$ is set to zero under the null hypothesis.

Thus, the test statistic is given by

$$LM_{LCC(L)^\star} = T^{-1} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial \phi_2'} \right) \left[ \widehat{\mathcal{I}}(\widehat{\theta}_T) \right]^{-1}_{(\phi_2,\phi_2)} \left( \sum_{t=1}^{T} \frac{\partial l_t(\widehat{\theta}_T)}{\partial \phi_2} \right), \tag{2.C.40}$$

where $LM_{LCC(L)^\star} \xrightarrow{D} \chi^2_1$ under $H_{02} : \phi_2 = 0$.

# Chapter 3

# Volatility spillovers across daytime and overnight information between China and world equity markets[1]

## 3.1 Introduction

Financial market integration has been a hotly debated issue in international finance given its importance for risk management and portfolio diversification to international investors. The existing empirical studies on the financial market integration have documented that stock markets around the world are becoming increasingly integrated over time, especially in emerging countries (Bekaert, 1995; Harvey, 1995; Liu and Pan, 1997; Masih and Masih, 1999).

Over the two last decades, Chinese stock market has strongly grown, according to statistics of the Chinese Securities Regulatory Commission (CSRC), two Chinese national stock exchanges, the Shanghai and Shenzhen stock exchanges respectively ranked sixth and fourteenth in terms of market capitalization, which was US$2.71 trillion and US$1.31 trillion at the end of 2010. The increased importance of Chinese stock market has attracted many researchers to pay more attention on its integration with the rest of the world.

Existing empirical works on Chinese financial integration show that China is not or weakly integrated among other international markets. Using the cointegration techniques, some studies show that the Chinese

[1]This chapter is part of joint work with Jian Hua.

mainland stock market is isolated from other markets. Zhu *et al.* (2004) test the cointegrating relationship and Granger causality among the Shanghai, Shenzhen, and Hong Kong stock markets by using daily data from 1993 to 2001, but they do not find any long-run relationships among these markets. Similar to Zhu *et al.* (2004), Cheng and Glascock (2005) argue that the three Greater China stock markets are neither cointegrated with each other nor with Japan and the US but there exist weak nonlinear relationships between these markets over the period from 1993 to 2001. Huyghebaert and Wang (2010) employ the standard cointegration analysis and suggest continued isolation of the Chinese mainland markets for the period from 1992 to 2003. Contrary to these studies, Girardin and Liu (2007) find a long-run relationship between the A-share and Hang Seng indices over 1997-2006; they suggest that the presence of identical fundamentals (multiple listing of Mainland firms) across markets helps to explain the evidence of long-run financial integration in spite of capital controls.

However, the cointegration analysis is designed for capturing the long-term relationship across markets. In addition, markets might be integrated not only in terms of returns but also in terms of volatility.

Wang and Firth (2004) study the linkages between the three emerging markets in Greater China (China, Hong Kong, Taiwan) and three developed markets (Japan, the US, and the UK) over the period of 1994 to 2001. Using daily open and close price data, they show that there exists only one unidirectional return spillover effect from Hong Kong and Japan to China. The authors use univariate GARCH framework; however, that kind of modelling doesn't permit to take into account the endogeneity of the conditional correlations. To address these issues, some papers use multivariate heteroskedastic models.

Since the last couple of decades, the multivariate GARCH models became very popular to analyse the conditional second moments of financial data (see Bauwens, Laurent and Rombouts (2006); Silvennoinen and Teräsvirta (2009a) for surveys). Johansson and Ljungwall (2009) investigate the relationship among the stock markets in Greater China from 1994 to 2005. Their empirical findings, by using weekly data, show that it does not exist long-run relationships among these markets, but the short-run spillover effects detected by an asymmetric constant conditional correlation GARCH model are present in both returns and volatility in the region. China and Hong Kong both have received mean spillover effects from Taiwan. Volatility of Hong Kong has spilled over into Taiwan, which in turn affects the volatility of Chinese mainland market. Chuang et al.

(2007) investigate the interdependence of returns and volatilities in East Asian equity markets, excluding China, from 1992 to 2006. Using weekly data in a six variates VAR-BEKK model, they show that Japan is weakly impacted by other markets, and has an important influence in terms of volatility over Hong Kong, Seoul, Singapore, Taiwan and Thailand. In a very recent and independent work, Allen, Amram and McAleer (2013) explore the volatility spillovers from the Chinese stock market to USA, Australia, Hong-Kong, Japan and Singapore. They use a step-by-step methodology starting from the univariate ARMA-GARCH modelling to the multivariate VARMA-CCC-AGARCH model developed by Ling and McAleer (2003). Their results show some evidence of volatility spillover effects prior to the crisis and weak evidence of spillover effects during the global financial crisis of 2007. However, they suspect non-constant correlations by using a "bimean equation" to model the conditional mean for each market, and an ARMA model to capture volatility spillovers; but they never test the constancy of the conditional correlations preferring to use a 120 days moving windows.

Although there are many studies focusing on the integration of China stock markets, few studies have been investigated with recent data, especially for the post-2005 period. In this chapter, we want to investigate the transmission of daytime and overnight information between China and other markets. Notice that empirical studies based on overnight returns remain relatively scarce.

In order to investigate the above question, we take a sample which consists of a group of Asian stock markets (China, Hong Kong, Japan, Korea, and Taiwan) and a group of non-Asian stock markets (the UK, Germany, Canada and the US) over the period 2005 to 2013. We divide our sample data into three sub-samples: before, during and after the global financial crisis of 2007. First, to avoid the possible problem of non-synchronous trading between Chinese and other stock markets, we separate the daily returns into two parts: daytime and overnight returns, which represent respectively daytime and overnight information proxies. Then, we set and test some basic hypotheses to specify the model's mean equation. Lastly, after testing the constancy of the conditional correlations, we adopt the bivariate extended constant conditional correlations GARCH model (ECCC-GARCH) of Bollerslev (1990) or the bivariate extended dynamic conditional correlations GARCH model (EDCC-GARCH) of Engle (2002) in order to measure the direction and the extent of the volatility spillovers. The test of constancy of conditional correlations permits to avoid any misspecification issue and yields to use the right

conditional correlation model.

As in our sample some markets have different trading hours, when we specify the model and interpret our econometrical results, we need to take into account the problem of non-synchronicity: for example, an opened market cannot influence a closed market. This is the case for Singapore, Hong Kong, Germany, the UK, the US and Canada toward China. Our split-sample analysis show that:

(a) There exists clear evidence of unidirectional and bidirectional contemporaneous and lagged dependence in the daytime returns of the stock indices between China and Asia markets before, during and after the crisis. The relationships between China and the non-Asian markets exist before and after the crisis. The overnight returns of China are exclusively explained by the Asian markets before the crisis, whereas there exist relationships during and after the crisis with North America and Europe.

(b) Prior to the global financial crisis, China is likely to be independent from other markets in term of daytime volatility, whereas the overnight volatility spillovers exist from China to the US and the UK.

(c) During the crisis, there exist unidirectional daytime volatility spillovers from China to the UK, whereas the overnight volatility spillovers only exist with Japan and the US.

(d) After the crisis, the China's daytime volatility spillovers are from Taiwan to China, while the overnight volatility spillover effects are from China to the US and the UK.

This chapter will be structured as follows. Section 3.2 reviews the institutional background about the Chinese equity markets. Section 3.3 contains the description of the data and the methodology. Section 3.4 provides the empirical analysis. Section 3.5 summarizes our conclusions.

## 3.2 Institutional background on the Chinese stock markets

China has two national stock exchanges: the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE), founded on November 26, 1990 and April 11, 1991, respectively. Both are nonprofit institutions directly governed by the China Securities Regulatory Commission (CSRC). At the end of 2010, the SSE and the SZSE are ranked sixth and fourteenth in terms of market capitalization, respectively. Although the Chinese stock market has strongly grown over the two last decades, it is still an emerging market. There are some key institutional characteristics which

we need to consider (Eun and Huang, 2007): (i) Chinese stock market is dominated by individual investors: the total number of individual investors' accounts is about 162 millions in 2010, 265 times of number of legal/institutional accounts (0.61 millions). (ii) Before the 2005 split-share reform, the majority of Chinese companies are controlled by government entities (this non-tradable share is about two-third on average). (iii) Few domestically listed companies have eligible foreign shares (such as B- and H-shares), the majority of Chinese-listed companies is prohibited to foreigners; overseas investors have been allowed limited access to the domestic A-share market through the QFII (Qualified Foreign Institutional Investor) program beginning in 2002. (iv) Under the capital controls, Chinese mainlanders can not invest oversea shares, excepted for QDII (Qualified Domestic Institutional Investor) since 2006.

Overall, under the restrictions against capital flows and cross settlement, Chinese stock market is still a highly imperfect emerging stock market. Recently, with the *de jure* financial liberalizations in China, it may enhance the level of the opening of Chinese stock market. Under the share ownership structure, the domestic shares can be classified in two types: restricted and freely-traded shares. The restricted shares (state shares and legal entity shares) are not tradable on the stock exchange. The freely-traded shares (or public shares) are the domestic ordinary A-shares that can be freely traded on the stock exchanges. As we documented above, before 2005 the majority of Chinese companies are controlled by government entities (state and legal entity shares). To eliminate those non-tradable shares and improve the infrastructure and the efficiency of Chinese capital markets, the CSRC launched the split-share structure reform on April 29, 2005.

In order to better control the equity market development, the CSRC decided that, according to the share ownership, the domestic shares are classified into three different categories: state shares, legal entity shares, and public shares. The state shares are shares owned by the state, which are not allowed for trading on the stock market. Legal entity shares are shares owned by other state entities, these other state entities include stock companies, non-bank financial institutions. Similar to state shares, legal entity shares are not tradable on the stock exchange. Public shares are the domestic ordinary A-shares that can be freely traded on the stock exchanges. According to the statistic from the CSRC, by the end of 2004, the total shares of Chinese listed companies were 714.9 billion, of which 454.3 billion shares, or 64% of the total, were non-tradable. For these non-tradable shares, 74% of them were state-owned shares.

However, this split-share structure leads to poor performance and inefficient corporate governance of listed firms and limits the development of the capital market. Thus, to improve the infrastructure and the efficiency of Chinese capital markets, the CSRC launched the split-share structure reform to eliminate those non-tradable shares by converting them into tradable shares on April 29, 2005. This reform successfully increased firm's liquidity and performance, especially for stocks with less attractive characteristics. By the end of 2007, 1298 listed firms which represented 98% of the market capitalization, had started or finished the reform. Finally, at the end of 2009, approximately 80% of non-tradable shares were converted into tradable shares.

On 21 July 2005, China announced a 2.1% appreciation of the RMB against the US dollar, a switch to a managed flexible exchange rate regime. The new exchange rate based more on market supply and demand, with reference to a basket of currencies (numerical weights unannounced), allowed a fluctuation limit of 0.3% per day. This relaxation of the exchange rate regime from fixed to managed floating can be interpreted as a more open Chinese capital market. But China turns to fix exchange rate regime as the financial crisis intensified in July 2008, and more recently, it announced a switch again to the managed floating exchange rate in June 2010. However, all these financial liberalizations would represent potential impacts on the opening connections with international markets, especially for post-2005 period.

## 3.3 Data and specification

### 3.3.1 Data

We use the daily opening (Figure 3.1) and closing (Figure 3.2) prices of stock market indices for Greater China market (China Securities Index 300 for China, the Hang Seng Index for Hong Kong, the Taiwan Weighted Index for Taiwan), the three other main Asian markets (the Nikkei 225 Index for Japan, the Straits Times Index for Singapore, and KOSPI Index for Korea), the two main European markets (the DAX index for Germany and the FTSE 100 index for the UK) and the North American market (the S&P500 for the USA and the Toronto Stock Exchange for Canada) from 2005 to 2013. Overall, our dataset contains ten stock indices of various countries or regions with 1712 daily data in each index. Our daily stock market data is provided by the database of Resset. Weekends' data are excluded from the sample, while for holidays' data, following the usual

practice in the literature, the previous day's return fills in the missing observations.

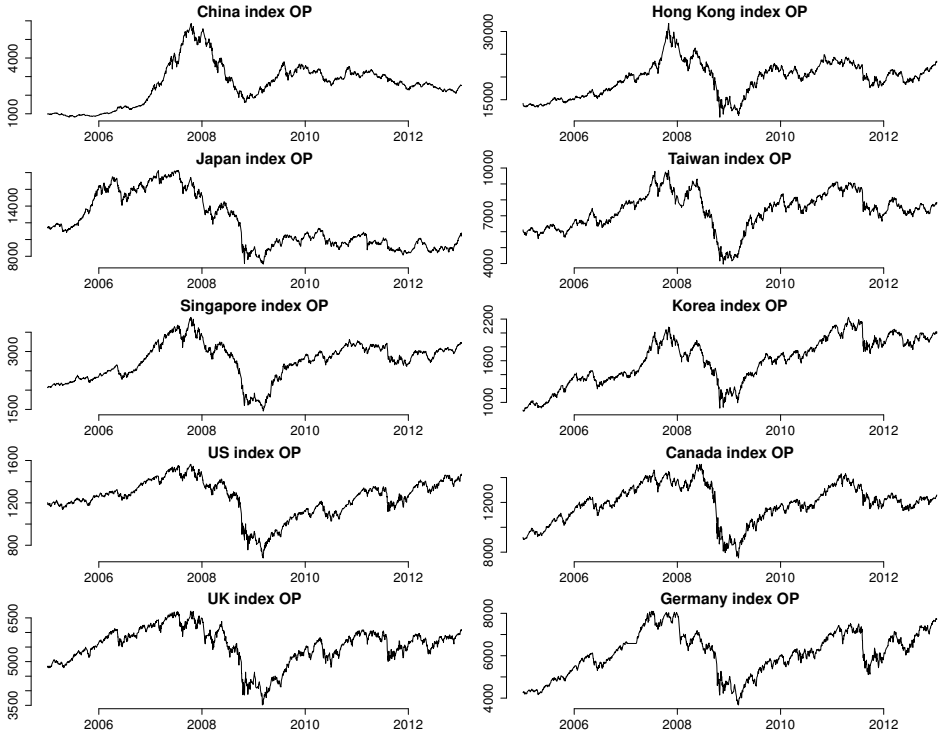



Figure 3.1: Opening prices of stock market indices

The full sample is separated into three sub-samples: prior to the crisis from January 3, 2005 to April 30, 2007 (500 observations), during the crisis from May 1, 2007 to December 31, 2009 (561 observations) and after the crisis from January 1, 2010 to January 1, 2013 (651 observations). As shown in Figure 3.3, the pre-crisis and post-crisis periods are characterised by low variability of returns, while the crisis period is characterised by high variability of returns and clusters of volatilities. Moreover, in April 2007, New Century Financial Corporation, a leading subprime mortgage lender, files for bankruptcy protection, as reported by www.stlouisfed.org/timeline/timeline.cfm.

Following Wang, Rui and Firth (2002) and Wang and Firth (2004), we split daily returns into two parts. Let $P_o$ and $P_c$ the opening and closing stock prices, respectively, such that

$$\begin{aligned} ln(P^i_{c,t}/P^i_{c,t-1}) &= ln(P^i_{c,t}/P^i_{o,t}) + ln(P^i_{o,t}/P^i_{c,t-1}) \\ R^i_t &= R^i_{DT,t} + R^i_{ON,t} \end{aligned} \tag{3.1}$$

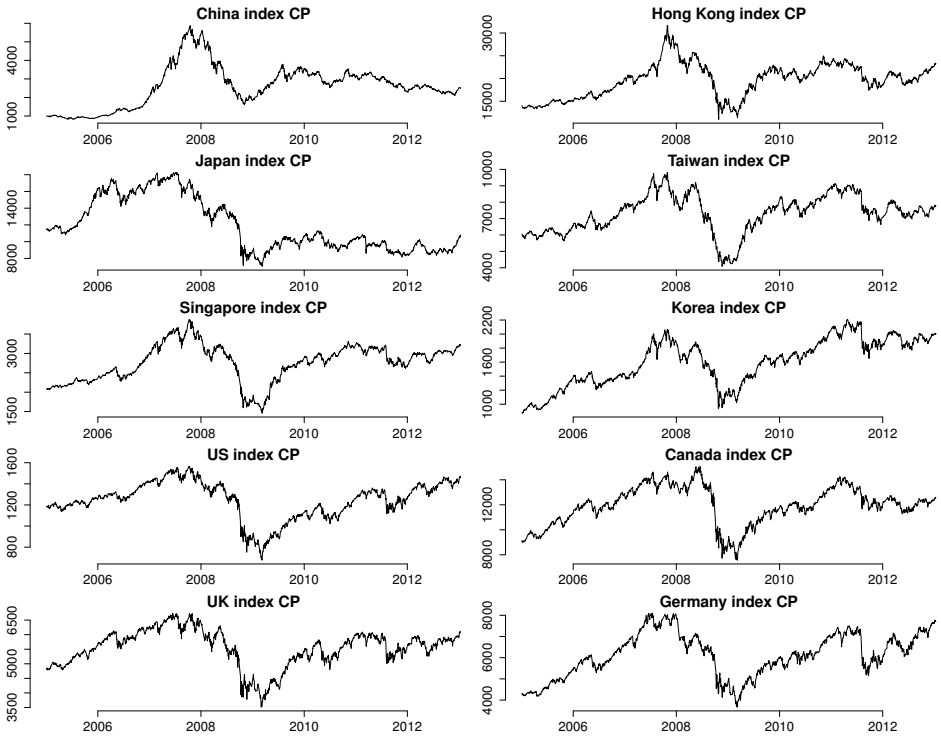



Figure 3.2: Closing prices of stock market indices

where $R_t^i$ are the returns, $R_{DT,t}^i$ are the daytime returns ($DT$), and $R_{ON,t}^i$ are the overnight returns. The superscript $i$ represents each country, and $t = 1, ..., T$. The daytime and overnight returns are plotted in Figures 3.3 and 3.4 and show high negative and positive returns during the period of crisis.

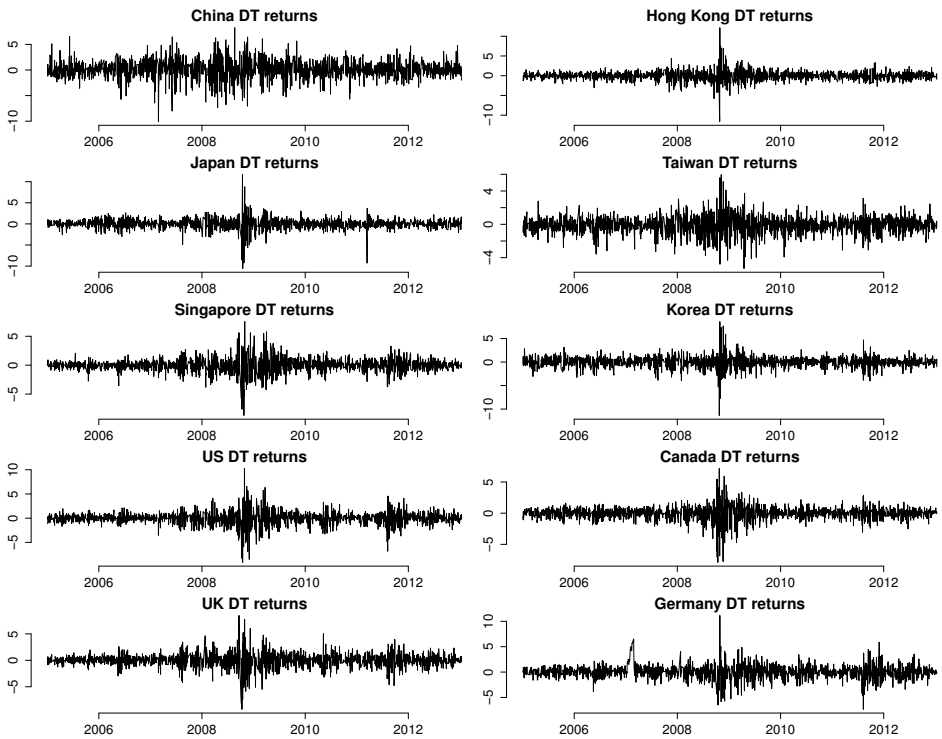



Figure 3.3: Daytime returns

Tables 3.1, 3.2 and 3.3 show the summary statistics of the daytime and overnight returns before, during and after the crisis, respectively.

Globally, during the crisis (Table 3.2), the returns indicate higher standard deviations (S D), minimum (Min), maximum (Max), kurtosis (Kurt) and skewness (Skew) than before (Table 3.1) and after (Table 3.3) the crisis for the daytime and overnight returns. We remark that overnight returns have higher excess of kurtosis than daytime returns whatever the period. As expected in financial time series, the Kolmogorov-Smirnov (KS) tests show evidence of non Gaussian distribution. We performed Ljung-Box tests with six (LB(6)) and twelve (LB(12)) lags on returns and squared returns ($LB^2(6)$ and $LB^2(12)$) which favour GARCH models.

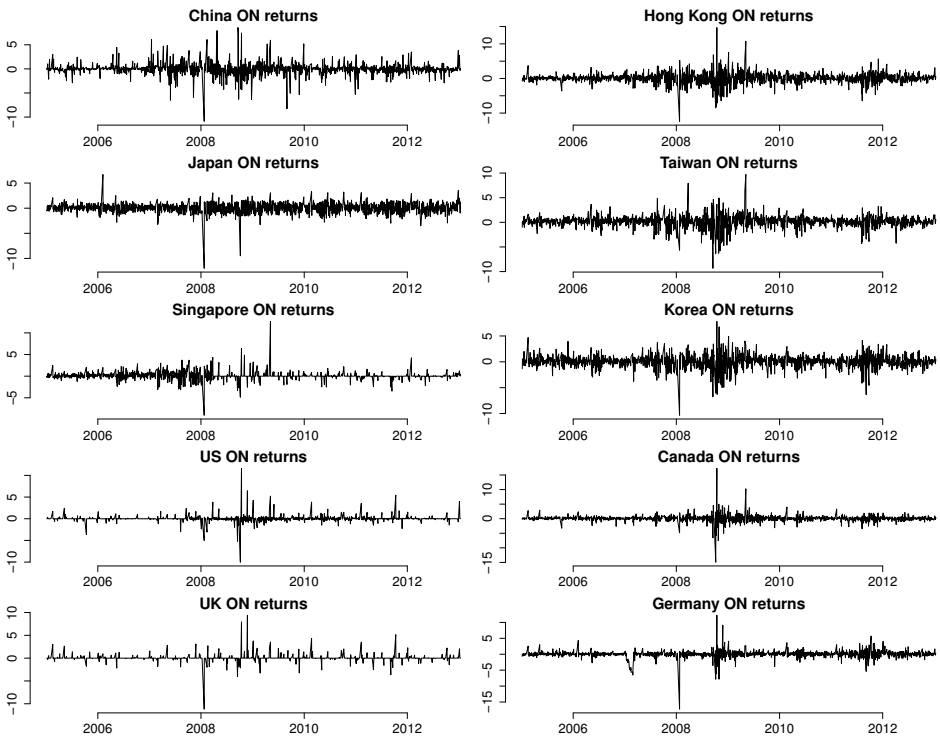



Figure 3.4: Overnight returns

Table 3.1: Summary statistics (before the crisis)

| Var. | Mean | S D | Median | Min | Max | Kurt | Skew | LB(6) | LB(12) | $LB^2(6)$ | $LB^2(12)$ | KS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Daytime | | | | | | |
| China | 0.00180 | 0.01519 | 0.00174 | -0.10069 | 0.06527 | 5.73495 | -0.69220 | 6.18708 | 11.02327 | 157.860*** | 297.151*** | 0.05740*** |
| Hong-Kong | -0.00004 | 0.00681 | 0.00023 | -0.03052 | 0.02041 | 1.43973 | -0.44610 | 8.48903 | 16.33319 | 715.248*** | 972.972*** | 0.06311*** |
| Taiwan | -0.00122 | 0.00778 | -0.00095 | -0.03572 | 0.02775 | 2.45840 | -0.52398 | 5.97318 | 13.01567 | 245.987*** | 380.417*** | 0.05579*** |
| Japan | 0.00011 | 0.00867 | 0.00006 | -0.02993 | 0.02625 | 0.82849 | -0.21745 | 9.17844 | 13.35723 | 1171.28*** | 1872.01*** | 0.05265*** |
| Korea | -0.00005 | 0.00910 | -0.00019 | -0.03272 | 0.03156 | 1.25003 | -0.29506 | 9.53217 | 10.64047 | 741.474*** | 1017.36*** | 0.04798*** |
| Singapore | -0.00060 | 0.00620 | -0.00013 | -0.03559 | 0.02060 | 3.45381 | -0.67205 | 10.9642* | 17.76425 | 953.136*** | 1763.43*** | 0.06550*** |
| UK | 0.00036 | 0.00700 | 0.00064 | -0.02963 | 0.02605 | 2.09205 | -0.35492 | 9.78134 | 23.1305** | 899.301*** | 1594.40*** | 0.05019*** |
| Germany | 0.00246 | 0.01113 | 0.00148 | -0.03835 | 0.06528 | 6.99311 | 1.76091 | 602.591*** | 959.368*** | 817.382*** | 1146.80*** | 0.14390*** |
| US | 0.00035 | 0.00651 | 0.00081 | -0.03526 | 0.02138 | 2.08584 | -0.32351 | 7.73979 | 9.82136 | 907.065*** | 1656.80*** | 0.04968*** |
| Canada | -0.00027 | 0.00640 | -0.00004 | -0.01886 | 0.01621 | 0.01837 | -0.26465 | 2.43260 | 19.8467* | 1049.02*** | 1853.05*** | 0.03185 |
| | | | | | | Overnight | | | | | | |
| China | 0.00071 | 0.00738 | 0.00000 | -0.02888 | 0.06121 | 18.7186 | 2.51004 | 5.79339 | 26.6377*** | 54.3182*** | 117.702*** | 0.20941*** |
| Hong-Kong | 0.00079 | 0.00690 | 0.00077 | -0.03616 | 0.03733 | 6.21073 | -0.12203 | 9.61431 | 17.94415 | 232.574*** | 353.259*** | 0.08559*** |
| Taiwan | 0.00176 | 0.00675 | 0.00190 | -0.02690 | 0.03650 | 3.97676 | -0.03838 | 13.5225** | 17.71621 | 235.208*** | 380.892*** | 0.09459*** |
| Japan | 0.00073 | 0.00704 | 0.00088 | -0.03240 | 0.06704 | 16.9874 | 1.25589 | 13.6408** | 20.8217* | 1.84965 | 4.61143 | 0.06222*** |
| Korea | 0.00116 | 0.00808 | 0.00141 | -0.03769 | 0.04713 | 5.15283 | 0.15491 | 10.07010 | 19.49922* | 202.178*** | 429.394*** | 0.09009*** |
| Singapore | 0.00160 | 0.00621 | 0.00139 | -0.03084 | 0.03016 | 5.32694 | 0.04468 | 22.5571*** | 33.7476*** | 5.00700 | 9.26323 | 0.09907*** |
| UK | 0.00024 | 0.00309 | 0.00000 | -0.02173 | 0.03022 | 41.8772 | 2.32330 | 3.13962 | 6.08061 | 0.27525 | 1.91131 | 0.47232*** |
| Germany | -0.00137 | 0.00996 | 0.00033 | -0.06528 | 0.04329 | 13.7903 | -2.80019 | 1119.21*** | 1729.5*** | 54.7589*** | 82.3505*** | 0.24774*** |
| US | 0.00008 | 0.00290 | 0.00000 | -0.03641 | 0.02431 | 68.1607 | -1.80691 | 2.51190 | 5.36109 | 261.493*** | 275.584*** | 0.43526*** |
| Canada | 0.00105 | 0.00531 | 0.00100 | -0.04267 | 0.02900 | 15.3098 | -0.97299 | 4.56098 | 8.36198 | 389.045*** | 458.916*** | 0.11676*** |

Table 3.2: Summary statistics (during the crisis)

| Var. | Mean | S D | Median | Min | Max | Kurt | Skew | LB(6) | LB(12) | $LB^2(6)$ | $LB^2(12)$ | KS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Daytime | | | | | | |
| China | 0.00138 | 0.02341 | 0.00207 | -0.08028 | 0.08249 | 0.73369 | -0.27182 | 9.06847 | 14.1313 | 157.860*** | 297.151*** | 0.06589*** |
| Hong-Kong | -0.00080 | 0.01725 | -0.00087 | -0.11616 | 0.12155 | 9.33044 | 0.27075 | 19.8190*** | 44.3864*** | 715.248*** | 972.972*** | 0.06856*** |
| Taiwan | -0.00114 | 0.01374 | -0.00093 | -0.05298 | 0.05957 | 2.22734 | 0.09879 | 5.07370 | 15.4203 | 245.987*** | 380.417*** | 0.06428*** |
| Japan | -0.00049 | 0.01753 | -0.00098 | -0.10563 | 0.11658 | 10.2079 | -0.31041 | 11.5312* | 25.0520** | 1171.28*** | 1872.01*** | 0.11215*** |
| Korea | -0.00055 | 0.01601 | 0.00005 | -0.11350 | 0.08597 | 8.21597 | -0.32191 | 12.6910** | 15.5294 | 741.474*** | 1017.36*** | 0.08814*** |
| Singapore | -0.00099 | 0.01684 | -0.00068 | -0.08696 | 0.07531 | 3.70513 | -0.18097 | 7.94274 | 21.6924** | 953.136*** | 1763.43*** | 0.07556*** |
| UK | -0.00062 | 0.01771 | -0.00016 | -0.09265 | 0.08469 | 4.48169 | -0.31985 | 32.0258*** | 46.3195*** | 899.300*** | 1594.40*** | 0.07870*** |
| Germany | -0.00030 | 0.01609 | -0.00002 | -0.06486 | 0.11141 | 6.05796 | 0.37790 | 28.7333*** | 43.3229*** | 817.382*** | 1146.80*** | 0.07569*** |
| US | -0.00050 | 0.01857 | 0.00079 | -0.09127 | 0.10246 | 4.91168 | -0.31504 | 26.3869*** | 36.1463*** | 907.065*** | 1656.80*** | 0.09335*** |
| Canada | -0.00118 | 0.01550 | 0.00000 | -0.07891 | 0.07154 | 4.80225 | -0.62402 | 4.30058 | 22.8097** | 1049.02*** | 1853.05*** | 0.08534*** |
| | | | | | | Overnight | | | | | | |
| China | -0.00135 | 0.01672 | -0.00035 | -0.10890 | 0.08577 | 9.45426 | -0.03611 | 9.42883 | 13.2277 | 54.3182*** | 117.702*** | 0.14612*** |
| Hong-Kong | 0.00087 | 0.02106 | 0.00120 | -0.12393 | 0.14706 | 8.14354 | 0.14075 | 9.06714 | 38.9493** | 232.574*** | 353.259*** | 0.08361*** |
| Taiwan | 0.00117 | 0.01646 | 0.00296 | -0.09336 | 0.09708 | 6.25755 | -0.34585 | 21.6522*** | 41.6128*** | 235.208*** | 380.892*** | 0.12335*** |
| Japan | -0.00041 | 0.01132 | 0.00059 | -0.11968 | 0.03057 | 29.2964 | -3.06884 | 6.39156 | 12.5495 | 1.84965 | 4.61143 | 0.06876*** |
| Korea | 0.00069 | 0.01607 | 0.00199 | -0.10335 | 0.07810 | 5.76674 | -0.71891 | 2.35979 | 6.98540 | 202.178*** | 429.394*** | 0.11217*** |
| Singapore | 0.00069 | 0.01159 | 0.00000 | -0.09030 | 0.12641 | 34.0754 | 1.85879 | 2.47893 | 5.07290 | 5.00700 | 9.26323 | 0.29796*** |
| UK | 0.00030 | 0.00889 | 0.00000 | -0.11178 | 0.09384 | 79.7933 | -0.16877 | 1.31232 | 4.67839 | 0.27525 | 1.91131 | 0.44910*** |
| Germany | -0.00008 | 0.01368 | 0.00057 | -0.17242 | 0.12223 | 63.4274 | -2.67639 | 15.1415** | 21.7026** | 54.7589*** | 82.3505*** | 0.20370*** |
| US | 0.00000 | 0.00954 | -0.00003 | -0.10023 | 0.11615 | 69.4428 | 1.67966 | 40.9137*** | 45.0788*** | 261.493*** | 275.584*** | 0.27837*** |
| Canada | 0.00091 | 0.01604 | 0.00060 | -0.14923 | 0.17262 | 40.8335 | 0.81506 | 86.9607*** | 98.0417*** | 389.045*** | 458.916*** | 0.15100*** |

Table 3.3: Summary statistics (after the crisis)

| Var. | Mean | S D | Median | Min | Max | Kurt | Skew | LB(6) | LB(12) | $LB^2(6)$ | $LB^2(12)$ | KS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Daytime | | | | | | |
| China | 0.00046 | 0.01306 | -0.00002 | -0.05691 | 0.04792 | 1.76459 | -0.06838 | 5.32201 | 6.76454 | 157.860*** | 297.151*** | 0.06537*** |
| Hong-Kong | -0.00035 | 0.00813 | -0.00033 | -0.02716 | 0.03306 | 0.81864 | 0.13409 | 16.4355** | 19.2299* | 715.248*** | 972.972*** | 0.03370* |
| Taiwan | -0.00121 | 0.00840 | -0.00080 | -0.03925 | 0.03139 | 1.50705 | -0.26505 | 17.3593*** | 23.6227** | 245.987*** | 380.417*** | 0.04861*** |
| Japan | -0.00033 | 0.00871 | -0.00031 | -0.09277 | 0.03657 | 20.2881 | -1.82937 | 6.07609 | 22.5676** | 1171.28*** | 1872.01*** | 0.06088*** |
| Korea | -0.00020 | 0.00908 | 0.00048 | -0.04020 | 0.04704 | 2.77015 | -0.40579 | 12.4801* | 22.1305** | 741.474*** | 1017.36*** | 0.07051*** |
| Singapore | 0.00018 | 0.00888 | 0.00015 | -0.03769 | 0.03290 | 1.70730 | -0.27911 | 18.4393*** | 24.8895** | 953.136*** | 1763.43*** | 0.05031*** |
| UK | -0.00002 | 0.01120 | 0.00002 | -0.04779 | 0.05032 | 1.91243 | -0.12924 | 6.05812 | 19.2493* | 899.300*** | 1594.40*** | 0.06562*** |
| Germany | -0.00035 | 0.01242 | 0.00047 | -0.07336 | 0.05879 | 3.67011 | -0.29507 | 15.0461** | 25.9862** | 817.382*** | 1146.80*** | 0.08474*** |
| US | 0.00001 | 0.01137 | 0.00041 | -0.06821 | 0.04563 | 4.09695 | -0.50580 | 30.0117*** | 46.3086*** | 907.065*** | 1656.80*** | 0.09906*** |
| Canada | -0.00100 | 0.00721 | 0.00000 | -0.03055 | 0.03037 | 1.39894 | 0.03540 | 26.4509*** | 42.6350*** | 1049.02*** | 1853.05*** | 0.06215*** |
| | | | | | | Overnight | | | | | | |
| China | -0.00101 | 0.00795 | -0.00061 | -0.05285 | 0.03837 | 8.19944 | -0.43123 | 7.28413 | 24.9291** | 54.3182*** | 117.702*** | 0.14327*** |
| Hong-Kong | 0.00047 | 0.01089 | 0.00043 | -0.06454 | 0.05644 | 4.95131 | -0.36210 | 8.38907 | 16.0598 | 232.574*** | 353.259*** | 0.08224*** |
| Taiwan | 0.00116 | 0.00837 | 0.00185 | -0.04354 | 0.03253 | 5.40328 | -1.09402 | 5.59668 | 25.4347** | 235.208*** | 380.892*** | 0.10365*** |
| Japan | 0.00036 | 0.00941 | 0.00027 | -0.03492 | 0.03511 | 0.78478 | -0.02794 | 5.52396 | 17.9822 | 1.84965 | 4.61143 | 0.03096 |
| Korea | 0.00046 | 0.01017 | 0.00100 | -0.06342 | 0.04135 | 4.97048 | -0.70820 | 10.7311* | 29.9929*** | 202.178*** | 429.394*** | 0.09780*** |
| Singapore | -0.00001 | 0.00432 | 0.00000 | -0.03458 | 0.04233 | 31.1699 | -0.49093 | 7.34616 | 15.4242 | 5.00700 | 9.26323 | 0.42435*** |
| UK | 0.00022 | 0.00470 | 0.00000 | -0.03645 | 0.05140 | 47.9746 | 2.92498 | 4.32088 | 8.88622 | 0.27525 | 1.91131 | 0.46147*** |
| Germany | 0.00075 | 0.00901 | 0.00085 | -0.05379 | 0.05720 | 6.72979 | 0.22756 | 9.15914 | 16.7796 | 54.7589*** | 82.3505*** | 0.08956*** |
| US | 0.00040 | 0.00464 | 0.00000 | -0.02267 | 0.05434 | 49.9480 | 5.00555 | 1.71044 | 8.64764 | 261.493*** | 275.584*** | 0.28673*** |
| Canada | 0.00111 | 0.00736 | 0.00084 | -0.03473 | 0.04828 | 6.71514 | 0.24574 | 22.2958*** | 52.9640*** | 389.045*** | 458.916*** | 0.08887*** |

### 3.3.2 Methodology and hypotheses

According to different trading hours among international stock markets, we can classify generally these markets into three categories: full overlapping, partially overlapping and non-overlapping markets. But, in this study, our sample markets are mainly in the partially overlapping and non-overlapping markets. Thus, we need to be careful to the possible non-synchronous trading problem. Some studies employ weekly or monthly data to avoid this non-synchronicity problem. However, the use of low frequency data requires an enough length of sample, and we need to bear some missing information (compared with daily or high-frequency data analyses). Moreover, these low frequency data cannot capture daily correlation dynamics.

In this chapter, we follow the intuition of Wang, Rui and Firth (2002), dividing the daily returns into two parts: daytime and overnight returns as shown in (3.1). Therefore, we can measure the return and volatility spillovers through daytime and overnight information. In a stock market, the daytime trading period is typically about half the length of the overnight non-trading period (see Table 3.4). Ineluctably, not all valuable financial information becomes available during trading hours (Tsiakas, 2008). For each stock market, during the trading period, all valuable information from local market and its overlapping markets is incorporated into the daytime return, while all information from its non-overlapping markets take place overnight, which is reflected by overnight return.

Table 3.4: Exchange trading hours

| Stock Exchange | Greenwich Time | Local Time |
|---|---|---|
| Shanghai | 1:30-7:00 | 9:30-15:00 |
| New York | 14:30-21:00 | 9:30-16:00 |
| Toronto | 14:30-21:00 | 9:30-16:00 |
| London | 8:00-16:30 | 8:00-16:30 |
| Frankfurt | 8:00-16:30 | 9:00-17:30 |
| Tokyo | 0:00-6:00 | 9:00-15:00 |
| Hong Kong | 1:15-8:00 | 9:15-16:00 |
| Taiwan | 1:00-5:30 | 9:00-13:30 |
| Seoul | 0:00-6:00 | 9:00-15:00 |
| Singapore | 1:00-9:00 | 9:00-17:00 |

We classify our sample into two groups noting:

- $i_{Part-overl} = \{HK, TW, JP, KR, SG\}$ for partially overlapping markets,
- $i_{Non-overl} = \{DE, UK, US, CA\}$ for non-overlapping markets.

In order to investigate the daytime and overnight information relationship between Chinese stock market and markets $i_{Part-overl}$ or $i_{Non-overl}$, we set some basic hypotheses:

**China and partially overlapping markets**

- Daytime relationship

  **(a1)** The Chinese *daytime* return may be affected by the *contemporaneous daytime* and/or *overnight* returns of market $i_{Part-overl}$. It may also depend on its *own contemporaneous overnight* and *lagged daytime* returns.

  **(a2)** Market $i_{Part-overl}$ *daytime* return could be affected by the *contemporaneous daytime* and/or *overnight* returns of Chinese market, and also by its *contemporaneous overnight* and/or *lagged daytime* returns.

- Overnight relationship

  **(a3)** The Chinese *overnight* return may be affected by the *contemporaneous overnight* and/or *lagged daytime* returns of market $i_{Part-overl}$, additionally, it could also depend on its *lagged overnight* and *daytime* returns.

  **(a4)** Market $i_{Part-overl}$ *overnight* return may be affected by the *contemporaneous overnight* and/or *lagged daytime* returns of Chinese market, and also by its *lagged overnight* and/or *daytime* returns.

**China and non-overlapping markets**

- Daytime relationship

  **(b1)** The Chinese *daytime* return may be highly related with its *contemporaneous overnight* and *lagged daytime* returns, and it can be also explained by the *contemporaneous daytime* and/or *lagged overnight* returns of market $i_{Non-overl}$.

**(b2)** Market $i_{Non-overl}$ *daytime* return may depend on the *contemporaneous daytime* and/or *overnight* returns of the Chinese market, and also on its *contemporaneous overnight* and *lagged daytime* returns.

- Overnight relationship

**(b3)** The Chinese *overnight* return may depend on the *lagged daytime* and/or *overnight* returns of market $i_{Non-overl}$, moreover, it may also depend on its *lagged overnight* and *daytime* returns.

**(b4)** Market $i_{Non-overl}$ *overnight* return may can be explained by the *contemporaneous daytime*, *overnight* and/or *lagged daytime* returns of the Chinese market, and also by its *lagged overnight* and *lagged daytime* returns.

### 3.3.3 Model specification

**Mean equations**

We parametrise the bivariate mean equations according to the hypotheses given in Section 3.3.2:

- The daytime relationship between China and partially overlapping markets is expressed as
  - for market $i = \{JP, KR, TW\}$:

$$\begin{aligned} R^{cn}_{DT,t} &= \omega_1 + \beta_{11} R^{cn}_{ON,t} + \beta_{21} R^{cn}_{DT,t-1} + \beta_{31} R^{i}_{DT,t} + \beta_{41} R^{i}_{ON,t} + \varepsilon^{cn}_{DT,t} \\ R^{i}_{DT,t} &= \omega_2 + \beta_{12} R^{i}_{ON,t} + \beta_{22} R^{i}_{DT,t-1} + \beta_{32} R^{cn}_{DT,t-1} + \beta_{42} R^{cn}_{ON,t} + \varepsilon^{i}_{DT,t} \end{aligned}$$

  - for market $i = \{HK, SG\}$:

$$\begin{aligned} R^{cn}_{DT,t} &= \omega_1 + \beta_{11} R^{cn}_{ON,t} + \beta_{21} R^{cn}_{DT,t-1} + \beta_{31} R^{i}_{DT,t-1} + \beta_{41} R^{i}_{ON,t} + \varepsilon^{cn}_{DT,t} \\ R^{i}_{DT,t} &= \omega_2 + \beta_{12} R^{i}_{ON,t} + \beta_{22} R^{i}_{DT,t-1} + \beta_{32} R^{cn}_{DT,t} + \beta_{42} R^{cn}_{ON,t} + \varepsilon^{i}_{DT,t} \end{aligned}$$

- The overnight relationship between China and partially overlapping markets ($i = \{HK, TW, JP, KR, SG\}$) is expressed as

$$\begin{aligned} R^{cn}_{ON,t} &= \omega_1 + \beta_{11} R^{cn}_{DT,t-1} + \beta_{21} R^{i}_{ON,t} + \beta_{31} R^{i}_{DT,t-1} + \beta_{41} R^{i}_{ON,t-1} + \varepsilon^{cn}_{ON,t} \\ R^{i}_{ON,t} &= \omega_2 + \beta_{12} R^{i}_{ON,t-1} + \beta_{22} R^{i}_{DT,t-1} + \beta_{32} R^{cn}_{ON,t-1} + \beta_{42} R^{cn}_{DT,t-1} + \varepsilon^{i}_{ON,t} \end{aligned}$$

- The daytime relationship between China and non overlapping markets ($i = \{DE, UK, US, CA\}$) is expressed as

$$\begin{aligned} R^{cn}_{DT,t} &= \omega_1 + \beta_{11} R^{cn}_{ON,t} + \beta_{21} R^{cn}_{DT,t-1} + \beta_{31} R^{i}_{ON,t-1} + \beta_{41} R^{i}_{DT,t-1} + \varepsilon^{cn}_{DT,t} \\ R^{i}_{DT,t} &= \omega_2 + \beta_{12} R^{i}_{ON,t} + \beta_{22} R^{i}_{DT,t-1} + \beta_{32} R^{cn}_{DT,t} + \beta_{42} R^{cn}_{t,ON} + \varepsilon^{i}_{DT,t} \end{aligned}$$

- The overnight relationship between China and non overlapping markets ($i = \{DE, UK, US, CA\}$) is expressed as

$$\begin{aligned} R^{cn}_{ON,t} &= \omega_1 + \beta_{11} R^{cn}_{DT,t-1} + \beta_{21} R^{i}_{ON,t-1} + \beta_{31} R^{i}_{DT,t-1} + \beta_{41} R^{cn}_{ON,t-1} + \varepsilon^{cn}_{ON,t} \\ R^{i}_{ON,t} &= \omega_2 + \beta_{12} R^{i}_{ON,t-1} + \beta_{22} R^{i}_{DT,t-1} + \beta_{32} R^{cn}_{ON,t} + \beta_{42} R^{cn}_{DT,t} + \varepsilon^{i}_{ON,t} \end{aligned}$$

The information transmission contained in the prices is determinant for an accurate interpretation of volatilities and conditional correlations. The basic hypotheses in the previous section define the overnight and daytime information through the overnight and daytime returns, respectively.

The bivariate framework is mainly used in order to deal with asynchronicity phenomenon. We set China stock market as benchmark.

**Robustness**

We apply a trivariate mean equation to see if the bivariate results are robust. Due to asynchronicity issue, we only can specify for partially overlapping markets these three markets:

- China ($cn$), Japan ($jp$) and Korea ($kr$)

$$\begin{aligned} R^{cn}_{DT,t} = \omega_1 &+ \beta_{11} R^{cn}_{ON,t} + \beta_{21} R^{cn}_{DT,t-1} + \beta_{31} R^{jp}_{DT,t} + \beta_{41} R^{kr}_{DT,t} \\ &+ \beta_{51} R^{jp}_{ON,t} + \beta_{61} R^{kr}_{ON,t} + \varepsilon^{cn}_{DT,t} \\ R^{jp}_{DT,t} = \omega_2 &+ \beta_{12} R^{jp}_{ON,t} + \beta_{22} R^{jp}_{DT,t-1} + \beta_{32} R^{kr}_{ON,t} + \beta_{42} R^{kr}_{DT,t-1} \\ &+ \beta_{52} R^{kr}_{DT,t} + \beta_{62} R^{cn}_{ON,t} + \beta_{72} R^{cn}_{DT,t-1} + \varepsilon^{jp}_{DT,t} \\ R^{kr}_{DT,t} = \omega_3 &+ \beta_{13} R^{kr}_{ON,t} + \beta_{23} R^{kr}_{DT,t-1} + \beta_{33} R^{jp}_{ON,t} + \beta_{43} R^{jp}_{DT,t-1} \\ &+ \beta_{53} R^{jp}_{DT,t} + \beta_{63} R^{cn}_{ON,t} + \beta_{73} R^{cn}_{DT,t-1} + \varepsilon^{kr}_{DT,t} \end{aligned}$$

**Constant conditional correlation test**

Many conditional covariance and correlation models have been developed during the last couple of decades (see Bauwens, Laurent and Rombouts (2006) and Silvennoinen and Teräsvirta (2009a) for extensive

reviews). Instead of choosing and estimating an arbitrary multivariate GARCH model, we use two constant conditional correlation tests in order to avoid any misspecification. The LM tests developed in Chapter 1 use an artificial neural network and a Taylor expansion function to determine whether or not the time series, obtained by the above mean specifications, are conditionally correlated over time.

**Conditional correlation GARCH models**

According to the results of the tests of constancy of conditional correlations, we choose to model the second conditional moment with an extended constant conditional correlation GARCH model (ECCC-GARCH) or an extended dynamic conditional correlation GARCH model (EDCC-GARCH).

We define the multivariate GARCH process such that $\boldsymbol{\varepsilon}_t|\Omega_{t-1} \sim N(0, \mathbf{H}_t)$, where $\Omega_t$ is the information set up to time $t$, and

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t, \quad \forall t = 1, \ldots, T \tag{3.2}$$

with $\boldsymbol{\eta}_t \sim iid(0, \mathbf{I}_2)$ for the bivariate analysis.

We introduce briefly the two conditional correlation models:

- <u>ECCC-GARCH</u>

From (3.2), Bollerslev (1990) defines the conditional covariances as follows:

$$\mathbf{H}_t = [h_{ij,t}]_{i,j=1,2} = \mathbf{S}_t\mathbf{P}\mathbf{S}_t, \quad \forall t = 1, \ldots, T \tag{3.3}$$

where $\mathbf{P} = [\rho_{12}]$ is the positive definite $(2 \times 2)$ constant conditional correlation matrix, and $\mathbf{S}_t = diag(h_{11,t}^{1/2}, h_{22,t}^{1/2})$.

The vector form (VEC) of the conditional variance can be written as

$$\mathbf{h}_t = \mathbf{a}_0 + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}^{(2)} + \mathbf{G}\mathbf{h}_{t-1} \tag{3.4}$$

for the one lag model, $\mathbf{a}_0 = [a_{i0}]_{i=1,2}$ is a $(2 \times 1)$ vector, $\mathbf{A} = [a_{ij}]_{i,j=1,2}$ and $\mathbf{G} = [g_{ij}]_{i,j=1,2}$ are $(2 \times 2)$ matrices, and $\boldsymbol{\varepsilon}_t^{(2)} = \boldsymbol{\varepsilon}_t \odot \boldsymbol{\varepsilon}_t$ is a $(2 \times 1)$ vector. When the conditional correlation matrix $\mathbf{P}$ is positive definite and the elements of $\mathbf{a}_0$, $\mathbf{A}$ and $\mathbf{G}$ are positive, the conditional covariance matrix $\mathbf{H}_t$ is positive definite.

Note that the strong consistency of the quasi-maximum likelihood estimator has been explored for the extended CCC-GARCH by Jeantheau (1998), and Ling and McAleer (2003) have proved its asymptotic normality (see (Francq and Zakoïan, 2010, Chap. 11) for a review on asymptotic theory).

- EDCC-GARCH

From (3.2), Engle (2002) defines the conditional covariances as follows:

$$\mathbf{H}_t = \mathbf{S}_t \mathbf{P}_t \mathbf{S}_t, \quad \forall t = 1, \ldots, T \tag{3.5}$$

where $\mathbf{P}_t$ is the $(2 \times 2)$ time-varying correlation matrix. The model provides an estimate of the conditional covariance matrix $\mathbf{H}_t$ which is usually estimated by the constant parameter matrix $\mathbf{\Gamma}$ (see Aielli (2013) for more details on $\mathbf{\Gamma}$), and the standardized residuals $\mathbf{z}_t = \mathbf{S}_t^{-1} \boldsymbol{\varepsilon}_t$.

Thus, we have:

$$\mathbf{P}_t = diag(\gamma_{11,t}^{-1/2}, \gamma_{22,t}^{-1/2}) \mathbf{\Gamma}_t diag(\gamma_{11,t}^{-1/2}, \gamma_{22,t}^{-1/2})$$

where $\mathbf{\Gamma}_t = [\gamma_{12,t}]$, and it can be written as:

$$\mathbf{\Gamma}_t = (1 - \alpha - \beta)\mathbf{\Gamma} + \alpha \mathbf{z}_{t-1} \mathbf{z}'_{t-1} + \beta \mathbf{\Gamma}_{t-1} \tag{3.6}$$

where $\alpha$ and $\beta$ are scalar parameters capturing the effects of previous standardized shocks and dynamic conditional correlations on current standardized shocks and dynamic conditional correlations, respectively. The model is mean reverting as long as $\alpha + \beta < 1$.

There are some attempts to prove asymptotic theory of the DCC (for example Aielli (2013)), but this is out of the scope of this chapter, and many empirical studies show interesting results using this model.

Note that at the opposite of the diagonal models which assume a uniform (constant or dynamic) structure for all cross-correlations, the extended ones capture the volatility spillovers for both directions. The off-diagonal parameters of $\mathbf{G}$ ($g_{12}$ and $g_{21}$) identify the volatility transmission. The coefficients obtained by quasi-maximum likelihood estimation, and their significances obtained by the robust standard errors, indicate the direction and the amplitude of volatility spillovers between stock markets.

## 3.4 Results of information transmission

To figure out how the markets behaved during the financial turmoil of 2007, we perform sub-samples analyses. The full data sample is divided in three sub-samples: before, during and after the crisis as described in Section 3.3.

### 3.4.1 Transmission of daytime and overnight information.

The transmission of the daytime and overnight information are given by the coefficients $\beta$ in Panel A of Tables 3.5 to 3.23.

Panel A of Tables 3.5 to 3.13 shows the daytime information transmission between China and the foreign markets when the dependent variables are the daytime returns. Before the crisis, daytime and overnight informations from Japan influence China. This relationship becomes bidirectional during and after the crisis. Taiwan seems to influence China before, during and after the crisis with mainly daytime informations. The return spillovers are bidirectional between Mainland China and Hong Kong before, during, and after the crisis. The relationship between China and Singapore, and China and the UK is bidirectional before and after the crisis. This linkage is unidirectional from China to Singapore and the UK during the crisis. We do not find any relationship between China and the US during the crisis, although the US is influenced by daytime information before and after the crisis.

These results indicate that, unsurprisingly, China is likely to be close to the Asian stock markets before the crisis and the linkages seem to be reinforced during and after the crisis. The relationships with the non-Asian markets are strong during the crisis and remain after the crisis. These relationships are in line with some important financial liberalizations of the Chinese capital market, such as the switch in of a managed floating exchange rate regime of July 2005, the split-share structure reform on the A-share market of 2005, and the increase of the QFII and QDII quota over time, which enhance the level of the opening of Chinese stock market.

Instead of having only the daytime returns as usually done in several empirical studies, it should be interesting to see what happens for the overnight returns. The overnight information transmission is presented in Panel A of Tables 3.15 to 3.23 between China and the foreign markets when the dependent variables are the overnight returns.

Before the crisis, China influences with overnight information Japan, Hong Kong, Taiwan and Singapore. There is also transmission of daytime information to the UK, while China receives lagged daytime information spillovers from Hong Kong, Taiwan and Singapore.

During and after the crisis, China shows new relationships with the non-Asian countries. The US, the UK, Canada and Germany are influenced by Chinese daytime information, while China is mainly influenced by lagged daytime information from the Asian markets.

Table 3.5: Japan daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00168** | 0.00137 | 0.00053 |
| $\beta_{11}$ | 0.08649 | -0.13721** | -0.08618 |
| $\beta_{21}$ | -0.04132 | -0.06414 | -0.04607 |
| $\beta_{31}$ | 0.14372* | 0.20367*** | 0.33282*** |
| $\beta_{41}$ | 0.20132** | -0.01516 | -0.0352 |
| $\omega_2$ | 0.00004 | -0.00007 | -0.00037 |
| $\beta_{12}$ | 0.10761* | 0.41236*** | 0.13683*** |
| $\beta_{22}$ | -0.08485* | -0.04996 | -0.0168 |
| $\beta_{32}$ | -0.03709 | -0.05797* | -0.05963** |
| $\beta_{42}$ | 0.07332 | 0.14205*** | 0.00481 |
| | Panel B | | |
| $a_{10}$ | 0.00021 | 0.00020** | 0.00011 |
| $a_{20}$ | 0.00001 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.02033 | 0.13659 | 0.00093 |
| $a_{21}$ | 0.00453 | 0.00001 | 0.00731 |
| $a_{12}$ | 0.00025 | 0.08462 | 0.00017 |
| $a_{22}$ | 0.14062*** | 0.17184*** | 0.16927*** |
| $g_{11}$ | 0.03121*** | 0.43912*** | 0.30571*** |
| $g_{21}$ | 0.00065 | 0.01672 | 0.18686 |
| $g_{12}$ | 0.00597 | 0.01412 | 0.03845 |
| $g_{22}$ | 0.66360*** | 0.80155*** | 0.35758 |
| $\alpha$ | – | – | – |
| $\beta$ | – | – | – |
| | Panel C | | |
| $LB(6)$ | 5.19354 | 6.08758 | 1.36411 |
| $LB(12)$ | 12.16868 | 10.97497 | 2.90571 |
| $LB^2(6)$ | 3.88472 | 9.80528 | 3.70642 |
| $LB^2(12)$ | 17.70334 | 15.28054 | 16.52685 |
| $KS$ | 0.06484*** | 0.06464*** | 0.05696*** |
| $LB(6)i$ | 7.08656 | 5.72332 | 1.27986 |
| $LB(12)i$ | 11.04941 | 12.82596 | 10.12525 |
| $LB^2(6)i$ | 4.1632 | 1.96804 | 19.20014*** |
| $LB^2(12)i$ | 14.12144 | 7.69816 | 25.09243** |
| $KSi$ | 0.04911*** | 0.05114*** | 0.04364*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.6: Hong Kong daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00169** | 0.00099 | 0.00024 |
| $\beta_{11}$ | 0.06374 | -0.15326** | -0.18098** |
| $\beta_{21}$ | -0.03919 | -0.02517 | -0.03551 |
| $\beta_{31}$ | 0.21448** | 0.11746** | 0.12757** |
| $\beta_{41}$ | -0.0201 | -0.14345** | -0.07747 |
| $\omega_2$ | -0.00016 | -0.00123* | -0.00054* |
| $\beta_{12}$ | 0.0273 | -0.01158 | -0.01904 |
| $\beta_{22}$ | -0.10656** | -0.08625** | -0.00927 |
| $\beta_{32}$ | 0.09143*** | 0.28729*** | 0.31810*** |
| $\beta_{42}$ | -0.07140* | 0.02256 | -0.05046 |
| | Panel B | | |
| $a_{10}$ | 0.00014 | 0.00008* | 0.00011 |
| $a_{20}$ | 0.00003 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.06871 | 0.12659*** | 0.00064 |
| $a_{21}$ | 0.00821 | 0.00001 | 0.01318 |
| $a_{12}$ | 0.28338 | 0.00000 | 0.30285 |
| $a_{22}$ | 0.00005 | 0.09445*** | 0.04565*** |
| $g_{11}$ | 0.28734*** | 0.68782*** | 0.05495*** |
| $g_{21}$ | 0.03996 | 0.00572 | 0.06373 |
| $g_{12}$ | 0.00624 | 0.04767*** | 0.82118*** |
| $g_{22}$ | 0.12884 | 0.89531*** | 0.68155* |
| $\alpha$ | 0.00556 | 0.0596 | – |
| $\beta$ | 0.97921*** | 0.91429*** | – |
| | Panel C | | |
| $LB(6)$ | 6.00941 | 5.20115 | 1.7879 |
| $LB(12)$ | 11.32478 | 9.36599 | 3.43311 |
| $LB^2(6)$ | 2.84699 | 5.60103 | 2.08759 |
| $LB^2(12)$ | 11.88274 | 8.11277 | 16.84598 |
| $KS$ | 0.06009*** | 0.06663*** | 0.06248*** |
| $LB(6)i$ | 1.44373 | 7.52629 | 4.59202 |
| $LB(12)i$ | 9.15603 | 16.90223 | 12.06115 |
| $LB^2(6)i$ | 4.45954 | 5.15198 | 1.19077 |
| $LB^2(12)i$ | 10.03343 | 13.24648 | 9.27243 |
| $KSi$ | 0.05854*** | 0.02621 | 0.02362 |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.7: Taiwan daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00171** | 0.00186* | 0.00121** |
| $\beta_{11}$ | 0.08107 | -0.07033 | -0.03585 |
| $\beta_{21}$ | -0.04107 | -0.06745 | -0.04188 |
| $\beta_{31}$ | 0.14987* | 0.45340*** | 0.55524*** |
| $\beta_{41}$ | 0.17856* | 0.03107 | -0.0964 |
| $\omega_2$ | -0.00120*** | -0.00099* | -0.00145*** |
| $\beta_{12}$ | -0.04772 | -0.13192*** | 0.12114** |
| $\beta_{22}$ | -0.04143 | -0.03734 | -0.0476 |
| $\beta_{32}$ | 0.01585 | -0.02706 | -0.03903 |
| $\beta_{42}$ | 0.00068 | 0.00056 | -0.05058 |
| | | Panel B | |
| $a_{10}$ | 0.00017 | 0.00001 | 0.00001 |
| $a_{20}$ | 0.00005 | 0.00007 | 0.00004 |
| $a_{11}$ | 0.0412 | 0.06854 | 0.04317 |
| $a_{21}$ | 0.02506 | 0.00009 | 0.0012 |
| $a_{12}$ | 0.00008 | 0.07404 | 0.00764 |
| $a_{22}$ | 0.00028*** | 0.07107*** | 0.05166*** |
| $g_{11}$ | 0.18523*** | 0.83757*** | 0.52371*** |
| $g_{21}$ | 0.00133 | 0.16275*** | 0.06102 |
| $g_{12}$ | 0.05709 | 0.17132 | 0.83964*** |
| $g_{22}$ | 0.07022 | 0.09247 | 0.3144 |
| $\alpha$ | 0.00000 | 0.00329 | – |
| $\beta$ | 0.8895 | 0.96721*** | – |
| | | Panel C | |
| $LB(6)$ | 5.77254 | 5.26881 | 2.48774 |
| $LB(12)$ | 11.6568 | 8.34103 | 7.15347 |
| $LB^2(6)$ | 3.7227 | 4.965 | 3.67617 |
| $LB^2(12)$ | 16.56552 | 6.90923 | 11.19476 |
| $KS$ | 0.05834*** | 0.06066*** | 0.05361*** |
| $LB(6)i$ | 4.46043 | 2.84779 | 14.35649** |
| $LB(12)i$ | 11.08535 | 11.92278 | 20.37447* |
| $LB^2(6)i$ | 8.12467 | 19.13463*** | 18.14798*** |
| $LB^2(12)i$ | 27.06818*** | 32.84446*** | 30.24286*** |
| $KSi$ | 0.05232*** | 0.05953*** | 0.04725*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.8: Singapore daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00145** | 0.00127 | 0.00048 |
| $\beta_{11}$ | 0.06210 | -0.09799 | -0.03260 |
| $\beta_{21}$ | -0.05107 | -0.07358* | -0.05776 |
| $\beta_{31}$ | 0.28620** | 0.06008 | -0.26104** |
| $\beta_{41}$ | 0.04040 | -0.03884 | -0.04905 |
| $\omega_2$ | -0.00085*** | -0.00087 | 0.00038 |
| $\beta_{12}$ | 0.06186 | -0.08134 | -0.18887** |
| $\beta_{22}$ | -0.09792** | -0.01526 | -0.02668 |
| $\beta_{32}$ | 0.07310*** | 0.15641*** | 0.20436*** |
| $\beta_{42}$ | -0.06243* | 0.22182*** | 0.29575*** |
| | | Panel B | |
| $a_{10}$ | 0.00021 | 0.00001 | 0.00012 |
| $a_{20}$ | 0.00002 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.00440 | 0.06596 | 0.00000 |
| $a_{21}$ | 0.01269 | 0.00000 | 0.00530 |
| $a_{12}$ | 0.07997 | 0.00801 | 0.11834 |
| $a_{22}$ | 0.04910*** | 0.12284*** | 0.07066*** |
| $g_{11}$ | 0.07678*** | 0.90247*** | 0.05995*** |
| $g_{21}$ | 0.00207 | 0.00409 | 0.00510 |
| $g_{12}$ | 0.00092 | 0.00009 | 0.37788 |
| $g_{22}$ | 0.39139 | 0.87565*** | 0.90284*** |
| $\alpha$ | – | 0.03440 | – |
| $\beta$ | – | 0.92075*** | – |
| | | Panel C | |
| $LB(6)$ | 5.80331 | 5.20603 | 1.51720 |
| $LB(12)$ | 11.58507 | 10.34198 | 3.83517 |
| $LB^2(6)$ | 4.16891 | 4.10911 | 4.07879 |
| $LB^2(12)$ | 19.78176* | 5.83278 | 16.34767 |
| $KS$ | 0.06209*** | 0.06132*** | 0.06410*** |
| $LB(6)i$ | 8.43853 | 4.54180 | 10.09095 |
| $LB(12)i$ | 14.18858 | 7.85699 | 16.91299 |
| $LB^2(6)i$ | 12.21424* | 9.54746 | 4.00109 |
| $LB^2(12)i$ | 15.18807 | 12.17673 | 17.60315 |
| $KSi$ | 0.05697*** | 0.03130 | 0.03642** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.9: Korea daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00171** | 0.00158 | 0.00041 |
| $\beta_{11}$ | 0.09722 | -0.07243 | -0.13074* |
| $\beta_{21}$ | -0.02866 | -0.05588 | -0.04268 |
| $\beta_{31}$ | 0.21039*** | 0.39328*** | 0.42645*** |
| $\beta_{41}$ | 0.10106 | -0.00677 | 0.09023 |
| $\omega_2$ | 0.00012 | -0.00041 | -0.00014 |
| $\beta_{12}$ | -0.09755* | -0.08860* | -0.07895* |
| $\beta_{22}$ | -0.02594 | -0.07662* | -0.10633*** |
| $\beta_{32}$ | -0.01404 | -0.05818* | -0.03220 |
| $\beta_{42}$ | -0.03798 | 0.02693 | 0.03507 |
| | Panel B | | |
| $a_{10}$ | 0.00010 | 0.00002 | 0.00008* |
| $a_{20}$ | 0.00007 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.10970 | 0.05930 | 0.01171 |
| $a_{21}$ | 0.01351 | 0.00005 | 0.00126 |
| $a_{12}$ | 0.00006 | 0.00727 | 0.28639 |
| $a_{22}$ | 0.00009 | 0.12947*** | 0.10323*** |
| $g_{11}$ | 0.42394*** | 0.88671*** | 0.16719*** |
| $g_{21}$ | 0.01969 | 0.01368 | 0.09623* |
| $g_{12}$ | 0.04326 | 0.00000 | 0.35520 |
| $g_{22}$ | 0.08446 | 0.84211*** | 0.70426** |
| $\alpha$ | 0.00425 | – | 0.00000 |
| $\beta$ | 0.97513*** | – | 0.98838 |
| | Panel C | | |
| $LB(6)$ | 4.57304 | 5.07652 | 1.60318 |
| $LB(12)$ | 9.58057 | 10.25847 | 4.14067 |
| $LB^2(6)$ | 3.04776 | 4.48347 | 8.51467 |
| $LB^2(12)$ | 11.20218 | 5.57166 | 19.80397* |
| $KS$ | 0.06924*** | 0.07051*** | 0.05420*** |
| $LB(6)i$ | 8.99944 | 3.94613 | 2.62316 |
| $LB(12)i$ | 10.03940 | 9.49686 | 4.35001 |
| $LB^2(6)i$ | 13.88550** | 5.93167 | 8.14293 |
| $LB^2(12)i$ | 24.26064** | 10.05583 | 21.55128** |
| $KSi$ | 0.04179** | 0.05139*** | 0.06282*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.10: US daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00174** | 0.00139 | 0.00037 |
| $\beta_{11}$ | 0.09549 | -0.04601 | -0.06063 |
| $\beta_{21}$ | -0.03681 | -0.08141* | -0.06295 |
| $\beta_{31}$ | 0.36396 | -0.00803 | 0.17750 |
| $\beta_{41}$ | 0.10505 | -0.08264 | -0.03481 |
| $\omega_2$ | 0.00033 | -0.00064 | -0.00011 |
| $\beta_{12}$ | 0.10225 | 0.18205** | 0.19945* |
| $\beta_{22}$ | -0.08017* | -0.14725*** | -0.07796* |
| $\beta_{32}$ | 0.06613*** | 0.02632 | 0.10500*** |
| $\beta_{42}$ | -0.05813 | -0.02218 | 0.01774 |
| | | Panel B | |
| $a_{10}$ | 0.00020 | 0.00002 | 0.00000 |
| $a_{20}$ | 0.00002 | 0.00000 | 0.00001 |
| $a_{11}$ | 0.03040 | 0.06275** | 0.11173 |
| $a_{21}$ | 0.00051 | 0.00141 | 0.00002 |
| $a_{12}$ | 0.01043 | 0.00689 | 0.07620 |
| $a_{22}$ | 0.00127** | 0.12217*** | 0.07368*** |
| $g_{11}$ | 0.06602*** | 0.90387*** | 0.61929*** |
| $g_{21}$ | 0.00308 | 0.00468 | 0.02705 |
| $g_{12}$ | 0.00281 | 0.00001 | 0.57206*** |
| $g_{22}$ | 0.37996 | 0.87056*** | 0.78866*** |
| $\alpha$ | 0.00000 | – | 0.00000 |
| $\beta$ | 0.95418 | – | 0.94782 |
| | | Panel C | |
| $LB(6)$ | 5.82135 | 5.21842 | 0.73759 |
| $LB(12)$ | 10.38213 | 10.31442 | 4.15509 |
| $LB^2(6)$ | 4.51759 | 4.53873 | 1.91193 |
| $LB^2(12)$ | 18.74426* | 6.64472 | 9.66929 |
| $KS$ | 0.06409*** | 0.06292*** | 0.07383*** |
| $LB(6)i$ | 6.98880 | 3.50507 | 8.05671 |
| $LB(12)i$ | 8.91283 | 6.66561 | 15.48083 |
| $LB^2(6)i$ | 14.05933** | 11.93937* | 45.40284*** |
| $LB^2(12)i$ | 32.89691*** | 18.05229 | 55.92199*** |
| $KSi$ | 0.05841*** | 0.05801*** | 0.06235*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.11: Canada daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00158** | 0.00131 | 0.00043 |
| $\beta_{11}$ | 0.09687 | -0.04551 | -0.11232 |
| $\beta_{21}$ | -0.03213 | -0.08230* | -0.07226* |
| $\beta_{31}$ | 0.22015 | -0.04035 | 0.00029 |
| $\beta_{41}$ | -0.08019 | -0.12928* | 0.13457* |
| $\omega_2$ | -0.00031 | -0.00116* | -0.00032 |
| $\beta_{12}$ | 0.08560 | -0.09872** | -0.01775 |
| $\beta_{22}$ | -0.04077 | -0.07663* | -0.01395 |
| $\beta_{32}$ | 0.06857*** | -0.00421 | 0.05026** |
| $\beta_{42}$ | -0.06217 | 0.00399 | -0.03360 |
| | Panel B | | |
| $a_{10}$ | 0.00005 | 0.00001 | 0.00008 |
| $a_{20}$ | 0.00004 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.11681 | 0.06379 | 0.00010 |
| $a_{21}$ | 0.00007 | 0.00112 | 0.00127 |
| $a_{12}$ | 0.00012 | 0.00556 | 0.00329 |
| $a_{22}$ | 0.09382*** | 0.11042*** | 0.13792*** |
| $g_{11}$ | 0.66819*** | 0.90686*** | 0.31438*** |
| $g_{21}$ | 0.00016 | 0.00328 | 0.06160 |
| $g_{12}$ | 0.10282*** | 0.00010 | 0.64917* |
| $g_{22}$ | 0.00168 | 0.87900 | 0.66402 |
| $\alpha$ | – | – | – |
| $\beta$ | – | – | – |
| | Panel C | | |
| $LB(6)$ | 5.20585 | 4.69930 | 2.40150 |
| $LB(12)$ | 9.36400 | 9.73025 | 3.82376 |
| $LB^2(6)$ | 1.44535 | 4.15194 | 5.63860 |
| $LB^2(12)$ | 6.06275 | 6.41746 | 17.56711 |
| $KS$ | 0.06482*** | 0.06327*** | 0.06385*** |
| $LB(6)i$ | 5.62425 | 5.49108 | 4.58186 |
| $LB(12)i$ | 17.07998 | 11.82488 | 12.86407 |
| $LB^2(6)i$ | 24.95257*** | 2.00865 | 9.28068 |
| $LB^2(12)i$ | 33.81754*** | 10.87063 | 15.34829 |
| $KSi$ | 0.02700 | 0.06608*** | 0.03515** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.12: UK daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00168** | 0.00136 | 0.00036 |
| $\beta_{11}$ | 0.09578 | -0.09667 | -0.09154 |
| $\beta_{21}$ | -0.03547 | -0.07997* | -0.06397 |
| $\beta_{31}$ | 0.39819* | 0.05616 | 0.19931* |
| $\beta_{41}$ | 0.10850 | 0.03427 | 0.02622 |
| $\omega_2$ | 0.00035 | -0.00047 | 0.00007 |
| $\beta_{12}$ | 0.03811 | -0.07761 | -0.19255** |
| $\beta_{22}$ | -0.11633*** | -0.10908** | -0.00060 |
| $\beta_{32}$ | 0.05049** | 0.05656* | 0.16197*** |
| $\beta_{42}$ | -0.06471 | 0.20099*** | 0.15320** |
| | Panel B | | |
| $a_{10}$ | 0.00018** | 0.00004 | 0.00006 |
| $a_{20}$ | 0.00001 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.02941 | 0.07085 | 0.00044 |
| $a_{21}$ | 0.00042 | 0.00000 | 0.00962 |
| $a_{12}$ | 0.68248 | 0.17377 | 0.01560 |
| $a_{22}$ | 0.16738*** | 0.03842*** | 0.07556*** |
| $g_{11}$ | 0.04404*** | 0.67259*** | 0.43648*** |
| $g_{21}$ | 0.00001 | 0.42354*** | 0.01008 |
| $g_{12}$ | 0.01711 | 0.25010 | 0.36802 |
| $g_{22}$ | 0.61754*** | 0.06266 | 0.87684 |
| $\alpha$ | – | 0.03774 | – |
| $\beta$ | – | 0.69755* | – |
| | Panel C | | |
| $LB(6)$ | 4.81127 | 3.78620 | 1.21358 |
| $LB(12)$ | 10.94666 | 7.48342 | 3.07278 |
| $LB^2(6)$ | 3.17461 | 2.93923 | 3.78191 |
| $LB^2(12)$ | 15.50734 | 9.48142 | 11.71777 |
| $KS$ | 0.05620*** | 0.16249*** | 0.06295*** |
| $LB(6)i$ | 4.07260 | 4.24988 | 4.45304 |
| $LB(12)i$ | 11.83362 | 9.48462 | 12.52136 |
| $LB^2(6)i$ | 4.25853 | 0.29922 | 8.06181 |
| $LB^2(12)i$ | 17.72758 | 3.89855 | 10.05605 |
| $KSi$ | 0.02987 | 0.26486*** | 0.04415*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.13: Germany daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | Panel A | | |
| $\omega_1$ | 0.00197*** | 0.00137 | 0.00042 |
| $\beta_{11}$ | 0.11604 | -0.08731 | -0.08869 |
| $\beta_{21}$ | -0.04406 | -0.08061* | -0.06705* |
| $\beta_{31}$ | 0.16810** | 0.00118 | 0.00500 |
| $\beta_{41}$ | 0.02992 | 0.00554 | 0.01685 |
| $\omega_2$ | 0.00121*** | -0.00029 | -0.00040 |
| $\beta_{12}$ | -0.46071*** | 0.03360 | -0.04941 |
| $\beta_{22}$ | 0.25036*** | 0.13762*** | 0.03095 |
| $\beta_{32}$ | 0.02471 | -0.01133 | 0.00987 |
| $\beta_{42}$ | -0.04199 | -0.03548 | -0.05632 |
| | Panel B | | |
| $a_{10}$ | 0.00005** | 0.00002 | 0.00013 |
| $a_{20}$ | 0.00000 | 0.00000 | 0.00000 |
| $a_{11}$ | 0.02840 | 0.07223* | 0.00029 |
| $a_{21}$ | 0.01412 | 0.00000 | 0.00001 |
| $a_{12}$ | 0.68357 | 0.01198 | 0.07043 |
| $a_{22}$ | 0.07713*** | 0.07335*** | 0.10053*** |
| $g_{11}$ | 0.49420*** | 0.89444*** | 0.17446*** |
| $g_{21}$ | 0.03275 | 0.00330 | 0.02764 |
| $g_{12}$ | 0.05301 | 0.00003 | 0.00547 |
| $g_{22}$ | 0.74168*** | 0.91667*** | 0.86684*** |
| $\alpha$ | – | – | – |
| $\beta$ | – | – | – |
| | Panel C | | |
| $LB(6)$ | 6.37073 | 5.15145 | 1.88508 |
| $LB(12)$ | 15.11991 | 10.32738 | 3.76161 |
| $LB^2(6)$ | 2.27227 | 4.50225 | 3.82059 |
| $LB^2(12)$ | 7.12511 | 6.24718 | 16.00868 |
| $KS$ | 0.04826*** | 0.06369*** | 0.06302*** |
| $LB(6)i$ | 10.97304* | 14.36497** | 10.25979 |
| $LB(12)i$ | 27.42105*** | 18.33490 | 17.62778 |
| $LB^2(6)i$ | 1.70978 | 1.85542 | 6.88232 |
| $LB^2(12)i$ | 5.74826 | 7.99210 | 11.37374 |
| $KSi$ | 0.05294*** | 0.04761*** | 0.06187*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.14: CN-JP-KR daytime stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00170** | 0.00171* | 0.00046 |
| $\beta_{11}$ | 0.10044 | -0.08251 | -0.12153 |
| $\beta_{21}$ | -0.04156 | -0.05405 | -0.03781 |
| $\beta_{31}$ | 0.05861 | 0.03423 | 0.14535** |
| $\beta_{41}$ | 0.16021* | 0.37303*** | 0.35208*** |
| $\beta_{51}$ | 0.20444* | 0.17065 | -0.02961 |
| $\beta_{61}$ | -0.00955 | -0.11210 | 0.08163 |
| $\omega_2$ | -0.00007 | -0.00046 | -0.00034 |
| $\beta_{12}$ | -0.06901 | -0.19054** | 0.14059*** |
| $\beta_{22}$ | 0.03756 | -0.03464 | 0.02284 |
| $\beta_{32}$ | 0.17215*** | 0.65824*** | 0.10743** |
| $\beta_{42}$ | -0.04779 | 0.16831*** | -0.07118* |
| $\beta_{52}$ | 0.79498*** | 0.83732*** | 0.74359*** |
| $\beta_{62}$ | 0.09523** | 0.00164 | -0.05163 |
| $\beta_{72}$ | -0.01547 | -0.01233 | -0.00997 |
| $\omega_3$ | 0.00010 | 0.00031 | 0.00025 |
| $\beta_{13}$ | -0.21349*** | -0.61377*** | -0.10559** |
| $\beta_{23}$ | 0.05299 | -0.15649*** | 0.01384 |
| $\beta_{33}$ | 0.12831** | 0.19295** | -0.14118*** |
| $\beta_{43}$ | -0.07755* | 0.00590 | -0.00152 |
| $\beta_{53}$ | 0.85684*** | 0.87613*** | 0.83664*** |
| $\beta_{63}$ | -0.09039* | 0.00352 | 0.06105 |
| $\beta_{73}$ | 0.00498 | -0.00412 | -0.00517 |
| | | Panel B | |
| $a_{10}$ | 0.00021 | 0.00003 | 0.00012 |
| $a_{20}$ | 0.00000 | 0.00000 | 0.00003 |
| $a_{30}$ | 0.00000 | 0.00001 | 0.00002 |
| $a_{11}$ | 0.01939 | 0.07449 | 0.08358 |
| $a_{21}$ | 0.00804 | 0.00036 | 0.03603 |
| $a_{31}$ | 0.00750 | 0.00000 | 0.01576 |
| $a_{12}$ | 0.00000 | 0.02958 | 0.02092 |
| $a_{22}$ | 0.02819*** | 0.11626*** | 0.01460*** |
| $a_{32}$ | 0.00080 | 0.08949*** | 0.02664** |
| $a_{13}$ | 0.00000 | 0.00000 | 0.01313 |
| $a_{23}$ | 0.00018 | 0.05658 | 0.03347 |
| $a_{33}$ | 0.00006 | 0.03316 | 0.09324 |
| $g_{11}$ | 0.05597 | 0.85353 | 0.12496 |
| $g_{21}$ | 0.20724 | 0.01385 | 0.01866 |
| $g_{31}$ | 0.24805*** | 0.00000 | 0.09829*** |
| $g_{12}$ | 0.00012 | 0.00000 | 0.00028 |
| $g_{22}$ | 0.07261 | 0.12420** | 0.00153 |
| $g_{32}$ | 0.00075 | 0.00071 | 0.00005 |
| $g_{13}$ | 0.03157 | 0.00000 | 0.00161 |
| $g_{23}$ | 0.07686 | 0.65318 | 0.00046 |
| $g_{33}$ | 0.09117 | 0.84103 | 0.00475 |

Table 3.15: Japan overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00063* | -0.00119* | -0.00123*** |
| $\beta_{11}$ | 0.03907* | 0.11596*** | 0.03968* |
| $\beta_{21}$ | 0.03230 | 0.01661 | -0.03426 |
| $\beta_{31}$ | -0.00939 | 0.70402*** | 0.41054*** |
| $\beta_{41}$ | 0.01632 | 0.00448 | 0.00010 |
| $\omega_2$ | 0.00065** | -0.00040 | 0.00033 |
| $\beta_{12}$ | 0.03468 | -0.02854 | 0.02692 |
| $\beta_{22}$ | 0.01921 | -0.01676 | -0.02852 |
| $\beta_{32}$ | 0.07351*** | -0.01976 | 0.02225 |
| $\beta_{42}$ | -0.09426** | -0.00285 | -0.01877 |
| | | Panel B | |
| $a_{10}$ | 0.00005 | 0.00008 | 0.00003 |
| $a_{20}$ | 0.00005 | 0.00000 | 0.00003 |
| $a_{11}$ | 0.00009 | 0.13886*** | 0.00017 |
| $a_{21}$ | 0.00679 | 0.00002 | 0.00440 |
| $a_{12}$ | 0.00031 | 0.00000 | 0.00034 |
| $a_{22}$ | 0.01589*** | 0.11844*** | 0.05998*** |
| $g_{11}$ | 0.07870*** | 0.00001 | 0.23660*** |
| $g_{21}$ | 0.00517 | 0.28330*** | 0.31556*** |
| $g_{12}$ | 0.00194 | 0.75577* | 0.08848 |
| $g_{22}$ | 0.02583 | 0.44754 | 0.38714 |
| $\alpha$ | 0.00000 | 0.00000 | 0.03862 |
| $\beta$ | 0.92933 | 0.91939 | 0.92592*** |
| | | Panel C | |
| $LB(6)$ | 4.68886 | 4.49163 | 4.40915 |
| $LB(12)$ | 23.82002** | 8.37565 | 15.08746 |
| $LB^2(6)$ | 0.91849 | 2.10060 | 8.09666 |
| $LB^2(12)$ | 14.65791 | 2.95400 | 31.46779*** |
| $KS$ | 0.23284*** | 0.15241*** | 0.16145*** |
| $LB(6)i$ | 12.74302** | 3.43595 | 6.19112 |
| $LB(12)i$ | 20.90392* | 7.04511 | 16.73748 |
| $LB^2(6)i$ | 1.83433 | 1.28578 | 4.46770 |
| $LB^2(12)i$ | 5.90381 | 1.53459 | 6.25979 |
| $KSi$ | 0.05647*** | 0.04935*** | 0.02666 |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.16: Hong Kong overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00050 | -0.00173*** | -0.00125*** |
| $\beta_{11}$ | 0.02600 | 0.09337*** | 0.02151 |
| $\beta_{21}$ | 0.04809 | 0.04423 | -0.01914 |
| $\beta_{31}$ | 0.17003*** | 0.46610*** | 0.44137*** |
| $\beta_{41}$ | 0.04554 | 0.11980*** | 0.08619** |
| $\omega_2$ | 0.00076** | 0.00117 | 0.00053 |
| $\beta_{12}$ | 0.00980 | -0.10780** | -0.05381 |
| $\beta_{22}$ | 0.04382 | 0.10819* | 0.03628 |
| $\beta_{32}$ | 0.04840** | -0.08342** | -0.01516 |
| $\beta_{42}$ | -0.06071 | 0.00030 | 0.02584 |
| | | Panel B | |
| $a_{10}$ | 0.00000 | 0.00012 | 0.00003 |
| $a_{20}$ | 0.00003 | 0.00003 | 0.00001 |
| $a_{11}$ | 0.00010 | 0.02253* | 0.00314 |
| $a_{21}$ | 0.00085 | 0.00018 | 0.06486 |
| $a_{12}$ | 0.11382 | 0.00003 | 0.00170 |
| $a_{22}$ | 0.01051*** | 0.20231*** | 0.21257*** |
| $g_{11}$ | 0.72775*** | 0.24553*** | 0.09973* |
| $g_{21}$ | 0.19581*** | 0.02454 | 0.37477*** |
| $g_{12}$ | 0.19271 | 0.00145 | 0.01415 |
| $g_{22}$ | 0.01059 | 0.72993*** | 0.58543 |
| $\alpha$ | 0.00000 | 0.00000 | 0.16301* |
| $\beta$ | 0.89004 | 0.95388 | 0.26194 |
| | | Panel C | |
| $LB(6)$ | 2.65997 | 11.33289* | 3.59270 |
| $LB(12)$ | 17.51907 | 15.28437 | 15.27608 |
| $LB^2(6)$ | 2.82392 | 5.19939 | 6.69724 |
| $LB^2(12)$ | 8.20149 | 10.91212 | 36.77620*** |
| $KS$ | 0.22462*** | 0.13091*** | 0.15275*** |
| $LB(6)i$ | 8.64093 | 3.11981 | 6.70604 |
| $LB(12)i$ | 16.44187 | 5.09646 | 12.46419 |
| $LB^2(6)i$ | 3.71872 | 1.11112 | 16.07746** |
| $LB^2(12)i$ | 6.53973 | 1.67054 | 23.05896** |
| $KSi$ | 0.08843*** | 0.06108*** | 0.07183*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.17: Taiwan overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00050 | -0.00207*** | -0.00160*** |
| $\beta_{11}$ | 0.03325 | 0.09650*** | 0.02818 |
| $\beta_{21}$ | 0.03797 | -0.01685 | -0.03697 |
| $\beta_{31}$ | 0.09537* | 0.52247*** | 0.52013*** |
| $\beta_{41}$ | 0.03579 | 0.03785 | 0.03555 |
| $\omega_2$ | 0.00171*** | 0.00115 | 0.00119*** |
| $\beta_{12}$ | 0.02038 | 0.10862** | 0.00978 |
| $\beta_{22}$ | 0.01751 | 0.08016 | 0.04306 |
| $\beta_{32}$ | 0.04735** | -0.01457 | 0.01427 |
| $\beta_{42}$ | -0.03983 | -0.00147 | -0.00604 |
| | | Panel B | |
| $a_{10}$ | 0.00004 | 0.00015** | 0.00004 |
| $a_{20}$ | 0.00000 | 0.00001 | 0.00002 |
| $a_{11}$ | 0.00007 | 0.09972*** | 0.00000 |
| $a_{21}$ | 0.00023 | 0.00000 | 0.00166 |
| $a_{12}$ | 0.30468 | 0.00000 | 0.00001 |
| $a_{22}$ | 0.00052*** | 0.15234*** | 0.17947*** |
| $g_{11}$ | 0.03140*** | 0.00049 | 0.00558 |
| $g_{21}$ | 0.39055*** | 0.00053 | 0.01108 |
| $g_{12}$ | 0.00045 | 0.10188 | 0.04907 |
| $g_{22}$ | 0.47902 | 0.80719*** | 0.46871 |
| $\alpha$ | 0.00000 | 0.00000 | 0.15052 |
| $\beta$ | 0.90977 | 0.94313 | 0.39425 |
| | | Panel C | |
| $LB(6)$ | 4.03614 | 17.49012*** | 3.04394 |
| $LB(12)$ | 22.33392** | 20.48877* | 10.64384 |
| $LB^2(6)$ | 1.54236 | 11.91407* | 17.37246*** |
| $LB^2(12)$ | 15.51872 | 17.20350 | 34.65068*** |
| $KS$ | 0.23476*** | 0.15885*** | 0.16274*** |
| $LB(6)i$ | 7.72592 | 9.55792 | 3.66176 |
| $LB(12)i$ | 14.12533 | 20.69288* | 22.27368** |
| $LB^2(6)i$ | 1.34513 | 3.55470 | 8.86726 |
| $LB^2(12)i$ | 11.40869 | 9.86414 | 18.88272* |
| $KSi$ | 0.08399*** | 0.08344*** | 0.09424*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.18: Singapore overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00032 | -0.00178*** | -0.00114*** |
| $\beta_{11}$ | 0.02978 | 0.09170*** | 0.01333 |
| $\beta_{21}$ | 0.04650 | -0.02280 | -0.08808** |
| $\beta_{31}$ | 0.17743*** | 0.57958*** | 0.48994*** |
| $\beta_{41}$ | -0.02851 | 0.12271*** | 0.14390*** |
| $\omega_2$ | 0.00142*** | 0.00072 | 0.00004 |
| $\beta_{12}$ | 0.08722* | 0.00270 | -0.02836 |
| $\beta_{22}$ | -0.00005 | -0.00438 | -0.01059 |
| $\beta_{32}$ | 0.06155*** | -0.01547 | 0.00978 |
| $\beta_{42}$ | -0.08891** | 0.00847 | 0.03478 |
| | | Panel B | |
| $a_{10}$ | 0.00002* | 0.00014 | 0.00003 |
| $a_{20}$ | 0.00000 | 0.00001 | 0.00000 |
| $a_{11}$ | 0.00022 | 0.06784*** | 0.00141 |
| $a_{21}$ | 0.00034 | 0.08434 | 0.04644 |
| $a_{12}$ | 0.34737 | 0.00000 | 0.01653 |
| $a_{22}$ | 0.02145*** | 0.84611*** | 0.01658*** |
| $g_{11}$ | 0.38761*** | 0.17932** | 0.39514*** |
| $g_{21}$ | 0.37666*** | 0.05012 | 0.03859 |
| $g_{12}$ | 0.21142 | 0.12989 | 0.36136 |
| $g_{22}$ | 0.34594 | 0.30497* | 0.74895 |
| $\alpha$ | 0.00000 | 0.00000 | 0.00000 |
| $\beta$ | 0.92048 | 0.91319 | 0.88407 |
| | | Panel C | |
| $LB(6)$ | 2.05537 | 14.79298** | 4.19450 |
| $LB(12)$ | 14.37133 | 15.42494 | 19.38692* |
| $LB^2(6)$ | 1.70581 | 7.09060 | 2.10338 |
| $LB^2(12)$ | 11.72315 | 7.92589 | 16.53672 |
| $KS$ | 0.24445*** | 0.13216*** | 0.12987*** |
| $LB(6)i$ | 7.79249 | 4.12244 | 10.98498* |
| $LB(12)i$ | 15.28422 | 11.48336 | 15.42357 |
| $LB^2(6)i$ | 1.05502 | 2.35334 | 18.65270*** |
| $LB^2(12)i$ | 6.83487 | 5.57450 | 22.96488** |
| $KSi$ | 0.08126*** | 0.25956*** | 0.36844*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.19: Korea overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00056 | -0.00183*** | -0.00124*** |
| $\beta_{11}$ | 0.03640* | 0.11040*** | 0.03276 |
| $\beta_{21}$ | 0.03965 | 0.02139 | -0.01399 |
| $\beta_{31}$ | 0.05393 | 0.56482*** | 0.41220*** |
| $\beta_{41}$ | 0.02793 | 0.05100 | 0.05259* |
| $\omega_2$ | 0.00128*** | 0.00072 | 0.00045 |
| $\beta_{12}$ | -0.05295 | 0.00128 | -0.04976 |
| $\beta_{22}$ | 0.01745 | 0.02048 | -0.07175 |
| $\beta_{32}$ | 0.01443 | -0.03751 | 0.02506 |
| $\beta_{42}$ | -0.06958 | -0.02555 | -0.02000 |
| | | Panel B | |
| $a_{10}$ | 0.00002 | 0.00016*** | 0.00003 |
| $a_{20}$ | 0.00004 | 0.00001 | 0.00001 |
| $a_{11}$ | 0.00007 | 0.15419*** | 0.00196 |
| $a_{21}$ | 0.02216 | 0.00000 | 0.04979 |
| $a_{12}$ | 0.01135 | 0.00392 | 0.00339 |
| $a_{22}$ | 0.00136*** | 0.09556*** | 0.11889*** |
| $g_{11}$ | 0.60832*** | 0.00860 | 0.19552*** |
| $g_{21}$ | 0.46289*** | 0.00001 | 0.29232*** |
| $g_{12}$ | 0.08632 | 0.00135 | 0.06681 |
| $g_{22}$ | 0.00086 | 0.88468*** | 0.56878 |
| $\alpha$ | 0.00000 | 0.00000 | 0.16163 |
| $\beta$ | 0.92426 | 0.93766 | 0.35517 |
| | | Panel C | |
| $LB(6)$ | 4.21897 | 15.68092** | 6.35326 |
| $LB(12)$ | 23.45372** | 16.64013 | 16.92552 |
| $LB^2(6)$ | 0.97737 | 11.69142* | 8.24171 |
| $LB^2(12)$ | 15.18335 | 13.60534 | 27.38902*** |
| $KS$ | 0.23513*** | 0.15651*** | 0.17782*** |
| $LB(6)i$ | 8.46445 | 4.45771 | 6.69598 |
| $LB(12)i$ | 18.42696 | 10.52829 | 20.15652* |
| $LB^2(6)i$ | 3.15353 | 1.36880 | 7.99262 |
| $LB^2(12)i$ | 6.86909 | 5.75038 | 31.63241*** |
| $KSi$ | 0.08285*** | 0.08500*** | 0.09444*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.20: US overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00062* | -0.00125* | -0.00107*** |
| $\beta_{11}$ | 0.03928* | 0.08666*** | 0.015180 |
| $\beta_{21}$ | 0.034060 | 0.018170 | -0.036470 |
| $\beta_{31}$ | -0.003430 | 0.094090 | 0.020070 |
| $\beta_{41}$ | -0.000720 | 0.37341*** | 0.30038*** |
| $\omega_2$ | 0.000100 | 0.000180 | 0.00060*** |
| $\beta_{12}$ | -0.025460 | -0.018800 | -0.022790 |
| $\beta_{22}$ | -0.029360 | -0.07724*** | -0.08353*** |
| $\beta_{32}$ | 0.001000 | 0.008090 | -0.003160 |
| $\beta_{42}$ | -0.010210 | 0.17177*** | 0.17396*** |
| | | Panel B | |
| $a_{10}$ | 0.000060 | 0.000060 | 0.000030 |
| $a_{20}$ | 0.000000 | 0.000000 | 0.000000 |
| $a_{11}$ | 0.000030 | 0.057210 | 0.006230 |
| $a_{21}$ | 0.000000 | 0.004150 | 0.000010 |
| $a_{12}$ | 0.001300 | 0.328440 | 0.000750 |
| $a_{22}$ | 0.000090 | 0.00016*** | 0.000000 |
| $g_{11}$ | 0.000150 | 0.25744*** | 0.38105*** |
| $g_{21}$ | 0.02377*** | 0.07330*** | 0.20662*** |
| $g_{12}$ | 0.000120 | 1.18876*** | 0.171910 |
| $g_{22}$ | 0.388790 | 0.729440 | 0.153260 |
| $\alpha$ | 0.018390 | 0.425580 | 0.000000 |
| $\beta$ | 0.94641** | 0.202570 | 0.975790 |
| | | Panel C | |
| $LB(6)$ | 4.801930 | 6.333820 | 5.417690 |
| $LB(12)$ | 23.77801** | 8.084370 | 17.502480 |
| $LB^2(6)$ | 0.904080 | 1.747460 | 8.920020 |
| $LB^2(12)$ | 14.580450 | 3.873910 | 32.13714*** |
| $KS$ | 0.23404*** | 0.18155*** | 0.17524*** |
| $LB(6)i$ | 1.979630 | 10.477070 | 5.991860 |
| $LB(12)i$ | 4.166490 | 17.374370 | 12.756690 |
| $LB^2(6)i$ | 0.312220 | 8.716980 | 0.429760 |
| $LB^2(12)i$ | 0.563700 | 20.71640* | 2.043760 |
| $KSi$ | 0.37006*** | 0.22925*** | 0.25137*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.21: Canada overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00061* | -0.00124* | -0.00114*** |
| $\beta_{11}$ | 0.03139 | 0.08957*** | 0.01345 |
| $\beta_{21}$ | 0.04138 | -0.01911 | -0.08090** |
| $\beta_{31}$ | 0.03052 | 0.15449*** | 0.30874*** |
| $\beta_{41}$ | 0.09149* | 0.32958*** | 0.31239*** |
| $\omega_2$ | 0.00095*** | 0.00126** | 0.00059** |
| $\beta_{12}$ | -0.01896 | -0.21676*** | -0.08442** |
| $\beta_{22}$ | 0.05739* | -0.11644*** | 0.05425 |
| $\beta_{32}$ | 0.03773*** | 0.07754*** | 0.05615*** |
| $\beta_{42}$ | -0.03711 | 0.30304*** | 0.18647*** |
| | | Panel B | |
| $a_{10}$ | 0.00005 | 0.00013 | 0.00004 |
| $a_{20}$ | 0.00002 | 0.00001 | 0.00001 |
| $a_{11}$ | 0.00001 | 0.00775 | 0.00012 |
| $a_{21}$ | 0.00772 | 0.01131 | 0.00000 |
| $a_{12}$ | 0.00011 | 0.00902 | 0.07272 |
| $a_{22}$ | 0.00045 | 0.35092*** | 0.12900*** |
| $g_{11}$ | 0.00712 | 0.23903*** | 0.17214*** |
| $g_{21}$ | 0.00036 | 0.00220 | 0.10104 |
| $g_{12}$ | 0.03593 | 0.38668 | 0.00012 |
| $g_{22}$ | 0.05436 | 0.60294 | 0.60750*** |
| $\alpha$ | 0.00000 | 0.31937** | 0.00647 |
| $\beta$ | 0.90966 | 0.53003 | 0.96214*** |
| | | Panel C | |
| $LB(6)$ | 5.55978 | 8.55756 | 9.31104 |
| $LB(12)$ | 24.96239** | 11.31175 | 23.24097** |
| $LB^2(6)$ | 0.91336 | 2.58937 | 15.56706** |
| $LB^2(12)$ | 15.04899 | 8.43424 | 42.90488*** |
| $KS$ | 0.23693*** | 0.16247*** | 0.16779*** |
| $LB(6)i$ | 3.72707 | 18.89085*** | 7.51392 |
| $LB(12)i$ | 5.95953 | 19.95601* | 15.02344 |
| $LB^2(6)i$ | 1.03903 | 8.52083 | 1.46586 |
| $LB^2(12)i$ | 2.19408 | 58.78924*** | 3.88249 |
| $KSi$ | 0.11738*** | 0.08644*** | 0.08795*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.22: UK overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00062* | -0.00141** | -0.00110*** |
| $\beta_{11}$ | 0.03617* | 0.08829*** | 0.00331 |
| $\beta_{21}$ | 0.03564 | -0.05004 | -0.07924** |
| $\beta_{31}$ | -0.11419 | 0.09975 | 0.05986 |
| $\beta_{41}$ | 0.07968* | 0.25339*** | 0.27141*** |
| $\omega_2$ | 0.00026* | 0.00051 | 0.00045** |
| $\beta_{12}$ | -0.00924 | 0.01229 | -0.01178 |
| $\beta_{22}$ | 0.01305 | -0.10535*** | -0.04599*** |
| $\beta_{32}$ | 0.00206 | -0.01161 | -0.02709** |
| $\beta_{42}$ | -0.03669* | 0.19566*** | 0.17833*** |
| | | Panel B | |
| $a_{10}$ | 0.00005 | 0.00022 | 0.00000 |
| $a_{20}$ | 0.00001 | 0.00005 | 0.00001 |
| $a_{11}$ | 0.00008 | 0.17053*** | 0.04839* |
| $a_{21}$ | 0.00003 | 0.12625 | 0.00008 |
| $a_{12}$ | 0.00003 | 0.00013 | 0.00010 |
| $a_{22}$ | 0.00011 | 0.00029 | 0.00014*** |
| $g_{11}$ | 0.01480*** | 0.00117 | 0.74045*** |
| $g_{21}$ | 0.02191** | 0.00001 | 0.11815*** |
| $g_{12}$ | 0.03305 | 0.00015 | 0.48278 |
| $g_{22}$ | 0.02634 | 0.00013 | 0.11907 |
| $\alpha$ | 0.00000 | 0.00000 | 0.01309 |
| $\beta$ | 0.90093 | 0.91829 | 0.89215** |
| | | Panel C | |
| $LB(6)$ | 4.59163 | 3.78620 | 1.17174 |
| $LB(12)$ | 22.59108** | 7.48342 | 15.12619 |
| $LB^2(6)$ | 1.05009 | 2.93923 | 2.84554 |
| $LB^2(12)$ | 13.83260 | 9.48142 | 11.95525 |
| $KS$ | 0.23394*** | 0.16249*** | 0.15337*** |
| $LB(6)i$ | 2.83502 | 4.24988 | 6.90315 |
| $LB(12)i$ | 5.35912 | 9.48462 | 10.17256 |
| $LB^2(6)i$ | 0.96945 | 0.29922 | 0.73611 |
| $LB^2(12)i$ | 1.96287 | 3.89855 | 2.88001 |
| $KSi$ | 0.41473*** | 0.26486*** | 0.30595*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

Table 3.23: Germany overnight stock market

| | Pre-crisis | During crisis | Post-crisis |
|---|---|---|---|
| | | Panel A | |
| $\omega_1$ | 0.00041 | -0.00140** | -0.00101*** |
| $\beta_{11}$ | 0.04217* | 0.10260*** | 0.04236* |
| $\beta_{21}$ | 0.02031 | -0.00917 | -0.03462 |
| $\beta_{31}$ | -0.04539 | 0.03324 | 0.03141 |
| $\beta_{41}$ | 0.06058* | 0.32854*** | 0.24014*** |
| $\omega_2$ | 0.00015 | 0.00025 | 0.00118*** |
| $\beta_{12}$ | 0.46357*** | -0.13041*** | -0.09280** |
| $\beta_{22}$ | -0.36547*** | -0.15592*** | -0.05050* |
| $\beta_{32}$ | 0.02961 | 0.05759** | 0.14091*** |
| $\beta_{42}$ | -0.04133 | 0.34496*** | 0.40164*** |
| | | Panel B | |
| $a_{10}$ | 0.00004* | 0.00020*** | 0.00004 |
| $a_{20}$ | 0.00001 | 0.00000 | 0.00001 |
| $a_{11}$ | 0.00000 | 0.10395 | 0.00185 |
| $a_{21}$ | 0.00000 | 0.02704 | 0.00250 |
| $a_{12}$ | 0.44827 | 0.11024* | 0.00001 |
| $a_{22}$ | 0.11849*** | 0.51961*** | 0.08859*** |
| $g_{11}$ | 0.00927 | 0.00000 | 0.06445*** |
| $g_{21}$ | 0.06905 | 0.00065 | 0.00060 |
| $g_{12}$ | 0.00367 | 0.02689 | 0.09267 |
| $g_{22}$ | 0.49725** | 0.70058*** | 0.80648 |
| $\alpha$ | 0.00000 | 0.00000 | 0.01455 |
| $\beta$ | 0.97735 | 0.90073 | 0.96673*** |
| | | Panel C | |
| $LB(6)$ | 4.53756 | 4.58618 | 4.37053 |
| $LB(12)$ | 16.31938 | 6.46444 | 18.58781* |
| $LB^2(6)$ | 1.76704 | 2.05131 | 13.08295** |
| $LB^2(12)$ | 14.14490 | 5.62518 | 37.63565*** |
| $KS$ | 0.22329*** | 0.15608*** | 0.15440*** |
| $LB(6)i$ | 17.26514*** | 8.37239 | 3.18195 |
| $LB(12)i$ | 23.40065** | 13.61590 | 12.70685 |
| $LB^2(6)i$ | 1.51395 | 2.15847 | 2.12210 |
| $LB^2(12)i$ | 2.60830 | 2.64981 | 7.10585 |
| $KSi$ | 0.07482*** | 0.13431*** | 0.09115*** |

Note: $i$ represents the foreign market.

* Significance at the 10%.

** Significance at the 5%.

*** Significance at the 1%.

### 3.4.2 Volatility spillovers among China and other stock markets.

We test the constancy of the conditional correlations for the daytime and overnight returns before, during and after the crisis. We use the artificial neural networks-based test ($NEURAL$) and the Taylor expansion-based test ($TAYLOR$) developed in Chapter 1. Tables 3.24 and 3.25 report the constant conditional correlation test results for the daytime and overnight returns, respectively. According to these results, we are now able to well specify the conditional correlations by choosing an EDCC-GARCH when at least one of the tests rejects the null hypothesis of constancy of conditional correlations, while we choose an ECCC-GARCH when both tests do not reject the constancy. Note that Table 3.25 shows clear evidence of dynamic conditional correlations for the overnight returns for each period, for both tests, except for the trivariate case (CN-JP-KR: China, Japan and Korea) before the crisis.

Tables 3.5–3.23 show the estimations of the extended conditional correlation models. These models enable to take into account the endogeneity of the data fitted by the hypotheses of relationship proposed in Section 3.3.2.

The own-GARCH effects ($g_{11}$) for the Chinese daytime and overnight returns are significant (almost) whatever the period. In other words, own-past volatility affects the conditional variance of the China stock market. The statistical significance of the off-diagonal parameters ($g_{12}$ and $g_{21}$) of the $\mathbf{G}$ matrix identifies the volatility spillovers.

**Volatility spillovers between China and Asia stock markets**

Before the crisis, we find insignificant daytime volatility spillover effects between Asia stock markets and China, except in the trivariate design reported in Table 3.14 where, surprisingly, China influences Korea. For the overnight volatility spillover effects, we find them significant from China to Hong Kong, Taiwan, Singapore and Korea. As expected, from 2005, just after the beginning of the liberalization of China stock markets, daytime relationships are very scarce with Asia stock markets.

However, during the crisis we find significant unidirectional daytime volatility spillovers from Hong Kong to China and from China to Taiwan. Whether there is no relationship between China and Japan, their own past volatilities are significant as before the crisis. The overnight spillover effects are significant between China and Japan. These results are in line with our expectations since volatility spillovers characterise periods of crises.

Table 3.24: Constant conditional correlation tests (daytime returns)

| Daytime | $NEURAL$ | $p$-value | $TAYLOR$ | $p$-value |
|---|---|---|---|---|
| | | Pre-crisis | | |
| Japan | 10.54817 | 0.10339 | 23.63942 | 0.31086 |
| Hong Kong | 10.95257 | 0.05233 | 15.62921 | 0.61841 |
| Taiwan | 2.91181 | 0.81983 | 35.22377 | 0.01894 |
| Singapore | 8.10285 | 0.15066 | 11.45090 | 0.83219 |
| Korea | 9.90906 | 0.07785 | 19.15258 | 0.57535 |
| US | 2.88258 | 0.71808 | 28.33026 | 0.05720 |
| Canada | 5.32480 | 0.37754 | 13.30415 | 0.77320 |
| UK | 7.62340 | 0.26701 | 19.96697 | 0.33468 |
| Germany | 4.96683 | 0.54807 | 10.28346 | 0.97499 |
| CN-JP-KR | 12.17142 | 0.73210 | 57.34031 | 0.19340 |
| | | During crisis | | |
| Japan | 3.00356 | 0.69944 | 21.96771 | 0.34227 |
| Hong Kong | 9.33692 | 0.15550 | 43.24582 | 0.00292 |
| Taiwan | 3.82828 | 0.69990 | 15.28143 | 0.76008 |
| Singapore | 13.00448 | 0.04296 | 26.83269 | 0.17646 |
| Korea | 9.29547 | 0.15763 | 21.64446 | 0.42024 |
| US | 3.96795 | 0.68101 | 17.11470 | 0.64551 |
| Canada | 8.19860 | 0.22391 | 16.79125 | 0.72368 |
| UK | 6.21712 | 0.39931 | 34.55228 | 0.02262 |
| Germany | 4.72836 | 0.57909 | 12.17593 | 0.90989 |
| CN-JP-KR | 12.89915 | 0.79753 | 47.32198 | 0.50053 |
| | | Post-crisis | | |
| Japan | 3.02529 | 0.69609 | 27.87449 | 0.11240 |
| Hong Kong | 5.78105 | 0.32811 | 18.12353 | 0.57927 |
| Taiwan | 3.20812 | 0.66794 | 29.57747 | 0.10081 |
| Singapore | 4.32502 | 0.50363 | 17.66515 | 0.60946 |
| Korea | 7.56120 | 0.27204 | 33.80775 | 0.03799 |
| US | 4.21005 | 0.64827 | 31.36155 | 0.08895 |
| Canada | 6.60280 | 0.35915 | 17.48244 | 0.55722 |
| UK | 4.90394 | 0.55619 | 27.55425 | 0.23311 |
| Germany | 6.95282 | 0.32523 | 22.31487 | 0.44123 |
| CN-JP-KR | 21.85391 | 0.14797 | 58.12152 | 0.20107 |

Table 3.25: Constant conditional correlation tests (overnight returns)

| Overnight | $NEURAL$ | $p$-value | $TAYLOR$ | $p$-value |
|---|---|---|---|---|
| | | Pre-crisis | | |
| Japan | 25.02317 | 0.00014 | 49.29815 | 0.00000 |
| Hong Kong | 30.28003 | 0.00001 | 82.31747 | 0.00000 |
| Taiwan | 10.12058 | 0.07189 | 27.82061 | 0.00959 |
| Singapore | 7.80475 | 0.16733 | 36.19116 | 0.00030 |
| Korea | 25.53725 | 0.00011 | 69.56339 | 0.00000 |
| US | 53.20630 | 0.00000 | 81.31485 | 0.00000 |
| Canada | 28.85492 | 0.00001 | 60.48871 | 0.00000 |
| UK | 80.27564 | 0.00000 | 175.18430 | 0.00000 |
| Germany | 39.90085 | 0.00000 | 43.90978 | 0.00002 |
| CN-JP-KR | 15.44784 | 0.56326 | 57.34031 | 0.19340 |
| | | During crisis | | |
| Japan | 16.80541 | 0.01003 | 53.91252 | 0.00006 |
| Hong Kong | 30.17898 | 0.00004 | 4.57931 | 0.00032 |
| Taiwan | 30.63558 | 0.00003 | 88.42859 | 0.00000 |
| Singapore | 31.03175 | 0.00002 | 97.93770 | 0.00000 |
| Korea | 17.02342 | 0.00446 | 58.74226 | 0.00001 |
| US | 23.16618 | 0.00012 | 81.98742 | 0.00000 |
| Canada | 40.04169 | 0.00000 | 149.46050 | 0.00000 |
| UK | 29.54666 | 0.00001 | 315.25420 | 0.00000 |
| Germany | 138.79090 | 0.00000 | 430.94530 | 0.00000 |
| CN-JP-KR | 28.38599 | 0.05642 | 47.32198 | 0.50053 |
| | | Post-crisis | | |
| Japan | 5.91933 | 0.31415 | 22.40781 | 0.09755 |
| Hong Kong | 4.86997 | 0.43195 | 24.06315 | 0.04502 |
| Taiwan | 11.18084 | 0.04791 | 44.85403 | 0.00004 |
| Singapore | 12.58320 | 0.01350 | 47.75840 | 0.00000 |
| Korea | 14.16488 | 0.01460 | 13.12520 | 0.51669 |
| US | 84.57796 | 0.00000 | 43.90176 | 0.00000 |
| Canada | 11.48298 | 0.04260 | 60.45705 | 0.00000 |
| UK | 89.54113 | 0.00000 | 120.67740 | 0.00000 |
| Germany | 16.25159 | 0.00616 | 66.86363 | 0.00000 |
| CN-JP-KR | 112.23690 | 0.00000 | 214.35080 | 0.00000 |

After the crisis, there exist significant daytime spillover effects from Hong Kong and Taiwan to China, and from China to Korea. The trivariate modelling reported in Table 3.14 confirms the relationship between China and Korea (through $g_{31}$). Moreover, the overnight spillover effects are significant from China to Japan and Hong Kong. We find evidence of spillover effects for Hong Kong in the daytime and overnight returns, which is different with the findings of Allen, Amram and McAleer (2013) mentioning that there is surprising no evidence of volatility spillovers between China and Hong Kong during the crisis. Note that we have used a dynamic correlation structure as suggested by the authors, and as found in the conditional correlation tests. The results indicate that the relationships that China has developed during the crisis remained.

**Volatility spillovers between China and North America markets**

Because China is weakly integrated in the global financial market, before the crisis, we find insignificant and significant daytime spillover effects between China and the US, and from Canada to China, respectively. There exist unidirectional overnight volatility spillovers from China to the US and the UK.

During the crisis, the off-diagonal parameters indicate no volatility spillovers between China and the US. When we consider the volatility of the overnight returns, there is clear evidence of volatility spillovers between these two markets. The return of the fixed exchange rate regime against the USD of July 2008 may give birth to an expectation of a retreat of the open market policy, and affects the integration between China and other markets.

After the crisis, the daytime volatility spillover effects are significant from the US and Canada to China, while the own-GARCH effects are significant only for the US. We find significant overnight volatility spillover effects from China to the US. Whether we had not considered overnight informations we would never have noticed that there exist linkages between China and the US in both direction, the US influences China after the crisis through daytime volatility, while China influence the US through overnight volatility.

**Volatility spillovers between China and main Europe markets**

Before the crisis, there are insignificant daytime volatility spillover effects between China and Europe markets. The overnight volatility spillovers are from China to the UK. Although the linkage vanishes after the crisis, we also find significant daytime volatility spillovers from China

to the UK. Whether the overnight volatility spillover effects do not exist during the crisis, they come back after the crisis from China to the UK. The relationships between China and Europe support that Chinese stock market is becoming more integrated among the world equity markets.

To summarize the results, firstly, the Chinese market is highly related with the Asia market in terms of return and volatility spillovers in the daytime estimations, as well as in the overnight estimations. Secondly, due to the economic and politic importance of Hong Kong for China, we find significant evidence of daytime and overnight spillover effects during and after the crisis. Similarly with the findings of Johansson and Ljungwall (2009), we find that the China Mainland stock market is less related to Hong Kong and Taiwan, regarding their different histories and developments before the crisis. During and after the crisis period, we show significant evidence of linkages between China and these two markets, confirming the regional integration and the growing importance of the Great China financial markets. Thirdly, we find that the US and Chinese investors for the portfolio strategies have taken into account the informational daytime from the Chinese and US markets. In our study, this relationship exists after the crisis and concerns the informational part, while before and after the crisis, the relationship is related to the non-informational part. However, these findings are slightly different with Wang and Firth (2004) (pre-crisis period) and Allen, Amram and McAleer (2013) (post-crisis period), which document that there is clear evidence of volatility spillover effects between China and the US.

Overall, the linkages and interactions between the Mainland China stock market and the other foreign stock markets have increased substantially during and after the crisis, suggesting that the China market has grown more interdependent with world stock markets.

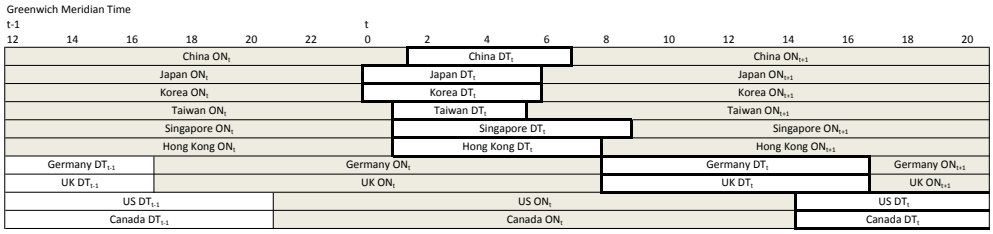
In the Panel C of Tables 3.5–3.23, we report the diagnostic tests. The Ljung-Box tests on the residuals and squared residuals show results indicating the good properties of the model used to analyse the existing relationships between China and the nine other stock markets. The Kolmogorov-Smirnov tests indicate, as usual for residuals of financial data, presence of non-normality.

### 3.4.3 Discussion

While many recent papers study the second conditional moment in multivariate frameworks, we draw attention to non-synchronicity issue between China and the rest of the world. Allen, Amram and McAleer (2013) use

daily data in a constant conditional correlation model, neglecting the non-synchronicity often occurring with partially and non-overlapping stock markets (in their case, China compared to the US, Australia, Hong-Kong, Japan and Singapore). Burdekin and Siklos (2012) apply short-run time series techniques to daily data on a broad group of Asia-Pacific stock markets and the US from 1995 to 2010. While most of the countries are in different time zones, their empirical work suggests the importance of crises in affecting the persistence of equity returns in the Asia-Pacific region and offers some support for contagion effects. Wang and Wang (2010) examined stock market linkages between Greater China and the US and Japan in terms of volatility and price spillovers. Whereas they established that volatility spillovers are stronger than price spillovers between the Greater China markets and the developed markets of the US and Japan, the authors have never considered that the studied markets are situated in different time zones. Even if they use open-to-close and close-to-open returns, they don't deal with non-synchronicity.

Table 3.26: Timezones



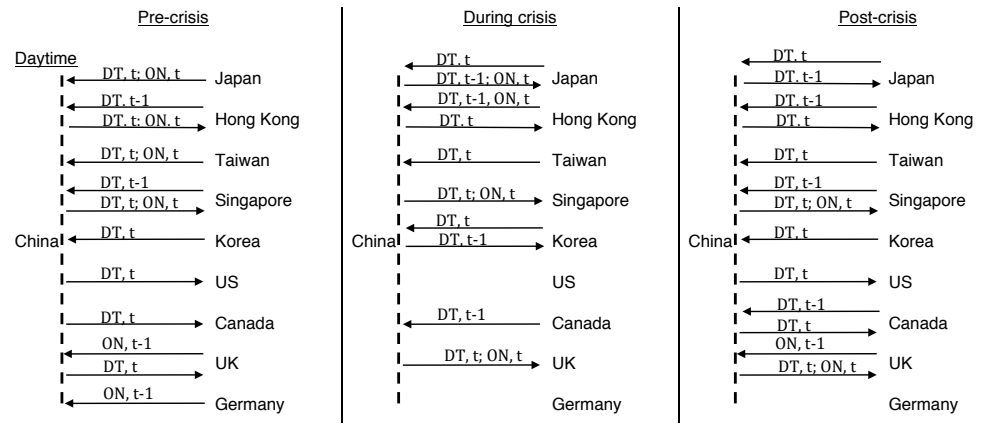




Figure 3.5: Diagrammatic representation of fitted daytime return spillovers

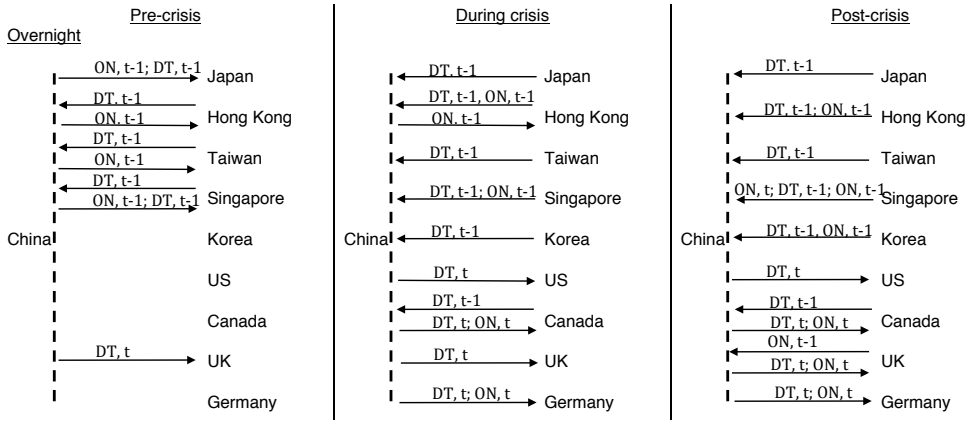



Figure 3.6: Diagrammatic representation of fitted overnight return spillovers

To investigate this issue, Figures 3.5 and 3.6 show the different return linkages where all the relationships take into account non-synchronicity. The arrow indicates the direction of the return spillovers, $DT$ are the daytime returns, and $ON$ are the overnight returns.

Figures 3.7 and 3.8 summarize all the existing linkages between China and the other markets before, during and after the crisis, considering only the feasible relationships as shown in Table 3.26. In other words, the closing price of a market still opened cannot influence the closing price of a market already closed. Whether the bivariate analysis of the conditional correlation yields to interesting results, the portfolio manager who needs to establish a portfolio diversification strategy have to adapt the results of estimation (given in Section 3.4) and its strategy subject to Table 3.26.

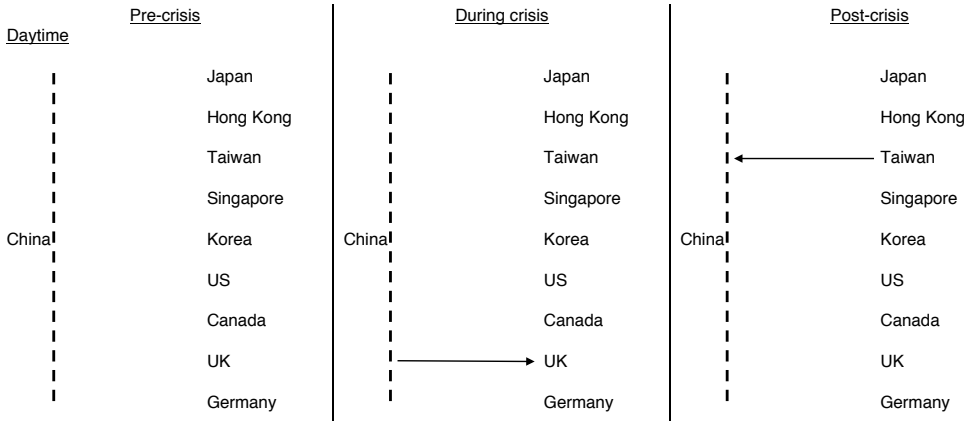



Figure 3.7: Diagrammatic representation of fitted daytime volatility spillovers

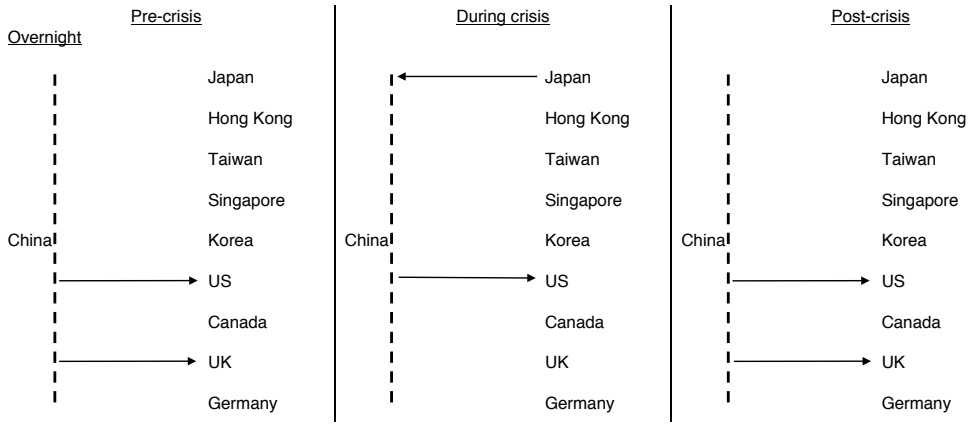



Figure 3.8: Diagrammatic representation of fitted overnight volatility spillovers

## 3.5 Conclusions

This Chapter investigates the transmission of daytime and overnight information in terms of returns and volatilities between China and other main stock markets. By using a very recent intra-day data for the period of 2005 to 2013, our main findings are as follows:

(i) China and other markets (with the test of constancy of conditional correlations) are differently conditionally correlated, a bivariate extended constant or dynamic conditional correlation models are adopted in this study. However, probably due to peculiar data for the overnight returns, the constancy of conditional correlation tests show that it is recommended to take a dynamic conditional correlation model.

(ii) There exists clear evidence of unidirectional and bidirectional contemporaneous and lagged dependence in the daytime returns of the stock indices between China and partially overlapping markets for the whole period. The relationships between China and the non-overlapping markets exist before and after the crisis. The overnight returns of China are principally explained by the Asian markets before the crisis, whereas the relationships during and after the crisis are also with North America and Europe.

Whether econometrically we obtain some evidence of volatility spillovers, we face to non-synchronicity issue. After controlling the different time zone, we only keep the results with economic interest:

(i) Before the global financial crisis, we don't find any evidence of daytime volatility spillover effects between China and the other markets, whereas the overnight volatility spillovers are from China to the US and the UK.

(ii) During the crisis, there exist daytime volatility spillovers from China to the UK, while there are unidirectional overnight volatility spillovers from Japan and to the US.

(iii) After the crisis, the daytime volatility spillovers are from Taiwan, whereas the overnight volatility spillover effects are from China to the US and the UK.

For future work, it should be interesting to investigate the relationship between the overnight information and the pre-opening trading. Does the foreign overnight information can affect the pre-opening trading in the Chinese stock market? The same question for inverse case. Of course, for this step we need the high-frequency intraday data.

# Conclusion générale et perspectives

Dans un contexte de globalisation financière toujours croissante et caractérisée par une augmentation des mouvements internationaux de capitaux, les modèles étudiant les seconds moments conditionnels des séries temporelles financières dans un cadre multivarié hétéroscédastique continuent de se développer.

Cette thèse a cherché à aborder orthogonalement l'aspect économétrique, par le développement de tests de constance des corrélations conditionnelles et de tests de non-linéarité dans les covariances conditionnelles, et l'aspect empirique, par l'étude des interactions que la Chine peut avoir avec le reste du monde en terme de transmission d'informations et de volatilités.

Le Chapitre 1 développe deux tests originaux de constance des corrélations conditionnelles dans les modèles GARCH multivariés. Ils sont construits selon la procédure du multiplicateur de Lagrange, où l'hypothèse nulle repose sur la constance des corrélations conditionnelles, tandis que l'hypothèse alternative propose une première spécification basée sur les réseaux de neurones artificiels et une seconde représentée par une forme fonctionnelle inconnue qui est linéarisée à l'aide d'un développement de Taylor. Ces deux nouveaux tests sont caractérisés par le fait qu'ils ne nécessitent que peu de connaissance sur la relation fonctionnelle déterminant les corrélations. Ils sont comparés aux tests de Tse (2000) et Silvennoinen and Teräsvirta (2014) et montrent de bonnes performances dans les simulations en échantillon fini. Les expériences de simulation montrent que ces tests ont des qualités un peu différentes. Le premier a de bonnes propriétés lorsque la variabilité des coefficients de la corrélation conditionnelle est faible ou moyenne, tandis que le second montre de bonnes performances lorsque la variabilité est élevée. De plus, les test développés dans ce chapitre sont robustes à la non-normalité que nous avons représentée par des modélisations originales basées sur le

Beta-$t$-EGARCH avec corrélation conditionnelle ou stochastique. Nos tests apparaissent donc très utiles dans l'investigation de corrélations conditionnelles dynamiques potentielles entre les séries économiques ou financières.

Les tests développés dans ce chapitre sont conditionnés à des hypothèses fortes d'hétéroscédasticité conditionnelle et il serait intéressant d'investiguer dans la direction des corrélations réalisées. En effet, si nos tests peuvent être appliqués dans le cadre des mesures *ex post*, cela devrait permettre d'étendre leur champs d'application et ainsi de considérer des problématiques propres à cette mesure.

Le Chapitre 2 propose deux tests originaux de non-linéarité des variances et covariances conditionnelles dans les modèles GARCH multivariés, et deux autres tests robustes aux effets de levier. Si l'hypothèse nulle repose sur une relation linéaire entre les innovations retardées au carré et les (co)variances conditionnelles, l'hypothèse alternative se caractérise quant à elle par une relation non-linéaire basée sur une fonction de transition exponentielle ou logistique. Par ailleurs, les propriétés asymptotiques et les moments marginaux d'ordre deux et quatre sont étudiés sous l'hypothèse nulle. Par une procédure très simple en deux étapes il est possible de déterminer si l'on est en présence de linéarité, de non-linéarité ou d'effets de levier dans les (co)variances conditionnelles. Les tests introduits dans ce chapitre montrent de bonnes performances dans nos simulations et dans les illustrations empiriques bivariés et multivariés. Nos tests semblent être très utiles pour conclure ou non à une modélisation non-linéaire des (co)variances conditionnelles.

Les séries temporelles, comme par exemple les rendements financiers, sont caractérisées par des sauts et de l'asymétrie. De nombreux modèles multivariés qui capturent les dynamiques dans les volatilités, co-volatilités et les corrélations conditionnelles ont été développés au cours des deux dernières décennies, occupant une place proéminente parmi la littérature en économétrie financière. Les modèles *Dynamic Conditional Score* (DCS) introduits par Harvey and Chakravarty (2008) – également appelés *Generalized Autoregressive Score* (GAS) et introduits par Creal, Koopman and Lucas (2013) – sont une classe très récente de modèles économétriques qui ont l'avantage de traiter les observations extrêmes d'une manière très naturelle. La littérature sur ce type de modèles est recensée en partie dans Harvey (2013) et mise à jour régulièrement sur gasmodel.com. À cela s'ajoutent les articles de Boudt, Cornelissen and Croux (2012); Laurent, Lecourt and Palm (2014); Sucarrat, Escribano and Grønneberg (2013) qui abordent différemment le traitement des sauts.

L'idée principale est d'étendre les tests de non-linéarité sur la base de cette spécification générale qui tend à réduire les persistances et les trop fortes variances et covariances conditionnelles après un choc (cf. Figures 3 et 4 dans l'Introduction générale de cette thèse pour remarquer qu'un choc dans les rendements a un impact important sur les volatilités et co-volatilités). Ainsi, nos tests seront robustes aux sauts qui sont une caractéristique bien connue des séries financières. De plus, le modèle développé dans le Chapitre 2 capture déjà les effets d'asymétrie des chocs négatifs et positifs dans les séries temporelles permettant des régimes intermédiaires en passant de l'un à l'autre de manière lisse. Cependant, il reste à estimer ce modèle en ajoutant une spécification qui atténuera ces chocs (par exemple, avec la distance de Mahalanobis).

Notons que les extensions des tests développés dans les Chapitres 1 et 2 se feront en utilisant pour le calcul de la matrice d'information la matrice sandwich, à savoir la combinaison du produit des gradients et de la hessienne. Ainsi, cette matrice d'information reconnaissant l'erreur de spécification devrait améliorer les performances des tests.

Le Chapitre 3 est une étude empirique sur la transmission d'information en séance (*daytime*) et hors séance (*overnight*) de cotation en termes de rendements et de volatilités entre la Chine et le reste du monde représenté par neuf marchés. À l'aide des tests développés dans le Chapitre 1 nous distinguons avec quels marchés la Chine a des corrélations conditionnelles dynamiques ou constantes. Bien que l'intégration financière de la Chine s'établit après 2005, les études récentes restent relativement faibles. Par ailleurs, l'asynchronicité est un phénomène qui n'est pas assez considéré dans ce type d'analyse. Trivialement, un marché encore ouvert ne peut pas influencer un marché déjà fermé. Si économétriquement les résultats montrent un intérêt particulier en terme de transmission financière, considérer le problème d'asynchronicité garantit une cohérence économique et financière. À travers les transmissions d'information en séance de cotation la Chine a des relations plus étroites avec les marchés asiatiques qu'avec les marchés nord américains et européens, tandis qu'à travers les transmissions d'information hors séance de cotation ces relations semblent inverses.

Pour de futures recherches il serait intéressant d'investiguer la relation entre les informations des cotations hors séance et les échanges de pré-ouverture. Par l'analyse des données intra-journalières il serait envisageable de déterminer si les informations des cotations hors séance provenant de l'étranger peut affecter les échanges de pré-ouverture au sein du marché boursier chinois, et inversement.

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